

GENERAL APTITUDE

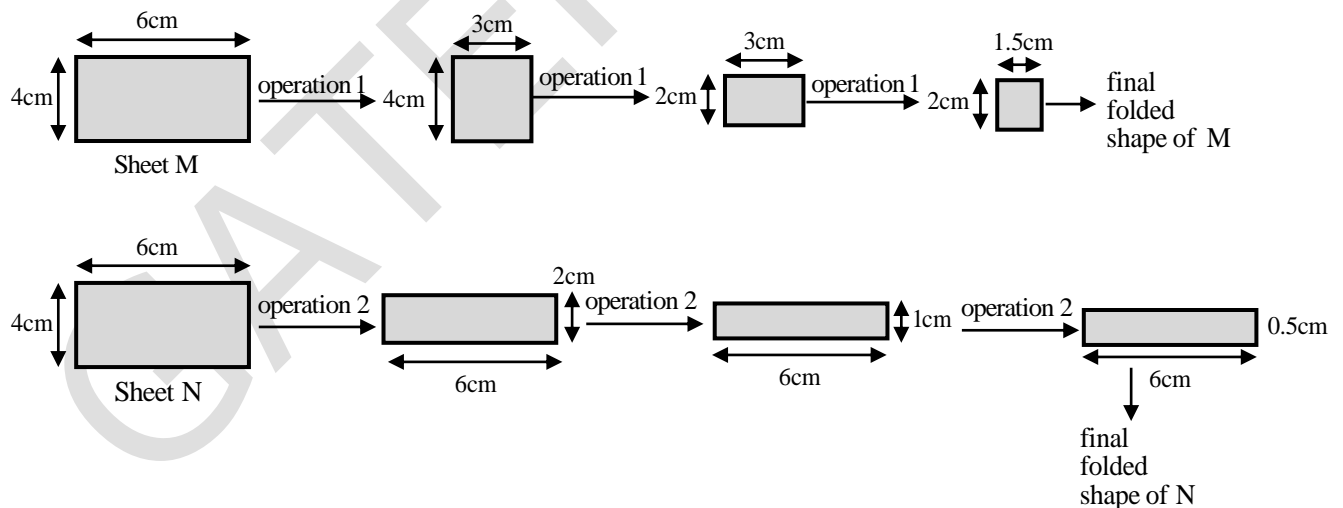
1. Getting to the top is _____ than staying on top.
(A) much easy (B) more easy (C) easiest (D) easier

Key: (D)

2. Consider two rectangular sheets, Sheet M and Sheet N of dimensions $6\text{cm} \times 4\text{cm}$ each
Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.
Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.
Folding operation 1 is carried out on Sheet M three times.
Folding operation 2 is carried out on Sheet N three times.
The ratio of perimeters of the final folded shape of Sheet N to the final folded shape of Sheet M is _____.
(A) 3:2 (B) 5:13 (C) 7:5 (D) 13:7

Key: (D)

Sol:

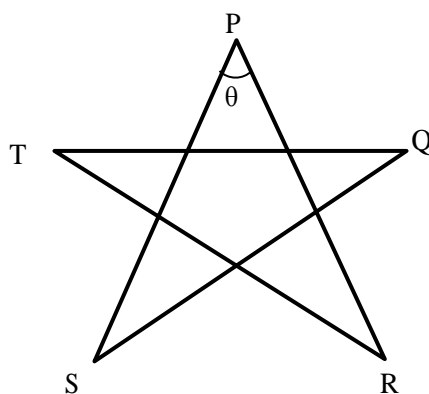


$$\therefore \text{Perimeter of final folded shape of N} = 2(6 + 0.5) = 13$$

$$\text{Perimeter of final folded shape of M} = 2(2 + 1.5) = 7$$

$$\therefore \text{Required ratio} = 13:7$$

3.



Five line segments of equal lengths, PR, PS, QS, QT and RT are used to form a star as shown in the figure above.

The value of θ , in degrees, is _____.

(A) 45

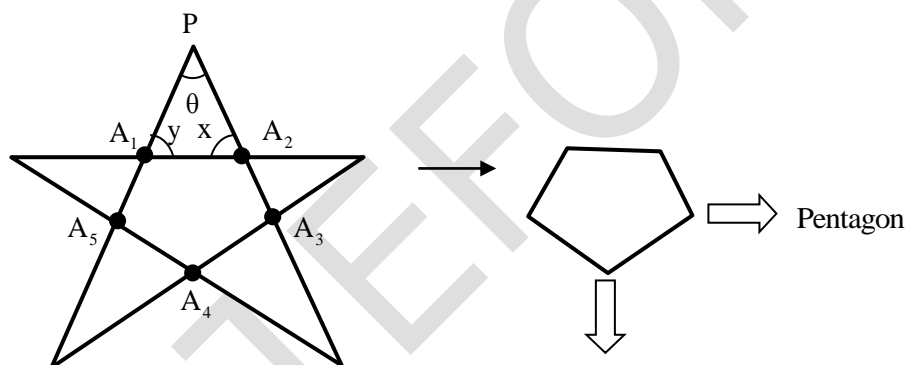
(B) 72

(C) 108

(D) 36

Key: (D)

Sol:



Sum of internal angles formed at pentagon $= (5 - 2) \times 180$
 $= 3 \times 180$

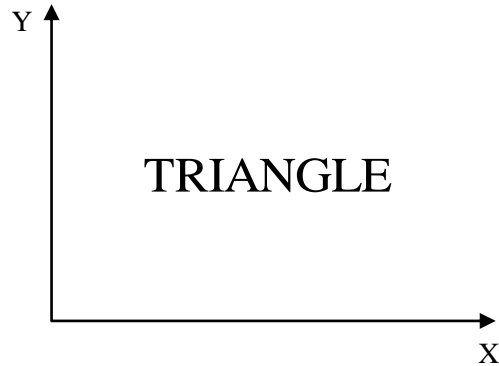
$$\Rightarrow \text{Each internal angle} = \frac{3 \times 180}{5} = 108^\circ$$

In $\triangle PA_1A_2$; $\angle\theta = ?$; $\angle x = 180 - 108 = 72$; $\angle y = 180 - 108 = 72$

$$\therefore \angle x + \angle y + \theta = 180$$

$$\Rightarrow \theta = 180 - 72 - 72 = 36^\circ$$

4.



The mirror image of the above text about X-axis is

- (A) TRIANGLE (B) TRIANGLE
(C) TRIANGLE (D) TRIANGLE

Key: (D)

Sol: The mirror image of TRIANGLE about the X-axis is

TRIANGLE

[\because Top and Bottom will interchange (or) symmetrical about X-axis]

5. Statement: Either P marries Q or X marries Y

Among the options below, the logical **NEGATION** of the above statement is:

- (A) Neither P marries Q nor X marries Y (B) X does not marry Y and P marries Q
(C) P does not marry Q and X marries Y (D) P marries Q and X marries Y

Key: (A)

Sol: Given statement: Either P marries Q or X marries Y.

The logical negation of the Either P marries Q or X marries Y

“above statement is Neither P marries Q nor X marries Y.”

6. A function, λ , is defined by

$$\lambda(p, q) = \begin{cases} (p - q)^2, & \text{if } p \geq q, \\ p + q, & \text{if } p < q. \end{cases}$$

The value of the expression $\frac{\lambda(-(-3+2), (-2+3))}{(-(-2+1))}$ is:

- (A) 16 (B) 0 (C) $\frac{16}{3}$ (D) -1

Key: (B)

Sol:
$$\frac{\lambda(-(-3+2), (-2+3))}{(-(-2+1))} = \frac{\lambda(-(-1), 1)}{1} = \lambda(1, 1) = (1-1)^2 = 0 \quad \left[\because \lambda(p, q) = (p-q)^2 \text{ if } p = q \right]$$

7. For persons P, Q, R and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R cannot sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is:

(A) 2 (B) 6 (C) 4 (D) 8

Key: (B)

Sol: According to the given information,

$$\frac{Q}{(P, R)} \times \frac{2}{1} \times \frac{S}{1} = 2 \text{ ways}$$

OR

$$\frac{2}{(P, R)} \times \frac{Q}{1} \times \frac{S}{1} = 2 \text{ ways}$$

OR

$$\frac{2}{(P, R)} \times \frac{Q}{1} \times \frac{S}{1} = 2 \text{ ways}$$

$$\Rightarrow \text{Total number of ways} = 2 + 2 + 2 = 6$$

\therefore The number of distinct seating arrangements possible is 6.

8. \oplus and \odot are two operators on numbers p and q such that

$$p \oplus q = \frac{p^2 + q^2}{pq} \text{ and } p \odot q = \frac{p^2}{q};$$

If $x \oplus y = 2 \odot 2$, then $x =$

(A) $\frac{3y}{2}$ (B) $2y$ (C) y (D) $\frac{y}{2}$

Key: (C)

Sol: $x \oplus y = 2 \odot 2$

$$\Rightarrow \frac{x^2 + y^2}{xy} = \frac{2^2}{2} \left[\because p \oplus q = \frac{p^2 + q^2}{pq} \text{ and } p \odot q = \frac{p^2}{q} \right]$$

$$\Rightarrow x^2 + y^2 = 2xy \Rightarrow x^2 + y^2 - 2xy = 0 \Rightarrow (x - y)^2 = 0 \Rightarrow x - y = 0 \Rightarrow \boxed{x = y}$$

9. Humans have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So far as we know, no other species possesses this ability. This skill is so important that we have different words to refer to its different flavors, such as imagination, invention and innovation.

Based on the above passage, which one of the following is TRUE?

- (A) We do not know of any species other than humans who possess the ability to construct mental worlds
- (B) imagination, invention and innovation are unrelated to the ability to construct mental worlds
- (C) No species possess the ability to construct worlds in their minds
- (D) The terms imagination, invention and innovation refer to unrelated skills

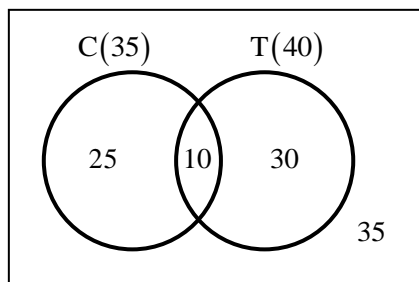
Key: (A)

10. In a company, 35% of the employees drink coffee, 40% of the employees drink tea and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

- (A) 35 (B) 15 (C) 40 (D) 25

Key: (A)

Sol:



$C \rightarrow$ Coffee \rightarrow % of employees drink Coffee = 35%

$T \rightarrow$ Tea \rightarrow % of employees drink Tea = 40%

\rightarrow % of employees drink both Tea and Coffee = 10%

% of employees drink neither Tea nor Coffee

$$= 100\% - (25\% + 10\% + 30\%)$$

$$= 35\% \quad (\because \text{From the venn diagram})$$

INSTRUMENTATION ENGINEERING

1. A single-phase transformer has a magnetizing inductance of 250 mH and a core loss resistance of 300Ω , referred to primary side. When excited with a 230V, 50Hz sinusoidal supply at the primary, the power factor of the input current drawn, with secondary on open circuit, is _____ (rounded off to two decimal places).

Key: (0.253)

Sol: $X_0 = 2\pi \times 50 \times 250 \times 10^{-3}$

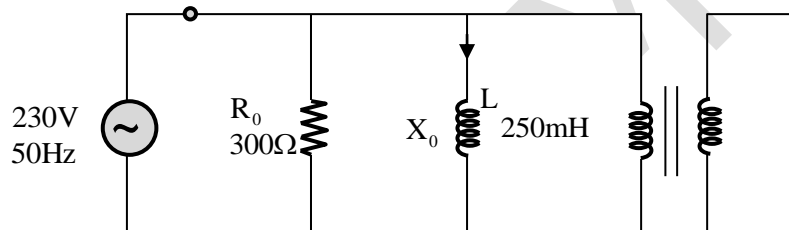
$$X_0 = j78.53\Omega$$

$$Z_0 = \frac{R_0 \times jX_0}{R_0 + jX_0} = \frac{300 \times j78.53}{300 + j78.53}$$

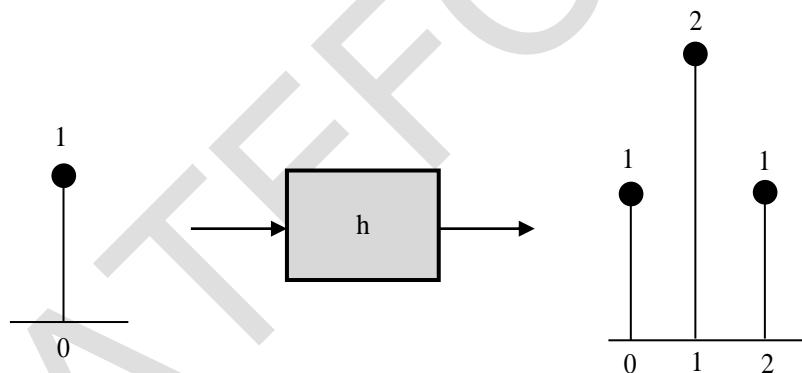
$$Z_0 = 19.23 + j73.49$$

$$Z_0 = 75.97 \angle 75.33^\circ$$

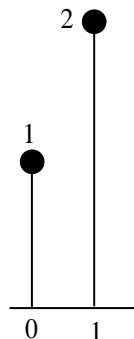
$$\cos \theta = \cos(75.33^\circ) = 0.253 \text{ lag}$$



2. The input-output relationship of an LTI system is given below



For an input $x[n]$ shown below



The peak value of the output when $x[n]$ passes through h is _____.

