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GATE 2017

Electronics Engineering

(Morning Session : 05-02-2017)

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SECTION A : ELECTRONICS ENGINEERING

Q.1 The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance is $V(l) = e^{-\gamma l + j\omega t}$ Volts, where l is the distance along the length of the cable in metres, $\gamma = (0.1 + j40)\ \text{m}^{-1}$ is the complex propagation constant, and $\omega = 2\pi \times 10^9$ rad/s is the angular frequency. The absolute value of the attenuation in the cable in dB/metre is

Ans. (0.868)

$$V(l) = V_o e^{-\alpha l} e^{-j\beta l} e^{j\omega t}$$

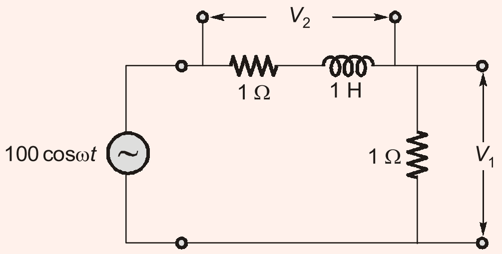
$$\text{Attenuation} = \frac{|\text{Input}|}{|\text{Output}|} = \frac{|V_o(0)|}{|V_o(l)|}$$

$$\text{Attenuation per meter} = \frac{|V_o|}{|V_o(1\text{m})|} = e^{\alpha}$$

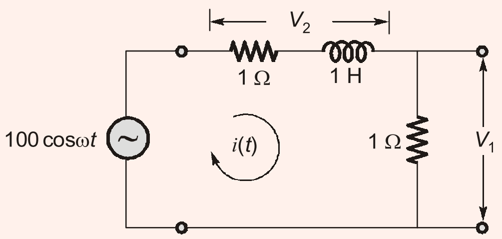
$$\text{Attenuation in dB/m} = (20\log e^{\alpha})\text{dB/m} = 20(0.1)\log e = 0.868 \text{ dB/m}$$

End of Solution

Q.2 In the circuit shown, the positive angular frequency ω (in radians per second) at which the magnitude of the phase difference between the voltages V_1 and V_2 equals $\frac{\pi}{4}$ radians, is _________.



Ans. (1)



Let, $i(t) = I_m \angle\theta_i$ and $Z_2 = 1 + j\omega = \sqrt{1+\omega^2}\ \angle\theta_2$

$$\theta_2 = \tan^{-1}\left(\frac{\omega}{1}\right)$$

$$V_1 = i(t)\ (1\ \Omega) = I_m \angle\theta_i$$

$$V_2 = i(t)Z_2 = I_m\sqrt{1+\omega^2}\ \angle\theta_2 + \theta_i$$

From the given data,

$$(\theta_i + \theta_2) - (\theta_i) = \frac{\pi}{4}$$

$$\theta_2 = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\omega}{1}\right) = \frac{\pi}{4}$$

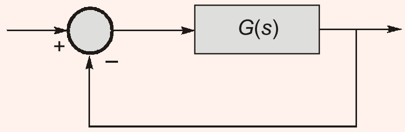
$$\omega = 1 \text{ rad/sec}$$

End of Solution

Q.3 The open loop transfer function

$$G(s) = \frac{(s+1)}{s^p(s+2)(s+3)}$$

where p is an integer, is connected in unity feedback configuration as shown in the figure.



Given that the steady state error is zero for unit step input and is 6 for unit ramp input, the value of the parameter p is ________.

Ans. (1)

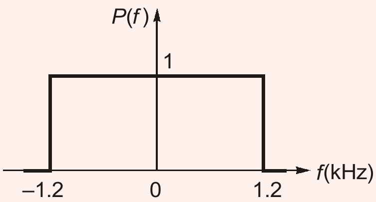
Steady state error of type 1 system, for step input is zero, for ramp input is $\frac{1}{K_v}$ and for parabolic input is infinity.

So, the given system must be of type-1 and p = 1.

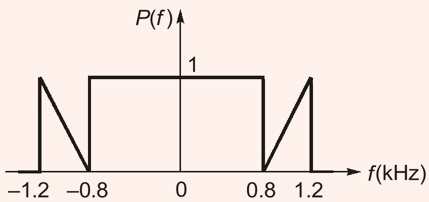
End of Solution

Q.4 In a digital communication system, the overall pulse shape $p(t)$ at the receiver before the sampler has the Fourier transform $P(f)$. If the symbols are transmitted at the rate of 2000 symbols per second, for which of the following cases is the inter symbol interference zero?

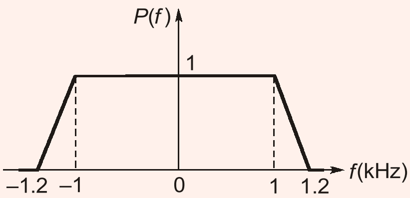
(a)



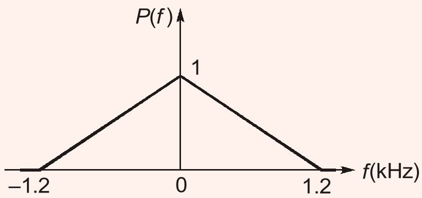
(b)



(c)



(d)

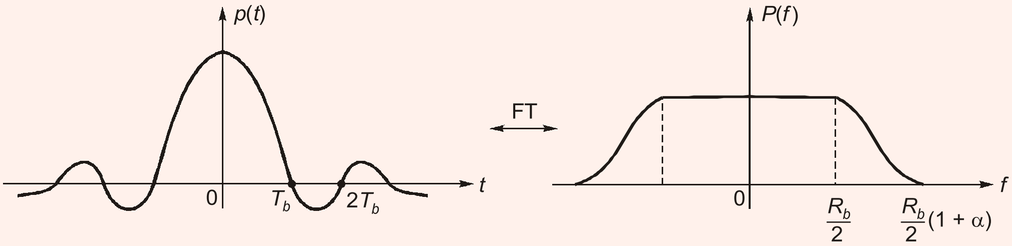


Ans. (c)

Given, symbol rate = 2000 symbol/sec

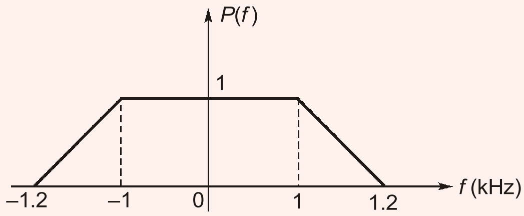
$$R_b = 2000 \text{ symbols/sec}$$

For zero ISI, raised cosine pulses will be used.



For ISI free transmission, zero crossings of $p(t)$ should occur at T_b, $2T_b$, ...

Option (c) given as,



$$R_b = 2000 \text{ symbols/sec}$$

$$\Rightarrow \quad \frac{R_b}{2} = 1 \text{ kHz}$$

Hence, option (c) is correct.

End of Solution

Q.5 The Miller effect in the context of a Common Emitter amplifier explains

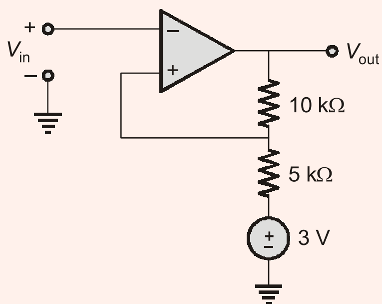
(a) an increase in the low-frequency cutoff frequency
(b) an increase in the high-frequency cutoff frequency
(c) a decrease in the low-frequency cutoff frequency
(d) a decrease in the high-frequency cutoff frequency

Ans. (d)

Miller effect increases input capacitance and thereby decreases the higher cut-off frequency.

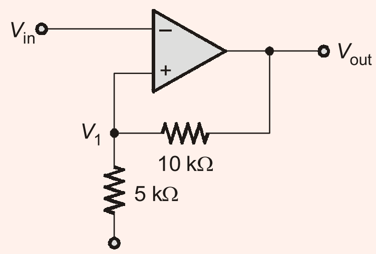
End of Solution

Q.6 For the operational amplifier circuit shown, the output saturation voltages are ± 15 V. The upper and lower threshold voltages for the circuit are, respectively,



(a) +5 V and –5 V
(b) +7 V and –3 V
(c) +3 V and –7 V
(d) +3 V and –3 V

Ans. (b)



$$V_1 = \frac{3\times10 + V_o \times 5}{15} = \frac{6+V_o}{3}$$

$$V_{UT} = \frac{6+15}{3} = 7\text{ V}$$

$$V_{LT} = \frac{6-15}{3} = -3\text{ V}$$

End of Solution

Q.7 Consider the 5 × 5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigenvalue. Then the real eigenvalue of A is

(a) –2.5
(b) 0
(c) 15
(d) 25

Ans. (c)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

Sum of all elements in any one row must be zero.

i.e., $15 - \lambda = 0$

$\lambda = 15$

End of Solution

Q.8 Consider a single input single output discrete-time system with $x[n]$ as input and $y[n]$ as output, where the two are related as

$$y[n] = \begin{cases} n\left|x[n]\right|, & \text{for } 0 \le n \le 10 \\ x[n] - x[n-1], & \text{otherwise.} \end{cases}$$

Which one of the following statements is true about the system?

