$\mu P + \mu C \longrightarrow 2Q$

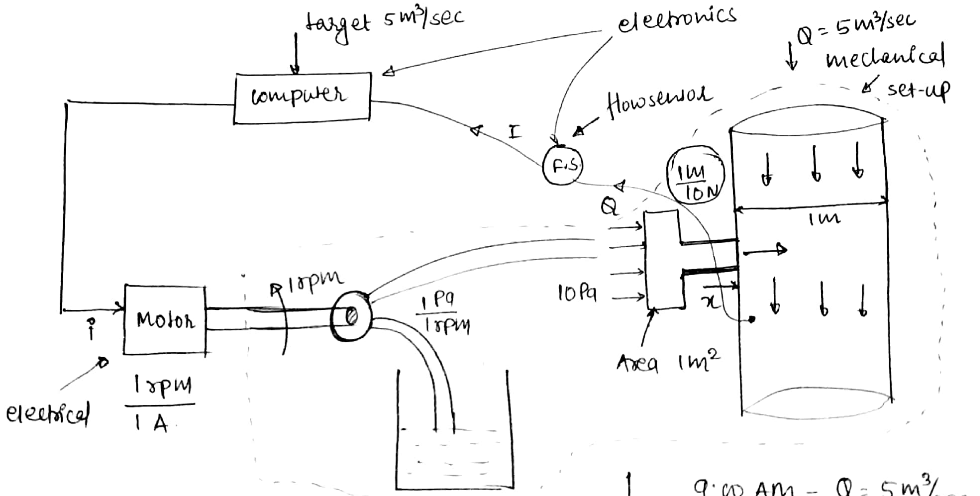
Sensors + Actuator + control system $\longrightarrow$ 5Q

Robotics $\longrightarrow$ 5Q

Mechatronics:

→ Integration of electronics and electrical devices for mechanical system. leads to the development of mechatronics engineering

→ Consider the example shown below, where the estabilished steady flow should be shut down, if the flow rate is more than ~~10~~ 5 m^3/sec.



To shut down flow

$x = 1m$

$F = 10N$

$P_i = 10 Pa$

9:00 AM – Q = 5 m^3/sec

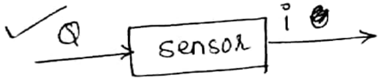
10:00 AM – Q = 5 m^3/se

11:00 AM – Q = 5 m^3/sec

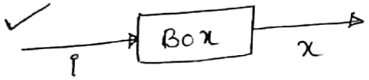
> 11:00 AM if Q = 5 m^3/sec

Shut down the flow

Q → x → F → Pin → speed of motor → i



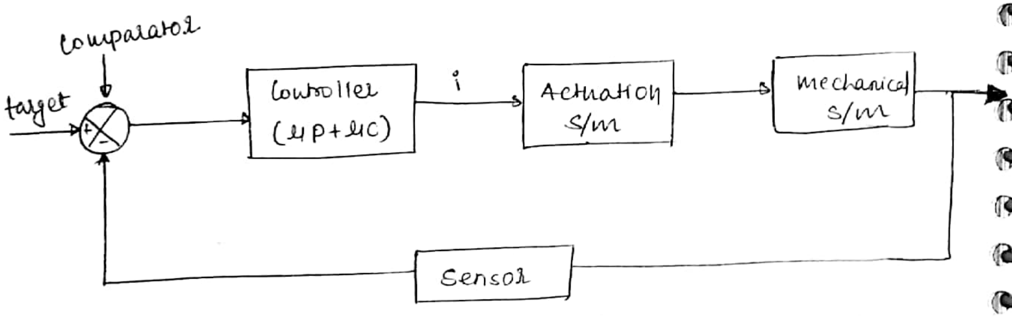
ME → E.E



E.E. → M.E. ⇒ Actuator

transducer = sensor + Actuator

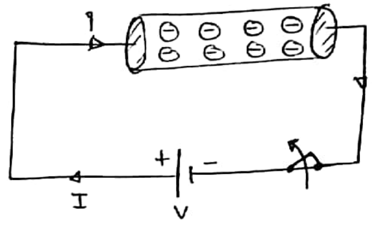
Block diagram of Mechatronics



Note

→ In every mechatronics system, usually we can find electronic controller (either microprocess or microcontroller), electronic sensor, and electrical or mechanical actuators

Basics of electricity

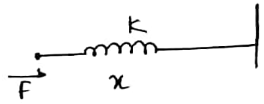


$V = RI$ (@ temp. constant)

$I = \frac{V}{R}$

R↑ → I↓

R↓ → I↑



$F = kx$

→ the direction of current can be reversed by reversing the polarities of potential source.

Strain Gauge

→ It is a metallic or semi-conductor based resistive sensor which changes its resistance because of the applied load.

→ Strain gauge is used to measure Force (F), all the associated variables with the force [T, a, σ]

→ Consider the metallic wire shown below

$R = \frac{\rho l}{A}$

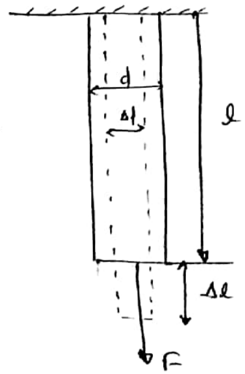
$R = f(\rho, l, A)$ (ρ → const)

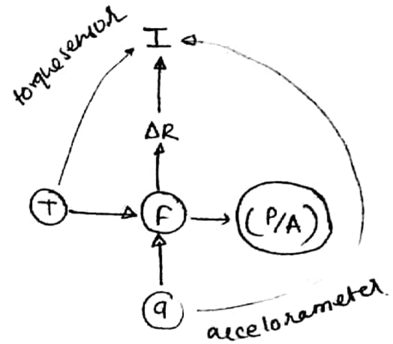
@ $F \neq 0$ (b)

$l' \rightarrow l + \Delta l$

$A' \rightarrow A - \Delta A$

$R' \rightarrow R + \Delta R$





$\Delta l \rightarrow$ [Strain Gauge] $\rightarrow \Delta R$

$\Rightarrow \Delta R \propto \Delta l$

$\frac{\Delta l}{l} \propto \frac{\Delta R}{R} \Rightarrow \frac{\Delta l}{l} = K \frac{\Delta R}{R}$

$\Rightarrow \frac{\Delta l / l}{\Delta R / R} = K$

Gauge factor

$\Rightarrow \boxed{\frac{\Delta R/R}{\Delta l/l} = \frac{1}{K} = G_f}$

$\Rightarrow \Delta R = R G_f \left(\frac{\Delta l}{l}\right)$

$\Rightarrow \Delta R = R G \frac{\cancel{A}}{AE}$

$\Rightarrow \Delta R = \left(\frac{R G_f}{AE}\right) F$

$\Rightarrow \Delta R \propto F$ (ΔA is very small)

Q IN-2007 A strain gauge which is attached to a specimen subjected to tensile force, the has a length of 20 cm is subjected to tensile force, the nominal resistance of strain gauge is 100 Ω at unstrained condition. the changes in the resistance and elongation in the gauge are given as 0.35 Ω and .2mm respectively then the gauge factor of strain gauge is.

$G_f = \frac{\Delta R/R}{\Delta l/l} = \frac{0.35/100}{.2/200} = \frac{0.35 \times \cancel{200}}{\cancel{.2} \times \cancel{100}} = 3.5$

2004

Q A strain gauge attached to specimen, to which an axial load of 10N is applied find the change in resistance of the Gauge for the applied load given that $E = 2\times10^{11}$ N/m² and unstrained resistance R = 100 Ω, area of X-S/C A affected because of the load is 10^{-6} m², gauge factor $G_f = 2.0$

Sol

$\Delta R = R G_f \frac{F}{AE}$

$= \frac{100 \times 2 \times 10}{10^{-6} \times 2 \times 10^{11} \times \cancel{100}}$

$= \frac{1000}{10^5}$

$\Delta R = 0.01\,\Omega$

2015

Q A p-type semiconductor strain gauge has a nominal resistance of 1000 Ω $G_f = +200$ at 25°C the change in resistance (ΔR) of strain gauge when it is subjected to strain of 10^{-4} m/m.

Sol

$\Delta R = R G_f \frac{\Delta l}{l}$

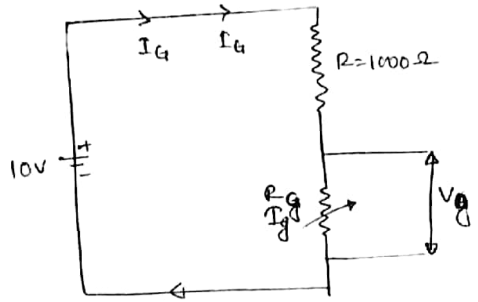
$= 1000 \times 200 \times 10^{-4}$

$\Delta R = 20\,\Omega$

Note:

- Semiconductor strain gauges generally has high gauge factor compare to metallic gauges
- the only problem with the semiconductor strain gauge is it is highly sensitive to external temp. variation.

extension to privious problem



$I_g = \frac{10V}{R+R_g}$, $V_G = R_g I_g$

F = 0N

$\frac{\delta l}{l} = 0$

$R_g = 1000$

$I_g = \frac{10}{2000}$

= 5mA

$V_g = R_g I_g$

$V_g = 5V$ → offset

F ≠ 0

$\frac{\Delta l}{l} = 10^{-4}$ m/m

$\Delta R = 20\Omega$

$R_g' = 1020$

$I_g' = \frac{10}{1000+1020}$

= 4.95 mA

$V_g' = I_g' R_g'$

= 5.05V

10^{-4} m/m = 50mV

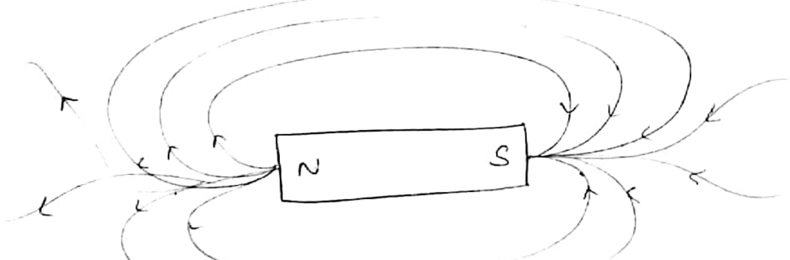
1 m/m = 500V

Strain	ΔR_g	ΔI_g	ΔV_g
10^{-4} m/m	20Ω	-50μA	50 mV

[sensitivity = $\frac{500V}{1 m/m}$]

—x—

Magnet — Permanent magnet / electromagnet



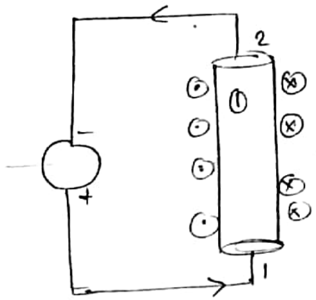
(i) magnetic flux (Φ) (scaler)

(ii) magnetic field density ($\vec{B}$) (vector)

Reluctance ↓ B ↑ → resistance to ~~electro~~ magnetic density

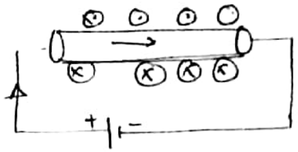
Remnance ↓ I ↑

Electro-magnet



θ = 90°: if $I_1 \neq 0A$, $B_1 \neq 0 \frac{Wb}{m^2}$; $I_2 = 0A$, $B_2 = 0 \frac{Wb}{m^2}$

θ = 180°: $I_1 = 0A$, $B_1 = 0 \frac{Wb}{m^2}$; $I_2 \neq 0A$, $B_2 \neq 0 \frac{Wb}{m^2}$



Stepper Motor

→ It is a digital electrical actuator which converts electrical form of energy mechanical form of energy

The rotor of the stepper motor make incremental angle in discrete manner when the stator poles are exited in programmed manner.

→ stepper motor are mainly classified into three types.

(i) Variable reluctance type stepper motor

(ii) Permanent magnet type stepper motor

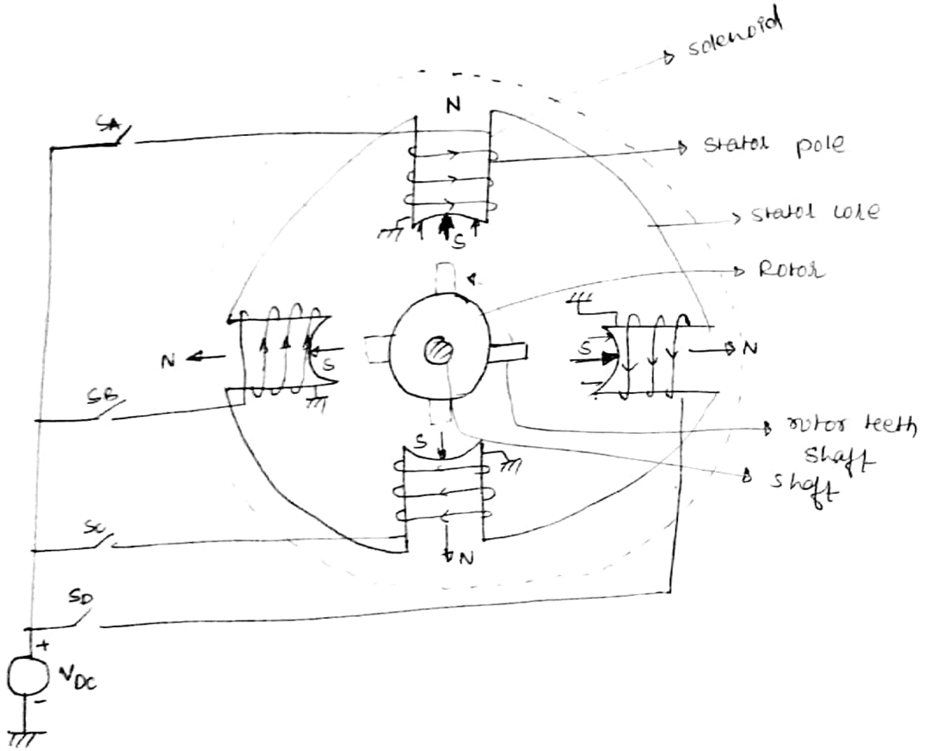
(iii) Hybrid-type stepper motor

Variable Reluctance type stepper motor

→ It consists of a stator which has poles as shown VRSM type stepper motor has the rotor in cylindrical shape and made up of iron core. The rotor of VRSM type stepper motor consists the teeth made up off magnetic material.

→ If the stator pole is energied with D.C current then the corresponding stator pole acts like electro-magnet

→ Because of the magnetic field density at the stator pole the rotor rotates and allings to ~~excess~~ axis of the stator



Operation for A.C.W. rotation

→ if S_D is on S_A, S_B, S_C are off; $I_D \neq 0$, $I_A, I_B, I_C = 0$ } $\theta = 0°$

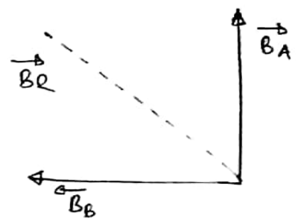
→ if S_A is on S_B, S_C, S_D are off; $I_A \neq 0$, $I_B, I_C, I_D = 0$ } $\theta = 90°$

→ if S_B is on S_A, S_C, S_D are off; $I_B \neq 0$, $I_A, I_C, I_D = 0$ } $\theta = 180°$

→ if S_C is on S_A, S_B, S_D are off; $I_C \neq 0$, $I_A, I_B, I_D = 0$ } $\theta = 270°$

→ if S_D is on S_A, S_B, S_C are off; $I_D \neq 0$, $I_A, I_B, I_C = 0$ } $\theta = 360°$

→ if S_A and S_B are on S_C, S_D are off; I_A & $I_B \neq 0$, $I_C, I_D = 0$ } $\theta = 135°$



Note:

→ To reverse the direction of rotation, we should reverse the switching sequence

Switching Sequence

S_A	S_B	S_C	S_D	θ
0	0	0	1	0°
1	0	0	1	45°
1	0	0	0	90°
1	1	0	0	135°
0	1	0	0	180°
0	1	1	0	225°
0	0	1	0	270°
0	0	1	0	315°
0	0	0	1	360°

half step angle

full step angle

Stator Pitch (θ_S):

→ The angular separation between two successive poles of the stator is called stator pitch

$$\boxed{\theta_S = \frac{360^\circ}{\text{no. of stator pole}}} = \frac{360^\circ}{4} = 90^\circ$$

Rotor Pitch (θ_R)

→ The angular separation between two successive teeth of the rotor is called rotor pitch

$$\boxed{\theta_R = \frac{360^\circ}{\text{no. of teeth on rotor}}} = \frac{360^\circ}{2} = 180^\circ$$

Full step angle (θ_{FS})

The angular rotation of the rotor, when only one switch is activated is called full step angle

$$\boxed{\theta_{FS} = \theta_R - \theta_S}$$

i.e. $\theta_{FS} = 180 - 90^\circ = 90^\circ$

half step angle (θ_{HS})

→ The angular rotation of the rotor, when two switches are activated at a time as shown in the switching sequence is called half step angle.

$$\boxed{\theta_{HS} = \frac{\theta_{FS}}{2} = \frac{\theta_R - \theta_S}{2}}$$

→ The design limitation in V.R.S.M is

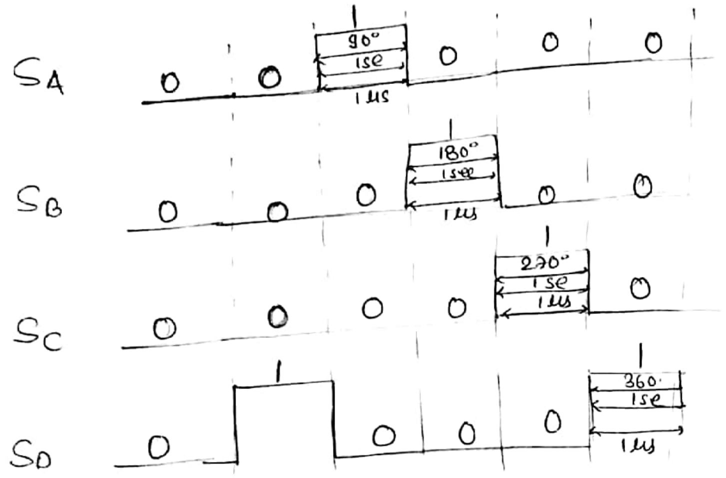
$$\boxed{\text{No. of teeth on Rotor} \geq \frac{\text{No. of pole on stator}}{2}}$$

→ min no of teeth on rotor = 2

→ min no. of poles on stator = 4

→ no. of ~~teet~~ teeth and no. poles should be always in even no.

