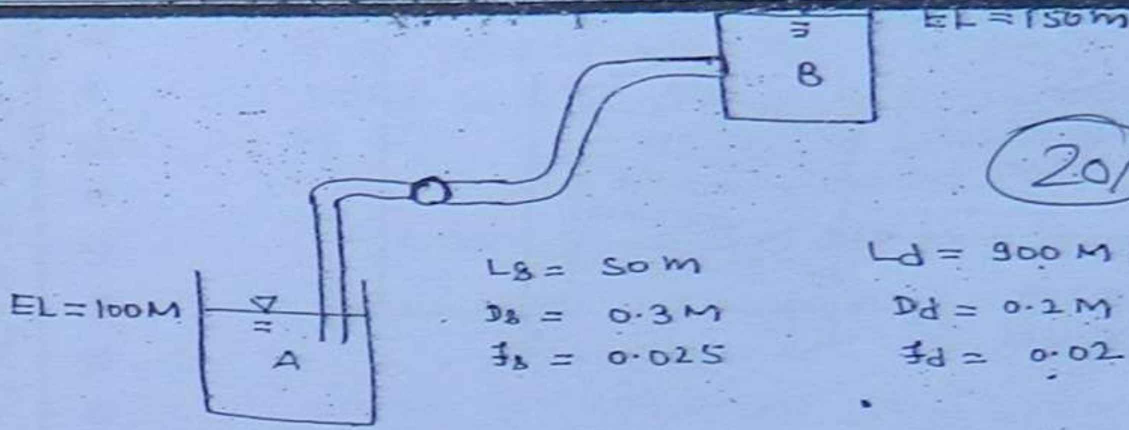


201

$L_s = 50\,m$ $\quad$ $L_d = 900\,M$

$D_s = 0.3\,M$ $\quad$ $D_d = 0.2\,M$

$f_s = 0.025$ $\quad$ $f_d = 0.02$

Head added by the pump.

$$H_p = \text{static head} + h_{fs} + h_{fd}$$

$$= (150-100) + \frac{f_s \cdot L_s Q^2}{12.1\, D_s^5} + \frac{f_d L_d Q^2}{12.1\, D_d^5}$$

$$\therefore h_{fs} = \frac{0.025 \times 50 \times Q^2}{12.1 \times (0.3)^5} = 42.5\,Q^2$$

$$h_{fd} = \frac{0.02 \times 900 \times Q^2}{12.1 \times (0.2)^5} = 4648.76\,Q^2$$

$$\therefore H_p = 50 + 42.5\,Q^2 + 4648.76\,Q^2$$

$$= 50 + 4691.26\,Q^2 = 80 - 7000\,Q^2$$

$$\therefore Q = 0.05\ M^3/sec$$

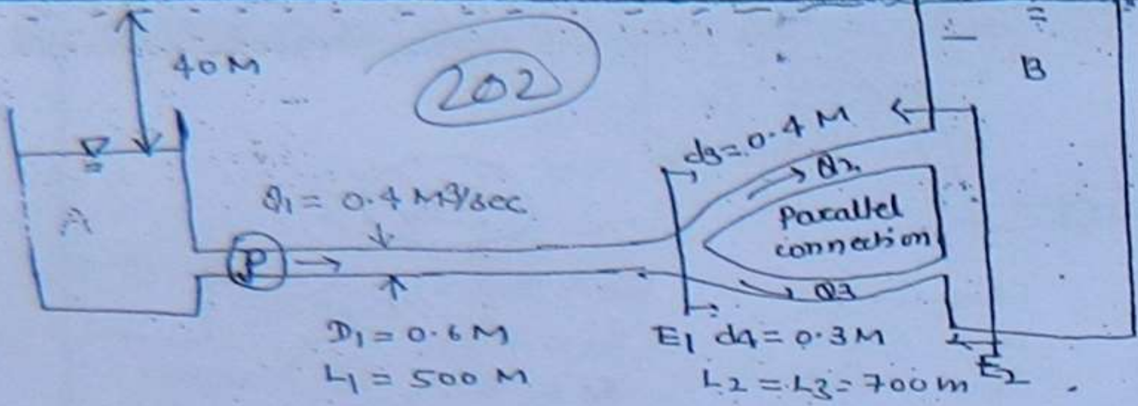
$$\therefore H_p = 80 - 7000 \times (0.05)^2$$

$$= 62.5\ m$$

$\therefore$ Power $= wQH_p$

Prob

Two reservoirs A & B are connected through a pipe system, consisting of one 60 cm pipe having 500M length which branches thereafter into two pipes of 40cm dia. & 30 cm dia. each 700 M Long. A pump situated near reservoir A discharges 0.4 m3/sec through the pipe system. The difference in the reservoir level is such that reservoir level B is 45 M above than reservoir Level A. Assuming f = .02 determine the power required for the pump. Assuming pump efficiency is 50%



$\boxed{h_{f2} = h_{f3}}$

$$\therefore \frac{f L Q_2^2}{12.1 D_2^5} = \frac{f L Q_3^2}{12.1 D_3^5} \Rightarrow \frac{Q_2^2}{Q_3^2} = \left(\frac{D_2}{D_3}\right)^5 = (4/3)^5$$

$$\therefore Q_2/Q_3 = \sqrt{4.21} \quad \text{— i)}$$

$$Q_2 + Q_3 = 0.4 \quad \text{— ii)}$$

$$\left.\begin{aligned} Q_2 &= 0.268 \\ Q_3 &= 0.13 \end{aligned}\right\} \Leftarrow$$

Total head added by the pump

$= 40 + h_L$

$[P = 309$ kW

NOTE: Head added by the pump is $\underset{(40)}{h_s} + \underbrace{h_{f1} + (h_{f2} \mp h_{f3})}_{(9.87)}$

However $h_{f2} = h_{f3}$

ESE 1999

A town of 2 Lakh population is to be supplied water from a source, 2500 M away. The Lowest water Level in the source is 15 M below the water works of the town. The demand of the water is estimated as 150 Lit/capital/day. A pump of 300 H.P. is oprated for 15 Hrs. & if the Max^m demand is 150 % of avg. demand and velocity of flow through pipe is 1.3 M/sec and Efficiency of the pump is 70 %. Determine the H.G. and friction factor. [Determine $\{P/\omega + Z\}$ the total water required]

$$\boxed{P = \frac{\omega Q H_{\text{added by the pump}}}{\eta} \quad \eta \to \text{Efficiency of pump}}$$

Water flows in a 80 MM pipe at reynolds No. 80,000. The pipe is estimated to have a equivalent sand grain roughness of size 0.16 mm. Determine the head Loss expected in 500 M Length of pipe. If pipe wear to act as smooth pipe, how much head Loss May be expected. ν of water $= 10^{-6}\,\frac{M^2}{sec}$

S/1992

The following explicit eqn of f May be used

$$\frac{1}{\sqrt{f}} = 1.14 - 2\,\text{Log}_{10}\left[\left(K/d\right) + \frac{21.25}{Re^{0.9}}\right]$$

Where K = Equivalent sand grain roughness = 0.16 MM

d = dia. of pipe = 80 MM

Re = Reynolds No. = 80,000

L = 500 M

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$$Re = \frac{Vd}{\nu} \Rightarrow V = \frac{80{,}000 \times 10^{-6}}{0.08} = 1\ \text{M/sec.}$$

For Rough pipe ↓

$$\frac{1}{\sqrt{f}} = 1.14 - 2\,\text{Log}_{10}\left[\left(\frac{0.16}{80}\right) + \frac{21.25}{(80{,}000)^{0.9}}\right]$$

$$\Rightarrow f = 0.0257$$

$$\therefore \text{Head Loss} = \frac{fLV^2}{2gD} = 8.15\ \text{M}$$

For smooth pipe ↓ $\boxed{K=0}$

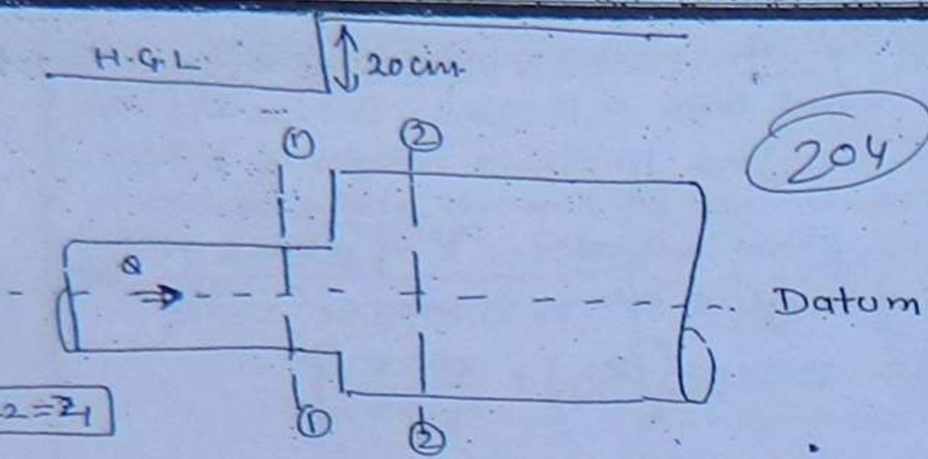
$$\frac{1}{\sqrt{f}} = 1.14 - 2\,\text{Log}_{10}\left[\frac{21.25}{(80{,}000)^{0.9}}\right]$$

$$\therefore f = 0.0187$$

$$\therefore \text{Head Loss} = \frac{8.15}{0.0257} \times 0.0187 = 5.96\ \text{M.}$$

S/1998

A horizontal pipe having diameter of 0.5 M expends at a junction to one Meter pipe having straight Length If hydraulic gradeline at junction rises by 20 cm. Find the flowrate in the Pipe.