(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal nor stable

Ans. (a)

Since present output does not depend upon future values of input, the system is causal and also every bounded input produces bounded output, so it is stable.

End of Solution

Q.9 Let (X_1, X_2) be independent random variables. X_1 has mean 0 and variance 1, while X_2 has mean 1 and variance 4. The mutual information I (X_1 ; X_2) between X_1 and X_2 in bits is ________.

Ans. (0)

Mutual information of two random variables is a measure of the mutual dependence of the two variables.

Given that, X and Y are independent. Hence, $I(X: Y) = 0$

End of Solution

Q.10 Consider the following statements about the linear dependence of the real valued functions $y_1 = 1$, $y_2 = x$ and $y_3 = x^2$, over the field of real numbers.

I. y_1, y_2 and y_3 are linearly independent on $-1 \le x \le 0$.
II. y_1, y_2 and y_3 are linearly dependent on $0 \le x \le 1$.
III. y_1, y_2 and y_3 are linearly independent on $0 \le x \le 1$.
IV. y_1, y_2 and y_3 are linearly dependent on $-1 \le x \le 0$.

Which one among the following is correct?

(a) Both I and II are true
(b) Both I and III are true
(c) Both II and IV are true
(d) Both III and IV are true

Ans. (b)

Any of the given three functions cannot be written as the linear combination of other two functions. Hence, the statements I and III are correct.

End of Solution

Q.11 A good transconductance amplifier should have

(a) high input resistance and low output resistance
(b) low input resistance and high output resistance
(c) high input and output resistances
(d) low input and output resistances

Ans. (c)

A good transconductance amplifier should have very high input resistance and very high output resistance.

End of Solution

Q.12 Consider a wireless communication link between a transmitter and a receiver located in free space, with finite and strictly positive capacity. If the effective areas of the transmitter and the receiver antennas, and the distance between them are all doubled, and everything else remains unchanged, the maximum capacity of the wireless link

(a) increases by a factor of 2
(b) decreases by a factor of 2
(c) remains unchanged
(d) decreases by a factor of $\sqrt{2}$

Ans. (c)

As per Friis free space propagation equation,

$$W_r = \frac{W_t A_{er} A_{et}}{(\lambda d)^2}$$

when A_{er} and A_{et} are doubled and d also doubled, W_r is same. Hence capacity is also same.

End of Solution

Q.13 Consider a stable system with transfer function

$$G(s) = \frac{s^p + b_1 s^{p-1} + \ldots + b_p}{s^q + a_1 s^{q-1} + \ldots + a_q}$$

where b_1, ..., b_p and a_1, ..., a_q are real valued constants. The slope of the Bode log magnitude curve of $G(s)$ converges to –60 dB/decade as $\omega \to \infty$. A possible pair of values for p and q is

(a) $p = 0$ and $q = 3$
(b) $p = 1$ and $q = 7$
(c) $p = 2$ and $q = 3$
(d) $p = 3$ and $q = 5$

Ans. (a)

Final slope = –60 dB/decade, which indicates that, $q - p = 3$

Among the given options, option (a) satisfies this condition.

End of Solution

Q.14 The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is

(a) 0
(b) 1
(c) 2
(d) 3

Ans. (c)

$$M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$:

$$\begin{bmatrix} 1 & 0 & 2 \\ 5 & 10 & 10 \\ 3 & 6 & 6 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 5R_1$ and $R_3 \leftarrow R_3 - 3R_1$:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 10 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 - \frac{6}{10}R_2$:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Which is in Echelon form

Rank of matrix M is,

$$\rho(M) = 2$$

End of Solution

Q.15 Consider the following statements for continuous-time linear time invariant (LTI) systems.

I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.

II. There is no causal and BIBO stable system with a pole in the right half of the complex plane.

Which one among the following is correct?

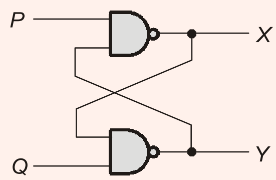
(a) Both I and II are true
(b) Both I and II are not true
(c) Only I is true
(d) Only II is true

Ans. (d)

- A BIBO stable system can have poles in right half of complex plane, if it is a non-causal system. So, statement-I is wrong.
- A causal and BIBO stable system should have all poles in the left half of complex plane. So, statement-II is correct.
- So, option (d) is correct.

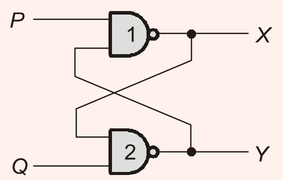
End of Solution

Q.16 In the latch circuit shown, the NAND gates have non-zero, but unequal propagation delays. The present input condition is: $P = Q =$ '0'. If the input condition is changed simultaneously to $P = Q =$ '1', the outputs X and Y are



(a) $X =$ '1', $Y =$ '1'
(b) either $X =$ '1', $Y =$ '0' or $X =$ '0', $Y =$ '1'
(c) either $X =$ '1', $Y =$ '1' or $X =$ '0', $Y =$ '0'
(d) $X =$ '0', $Y =$ '0'

Ans. (b)



Present input condition : $P = Q = 0 \Rightarrow$ Corresponding outputs are $X = Y = 1$

When input condition is changed to $P = Q = 1$ from $P = Q = 0$:

Possibility - 1 :

Let gate-1 is faster than gate-2, then the possible outputs are $X = 0$, $Y = 1$

Possibility - 2 :

Let gate-2 is faster than gate-1, then the possible outputs are $X = 1$, $Y = 0$

So, option (b) is correct.

End of Solution

Q.17 The clock frequency of an 8085 microprocessor is 5 MHz. If the time required to execute an instruction is 1.4 μs, then the number of T-states needed for executing the instruction is

(a) 1
(b) 6
(c) 7
(d) 8

Ans. (c)

Given than, $f_{CLK} = 5$ MHz

Execution time = 1.4 μs

Execution time = n(T - state)

n = number of T-states required to execute the instruction

$$\text{T - state (or) } T_{CLK} = \frac{1}{f_{CLK}} = 0.2\ \mu s$$

So,
$$n = \frac{1.4\ \mu s}{T_{CLK}} = \frac{1.4}{0.2} = 7$$

End of Solution

Q.18 Which one of the following statements about differential pulse code modulation (DPCM) is true?

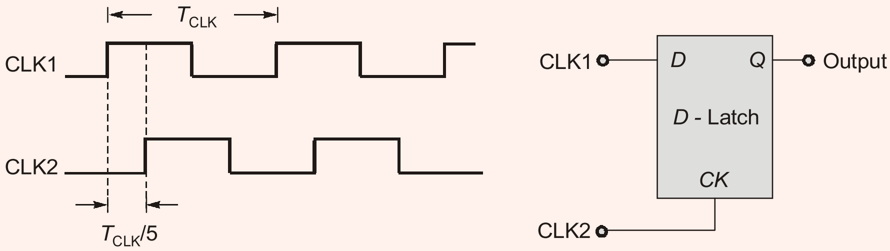
(a) The sum of message signal sample with its prediction is quantized.
(b) The message signal sample is directly quantized, and its prediction is not used.
(c) The difference of message signal sample and a random signal is quantized.
(d) The difference of message signal sample with its prediction is quantized.

Ans. (d)

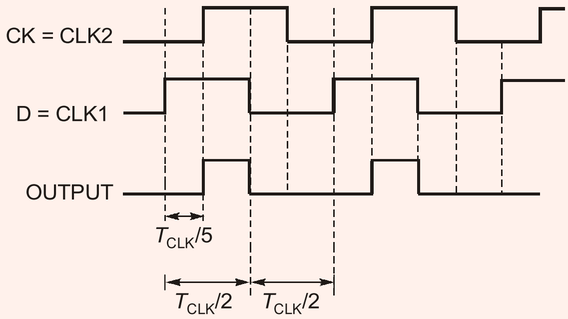
In DPCM, difference of message signal sample with its prediction is quantized.

End of Solution

Q.19 Consider the D-Latch shown in the figure, which is transparent when its clock input CK is high and has zero propagation delay. In the figure, the clock signal CLK1 has a 50% duty cycle and CLK2 is a one-fifth period delayed version of CLK1. The duty cycle at the output of the latch in percentage is ____________.



Ans. (30)

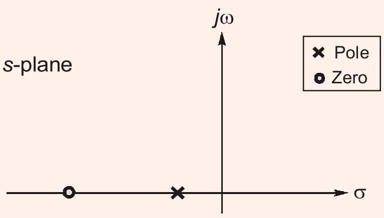


$$\text{Duty cycle of output} = \frac{\frac{T_{CLK}}{2} - \frac{T_{CLK}}{5}}{T_{CLK}} \times 100 = \frac{3}{10} \times 100 = 30\%$$

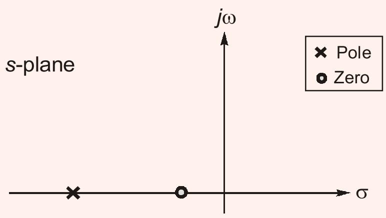
End of Solution

Q.20 Which of the following can be the pole-zero configuration of a phase-lag controller (lag compensator)?