Digital pulse logic



$\boxed{1 \text{ pulse} \rightarrow 1 \text{ step}}$

if 4 sec → 4 pulse → 360° → 1 rev.
1 sec → 1 pulse → 90° → 1/4 rev

1 sec = 1/4 rev.
1 min = 15 Rev
speed = 15 rev per min.

if 4 μsec → 1 rev
4 sec → 1×10^6 rev
1 min → 15×10^6 rev.
speed → 15×10^6 RPM

Q A stepper motor has 130 steps per rev. find the input digital pulse rate that should be applied to produce shaft speed of 10.5 rev/sec.

Sol

130 steps → 1 rev.

$\boxed{1 \text{ pulse} \rightarrow 1 \text{ step}}$

130 pulse → 130 steps → 1 rev
10.5 rev → 1 sec
(10.5 × 130) pulse → 1 sec
1365 pulse → 1 sec

pulse rate = 1365 pulse/sec

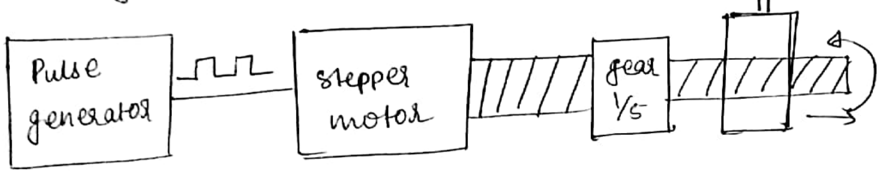
Q A stepper motor having a sensitivity of 300 steps per rev and running at 2400 rpm, required a input pulse rate.

Sol

300 steps → 1 rev
300 pulse → 1 rev.

2400 rev → 1 min
2400 × (300 pulse) → 60 sec
$\frac{2400 \times 300}{60}$ ← 1 sec
12,000 pulse ← 1 sec

Q In the lead drive point to point open loop CNC drive a stepper motor rotating at const. speed drives a table through a gear box and screw nut mechanism (pitch = 5 mm) and gear ratio = 1/5 if the stepper motor is driven by pulse generator of frequency 10,000 pulse/min the table if the stepper has a speed of 200 steps/rev. the table moment corresponding 1 pulse of the pulse generator is ___ key concept $\boxed{1 \text{ pulse} = 1 \text{ step}}$



10000 pulse → 1 min   200 steps → 1 rev

200 pulses → 1 rev.

1 min → 10000 pulse → 50 rev.

1 rev — 5 mm

1 min → 10 rev → 50 mm

1 min → 10,000 pulses → 50 mm

10,000 pulse → 50 mm

1 pulse → $\frac{50\,mm}{10^4}$

1 pulse → 5 μm

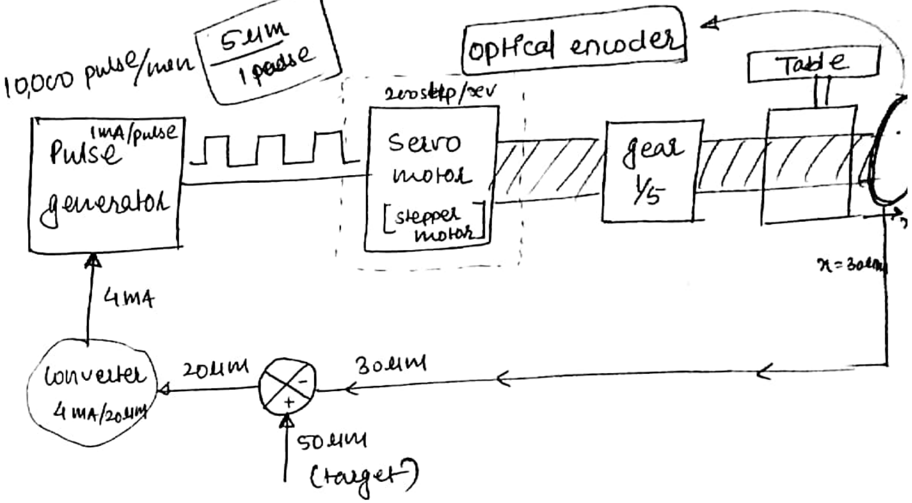
Note

→ In mechanical application we should have a stepper motor which provides lower step angle if the step angle of stepper motor is low, then the movement will be highly precised.

~~Closed loop~~ Servo Mechanism

→ A closed loop control system alogorithum which makes the comparator output equals to zero is called servo mechanism.

→ Any motor which is used in servo-mechanism, refered as servomotor.



Q. 30) A variable reluctance type stepper motor, which has 8 poles on stator and 6 teeth on rotor has full step angle and half step angle are.

$\theta_s = \frac{360}{8}$ $\theta_{FS} = \theta_R - \theta_s$

$\theta_R = \frac{360}{6}$ $= 360\left[\frac{1}{6} - \frac{1}{8}\right]$

$= \frac{\cancel{360}^{30} \times \cancel{2}}{\cancel{48}_{\cancel{24}_{2}}} = \underline{15°}$

θ_{HS} angle $= \underline{7.5°}$

ESE 14 A stepper motor is to be used to drive the linear axis of a certain mechatronics system the motor output shaft is connected to screw thread with 30mm pitch. linear resolution of 0.5 mm is required, what should be the step angle.

30 mm — 1 rev

0.5

$\frac{1}{30} \times 0.5$ $\frac{5}{300}$ $\frac{1}{60}$ rev

~~300~~ 30 mm — 360°

0.5 mm — $\frac{\cancel{360}^{12}}{\cancel{30}} \times 0.5$

$\underline{6.0°}$

$\underline{6°}$

E-17

Q. Consider the following statement regarding a stepper motor

1. The rotation angle of the motor is proportional to the input pulse
2. The motor has full torque under no load
3. Speed and electrical control signal of the motor varies linearly.

Permanent magnet type stepper motor:

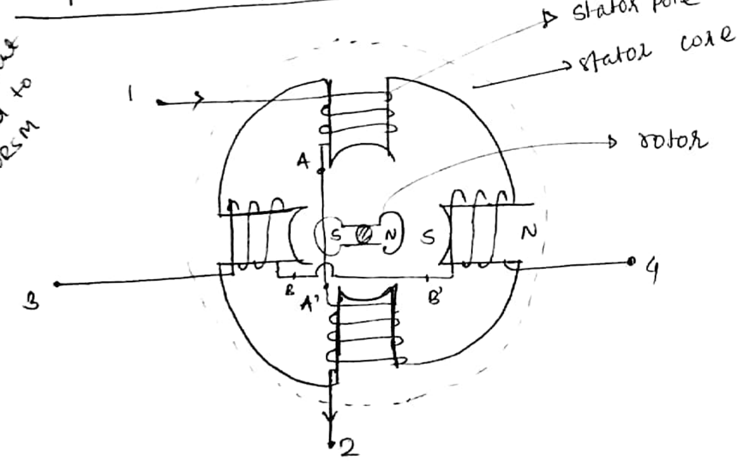
→ These stepper motor has construction of stator similar to V.R.S.M type stepper motor

→ The rotor of the permanent magnet type stepper motor is permanent magnet it self

(imp) The holding torque of permanent magnet type stepper motor is more than that of variable reluctance type motor is more than that of variable reluctance type stator motor it is generally used for low speed high torque applications

2-phase - 4/2 permanent magnet type stepper motor

Low current as compared to VRSM



No of stator pole = 4

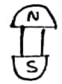
No of rotor pole (m) = 2

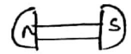
No of phase (N) = 2

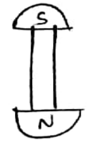
4/2 → no of stator pole, → no of rotor pole

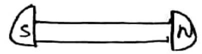
$$\boxed{\text{Step angle } \theta_{PS} = \frac{360^\circ}{N * m}}$$

N → no of phase.
m → no of poles of rotor

$i \neq 0A$ & $i_B = 0A$ [Current flow from A to A']  $\theta = 90^\circ$

$i = 0A$ & $i_B \neq 0A$ [Current from B to B']  $\theta = 180^\circ$

$i_B = 0A$ & $I_A \neq 0A$ [Current flow from A' to A]  $\theta = 270^\circ$

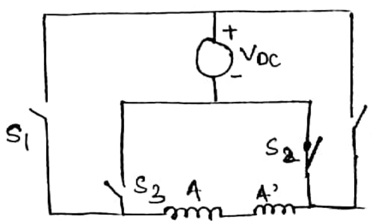
$i_A = 0A$ & $I_B \neq 0A$ [Current flow from B' to B]  $\theta = 360^\circ$

(imp) → Permanent magnet type stepper motor consumes low power compared to variable relactance type stepper motor

Switching sequence

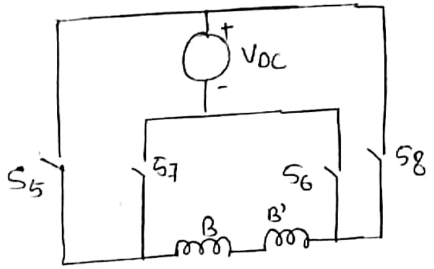
for phase A

if S_1 & S_2 are closed & S_3 & S_4 are opened
A → A'



if S_4 & S_3 are closed
S_2 & S_1 also open.
A' → A

for phase B



for 45°

phase A	$S_1 = 1$	$S_3 = 0$	
	$S_2 = 1$	$S_4 = 0$	
phase B	$S_5 = 0$	$S_7 = 1$	
	$S_6 = 0$	$S_8 = 1$	

S_8	S_7	S_6	S_5	S_4	S_3	S_2	S_1
1	1	0	0	0	0	1	1

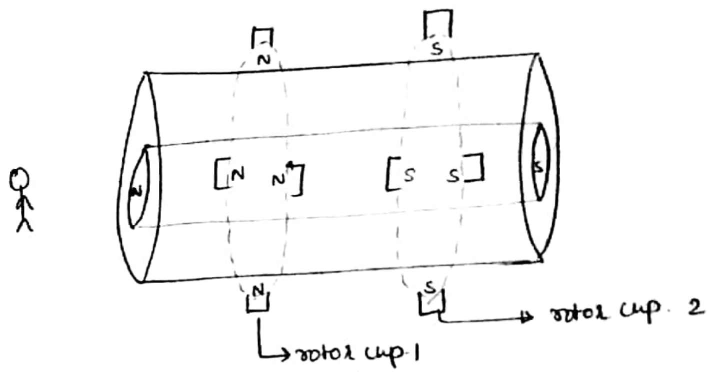
8 bit data

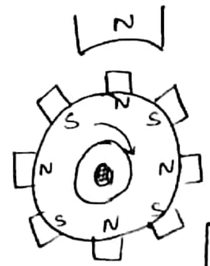
$\underline{\theta = 45°}$

Hybrid type stepper motor.

→ It consists constructional and operational advantages of both variable reluctance type stepper motor as well permanent magnet type stepper motor hybrid type stepper motor is exclusively used fu the applications where we require minimum step angle.

→ The rotor of hybrid type stepper motor consists of a permanent magnet, 2 rotor cups which has teeth made up off magnetic material as shown in the figure





$\theta_{FS} = 45°$

No of teeth rotor cup 1 = 100
No of teeth rotor cup 2 = 100
200

$$\boxed{\theta_s = \frac{360°}{\text{total no of teeth}}} = \frac{360°}{200} = 1.8°$$

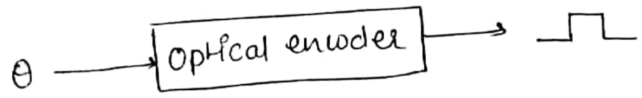
Minimum step angle

Hybrid < Variable reluctance < Permanent magnet

Optical encoder

→ It is digital transducer (sensor), which is used to measure angular position (θ) of the shaft and also used to measure RPM of the shaft

→ Optical encoders are mainly classified into two-types
(i) Incremental encoder
(ii) Absolute encoder



incremental Encoder

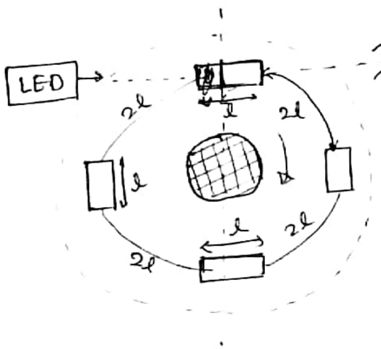
→ In these encoder, ~~a~~ it consist a disc on which equal length holes is separated at equidistance as shown in the figue.

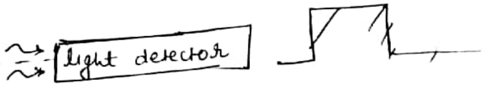
→ incremental encoders are mainly of 2-types, ~~single track~~

(i) single track incremental encoder

(ii) Multi track incremental encoder

Single track incremental encoder

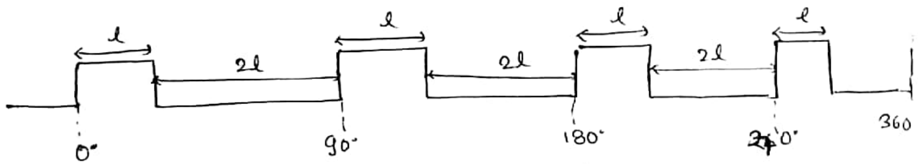




$\boxed{1 \text{ hole} \rightarrow 1 \text{ pulse}}$

4 pulse ⟶ 1 rev

$$\boxed{\text{Resolution} = \frac{360}{\text{no. of hole}}} = \frac{360}{4} = 90^\circ$$



$12l = 360$

$l = \frac{360}{12} = 30^\circ$

Resolution = 90° (or) $3l$

if $l = 1$ cm

Resolution = $3l$ = 3cm

Pulse counter

Note

→ If the incremental encoder consist N holes on the disc then the resolutions equals to

$$\boxed{\text{resolution} = \frac{360^\circ}{\text{no of holes on disc (N)}}}$$

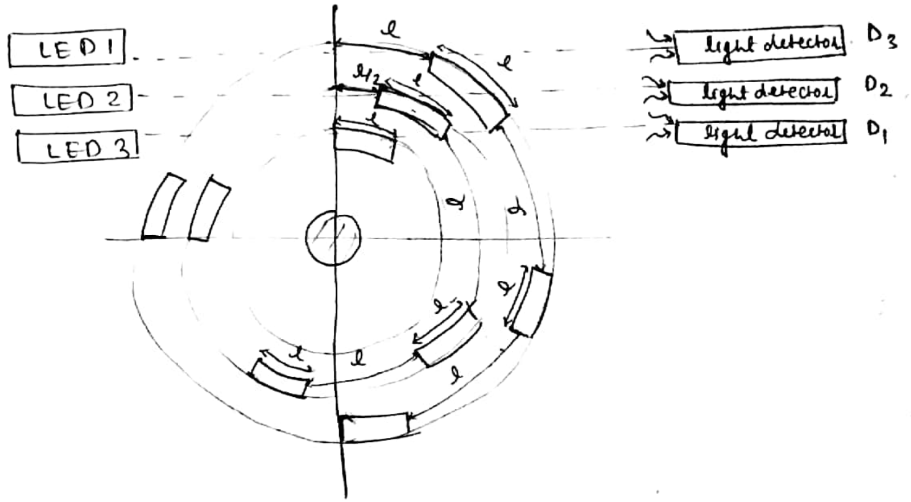
→ By using single track Incremental encoder we can find the find the shaft but we can't indentify the direction of rotation. to find the direction of rotation as well as shaft speed both we generally prefer multi-track incremental encoder.

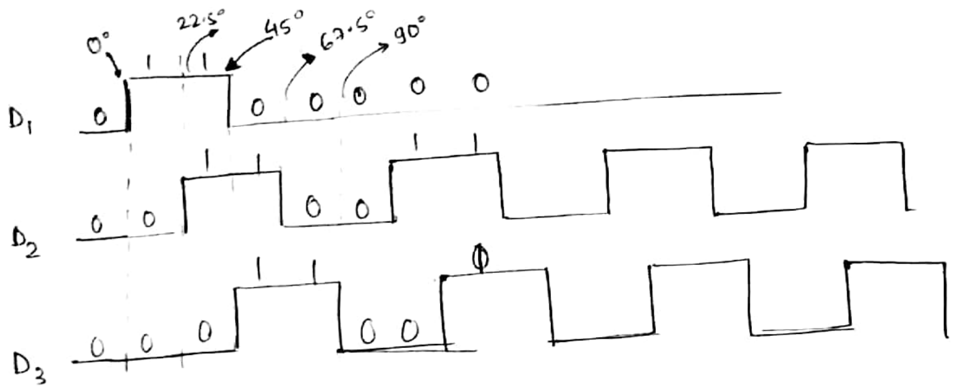
3-track Incremental encoder

Note

→ By using more than one track encoder we can detect both speed of the shaft as well as direction of rotation.

