Number of printed pages – 8

3rd Semester Examination – 2011

Mathematics-III

Code : 211303

Full Marks – 70

Time : 3 Hours

Instructions :

There are NINE questions in this paper. All questions carry equal marks.

Attempt Five questions in all.

Question No. 1 is compulsory.

1. Choose the correct or best alternative of any seven of the following :

(a) The value of the $\lim_{(x,y)\to(0,0)}\left[\tan^{-1}\left(y/x\right)\right]$ is

(i) 0 (ii) $\frac{\pi}{2}$

(iii) – (iv) does not exist

P.T.O.

(b) The solution of $(D^2+2D+2)y=0$, $y(0)=0$, $y'(0)=1$ is

(i) $e^{x}\cdot\sin x$ (ii) $e^{-x}\cdot\cos x$

(iii) $e^{-x}\cdot\sin x$ (iv) $e^{x}\cdot\cos x$

(c) The solution of $y'+y\tan x=\cos x$, $y(0)=0$ is

(i) $\sin x$ (ii) $\cos x$

(iii) $x\cdot\sin x$ (iv) $x\cdot\cos x$

(d) The differential equation $M(x.y)dx+N(x.y)dy = 0$ is an exact differential equation if

(i) $\frac{\partial M}{\partial y}+\frac{\partial N}{\partial x}$ (ii) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$ (iv) $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=1$

(e) The value of Bessel's function $J_2(x)$ in terms of $J_1(x)$ and $J_0(x)$ is

(i) $2J_1(x)-xJ_0(x)$ (ii) $\frac{4}{x}J_1(x)-J_0(x)$

(iii) $\frac{2}{x}J_1(x)-\frac{2}{x}\cdot J_0(x)$ (iv) None of these

(f) The value of z for which e^z is real, is

(i) any multiple of π

(ii) odd multiple of

(iii) all multiple of $\frac{\pi}{2}$

(iv) any real number

(g) The inequality between mean and variance of binomial distribution which is true is

(i) mean < variance

(ii) mean = variance

(iii) mean > variance

(iv) (mean)·(variance) = 1

(h) Let $f(x) = e^{-x}$ for $x \geq 0$

$= 0$ for $x < 0$

then the value of the probability distribution function $x=2$ is

(i) $1+e^{-2}$

(ii) $1-e^{-2}$

(iii) $1+e^{2}$

(iv) $1+e^{-2t}$

(i) The necessary condition for $f(z)=(u+iv)$ to be analytic at all points in a region R are

(i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ (ii) $-\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

(iii) $-\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ (iv) None of these

(j) The partial differential equation

$$x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$ is an example of

(i) hyperbolic equation

(ii) elliptic equation

(iii) parabolic equation

(iv) None of the above

2 (a) Solve in series the equation

$$(1-x^2) \cdot y_2 - 2xy_1 + 2y = 0$$

(b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

3. (a) Prove that :

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n \cdot J_n(x)$$

(b) Prove that :

(i) $(n+1)\cdot P_{n+1}(x) =$

$(2n+1)x \cdot P_n(x) - n \cdot P_{n-1}(x)$

(ii) $P_n(-x) = (-1)^n \cdot P_n(x)$

Symbol has usual meaning.

4. Prove that :

(a) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

(b) $x \cdot J_n'(x) = n \cdot J_n(x) - x \cdot J_{n+1}(x)$

[illegible]

[illegible] partial differential equation [illegible] the arbitrary function from

[illegible] $x^2+y^2+z^2)=0$.

(b) Solve

$$(y+z)\cdot\frac{\partial z}{\partial x} + (z+x)\frac{\partial z}{\partial y} = x+y$$

(c) Solve

$$[illegible]\ y^2)\,(p^2+q^2)=1$$

[illegible] using the method of separaton of variables solve

912a2

$$\frac{\partial^2 z}{\partial x^2} - 2\cdot\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

[illegible] Find the D'Alembert's solution of the wave equation

929 (M)

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$$

7. (a) Define analytic function. Test the analyticity of the function $w = \sin z$ and hence derive that $\frac{d(\sin z)}{dz} = \cos z$.

$$= 0 \qquad z = 0$$

satisfy the Cauchy-Riemann equation at $z = 0$. Is the function analytic at $z = 0$? Justify your answer.

8. (a) Define harmonic function. Prove that $u = x^2 - y^2$ and $v = \dfrac{y}{x^2 + y^2}$ are harmonic functions of (x, y), but are not harmonic conjugate.

(b) Find the Taylor series expansion of a function of the complex variable $f(z) = \dfrac{1}{(z-1)(z-3)}$ about the point $z = 4$. Find its region of convergence.

9. (a) Three urns contains 6 red, 4 black; 4 red, 6 black; 5 red, 5 black balls respectively. One of the urn is selected at

random and a ball is drawn from it. If the ball drawn is red find the probability that drawn from the 1st urn

(b) Establish a relation between [moments] about mean and moments about any point.