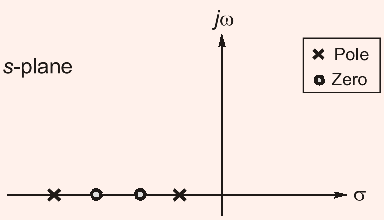
(a)



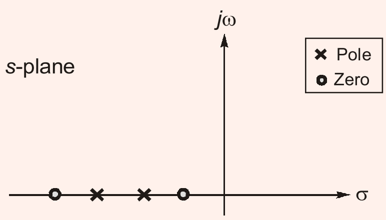
(b)



(c)



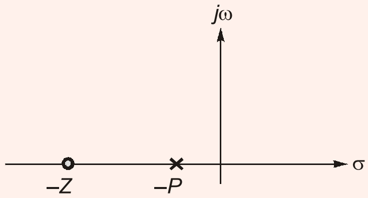
(d)



Ans. (a)

Phase lag controller transfer function is

$$G_C(s) = \frac{s+Z}{s+P};\ |Z|>|P|$$



End of Solution

Q.21 A periodic signal $x(t)$ has a trigonometric Fourier series expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

If $x(t) = -x(-t) = -x(t - \pi/\omega_0)$, we can conclude that

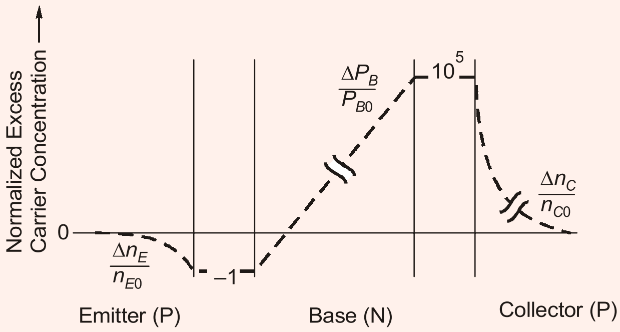
(a) a_n are zero for all n and b_n are zero for n even
(b) a_n are zero for all n and b_n are zero for n odd
(c) a_n are zero for n even and b_n are zero for n odd
(d) a_n are zero for n odd and b_n are zero for n even

Ans. (a)

Signal has odd and half wave symmetries. So, all a_n are zero and b_n are zero for n even.

End of Solution

Q.22 For a narrow base PNP BJT, the excess minority carrier concentrations (Δn_E for emitter, Δp_B for base, Δn_C for collector) normalized to equilibrium minority carrier concentrations (n_{E0} for emitter, p_{B0} for base, n_{C0} for collector) in the quasi-neutral emitter, base and collector regions are shown below. Which one of the following biasing modes is the transistor operating in?



X and Y axes are not to scale

(a) Forward active
(b) Saturation
(c) Inverse active
(d) Cutoff

Ans. (c)

Emitter-base junction (J_E) is in RB

Collector-base junction (J_C) is in FB

Hence, inverse active mode.

End of Solution

Q.23 A bar of Gallium Arsenide (GaAs) is doped with Silicon such that the Silicon atoms occupy Gallium and Arsenic sites in the GaAs crystal. Which one of the following statements is true?

(a) Silicon atoms act as *p*-type dopants in Arsenic sites and *n*-type dopants in Gallium sites.

(b) Silicon atoms act as *n*-type dopants in Arsenic sites and *p*-type dopants in Gallium sites.

(c) Silicon atoms act as *p*-type dopants in Arsenic as well as Gallium sites.

(d) Silicon atoms act as *n*-type dopants in Arsenic as well as Gallium sites.

Ans. (a)

Si acts as *p*-type dopant in As sites.

Si acts as *n*-type dopant in GA sites.

End of Solution

Q.24 An n^+ - n Silicon device is fabricated with uniform and non-degenerate donor doping concentrations of $N_{D1} = 1 \times 10^{18}$ cm^{-3} and $N_{D2} = 1 \times 10^{15}$ cm^{-3} corresponding to the n^+ and n regions respectively. At the operational temperature T, assume complete impurity ionization, $kT/q = 25$ mV, and intrinsic carrier concentration to be $n_i = 1 \times 10^{10}$ cm^{-3}. What is the magnitude of the built-in potential of this device?

(a) 0.748 V
(b) 0.460 V
(c) 0.288 V
(d) 0.173 V

Ans. (d)

$$V_0 = V_T \ln\frac{n_1}{n_2} = V_T \ln\frac{N_{D1}}{N_{D2}}$$

$$= 0.25 \ln\frac{10^{18}}{10^{15}} = 0.025 \ln(1000)$$

$$= 0.173 \text{ V}$$

End of Solution

Q.25 Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) _______.

Ans. (0.028)

When three dice are thrown

Total number of possible cases = 6 × 6 × 6 = 216

Favourable cases of all three dice have same number are,

$$\begin{Bmatrix} (1,1,1) & (2,2,2) & (3,3,3) \\ (4,4,4) & (5,5,5) & (6,6,6) \end{Bmatrix}$$

Number of favourable cases = 6

$$\text{Required probability} = \frac{6}{216} = \frac{1}{36} = 0.028$$

End of Solution

Q.26 Let h[n] be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$h[0] = \frac{1}{3};\ h[1] = \frac{1}{3};\ h[2] = \frac{1}{3};\ \text{and } h[n] = 0 \text{ for } n < 0 \text{ and } n > 2.$$

Let $H(\omega)$ be the discrete-time Fourier transform (DTFT) of $h[n]$, where ω is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to ________.

Ans. (2.10)

Since $$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

So, $$H(e^{j\omega}) = \frac{1}{3}e^{j\omega}[1 + 2\cos\omega]$$

$$H(e^{j\omega_0}) = 0, \quad \text{for } (1 + 2\cos\omega_0) = 0$$

or $$\cos\omega_0 = -\frac{1}{2}$$

or $$\omega_0 = \frac{2\pi}{3} = 2.10 \text{ radians}$$

End of Solution

Q.27 Two discrete-time signals $x[n]$ and $h[n]$ are both non-zero only for n = 0, 1, 2 and are zero otherwise. It is given that

$x[0] = 1$, $x[1] = 2$, $x[2] = 1$, $h[0] = 1$.

Let $y[n]$ be the linear convolution of $x[n]$ and $h[n]$. Given that $y[1] = 3$ and $y[2] = 4$, the value of the expression $(10y[3] + y[4])$ is ________.

Ans. (31)

$$x[n] = \{1, 2, 1\}$$
$$h[n] = \{1, a, b\}$$
$$y[n] = \{A, 3, 4, B, C\}$$

x \ h	1	a	b
1	1	a	b
2	2	2a	2b
1	1	a	b

$y[0] = 1$; $y[1] = 2 + a$; $y[2] = 1 + 2a + b$; $y[3] = a + 2b$; $y[4] = b$

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MADE EASY Students Top in ESE-2016

84% Selections in ESE 2016

AIR 1 — Civil — Jatin Kumar

AIR 1 — Mechanical — M. Idul Ahmad

AIR 1 — Electrical — Gaurav

AIR 1 — Electronics & Tel. — Naveen Sharma

4 Streams, 4 Toppers all 4 MADE EASY Students

CE — 10 Selections in Top 10

AIR-1	AIR-2	AIR-3	AIR-4	AIR-5	AIR-6	AIR-7	AIR-8	AIR-9	AIR-10
Jatin Kumar	Bivek Joishi	Rachit Jain	Adarsh R.S.	Mahipal Singh	Nitish Garg	Shivam D.	Amrit Anand	Avdhesh	Himanshu Tiwari

ME — 9 Selections in Top 10

AIR-1	AIR-3	AIR-4	AIR-5	AIR-6	AIR-7	AIR-8	AIR-9	AIR-10
Mohd Idul Ahmad	Chirag Srivastav	Taronjot Singh	Deepak Vijay	Dheeraj Kumar	Tarun Kumar	JGMV Pramod	Gaurav Kant	Jivitesh Anand

EE — 10 Selections in Top 10

AIR-1	AIR-2	AIR-3	AIR-4	AIR-5	AIR-6	AIR-7	AIR-8	AIR-9	AIR-10
Gaurav	B Venkatesh	Tanuj K Sharma	Varsha	Ashish Verma	Mufeed Khan	Shyamji D.	Sk Taushifur R.	Arvind Biswal	Gaurav Tyagi

E&T — 10 Selections in Top 10

AIR-1	AIR-2	AIR-3	AIR-4	AIR-5	AIR-6	AIR-7	AIR-8	AIR-9	AIR-10
Naveen B. Sharma	Amit Rawat	Aswathy S.	Thaduri Naveen	Vinit Ranjan	Harshit Jain	Akash Chhikara	Vivek Jain	Jadugurla N.	Prabhakar