$\boxed{Z_2 = Z_1}$

$D_1 = 0.5\,M \quad D_2 = 1.0\,m$

$$H.G.L \Rightarrow \left(\frac{p_2}{w} + \cancel{Z_2}\right) - \left(\frac{p_1}{w} + \cancel{Z_1}\right) = 0.2$$

$$\therefore \frac{p_2}{w} - \frac{p_1}{w} = 0.2\,M \quad \text{--- (1)}$$

Head loss due to sodden expansion

Apply B. eqn b/w (1)-(1) & (2)-(2)

$$\cancel{Z_1}^{0} + p_1/w + \frac{V_1^2}{2g} = \cancel{Z_2}^{0} + \frac{p_2}{w} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{(V_1 - V_2)^2}{2g} - \frac{V_2^2}{2g}$$

$$0.2 = \cancel{\frac{V_1^2}{2g}} - \cancel{\frac{V_1^2}{2g}} - \frac{V_2^2}{2g} + \frac{2V_1V_2}{2g} - \frac{V_2^2}{2g}$$

$$\Rightarrow 0.2 = \frac{2V_1V_2 - 2V_2^2}{2g} \Rightarrow 2V_1V_2 - 2V_2^2 = 0.4 \times 9.81$$

$$\Rightarrow \boxed{V_1V_2 - V_2^2 = 0.2 \times 9.81}$$

$$V_1 = Q/A_1 = \frac{Q}{\frac{\pi}{4} D_1^2}$$

$$V_2 = Q/A_2 = \frac{Q}{\frac{\pi}{4} D_2^2}$$

$\Downarrow$

$$\boxed{Q = 0.635\ M^3/sec}$$

→ To know direction of flow, know TE_1 & TE_2

If $TE_1 > TE_2$ than flow from (1)-(1) to (2)-(2)

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1) channel with hump:

Probl (a) A Rect. channel has a width of 2.0 m and carries a discharge of 4.80 m³/sec with a depth of 1.60 m. At a certain section a small, smooth hump with a Flat top and of height 0.10 m is proposed to be built. Calculate the Likely change in the water surface. Neglect the Energy Loss.

Step (1) Determine the U/s Flow conditions i.e. whether Flow is subcritical or super critical.

$$q = \frac{4.80}{B} = \frac{4.80}{2.0} = 2.40 \text{ m}^3/\text{sec/m}$$

$$V_1 = \frac{2.40}{1.6 = y_1} = 1.50 \text{ m/sec}$$

$$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{1.5}{\sqrt{9.81 \times 1.6}} = 0.378 < 1$$

[$D = y_1$ since channel is rectangular]

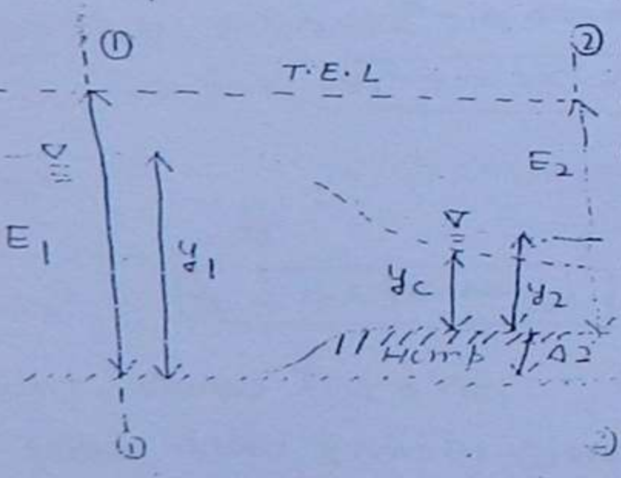
U/s Flow is subcritical so the hump will cause a drop in the water surface elevation.

step 2)

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 1.6 + \frac{(1.5)^2}{2 \times 9.81}$$

$$= 1.715 \text{ m}$$



Similarly at section (2)

$$E_2 = E_1 - \Delta z$$

$$= 1.715 - 0.10$$

$$= 1.615 \text{ m}$$

step 3) check of the flow condition at section (2) is critical.

$$y_c = \left[\frac{q^2}{g}\right]^{1/3} = \left[\frac{(2.4)^2}{9.81}\right]^{1/3} = 0.837 \text{ m}$$

$$E_c = 1.5\, y_c = 1.255 \text{ m}$$

Since Min sp. Energy at section (2) ... therefore the available Energy at that section

i.e. $E_{c2} < E_2$ hence $y_2 > y_c$ and depth y_1 will be remain unchanged.

step 4> calculation of depth y_2

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$$: E_2 = y_2 + \frac{v_2^2}{2g} \qquad \left[v_2 = q/y_2\right.$$

$$\therefore E_2 = y_2 + \frac{q^2}{2g y_2^2}$$

By trial & error method

$$1.615 = y_2 + \frac{(2.4)^2}{2\times 9.81 \times y_2^2} \Rightarrow \boxed{y_2 = 1.48\,m}$$

(b) If the height of the hump is 0.5m. Estimate the water surface elevation on the hump and at a section of the hump.

$v_1 = 1.50$ m/sec
$F_1 = 0.378 < 1$
$E_1 = 1.715$ m
$y_2 = y_{c2} = 0.837$m

Available sp. energy at section (2)

$\Rightarrow E_2 = E_1 - \Delta Z$

$E_2 = 1.715 - 0.5 = 1.215$ m

$E_{c2} = 1.5\, y_{c2} = 1.256$m

→ Since sp. energy at section (2) is greater than E_2, the available sp. Energy at that section.
Hence the depth at section (2) will be at the critical depth.
Hence $y_2 = y_{c2} = 1.256$ m
The U/s depth y_1 will increase to a depth y_1' such that new sp. energy at the U/s section 1 is

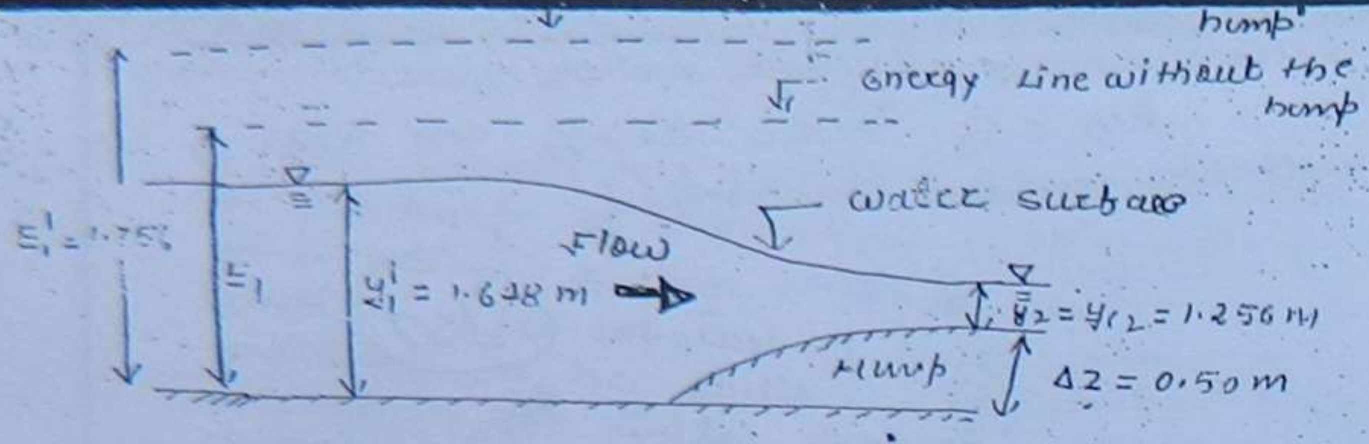
$$E_1' = E_{c2} + \Delta Z$$

$$E_1' = y_1' + \frac{v_1'^2}{2g} = E_{c2} + \Delta Z$$

$$\Rightarrow y_1' + \frac{q^2}{2g y_1'^2} = 1.256 + 0.5$$
$$= 1.756$$

$$\Rightarrow y_1' + \frac{(2.4)^2}{2\times 9.81 \times y_1'^2} = 1.756$$

$$\Rightarrow \boxed{y_1' = 1.648} \Rightarrow \boxed{y_1' > y_2} ***$$



Prob2

A Rect. channel 2.5 m wide carries 6.0 m³/sec of flow at a depth of 0.50m. calculate the height of a flat topped hump required to be placed at a section to cause critical Flow. The energy Loss due to the obstruction by the hump can be taken as 0.1 times the U/s velocity head.

step 1>

$q = 6.0/2.5 = 2.4$ m³/sec/m

$V_1 = 2.4/0.5 = 4.8$ m/sec

$$Fr_1 = \frac{4.80}{\sqrt{9.81 \times 0.5}} = 2.167 > 1 \Rightarrow \text{U/s Flow is supercritical.}$$

step2>

$$E_1 = 0.50 + \frac{(4.8)^2}{2 \times 9.81} = 1.674 \text{ m}$$

step3> since Flow at section (2) is critical

so $$y_c = (q^2/g)^{1/3} = \left[\frac{(2.4)^2}{9.81}\right]^{1/3} = 0.837 = y_2$$

Now $$\frac{V_c^2}{2g} = \frac{y_c}{2} = 0.419 = \frac{V_2^2}{2g}$$

∴ Applying Energy Equation b/w section (1) and (2)

$$E_1 - E_L = y_2 + \left(\frac{V_2^2}{2g}\right) + \Delta z$$

Δz → height of hump

$$E_L = 0.1 \frac{V_1^2}{2g} = 0.117 \text{ m}$$

$$\Rightarrow 1.674 - 0.117 = 0.837 + 0.419 + \Delta z$$

$$\Rightarrow \boxed{\Delta z = 0.301 \text{ m}}$$

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[illegible]

of $15.0\ m^3/sec$. at a depth of 2.0 m. It is proposed to reduce the width of the channel at a hydraulic structure. Assuming the transition to be horizontal and flow to be frictionless determine the water surface elevation U/s and D/s of the constriction when the constricted width is 2.50 m and 2.20 m.

Step 1 check for flow (subcritical or supercritical)

$$\therefore F_1 = \frac{V_1}{\sqrt{g y_1}} \qquad \left[V_1 = \frac{Q}{B_1 y_1} = \frac{15.0}{3.5 \times 2} = 2.143\ m/sec \right.$$

$$\therefore F_1 = \frac{2.143}{\sqrt{9.81 \times 2.0}} = 0.484 < 1$$

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The U/s flow is subcritical and the transition will cause a drop in the water surface.