→ To have a good resolution we should have more no. of tracks with appropriate hole location on the disc





$8l = 360°$

$l = 45°$

θ	D_1	D_2	D_3
0°	0	0	0
0 < 22.5°	1	0	0
22.5° < θ < 45°			
45° < θ < 67.5°	1	1	0
67.5° < θ < 90°	0	1	1
90 < θ < 112.5°	0	0	1
112.5° < θ < 135°	0	0	0
	0	1	0
135° < θ < 157.5°	0	1	1

$\boxed{\text{Resolution} = 22.5°}$

Disadvantage

→ By using multi-track encoder we may not able to find exact posn of the shaft (θ) and also continuous observation is required to overcome these problem we generally prefer absolute ~~Enco~~ encoder

Absolute encoder:

→ It provides the information about position of the shaft, more accurately compared to incremental encoders

→ In absolute encoder the information of θ will be converted to ~~bina~~ BCD logic (binary coded decimal logic)

Binary logic
- → single bit binary logic [b_0]
- → Two bit binary logic [b_1, b_0]
- → three bit binary logic [b_2, b_1, b_0]
- ⋮
- → 'N' bit binary logic [$b_{n-1}, b_{n-2} \dots b_0$]

b_{n-1}: most significant binary digit

b_0: least significant binary digit

maximum decimal display $(2^N - 1)$

maximum no of combination (2^N)

(i) single bit binary logic [b_0]

$\boxed{D = b_0 2^0}$

D	b
0	0
1	1

(ii) Two bit binary logic [b_1, b_0]

$\boxed{D = b_1 2^1 + b_0 2^0}$

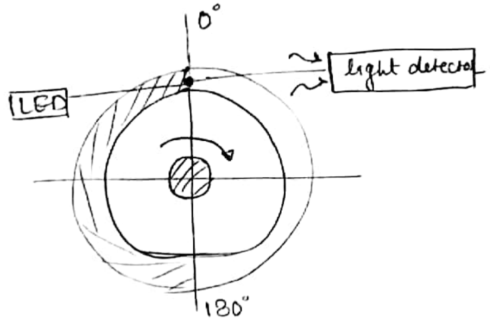
D	b_1	b_0
0	0	0
1	0	1
2	1	0
3	1	1

(iii) Three bit binary logic [b_2, b_1, b_0]

$\boxed{D = b_2 2^2 + b_1 2^1 + b_0 2}$

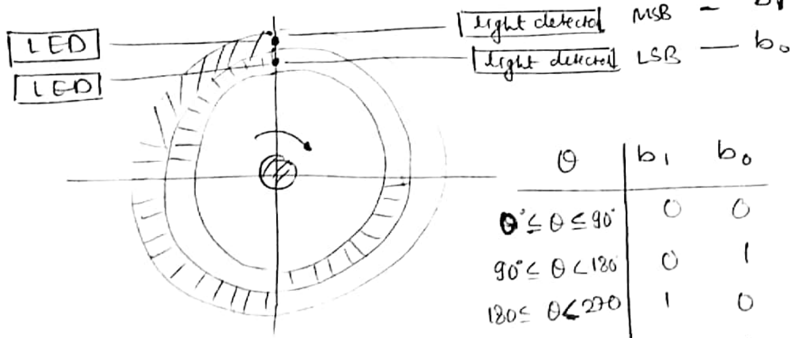
D	b_2	b_1	b_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Single track Absolute encoder



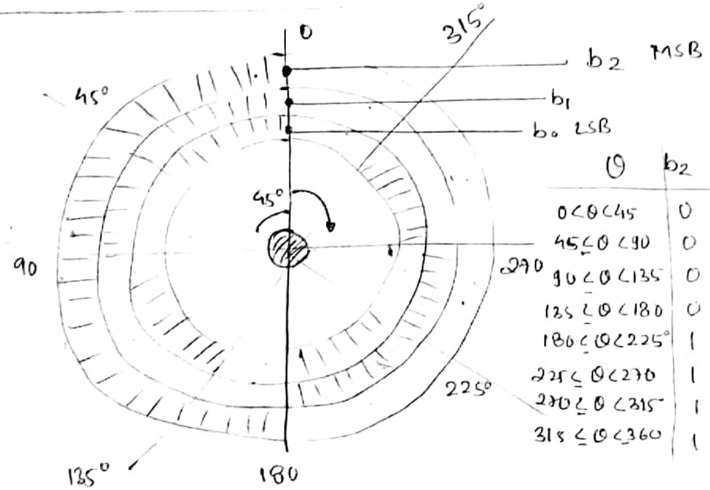
θ	b_0
$0° \le \theta < 180°$	0
$180° \le \theta < 360°$	1

two track Absolute encoder



θ	b_1	b_0
$0° \le \theta \le 90°$	0	0
$90° \le \theta < 180$	0	1
$180 \le \theta < 270$	1	0
$270° \le \theta < 360°$	1	1

Three track Absolute encoder



θ	b_2	b_1	b_0
$0 < \theta < 45$	0	0	0
$45 \le \theta < 90$	0	0	1
$90 < \theta < 135$	0	1	0
$135 \le \theta < 180$	0	1	1
$180 \le \theta < 225°$	1	0	0
$225 \le \theta < 270$	1	0	1
$270 \le \theta < 315$	1	1	0
$315 \le \theta < 360$	1	1	1

Note

→ In absolute encoder, we generally use the coded disc as a result we will get only coded output when the absolute encoded is used for shaft ~~sot~~ position measurement

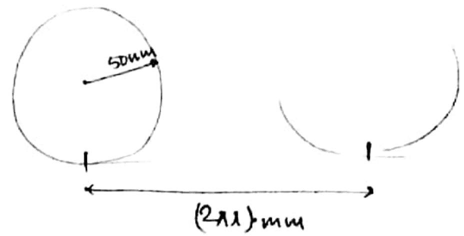
→ If the absolute encoder consists of 'N' tracks (or) 'N' bits then the resolution

$$\boxed{\text{Resolution} = \frac{360°}{2^N}}$$

Q A shaft encoder used with 50 mm radius tracking wheel to moniter the linear displacement if the encoder ~~ten~~ produce 256 pulse per rev. what will be the no. of pulse produced for the linear displacement of 200 mm.

Sol

1 rev



256 pulse/rev

1 rev ⟶ 256 pulses

$2\pi r$ mm ⟶ 256 pulse

314 mm ⟶ 256 pulse

200 mm ⟶ ~~162.97 pulse~~ 162.97 pulse ⟶ 162 pulse

Q A shaft encoder which is attached to a wheel has a ~~sensitity~~ sensitivity of 500 pulse/rev a digital pulse counter is connected to the encoder indicates 5500 pulses in one sec. the speed of the shaft in rpm is ___

Sol

500 — 1

5500 — $\frac{5500}{500} = 11$ rev.

11 rev — 1 sec.

11×60 — 60 sec -

⟹ shaft speed = 660 rpm.

Q. A stepper motor which is rotating at 200 steps per rev. is subjected ~~that~~ with a input pulse rate of 1000 pulse per sec. If a 3-track absolute optical encoder is attached to the shaft, then the binary display just after 1.25 sec is —

Sol:

200 step — 1 rev.

1 pulse — 1 step

200 pulse — 1 Rev.

1000 pulse — 1 sec

1 sec — 1000 pulse — 5 rev.

at $t = 1\text{ sec}^+ \longrightarrow 5\text{ rev}^+ \longrightarrow (5\times 360)^+ \longrightarrow 0^+ \longrightarrow 000$

at $t = 1.25\text{ sec}^+ \longrightarrow (6.25)\text{ rev.}$

$$\left.\begin{matrix} 6\text{ rev} + 0.25\text{ rev} \\ 0 + 90^+ \end{matrix}\right\} 90^+ \longrightarrow \boxed{010}$$

Q. A shaft is rotating at a speed of 600 rpm is connected to a gear box, which has a ratio of 1/10 find the binary display indicated at the output side of absolute encoder just after 0.25 sec is —

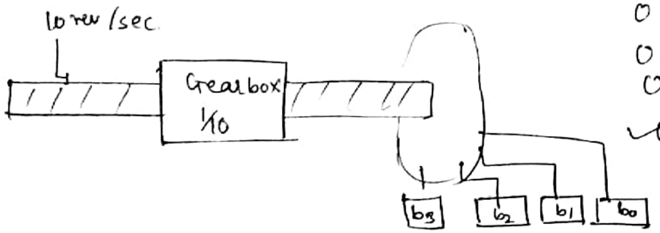
60 rev — 1 min

1 rev — 1 sec

0.25 rev — 0.25 sec

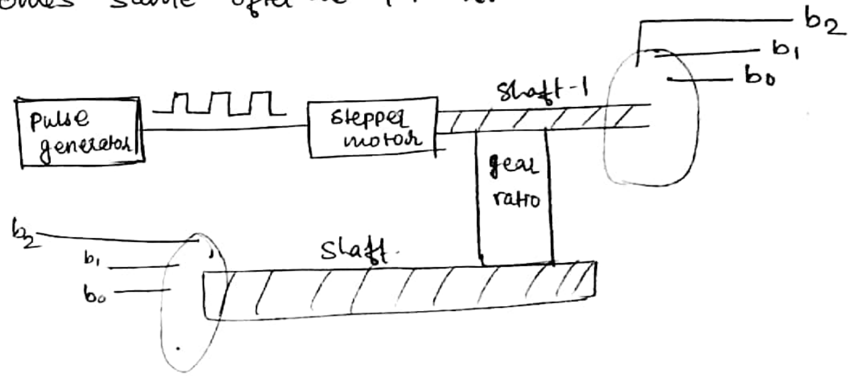
$$\frac{360}{2^4} = \frac{360}{16} = 22.5$$

b_3	b_2	b_1	b_0	θ
0	0	0	0	$0 < \theta < 22.5$
0	0	0	1	$22.5 < \theta < 45$
0	0	1	0	$45 < \theta < 67$
0	0	1	1	$67 < \theta < 90$
0	1	0	0	$90 < \theta < 112.5$



Q. A stepper motor, which has a sensitivity of 200 steps/rev. is supplied with pulses from pulse generator at a rate of 1200 pulse per min. If the stepper motor shaft is connected to 3-track absolute optical encoder and shaft two gear of ratio = 1/6 as shown in the fig. shaft two is passed through gear and connected to 2nd 3-track absolute encoder as shown find the min. time required for which the binary display of both the encoder becomes same after the 1st sec.

Sol:



1 rps   60 rpm

1/6 rps   10 rpm

000

Shaft-1

$1\text{ sec}^+ \longrightarrow 1\text{ rev}^+ \longrightarrow 360^+ \longrightarrow 000$

$1\text{ sec} + \frac{1}{8}\text{ sec} = 1.125\text{ sec} \longrightarrow 360^\circ + 45^+ \longrightarrow \boxed{001}$

1 sec

Shaft-2

$1\text{ sec} \longrightarrow \frac{1}{6}\text{ rev} \longrightarrow 60^\circ$

at $t = 1.125\text{ sec} \longrightarrow 67.5^\circ \longrightarrow \boxed{001}$

From 1.125 to 1.25 the display will be same

after 1st sec so, Ans will be at 0.125 sec

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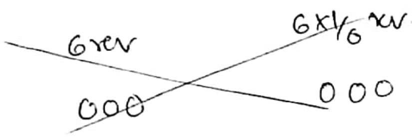
	Shaft 1	Shaft 2
t= 1sec	1rev +	1/6 rev +
	360 +	60 +
	0° = 000	001

t = 6sec



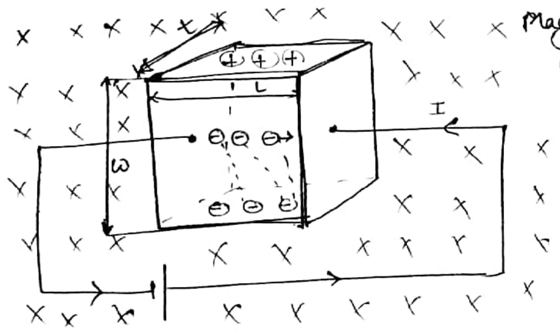
Hall Sensor

→ hall sensers works on the principle hall-effect

Hall- Effect

→ if a conductor or semiconductor which carries the current in one perticular direction, if it is placed in perpendicular magnetic field then voltage will be generated on the surface of the conductor

→ Hall effect can be observed in both conductors as well as semiconductors.



Magnetic field into the board

(i) Current density (J)

$$J = \frac{I}{A} = \frac{I}{w \times t} = \frac{Ne \times l}{\underbrace{w \times t \times l}_{V} \times \text{time}}$$

drift velocity

$$I = \frac{dq}{dt} = \frac{Q}{\text{time}} = \frac{Ne}{\text{time}}$$

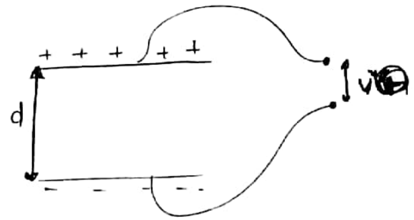
$$\boxed{J = \frac{N}{V} e v_d = \frac{I}{A} = n e v_d} \quad \text{—— (1)}$$

(ii) force acting on charge particle

$$\vec{F_B} = q(\vec{v_d} \times \vec{B})$$

$$F_B = q v_d B = F_B = e v_d B \quad \text{—— (2)}$$

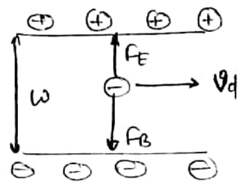
(iii) Force acting on charge particle because of electric field



$$E = \frac{V_H}{d}$$

$$F_E = qE = q\left[\frac{V_H}{d}\right] \quad \text{—— (3)}$$

(iv) At steady state



$$F_E = F_B$$

$$e v_d B = e \frac{V_H}{w}$$

$$V_H = w v_d B$$

$$V_H = w\left[\frac{I}{A n e}\right] B$$

$$= w\left[\frac{I}{w \times t \times n \times e}\right] B$$

$$V_H = \frac{1}{ne} \cdot \frac{IB}{t}$$

K_H = hall coefficient

V_H = hall voltage (volt)

$$\boxed{V_H = k_H \cdot \frac{IB}{t}}$$

Note

→ In hall sensor intentionaly we will apply magnetic field perpendicular to the direction of current

→ The developed hall-voltage will be always ⊥ to the both applied current and magnetic field density

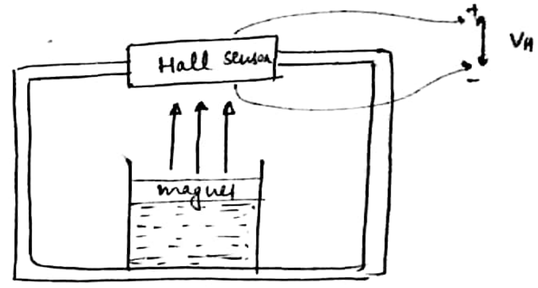
→ (Imp) It has no. of applications (i) wt measurement
(ii) shaft speed measurment
(iii) water level

Q A unit magnetic field of 0.1 W/m² is passed through a current carrying conductor in perpendicular direction produces a voltage of 10 V on the surface of sensor if the thickness of conductor is 0.1 m and the magnitude of current is 10A., then the magnitude of hall coefficient of conductor is ____

$$10 = K_H \times \frac{10 \times 0.1}{0.1}$$

$$K_H = 1 \left(\frac{V \cdot m^3}{A \cdot Wb}\right)$$

Q For the liquid level measurement hall sensor setup is used as shown below the sensor carries a current of 2A perpendicular to the magnetic field the magnetic field associated with the sensor changes with the liquid level as $B(h) = 0.2h + 0.1$ Wb/m² if output voltage of hall sensor is ⊥ to both applied magnetic field and current then find the change in output voltage when the water level in the tank increased from 1m to 3m. given that thickness of sensor is 0.1 m and hall coefficient is 1 unit.



$$B(h) = 0.2h + 1$$

$$V_H = K_H \frac{I\, B(h)}{t}$$

$$V_H = 20\, B(h)$$

at $h = 0m$ (tank is empty)

$$V_H(0) = 20\, B(0)$$

$$(V_H)_3 = 2V \longrightarrow \text{Offset}$$

at $h = 1m$

$$V_H(1) = 20\, B(1) = 20 \times 0.3 = 6V$$

at $h = 3m$

$$V_H(3) = 20\, B(3) = 20 \times 0.7 = 14V$$

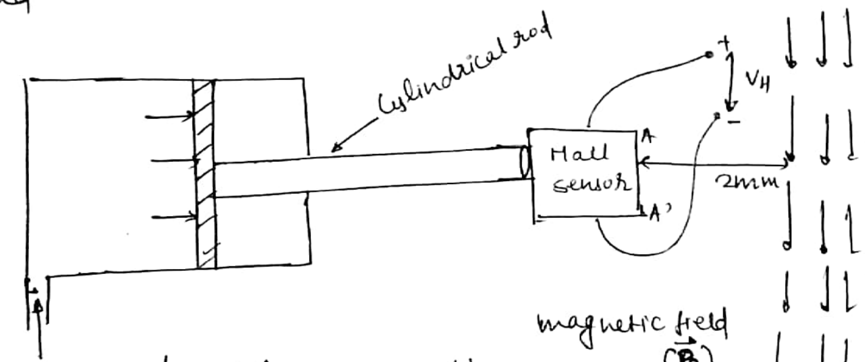
$$\Delta V_H = 14V - 6V$$

$$\Delta V_H = 8V$$

Note:

$Y = mx$, $Y = mx + C$, both are straight lines but 2nd relation won't follow principle of homogeneity because of these $y = mx + C$ is treated as non-linear relation for a given system.