39 Selections in Top 10 | **76** Selections in Top 20 | **505** Selections out of total 604 vacancies

CE	Selections in Top 10 **10**	MADE EASY Selections **182** Out of **225** Vacancies	MADE EASY Percentage **81%**
ME	Selections in Top 10 **9**	MADE EASY Selections **159** Out of **179** Vacancies	MADE EASY Percentage **89%**
EE	Selections in Top 10 **10**	MADE EASY Selections **86** Out of **106** Vacancies	MADE EASY Percentage **81%**
E&T	Selections in Top 10 **10**	MADE EASY Selections **78** Out of **94** Vacancies	MADE EASY Percentage **83%**

"2 out of every 3 selected students, are from CLASSROOM COURSE"

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Given, $y[1] = 2 + a = 3 \Rightarrow a = 1$

Given, $y[2] = 1 + 2a + b = 4 \Rightarrow b = 1$

So, $y[3] = a + 2b = 3$

and $y[4] = b = 1$

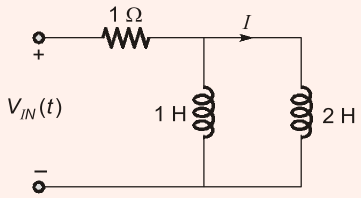
So, $10y[3] + y[4] = 10 \times 3 + 1 = 31$

End of Solution

Q.28 In the circuit shown, the voltage $V_{IN}(t)$ is described by:

$$V_{IN}(t) = \begin{cases} 0, & \text{for } t < 0 \\ 15 \text{ Volts}, & \text{for } t \geq 0 \end{cases}$$

where t is in seconds. The time (in seconds) at which the current I in the circuit will reach the value 2 Amperes is ________.



Ans. **(0.34)**

$$i_s(t) = \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$$

$$i_s(t) = \frac{15}{1}\left[1 - e^{\frac{-3t}{2}}\right] \text{A}$$

current through 2 H, $i(t) = i_s(t)\frac{1}{1+2}$

$$i(t) = 5\left[1 - e^{\frac{-3t}{2}}\right] \text{A}$$

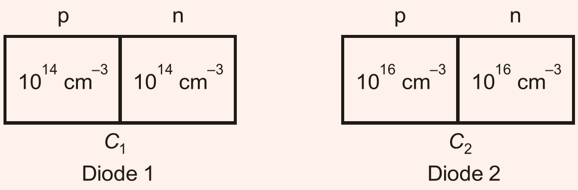
at $i(t) = 2$ A,

$$2 = 5\left[1 - e^{\frac{-3t}{2}}\right]$$

By solving, $t = 0.3405$ sec

End of Solution

Q.29 As shown, two Silicon (Si) abrupt p-n junction diodes are fabricated with uniform donor doping concentrations of $N_{D1} = 10^{14}$ cm^{-3} and $N_{D2} = 10^{16}$ cm^{-3} in the n-regions of the diodes, and uniform acceptor doping concentrations of $N_{A1} = 10^{14}$ cm^{-3} and $N_{A2} = 10^{16}$ cm^{-3} in the p-regions of the diodes, respectively. Assuming that the reverse bias voltage is >> built-in potentials of the diodes, the ratio C_2/C_1 of their reverse bias capacitances for the same applied reverse bias, is ________.



Ans. (10)

$$W = \sqrt{\frac{2\varepsilon V_R}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)} \qquad \because V_R >> V_{bi}$$

Under reverse biased condition,

$$C = \frac{\varepsilon A}{W} \qquad \Rightarrow C \propto \frac{1}{W}$$

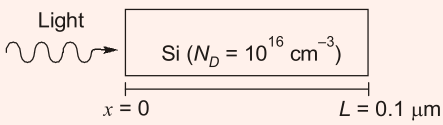
So,

$$\frac{C_2}{C_1} = \frac{W_1}{W_2}$$

$$= \sqrt{\frac{\frac{2\varepsilon V_R}{e}\left(\frac{2}{10^{14}}\right)}{\frac{2\varepsilon V_R}{e}\left(\frac{2}{10^{16}}\right)}} = 10$$

End of Solution

Q.30 As shown, a uniformly doped Silicon (Si) bar of length L = 0.1 μm with a donor concentration $N_D = 10^{16}$ cm^{-3} is illuminated at x = 0 such that electron and hole pairs are generated at the rate of $G_L = G_{L0}\left(1-\frac{x}{L}\right), 0 \le x \le L$, where $G_{L0} = 10^{17}$ cm^{-3} s^{-1}. Hole lifetime is 10^{-4} s, electronic charge $q = 1.6 \times 10^{-19}$ C, hole diffusion coefficient D_p = 100 cm^2/s and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at $x = L$, the magnitude of the diffusion current density at $x = L/2$, in A/cm^2, is ________.



Ans. (16)

Net hole density varying in the direction of x is,

$$p_n(x) = p_{no} + \Delta p = p_{no} + G_L\tau_p$$

$$= p_{no} + G_{Lo}\,\tau_p\left(1-\frac{x}{L}\right)$$

$$J_{p,\,diff} = -eD_p\frac{dp}{dx} = -eD_p\left[\frac{-G_{Lo}\,\tau_p}{L}\right]$$

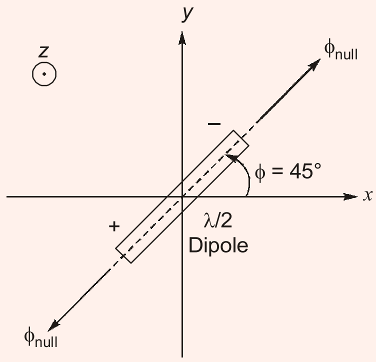
$$= \frac{1.6\times10^{-19}\times100\times10^{17}\times10^{-4}}{0.1\times10^{-4}}\text{ A/cm}^2 = 16\text{ A/cm}^2$$

End of Solution

Q.31 A half wavelength dipole is kept in the x-y plane and oriented along 45° from the x-axis. Determine the direction of null in the radiation pattern for $0 \le \phi \le \pi$. Here the angle $\theta(0 \le \theta \le \pi)$ is measured from the z-axis, and the angle $\phi(0 \le \phi \le 2\pi)$ is measured from the x-axis in the x - y plane.

(a) $\theta = 90°, \phi = 45°$
(b) $\theta = 45°, \phi = 90°$
(c) $\theta = 90°, \phi = 135°$
(d) $\theta = 45°, \phi = 135°$

Ans. (a)



Since the dipole is in X-Y plane, the radiation pattern has nulls for $\theta = 90°$, $\phi = 45°$ and 225°

for $0 \le \phi \le \pi$, $\phi = 45°$

End of Solution

Q.32 Which one of the following options correctly describes the locations of the roots of the equation $s^4 + s^2 + 1 = 0$ on the complex plane?

(a) Four left half plane (LHP) roots.
(b) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis.
(c) Two RHP roots and two LHP roots.
(d) All four roots are on the imaginary axis

Ans. (c)

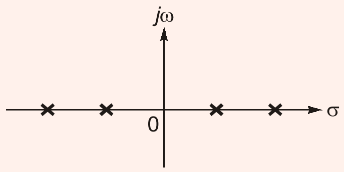
$$q(s) = s^4 + s^2 + 1 = 0$$

s^4	1	1	1
s^3	~~0~~ 4	~~0~~ 2	0
s^2	0.5	1	0
s^1	−6	0	0
s^0	1	0	0

$$\frac{dA(s)}{ds} = 4s^3 + 2s^1$$

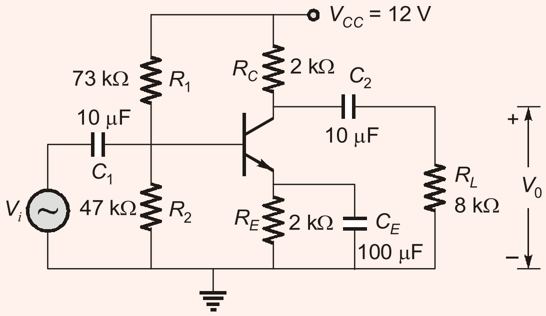
There are two sign changes in the first column of the R-H table and the order of auxillary equation is 4. So, four poles are symmetric about origion.

∴ 2 RHP roots and 2 LHP roots.



End of Solution

Q.33 For the DC analysis of the Common-Emitter amplifier shown, neglect the base current and assume that the emitter and collector currents are equal. Given that V_T = 25 mV, V_{BE} = 0.7 V, and the BJT output resistance r_o is practically infinite. Under these conditions, the midband voltage gain magnitude, $A_v = |v_o/v_i|$ V/V, is ________.