- (A) 5 (B) 4 (C) 2 (D) 6

Key: (A)

Sol: From the given information it is clear that the unit impulse response of a Discrete time LTI is given i.e.,

$$h(n) = \left[\underset{\uparrow}{1}, 2, 1 \right], \text{ we need to obtain the peak value of output } y(n) \text{ for input } x(n) = \left[\underset{\uparrow}{1}, 2 \right]$$

$$y(n) = x(n) * h(n) = \left[\underset{\uparrow}{1}, 2, 1 \right] * \left[\underset{\uparrow}{1}, 2 \right]$$

$$\begin{array}{r} 1 \quad 2 \quad 1 \\ 1 \quad 2 \\ \hline 1 \quad 2 \quad 1 \\ \quad 2 \quad 4 \quad 2 \\ \hline 1 \quad 4 \quad 5 \quad 2 \end{array}$$

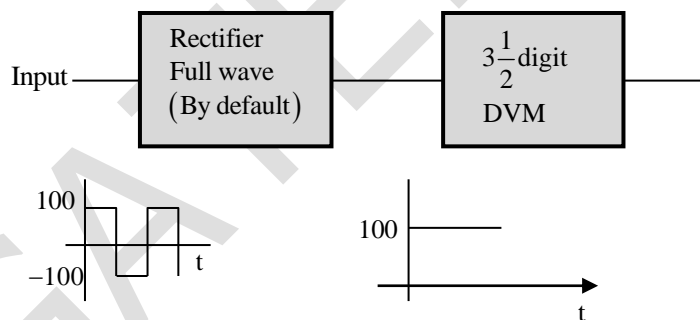
$$\text{So } y(n) = \left[\underset{\uparrow}{1}, 4, 5, 2 \right]$$

$$y_{\text{peak}} = 5$$

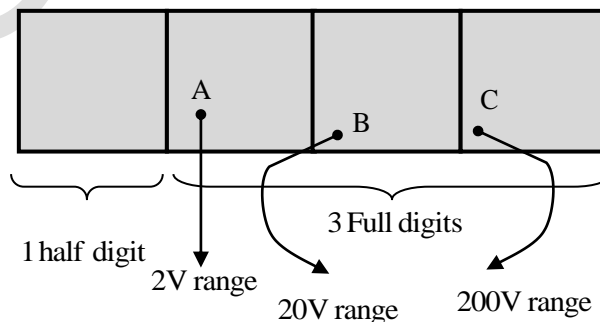
3. A $3\frac{1}{2}$ digit, rectifier type digital meter is set to read in its 2000V range. A symmetrical square wave of frequency 50Hz and amplitude $\pm 100\text{V}$ is measured using the meter. The meter will read _____.

Key: (100)

Sol: It is given that a $3\frac{1}{2}$ digit rectifier type digital meter is set at 2000V range. The input to meter is square wave of amplitude $\pm 100\text{V}$ with frequency 50Hz. We need to obtain meter reading.



We know DVM reads the average value, and average of 100V, D.C is 100.



We can see that in $3\frac{1}{2}$ digital display the maximum range possible is 200V, however in question it is given 2000V. Let's take the highest range i.e., 200V. In this case out of A, B, C only C LED will be selected.

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
|---|---|---|---|

When it has to read 100V, its value which will be displayed is 100.0

4. Consider the row vectors $v = (1, 0)$ and $w = (2, 0)$. The rank of the matrix $M = 2v^T v + 3w^T w$, where the superscript T denotes the transpose, is
- (A) 4 (B) 2 (C) 3 (D) 1

Key: (D)

Sol: $v^T v = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $w^T w = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore M = \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix}$ is Row-Echelon form

\therefore Rank of M is 1 (i.e., number of non-zero rows)

5. In an ac main, the rms voltage V_{ac} , rms current I_{ac} power W_{ac} are measured as: $V_{ac} = 100V \pm 1\%$, $I_{ac} = 1A \pm 1\%$ and $W_{ac} = 50W \pm 2\%$ (errors are with respect to readings). The percentage error in calculating the power factor using these readings is
- (A) 4% (B) 3% (C) 2% (D) 1%

Key: (A)

Sol: In some Power measurement, it is given that

$V_{ac} = 100V \pm 1\%$

$I_{ac} = 1A \pm 1\%$

$W_{ac} = 50W \pm 2\%$

We need to compute % error in power factor

$W = VI \cos \phi$

$P.F = \cos \phi = \frac{W}{VI} = \frac{(50 \pm 2\%)}{(100 \pm 1\%)(1 \pm 1\%)} = \frac{50}{100 \times 1} \pm [1\% + 1\% + 2\%] = 0.5 \pm 4\%$

So % error in power factor is 4.

6. A Boolean function F of the three variables X, Y and Z is given as

$$F(X, Y, Z) = (X' + Y + Z) \cdot (X + Y' + Z') \cdot (X' + Y + Z') \cdot (X'Y'Z' + X'YZ' + XYZ')$$

Which one of the following is true?

- (A) $F(X, Y, Z) = X'Y'Z + XYZ$ (B) $F(X, Y, Z) = (X' + Y) \cdot (X + Y' + Z')$
(C) $F(X, Y, Z) = X'Z' + YZ'$ (D) $F(X, Y, Z) = (X + Y + Z') \cdot (X' + Y' + Z')$

Key: (C)

Sol: We need to simplify the following Boolean expression,

$$\begin{aligned} F(X, Y, Z) &= (\overline{X} + Y + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + Y + \overline{Z})(\overline{X}\overline{Y}\overline{Z} + \overline{X}Y\overline{Z} + XY\overline{Z}) \\ &= (\overline{X} + Y)(X + \overline{Y} + \overline{Z})[\overline{X}\overline{Z}(\overline{Y} + Y) + XY\overline{Z}] \\ &= [YX + \overline{X}(\overline{Y} + \overline{Z})][\overline{X}\overline{Z} + XY\overline{Z}] \quad \left[\begin{array}{l} \because (A+B)(A+C) = A+BC \\ \because (A+B)(\overline{A}+C) = \overline{A}B+AC \\ \because \overline{X} + XY = \overline{X} + Y \end{array} \right] \\ &= (XY + \overline{X}\overline{Y} + \overline{X}\overline{Z})[\overline{Z}(\overline{X} + Y)] \\ &= (XY + \overline{X}\overline{Y} + \overline{X}\overline{Z})(\overline{X}\overline{Z} + Y\overline{Z}) \\ &= \overbrace{XY \cdot \overline{X}\overline{Z}}^0 + XY \cdot Y\overline{Z} + \overbrace{\overline{X}\overline{Y} \cdot \overline{X}\overline{Z}}^0 + \overline{X}\overline{Y} \cdot Y\overline{Z} + \overline{X}\overline{Z} \cdot \overline{X}\overline{Z} + \overline{X}\overline{Z} \cdot Y\overline{Z} \\ &= 0 + XY\overline{Z} + \overline{X}\overline{Y}\overline{Z} + 0 + \overline{X}\overline{Z} + \overline{X}Y\overline{Z} \\ &= \overline{X}\overline{Z} + \overline{X}\overline{Y}\overline{Z} + Y\overline{Z}(\overline{X} + X) \\ &= \overline{X}\overline{Z} + \overline{X}\overline{Y}\overline{Z} + Y\overline{Z} \\ &= \overline{X}\overline{Z}(1 + \overline{Y}) + Y\overline{Z} \\ &= \overline{X}\overline{Z} + Y\overline{Z} \end{aligned}$$

7. A 300V, 5A, LPF wattmeter has a full scale of 300W. The wattmeter can be used for loads supplied by 300V ac mains with a maximum power factor of _____ (rounded off to one decimal place).

Key: (0.2)

Sol: It is given that 300V, 5A wattmeter has full scale value 300W. We need to obtain the maximum power factor of load this wattmeter can measure.

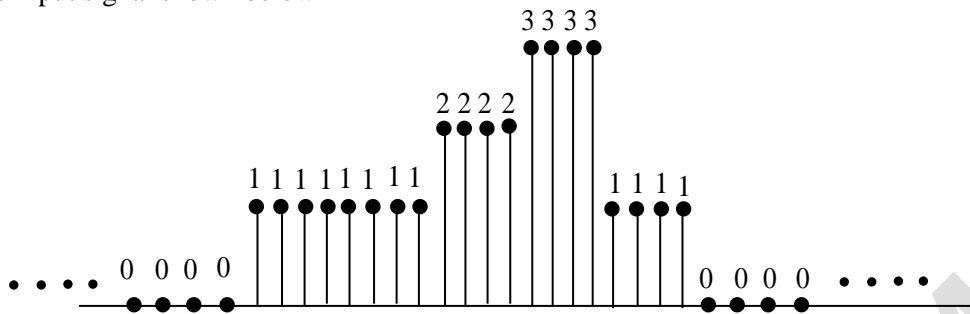
$$P = VI \cos \phi$$

$$300 = 300 \times 5 \times \cos \phi$$

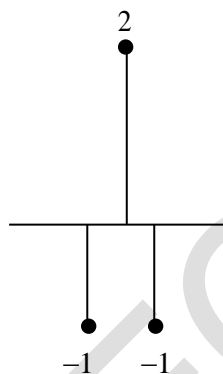
$$\Rightarrow \cos \phi = \frac{300}{300 \times 5} = 0.2$$

It means on full scale operating condition power factor of load can be upto 0.2.

8. The input signal shown below



is passed through the filter with the following taps



The number of non-zero output samples is _____.

Key: (10)

Sol: For a LTI system the impulse response $h(n) = [-1, 2, -1]$

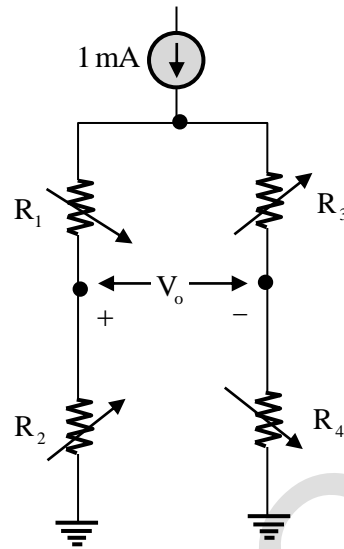
Input signal $x(n) = [1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 1, 1, 1, 1]$

We need to obtain number of non zero samples at output $y(n) = x(n) * h(n)$

| | | | | | | | | | | | | | | | | | | | | | | |
|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | | | |
| -1 | 2 | -1 | | | | | | | | | | | | | | | | | | | | |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -2 | -2 | -2 | -2 | -3 | -3 | -3 | -3 | -1 | -1 | -1 | -1 | | | |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 6 | 6 | 6 | 6 | 2 | 2 | 2 | 2 | | |
| | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -2 | -2 | -3 | -3 | -3 | -3 | -3 | -3 | -1 | -1 | -1 | -1 | |
| $y(n) =$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | 2 | -2 | 0 | 0 | 1 | -1 |

We can see in $y(n)$ we have 10 number of non zero samples which are circled.

9. For the full bridge made of linear strain gages with gage factor 2 as shown in the diagram, $R_1 = R_2 = R_3 = R_4 = 100\Omega$ at 0°C and strain is 0. The temperature coefficient of resistance of the strain gages used is 0.005 per $^\circ\text{C}$. All strain gages are made of same material and exposed to same temperature. While measuring a strain of 0.01 at a temperature of 50°C , the output V_o in millivolt is _____ (rounded off to two decimal places).