Q Consider the following fig. where the gap between the cylinder and hall sensor is assumed to be zero. If the cylindrical rod mechanism has a sensitivity of 1mm/10N then find the minimum input pressure of the oil which should act on piston sheet to generate output voltage. The piston diameter of piston is 100mm and the hall-sensor carries a const current of 2A ⊥r to magnetic field as shown in the figure.



to get output voltage

$x = 2mm$

$10N - 1mm$

Required force $= 20N - 2mm$

$$P_{in} = \frac{F}{\pi/4 \times d^2} = \frac{20}{\pi/4 \times (0.1)^2}$$

$$= 2.56\,kPa$$

Q In the above setup if volumetric flow rate of oil which inters into cylindrical chamber is 10ml/sec then find the min time required after which voltage will be generated is

Sol

$1\,l = 10^{-3}\,m^3$

$1000\,ml = 10^{-3}\,m^3$

$1\,ml = 10^{-6}\,m^3$

1.57 sec

$1\,ml - 10^{-6}\,m^3$

$10\,ml = 10^{-5}\,m^3 -$ 1sec.

$\cancel{2\times10^{-3}\,m^3} -$ $\frac{2\times10^{-3}}{10^{-5}}$

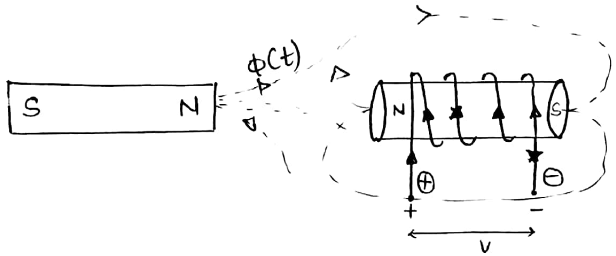
$$\text{Speed} = V = \frac{10^{-5}}{A} = \frac{10^{-5}}{\pi/4 \times (0.1)^2} = 1.2732 \times 10^{-3}\,m \;-\; 1sec$$

$2\times10^{-3}\,m - \underline{1.57\,sec}$

---X---

Electromagnetic Induction

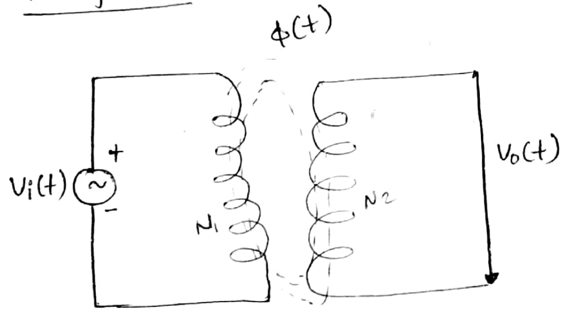
If the magnetic flux associated with the coil change with time then voltage will be introduced in coil and ~~they~~ then voltage is called induced emf. and magnitude of induced emf is equaly to rate of flux.



The magnitude of output voltage

$$\boxed{V = \frac{d\phi(t)}{dt}}$$

Transformer



$$V_i(t) = V_i \sin \omega t$$

$$V_o(t) = V_o \sin(\omega t - \phi)$$

$$\frac{V_o}{V_i} = \frac{N_2}{N_1} = K = \text{transformation ratio}$$

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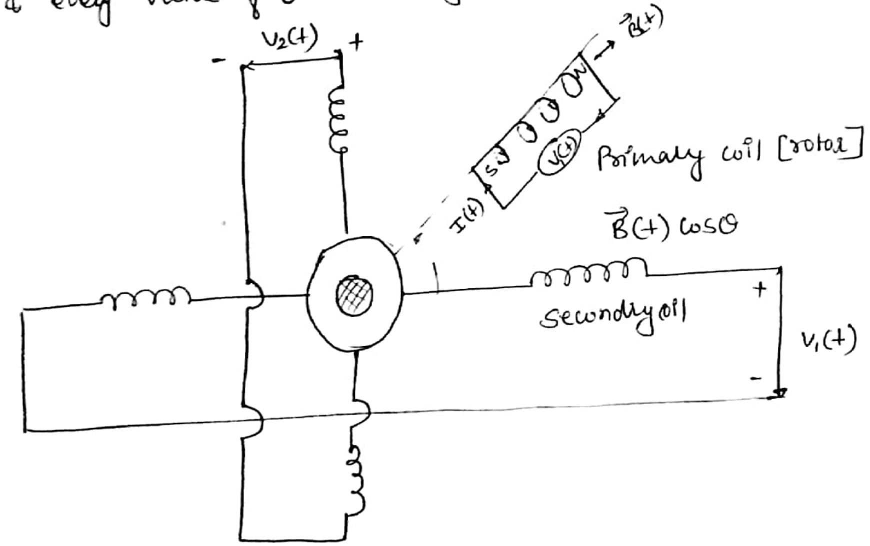
if $N_2 < N_1$; $K < 1$ $V_0 < V_1$ step-down tran

if $N_2 > N_1$ $K > 1$ $V_0 > V_1$ step-Up

Resolver

- It is a position sensor, which is use to measure the angular position of the shaft (θ), angular velocity
- Resolver operates on Electromagnetic Induction principle (Mutual Inductance variation)
- The Resolution of resolver is very much better than optical encoder but the only problem it generates analogue voltage which may require ADC (Analoge to digital converter) in latter stages
- It consists of a primarycoil which acts like a rotor and two secondry coil (4-windings) which acts like stator
- In resolver the input voltage applied to the primary coil will be resolved into 2 component, thereby for each a every value of θ we will get two component of output.



$V_1(t) = K V_i(t) \cos\theta$

$V_1(t) = K V_i \sin\omega t \cos\theta$

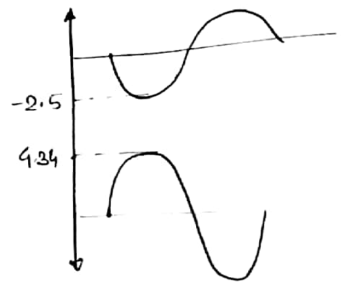
$V_1(t, \theta) = (K V_i \cos\theta) \sin\omega t \Rightarrow$ Amplitude of $V_1(t) = V_1 = K V_i \cos\theta$

$V_2(t) = K V_i(t) \sin\theta$

$V_2(t, \theta) = K V_i \sin\theta \sin\omega t$

Amplitude of $V_2(t) = V_2 = K V_i \sin\theta$

Cathod Ray oscillator



$K = 0.5$, $V_i(t) = 10\sin\omega t$ $\theta = 120$

$V_1(t, \theta) = \left(0.5 \times 10 \times \frac{1}{2}\right) \sin\omega t$

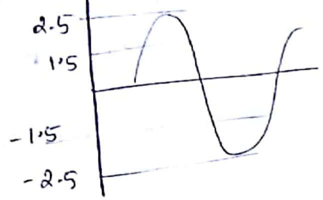
$= -2.5 \sin\omega t$

$V_2(t, \theta) = \left(0.5 \times 10 \times \frac{\sqrt{3}}{2}\right) \sin\omega t$

$= 4.34 \sin\omega t$

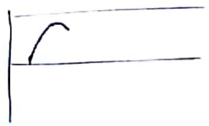
Q A resolver, which is use to measure angular position of the shaft θ generates 2 components of output. the cosine output of resolver is connected to CRO (wave form analyser) which display the output as shown in the figue given that transformation ratio $K = 0.5$ and input voltage is $V_i(t) = 10\sin\omega t$ and frequency of the supply is very high compared to the shaft speed then the angular position of the shaft is a

a) 60° b) 30° c) 90° d) 120°



In the application, where we use resolver we should ensure that frequency of the supply should be more than shaft speed. Otherwise we will loose the information of θ

$$\boxed{(fr.)_{shaft} << (f)_{wave}}$$



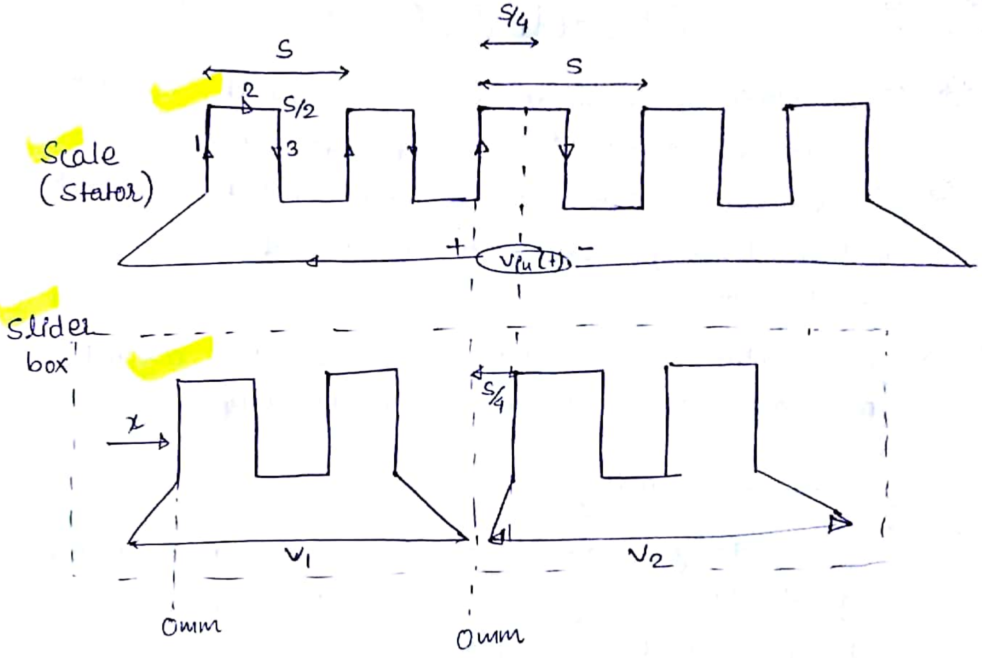
Inducto syn

→ It is a position sensor, which is used to measure linear displacement x as well as angular displacement θ. Inducto syn operates on the principal of electromagnetic induction and it works similar to resolver the resolution and sensitivity of inducto syn is even better than resolver

→ linear inducto syn generally has 2 parts

(i) stator (scale) it has printed conductive material in regular rectangular shape as shown.

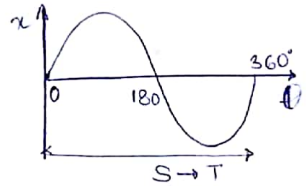
(ii) Slider : It also has printed conductive ~~trays~~ trace and exposed to material, scale of inducto syn.



$$V_1(t,\theta) = kV_{in}(t)\cos\theta$$
$$= kV_{in}(t)\cos\omega t$$
$$= kV_{in}(t)\cos(2\pi f t)$$
$$= kV_{in}(t)\cos\left(2\pi \frac{t}{T}\right)$$

x	V_1
0mm	Vmax
$\frac{S}{2}$ mm	−Vmax
S mm	Vmax
1.5 Smax	−Vmax

$$V_1(t,x) = kV_{in}(t)\cos\left(2\pi \frac{x}{S}\right)$$
$$V_2(t,x) = kV_{in}(t)\sin\left(2\pi \frac{x}{S}\right)$$



$$\boxed{\begin{aligned} S &= 360^\circ \\ S/4 &= 90^\circ \end{aligned}}$$

T → S → 360°
t → x → θ

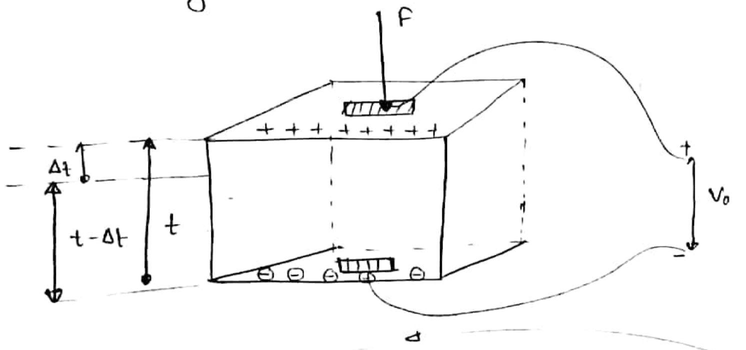
$$\boxed{\frac{t}{T} = \frac{x}{S} = \frac{\theta}{360^\circ}}$$

Piezoelectric Accelerometer

- → Piezoelectric accelerometer is a sensor which is use to measure accelaration of a moving object.
- → It generates output voltage and amplitude of output voltage linearly depends on variation of the input accelaration.
- → Piezoelectric accelerometer has piezoelectric crystal which operates on piezoelectric property.

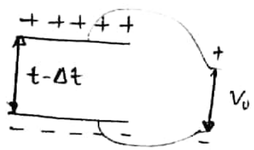
Piezoelectric property

→ If we apply mechanical stress on piezoelectric crystal because of the deformation of crystal charge will be developed on the surface of the crystal. the developed charge will be converted to voltage with the help of capacitance of the crystal.



$$F \longrightarrow F/A \longrightarrow \Delta t \longrightarrow q \longrightarrow V$$

↑ Polarization



$q \propto F$; another relation ($q \propto \Delta t$)

$q = dF$ —— ①

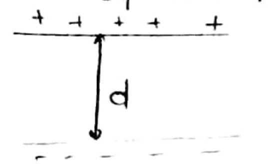
d → Charge sensitivity (C/N)

$q = CV_0$ —— ②

$CV_0 = dF$

$V_0 = \left(\frac{d}{C}\right) F$

Capacitor concept



$q = CV_c$

$C = \frac{\epsilon A}{d}$

if $t + \Delta t \simeq t$

$V_0 = \left(\frac{d}{\frac{\epsilon A}{t}}\right) \times F$

$V_0 = \frac{d}{\epsilon} \times t \times \frac{F}{A}$

d → Charge sensitivity

$$\boxed{\frac{\left(\frac{V_0}{t}\right)}{\left(\frac{F}{A}\right)} = \frac{d}{\epsilon} = g} = \text{sensitivity of piezoelectric crystal}$$

Q A piezoelectric crystal which has Youngs modulus $E = 90\,GPa$ has a diameter of 10 mm and thickness of 2 mm if voltage sensitivity is 4500 V/μm and output voltage generated is 127.3 V then the applied load is

1 μm — 4500 V

1×10^{-6} m — 4500 V

$\frac{127.3 \times 10^{-6}}{4500}$ — 127.3 V

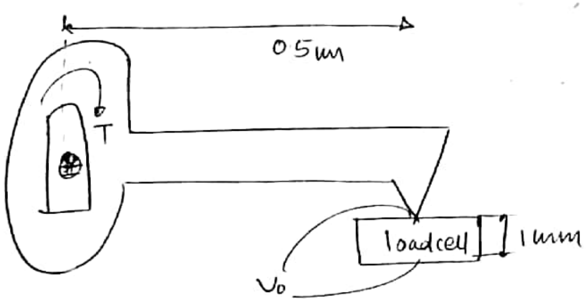
$= 0.028 \times 10^{-6}$ m. ← Δt

$$E = \frac{F/A}{\Delta t/t}$$

$$90\times10^{9} \times \frac{0.028\times10^{-6}}{2\times10^{-3}} \times \frac{\pi}{4} \times 10^{2}\times10^{-6} = F$$

$$F \approx \cancel{1000}\ 100N$$

Q A dynamometer wring makes contact with piezoelectric load cell as shown the ~~g~~ g const of the piezoelectric material is $50\times10^{-3}\left(\frac{Vm}{N}\right)$ and the surface area of the load cell is $4cm^2$ if a torque of 20 Nm is is applied to the dynamometer as shown in the figure then the output voltage V_o accross the load is ____ volt



torque sensor

$$F = \frac{20}{0.5} = 40N$$

$$50\times10^{-3} = \frac{\frac{V_o}{1\times10^{-3}}}{\frac{40}{4\times10^{-4}}}$$

$$50\times\cancel{10^{-3}} \times \frac{\overset{10}{\cancel{40}}}{\cancel{4\times10^{-4}}} \times \cancel{10^{-3}} = V_o$$

$$V_o = 5V$$

Q A piezoelectric crystal with dimension 6mm×6mm×2mm the voltage sensitivity of crystal is $0.65\left(\frac{VM}{N}\right)$ used for force measurement find the amplitude of applied force if the voltage developed is 300V

Sol

$$\frac{\frac{V_o}{t}}{\frac{F}{A}} = g$$

$$\frac{300 \times 6\times6\times10^{-6}}{2\times10^{-3}\times F} = 0.65$$

$$\frac{\overset{30}{\cancel{300}}\times6\times6\times10^{-6}}{2\times10^{-3}\times \underset{0.13}{\cancel{0.65}}} = F$$

$$F = 83N$$

Q A quartz crystal of dimension 10mm×10mm×1mm is subjected to a deformation of $10^{-8}\sin 100t$ m then find the amplitude of voltage ~~dg~~ generated if the charge sensitivity is 2×10^{-12} C/N and $E = 86\times10^{10}$ (N/m²) and permitivity $\varepsilon = 42\times10^{-12}$ F/m.