Step 2>

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.0 + \frac{(2.143)^2}{2 \times 9.81} = 2.234\ m$$

Let B_{2m} = minimum width at section (2) which does not cause choking

than $E_{cm} = E_1 = 2.234\ m$

$$\therefore y_{cm} = \tfrac{2}{3} E_{cm} = \tfrac{2}{3} \times 2.234 = 1.489\ m \quad **$$

But
$$y_{cm}^3 = \left[\frac{Q^2}{g B_{2m}^2}\right]$$

$$\Rightarrow B_{2m} = \left[\frac{Q^2}{g\, y_{cm}^3}\right]^{1/2} = 2.636\ m$$

Step (3)>

Since $B_2 = 2.50 < B_{2m}$

Hence choking conditions would prevail

$\therefore$ The depth at section (2) = $y_{c2} = y_2$

$\therefore$ U/s depth y_1 will increase to y_1'

$$\therefore q_2 = \frac{15.0}{B_2} = \frac{15.0}{2.5} = \underline{6.0\ m^3/sec/m}$$

—

$y_{c2} = [q^2/g]^{1/3} = \left[\frac{(6)^2}{9.81}\right]^{1/3} = 1.542\ m$ **

$\therefore E_{c2} = 1.5\ y_{c2} = 1.5 \times 1.542 = 2.3136\ m$

$E_1' = E_{c2} = 2.31$ with new u/s depth of y_1' such that $q_1 = y_1' v_1' = 15/3.5$

$= 4.2857\ m^2/s$

$y_1' + \frac{v_1^2}{2g} = 2.3136$

$y_1' + \frac{(4.2857)^2}{2 \times 9.81 \times y_1'^2} = 2.3136$

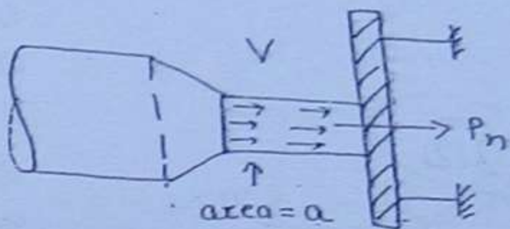
(210)

$\Rightarrow \boxed{y_1' = 2.122\ m}$

NOTE: For the same discharge when $B_2 < B_{2m}$ (under choking conditions) the depth at critical section will be different from y_{cm} and depends upon value of B_2.

1.) Impact of JETS:

<u>case Ist</u>: when Jet of water strikes normally to a stationary Flat plate



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If Loss of energy due to impact is negligible & surface is smooth so that friction Loss is negligible, Force exerted by the jet on the plate is

$$P_n = \rho Q \left[V - 0 \right]$$

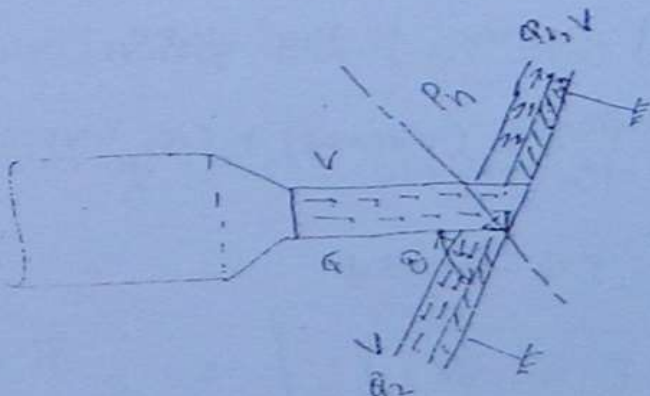
$$\boxed{P_n = \rho a V^2}$$

a → area of Jet
V → initial velocity in the direction of flow

∴ work done by the Jet on plate = zero.

[b'coz plate remains stationary]

case IInd: Jet strikes on an Inclined Flat stationary plate



P_n = Force exerted by Jet normal to plate

$$= \rho Q \left[V \sin\theta - 0 \right]$$

$$\boxed{P_n = \rho a V^2 \sin\theta}, \quad \boxed{\text{work-done} = 0}$$

Force exerted along the plate $= 0$

$$F = \rho Q V\cos\theta - \rho Q_1 V - \left(\rho Q_2(-V)\right) = 0$$

$$\Rightarrow \quad Q\cos\theta - Q_1 + Q_2 = 0 \text{ --- (ii)}$$

By eqn (i) & (ii)

Imp
$$\boxed{\begin{aligned} Q_1 &= Q/_2\left[1+\cos\theta\right] \\ Q_2 &= Q/_2\left(1-\cos\theta\right) \end{aligned}}$$

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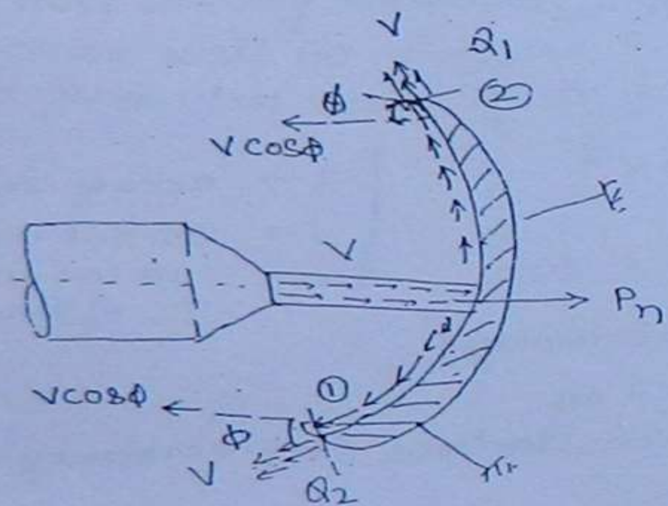
$$\therefore \boxed{\frac{Q_1}{Q_2} = \frac{1+\cos\theta}{1-\cos\theta}} \text{ Imp}$$

Case IIIrd: Force exerted by Jet on a stationary curved plate



Force exerted by Jet normal to the plate

$$P_n = \rho Q V - \left[\rho Q_1(-V\cos\phi) + \rho Q_2(-V\cos\phi)\right]$$

$$= \rho Q V + \rho Q \cdot V\cos\phi$$

$$\therefore \boxed{\begin{aligned} P_{n_1} &= \rho Q V\left(1+\cos\phi\right) \\ P_n &= \rho a V^2\left(1+\cos\phi\right) \end{aligned}}$$

$\boxed{\text{work done} = 0}$

$$= \rho Q$$

$$= \rho . a [V - u]$$

∴ Force exerted by the jet on the plate

$$P_n = \rho . a (V-u) [V-u]$$

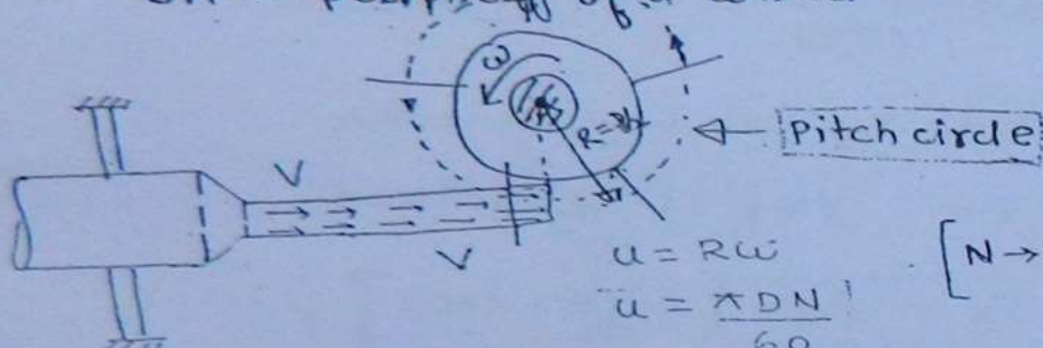
$$\boxed{P_n = \rho a (V-u)^2}$$

∴ Work-done per sec. by the Jet.

$$\boxed{W = \rho a (V-U)^2 \times u}$$

2/4

ase VI^{th}: Jet strikes on a series of Vanes Mounted on a periphery of a wheel



$u = R\omega$

$u = \frac{\pi D N}{60}$ [N → R.P.M

Mass of water strikes the Vane / Per sec.

$$= \rho a V = \rho Q \qquad [Q = aV$$

Force exerted by Jet on vane will be

$$= \rho a V [V - u]$$

$$= \rho a [V^2 - Vu]$$

Work done by Jet per sec. $= \rho a [V^2 - Vu] \times u$

$$= \rho a [V^2 u - u^2 V]$$

$$W = \rho Q [V - u] \times u$$

K.E / sec of Jet = $\frac{1}{2} \times$ Mass Flowing / sec $\times V^2$

$$K.E. = \frac{1}{2} \rho a V^3$$

$$\boxed{\eta = \frac{\text{work-done/sec}}{\text{K.E./sec}}}$$

$$\therefore \eta = \frac{\rho a V[V-u]\cdot u}{\tfrac{1}{2}\rho a V^3} = \frac{2[Vu-u^2]}{V^2}$$

For Maxm. Efficiency of Jet, $\frac{d\eta}{du} = 0$ 215

$$\therefore \frac{2}{V^2}[V-2u] = 0$$

$$\Rightarrow \boxed{u = V/2}$$ imp

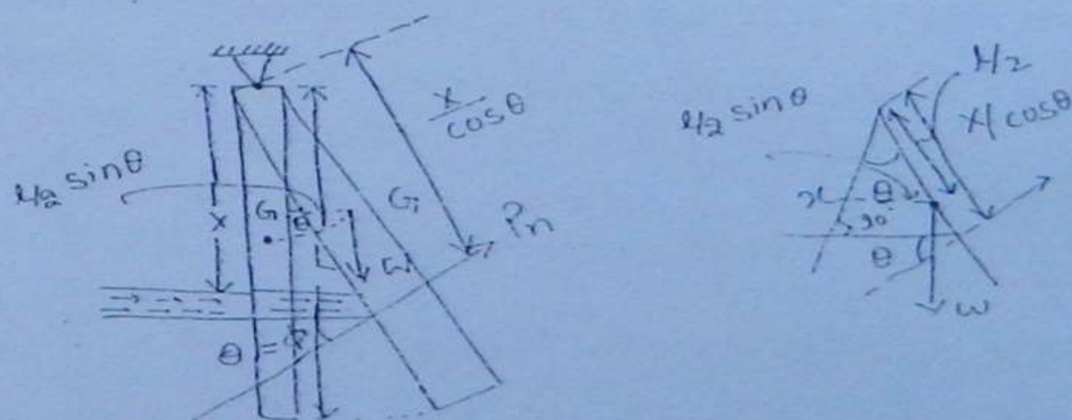
$\boxed{\text{tangential vel.} = \tfrac{1}{2}\ \text{Jet velocity}}$

$$\therefore \eta_{Max} = \frac{2\left[V\times V/2 - (V/2)^2\right]}{V^2} = 0.5$$

$$= 50\%$$ imp

NOTE :- In above analysis vanes are flat & If vanes are made curved than Efficiency may be further increased as in case of Platon wheel.