Ans. (128)

$$V_{Th} = \frac{12 \times 47}{73 + 47} = 4.7 \text{ V}$$

$$I_C = \frac{V_{Th} - V_{BE}}{R_E} = \frac{4}{2} = 2 \text{ mA} \qquad \because I_C = I_E,\ I_B = 0$$

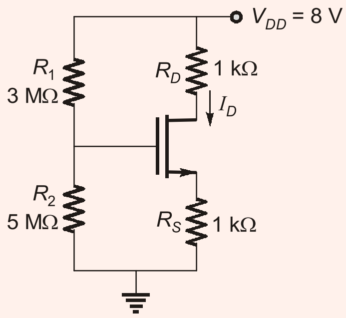
$$g_m = \frac{I_C}{V_T} = \frac{2}{25} = 80 \text{ m℧}$$

$$A_V = -g_m R'_L = -80\,(2 \,||\, 8) = -128$$

$$|A_V| = 128$$

End of Solution

Q.34 For the circuit shown, assume that the NMOS transistor is in saturation. Its threshold voltage V_{tn} = 1 V and its transconductance parameter $\mu_n C_{ox}\left(\frac{W}{L}\right) = 1\text{mA/V}^2$. Neglect channel length modulation and body bias effects. Under these conditions, the drain current I_D in mA is ___________.



Ans. (2)

$$V_{GS} = \frac{8\times 5}{8} - 1\times I_D = 5 - I_D \qquad \text{(Here } I_D \text{ is numerically in mA)}$$

$$I_D = 5 - V_{GS} \qquad ...(i)$$

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2$$

$$5 - V_{GS} = \frac{1}{2}(V_{GS} - 1)^2$$

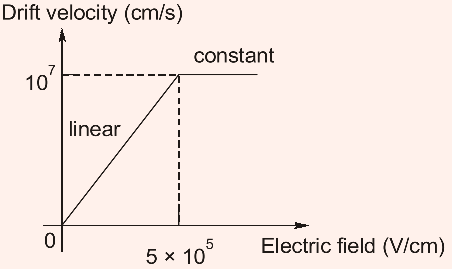
$$10 - 2V_{GS} = V_{GS}^2 + 1 - 2V_{GS}$$

$$V_{GS}^2 = 9 \quad \Rightarrow V_{GS} = 3 \text{ V}$$

$$I_D = 5 - 3 = 2 \text{ mA}$$

End of Solution

Q.35 The dependence of drift velocity of electrons on electric field in a semiconductor is shown below. The semiconductor has a uniform electron concentration of $n = 1 \times 10^{16}$ cm^{-3} and electronic charge $q = 1.6 \times 10^{-19}$ C. If a bias of 5 V is applied across a 1 μm region of this semiconductor, the resulting current density in this region, in kA/cm^2, is ________.



Ans. (1.6)

$$E = \frac{V}{d} = \frac{5}{10^{-4}} = 5\times 10^4 \text{ V/cm}$$

slope of the curve, $m = \frac{10^7 - 0}{5\times 10^5} = 20$ $\qquad y = mx$

$$v_d = 20 \times E = 20 \times 5 \times 10^4 = 10^6 \text{ V/cm}$$

$$J = nev_d = 1 \times 10^{16} \times 1.6 \times 10^{-19} \times 10^6$$

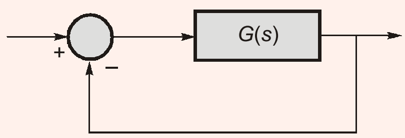
$$= 1.6 \times 10^3 \text{ A/cm}^2 = 1.6 \text{ kA/cm}^2$$

End of Solution

Q.36 A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

is connected in unity feedback configuration as shown in the figure.



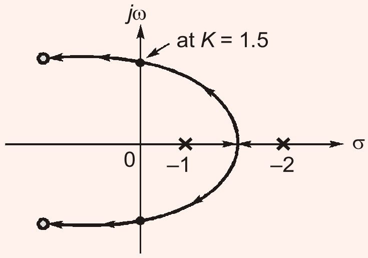
For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for $K = 1.5$. The closed loop system is stable for

(a) $K > 1.5$
(b) $1 < K < 1.5$
(c) $0 < K < 1.5$
(d) no positive value of K

Ans. (a)

Given that, $G(s) = \dfrac{K(s^2 + 2s + 2)}{s^2 - 3s + 2}$

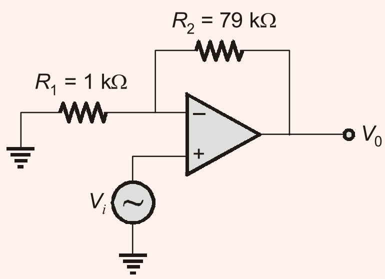
The root locus plot of the given system is as follows:



∴ System is stable for $K > 1.5$.

End of Solution

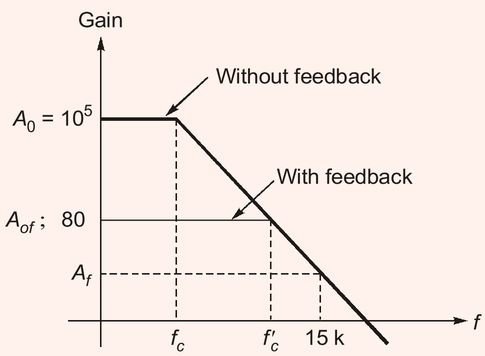
Q.37 The amplifier circuit shown in the figure is implemented using a compensated operational amplifier (op-amp), and has an open-loop voltage gain, $A_0 = 10^5$ V/V and an open-loop cut-off frequency, $f_c = 8$ Hz. The voltage gain of the amplifier at 15 kHz, in V/V, is ___________.



Ans. (44.4)

- In the given circuit,

Feedback factor, $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{1}{80}$



- $$A_{of} = \frac{A_o}{1+A_o\beta} \simeq 80$$
- $$f'_c = f_c(1+A_o\beta) = 8\left(1+\frac{10^5}{80}\right)\text{Hz} = 10{,}008\text{ Hz}$$
- Gain at f = 15 kHz = 15000 Hz is,

$$A_f = \frac{A_{of}}{\sqrt{1+\left(\frac{f}{f'_c}\right)^2}} = \frac{80}{\sqrt{\left(1+\left(\frac{15000}{10008}\right)^2\right)}} \simeq 44.4$$

End of Solution

Q.38 The expression for an electric field in free space is $E = E_0(\hat{x}+\hat{y}+j2\hat{z})e^{-j(\omega t - kx + ky)}$, where x, y, z represent the spatial coordinates, t represents time, and ω, k are constants. This electric field

(a) does not represent a plane wave.
(b) represents a circularly polarized plane wave propagating normal to the z-axis.
(c) represents an elliptically polarized plane wave propagating along the x - y plane.
(d) represents a linearly polarized plane wave.

Ans. (c)

E field direction $\Rightarrow (\hat{a}_x + \hat{a}_y + j2\hat{a}_z)$

Propagation direction $\Rightarrow k\hat{a}_x - k\hat{a}_y$

E is perpendicular to propagation

$$E \cdot P = 0$$

Component in $\hat{a}_z$ has magnitude of 2.

Component in X-Y plane has magnitude of $\sqrt{2}$.

These two components are out of phase by 90° and have unequal amplitudes. So, it is elliptically polarized wave.

End of Solution

Q.39 Starting with $x = 1$, the solution of the equation $x^3 + x = 1$, after two iterations of Newton-Raphson's method (up to two decimal places) is ________.

Ans. (0.68)

$$f(x) = x^3 + x - 1$$
$$f(1) = 1$$
$$f'(x) = 3x^2 + 1$$
$$f'(1) = 4$$

By Newton-Raphson method,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

For $x_0 = 1$, $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1}{4} = 0.75$$

For $x_1 = 0.75$,

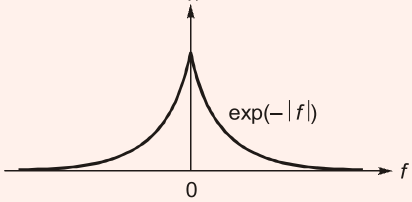
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.75 - \frac{0.171875}{2.6875} = 0.686$$

End of Solution

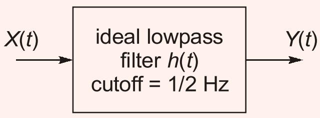
Q.40 Let $X(t)$ be a wide sense stationary random process with the power spectral density $S_X(f)$ as shown in figure (a), where f is in Hertz (Hz). The random process $X(t)$ is input to an ideal lowpass filter with the frequency response

$$H(f) = \begin{cases} 1, & |f| \le \frac{1}{2}\text{ Hz} \\ 0, & |f| > \frac{1}{2}\text{ Hz} \end{cases}$$

as shown in figure (b). The output of the lowpass filter is $Y(t)$.