Key: (2)

Sol: $R_{50} = R_o (\alpha \Delta T)$
 $= 100 [0.005 \times (50 - 0)] = 25\Omega$

$\Delta R = R_o (GF) \times \epsilon$
 $= 100 \times 2 \times 0.01 = 2\Omega$

$\therefore R_1 = 100 + 25 - 2 = 123\Omega$

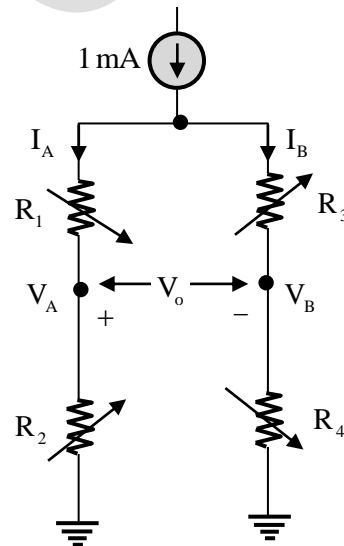
$R_2 = 100 + 25 + 2 = 127\Omega$

$R_3 = 100 + 25 + 2 = 127\Omega$

$R_4 = 100 + 25 - 2 = 123\Omega$

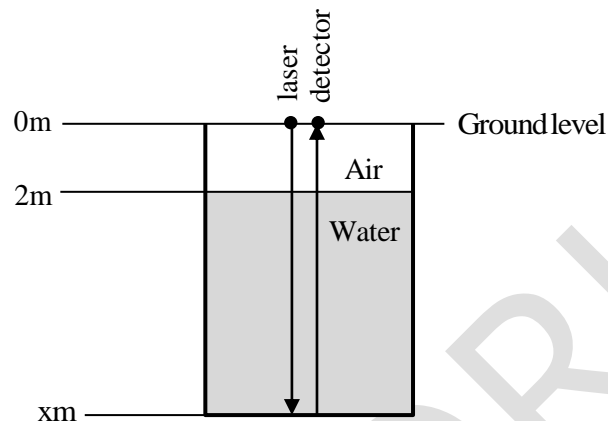
$I_A = I_B = (0.5\text{mA})$ as $(R_1 + R_2 = R_3 + R_4)$

$V_o = V_A - V_B$
 $= (127 - 123) \times 0.5 \times 10^{-3}$
 $= 2\text{mV}$



10. A laser pulse is sent from ground level to the bottom of a concrete water tank at normal incident. The tank is filled with water up to 2 m below the ground level. The reflected pulse from the bottom of the tank travels back and hits the detector. The round-trip time elapsed between sending the laser pulse, the pulse hitting the bottom of the tank, reflecting back and sensed by the detector is 100 ns. The depth of the tank from ground level marked as x in metre is _____.

(Refractive index of water $n_{\text{water}} = 1.3$ and velocity of light in air $c_{\text{air}} = 3 \times 10^8$ m/s)



- (A) 9 (B) 10 (C) 11 (D) 12

Key: (D)

Sol: $t_A = t_d$, $t_B = t_c$

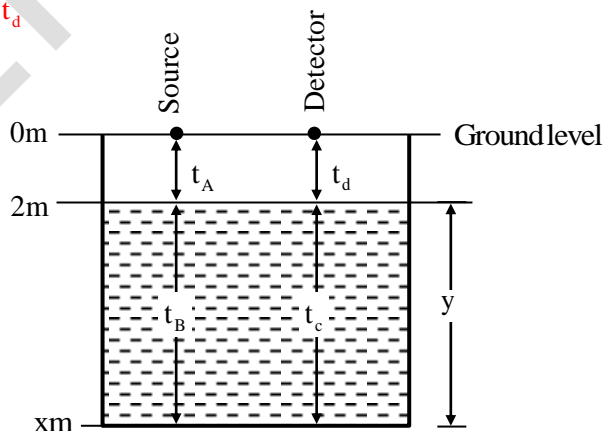
$$\text{Total time taken} = 100 \text{ nsec} = t_A + t_B + t_c + t_d \\ = 2(t_A + t_B)$$

$$\therefore 100 \text{ nsec} = 2 \left[\frac{2}{c} + \frac{y}{c/1.3} \right] = 2 \left[\frac{2}{c} + \frac{1.3y}{c} \right]$$

$$100 \text{ nsec} = \frac{4 + 2.6y}{c}$$

Solving $y = 10$, $[c = 3 \times 10^8 \text{ m/sec}]$

$$x = (y + 2) \\ = 10 + 2 = 12 \text{ m}$$

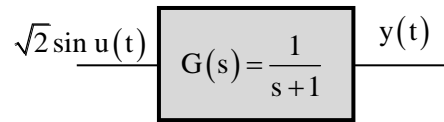


11. A sinusoid $(\sqrt{2} \sin t) \mu(t)$, where $\mu(t)$ is the step input, is applied to a system with transfer-function

$$G(s) = \frac{1}{s+1}. \text{ The amplitude of the steady state output is } \underline{\hspace{2cm}}.$$

Key: (1)

Sol: As per given information we need to obtain amplitude of $y(t)$ in the following diagram.



$$G(s) = \frac{1}{s+1}$$

$$G(j\omega) = \frac{1}{j\omega+1}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}, \angle G(j\omega) = -\tan^{-1}(\omega)$$

$$|G(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}}, \angle G(j\omega)_{\omega=1} = -45^\circ$$

$$\begin{aligned} y(t) &= \sqrt{2} |G(j\omega)|_{\omega=1} \sin\left(t - \angle G(j\omega)_{\omega=1}\right) u(t) \\ &= \sqrt{2} \times \frac{1}{\sqrt{2}} \sin(t - 45^\circ) u(t) \\ &= \sin(t - 45^\circ) u(t) \end{aligned}$$

So, amplitude of steady state output is 1.

12. Taking N as positive for clockwise encirclement, otherwise negative, the number of encirclement N of $(-1, 0)$ in the Nyquist plot of $G(s) = \frac{3}{s-1}$ is _____.

Key: (-1)

Sol: We need to obtain number of encirclement about $(-1, 0)$ for

$$G(s) = \frac{3}{s-1}$$

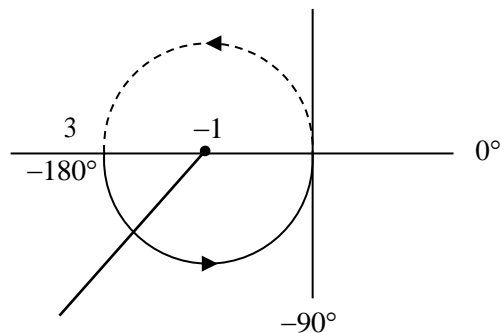
$$G(j\omega) = \frac{3}{\sqrt{\omega^2+1}} \angle -180^\circ + \tan^{-1}(\omega)$$

$$\text{When } \omega = 0, \quad G(j0) = 3 \angle -180^\circ$$

$$\omega = 1, \quad G(j1) = \frac{3}{\sqrt{2}} \angle -135^\circ$$

$$\omega = \infty, \quad G(j\infty) = 0 \angle -90^\circ$$

Based on above data the Nyquist plot for $G(s)$ will be



[Dotted part for $\omega: -\infty$ to 0 and Solid part for $\omega: 0$ to ∞]

Net encirclement about $-1, 0$ is 1 in anticlockwise direction.

As per question statement N is +ve for clockwise encirclement it means N is -ve for anticlockwise encirclement.

Hence $N = -1$, in this case.

13. A bar primary current transformer of rating 1000/1 A, 5VA, UPF has 995 secondary turns. It exhibits zero ratio error and phase error of 30 minutes at 1000A with rated burden. The watt loss component of the primary excitation current in ampere is _____ (rounded off to one decimal place).

Key: (5)

Sol: Given, $N_1 = 1$, $N_2 = 995$

$$P_f = 1 \Rightarrow \delta = 0$$

Rating : 1000 / 1A, 5VA

$$\text{Ratio error} = 0 = \frac{\text{Nominal Ratio}(K_n) - \text{Actual Ratio}(R)}{\text{Actual Ratio}(R)} \times 100$$

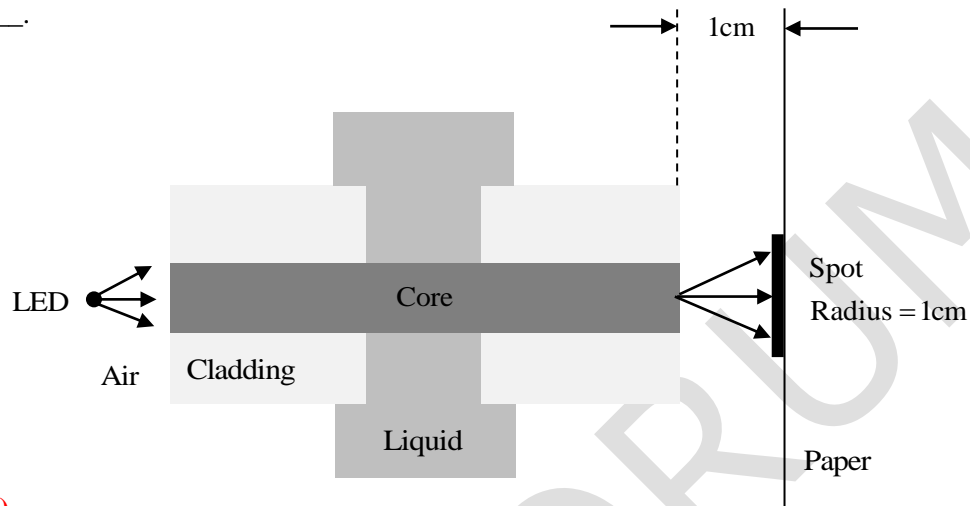
$$\Rightarrow K_n = R$$

$$\Rightarrow \frac{1000}{1} = n + \left(\frac{I_w \cos \delta + I_\mu \sin \delta}{I_s} \right) \left\{ \because K_n = \frac{I_p}{I_s} = \frac{1000}{1} \right\}$$

$$\Rightarrow 1000 = \frac{995}{1} + \frac{I_w \times \cos 0 + I_\mu \sin 0^\circ}{I_s} \left\{ \because n = \frac{N_s}{N_p} \right\}$$

$$\Rightarrow 5 = \frac{I_w}{I_s} \Rightarrow I_w = 5 \times I_s = 5 \times 1A$$

14. In the figure shown, a larger multimode fiber with $n_{\text{core}} = 1.5$ and $n_{\text{load}} = 1.2$ is used for sensing. A portion with the cladding removed passes through a liquid with refractive index n_{liquid} . An LED is used to illuminate the fiber from one end and a paper is placed on the other end, 1 cm from the end of the fiber. The paper shows a spot radius 1 cm. The refractive index n_{liquid} of the liquid (rounded off to two decimal places) is _____.



Key: (1.32)

Sol: Applying Snell's law at point 'X'

$$n_1 \sin \theta_i = n_o \sin \theta$$

$$\therefore \sin \theta_i = \frac{1 \times \sin 45^\circ}{1.5} = 0.47$$

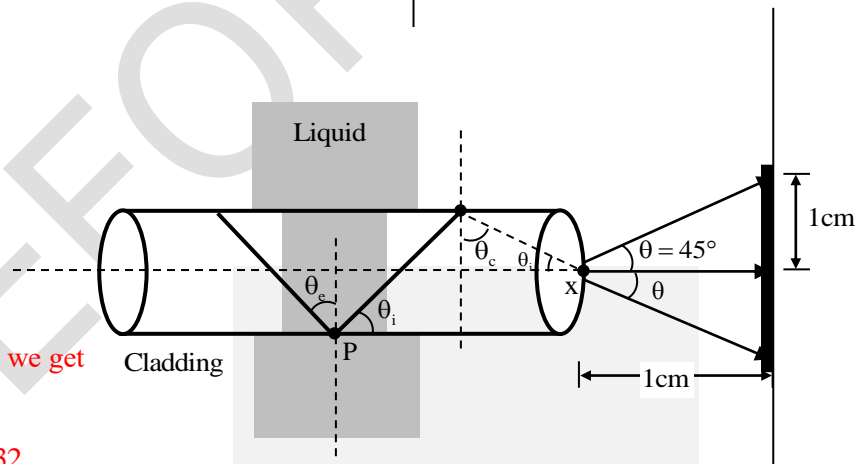
$$\therefore \theta_i = \sin^{-1}(0.47) = 28.12^\circ$$

$$\theta_c = 90 - \theta_i = 90 - 28.12 = 61.87^\circ$$

Now applying Snell's law at point P, we get

$$n_1 \sin \theta_c = n_{\text{liquid}}$$

$$\therefore n_{\text{liquid}} = n_1 \sin \theta_c = 1.5 \sin(61.87) = 1.32$$



15. A 10 bit ADC has a full-scale of 10.230V, when the digital output is $(11\ 1111\ 1111)_2$. The quantization error of the ADC in millivolts is _____.

Key: (+5)

Sol: • It is given that in a 10 bit ADC F.S.O is $\pm 0.25V$,

We need to compute the quantization error when output $(1111111111)_2$

$$\bullet \text{ Step size } (\Delta) = \frac{\text{F.S.O}}{2^n - 1} = \frac{10.23}{2^{10} - 1} = \frac{10.23}{1023} = 10\text{mV}$$

$$\bullet \text{ Quantization error: } \pm \frac{\Delta}{2} = \pm \frac{10\text{mV}}{2} = \pm 5\text{mV}$$

16. The determinant of the matrix M shown below is _____.

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Key: (4)

Sol: $|M| = \begin{vmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{vmatrix}$

Apply $R_2 - 3R_1$, we get $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{vmatrix} = 1 \times \begin{vmatrix} -2 & 0 & 0 \\ 0 & 4 & 3 \\ 0 & 2 & 1 \end{vmatrix} = 1 \times (-2) \times \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = (-2) \times (4 - 6) = 4$

17. An infinitely long line, with uniform positive charge density, lies along the z-axis. In cylindrical coordinates (r, ϕ, z) , at any point \vec{P} not on the z-axis, the direction of the electric field is

(A) \hat{z} (B) $\frac{(\hat{r} + \hat{z})}{\sqrt{2}}$ (C) \hat{r} (D) $\hat{\phi}$

Key: (C)

Sol: For an infinite line, that lies along z-axis and carries a uniform charge density ρ_ℓ C/m, the electric field intensity in cylindrical coordinate system is given by

$$\vec{E} = \frac{\rho_\ell}{2\pi\epsilon r} \hat{a}_r = \frac{\rho_\ell}{2\pi\epsilon r} \hat{r} \text{ V/m}$$

Thus, the electric field intensity will be along \hat{r} direction.