case VIIth: Jet of water strikes normal to a hanging vertical plate



$$P_n = \rho a\left[V\cos\theta - 0\right]$$

$$P_n = \rho a V^2 \cos\theta$$

$a \rightarrow$ area of Jet

$\cos\theta$ ... /2

$\hookrightarrow$ wt. of Plate

$$\therefore (\rho a v^2 \cos\theta)\left(\frac{x}{\cos\theta}\right) = \omega\left(\frac{L}{2}\sin\theta\right)$$

$$\therefore \boxed{\sin\theta = \frac{2\rho a v^2 . x}{WL}}$$

Special case:

When $x = L/2$ [Jet strikes on the C.G.]

$$\therefore \boxed{\sin\theta = \frac{\rho a v^2}{W}}$$

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Prob. 1

A Jet of water of dia. 25 MM dia strikes to a 20cm×20cm square plates of uniform thickness with a vel. of 10 M/sec at the centre of the plate which is suspended vertically by a hinge on its top edge. The weight of the plate is 98.1N/m. The Jet strikes normal to the plate. What Force must be applied at lower edge of plate so that plate is kept vertical. If the plate is allowed to deflect freely what will be the angle of defln with vertical due to the force exerted by the Jet of water.

Ans: $\theta = 30$, $F = 24.5$ N

Turbines

* used to convert hydraulic Energy into Mechanical Energy which is further converted into Electric Energy by Means of generators.
* It extracts energy

Pumps

/ Fuel energy

* First electric Energy is converted into Mechanical energy by Motor & Mech. Energy is converted into Pressure energy or Hydraulic Energy by pumps.
* It adds energy.

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Turbines:

Energy
- → Thermal power plant ≃ 71 to 72 %
- → Hydral Power plant ≃ 23 to 24 %
- → Atomic " " ≃ 1.5 to 2 %
- → Minor
  - → Tidal Energy
  - → wind "
  - → solar "
  - → Biogas "

classification of Turbnies:

a) classification on the basis of available head

- → High head Turbines { Suitable when $H > 250\ m$ }
  - Ex: Pelton wheel
  - Turgo Impluse wheel
- → Medium head Turbines { $60M \leq H \leq 250M$ }
  - Ex: Francies turbines.
- → Low head Turbines [$H \leq 60M$]
  - ex: Kaplan & prop Turbines.

- → High [V_1 = 300 to 1000]
  - ex: Kaplan & propeller
- → Medium sp. speed [V_2 = 60 to 300
- → Low sp. speed [V_3 < 60 m/sec.
  - Ex: Pleton wheel

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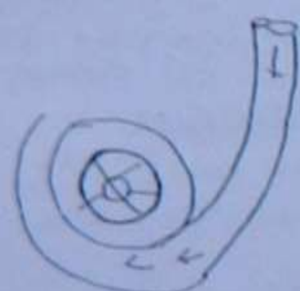
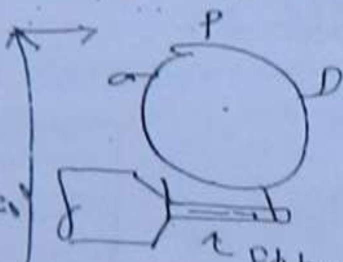
c> classification on the basis of available energy at the inlet of turbines

- → Reaction Turbine (Pressure Turbine) * at the inlet of turbine Energy available is pressure energy & K.E.
- → Impluse turbine (velocity turbine)
  - → only K.E. is available at the inlet & pressure is atmospheric



Ex: Reaction turbine
- Propeller
- Francis T.
- Kaplan



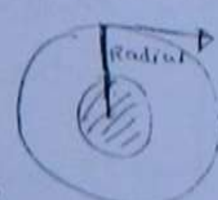
Impluse turbine

Ex: Pelton wheel
Turgo Impluse wheel

- Reaction turbnies have closes casing

d> classification on the basis of type of flow with in the turbine

- → Radial Flow
  - Inward Radial flow → Francis Turbines
  - outward Radial flow
    - → centrifugal Pumps commonly but some turbines may also be designed
- → Tangential Flow (Parallel Flow) → Ex: (a) Pleton wheel (b) Turgo Impluse wheel
- → Axial Flow → Ex: Kaplan, Propeller
- → Mixed Flow (water may enter in Radial dirn & may leave in axial direction)
  - Ex: Modern FrancisT.



~~c> classification on the basis of which is in practical use)~~

is combination of Last two classification.

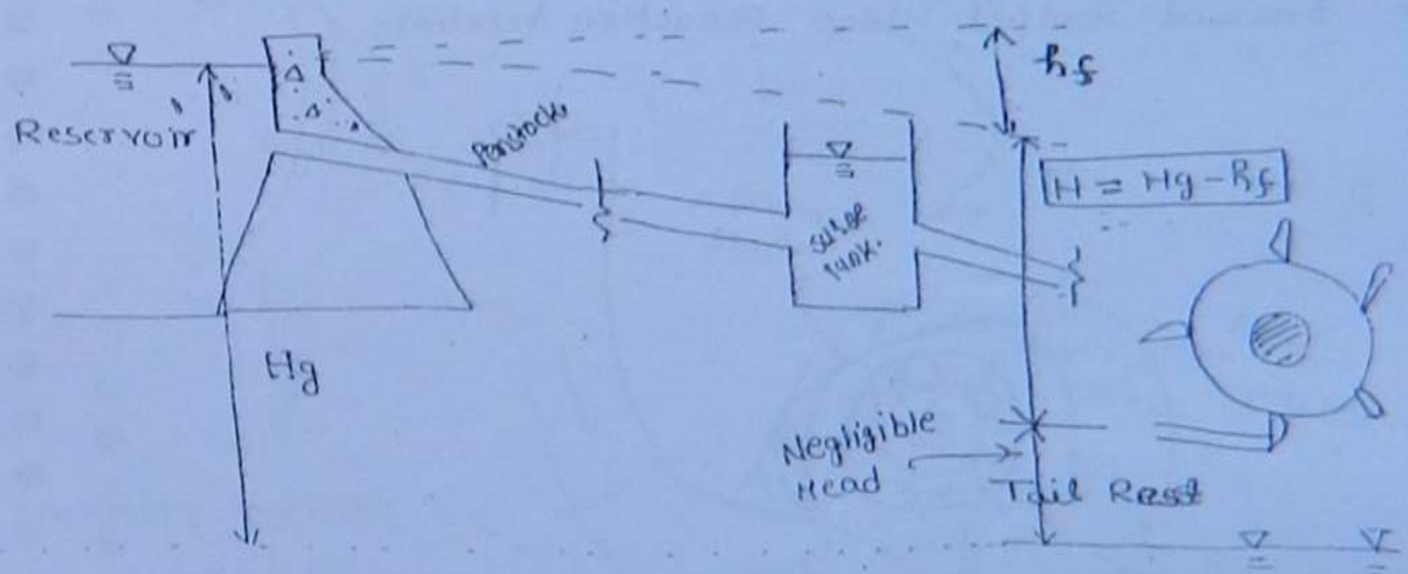
i> Francis Turbines → Inward Radial flow Reaction turbine.

ii> Pelton wheel / Turgo Impluse wheel → Tangential flow Impluse turbine.

iii> Kaplan / Propeller → . Axial flow Reaction turbines

iv> Modern Francis → Mixed Flow Reaction turbines

Important units of hydropower plant: ⤓ 2/9



a> Surge Tank: ⤓

Betn the reservoir & turbine house, surge tank is provided inorder to minimize water hammer pressure problem in penstock. Surge tank also helps in maintaining constant head at turbine.

b> Penstock: ⤓

It is the pipe through which water is brought from reservoir or from surge tank to the turbine chamber. The pipe always takes head loss at the turbine energy is used to convert Into work Pressure at exit of turbine May fall below atmospheric therefore disposal of exit water directly into atmosphere is not safe hence a tube of gradually diverging section is used to carry exit water of turbine to tail rest & this is always submerged at some depth below tail Rest Level.

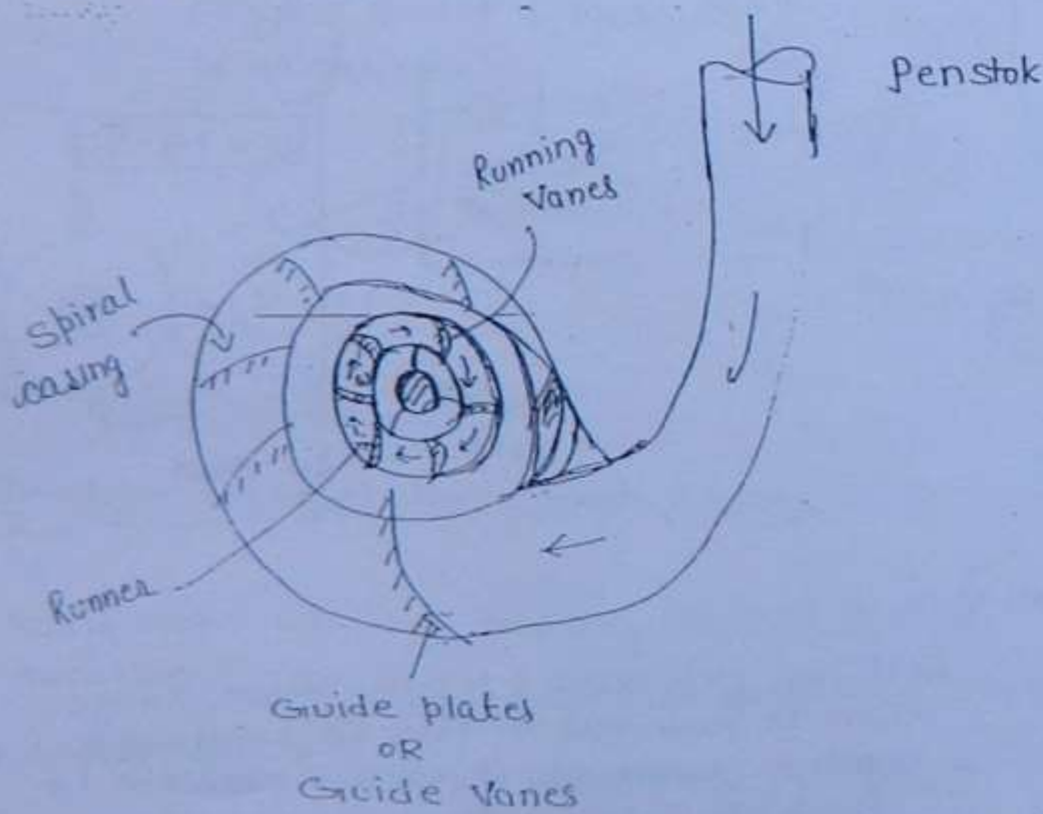
2. In case of Impluse turbines, pressure remains const. hence draft tubes is not essential.