(a)



(b)

Let E be the expectation operator and consider the following statements:

I. $E(X(t)) = E(Y(t))$

II. $E(X^2(t)) = E(Y^2(t))$

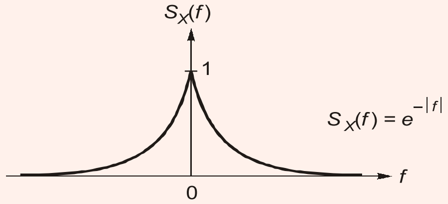
III. $E(Y^2(t)) = 2$

Select the correct option:

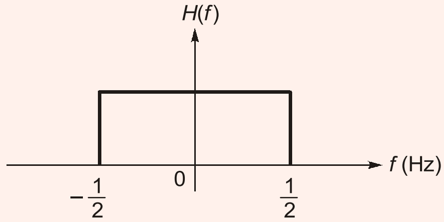
(a) only I is true
(b) only II and III are true
(c) only I and II are true
(d) only I and III are true

Ans. (a)

The given input power spectral density is as follows:



Frequency response of the low pass filter is as follows:



- $E[Y(t)] = H(0)\, E[X(t)]$

$$H(0) = 1$$

So, $$E[Y(t)] = E[X(t)]$$

- $$E[Y^2(t)] \neq E[X^2(t)]$$

  Since, LPF does not allow total power from input to output.

- $$E[X^2(t)] = \int_0^{\infty} S_X(f)\,df = 2\text{ W}$$

  As $E[Y^2(t)] \neq E[X^2(t)]$,

  $$E[Y^2(t)] \neq 2$$

- So, only statement - I is correct.

End of Solution

Q.41 Let $I = \int_C (2zdx + 2ydy + 2xdz)$ where x, y, z are real, and let C be the straight line segment from point A: (0, 2, 1) to point B: (4, 1, –1). The value of I is ______

Ans. **(–11)**

$A(0, 2, 1)$ and $B(4, 1, -1)$

The equation of the line AB is

$$\frac{x-0}{4-0} = \frac{y-2}{1-2} = \frac{3-1}{-1-1} = t \quad \text{say}$$

$$x = 4t \;;\quad y = -t + 2 \;;\quad z = -2t + 1$$

$$dx = 4dt \;;\quad dy = -dt \;;\quad dz = -2dt$$

t varies from 0 to 1

$$I = \int_0^1 2(-2t+1)\,4dt + 2(-t+2)\,(-dt) + 2(4t)\,(-2dt)$$

$$= \int_0^1 (-16t + 8 + 2t - 4 - 16t)dt = \int_0^1 (-30t + 4)dt$$

$$= \left(-30\frac{t^2}{2} + 4t\right)\Bigg|_0^1 = -15 + 4 = -11$$

End of Solution

Q.42 The Nyquist plot of the transfer function

$$G(s) = \frac{K}{(s^2 + 2s + 2)\,(s+2)}$$

does not encircle the point $(-1 + j0)$ for $K = 10$ but does encircle the point $(-1 + j0)$ for $K = 100$. Then the closed loop system (having unity gain feedback) is

(a) stable for $K = 10$ and stable for $K = 100$
(b) stable for $K = 10$ and unstable for $K = 100$
(c) unstable for $K = 10$ and stable for $K = 100$
(d) unstable for $K = 10$ and unstable for $K = 100$

Ans. (b)

Given that,

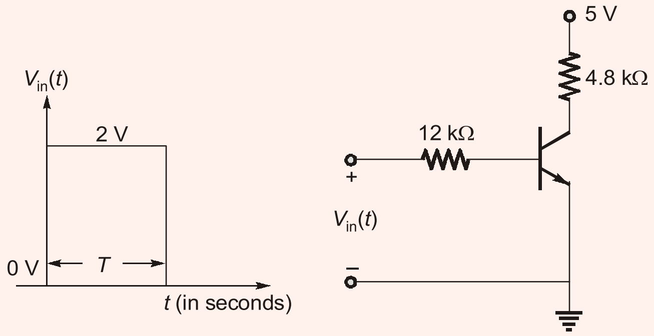
$$G(s) = \frac{K}{(s^2 + 2s + 2)(s + 2)}$$

P = open loop poles in RHS of s-plane
Z = closed loop poles in RHS of s-plane
N = number of encirclements about the point $(-1 + j0)$

For $K = 10$; $N = P - Z = 0 \Rightarrow$ $Z = 0$: stable
For $K = 100$; $N = P - Z = 1 \Rightarrow$ $Z \neq 0$: unstable

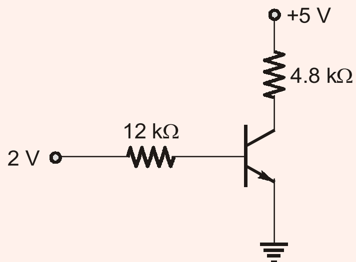
End of Solution

Q.43 In the figure shown, the *npn* transistor acts as a switch



For the input $V_{in}(t)$ as shown in the figure, the transistor switches between the cut-off and saturation regions of operation, when T is large. Assume collector-to-emitter voltage at saturation $V_{CE(sat)} = 0.2$ V and base-to-emitter voltage $V_{BE} = 0.7$ V. The minimum value of the common-base current gain (α) of the transistor for the switching should be __________.

Ans. (0.902)



$$I_B = \frac{2 - 0.7}{12} = 0.10833\text{ mA}$$

$$I_{C(sat)} = \frac{5 - 0.2}{4.8} = 1\text{ mA}$$

$$I_B \geq I_{B(min)} = \frac{I_{C(sat)}}{\beta}$$

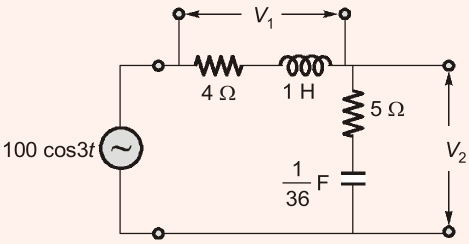
$$I_B \geq \frac{1\text{mA}}{\beta}$$

$$\beta \geq \frac{1}{0.10833} \text{ and } \beta_{min} = 9.23$$

$$\alpha_{min} = \frac{\beta_{min}}{1+\beta_{min}} = 0.902$$

End of Solution

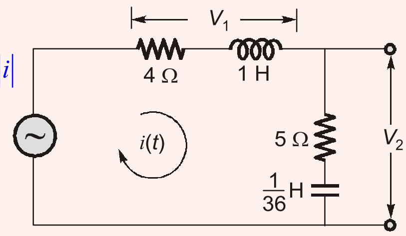
Q.44 The figure shows an RLC circuit excited by the sinusoidal voltage 100 cos(3t) Volts, where t is in seconds. The ratio $\frac{\text{amplitude of } V_2}{\text{amplitude of } V_1}$ is ________.



Ans. (2.6)

$$\omega = 3 \text{ rad/sec} \ ; \ Z_1 = (4 + j3)\ \Omega$$

and $Z_2 = (5 - j12)\ \Omega$



$$|V_2| = |i|\,|z_2| = |i|\sqrt{5^2+12^2} = 13|i|$$

$$|V_1| = |i|\,|z_1| = |i|\sqrt{4^2+3^2} = 5\,|i|$$

$$\frac{|V_2|}{|V_1|} = \frac{13|i|}{5|i|} = \frac{13}{5} = 2.6$$

End of Solution

Q.45 An optical fiber is kept along the $\hat{z}$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ directions in the fiber are n_x = 1.5000 and n_y = 1.5001, respectively ($n_x \neq n_y$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is 1.5 μm. If the lightwave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized, in centimetres, is ________.

Ans. (0.375)

Initially the wave is circularly polarized. So, the initial phase difference between field components in $\hat{a}_x$ direction and $\hat{a}_y$ direction is $\frac{\pi}{2}$.

To become linearly polarized, the wave must travel a minimum distance, such that, the phase difference at that point between the field components in $\hat{a}_x$ direction and $\hat{a}_y$ direction is π (i.e., the travel of this minimum distance should provide an additional phase difference of $\pi/2$ between $\hat{a}_x$ and $\hat{a}_y$ field components).

$$\text{So,} \quad z_{\min} k_x \sim z_{\min} k_y = \frac{\pi}{2}$$

$$z_{\min}\left[\frac{\omega}{v_{px}} \sim \frac{\omega}{v_{py}}\right] = \frac{\pi}{2}$$

$$2\pi z_{\min}\left[\frac{f}{C}\sqrt{\varepsilon_{rx}} \sim \frac{f}{C}\sqrt{\varepsilon_{rx}}\right] = \frac{\pi}{2}$$

$$\frac{4 z_{\min}}{\lambda_0}\left[n_x \sim n_y\right] = 1$$

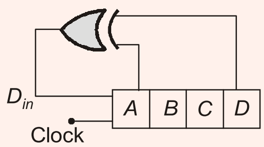
$$z_{\min} = \frac{\lambda_0}{4(n_x \sim n_y)} = \frac{1.5}{4(1.5 \sim 1.5001)}\,\mu\text{m}$$

$$= \frac{1.5}{4(0.0001)}\,\mu\text{m} = \frac{1.5}{4}\,\text{cm}$$

$$z_{\min} = 0.375 \text{ cm}$$

End of Solution

Q.46 A 4-bit shift register circuit configured for right-shift operation, i.e. $D_{in} \to A$, $A \to B$, $B \to C$, $C \to D$, is shown. If the present state of the shift register is $ABCD$ = 1101, the number of clock cycles required to reach the state $ABCD$ = 1111 is _______



Ans. (10)

Clock Number	$D_{in} = A \oplus D$ Just before clock	A	B	C	D
Initial	-	1	1	0	1
1	0	0	1	1	0
2	0	0	0	1	1
3	1	1	0	0	1
4	0	0	1	0	0
5	0	0	0	1	0
6	0	0	0	0	1
7	1	1	0	0	0
8	1	1	1	0	0
9	1	1	1	1	0
10	1	1	1	1	1

So, 10 clock cycles are required.