18. An amplitude modulation (AM) scheme uses tone modulation, with modulation index of 0.6. The power efficiency of the AM scheme is _____% (rounded off to one decimal place).

Key: (15.25)

Sol: It is given that in A.M. scheme, the modulation index is $\mu = 0.6$, we need to obtain power efficiency

$$\begin{aligned} \% \eta &= \frac{\mu^2}{2 + \mu^2} \times 100 \quad (\text{for A.M}) \\ &= \frac{0.6^2}{2 + 0.6^2} \times 100 = 15.25\% \end{aligned}$$

19. For a 4-bit Flash type Analog to Digital Converter (ADC) with full scale input voltage range “V”, which of the following statement(s) is/are true?

- (A) A change in the input voltage by $\frac{V}{16}$ will always flip the LSB of the output
- (B) The ADC requires one 4 to 2 priority encoder and 4 comparators
- (C) A change in the input voltage by $\frac{V}{16}$ will always flip MSB of the output
- (D) The ADC requires 15 comparators

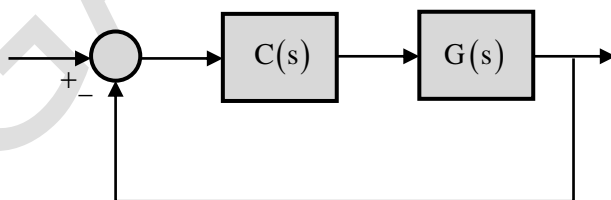
Key: (A,D)

Sol: We need to pick the correct statement regarding 4 bit flash type ADC.

1. A change in input voltage by $\frac{V}{16}$ will always flip LSB of output it is true. If we fix the step size at $\frac{V}{16}$, then when the input changes by this amount then at output the binary number increments or decrement by one step and when it happens there is always flip in LSB bit.
2. The ADC requires one 4×2 priority encodes and 4 comparators it is false, since for n bit flash type ADC need one $2^n \times n$ priority encoded and $2^n - 1$ number of comparators. So, a 4 bit structure need 16×4 priority encoder and 15 number of comparator.
3. This statement is false as it is opposite 1st statement k. Output change happens through LSB not MSB.
4. This statement is true as it need 15 comparators.

20. Consider a unity feedback configuration with a plant and a PID controller as shown in the figure.

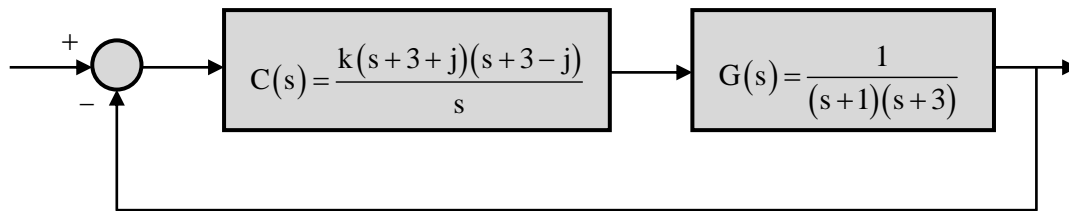
$G(s) = \frac{1}{(s+1)(s+3)}$ and $C(s) = K \frac{(s+3-j)(s+3+j)}{s}$ with K being scalar. The closed loop is



- (A) only stable for $K < 0$
- (B) stable for all value of K
- (C) only stable for $K > 0$
- (D) only stable for K between -1 and $+1$

Key: (C)

Sol: The given block diagram is as follows



The closed loop transfer function is $T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$

Characteristic equation is $1 + C(s)G(s) = 0$

$$\Rightarrow s(s+1)(s+3) + k[(s+3)^2 - (j)^2] = 0$$

$$\Rightarrow s(s^2 + 4s + 3) + k[s^2 + 6s + 9 + 1] = 0$$

$$\Rightarrow s^3 + s^2(k+4) + s(3+6k) + 10 = 0$$

For stability we need

$$(k+4)(3+6k) > 10$$

$$\Rightarrow 3k + 6k^2 + 12 + 24k > 10$$

$$\Rightarrow 6k^2 + 27k + 2 > 0$$

$$[\because as^3 + bs^2 + cs + d = 0 \text{ for stability } bc > ad]$$

- If $k=1$ then above equation is valid, hence option A is wrong
- If $k=-1$ then above equation is invalid, hence option B is wrong.
- If $k=2$ then also above equation is valid, hence option D is wrong
- If $k > 0$ then always above equation is valid, hence option C is correct.

21. A strain gage having nominal resistance of 1000Ω has a gage factor of 2.5. If the strain applied to the gage is $100\text{ }\mu\text{m/m}$, its resistance in ohm will change to _____ (rounded off to two decimal places).

Key: (1000.25)

Sol: $\frac{\Delta R}{R} = G.F \times \epsilon$

$$\begin{aligned} \Delta R &= 1000 \times 2.5 \times 100 \times 10^{-6} \\ &= 0.25\Omega \end{aligned}$$

Resistance of the gauge changes to $1000 + 0.25 = 1000.25\Omega$.

22. Let $u(t)$ denote the unit step function. The bilateral Laplace transform of the function $f(t) = e^t u(-t)$ is _____.

(A) $\frac{1}{s-1}$ with real part of $s > 1$

(B) $\frac{-1}{s-1}$ with real part of $s > 1$

(C) $\frac{1}{s-1}$ with real part of $s < 1$

(D) $\frac{-1}{s-1}$ with real part of $s < 1$

Key: (D)

Sol: Here we need to obtain Bilateral Laplace transform of $e^t u(-t)$

Laplace of $e^{-t} u(t)$ is $\frac{1}{s+1}$; $\sigma > -1$ (standard result)

$$e^t u(t) \rightarrow \frac{1}{-s+1}; \sigma < 1 \text{ (time reversal property)}$$

$$e^t u(-t) \leftrightarrow \frac{-1}{s-1}; \operatorname{Re}(s) < 1$$

23. Input-output characteristic of a temperature sensor is exponential for a

(A) Mercury thermometer

(B) Thermistor

(C) Thermocouple

(D) Resistive Temperature Device (RTD)

Key: (B)

Sol: In case of thermistors,

$$R_{T_1} = R_{T_2} \exp \beta \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

So, the input – output characteristics of a temperature sensor is exponential for a thermistor.

24. The signal $\sin(\sqrt{2\pi}t)$ is

(A) periodic with period $T = 4\pi^2$

(B) periodic with period $T = 2\pi$

(C) periodic with period $T = \sqrt{2\pi}$

(D) not periodic

Key: (D)

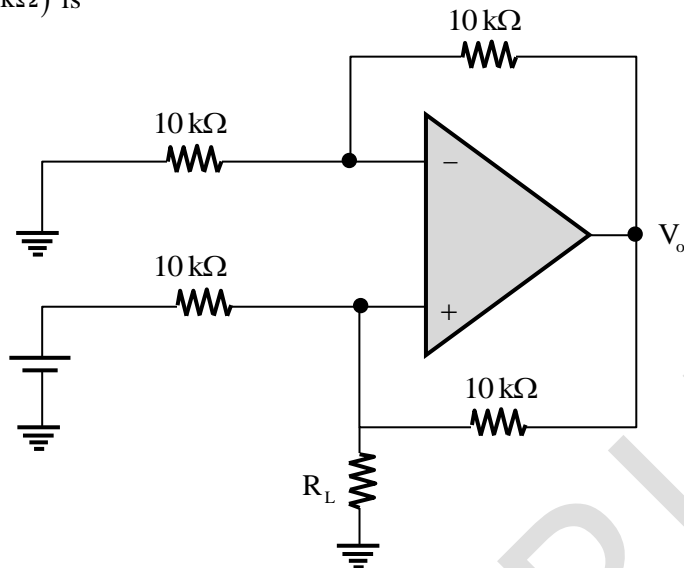
Sol: We need to obtain period of $x(t) = \sin \sqrt{2\pi}t$

For a signal to be periodic we need $x(t) = x(t \pm T_0)$

$$x(t \pm T_0) = \sin(\sqrt{2\pi}(t \pm T_0))$$

We can see no value of T_0 can make the above expression to $\sin \sqrt{2\pi}t$ i.e., $x(t)$, hence it's a non-periodic signal.

25. The output V_o of the ideal OpAmp used in the circuit shown below is 5V. Then the value of resistor R_L in kilo ohm ($k\Omega$) is



- (A) 25 (B) 5 (C) 2.5 (D) 50

Key: (A)

Sol: By the use of voltage division rule:

$$V_- = \frac{10k}{10k + 10k} \times V_o \quad \left(\begin{array}{l} \text{Op-amp is ideal so input} \\ \text{current to the op-amp is zero} \end{array} \right)$$

$$= \frac{1}{2} \times 5V = 2.5V$$

By the use of virtual short concept for Ideal op-amp

$$V_+ = V_- = 2.5V$$

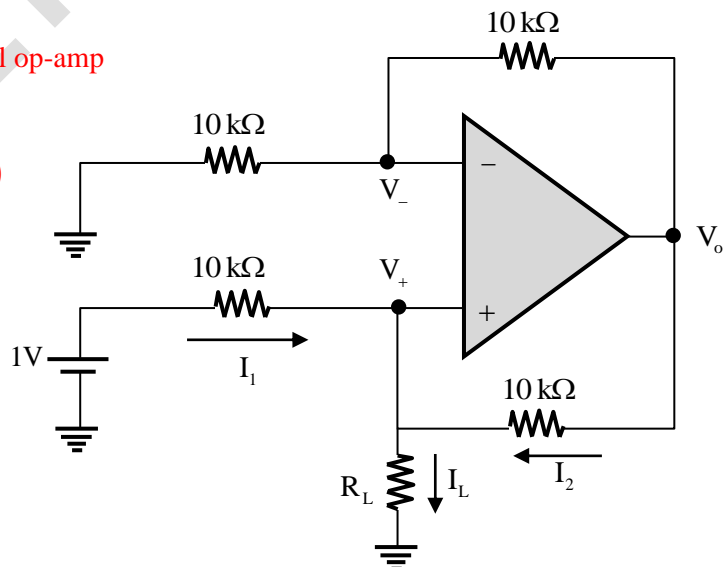
Apply KCL at non-inverting terminal (V_+)

$$\frac{1 - V_+}{10k} + \frac{V_o - V_+}{10k} = \frac{V_+}{R_L}$$

$$\frac{1V - 2.5V}{10k} + \frac{5V - 2.5V}{10k} = \frac{2.5V}{R_L}$$

$$\frac{1V}{10k} = \frac{2.5V}{R_L}$$

$$\therefore R_L = 25k\Omega$$

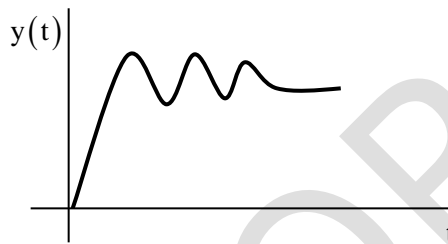


26. The step response of a circuit is seen to have an oscillatory behavior at the output with oscillations dying down after some time. The correct inference (s) regarding the transfer function from input to output is/are
- (A) that it does not have a real pole
 - (B) that it is first order system
 - (C) that it is of at least second order
 - (D) that it has at least one pole-pair that is underdamped

Key: (C,D)

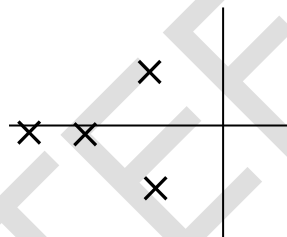
Sol: It is given that step response of a circuit is oscillatory for some time and then it dies after some time. It means the response should be (figure 1)

We need to comment about the nature of transfer function.



(figure 1)

Statement: 1



(figure 2)

If the pole pattern of transfer function as shown in figure 2, still the response will be similar to figure 1.

So, statement 1 is false.

Statement 2:

If the input is step, then to have oscillatory response the order should be at least 2 such that complex pole possibility will come into picture, in other words for step input oscillatory response will never happen in first order system.

Statement 3:

It is correct, as per the explanation of statement 2.

Statement 4:

It is correct since for oscillator response complex pole is must i.e., if the system will have only real pole for step input oscillation will never come it will be either critical or over damped response. Presence of complex pole indicate underdamped nature.