Sol

$$\frac{F}{A} = \frac{\Delta t}{t}\times E$$

$$F = \frac{10\times10\times10^{-6}\times10^{-8}\times\sin 100t}{1\times10^{-3}}\times 8.6\times10^{10}$$

$$\frac{\frac{V_o}{t}}{\frac{F}{A}} = \frac{d}{\varepsilon}$$

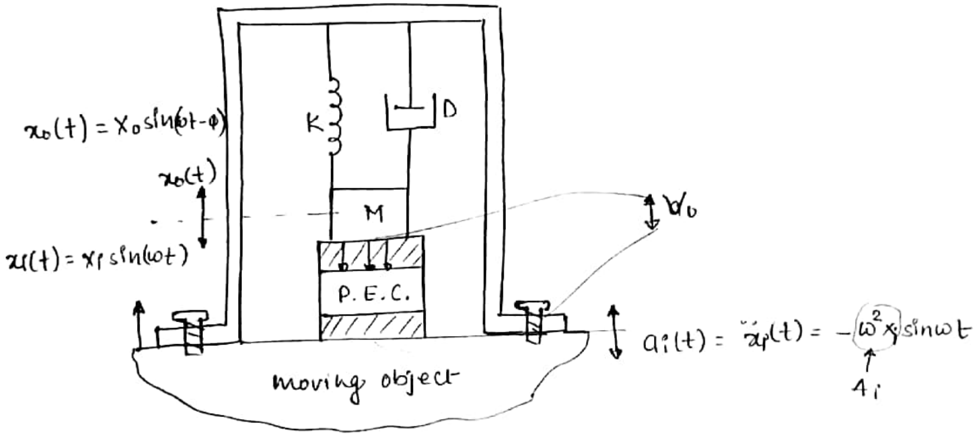
$$V_o = t\times\frac{d}{\varepsilon}\times\frac{F}{A}$$

$$= \cancel{t}\times\frac{d}{\varepsilon}\times\frac{\Delta t}{\cancel{t}}\times E$$

$$V_o = \frac{2\times10^{-12}\times10^{-8}\times86\times10^{10}}{42\times10^{-12}} = 40.95V$$

Accelerometer

- In general accelerometer works on inertial affects of the mass.
- every accelerometer consists of mass supported by spring and damper if the mass damper spring setup is attached to any moving object then their exist a relative displacement in the mass w.r.t. moving object.
- The relative displacement of the mass will be converted to voltage by piezoelectric crystal which is shown in the figure.



From mass damper spring system

$$M \cdot \frac{d^2 x_o(t)}{dt^2} + D\frac{d x_o(t)}{dt} + K x_o(t) = M a_i(t)$$

at steady state

$$x_o(t) = \frac{M}{K} a_i(t)$$

Natural frequency of the system

$$\omega_n = \sqrt{\frac{K}{m}} \quad \left(\frac{rad}{sec}\right)$$

$$x_o(t) = \frac{1}{\omega_n^2} a_i(t)$$

$$x_o(t) = X_o \sin(\omega t - \phi)$$

$$\frac{1}{\omega_n^2} \cdot A_i \sin\omega t = X_o \sin(\omega t - \phi)$$

$$\boxed{X_o = \frac{1}{\omega_n^2} A_i}$$

from Piezoelectric crystal

$$q \propto \Delta t$$

$$q \propto X_o$$

$$\Rightarrow q = k' x_o$$

$$q = k'\left[\frac{1}{\omega_n^2} \cdot A_i\right]$$

$$q = \frac{k'}{\omega_n^2} \cdot A_i$$

$$q = C V_o$$

$$C V_o = \frac{k'}{\omega_n^2} \cdot A_i$$

$$\boxed{V_o = \frac{k'}{C\omega_n^2} \cdot A_i}$$

amplitude of voltage → $\boxed{V_o \propto A_i}$

~~Q which one of the following statement is~~

Note:

Piezoelectric crystal can't be used for low frequency input and the frequency of operation should be as high as possible for all it's applications

Q. which one of the following statement is correct. accelerometer working in displacement mode (displacement pickup) should have.

a) weak spring and heavy mass

b) stiff spring and light mass

c) weak spring and light mass

d) stiff spring and heavy mass

$$x_0 = \left(\frac{\omega}{\omega_n}\right)^2 \cdot x_i$$

$\omega_n \downarrow$ $\quad \omega_n = \sqrt{\frac{K}{m}}$ $\quad \left\{ \begin{matrix} K \downarrow \\ m \uparrow \end{matrix} \right\}$

Q ideally accelerometer should have ____ string and ____ mass

Q Piezoelectric accelometer should have ____ string and ____ mass.

x

Control System

Group of components connected together and working together to perform a specific task ~~designed~~ desired task forms control system.

→ Automatic control system is mainly classified into 2 types

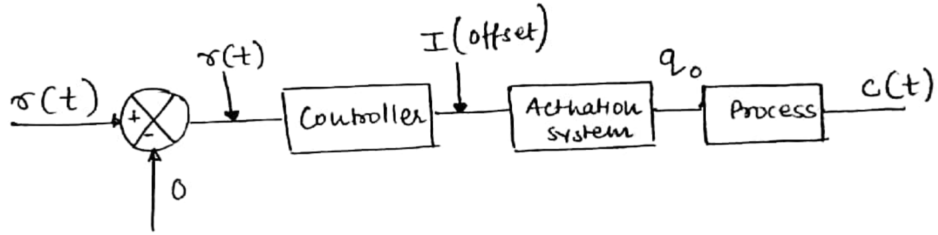
(i) Open loop control system

(ii) Closed loop control system.

→ $\boxed{\text{Open loop control system.}}$

In these control system controller doesn't get information of the process variable as it doesn't consists feedback.

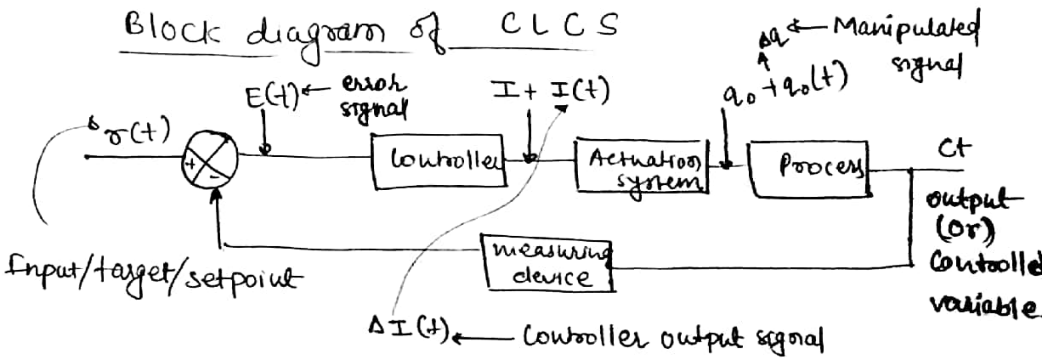
Block diagram of O.L.C.S

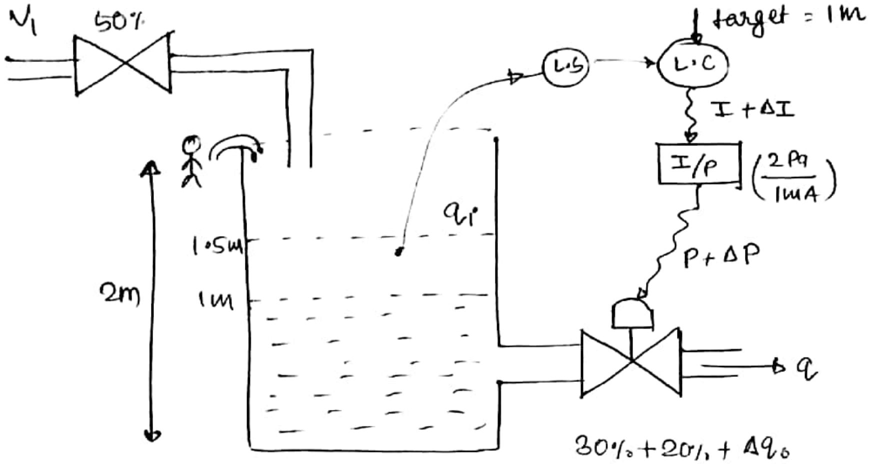


$\boxed{\text{Closed loop control system}}$

In these control system controller gets the information of process variable time to time as it has feedback mechanism and feedback may be sensor or transducer or transmittor

Block diagram of CLCS





→ In open loop control system as well as closed loop control system ideally c(t) should be equal to r(t) at steady state.

→ Open loop control system is very much accurate in the absense of disturbance and if the calibration is perfect but closed loop control system is accurate in the absense of disturbance as well as in the presense of disturbance.

→ Open loop control system can't perform regulatory mechanism as it doesn't consists feedback ~~back~~ C.L.C.S. can perform servo mechanism as well as regulatory mechanism very efficiently if the sensor is accurate

→ In open loop control system as well as closed loop control system we should develop a program with the help of mathematical expression of the physical system.

Mathematical Modeling

→ The process of developing mathematical expression to a physical system is called modeling.

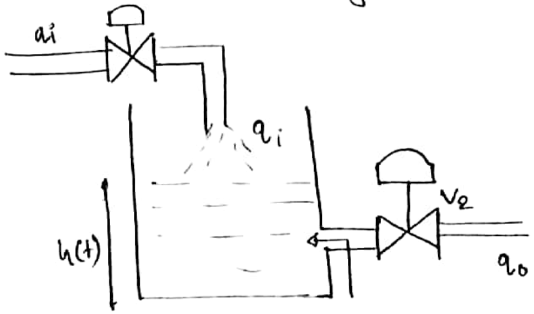
→ Apply conservation of energy or force or flow rate or voltage or current depending upon the nature of the system
step 1

→ Rearrange the step 1 and find state diff equation if possible
step 2

→ Apply the concepts of diff. equation or laplace transform, numerical method to find the variable w.r.t. time
step 3

→ The accuracy of control system and controller design depends on the accuracy of the modeling.

<i> Find the value of water level in the tank w.r.t time in the system shown below.



input flow rate − output flow rate = Accumulation of water in tank

$$q_i(t) - q_o(t) = \frac{A\,dh(t)}{dt}$$

$$q_i(t) - \frac{h(t)}{R} = \frac{A\,dh(t)}{dt}$$

R → resistance at V_2

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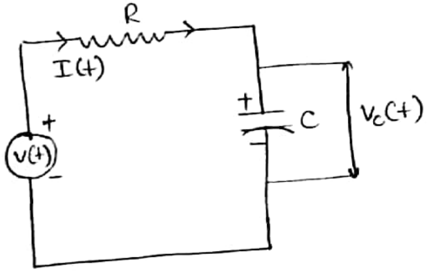
$$\boxed{\frac{dh(t)}{dt} + \frac{1}{AR}h(t) = \frac{1}{A}q_i(t)}$$ Ord. diff eqn.

method to solve
i) D.E
ii) Laplace
iii) Numerical method

(ii) Develop the voltage across capacitor w.r.t. time by using modeling approach.



Input voltage - Output voltage = Accumulation

$V_i(t) - V_c(t) = V_R(t)$

$V_i(t) - V_c(t) = RI(t)$

$I(t) = I_R(t) = I_c(t)$

$V_i(t) - V_c(t) = RI_c(t)$

$V_i(t) - V_c(t) = RC\frac{dV_c(t)}{dt}$

$q(t) = C\,V_c(t)$

$I_c(t) = \frac{dq(t)}{dt} = \frac{C\,dV_c(t)}{dt}$

$$\boxed{\frac{dV(t)}{dt} + \frac{1}{RC}V_c(t) = \frac{1}{RC}V_i(t)}$$

DE, L.T. NM

(iii)

State-Space Representation

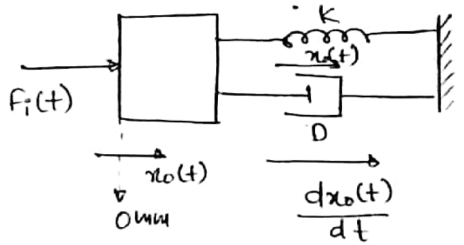
(iii)

→ Representing mathematical model (diff. eqn) for matrix form is the one of the main purpose of state-space representation.

→ Every physical system can be modelled as diff. eqn and every diff. eqn can be converted into matrix representation (state space representation)

→ The no. of state variables of a physical system is equals to order of the system.

(iii) Develop the mathematical model of mass damper spring system and convert the model into state space representation (matrix form)



$x_0(t) \rightarrow$ state variable 1 $\rightarrow P_1(t)$

$\frac{dx_0(t)}{dt} \rightarrow$ state variable 2 $\rightarrow P_2(t)$

$$M\frac{d^2x_0(t)}{dt} = F_i(t) - D\frac{dx_0(t)}{dt} - Kx_0(t)$$

$$\frac{d^2x_0(t)}{dt^2} + \frac{D}{M}\frac{dx_0(t)}{dt} + \frac{K}{M}x_0(t) = \frac{1}{M}F_i(t)$$

$x_0(t) \rightarrow P_1(t)$

$\frac{dx_0(t)}{dt} \rightarrow P_2(t) = \dot{P}_1(t)$

$\frac{d^2x_0(t)}{dt^2} \rightarrow$ $\dot{P}_2(t) + \frac{D}{M}P_2(t) + \frac{K}{M}P_1(t) = \frac{1}{M}F_i(t)$

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$$\dot{P}_1(t) = 0P_1(t) + 1\cdot P_2(t) + 0\,F_i(t)$$

$$\dot{P}_2(t) = \frac{-k}{M}P_1(t) - \frac{D}{M}P_2(t) + \frac{1}{M}F_i(t)$$

$$x_0(t) = 1\cdot P_1(t) + 0\cdot P_2(t) + 0\cdot F_i(t)$$

$$\begin{bmatrix}\dot{P}_1(t)\\ \dot{P}_2(t)\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ \frac{-k}{M} & \frac{-D}{M}\end{bmatrix}_{2\times2}}_{\text{System Matrix }[A]}\underbrace{\begin{bmatrix}P_1(t)\\ P_2(t)\end{bmatrix}}_{\text{State variable Matrix}} + \underbrace{\begin{bmatrix}0\\ 1/M\end{bmatrix}}_{\text{Input matrix }[B]}F_i(t)$$

$$[x_0(t)] = \underbrace{[1\ \ 0]}_{\text{Output Matrix }[C]}\begin{bmatrix}P_1(t)\\ P_2(t)\end{bmatrix} + [0]\,F_i(t)$$

Q A physical system is mathematically modeled as 3rd order diff. equation

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$x(t)$: input of system
$y(t)$: output of the system

Convert the above model into matrix representation

Sol

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$y(t) \rightarrow P_1(t)$

$\frac{dy(t)}{dt} \rightarrow P_2(t) = \dot{P}_1(t)$

$\frac{d^2y(t)}{dt^2} \rightarrow P_3(t) = \dot{P}_2(t)$

$\frac{d^3y(t)}{dt^3} \rightarrow \dot{P}_3(t) + 4P_3(t) + 3P_2(t) + 2P_1(t) = x(t)$

$$\begin{bmatrix}\dot{P}_1(t)\\ \dot{P}_2(t)\\ \dot{P}_3(t)\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -3 & -4\end{bmatrix}_{3\times3}\begin{bmatrix}P_1(t)\\ P_2(t)\\ P_3(t)\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}}_{\text{output vector}}x(t)$$

$$y(t) = [1\ \ 0\ \ 0]\begin{bmatrix}P_1(t)\\ P_2(t)\\ P_3(t)\end{bmatrix}$$

Shortcut

shift ~~to~~ one unit
1 above max
first ~~coe~~ coeff. from right negative of earlier

Controllability

→ If the status of the system is changed from one value to another or one position to another position in the finite time with the finite inputs then we can say system is controllable.

→ for a controllable systems only we can design controller.

→ To check controllability consider controllability matrix

$$\boxed{[\phi_c]_{2\times2} = [A^0B \qquad AB]}$$

for 3rd order system

$$\boxed{[\phi_c] = [\, A^0B \quad A^1B \quad A^2B \,]_{3\times3}}$$

if $|\phi_c| = 0$ then the system is uncontrollable

$|\phi| \neq 0$ then " " controllable

Observability

→ If we can calculate the state variables of the system at any perticular time from the output of the system at that perticular time then we can say the system is observable.

→ In mass-damper arrangement if we get $x(t)$ function then the system is observable

To check observability

Observability Matrix $= [\phi_o] = [\, A^0C^T \quad AC^T \,]$

$= [\, A^0C^T \quad AC^T \quad A^2C^T \,]$

if $|\phi_o| = 0$ then the s/m is non-observable

$|\phi_o| \neq 0$ " " " " observable

EC-2015 A system is represented in state space model with the system matrix $[A] = \begin{bmatrix} 1 & 2 \\ \alpha & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ the value of α for which the system is uncontrollable is

Sol

$|\phi_c| = [\, B \quad AB \,]$

$|\phi_c| = 0$

$= \begin{vmatrix} 1 & 3 \\ 1 & \alpha+6 \end{vmatrix} = 0$

$\Rightarrow \alpha + 6 - 3 = 0$

$\Rightarrow \alpha = -3$

EC-13 The state space representation of a third order system is given by

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$Y(t) = [\, 1 \quad 0 \quad 0 \,] \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

where $u(t)$ is input to the system $Y(t)$ is output of the system. then the system is controlled for which of the following condition

$\Phi_c = [B \; AB \; A^2B]$

$A^2 = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ a_2 a_3 & 0 & 0 \\ 0 & a_3 a_1 & 0 \end{bmatrix}$

$|\Phi_c| \neq 0$

$\begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{vmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$

$-a_1 a_2^2 \neq 0$

$a_1 \neq 0, a_2 \neq 0 \quad a_3 = 0$

EC-09 2M

Q the state - space representation of s/m matrix

$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C \begin{bmatrix} 1 & 1 \end{bmatrix}$

$\Phi_c = [B \quad AB]$

$= \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $\quad \Phi_c = 0$ uncontrollable.