d) Turbine units:]

(220)

1) Francis Turbine:]

* Inward Radial Flow Reaction turbine



- Casing is a spiral chamber which is of gradually decreasing area inorder to keep constant velocity at inlet of vanes. [as decreases, as A is decreases] somewater entered into runner
- Guide plates are parmanently attached in casing which ... the water to enter into runner
- Runner is rotating unit on which curved vanes are

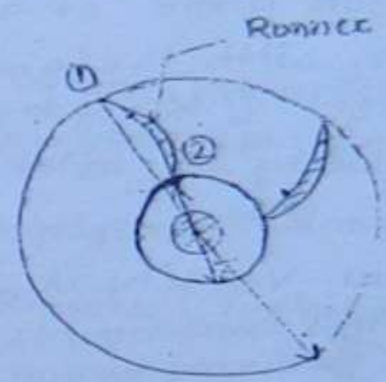
∴ Rotating Runner transfers Energy to shaft & finally the shaft may be connected to generator.

29/10/04

1> FRANCIS TURBINE: ↓

(24)

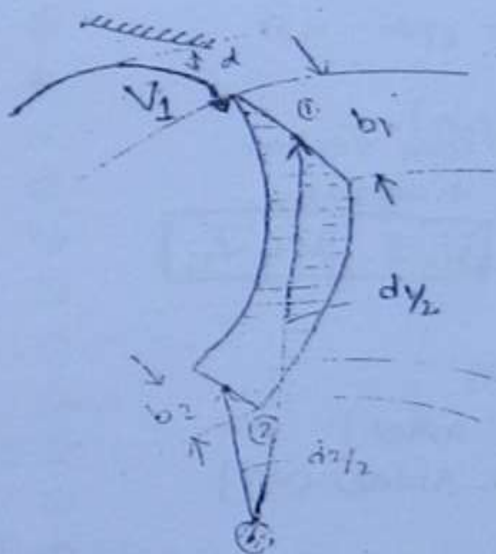
Runner ↓

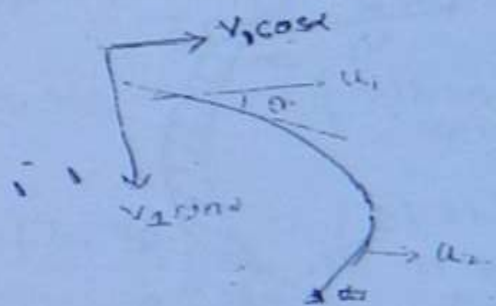


① → Entry point
② → Exit point

Inner dia. = Dia. at exit
$= d_2$

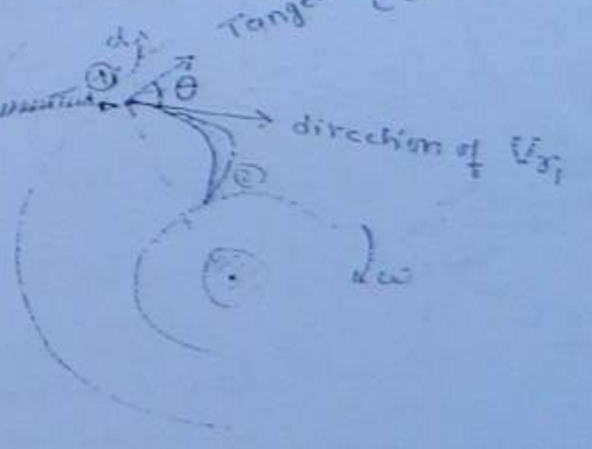
outer diameter
= Dia. at Inlet
$= d_1$





Velocity triangle : ↓

Guide Blade (Fixed to the casing)



u = Tangential velocity / Peripherial velocity of Runner at Inlet

$= r_1\omega$

$= d_1/2\ \omega$

$= d_1/2 \left(\frac{2\pi N}{60}\right)$

$\boxed{u_1 = \frac{\pi d_1 N}{60}}$ m/sec.

θ = Vane Angle at inlet of
Runner

V_{r1} = Relative velocity at inlet (Rel. vel. of fluid with respect to Blade.

$\boxed{\theta > 90^\circ, = 90^\circ, < 90^\circ}$

If $\theta = 90^\circ$, Vanes are said to be radial at inlet

α = Guide blade angle or angle b/w u_1 & V_1

α = Angle of V_1 with the tangent of wheel / Runner at inlet [b/w 10 to 90°]

V_1 = Abs. velocity of fluid at inlet

$V_1 \cos\alpha$ = Tangential component of abs. velocity [whirl velocity at inlet V_{w1}). This component is responsible for producing torque

Exit Point

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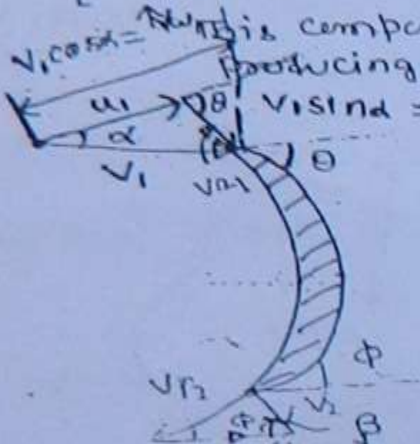
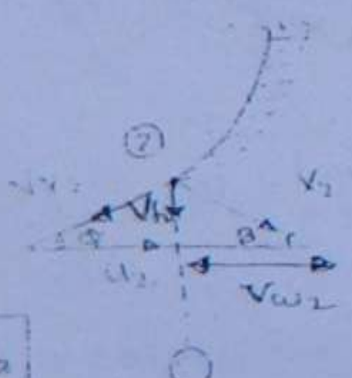


From vector diagram

$\boxed{\overline{V_1} = \overline{u_1} + \overline{V_{r_1}}}$

$V_1 \sin\alpha$ = Radial component of V_1
(Radial velocity at inlet)
(velocity of flow at inlet) [V_{f1}]



$\boxed{\overline{V_{r_2}} + \overline{u_2} = \overline{V_2}}$

V_{r2} = Relative velocity at exit which is in the direction of vane. In Rxn. turbine V_{r1} need not necessarily to be equal V_{r2}

u_2 = Tangential velocity of the Runner at exit

V_2 = Abs. velocity at exit

→ $\boxed{\beta \text{ May be } > 90^\circ, = 90^\circ, < 90^\circ}$

$V_2 \sin \beta = V_{f_2} =$ Radial (Flow) velocity at exit

NOTE: If β is 90° than vanes are said to be radial at exit or if turbines discharges radially outward than $\beta = 90°$

- In case of Francis Turbine in order to increase the efficiency β is purposely made 90°.

- Following points may be noted w.r.t. Francis Turbines

(i) Discharge through Runner (Turbine) : 223

$$Q = A_{f_1} \times V_{f_1}$$

$$= (\pi d_1 b_1) \times V_{f_1} \text{ ------ } \{\text{When thickness of vane is negligible}\}$$

$$= [\pi d_1 - nt]\, b_1 V_{f_1} \text{ ------ } \{\text{If there are } n \text{ vanes having } t \text{, thickness each}\}$$

$$= K . \pi d_1 b_1 V_{f_1} \text{ ------ } \{K \text{ is coeff. which is account for reduced area, occupied vanes thickness}\}$$

For eg: 5% area at circumference is occupied by vane thickness than $K = 0.95$

At exit $\quad Q = V_{f_2} \times A_{f_2}$

$$= (\pi d_2 b_2) \times V_{f_2}$$

$$= [\pi d_2 - nt] \times b_2 V_{f_2}$$

$$= K \pi d_2 . b_2 . V_{f_2}$$

$$\boxed{Q = \pi d_1 b_1 V_{f_1} = \pi d_2 b_2 V_{f_2}}$$

$$u_1 = \frac{\pi d_1 N}{60} = r_1 \omega$$

$$u_2 = \frac{\pi d_2 N}{60} = r_2 \omega$$

$$\therefore \boxed{u_1/u_2 = d_1/d_2}$$

(3) velocity triangle at inlet.

case A) when $\theta < 90°$

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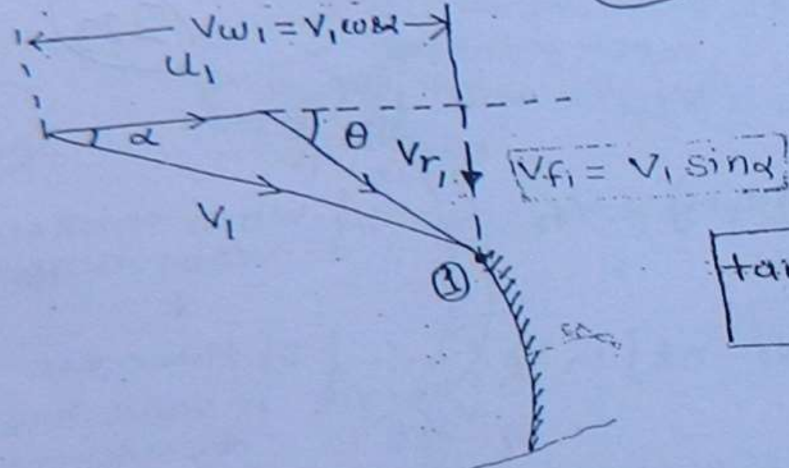


$$\boxed{\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}}$$

NOTE: If θ is not given than in order to obtain velocity triangle compare magnitudes of u_1 & V_{w_1}. If