End of Solution

Q.47 Which one of the following gives the simplified sum of products expression for the Boolean function $F = m_0 + m_2 + m_3 + m_5$, where m_0, m_2, m_3 and m_5 are minterms corresponding to the inputs A, B and C with A as the MSB and C as the LSB?

(a) $\bar{A}B + \bar{A}\bar{B}\bar{C} + A\bar{B}C$ (b) $\bar{A}\bar{C} + \bar{A}B + A\bar{B}C$

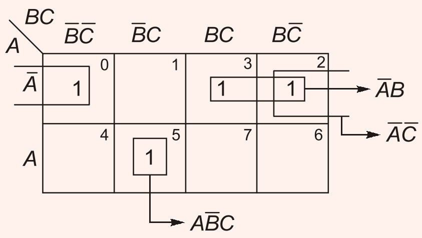
(c) $\bar{A}\bar{C} + A\bar{B} + A\bar{B}C$ (d) $\bar{A}BC + \bar{A}\bar{C} + A\bar{B}C$

Ans. (b)

Given Boolean function is,

$$F = m_0 + m_2 + m_3 + m_5$$

It can be minimized by using K-map as given below.



$$F = \bar{A}\bar{C} + \bar{A}B + A\bar{B}C$$

End of Solution

Q.48 Let $x(t)$ be a continuous time periodic signal with fundamental period $T = 1$ seconds. Let $\{a_k\}$ be the complex Fourier series coefficients of $x(t)$, where k is integer valued. Consider the following statements about $x(3t)$:

I. The complex Fourier series coefficients of $x(3t)$ are $\{a_k\}$ where k is integer valued.

II. The complex Fourier series coefficients of $x(3t)$ are $\{3a_k\}$ where k is integer valued.

III. The fundamental angular frequency of $x(3t)$ is 6π rad/s.

For the three statements above, which one of the following is correct?

(a) only II and III are true (b) only I and III are true

(c) only III is true (d) only I is true

Ans. (b)

Initially $T = 1$ sec, so $\omega_0 = 2\pi$ rad/sec. When $x(t)$ is compressed by 3, frequency will expand by same factor but there is no change in values of a_k.

So, both statement I and III are correct.

End of Solution

Q.49 A continuous time signal $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$, where t is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response

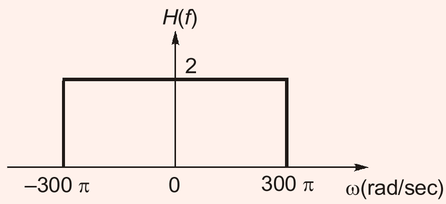
$$h(t) = \begin{cases} \dfrac{2\sin(300\pi t)}{\pi t}, & t \neq 0 \\ 600, & t = 0 \end{cases}$$

Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is ________.

Ans. (8)

$$x(t) = 4\cos 200\pi t + 8\cos 400\pi t$$

$$h(t) = \frac{2\sin 300\pi t}{\pi t} \xleftrightarrow{F.T}$$



So, $y(t) = 8\cos 200\pi t$

$\therefore$ $|y(t)|_{max} = 8$

End of Solution

Q.50 The following FIVE instructions were executed on an 8085 microprocessor.

MVI A, 33 H
MVI B, 78 H
ADD B
CMA
ANI 32 H

The accumulator value immediately after the execution of the fifth instruction is

(a) 00 H
(b) 10 H
(c) 11 H
(d) 32 H

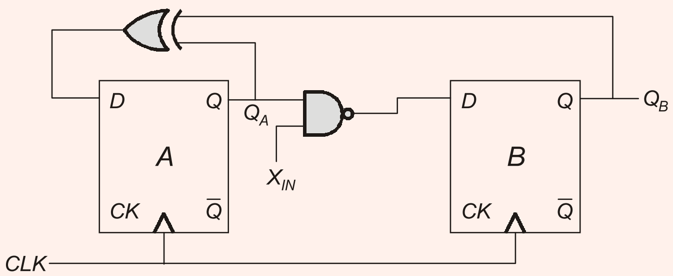
Ans. (b)

MVI A, 33 H : (A) = 33 H
MVI B, 78 H : (B) = 78 H
ADD B : (A) ← (A) + (B) = 33 H + 78 H = AB H ⇒ (A) = AB H
(A) = 1010 1011
CMA : Complement Accumulator ⇒ (A) = 0101 0100
ANI 32 H : (A) ← (A) AND 32 H

0101 0100
<u>0011 0010</u>
<u>0001 0000</u> ⇒ (A) = 10 H

End of Solution

Q.51 A finite state machine (FSM) is implemented using the D flip-flops A and B, and logic gates, as shown in the figure below. The four possible states of the FSM are Q_AQ_B = 00, 01, 10, and 11.



Assume that X_{IN} is held at constant logic level throughout the operation of the FSM. When the FSM is initialized to the Q_AQ_B = 00 and clocked, after a few clock cycles, it starts cycling through

(a) all of the four possible states if $X_{IN} = 1$
(b) three of the four possible states if $X_{IN} = 0$
(c) only two of the four possible states if $X_{IN} = 1$
(d) only two of the four possible states if $X_{IN} = 0$

Ans. (d)

In the given diagram,

$$D_A = Q_A \oplus Q_B \text{ and } D_B = \overline{Q_A X_{IN}}$$

For $X_{IN} = 0$:

$$D_A = Q_A \oplus Q_B \text{ and } D_B = 1$$

Present State		D_A	D_B	Next State	
Q_A	Q_B			Q_A^+	Q_B^+
0	0	0	1	0	1
0	1	1	1	1	1
1	1	0	1	0	1
0	1	1	1	1	1

So, for $X_{IN} = 0$, Number of possible states = 2

For $X_{IN} = 1$:

$$D_A = Q_A \oplus Q_B \text{ and } D_B = \bar{Q}_A$$

Present State		D_A	D_B	Next State	
Q_A	Q_B			Q_A^+	Q_B^+
0	0	0	1	0	1
0	1	1	1	1	1
1	1	0	0	0	0
0	0	0	1	0	1

So, for $X_{IN} = 1$, Number of possible states = 3

So, the option (d) is correct.

End of Solution

Q.52 Let $f(x) = e^{x + x^2}$ for real x. From among the following, choose the Taylor series approximation of $f(x)$ around $x = 0$, which includes all powers of x less than or equal to 3,

(a) $1 + x + x^2 + x^3$
(b) $1 + x + \frac{3}{2}x^2 + x^3$
(c) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$
(d) $1 + x + 3x^2 + 7x^3$

Ans. (c)

About $x = 0$

$$f(x) = e^x e^{x^2}$$

$$= \left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....\right)\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+....\right)$$

$$= 1+x^2+\frac{x^4}{2}+\frac{x^6}{6}+....+x+x^3+\frac{x^5}{2}+\frac{x^7}{6}+.....$$

$$+\frac{x^2}{2}+\frac{x^4}{2}+\frac{x^6}{4}+\frac{x^8}{12}.....+\frac{x^3}{6}+\frac{x^5}{6}+\frac{x^7}{12}+\frac{x^9}{36}+....$$

$$= 1+x+\frac{3}{2}x^2+\frac{7}{6}x^3$$

End of Solution

Q.53 A three dimensional region R of finite volume is described by

$$x^2 + y^2 \le z^3 \ ; \ 0 \le z \le 1,$$

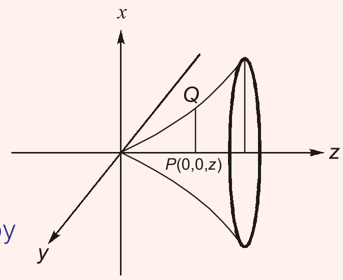
where x, y, z are real. The volume of R (up to two decimal places) is ________.