$\Phi_o = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$

$\Phi_o \neq 0 \quad (-2+1) = \underline{\underline{-1}}$

Observable.

Consider a 2^{nd} order system which has state space representation in the form

$$[\dot{P}(t)]_{2\times1} = [A]_{2\times2} [P(t)] + [B]_{2\times1} U(t)$$

If suppose $P_1(t) = P_2(t)$ then the system is

Sol

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} U(t) \quad —①$$

$\dot{P}_1(t) = a_{11} P_1(t) + a_{12} P_2(t) + b_{11} U(t)$

$\dot{P}_2(t) = a_{21} P_1(t) + a_{22} P_2(t) + b_{21} U(t)$

$\underline{P_1(t) = P_2(t)}$

$\dot{P}_1(t) = a_{11} P_1(t) + a_{12} P_1(t) + b_{11} U(t)$

$\dot{P}_1(t) = a_{11} P_1(t) + a_{22} P_1(t) + b_{21} U(t)$

$\left\{ \begin{array}{l} P_1(t) = P_2(t) \\ P_1(t) = \dot{P}_2(t) \end{array} \right\}$

$[\Phi_c] = [B \quad AB]$

assign temporary value in ① & get the answer

here the system is $\underline{\text{uncontrollable}}$.

Actuators

Hydraulic actuators

→ These actuators converts liquid pressure energy into mechanical power (mechanical energy)

→ The amount of output mechanical power depends on the pressure applied at piston and volumetric flow rate of the liquid

→ Hydraulic actuator are mainly classified into two types
  (i) hydraulic cylinders (linear actuators)
  (ii) hydraulic motors (rotatory actuators)

Hydraulic cylinders

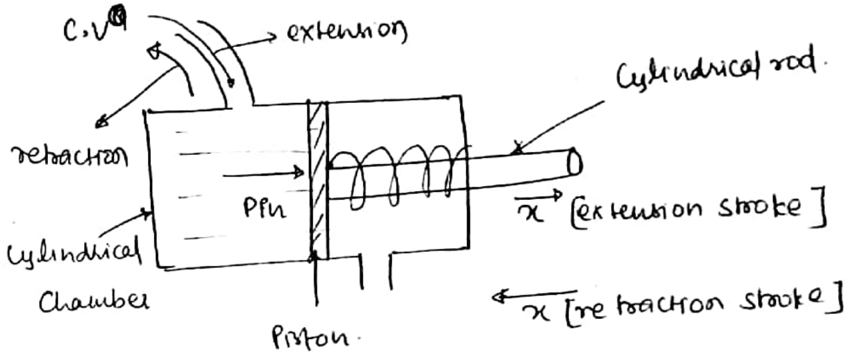
→ As the name indicates these actuators converts liquid pressure energy into linear motion hydraulic cylinder are classified in four type

(i) single acting type hydraulic cylinder
(ii) Double acting " " "
(iii) Telescopic type " " "
(iv) Tendon " " " "

hydraulic single acting type

→ In this actuator we can perform either extension or retraction with the help of liquid pressure energy

→ Usually we prefer extension stroke with liquid and retraction stroke with the help of spring as in figure.



Velocity of cylinder rod $\boxed{(v) = \frac{Q}{A_p}\ (m/s)}$

Force transmitted $\boxed{(F) = P_{in} \times A_p\ (N)}$

Mechanical power transmitted $= F \times V$

$= P_{in} \times \cancel{A_p} \times \frac{Q}{\cancel{A_p}}$

$$\boxed{(P) = P_{in} \times Q \quad (W)}$$

ES 2017 What will be the velocity of the piston movement for a single acting hydraulic actuator, when the liquid pressure applied is 100 bar the diameter of the piston is 50 mm and flow rate of liquid is 0.3 m³/min.

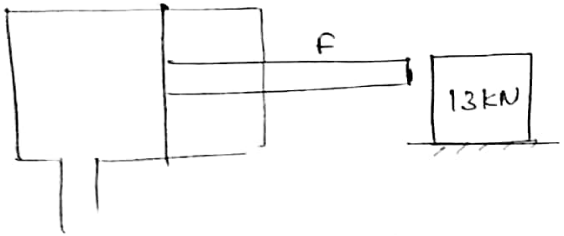
$$V = \frac{0.3 \times 4}{\pi \times 50^2 \times 10^{-6}}$$

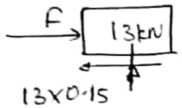
$$= \frac{1.2}{3.14 \times 2500} \times 10^6$$

$$= \frac{1200000}{3.14 \times 2500 \times 60}$$

$$= 2.$$

Q A hydraulic cylinder has to move a weight of 13 kN the speed of cylinder is to be accelerated up to a velocity of 0.13 m/s in 0.5 sec assume the coeff of sliding friction 0.15 and the dia of piston is 50 mm find the input pressure that should be applied at piston.





$$F - 13\times10^3 \times 0.15 = \frac{13\times10^3}{10} \times \frac{0.13}{0.5}$$

$$= 13\times100 \times \frac{0.13}{0.5}$$

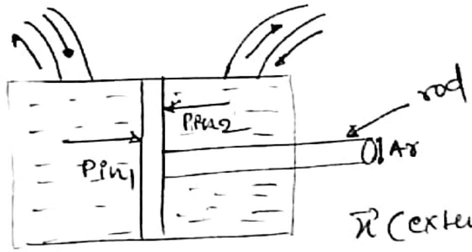
$$F = 2288\,N$$

$$\text{Pressure} = \frac{F}{A}$$

$$= \frac{2288}{\pi/4 \times 0.05^2} = 1.16\,MPa$$

Double acting type hydraulic cylinder

→ In these actuators both extension as well as retraction will be done with the help of liquid pressure



$\vec{x}$ (extension stroke)

$\overleftarrow{x}$ (retraction stroke)

During extension stroke

→ velocity of cylinder

$$V_1 = \frac{Q_1}{A_p} \text{ (m/sec)}$$

→ Force transmitted

$$F_1 = P_{in} \times A_p$$

→ Mechanical power transmitted (P_1)

$$= F\times V$$

$$= P_{in} \times A_p \times \frac{Q}{A_p}$$

$$= P_{in} \times Q \text{ (W)}$$

During retraction stroke

→ velocity of cylinder

$$V_2 = \frac{Q_2}{(A_p - A_r)}$$

→ Force transmitted

$$F_2 = P_{in_2} \times (A_p - A_r)$$

→ Mechanical power transmitted

$$P_2 = F_2 \times V_2$$

$$= P_{in}(A_p - A_r) \times \frac{Q}{A_p - A_r}$$

$$= P_{in} Q \text{ (W)}$$

Q A hydraulic pump delivers 0.003 m³/sec of oil to a double acting cylinder having a 6cm piston dia and 2cm rod diameter, it is assumed that the cylinder supports load of 5000 N in both the

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strokes it moves the wt. horizontally on the floor calculate the pressure applied and velocity of cylindrical rod and power transmitted in both stroke.

sol

extension

$$V_1 = \frac{0.003 \times 4}{\pi \times 0.06^2}$$

$$= 1.06 \, m/sec$$

$P_1 = 1770 \, kPa$

Power = 5.3 kW

Retraction

$$V_2 = \frac{0.003 \times 4}{\pi \times (0.06^2 - 0.02^2)}$$

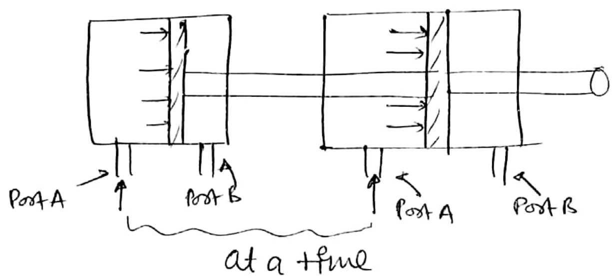
$V_2 = 1.19 \, m/s$

$P_2 = 1990 \, kPa$

Power = ~~5.3 kW~~ 6.0 kW

Tendon type

→ This type of cylinder is used for larger mechanical motion and we have operational constraint that the liquid should be sent in both the ports at a time.

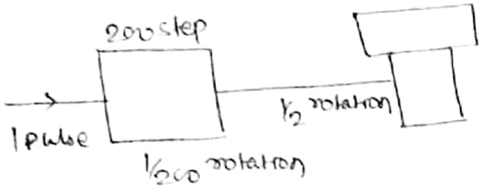


Basic length unit

movement of the table corresponding to 1 Pulse received by the motor.

अगर हम मोटर को 1 Pulse भेजेगे, तो Table जितना Move करेगा

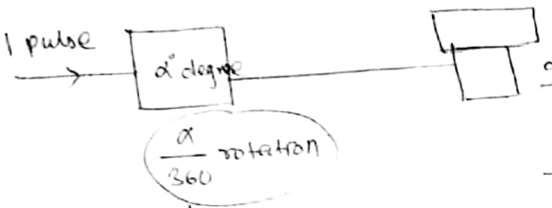
(1)



$\frac{P}{200} = BLU$

$\frac{P}{200} \times 1000\,\mu m = BLU$

(2)



$\frac{\alpha P}{360}\ mm = BLU$

$\frac{\alpha P}{360} \times 1000\,\mu m = BLU$

↳ reciprocal of this = no of step

(3) If frequency of pulse is 1000HZ

1 s के अंदर 1000 pulse भेजा

1 min " " $1000 \times 60\ \frac{pulse}{min}$

no. of step of motor → $\frac{1000 \times 60}{200}\ \frac{rotation}{min}$ = rpm

Table speed or feed $= \frac{1000 \times 60}{200} \times P\ \frac{mm}{min}$

$= 1000 \times 60 \times BLU\ \frac{mm}{min}$

(4) If motor rotates with N (~~5000~~ 500 r

1 मिनट के अंदर 500 rotation किया मतलब $500 \times 200\ \frac{pulse}{min}$ भेजा

Table speed = feed $= 500 \times P\ \frac{mm}{min} = 500 \times 200 \times BLU\ \frac{mm}{min}$

1 s के अंदर — $\frac{500 \times 200}{60}$

(5) अगर Answer pulse मे होता है तो उलटा गीत माना है |

let BLU = 0.005 mm

0.005 mm movement के लिए 1 pulse भेजना पडता है

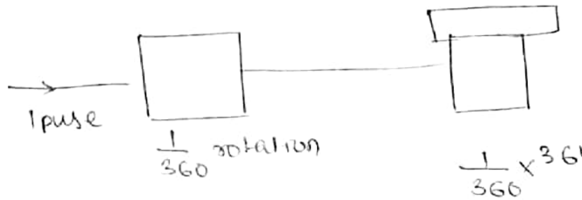
∴ 1 mm " " " $\frac{1}{0.005}$ pulse

∴ x m " " " $\frac{x}{0.005}$ pulse

1 min — $100\ \frac{mm}{min}$ — $\frac{100}{0.005}\ \frac{pulse}{min}$

1 s — $\frac{100}{0.005 \times 60}\ \frac{pulse}{s}$ Hz

G97

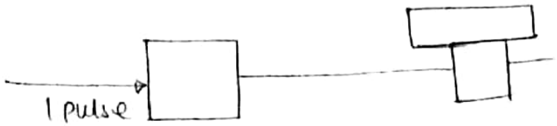


$\frac{1}{360} \times 36mm = BLU$

$\frac{3.6}{360} \times 1000\,\mu m = BLU$

$\frac{360}{36} = 10\,\mu m$

$\boxed{G-07}$ (PC)



1 pulse

$\frac{1.8}{360}$

$\frac{1.8}{3600} \times 2\ mm = BLU$

$10\ \mu m = BLU$

$\boxed{E-17}$

1 s ———— 200 pulse

1 m ———— $200 \times 60 \frac{pulse}{min}$

$RPM = \frac{200 \times 60}{150}\ rpm$

1 rpm — 4 m

$\frac{200 \times 60}{150}\ rpm = \frac{200 \times \cancel{60}^{4}}{15\cancel{0}} \times 4$

$= 320\ mm/min.$

2nd method

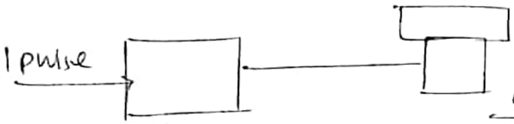
1 pulse

$\frac{1}{150}$

$4/150 = BLU$

$200 \times 60 \times \frac{4}{150} = 320 \frac{mm}{min}$

$\boxed{Example}$



1 pulse

$1/500$

$\frac{5}{500} = BLU$

(a) linear velocity = RPM × BLU = $\frac{600 \times 5}{500}$

$= 600 \times 5$

$= 3000\ mm/min = 6\ mm/min$

(b) $BLU = \frac{5}{500} = \frac{1}{100}$

(c)

~~500 pulse~~

1 rev - 500 pulse

600 rev = $500 \times 600 \frac{pulse}{min}$

$= \frac{500 \times \cancel{600}^{10}}{\cancel{60}} \frac{pulse}{sec}$

$= 5000\ Hz$

$\boxed{E-11}$

(2) $500 \times 6 = 3000\ mm/min$

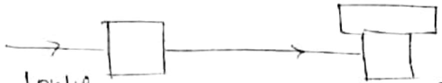
(1) ~~BLU = 6~~ 1 s ———— 1000

1 min ———— 1000×60 pulse

1000×60 pulse — 500 rotation

1 pulse — $\frac{500}{1000 \times 60}$ rotation

$= \frac{1}{200}$ rotation



$\frac{6}{120}$ mm

1 pulse $\frac{1}{200}$ rotation

$\frac{6}{120} \times 1000\ \mu m = 50\ \mu m$

(3) 1000 Hz

$\boxed{G-10}$ (PC) 0.005 mm ———— 1

9 mm ———— $\frac{1}{0.005} \times 9$ pulse

$= \frac{9000}{5}$

$= 1800$

G-16 | PI

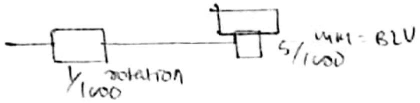
$M \times 5 = 6000 \frac{mm}{min}$

$M = 1200 \frac{Rev}{min} = \frac{1200}{60} \frac{Re}{sec}$

1 rev — 1000 pulse

1200 rev $= \frac{1000 \times 60}{\cancel{1200}}$ 20

$= 50$ HZ



0.005 mm ⟶ 1 pulse

1 mm — $\frac{1}{0.005}$ pulse

$6000 \frac{mm}{min} = \frac{6000}{0.005} \frac{Pulse}{min}$

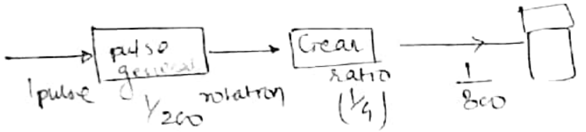
$= \frac{6000}{0.005 \times 60} \frac{Pulse}{sec}$

$= 20000$ HZ

$= 20$ kHz

G-14 PI - C

G-8 - (i)



BLU $= \frac{4}{800}$

$\frac{1}{200}$

.005 mm

= 5 μm

(b)

$\frac{4}{800}$

(ii)

(a)

1 min के अंदर 1000 pulse भेजा था तो $1000 \times 5 \frac{\mu m}{min}$ vel था

1 min के अंदर 1000 pulse ~~भेजता है~~ भेजता तो $1000 \times 10 \frac{\mu m}{min}$ vel होगा

1 min के अंदर 500 pulse भेजा तो $500 \times 10 \frac{\mu m}{min}$ vel होगा

G-09 - (9)

E-17

30 mm ——— 360

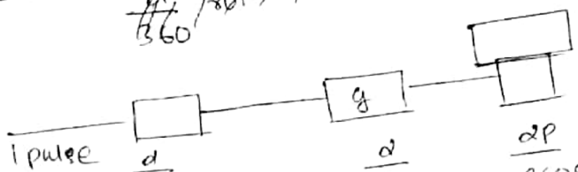
1 mm ——— 360/30

0.5 mm ——— $\frac{360}{30} \times 0.5$

$= 6°$

G-14

$\frac{\alpha}{360}$ / rot / ——— / 1



$\frac{\alpha}{360}$ $\frac{\alpha}{360 g}$ $\frac{\alpha P}{360 g}$ = BLU

$\frac{\alpha P}{360 g}$ mm ——— 1 pulse

1 mm ——— $\frac{360 g}{\alpha P}$ pulse

x mm - $\frac{360 g}{\alpha P} x$

A-10

(i) BLU $= \frac{3}{200} \frac{mm}{pulse} = \frac{3}{200} \times 1000 = 15 \mu m$

(ii) ~~1 pulse~~ $\frac{3}{200}$ mm — 1 pulse

100 mm — $\frac{200 \times 100}{3} \frac{pulse}{min}$

$\frac{200 \times 100}{3 \times 200}$ 33.33 rpm

$= \frac{\cancel{200}^{10} \times 600}{3 \times \cancel{60}_{3}} \frac{pulse}{sec}$