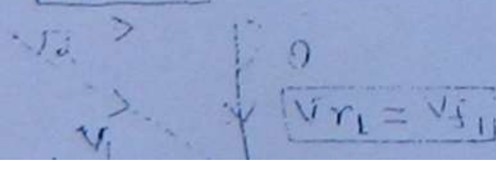
$$\boxed{u_1 < V_{w_1}} \Rightarrow \boxed{\theta < 90°}$$

If $u_1 = V_{w_1} \Rightarrow \theta = 90°$

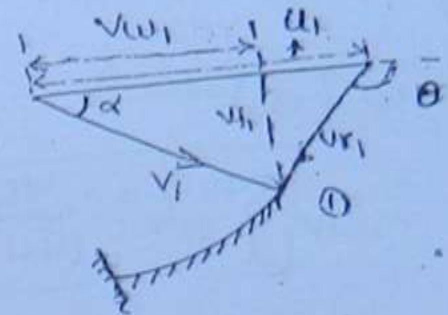
If $u_1 > V_{w_1} \Rightarrow \theta > 90°$

case B) when $\theta = 90°$ means vanes are set Radially at inlet

$$\boxed{u_1 = V_{w_1}}$$



case C> when $\theta > 90°$



$$\boxed{\tan(180-\theta) = \frac{V_{f_1}}{u_1 - V_{w_1}}}$$

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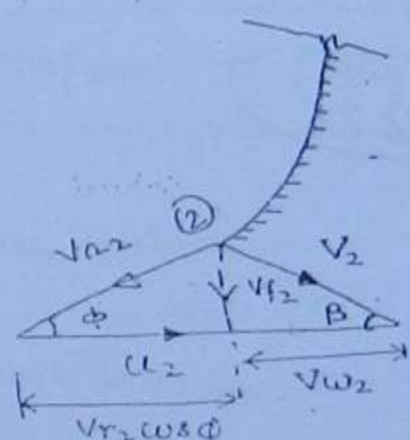
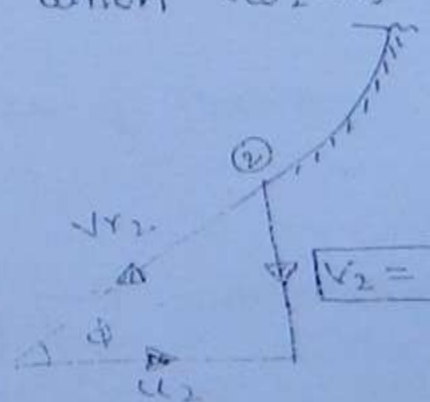
4> velocity triangle at exit

case A> when V_{w_2} is in direction of V_{w_1}



$$\boxed{|u_2| = |V_{r_2}\cos\phi| + |V_{w_2}|}$$

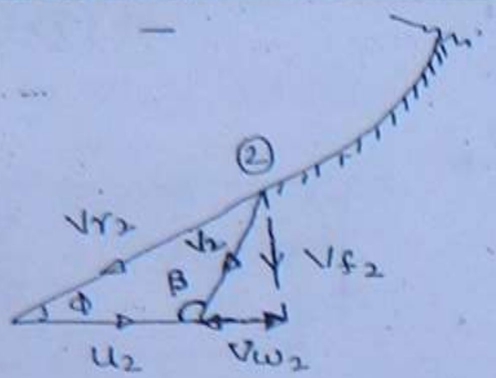
V.IMP case B> when $V_{w_2} = 0$, whirl component at outlet is zero or turbine discharges radially outward.



$$\boxed{V_2 = V_{f_2}}$$

- $\beta = 90°$, $V_{w_2} = 0$
- $|u_2| = |V_{r_2}\cos\phi|$
- $\tan\phi = \dfrac{V_{f_2}}{u_2}$

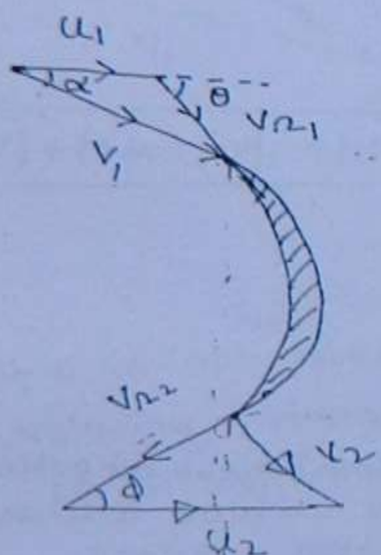
- commum case of Francis Turbine



$$\boxed{\tan\phi = \frac{V_{f_2}}{|u_2| + |V_{w_2}|}}$$

$$\therefore \boxed{|V_{r_2}\cos\phi| = |u_2| + |V_{w_2}|}$$

5> Work-done per sec by the water on Runner (Runner Power)



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Rate of change of Angular momentum = T

$$\text{Work-done/sec.} = T \times \omega$$

T = Angular momentum/sec at inlet − Angular Momentum/sec at outlet

Ang. Momentum ...

$\therefore$ Ang. Moment/sec $= \left(\frac{\text{Mass}}{\text{sec}}\right) \cdot V r$

$$\therefore T = (\text{Mass/sec})\, V_{w_1} r_1 - \left(\frac{\text{Mass}}{\text{sec}}\right) V_{w_2} r_2$$

$$\therefore T = \rho Q \left[V_{w_1} r_1 - V_{w_2} r_2 \right]$$

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$$\therefore \text{work-done/sec.} = T \times \omega$$

$$= \rho Q \left[V_{w_1} r_1 \omega - V_{w_2} r_2 \omega \right]$$

$$= \rho Q \left[V_{w_1} u_1 - V_{w_2} u_2 \right].$$

$$\therefore \boxed{\text{Work-done/sec.} = \left(\frac{w}{g}\right) Q \left[V_{w_1} u_1 - V_{w_2} u_2 \right]}$$

[$w \rightarrow$ unit wt. of fluid]

• For Francis Turbine

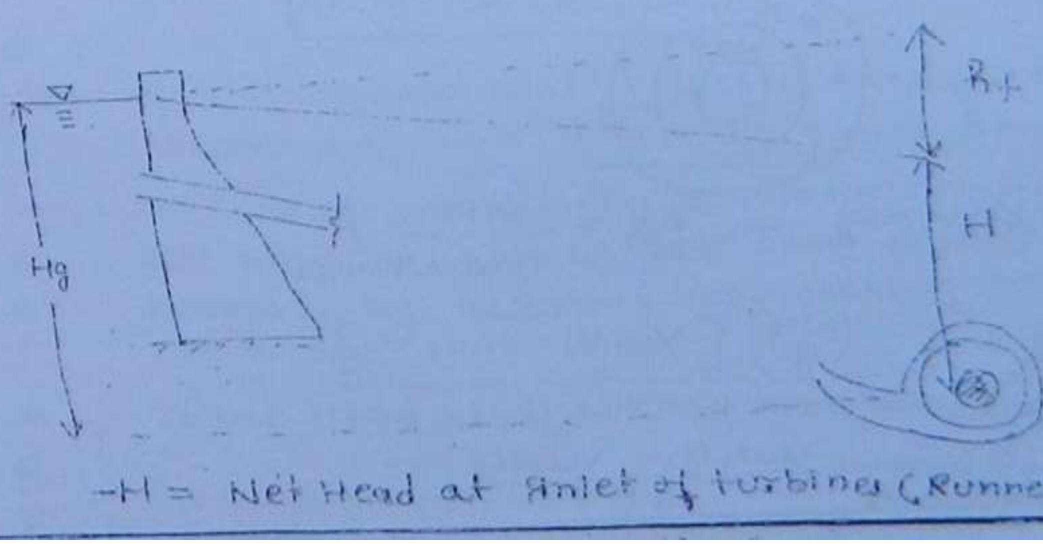
$$V_{w_2} = 0$$

$\therefore$ Work done/sec. = R.P. (Runner Power) :

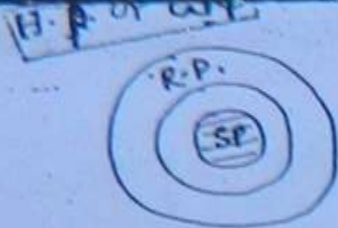
$$= \rho Q \left[V_{w_1} u_1 \right]$$

$$= \frac{wQ}{g} \left[V_{w_1} u_1 \right]$$

6> Powers of Turbine



-H = Net Head at inlet of turbines (Runner = $H_g - h_f$



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a> Hydraulic Power (Water Power): It is the power available at the inlet of turbine

$$H.P.\,(W.P.) = \omega \cdot Q H \rightarrow KW \quad \left[\omega = 9.81\ KN/m^3\right]$$

$$= \frac{\rho Q H}{75} \rightarrow H.P.\ \{\text{Horse Power}\}$$

b> Runner Power = work-done by water on Runner

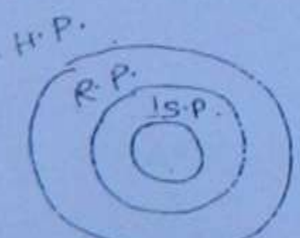
$$= \frac{\omega Q}{g}\left[V_{\omega_1} u_1 - V_{\omega_2} u_2\right]$$

c> Shaft-Power (Break Power): This is final power available at the shaft of turbine.

$$\therefore S.P. = R.P. - \text{Transmission Losses}\ \{\text{Mechanical Losses}\}$$

7> Efficiency of Turbines: ↓

(a)



$$\text{Hydraulic efficiency} = \eta_h = \frac{R.P.}{\text{Water Power}}$$

$$\therefore \eta_h = \frac{\left(\frac{\omega Q}{g}\right)\left[V_{\omega_1} u_1 - V_{\omega_2} u_2\right]}{\omega Q H}$$

$$\therefore \boxed{\eta_h = \frac{V_{\omega_1} u_1 - V_{\omega_2} u_2}{g H}}$$

$$\boxed{\eta_H = \frac{V_{w_1} u_1}{gH}}$$

2> Mechanical efficiency

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$$\boxed{\eta_M = \frac{S.P.}{R.P.}}$$

3> overall efficiency

$$\boxed{\eta_0 = \frac{S.P.}{H.P.}} = \frac{S.P.}{R.P.} \times \frac{R.P}{H.P} = \eta_H \times \eta_M$$

$$\therefore \boxed{\eta_0 = \eta_H \times \eta_M}$$

NOTE: If Leakage in the turbine chamber are also considered than Volumetric efficiency may be cosidered as

$$\boxed{\eta_V = \frac{Q'}{Q}}$$

$Q' \rightarrow$ Discharge at exit
$Q \rightarrow$ Discharge at inlet

However this is more Imp. for centrifugal pump & Negligible for turbines

If it is also accounted than overall efficiency

$$\boxed{\eta_0 = \eta_H \times \eta_M \times \eta_V}$$

NOTE: (a) If it is given that velocity through the runner is constant than $\boxed{V_{f_1} = V_{f_2}}$ and hence $A_{f_1} = A_{f_2}$

(b) In case of some data is missing than Bernoulli's eqn may be applied b/w inlet & outlet of the runner by assuming NO-Losses of head through the runner.