27. A single-phase transformer has maximum efficiency of 98%. The core losses are 80W and the equivalent winding resistance as seen from the primary side is 0.5Ω . The rated current on the primary side is 25A. The percentage of the rated input current at which the maximum efficiency occurs is
(A) 100% (B) 50.6% (C) 80.5% (D) 35.7%

Key: (B)

Sol: $\eta_{\max} = 98\%$

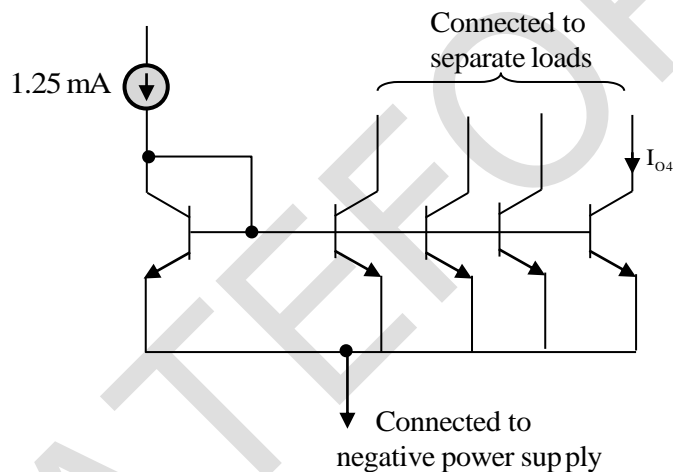
At maximum efficiency

$$(xI_2)^2 R_{02} = W_i$$

$$x = \sqrt{\frac{80}{25^2 \times 0.5}}$$

$$x = 50.59\%$$

28. All the transistors used in the circuit are matched and have a current gain β of 20. Neglecting the Early effect, the current I_{O4} in milliamperes is _____.



Key: (1)

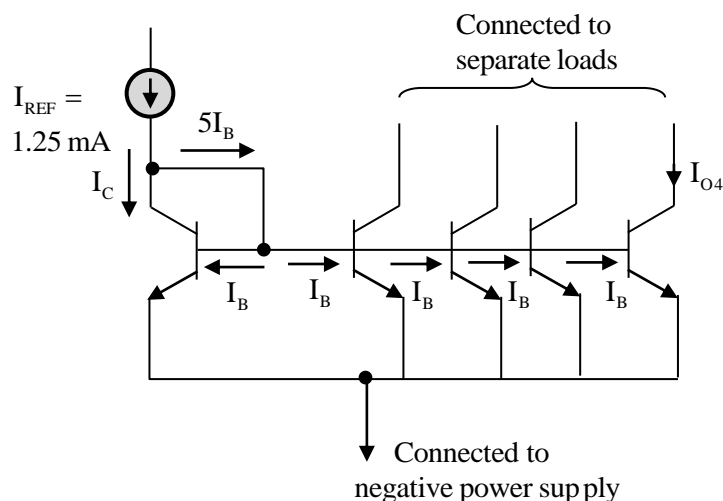
Sol: $I_{\text{REF}} = I_C + 5I_B$
 $= I_C + 5 \frac{I_C}{\beta} = \left(1 + \frac{5}{\beta}\right) I_C$

$$1.25\text{mA} = \left(1 + \frac{5}{20}\right) I_C$$

$$\therefore I_C = 1\text{mA}$$

All transistors are matched

$$\text{So, } I_{O4} = I_C = 1\text{mA}$$



29. Given $A = \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix}$. The value of the determinant $|A^4 - 5A^3 + 6A^2 + 2I| = \underline{\hspace{2cm}}$.

Key: (4)

Sol: Characteristic equation of A is $\begin{vmatrix} 2-\lambda & 5 \\ 0 & 3-\lambda \end{vmatrix} = 0$

$$\Rightarrow (2-\lambda)(3-\lambda) = 0 \Rightarrow \lambda^2 - 5\lambda + 6 = 0$$

$$\Rightarrow A^2 - 5A + 6I = 0 \quad (\text{Using Cayley - Hamilton theorem})$$

$$\text{Now } A^4 - 5A^3 + 6A^2 + 2I = A^2(A^2 - 5A + 6I) + 2I$$

$$= A^2 \times 0 + 2I \quad (\text{Using equation (1)})$$

$$= 2I$$

$$\therefore |A^4 - 5A^3 + 6A^2 + 2I| = |2I| = 2^2 |I| = 4 \quad (\because |kA| = k^n |A|)$$

30. Consider the function $f(x) = -x^2 + 10x + 100$. The minimum value of the function in the interval $[5, 10]$ is $\underline{\hspace{2cm}}$.

Key: (100)

Sol: $a = 5$, $b = 10$ i.e., Absolute minimum value required

$$f'(x) = 0 \Rightarrow -2x + 10 = 0 \Rightarrow x = 5 \in [5, 10] \text{ is a stationary point.}$$

$$f''(x) = -2 < 0 \Rightarrow f(x) \text{ has local maximum at } x = 5$$

$$\text{Now } f(5) = -25 + 50 + 100 = 125 \quad \text{and local minimum value does not exist.}$$

$$f(10) = -100 + 100 + 100 = 100$$

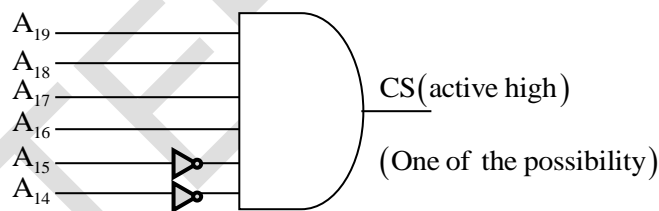
$$\therefore \text{Minimum value in } [5, 10] \text{ is } \min \{f(5), f(10)\} = \min(125, 100) = 100$$

31. A 16-bit microprocessor has twenty address lines (A_0 to A_{19}) and 16 data lines. The higher eight significant lines of the data bus of the processor are tied to the 8-data lines of a 16 Kbyte memory that can store one byte in each of its 16K address locations. The memory chip should map onto contiguous memory locations and occupy only 16 Kbyte of memory space. Which of the following statement(s) is/are correct with respect to the above design?

- (A) If the 16 Kbyte of memory chip is mapped with a starting address of 80000H, then the ending address will be 83FFFH.
- (B) The above chip cannot be interfaced as the width of the data bus of the processor and the memory chip differs.
- (C) The active high chip-select needed to map the 16 Kbyte memory with a starting address at F0000H is given by the logic expression $(A_{19} \cdot A_{18} \cdot A_{17} \cdot A_{16})$.
- (D) The 16 Kbyte memory cannot be mapped with contiguous address locations with a starting address as 0F000H using only A_{19} to A_{14} for generating chip select.

Key: (A, D)

- Sol:**
- It is given that a 16-bit microprocessor has 20 address the i.e., D_{15} to D_0 and A_{19} to A_0
 - It is also given D_{15} to D_8 are tied to 8 data line of a 16kB memory chip.
 - Minimum number of address line needed to access 16KB chip is 19 $[2^4 \times 2^{10} \times 8 = 2^{14} \times 8]$, then out of A_{14} to A_0 , for memory chip access A_{13} to A_0 will be renewed and A_{19} to A_{14} will be used for chip selection.
 - Line difference $= (11\ 1111\ 1111\ 1111)_2 = (3FFF)_{16}$
i.e., all value of address lines A_{13} to A_0 are taken 1.
 - If starting address is $(80000)_H$ then
Ending address = starting address + Line difference
 $= (80000)_{16} + (3FFF)_{16} = (83FFF)_{16}$
So statement (A) is correct.
 - Since 8 data lines of processor are tied with data line of memory chip, then 8 bit or 1 byte transfer can happen between processor and chip without any issue.
So statement (B) is false.
 - If starting address desired is F0000 then value of A_{19} to A_4 is 111100. Since chip select is active high



So expression of chip select is

$$CS = A_{19}A_{18}A_{17}A_{16}A_{15}A_{14}$$

But statement (C) says $CS = A_{19}A_{18}A_{17}A_{16}$

So statement (C) is wrong.

If starting address is 0F000 then ending address is $[0F000 + 03FFF = 12FFF]_{16}$

| | A_{19} | A_{18} | A_{17} | A_{16} | A_{15} | A_{14} | A_{13} | A_{12} | A_{11} | A_{10} | A_9 | A_8 | A_7 | A_6 | A_5 | A_4 | A_3 | A_2 | A_1 | A_0 |
|---|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Starting address (0F000) ₁₆ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ending address (12FFF) ₁₆ | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

If we see the bit pattern of starting and ending address all zero to all 1's variation happening only for A_{11} to A_0 it means in chip selection A_{19} to A_{14} will not be sufficient.

So, statement (D) is true.

32. $f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + \dots$ is the series expansion of

(A) $\frac{-1}{(z-1)}$ for $|z-1| < 1$

(B) $\frac{-1}{z(z-1)}$ for $|z-1| < 1$

(C) $\frac{1}{z(z-1)}$ for $|z-1| < 1$

(D) $\frac{1}{(z-1)^2}$ for $|z-1| < 1$

Key: (C)

Sol: Clearly, the expansion is power of (z-1)

(A) $f(z) = \frac{-1}{z-1} = -(z-1)^{-1}$

(B) $f(z) = \frac{-1}{z(z-1)} = \frac{-1}{(z-1)(1+z-1)} = \frac{-1}{z-1} \times [1+(z-1)]^{-1} = \frac{-1}{z-1} \times [1-(z-1)+(z-1)^2 - \dots]$
 $= -(z-1)^{-1} + 1 - (z-1) + (z-1)^2 - \dots$ if $|z-1| < 1$

(C) $f(z) = \frac{1}{z(z-1)} = \frac{1}{(z-1)} \times [1+(z-1)]^{-1} = \frac{1}{z-1} \times [1-(z-1)+(z-1)^2 - \dots]$
 $= (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + \dots$ if $|z-1| < 1$

(D) $f(z) = \frac{1}{(z-1)^2} = (z-1)^{-2}$

\therefore Option (C) is correct

33. A signal having a bandwidth of 5 MHz is transmitted using the Pulse code modulation (PCM) scheme as follows. The signal is sampled at a rate of 50% above the Nyquist rate and quantized into 256 levels. The binary pulse rate of the PCM signal in Mbits per second is _____.

Key: (120)

Sol: A signal having bandwidth $f_m = 5\text{MHz}$

Signal is sampled at a rate of 50% above Nyquist rate it means the sampling frequency.

$$f'_s = 1.5 f_s = 1.5 (2f_m) = 3f_m = 3 \times 5 \text{ MHz} = 15 \text{ MHz}$$

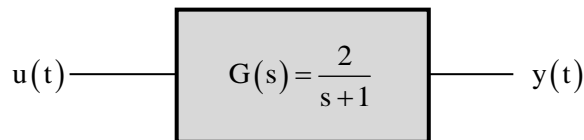
Number of levels given is $256 \Rightarrow 2^n = 256 \Rightarrow n = 8$

$$\text{Pulse rate: } nf'_s = 8 \times 15 \times 10^6 = 120 \text{ Mbps}$$

34. Consider a system with transfer-function $G(s) = \frac{2}{s+1}$. A unit step function $\mu(t)$ is applied to the system, which results in an output $y(t)$. If $e(t) = y(t) - \mu(t)$, then $\lim_{t \rightarrow \infty} e(t)$ is _____.

Key: (1)

Sol: As per the information given



We need to obtain $\lim_{t \rightarrow \infty} y(t) - u(t)$

$$Y(s) = G(s) \frac{1}{s} \quad \left(\because u(t) \rightarrow \frac{1}{s} \right)$$

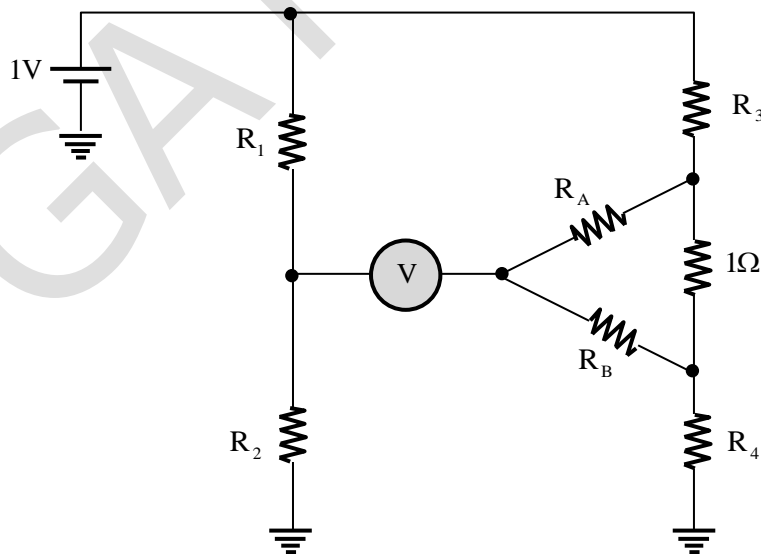
$$= \frac{2}{s(s+1)} = \frac{2}{s} + \frac{-2}{s+1}$$

$$y(t) = 2u(t) - 2e^{-t}u(t)$$

$$e(t) = y(t) - u(t) = 2u(t) - 2e^{-t}u(t) - u(t) = u(t) - 2e^{-t}u(t)$$

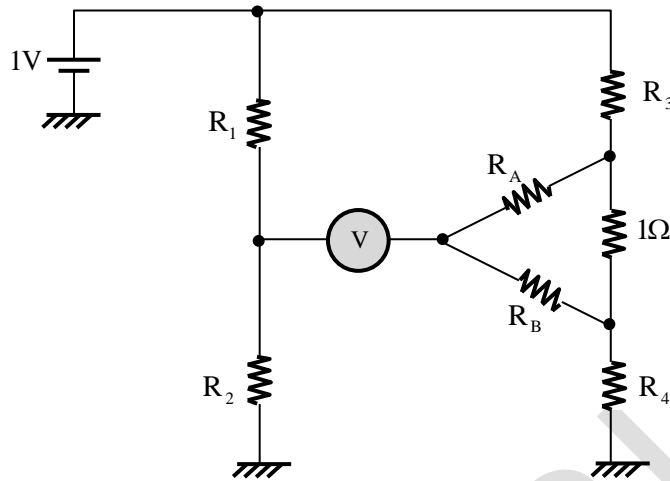
$$\Rightarrow e(\infty) = u(\infty) - 2e^{-\infty}u(\infty) = 1 - 0 = 1$$

35. In the bridge circuit shown, the voltmeter V showed zero when the value of the resistors are: $R_1 = 100\Omega$, $R_2 = 110\Omega$, and $R_3 = 90\Omega$. If $(R_1/R_2) = (R_A/R_B)$, the value of R_4 in ohm is _____.