$= 1000/9$ HZ

$= 111.11$ HZ

A-13

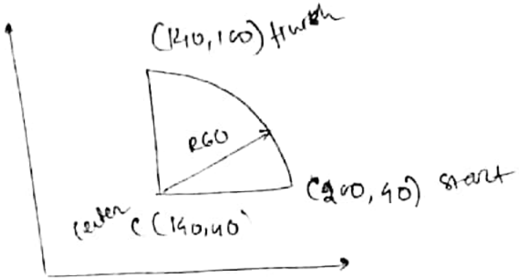
(i) BLU $= \frac{0.9}{360} \times 4$

$\frac{9 \times 4}{3600}$ mm — 1

2.87 mm ——— $\frac{3600}{4 \times 9} \times 2.87 = 287$

(ग) 100HZ

Slide 113



N_G03 X140.0 Y100.0 R60.0 F100.0

N_G03 X140.0 Y100.0 I-140.0 J40.0 F100.0

नुकता : start point का co-ordinate के साथ अगर I J,k जोड़ेंगे तो center का co-ordinate ~~मा~~ मिल जायेगा

$200 + I = 140 \Rightarrow I = -60$

$40 + J = 40 \Rightarrow J = 0.0$

N_G03 X140.0 Y100.0 I-60.0 ~~J0.0~~ F100.0

लिखने की जरूरत नहीं है

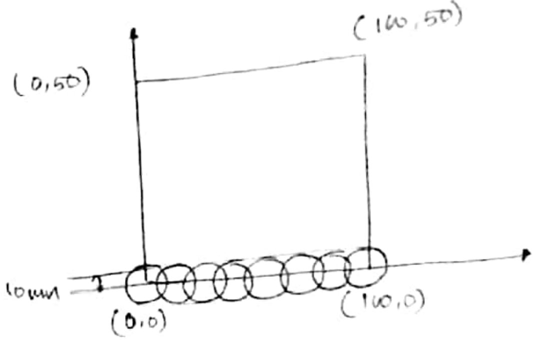
Slide 126

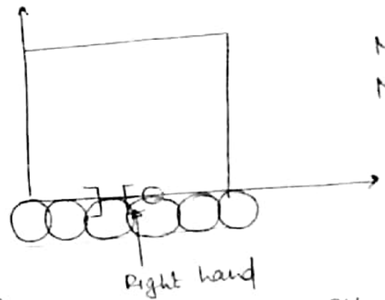
tool radius compensation

G40 radius compensation cancel

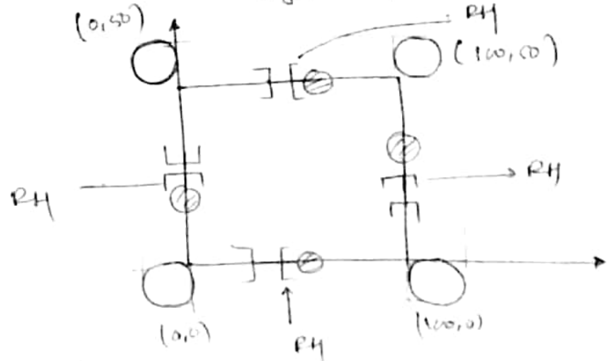
G41 left hand compensation

G42 Right hand compensation.



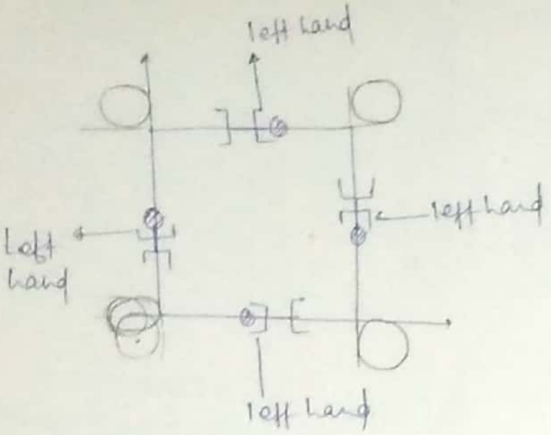


N_G42 D10.0
N_G01 X100.0 Y0.0 F100.0



G-a

N_G42 D10.0
N_G01 X100.0 Y0.0 F100.0
N_G01 X100.0 Y50 F100.0
N_G01 X0.0 Y50 F100
N_G01 X0.0 Y0.0 F100
M G40



N-G41 D100
N-G01X00Y500F100
N-~~G01~~ X1000Y~~500~~ F100
N-~~G01~~ X~~1000~~ Y0 F100
N-~~G01~~X00 Y~~0~~ F100
N-G40

Height Compensation

G-43- Positive compensation
G-44 Negative compensation
G 49 Height compensation cancel

Slide 143

Magic three coad (था अभी नहीं है)

The first digit of the coad is obtained by adding 3 to no of digits before the decimal point of rpm.
last 2- digit in the magic three code is first 2 digits of rpm.

eg. 10 rpm = (2+3)10 = 5510
100 rpm = (3+3)10 = 5610
3257 rpm = (4+3) 32 = 5732
840 rpm = (3+3)84 = 5684

Robotics

Transformations

Matrix Multiplication का जुगाड़

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \end{bmatrix} \begin{bmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{bmatrix} =$$

Step 1

2nd matrix मे जितना column है first matrix को उतना बार लिखना है।

$$\begin{bmatrix} a_1 & b_1 & c_1 & a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 & a_4 & b_4 & c_4 \end{bmatrix}$$

Step 2

2nd Matrix के column मे vertically जो जो element मिलेगा उसको horizontally multiply करना है।

$$\begin{bmatrix} a_1 l_1 + b_1 m_1 + c_1 n_1 & a_1 l_2 + b_1 m_2 + c_1 n_2 \\ a_2 l_1 + b_2 m_1 + c_2 n_1 & a_2 l_2 + b_2 m_2 + c_2 n_2 \\ a_3 l_1 + b_3 m_1 + c_3 n_1 & a_3 l_2 + b_3 m_2 + c_3 n_2 \\ a_4 l_1 + b_4 m_1 + c_4 n_1 & a_4 l_2 + b_4 m_2 + c_4 n_2 \end{bmatrix}$$

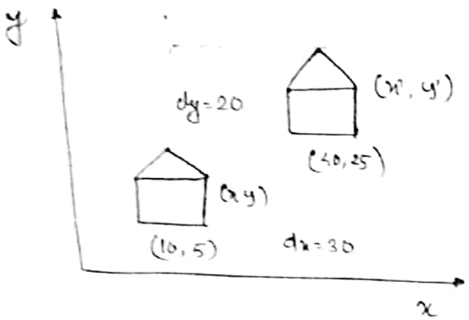
Step (3)

addition

2D-transformation

(1) Translation (2) Rotation (3) scaling (4) Shear
(5) Reflection

(1) Translation



$x' = x+30 = x+dx$

$y' = y+20 = y+dy$

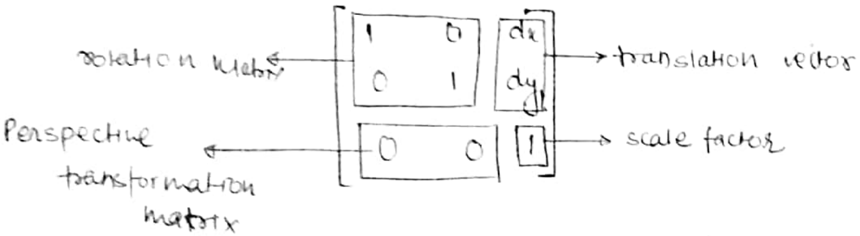
$x_{new} = x_{old} + dx$

$x_{new} = y_{old} + dy$

In matrix form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

homogeneous transformation matrix



$$\begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times10 & 0\times0 & 30\times1 \\ 0 & 1\times5 & 20\times1 \\ 0\times10 & 0\times5 & 1\times1 \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \\ 1 \end{bmatrix}$$

aayega

bekar

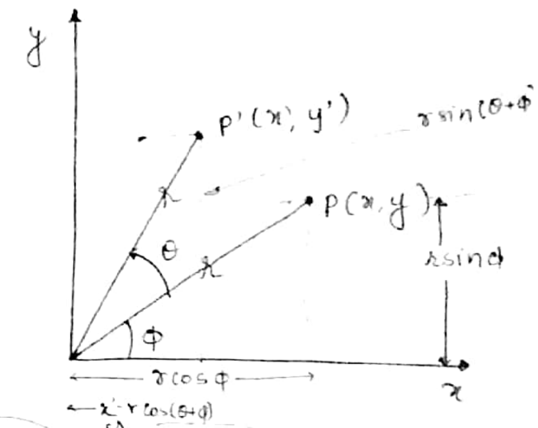
(2) Rotation about origin

known x, y, θ

Unknown x', y'

$\theta \rightarrow$ CCW $\rightarrow$ +ve

$\theta \rightarrow$ CW $\rightarrow$ −ve



$$x' = r\cos(\theta+\phi) = r\cos\theta\cos\phi - r\sin\theta\sin\phi$$

$$y' = r\sin(\theta+\phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\cos\theta + y\sin\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

***** Rotation matrix = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

If θ in clockwise direction

$$R = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

Note

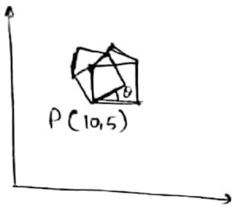
Que मे अगर Clockwise mentioned है तभी θ clockwise मानेंगे)

S-109

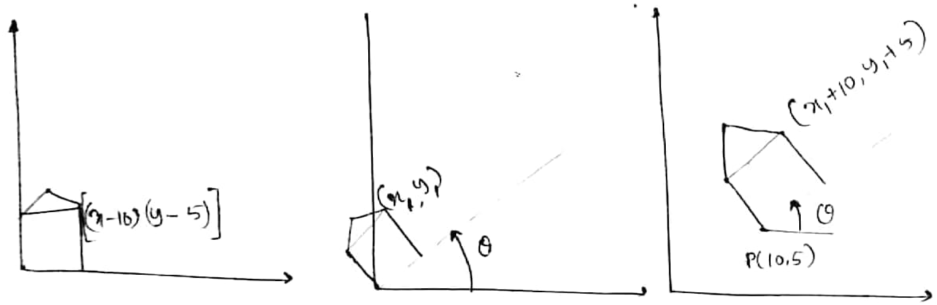
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$ (b)

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$

Rotation about any arbitary point P



1st translate to origin — 2nd Rotate θ about origin — 3rd translate back to P



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

S-110

step-1

P ⟶ origin

Q (4-1, 5-3)

Q (3, 2)

Step (2)

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & -0.6 \\ -0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2.4 + 1.2 \\ -1.8 + 1.6 \end{bmatrix}$$ (9)

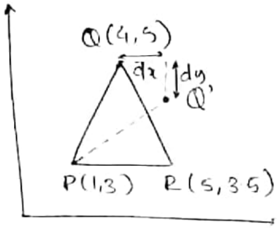
$$= \begin{bmatrix} 3.6 \\ -0.2 \end{bmatrix}$$

Step 3

(3.6+1, -0.2+3)

(4.6, 2.8)

अनुसार ले



(9)

Transformation matrix

Step (1)

$$T_1 = \begin{bmatrix} 1 & 0 & -P_x \\ 0 & 1 & -P_y \\ 0 & 0 & 1 \end{bmatrix}$$

Step (2)

$$T_r = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step (3)

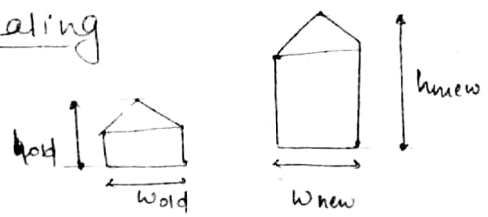
$$T_2 = \begin{bmatrix} 1 & 0 & P_x \\ 0 & 1 & P_y \\ 0 & 0 & 1 \end{bmatrix}$$

Final transformation matrix

$\cancel{T = T_1 \times T_r \times T_2}$

$\boxed{T = T_2 \times T_r \times T_1}$

③ Scaling



$$S_x = \frac{w_{new}}{w_{old}}$$

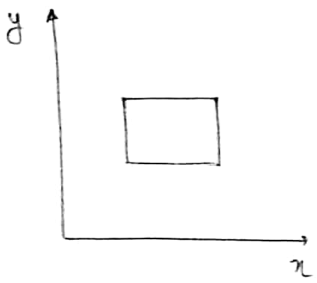
$$S_y = \frac{h_{new}}{h_{old}}$$

$$x_{new} = S_x \cdot x_{old}$$

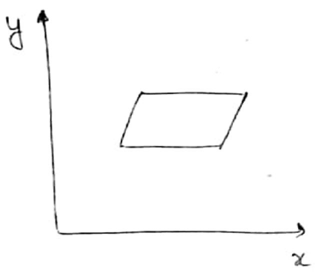
$$y_{new} = S_y \cdot y_{old}$$

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} S_x x + 0 \cdot y \\ 0x + S_y \cdot y \end{bmatrix}$$

④ Shear

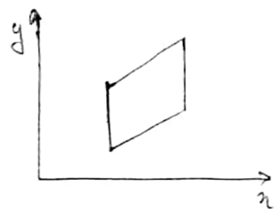


original



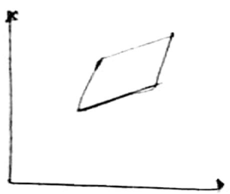
x - shear

$$T_x = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



y-shear

$$T_y = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Txy shear

$$T_{xy} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⑤ Reflection

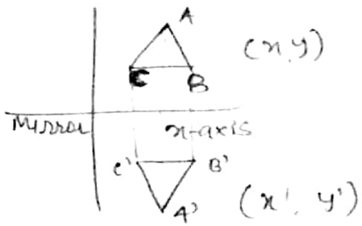
→ Reflection is mirror image of orginal obj

→ In other words we may say that it is rotatio of 180°

ⓐ x- axis reflection

$$T_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

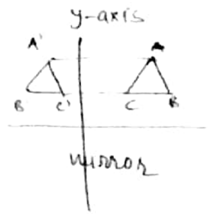
$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$x' = x$
$y' = -1$

ⓑ Y- axis reflection

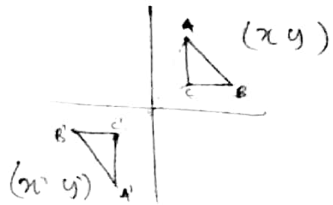
$$T_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



ⓒ Reflection about origin

→ origin will behaves like a point mirror

$$T_O = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



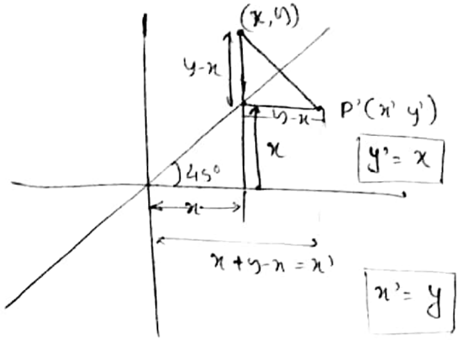
$x' = -x$
$y' = -y$

Note

हर type के que मे object हमेशा 1st quadrant मे होगा

(d) Reflection about $y = x$

$$T_{y=x} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

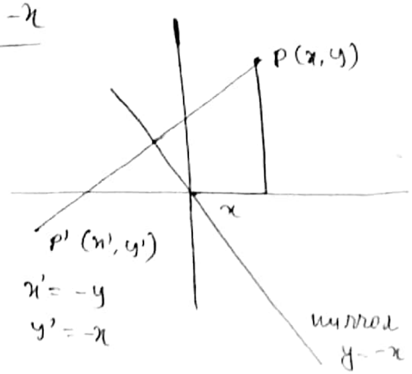


$\boxed{y' = x}$

$\boxed{x' = y}$

S-108 — (b)

(e) Reflection about $y = -x$



$x' = -y$

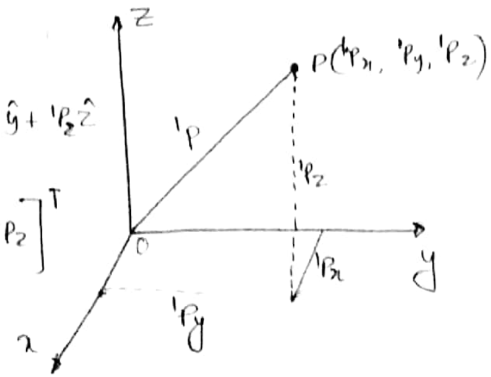
$y' = -x$

3-D Transformation

Co-ordination frame

$\vec{OP} = {}^1P = {}^1P_x\hat{x} + {}^1P_y\hat{y} + {}^1P_z\hat{z}$

$$\begin{bmatrix} {}^1P_x \\ {}^1P_y \\ {}^1P_z \end{bmatrix} = \begin{bmatrix} {}^1P_x & {}^1P_y & {}^1P_z \end{bmatrix}^T$$



→ A frame space notation is introduced as ${}^1P$ to refer to the point P or vector $\vec{OP}$ with respect to frame{1} (or) {x, y, z} with its components in the frame as $P({}^1P_x, {}^1P_y, {}^1P_z)$

Mapping

→ It refers to changing the description of the point or vector in space from one frame to another frame

→ mapping changes the description of point and not the point itself

→ The 2nd frame 3 possibilities in relation to the 1st frame

Possibility (1) Mapping involving translated frames

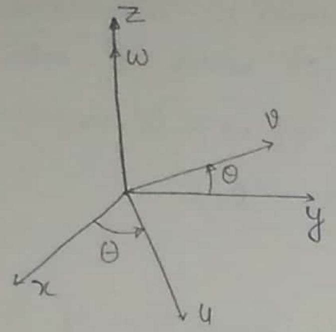
2nd frame is moved away from the first the axes of both frame is remain parallel

x ∥ u , v ∥ y , z ∥ w

Possibility (2) Rotated frame

→ 2nd frame is rotated w.r.t to the first the origin of both the frames is same

→ In robotics this is reffered as changing the orientation



$\{1\} = \{x\ y\ z\}$

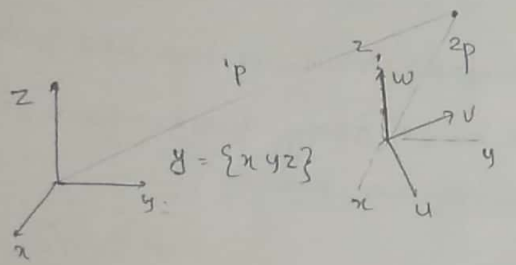
$\{2\} = \{u\ v\ w\}$

rotated about z-axis

Possibility (3)

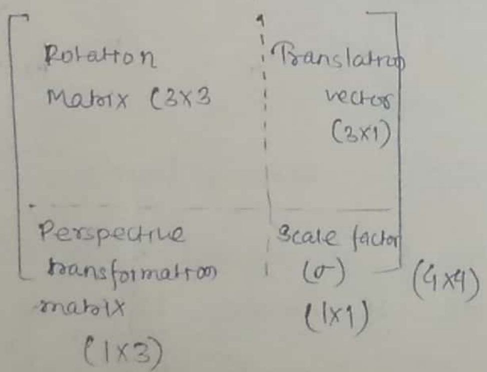
→ 2^{nd} frame is rotated w.r.t 1^{st} and moved away from it

i.e the 2^{nd} frame is translated and it's orientation is also change



$\{2\} = \{u\ v\ w\}$

Homogeneous transformation matrix



→ There is generalised homogeneous transformation matrix has above 4-submatrix.