[N-wt.]

(c) Total Head at inlet = work done/sec/unit wt of water
+ Energy head at outlet + Losses.

$$= \frac{\left(\frac{\omega Q}{g}\right)\left[V_{w_1}u_1 - V_{w_2}u_2\right]}{\omega Q} + \frac{V_2^2}{2g} + h_L$$

If $V_{w_2} = 0$ & Losses are Negligible than

[pressure energy is Negligible.

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$$\boxed{H = \frac{V_2^2}{2g} + \frac{V_{w_1}u_1}{g}}$$ .... Master Formula.

(d) If speed ratio is given than It Means

$$\phi = \frac{u_1}{\sqrt{2gH}}$$

[It is only significant for Inlet point
ϕ Lies between 0.75 to 0.85

(e) If flow ratio is given than

$$\psi = \frac{V_{f_1}}{\sqrt{2gH}}$$

[Also Valid at Inlet
ψ Lies b/w 0.15 to 0.35

(f) If width ratio is given than

$$= b/d \qquad \left\{ b_1/d_1 \right\}$$

An inward flow reaction turbine [illegible] have an external diameter of 700mm & a width of 180mm. If the guide vanes are at 20° to the wheel tangent and the abs. velocity of water at inlet is 25 M/sec. Then find

a> Discharge through the turbine

b> Runner vane angle at inlet

$N = 500$ r.p.m.

$d_1 = 700$ mm $= 0.7$ m

$b_1 = 150$ mm $= 0.15$ M

$\alpha = 20°$

$V_1 = 25$ M/sec.

$\theta = ?$

$Q = ?$

$$u_1 = \frac{\pi d_1 N}{60}$$

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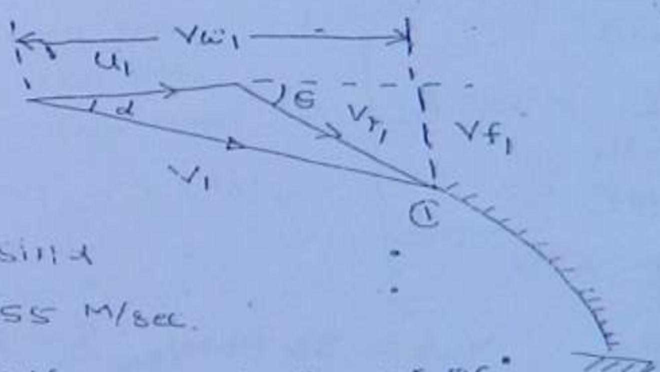
$u_1 = 18.326$ M/sec

$V_{w_1} = V_1 \cos\alpha$

$= 25 \cos 20°$

$= 23.49$ M/sec

since $u_1 < V_{w_1} \Rightarrow \theta < 90°$



$V_{f_1} = V_1 \sin\alpha$

$= 8.55$ M/sec.

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow \theta = 58.86°$$

$Q = A_{f_1} \cdot V_{f_1}$

$= (\pi d_1 b_1) V_{f_1}$

$= \pi \times 0.7 \times 0.15 \times 8.55$

$= 2.834$ M³/sec

Prob. 2. A reaction turbine works at 480 r.p.m. under a head of 120 M. If the dia. at inlet is 1.2 M & flow area at inlet is 0.4 M². The angle made by absolute and relative velocities at the inlet with the tangent at wheel is 60° & 20° respectively then determine

a> Flow rate

b> Runner power developed

c> Hydraulic efficiency

→ $N = 450$ R.P.M.

$H = 120$ M

$d_1 = 1.2$ M

$A_{f_1} = 0.4 \ M^2$

$\alpha = 20°$

$\theta = 60°$

$V_{w_2} = 0$

$Q = ?$

R.P = ?

$\eta_R = ?$

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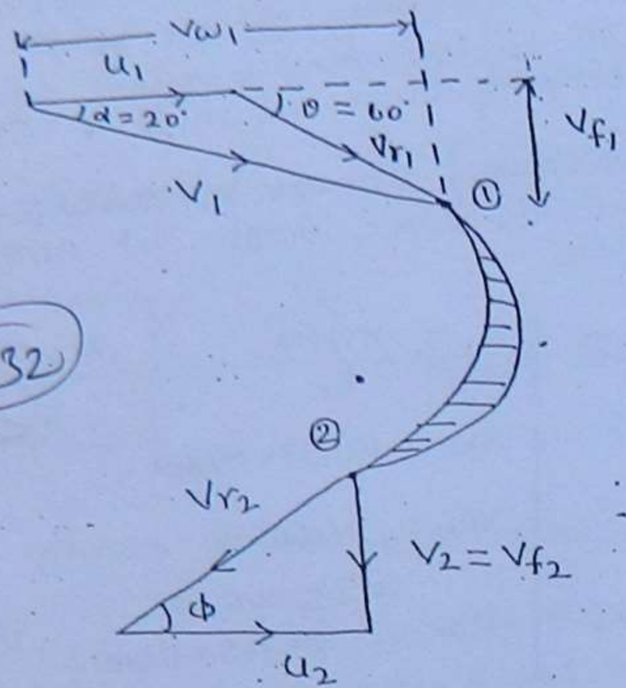

$$u_1 = \frac{\pi d_1 N}{60} = 28.27 \ M/sec.$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \quad \text{——①}$$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \quad \text{——②}$$

By eqn (1) & (2) $V_{w_1} = 35.78$ M/sec

$V_{f_1} = 13.03$ M/sec.

$$Q = A_{f_1} \cdot V_{f_1}$$

$$= 0.4 \times 13.03$$

$$Q = 5.21 \ m^3/sec$$

$$\eta_H = \frac{V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}}{gH} = 85.96\%$$

$$\text{Runner power} = \frac{wQ}{g}\left[V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}\right]$$

$$= 5270 \ KN$$

Prob 3. An inward reaction flow turbine runs at 192 r.p.m. The dia & width at inlet are 600 mm & 150 mm while the outlet dia. is 300 mm. The velocity of flow through the runner is constant at 1.5 m/sec. If the guide blades are 10° to the wheel tangent. Draw the inlet & outlet velocity diagram, if velocity of whirl at outlet is zero. Determine

(i) Runner Blade Angle
(ii) Abs. Velocity of water leaving the guide blade
(iii) Rel. velocity of water at inlet
(iv) width of the wheel at outlet
(v) Discharge through turbine
(vi) Head supplied
(vii) R.P. supplied (developed)
(ix) Hydraulic Efficiency.

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Given

$N = 192$ r.p.m.

$d_1 = 600$ mm

$b_1 = 150$ mm

$d_2 = 300$ mm

$V_{f1} = V_{f2} = 1.5$ m/sec

$\alpha = 10°$

$V_{w2} = 0$

θ & $\phi = ?$

$$u_1 = \frac{\pi d_1 N}{60} = 6.03 \text{ m/sec}$$

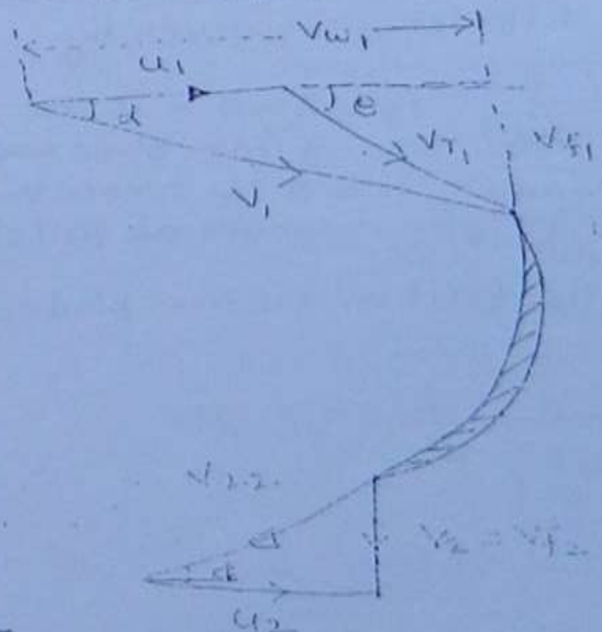
$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$\Rightarrow V_{w1} = 8.507 \text{ m/sec}$$

$$V_1 = \sqrt{V_{f1}^2 + V_{w1}^2}$$

$$= 8.63 \text{ m/sec}$$

Since $u_1 < V_{w1} \Rightarrow \theta < 90°$



$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\Rightarrow \theta = 31.19°$$

$$\frac{u_1}{u_2} = d_1/d_2$$

$$\Rightarrow u_2 = u_1 \times d_2/d_1$$

$$= 3.015 \text{ m/sec}$$

$$\tan\phi = \frac{V_{f2}}{u_2}$$

$$\Rightarrow \phi = 26.45°$$

$Q = \pi d_1 b_1 \cdot V_{f_1}$

$= \pi \times 0.6 \times 0.15 \times 1.5$

$= 0.424 \; m^3/sec$

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$= \pi d_2 . b_2 . V_{f_2}$

$\Rightarrow b_2 = 0.3 m = 300 mm$

$\sin\theta = \dfrac{V_{f_1}}{V_{r_1}} \Rightarrow V_{r_1} = V_{f_1} \operatorname{cosec}\theta$

