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Code : 211303

2012

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) All questions carry equal marks.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer (any *seven*) :

(a) Which of the following is an entire solution?

(i) $\frac{z}{1+z^2}$

(ii) $z\bar{z}$

(iii) e^{-z^2}

(iv) $e^{z^{-2}}$

(b) The value of $\int_C \frac{dz}{z+2}$, $C: |z|=1$ is

(i) $2\pi i$

(ii) $-2\pi i$

(iii) $4\pi i$

(iv) 0

(Turn Over)

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(c) $J_{\frac{1}{2}}(x)$ is given by

(i) $\sqrt{\frac{2\pi}{n}}\sin x$

(ii) $\sqrt{\frac{2\pi}{n}}\cos x$

(iii) $\sqrt{\frac{\pi}{2n}}\cos x$

(iv) $\sqrt{\frac{2}{\pi n}}\sin x$

(d) The polynomial $2x^2+x+3$ in terms of Legendre polynomial is

(i) $\frac{1}{3}(4P_2-3P_1+11P_0)$

(ii) $\frac{1}{3}(4P_2+3P_1-11P_0)$

(iii) $\frac{1}{3}(4P_2+3P_1+11P_0)$

(iv) $\frac{1}{3}(4P_2-3P_1-11P_0)$

(e) In the equation $P_0y''+P_1y'+P_2y=0$; $x=\alpha$ is singular point, if

(i) $P_0=0$

(ii) $P_0\neq 0$

(iii) $P_1=0$

(iv) $P_1\neq 0$

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(f) The solution of $z(x, y)$ of the equation $\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$ is

(i) $f(x + \log_e y, z) = 0$

(ii) $f(y + \log_e x, z) = 0$

(iii) $f(x + \log_e z, y) = 0$

(iv) $f(z + \log_e x, y) = 0$

(g) The particular integral of $(D^2 - D'^2)z = \cos(x+y)$ is

(i) $\frac{x}{2}\sin(x+y)$ (ii) $x\sin(x+y)$

(iii) $x\cos(x+y)$ (iv) $\frac{x}{2}\cos(x+y)$

(h) The partial differential equation from $z = (a+x)^2 + y$ is

(i) $z = \frac{1}{4}\left(\frac{\partial z}{\partial x}\right)^2 + y$

(ii) $z = \frac{1}{4}\left(\frac{\partial z}{\partial y}\right)^2 + y$

(iii) $z = \left(\frac{\partial z}{\partial x}\right)^2 + y$

(iv) $z = \left(\frac{\partial z}{\partial y}\right)^2 + y$

(i) The probability of getting a king when 1 card is drawn from a pack of 52 cards is

(i) $\frac{4}{13}$

(ii) $\frac{1}{3}$

(iii) $\frac{8}{13}$

(iv) $\frac{9}{52}$

(j) A coin is tossed 6 times in succession. The probability of getting at least one head is

(i) $\frac{63}{64}$

(ii) $\frac{3}{32}$

(iii) $\frac{1}{64}$

(iv) $\frac{1}{2}$

2. (a) What is a singular point? Find regular singular point of the equation

$$2x^2y'' + 3xy' + (x^2 - y)y = 0$$

(b) Solve in series the differential equation

$$3xy'' + 2y' + y = 0$$

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3. (a) Prove :

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \begin{cases} 0 & m \neq n \\ \dfrac{2}{2n+1} & m = n \end{cases}$$

(b) Explain in terms of Legendre polynomials the expression :

$$x^4 + x^3 + x^2 + x + 1$$

4. (a) Form the partial differential equation from $ax^2 + by^2 + z^2 = 1$.

(b) Solve :

$$(D^2 + 2DD' + D'^2)z = 2\cos y - x \sin y$$

5. (a) By separation of variables, solve

$$2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$$

(b) Find the solution of

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$

subject to boundary conditions

$$y(0, t) = 0, \; y(l, t) = 0, \; y(x, 0) = \phi(x);$$

$$\frac{\partial y}{\partial t}(x, 0) = \psi(x)$$

6. (a) What are the necessary conditions for a function $f(z)$ to be analytic, where $f(z) = 2xy + i(x^2 - y^2)$?

(b) Find the point where the function $f(z) = |z|^2$ is differentiable.

7. (a) Discuss the Cauchy integral formula and hence find the value of

$$\int_C \frac{2z^2 + z}{z^2 - 1}\, dz$$

where C is circle of unit radius with centre at $z = 1$.

(b) Find first 3 terms of Taylor series expansion of $f(z) = \dfrac{1}{z^2 + 4}$ about $z = -2$. Also find the region of convergences.

8. (a) Establish a relation between moment about mean and moment about any point.

(b) Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means equal to 160 inches^2 and 91 inches^2 respectively. Can these be regarded as drawn from the same normal populations?

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9. *(a)* Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.

(b) The probability that a man aged 60 will live to be 70 is 0·65. What is the probability that out of 10 men now aged 60 at least 7 will live to be 70?

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