Key: (99)

Sol: The given bridge circuit is as shown below



It is given that $\frac{R_1}{R_2} = \frac{R_A}{R_B}$ and voltmeter reading is 0.

The given circuit is standard Kelvin double bridge, and it is in balance condition since $\frac{R_1}{R_2} = \frac{R_A}{R_B}$ (given)

When this bridge is in balance then it always satisfies $R_1 R_4 = R_2 R_3$

$$\Rightarrow R_4 = \frac{R_2 R_3}{R_1} = \frac{110 \times 90}{100} = 99 \Omega$$

36. A piezoresistive pressure sensor has a sensitivity of 1 (mV/V)/kPa. The sensor is excited with a dc supply of 10V and the output is read using a 3½ digit 200 mV full-scale digital multimeter. The resolution of the measurement set-up, in pascal is _____.

Key: (10)

Sol: Given, Sensitivity = 1 (mV/V)/kPa

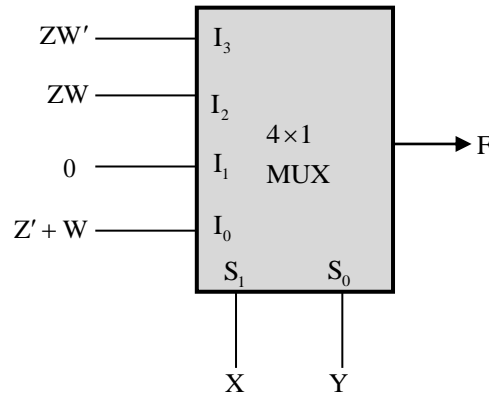
$$\text{Here also sensitivity} = \frac{\text{Full scale output}}{\text{input voltage}} \div \text{input pressure (in pascal)}$$

$$\therefore \frac{1 \text{ mV}}{\text{V}} = \frac{200 \text{ mV}}{10 \text{ V}} \div \text{Pressure input}$$

$$\therefore \text{Pressure input} = 20 \text{ k pascal}$$

$$\text{Resolution} = \frac{20 \text{ k pascal}}{\text{number of counts}} = \frac{20 \times 1000}{2000} = 10 \text{ pascals}$$

37. A 4×1 multiplexer with two selector lines is used to realize a Boolean function F having four Boolean variables X, Y, Z and W as shown below. S_0 and S_1 denote the least significant bit (LSB) and most significant bit (MSB) of the selector lines of the multiplexer respectively. I_0, I_1, I_2, I_3 are the input lines of the multiplexer.

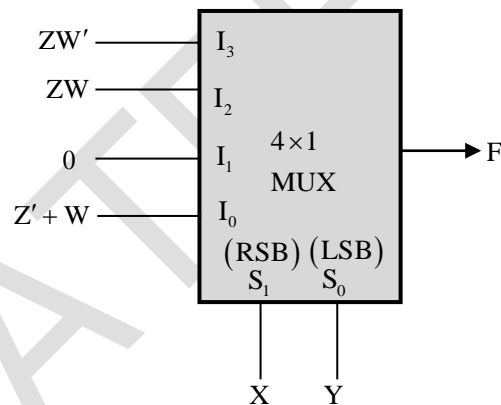


The canonical sum of product representation of F is

- (A) $F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$ (B) $F(X, Y, Z, W) = \sum m(0, 1, 3, 14, 15)$
(C) $F(X, Y, Z, W) = \sum m(2, 5, 9, 11, 14)$ (D) $F(X, Y, Z, W) = \sum m(1, 3, 7, 9, 15)$

Key: (A)

Sol: In this case we need to obtain the minterm combination of F .

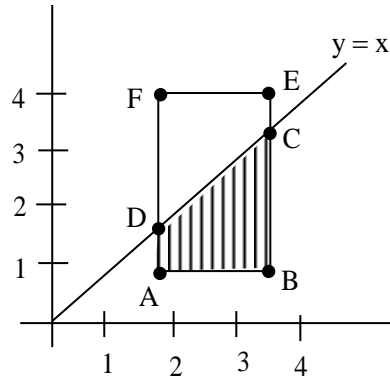


$$\begin{aligned}
 F &= \bar{S}_1 S_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3 \\
 &= \bar{X} \bar{Y} (\bar{Z} + W) + \bar{X} \bar{Y} \cdot 0 + X \bar{Y} (ZW) + XY (Z\bar{W}) \\
 &= \bar{X} \bar{Y} \bar{Z} + \bar{X} \bar{Y} W + X \bar{Y} ZW + XY Z\bar{W} \\
 &\quad \downarrow \quad \downarrow \quad \quad \quad \downarrow \quad \downarrow \\
 &\quad m_0, m_1 \quad m_3, m_1 \quad \quad m_{11} \quad m_{14} \\
 &= m_0 + m_1 + m_3 + m_{11} + m_{14} \\
 \Rightarrow F(X, Y, Z, W) &= \sum m(0, 1, 3, 11, 14)
 \end{aligned}$$

38. Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2,3)$ and $Y \sim U(1,4)$. Then $P(Y \leq X)$ is equal to _____ (rounded off to two decimal places).

Key: (0.5)

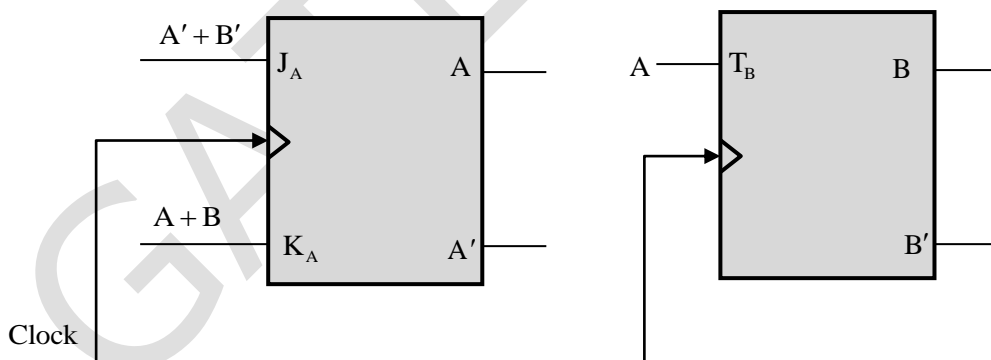
Sol: Let us take the value of x and y along X -axis, Y -axis respectively, we get the diagram as



Here rectangle is total possible outcomes (i.e., Sample space and Trapezium is favorable outcomes

$$\begin{aligned} \therefore P(Y \leq X) &= \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}} \\ &= \frac{\text{Area of ABCD}}{\text{Area of rectangular}} = \frac{\frac{1}{2}(AD + BC) \times AB}{AB \times AF} = \frac{\frac{1}{2}(1 + 2)}{3} = \frac{1}{2} = 0.5 \end{aligned}$$

39. Given below is the diagram of a synchronous sequential circuit with one J-K flip-flop and one T flip-flop with their outputs denoted as A and B respectively, with $J_A = (A' + B')$, $K_A = (A + B)$, and $T_B = A$.

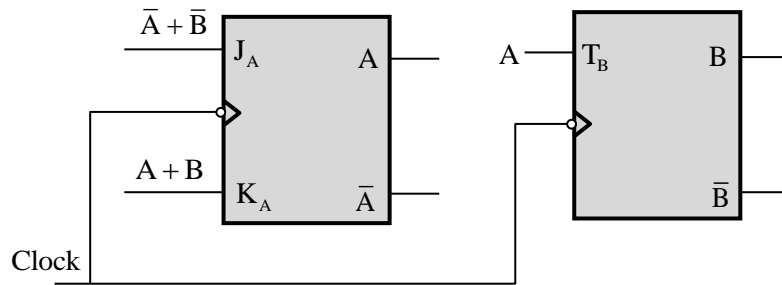


Starting from the initial state ($AB = 00$), the sequence of states (AB) visited by the circuit is

- (A) $00 \rightarrow 01 \rightarrow 11 \rightarrow 00 \dots$ (B) $00 \rightarrow 10 \rightarrow 11 \rightarrow 01 \rightarrow 00 \dots$
(C) $00 \rightarrow 01 \rightarrow 10 \rightarrow 11 \rightarrow 00 \dots$ (D) $00 \rightarrow 10 \rightarrow 01 \rightarrow 11 \rightarrow 00 \dots$

Key: (D)

Sol: In the following circuit, we need to obtain the counting sequence of the counter circuit given



| Present state | | Flip flop Inputs | | | Next state | |
|---------------|---|-------------------|-----------|-------|------------|-------|
| A | B | J_A | K_A | T_B | A^+ | B^+ |
| | | (\overline{AB}) | $(A + B)$ | A | | |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 |

Using J_A, K_A, A we obtained A^+ using characteristic equation of JK flip flop, similarly, using T_B, B we obtained B^+

If the initial state is 00 then the counting sequence is $00 \rightarrow 10 \rightarrow 01 \rightarrow 11 \rightarrow 00$ from the above table.

40. A toroid made of CRGO has an inner diameter of 10 cm and an outer diameter of 14cm. The thickness of the toroid is 2cm. 200 turns of copper wire is wound on the core, $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$ and μ_R of CRGO is 3000. When a current of 5mA flows through the winding, the flux density in the core in millitesla is _____.

Key: (10)

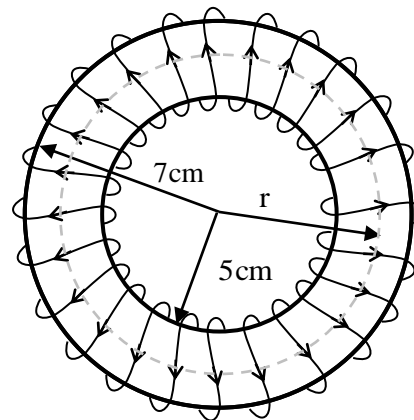
Sol: Applying Ampere's Law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu Ni$$

$$B \oint dl = \mu Ni$$

$$B(2\pi r) = \mu Ni$$

$$B = \frac{\mu_r \mu_o Ni}{2\pi r} = \frac{3000 \times 4\pi \times 10^{-7} \times 200 \times 5 \times 10^{-3}}{2\pi \times 6 \times 10^{-2}} = 10 \text{ mT}$$



$$r = \text{mean radius} = \frac{7 + 5}{2} = 6 \text{ cm}$$

41. Let $f(z) = \frac{1}{z^2 + 6z + 9}$ defined in the complex plane. The integral $\oint_C f(z) dz$ over the contour of a circle C with centre at the origin and unit radius is _____.