Ans. (0.7853)

Let $x^2 + y^2 = t^2$

$t^2 = z^3$

Here revolution is about z axis



volume of region $R = \int_0^1 \pi (PQ)^2 dz$

Here PQ is radius of circle at some z, which is given by

$$PQ = \sqrt{x^2+y^2}$$

$$(PQ)^2 = x^2 + y^2 = z^3$$

so, volume of region $R = \int_0^1 \pi t^2 dz = \int_0^1 \pi z^3 dz = \left.\frac{\pi z^4}{4}\right|_0^1$

$$= \frac{\pi}{4} = 0.7853$$

End of Solution

Q.54 In binary frequency shift keying (FSK), the given signal waveforms are

$$u_0(t) = 5\cos(20000\pi t) \ ; \ 0 \le t \le T, \text{ and}$$

$$u_1(t) = 5\cos(22000\pi t) \ ; \ 0 \le t \le T,$$

where T is the bit-duration interval and t is in seconds. Both $u_0(t)$ and $u_1(t)$ are zero outside the interval $0 \le t \le T$. With a matched filter (correlator) based receiver, the smallest positive value of T (in milliseconds) required to have $u_0(t)$ and $u_1(t)$ uncorrelated is

(a) 0.25 ms
(b) 0.5 ms
(c) 0.75 ms
(d) 1.0 ms

Ans. (b)

$$u_0(t) = 5\cos 20000\pi t$$
$$u_1(t) = 5\cos 22000\pi t$$
$$f_1 = 11000 \text{ Hz and } f_2 = 10000 \text{ Hz}$$

For FSK waveforms to be uncorrelated,

$$f_1 - f_2 = n\frac{R_b}{2} \;;\; n = 1, 2, 3, \ldots.$$
$$R_b = \frac{2(f_1 - f_2)}{n} = \frac{2000}{n} \text{ bits/sec}$$
$$R_{b\,(max)} = 2000 \text{ bits/sec} \qquad \because \text{ minimum value of } n = 1$$
$$T_{b\,(min)} = \frac{1}{R_{b(max)}} = 0.5 \text{ ms}$$

End of Solution

Q.55 Which one of the following is the general solution of the first order differential equation

$$\frac{dy}{dx} = (x + y - 1)^2,$$

where x, y are real?

(a) $y = 1 + x + \tan^{-1}(x + c)$, where c is a constant.
(b) $y = 1 + x + \tan(x + c)$, where c is a constant.
(c) $y = 1 - x + \tan^{-1}(x + c)$, where c is a constant.
(d) $y = 1 - x + \tan(x + c)$, where c is a constant.

Ans. (d)

$$\frac{dy}{dx} = (x + y - 1)^2 \qquad \ldots(i)$$

Let $x + y - 1 = t \qquad \ldots(ii)$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$
$$\frac{dy}{dx} = \frac{dt}{dx} - 1 \qquad \ldots(iii)$$

Substituting equations (ii) and (iii) in equation (i)

$$\frac{dt}{dx} - 1 = t^2$$
$$\frac{dt}{t^2 + 1} = dx$$

Integrating both side

$$\int \frac{1}{t^2 + 1} dt = \int dx$$
$$\tan^{-1} t = x + c$$

Since, $t = x + y - 1$

$\therefore$
$$\tan^{-1}(x + y - 1) = x + c$$
$$x + y - 1 = \tan(x + c)$$
$$y = 1 - x + \tan(x + c)$$

End of Solution

SECTION B : GENERAL APTITUDE

Q.1 In the summer, water consumption is known to decrease overall by 25%. A Water Board official states that in the summer household consumption decreases by 20%, while other consumption increases by 70%.

Which of the following statements is correct?

(a) The ratio of household to other consumption is 8/17.
(b) The ratio of household to other consumption is 1/17.
(c) The ratio of household to other consumption is 17/8.
(d) There are errors in the official's statement.

Ans. (d)

Let H is household consumption and P be other consumption.

$$H \times 0.8 + P \times 1.7 = (H + P) \times 0.75 \qquad \text{(According to given condition)}$$

$\therefore$ from here ratio is negative.

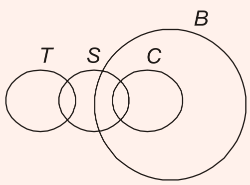
End of Solution

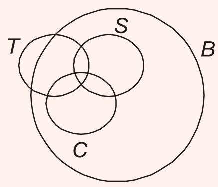
Q.2 Some tables are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?

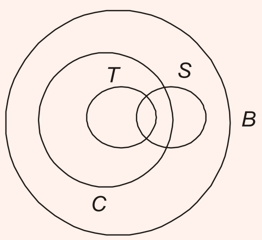
(i) At least one bench is a table
(ii) At least one shelf is a bench
(iii) At least one chair is a table
(iv) All benches are chairs

(a) only (i)
(b) only (ii)
(c) only (ii) and (iii)
(d) only (iv)

Ans. (b)







Only conclusion (ii) follows.

End of Solution

Q.3 I ___________ made arrangements had I ___________ informed earlier.
(a) could have, been
(b) would have, being
(c) had, have
(d) had been, been

Ans. (a)
Could have, been
I could have made arrangements had I been informed earlier. Use of conditional sentence based on past participle form.

End of Solution

Q.4 40% of deaths on city roads may be attributed to drunken driving. The number of degrees needed to represent this as a slice of a pie chart is
(a) 120
(b) 144
(c) 160
(d) 212

Ans. (b)

$$40\% \text{ of } 360° = \frac{40}{100} \times 360° = 144°$$

So, the angle subtended on pie chart will be = 144°.

End of Solution

Q.5 She has a sharp tongue and it can occasionally turn ___________.
(a) hurtful
(b) left
(c) methodical
(d) vital

Ans. (a)
Hurtful
Hurtful means causing pain or suffering or something that is damaging or harmful. The expression 'sharp tongue' defines a bitter or critical manner of speaking.

End of Solution

Q.6 Trucks (10 m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20 m after each truck and a gap of at least 15 m after each car. Trucks and cars travel at a speed of 36 km/h. If cars and trucks go alternately. What is the maximum number of vehicles that can use the bridge in one hour?
(a) 1440
(b) 1200
(c) 720
(d) 600

Ans. (a)
Length of Truck + gap required = 10 + 20 = 30 m
Length of Car + gap required = 5 + 15 = 20 m
Alternative pairs of Truck and Car needs

$$30 + 20 = 50 \text{ m}$$

Let 'n' be the number of repetition of (Truck + Car) in 1 hour (3600 secs)

$$\text{Given speed} = 36 \text{ km/hr} = 10 \text{ m/sec}$$

$$\frac{50\text{ m} \times n}{3600\text{ secs}} = 36\text{ km/hr}$$

$$\frac{50n}{3600}\text{ m/sec} = 10\text{ m/sec}$$

$$n = \frac{36000}{50} = 720\text{ (Trucks + Cars)}$$

So, 720 (Trucks + Cars) passes = 720 × 2 = 1440 vehicles

End of Solution

Q.7 "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".

Here, the word 'antagonistic' is closest in meaning to

(a) impartial
(b) argumentative
(c) separated
(d) hostile

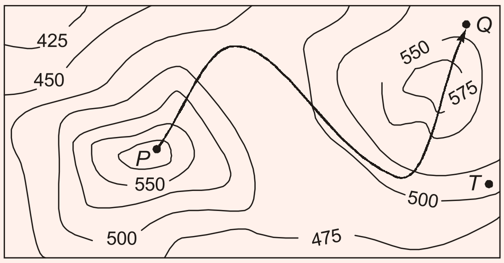
Ans. (d)

Hostile

Antagonist is a adversary or one who opposes/contends against another.

End of Solution

Q.8 A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.

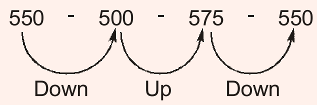


The path from P to Q is best described by

(a) Up-Down-Up-Down
(b) Down-Up-Down-Up
(c) Down-Up-Down
(d) Up-Down-Up

Ans. (c)

Contour lines can be observed to cross region with height



Down-Up-Down satisfies.

End of Solution

Q.9 There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?

(a) 56
(b) 52
(c) 48
(d) 44

Ans. (a)

Subgroups containing only Indians = ${}^3C_1 + {}^3C_2 + {}^3C_3$ = 3 + 3 + 1 = 7

Subgroups containing one Indian and rest Chinese = ${}^3C_1\left[{}^3C_1 + {}^3C_2 + {}^3C_3\right]$

= 3[3 + 3 + 1] = 21

Subgroups containing two Indian and remaining Chinese = ${}^3C_2\left[{}^3C_1 + {}^3C_2 + {}^3C_3\right] = 21$

Subgroups containing three Indian and remaining Chinese = ${}^3C_3\left[{}^3C_1 + {}^3C_2 + {}^3C_3\right] = 7$

Total number of subgroups = 7 + 21 + 21 + 7 = 56.

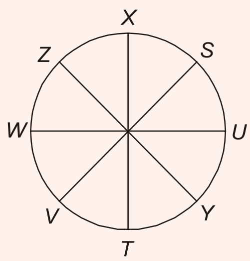
End of Solution

Q.10 *S*, *T*, *U*, *V*, *W*, *X*, *Y* and *Z* are seated around a circular table. *T*'s neighbours are *Y* and *V*. *Z* is seated third to the left of *T* and second to the right of *S*. U's neighbours are *S* and *Y*; and *T* and *W* are not seated opposite each other. Who is third to the left of *V*?

(a) *X*
(b) *W*
(c) *U*
(d) *T*

Ans. (a)

Following circular seating arrangement can be drawn



Only one such arrangement can be drawn.
The person on third to the left of *V* is *X*.

End of Solution

■■■■