Code : 211303

B.Tech 3rd Semester Exam., 2014

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *All questions carry equal marks.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt* ***FIVE*** *questions in all.*

(iv) *Question No.* ***1*** *is compulsory.*

1. Choose the correct answer/Fill in the blanks of any *seven* of the following :

(a) $J_{1/2}(x)$ is given by

(i) $\sqrt{\frac{2\pi}{x}} \sin x$

(ii) $\sqrt{\frac{2\pi}{x}} \cos x$

(iii) $\sqrt{\frac{\pi}{2x}} \cos x$

(iv) $\sqrt{\frac{2}{\pi x}} \sin x$

(b) The incorrect equation among the following is

(i) $P_0(x) = 1$

(ii) $P_1(u) = x$

(iii) $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$

(iv) $P_n(-x) = (-1)^{n+1} P_n(x)$

(c) The solution of $p + q = z$ is

(i) $f(x+y, y+\log z) = 0$

(ii) $f(xy, y\log z) = 0$

(iii) $f(x-y, y-\log z) = 0$

(iv) None of the above

(d) The solution of $zxp - zyq = y^2 - x^2$ is ——.

(e) The inverse transformation $w = \frac{1}{2}$ transforms the straight line $ay + bx = 0$ into

(i) circle

(ii) straight line through the origin

(iii) parabola

(iv) None of the above

(f) If $f(z)$ is analytic and equals $u(x, y) + iv(x, y)$, then $f'(z)$ equals

(i) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

(ii) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

(iii) $\frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$

(iv) None of the above

(g) The zeros and singularities of $\frac{z^2+1}{1-z^2}$ are 1, −1.

(h) The area under the whole normal curve is 1.

(i) If the mean of a Poisson distribution is m, then SD of this distribution is

(i) m^2

(ii) $\sqrt{m}$

(iii) m

(iv) None of the above

(j) If A and B are mutually exclusive events, then $P(A \cup B) =$ ——. P(A) + P(B)

2. (a) Solve in series, using Frobenius method, the equation

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

(b) Prove that

$$e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$$

3. (a) State and prove Rodrigues formula.

(b) Prove that

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

4. (a) Solve :

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$$

(b) Solve :

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

5. Solve the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial u^2}$ for the conduction of heat along a rod without radiation subject to the conditions

$$\frac{\partial u}{\partial x} = 0 \text{ for } x = 0 \text{ and } x = l$$

u is not infinite for $t \to \infty$, $u = lx - x^2$ for $t = 0$, between $x = 0$ and $x = l$.

6. Show that $v(x, y) = -\sin x \sinh y$ is harmonic. Find the conjugate harmonic of v.

7. State and prove Cauchy's integral formula.

8. *(a)* Find the mean and variance of a binomial distribution.

(b) Determine the probability p that there are 3 defective items in a sample of 100 items if 2% of items made in this factory are defective.

9. *(a)* Show that the area under the normal curve is unity.

(b) A box contains 8 items of which 2 are defective. A person draws 3 items from the box. Determine the expected number of defective items he has drawn.

★★★