Key: (0)

Sol: C is $|z| = 1$ and $z^2 + 6z + 9 = 0 \Rightarrow z = -3, -3$ are the singular points, which lies outside the curve C

$\therefore f(z)$ is analytic in and on ' C '

\therefore By Cauchy integral theorem, $\oint_C f(z) dz = 0$

42. A $10\frac{1}{2}$ digit Counter-timer is set in the 'frequency mode' of operation (with $T_s = 1s$). For a specific input, the Counter-timer is changed to operate in the 'Period mode' and the range selected is microseconds (μs , with $f_2 = 1$ MHz). The counter will then display

(A) 0 (B) 1000 (C) 100 (D) 10

Key: (B)

Sol: Counting number of cycle of unknown signal in 1 sec interval method is known as frequency mode. If

timer output is N , then $f_{\text{unknown signal}} = N_1, T_{\text{unknown signal}} = \frac{1}{N_1}$

Calculating 1 period of unknown signal (T_{signal} in terms of multiple of a known signal period is periodic mode).

$$T_{\text{unknown signal}} = N_2 T_{\text{known signal}}$$

As per the data given $N_1 = 1000 = f_{\text{unknown signal}}$

As per the data given $N_1 = 1000 = f_{\text{unknown signal}}$

$$\Rightarrow T_{\text{unknown signal}} = \frac{1}{N_1} = 10^{-3}$$

$$N_2 = \frac{T_{\text{unknown signal}}}{T_{\text{known signal}}} \left[\because f_s = 1 \text{ MHz given means, i.e., known signal } T_{\text{signal}} = \frac{1}{f_s} = 10^{-6} \right]$$

$$= \frac{10^{-3}}{10^{-6}} = 10^3 = 1000$$

43. Given $y(t) = e^{-3t}u(t) * u(t+3)$, where $*$ denotes convolution operation. The value of $y(t)$ as $t \rightarrow \infty$ is _____ (rounded off to two decimal places).

Key: (0.333)

Sol: In this case we need to obtain $y(\infty)$ where

$$y(t) = e^{-3t}u(t) * u(t+3)$$

$$e^{-at}u(t) * u(t) \xLeftrightarrow{\quad} \frac{1}{a}(1 - e^{-at})u(t) \text{ (Standard result)}$$

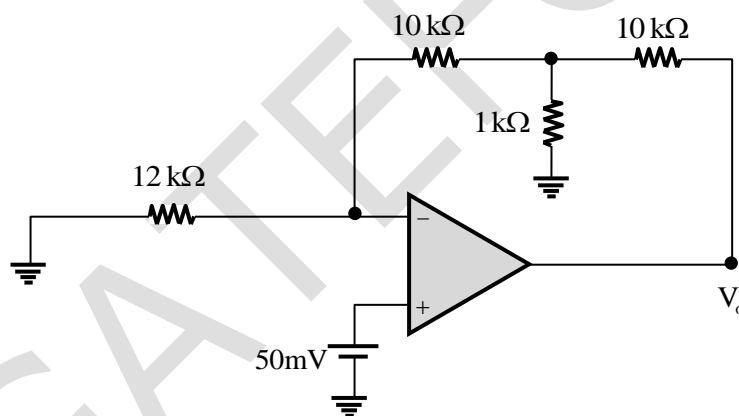
$$e^{-3t}u(t) * u(t) \xLeftrightarrow{\quad} \frac{1}{3}(1 - e^{-3t})u(t)$$

$$e^{-3t}u(t) * u(t+3) = \frac{1}{3}(1 - e^{-3(t+3)})u(t+3) \text{ (By time shifting properly)}$$

$$\text{So, } y(t) = \frac{1}{3}[1 - e^{-3(t+3)}]u(t+3)$$

$$y(\infty) = \frac{1}{3} \times (1 - e^{-\infty})u(\infty) = \frac{1}{3} \times (1 - 0) \times 1 = \frac{1}{3} = 0.333$$

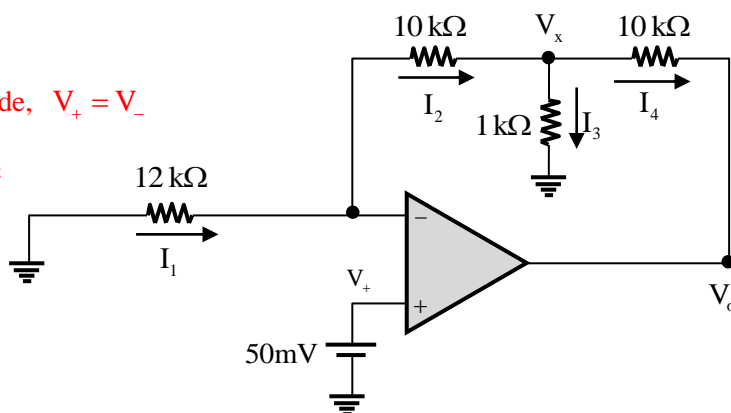
44. The circuit shown below uses an ideal OpAmp. Output V_o in volt is (rounded off to one decimal place).



Key: (1.05)

Sol: As per virtual short concept for ideal diode, $V_+ = V_-$

Apply KCL at inverting terminal, $I_1 = I_2$



$$\frac{0 - V_-}{12k} = \frac{V_- - V_x}{10k}$$

$$\therefore V_x = V_- + \frac{V_- \times 10}{12}$$

$$= 50\text{mV} + \frac{50\text{mV} \times 10}{12} (\because V_+ = V_- = 50\text{mV}) = 91.67\text{mV}$$

Apply KCL at node V_x , $I_2 = I_3 + I_4$

$$\frac{V_- - V_x}{10k} = \frac{V_x}{1k} + \frac{V_x - V_o}{10k}$$

$$\frac{V_- - V_x}{10} = V_x + \frac{V_x - V_o}{10}$$

$$= \frac{50\text{mV} - 91.67\text{mV}}{10} = 91.67\text{mV} + \frac{91.67\text{mV} - V_o}{10} (\because V_+ = V_- = 50\text{mV})$$

$$= V_o = 1.05\text{ Volts}$$

45. The transistor Q_1 has a current gain $\beta_1 = 99$ and the transistor Q_2 has a current gain $\beta_2 = 49$. The current I_{B2} in microampere is _____.

Key: (10)

Sol: Given: $\beta_1 = 99$ & $\beta_2 = 49$

$$I_{E1} = (1 + \beta_1) I_{B1}$$

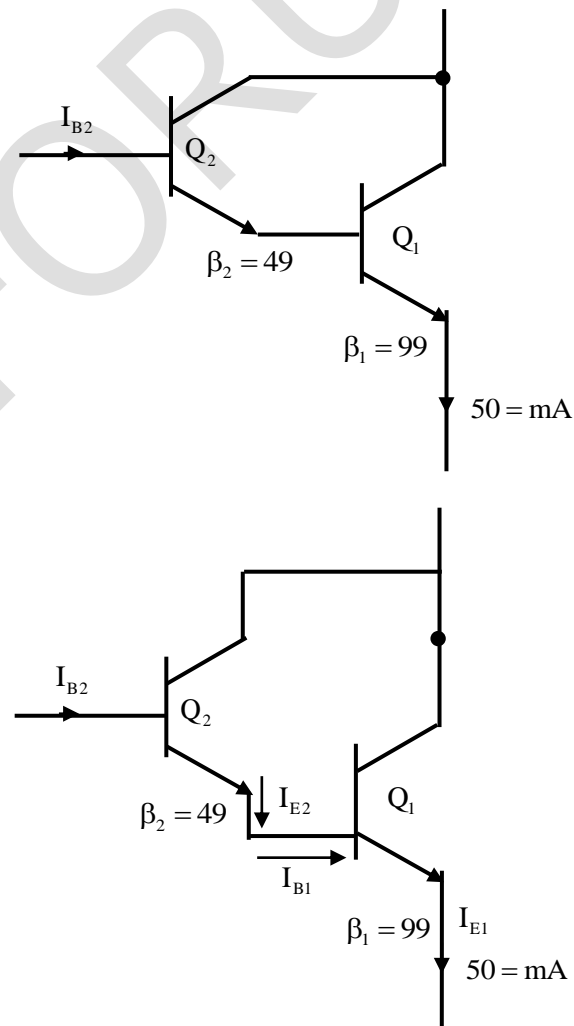
$$50\text{mA} = (1 + 99) \times I_{B1}$$

$$\therefore I_{B1} = \frac{50}{100}\text{mA}$$

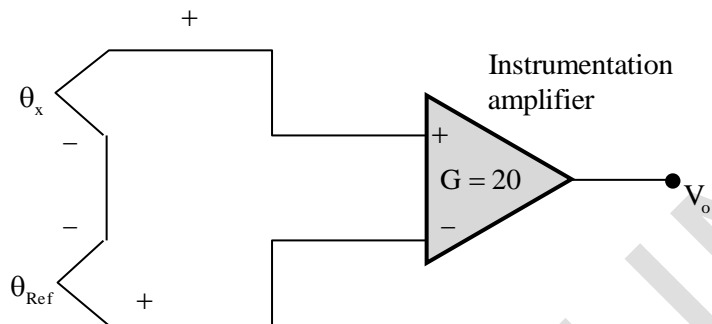
From Circuit $I_{E2} = I_{B1} = (1 + \beta_2) I_{B2}$

$$\frac{50}{100}\text{mA} = 50 \times I_{B2}$$

$$\therefore I_{B2} = 10\mu\text{A}$$



46. A J-type thermocouple has an output voltage $V_\theta = (13650 + 50\theta_x)\mu\text{V}$, where θ_x is the junction temperature in Celsius ($^\circ\text{C}$). The thermocouple is used with reference junction compensation, as shown in the figure. The instrumentation amplifier used to gain $G = 20$. If θ_{Ref} is 1°C , for an input θ_x of 100°C , the output V_o if instrumentation amplifier in millivolt is



- (A) 101 mV (B) 98mV (C) 99 mV (D) 100mV

Key: (C)

Sol: $V_A = V_x - V_{\text{Ref}}$

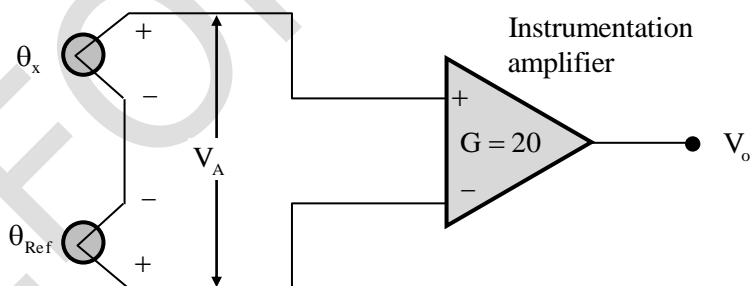
$$= [13650 + 50\theta_x] - [13650 + 50\theta_{\text{Ref}}]$$

$$= 50[\theta_x - \theta_{\text{Ref}}]$$

$$= 50[100 - 1] = 4950\mu\text{V}$$

$$V_o = G \times V_A$$

$$= 20 \times 4950 \times 10^{-6} \text{ V} = 99\text{mV}$$



47. Consider the sequence $x_n = 0.5x_{n-1} + 1, n = 1, 2, \dots$ with $x_0 = 0$. Then $\lim_{n \rightarrow \infty} x_n$ is

- (A) 2 (B) 1 (C) ∞ (D) 0

Key: (A)

Sol: Suppose $\lim_{n \rightarrow \infty} (x_n) = \alpha$, where α is finite

then $\lim_{n \rightarrow \infty} (x_{n-1}) = \alpha$, also

Given sequence $x_n = 0.5x_{n-1} + 1$

$$\Rightarrow \lim_{n \rightarrow \infty} x_n = (0.5) \lim_{n \rightarrow \infty} (x_{n-1}) + 1$$

$$\alpha = (0.5)\alpha + 1$$

$$\Rightarrow (0.5)\alpha = 1 \Rightarrow \alpha = 2$$

48. A slip-ring induction motor is expected to be started by adding extra resistance in the rotor circuit. The benefit that is derived by adding extra resistance in the rotor circuit in comparison to the rotor being shorted is
- (A) The losses at starting would be lower (B) The power factor at start will be lower
(C) The starting torque would be higher (D) The starting current is higher

Key: (C)

Sol: $\downarrow I_{st} = \frac{V}{\uparrow (R_2 + R_{ex} + jX)}$

As $R_{ex} \uparrow \rightarrow I_{st} \downarrow$ (starting current decreases)

1. As I_{st} is reduced, losses are reduced

2. $\uparrow p.f = \frac{(R + R_{ex})}{Z} \uparrow$ ($\cos \phi$ increases)

3. $\uparrow T_{st} \propto (R_2 + R_{ex}) \uparrow$ (T_{st} increases)

Option (C) is incorrect.

49. A household fan consumes 60W and draw a current of 0.3125 A (rms) when connected to a 230V (rms) ac, 50Hz single phase mains. The reactive power drawn by the fan in VAR is _____ (rounded off to the nearest integer).