→ Perspective transformation matrix is useful in vision system and is set to zero vector wherever no perspective view are involved

→ The scale factor has non-zero +ve value is called global scaling parameter

$\sigma > 1$ is used for reducing

$0 < \sigma < 1$ " " " enlarging

For robotics $\sigma = 1$ is used

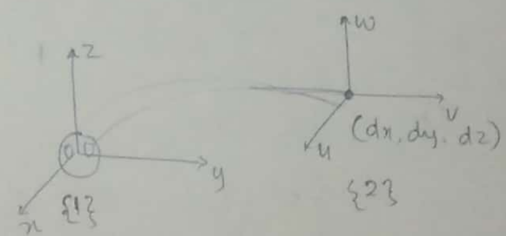
$$^1T_2 = \begin{bmatrix} ^1R_2 & ^1D_2 \\ 000 & 1 \end{bmatrix}$$

where

$\{1\} = \{x\,y\,z\}$, $\{2\} = \{u, v\, w\}$

$$^1R_2 = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

$$^1D_2 = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = \begin{bmatrix} d_x & d_y & d_z \end{bmatrix}^T$$



frame $\{1\} = \{x\ y\ z\}$ का origin मे बैठ कर frame $\{2\}$ के origin को देखना लेकिन वहा से उठना नही है।

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* * * * * Funda

If ^2p known $\;\; ^1p = {}^1T_2 \cdot {}^2p$

If ^1p known $\;\; ^2p = {}^2T_1 \cdot {}^1p$ $\;\; \left[\text{where } {}^2T_1 = [{}^1T_2]^{-1}\right]$

If vector involved

$$[\text{New vector}] = \underset{\substack{\uparrow \\ {}^1T_2}}{[HTM]}\,[\text{old vector}]$$

Pure translation

$$^1T_2 = \begin{bmatrix} ^1R_2 & ^1D_2 \\ 0\,0\,0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Eg consider a frame {2} which is obtained from the frame {1} by translating it 2 units along y and 1 unit along z, find HTM & ^1p if $^2p = [0\ 2\ 3]^T$

Sol

$$HTM = {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1p = {}^1T_2 \cdot {}^2p$$

$$= \underset{4\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 0 \\ 2 \\ 3 \\ 1 \end{bmatrix}}$$

$$^1p = \begin{bmatrix} 0 \\ 2+2 \\ 3+1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \\ 1 \end{bmatrix}$$

$$^1p = [0\ \ 4\ \ 4]$$

जुगाड से

^2p	0	2	3
translation	+0	+2	+1
^1p	[0	4	4]

if vector involved

for the vector $\vec{V} = 25\hat{i} + 10\hat{j} + 20\hat{k}$ perform a translation by a distance of 8 in the x-direction, 5 in y-direction and 0 in z-direction find HTM and new vector

$$[\text{New vector}] = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{HTM} \underbrace{\begin{bmatrix} 25 \\ 10 \\ 20 \\ 1 \end{bmatrix}}_{\text{old vector}}$$

$$= \begin{bmatrix} 25+8 \\ 10+5 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 33 \\ 15 \\ 20 \\ 1 \end{bmatrix}$$

1 — bekar

$$\vec{V}_{new} = 33\hat{i} + 15\hat{j} + 20\hat{k}$$

जुगाड से

$\vec{V} =$	$25\hat{i}$	$+10\hat{j}$	$+20k$
translation	8	5	0
$\vec{V}_{new}$	$33\hat{i}$	$+ 15\hat{j}$	$+ 20\hat{k}$

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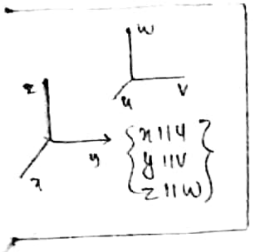
हुआ कैसे

$$^1R_2 = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

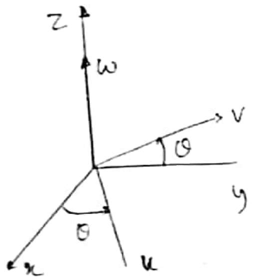


$$= \begin{bmatrix} \cos 0^\circ & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 0^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Pure rotation

Principal axis rotation



$$^1R_2 = R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^1R_2 = R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$^1R_2 = R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

bich मे आने से sin का sign change हो जायेगा

$$^1T_2 = \left[\begin{array}{c|c} ^1R_2 & ^1d_2 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Que Frame {2} is obtained by frame {1} by rotating it about z axis by angle of 30° find HTM

$$R_z(\theta) = \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$HTM = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Que The co-ordinates of point P in frame 1 are $[3\ 2\ 1]^T$. The position vector P is rotated about the z-axis by 45° find the co-ordinates of point Q, the new position of P

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt2 & -1/\sqrt2 & 0 & 0 \\ 1/\sqrt2 & 1/\sqrt2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt2 - 2/\sqrt2 \\ 3/\sqrt2 + 2/\sqrt2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt2 \\ 5/\sqrt2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 3.535 \\ 1 \\ 1 \end{bmatrix}$$

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translated → $P_{new} = 1\hat{i} + 3\hat{j} - 5\hat{k}$

$2\hat{i} + 3\hat{j} - 4\hat{k}$

(a) $\quad P_{new} \rightarrow [3 \quad 6 \quad -9]$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ -9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 3 \\ -9 \\ 1 \end{bmatrix}$$

Translation & rotation combined

Ex. Frame {2} is rotated w.r.t frame {1} about x-axis by an angle of 60 the position of the origin of frame {2} as seen from {1} is ${}^1D_2 = [7 \ 5 \ 7]^T$. Obtain ${}^1T_2$ and ${}^1p$ if ${}^2p = [2 \ 4 \ 6]^T$

$${}^1p = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1/2 & -\sqrt3/2 & 5 \\ 0 & \sqrt3/2 & 1/2 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{HTM} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 1.804 \\ 13.464 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 2 - 3\sqrt3 + 5 \\ 2\sqrt3 + 3 + 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \times 0.732 \\ 10 + 2\sqrt3 \\ 1 \end{bmatrix}$$

Combined Rotation

→ Fundamental rotation matrix can be multiplied together to represent a sequence of finite rotation.

Ex. The overall rotation matrices representing a rot^n of angle θ_1 about x-axis followed by a rotation of angle θ_2 about Y-axis can be obtained by

$${}^1R_2 = R_y(\theta_2) \cdot R_x(\theta_1) \leftarrow \text{ulta sequence hoga}$$

$$= \begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_1 & -S_1 \\ 0 & S_1 & C_1 \end{bmatrix}$$

$C_2 - \cos\theta_2$

$C_1 = \cos\theta_1$

$S_1 - S$

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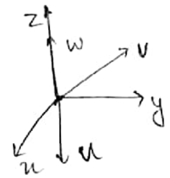


$${}^1R_2 = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \cos(90+\theta) & \cos 90^\circ \\ \cos(90-\theta) & \cos\theta & \cos 90^\circ \\ \cos 90 & \cos 90^\circ & \cos 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotated about Z-axis

$^2T_1$

$$\boxed{^2T_1 = [^1T_2]^{-1}}$$

imp

$$^2T_1 = [^1T_2]^{-1} = \left[\begin{array}{ccc|c} & [^1R_2]^T & & -[^1R_2]^T\,{}^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$^1T_2 = \left[\begin{array}{ccc|c} & ^1R_2 & & ^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$^2T_1 = \left[\begin{array}{ccc|c} & ^2R_1 & & ^2D_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

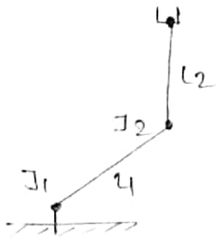
$$^1R_2 = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{z}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \end{bmatrix}$$

$$^2R_1 = \begin{bmatrix} \hat{u}\cdot\hat{x} & \hat{u}\cdot\hat{y} & \hat{u}\cdot\hat{z} \\ \hat{v}\cdot\hat{x} & \hat{v}\cdot\hat{y} & \hat{v}\cdot\hat{z} \\ \hat{w}\cdot\hat{x} & \hat{w}\cdot\hat{y} & \hat{w}\cdot\hat{z} \end{bmatrix}$$

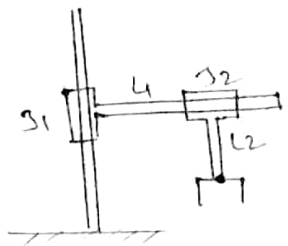
Direct and Indirect kinematics

Position representation

(1) The kinematics of RR robot is more difficult is analysis than PP robot



RR Robot



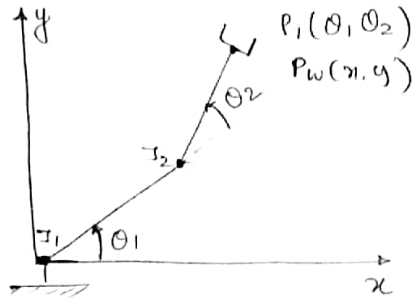
PP Robot

→ The position of the end of the arm may be represent in a no. of ways.

→ One way 1 is to utilise the joint angles the θ_1 & θ_2 these is known as joint space representation $P_j(\theta_1, \theta_2)$

→ Another to define the arm position in the world space. there involves the use of cartesian co-ordinate system i.e external to the robot. ~~The arm~~

→ The origin of cartesian co-ordinate system often located in the robot base $P_w(x, y)$



→ world space is useful when the robot must communicate with other M/c because other M/c may not have a detailed understanding of the robots kinematics

→ In order to use both representation we must be able to transform from one to other

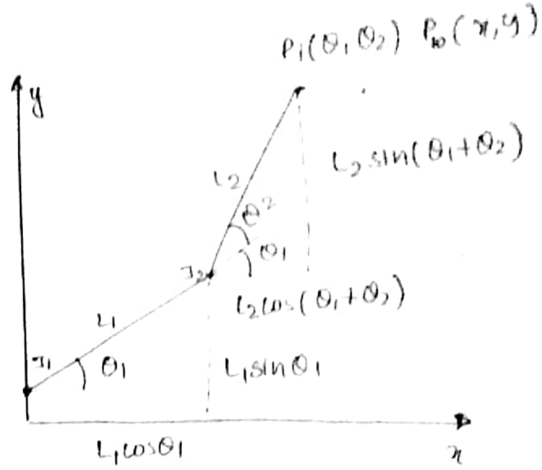
→ Going from joint space to world space is called follower transformation (or) direct kinematics

→ Going from world space to joint space is called the reverse transformation (or) Inverse kinematics

Direct kinematics of 2 degree of freedom

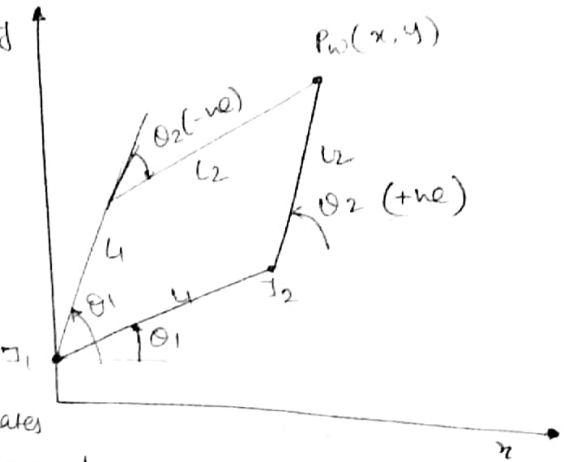
$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$



Inverse kinematics of 2 DOF Arm

→ For the tooling manipulator we have developed, there are 2 possible configurations for reaching the pt. (x, y)



→ These is so because the relation b/w the joints angle and the end factor co-ordinates involve sine & co-sine term hence we get solution when we solve the 2-eqn as given before.

→ Some strategy must be developed to select the appropriate configuration

Ex In the PUMA Robot. control language VAL there is set of commands called 'ABOVE and BELOW' that determine whether the elbow is to make angle θ_1

re greater (or) less than zero.

let θ_2 is (+)ve

$$x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) = L_1C_1 + L_2C_1C_2 - L_2S_1S_2$$

$$y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) = L_1S_1 + L_2S_1C_2 + L_2C_1S_2$$

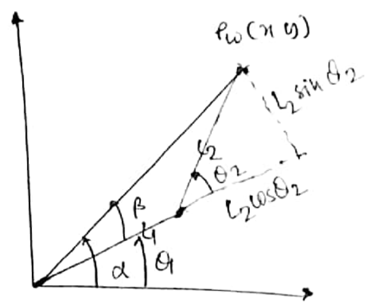
$$x^2 = L_1^2C_1^2 + L_2^2C_1^2C_2^2 + L_2^2S_1^2S_2^2 + 2L_1C_1^2L_2C_2 - 2L_2^2C_1C_2S_1S_2 - 2L_1L_2C_1S_1S_2$$

$$y^2 = L_1^2S_1^2 + L_2^2S_1^2C_2^2 + L_2^2C_1^2S_2^2 + 2L_1S_1^2L_2C_2 + 2L_2^2S_1C_2C_1S_2 + 2L_1S_1L_2C_1S_2$$

$$x^2+y^2 = L_1^2 + L_2^2C_2^2 + L_2^2S_2^2 + 2L_1L_2C_2$$

$$x^2+y^2 = L_1^2 + L_2^2 + 2L_1L_2C_2$$

$$C_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} \Rightarrow \theta = \text{known}$$



$$\theta_1 = \alpha - \beta$$

$$\tan\theta_1 = \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

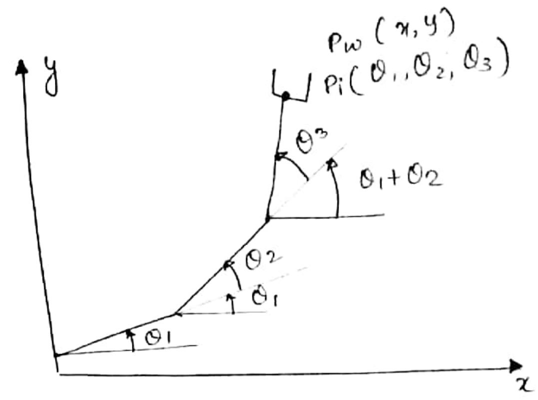
$$\tan\theta_1 = \frac{y/x - \frac{L_2S_2}{L_1+L_2C_2}}{1+\frac{y}{x}\times\frac{L_2S_2}{L_1+L_2C_2}}$$

$$\tan\alpha = \frac{y}{x} \qquad \tan\beta = \frac{L_2\sin\theta_2}{L_1+L_2\cos\theta_2}$$

$$\boxed{\tan\theta_1 = \frac{yL_1 + yL_2C_2 - xL_2S_2}{xL_1 + xL_2C_2 + yL_2S_2}}$$

Adding orientation

Direct



2 eq^n, 3 unknown $(\theta_1\,\theta_2\,\theta_3)$

$$\begin{cases} x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) + L_2\cos(\theta_1+\theta_2+\theta_3) \\ y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3) \end{cases}$$

↳ Denavit Hartenberg Notation (or) DH parameter (or) DH ~~parameter~~ notation

→ The defination of a manipulator ~~for~~ & with four joint link parameters one for each link and a systamatic procedure for assigning right handed orthonormal co-ordinate frames, one to each link in an open kinematic chain is DH notion

→ n − DOF will have (n+1) frames with the frame ~~zero~~ {0} (or) base frame acting as the reference inertial frame and frame {n} being the tool frame.

(ab) DH parameters

1. link length
2. link twist
3. joint distance
4. joint angle.

2 length
2 angle