Key: (39.57)

Sol: It is given that for a fan

$$I = 0.3125 \text{ A (r.m.s)}$$

$$V = 230 \text{ V (r.m.s)}$$

$$f = 50 \text{ Hz}$$

Average power $P = 60\text{W}$

We need to obtain reactive power Q

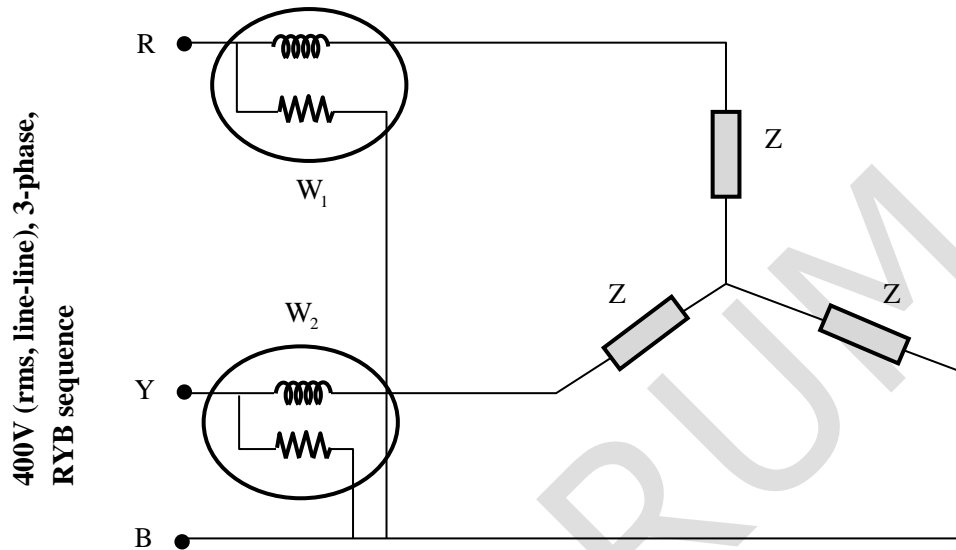
$$P = V_{rms} I_{rms} \cos \phi$$

$$\Rightarrow \cos \phi = \frac{P}{V_{rms} I_{rms}} = \frac{60}{230 \times 0.3125} = 0.8347$$

$$\Rightarrow \sin \phi = \sqrt{1 - \cos^2 \phi} = 0.5505$$

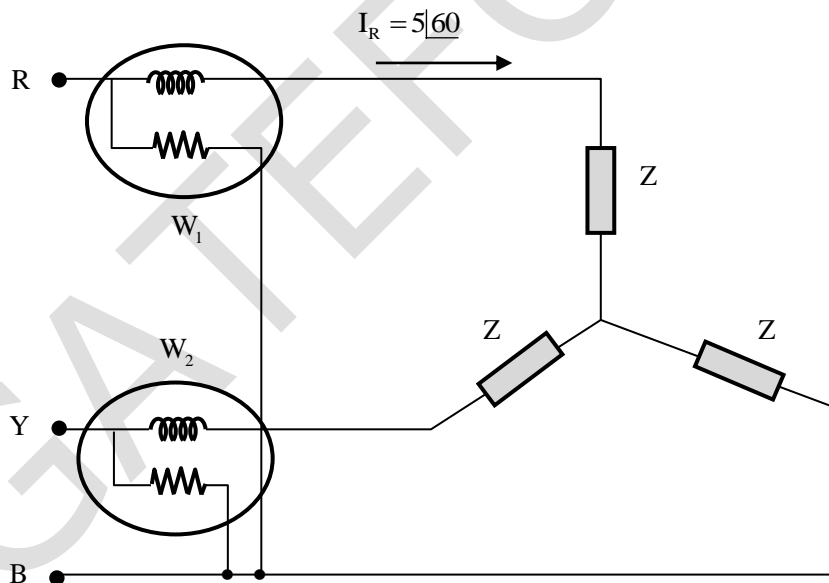
$$\Rightarrow \text{Reactive power} = V_{rms} I_{rms} \sin \phi = 230 \times 0.3125 \times 0.5505 = 39.57 \text{ VAR} \approx 40\text{VAR}$$

50. The power in a 400V (rms, line-line) three-phase, three-wire RYB sequence system is measured using the two wattmeters, as shown. The R-line current is $5\angle 60^\circ\text{A}$. Wattmeter W_1 in the R-line will read (in watt) _____.



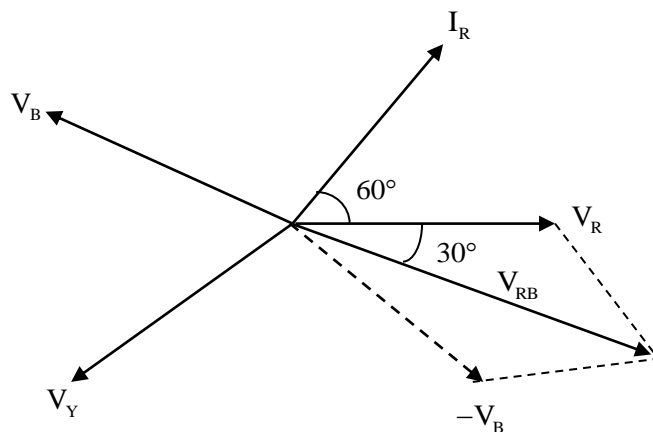
Key: (0)

Sol: The given circuit is as follows



We need to obtain reading of W_1

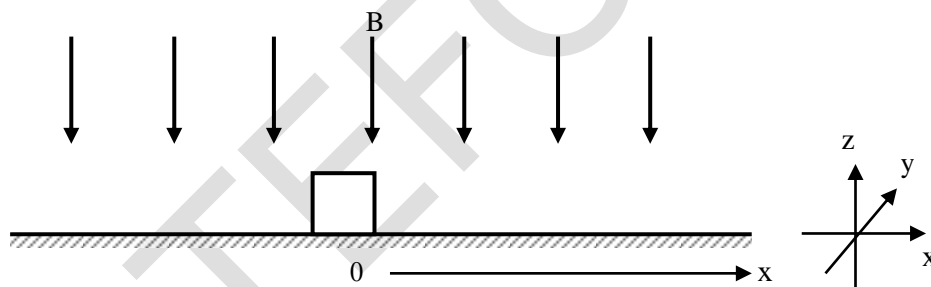
$$W_1 = V_{RB} I_R \cos[\theta_{V_{RB}} - \theta_{I_R}]$$



We can see the angle between I_R and V_{RB} is 90° from the phasor diagram.

$$\Rightarrow W_i = V_{RB} I_R \cos 90^\circ = 0W$$

51. The figure below shows an electrically conductive bar of square cross-section resting on a plane surface. The bar of mass of 1 kg has depth of 0.5m along the y direction. The coefficient of friction between the bar and the surface is 0.1. Assume the acceleration due to gravity to be 10 m/s^2 . The system faces a uniform flux density $B = 1\hat{z}\text{T}$. A time $t = 0$, a current of 10A is switched onto the bar and is maintained.



When the bar has moved by 1m, its speed in metre per second is _____ (rounded off to one decimal place).

Key: (2.8)

Sol: Given, $\mu = 0.1$

Frictional force, $F_f = \mu N = 0.1 \times (1 \times 10) = 1\text{N}$

Motion of Bar caused by force, $F = BIL = -1 \times 10 \times 0.5 = 5\text{N}$

\therefore Net force $= 5 - 1 = 4\text{N}$

As frictional force is the opposing force to motion,

Force, $F = ma$

$4 = 1 \times a = 4\text{m/sec}^2$

$$v^2 = u^2 + 2as, \text{ where } u = \text{initial velocity} = 0, \text{ distance, } s = 1\text{m}$$

$$\therefore V^2 = 2 \times 4 \times 1 + 0 = 8$$

$$\therefore V = \sqrt{8} = 2.8\text{m/sec}$$

52. Given: Density of mercury is $13,600 \text{ kg/m}^3$ and acceleration due to gravity is 9.81 m/s^2 . Atmospheric pressure is 101 kPa . In a mercury U-tube manometer, the difference between the heights of the liquid in the U-tube is 1 cm . The differential pressure being measured in pascal is _____ (rounded off to the nearest integer).

Key: (1334.16)

Sol: Density of manometric fluid (ρ) = $13,600 \text{ kg/m}^3$

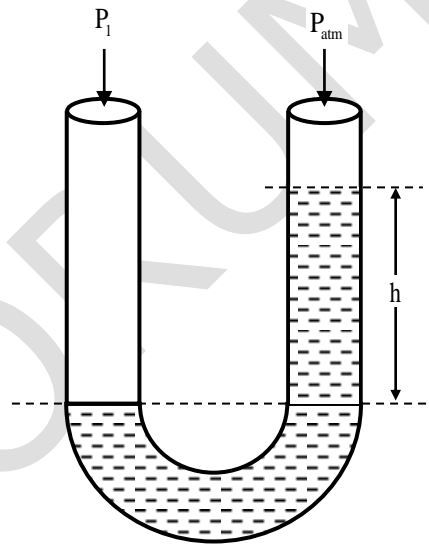
$$g = 9.81 \text{ m/sec}^2$$

$$P_1 = P_{\text{atm}} + \rho gh$$

$$P_1 - P_{\text{atm}} = \rho gh$$

$$= 13,600 \times 9.81 \times 1 \times 10^{-2}$$

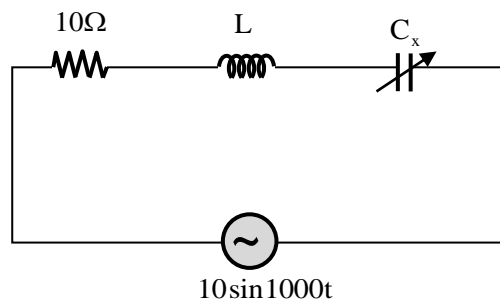
$$= 1334.16 \text{ Pa}$$



53. An air cored coil having a winding resistance of 10Ω is connected in series with a variable capacitor C_x . The series circuit is excited by a 10V sinusoidal voltage source of angular frequency 1000 rad/s . As the value of the capacitor is varied, a maximum voltage of 30V was observed across it. Neglecting skin-effect, the value of the inductance of the coil in millihenry is _____.

Key: (30)

Sol: As per given information, we can draw the following circuit



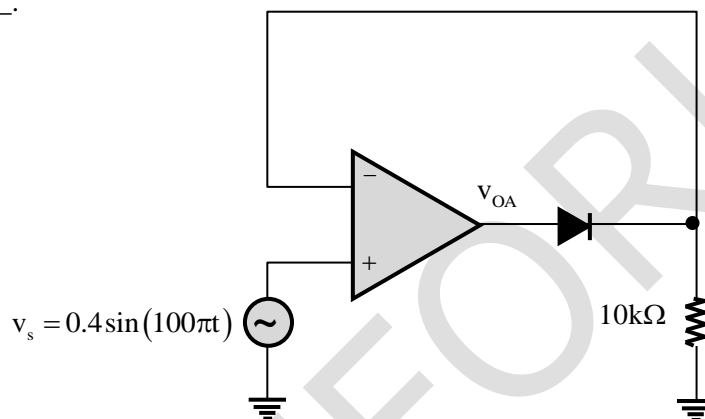
We know as response $|V_c| = Q|V_{in}|$, assuming the source sequence to be resonant frequency.

$$30 = Q \times 10 \Rightarrow Q = 3$$

$$Q = \frac{\omega_o L}{R}$$

$$L = \frac{QR}{\omega_o} = \frac{3 \times 10}{1000} = 30 \text{ mH}$$

54. The diode used in the circuit has a fixed voltage drop of 0.6V when forward biased. A signal v_s is given to the ideal OpAmp as shown. When v_s is at its positive peak, the output (v_{OA}) of the OpAmp in volts is _____.



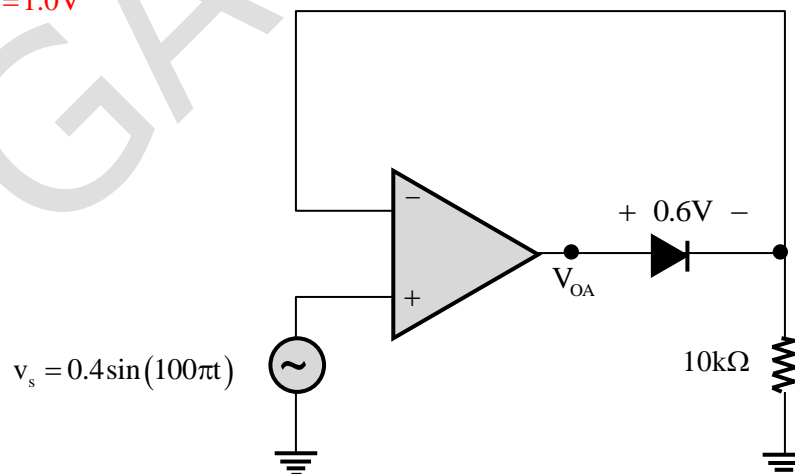
Key: (1)

Sol: In the half-cycle of the input op-amp output V_o will approach positive saturation voltage and diode will be in conduction mode. So, at positive peak of input $V_s = 0.4V$ and as per the polarity of diode,

$$V_{oA} = V_D + 0.4V$$

$$= 0.6V + 0.4V \quad (\text{Given: } V_D = 0.6V)$$

$$V_{oA} = 1.0V$$



55. When the movable arm of a Michelson interferometer in vacuum ($n = 1$) is moved by $325 \mu\text{m}$, the number of fringe crossings is 1000. The wavelength of the laser used in nanometers is _____.

Key: (650)

Sol: Given, $d = 325\text{mm}$

Path difference, $= 2d = 2 \times 325$

$$n\lambda = 2d$$

$$n = \frac{2d}{\lambda} = \frac{2 \times 325}{1000} = 650\text{nm}$$



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