$= 2.89 \; m/sec.$

$R.P. = \dfrac{wQ}{g}\left[V_{w_1}u_1 - \cancel{V_{w_2}}^{0} u_2\right]$

$= 21.74 \; KW$

- Apply Master formula {Apply B.Eqn b/w ① & ②}

$H = \dfrac{V_2^2}{2g} + \dfrac{V_{w_1}u_1}{g}$ [Assuming Minor Losses to be neglected]

$= 5.34 \; M$

$\eta_h = \dfrac{V_{w_1}u_1}{gH}$

$= 95.1 \%$

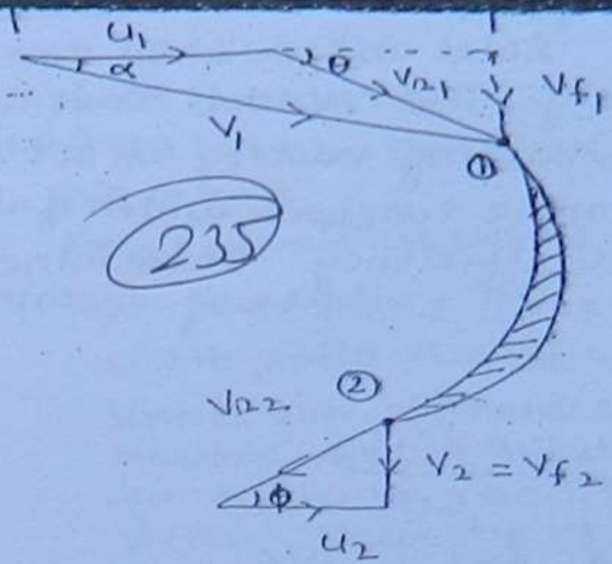
Prob.4
CS/2003

An inward flow reaction turbine discharges radially the velocity of flow through the runner is const. So that hydraulic efficiency is given by

$$\eta_h = \frac{1}{1 + \dfrac{\frac{1}{2}\tan^2\alpha}{1 - \left(\dfrac{\tan\alpha}{\tan\theta}\right)}}$$

, where α is the guide blade angle & θ is runner vane angle at inlet.

Assume there is no friction on the blades.



$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$\Rightarrow V_{f1} = V_{w1}\tan\alpha$$

$$V_2 = V_{f2} = V_{f1} = V_{w1}\tan\alpha \quad \text{— (1)}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1}-u_1}$$

$$\therefore \tan\theta = \frac{V_{w1}\tan\alpha}{V_{w1}-u_1}$$

$$\therefore V_{w1}-u_1 = V_{w1}\frac{\tan\alpha}{\tan\theta}$$

$$\therefore u_1 = V_{w1}\left[1-\frac{\tan\alpha}{\tan\theta}\right] \quad \text{— (2)}$$

$$\eta_R = \frac{V_{w1}u_1}{gH} \quad \text{— (iii)}$$

Apply B. Eqn b/w (1) & (2)

$$H = \frac{V_2^2}{2g} + \frac{V_{w1}u_1}{g}$$

$$= \frac{V_2^2}{2g} + \frac{V_{w1}\cdot u_1}{g}$$

$$\therefore \frac{V_{w1}u_1}{g\eta_R} = \frac{V_2^2}{2g} + \frac{V_{w1}\cdot u_1}{g}$$

$$\Rightarrow \frac{1}{\eta_R} = \frac{V_2^2}{2u_1V_{w1}} + 1$$

$$\therefore \eta_R = \frac{1}{1+\left(\frac{V_2^2}{2u_1V_{w1}}\right)}$$

$$= \frac{1}{1+\dfrac{V_{w1}^2\tan^2\alpha/2}{V_{w1}\times V_{w1}\left[1-\frac{\tan\alpha}{\tan\theta}\right]}}$$

$$\therefore \eta_R = \frac{1}{1+\left[\dfrac{\tan^2\alpha/2}{1-\frac{\tan\alpha}{\tan\theta}}\right]}$$

5/2001 at an avg. head of 160 m with a discharge of 80 m³/sec. The inlet and outlet dia. are 4 m & 2 m respectively. The runner blade angle is 120°. Radial discharging velocity at outlet is 15 m/sec. Assuming constant width of wheel and 90% hydraulic efficiency. Determine H.P. produced, in MW & R.P.M. of machine.

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Given

$H = 160$ m

$Q = 80\ m^3/sec$

$d_1 = 4$ m

$d_2 = 2$ m

$\theta = 120°$

$V_{f_2} = V_2 = 15$ m/sec.

$V_{w_2} = 0$

$b_1 = b_2 = $ const.

$\eta_h = 0.9$

H.P. = ?

N = ?

$$u_1 = \frac{\pi d_1 N}{60}$$

$$Q = \pi d_2 b_2 V_{f_2}$$

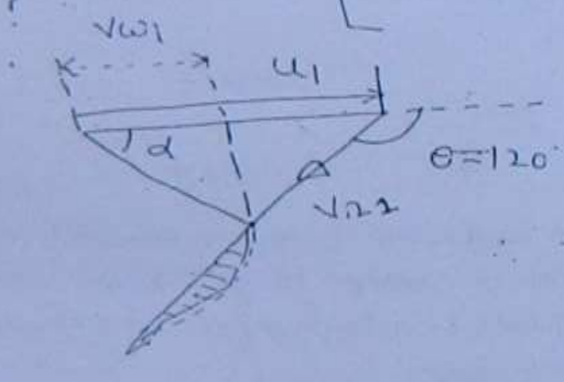
$$80 = \pi \times 2 \times b_2 \times 15$$

$$\Rightarrow b_2 = 0.84 \text{ m}$$

$$b_1 = b_2 = 0.84 \text{ m}$$

$$Q = V_{f_1} \cdot \pi d_1 b_1$$

$$\Rightarrow V_{f_1} = 7.5 \text{ m/sec.}$$



$$\tan(180-\theta) = \frac{V_{f_1}}{u_1 - V_{w_1}} \quad \text{---(I)}$$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$\Rightarrow V_{w_1} \cdot u_1 = 0.9 \times 9.81 \times 160 \quad \text{---(II)}$$

By solving eqn (I) & (II)

$$u_1 = 39.81 \text{ m/sec} = \frac{\pi d_1 N}{60}$$

$$V_{w_1} = 35.96 \text{ m/sec} \Rightarrow N = 190 \text{ r.p.m.}$$

∴ Hydraulic Power = $\rho Q g H$

$= 9.81 \times 80 \times 160$ kW

∴ H.P. = 125.5 MW.

Sheet 1 P.U. Poub17

Q. 1 Design a Francis turbine runner with the following data:

Net head 68 m

Speed $N = 750$ r.p.m

Output power $= 330$ kW (S.P.)

$\eta_h = 94\%$

η_o (overall efficiency) $= 85\% = \eta_o$

ψ = Flow ratio $= 0.15 \Rightarrow \psi = \frac{V_{f1}}{\sqrt{2gH}} = 0.15$

Width ratio $n = b/d = 0.1$

Inner dia. of runner is ½ of outer dia. Also assume 6% of the circumferential area of the runner to be occupied by the thickness of vanes. Velocity of flow remains constant & flow is radial at exit.

(237)

Given $K = 0.94$, $d_2 = d_1/2$, $b_1/d_1 = 0.1$, $V_{f1} = V_{f2}$

$$\boxed{\eta_o = \frac{S.P}{H.P.}} = \frac{330}{H.P} \Rightarrow H_P = 388.235 \text{ kW}$$

$$= \omega Q H, \quad \left[\omega = 10\,\frac{kN}{m^3} = 9.81\,\frac{kN}{m^3} \right.$$

$$\Rightarrow Q = 0.58 \text{ m}^3/\text{sec}$$

$\therefore \frac{V_{f1}}{\sqrt{2gH}} \Rightarrow V_{f1} = 5.48$ m/sec

$$Q = K.(\pi d_1 b_1) \times V_{f1} = 0.94 \times \pi \times d_1 b_1 \times 5.48 = 0.58$$

$b_1 d_1 = 0.036 \text{ m}^2$ — (II)

$b_1/d_1 = 0.1$ — (I)

$b_1 = 0.1 d_1$

$\Rightarrow d_1 \times 0.1 d_1 = 0.036$

$\Rightarrow 0.1 d_1^2 = 0.036$

$\Rightarrow d_1 = 0.6$ M $= 600$ mm

$b_1 = 0.06$ M $= 60$ mm

$Q = 0.94 \pi d_1 b_1 V_{f1} = 0.94 \pi d_2 b_2 V_{f2}$

$\therefore b_2 = \frac{d_1 b_1}{d_2} \left(\frac{V_{f1}}{V_{f2}}\right)$

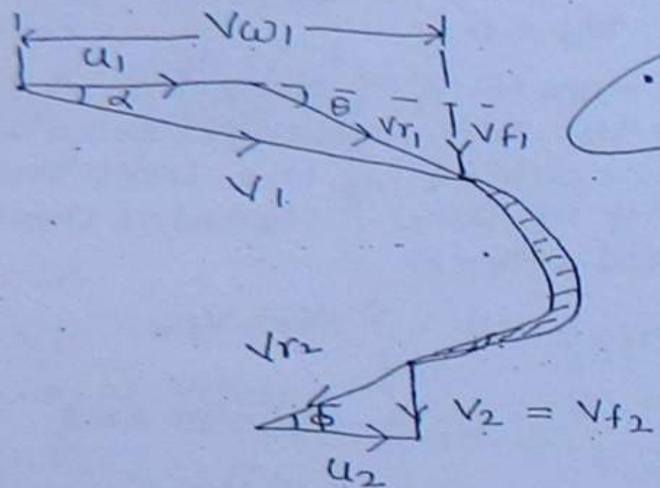
$\Rightarrow \boxed{b_2 = 120 \text{ mm}}$

$$\frac{\quad}{60} = \frac{\quad}{60} = 23.56 \text{ m/sec.}$$

$$\therefore \eta_H = \frac{V_{w_1} u_1}{gH} \Rightarrow V_{w_1} = \frac{0.94 \times 9.81 \times 68}{23.56}$$

$$= 26.61 \text{ m/sec.}$$

since $u_1 < V_{w_1} \Rightarrow \theta < 90^\circ$



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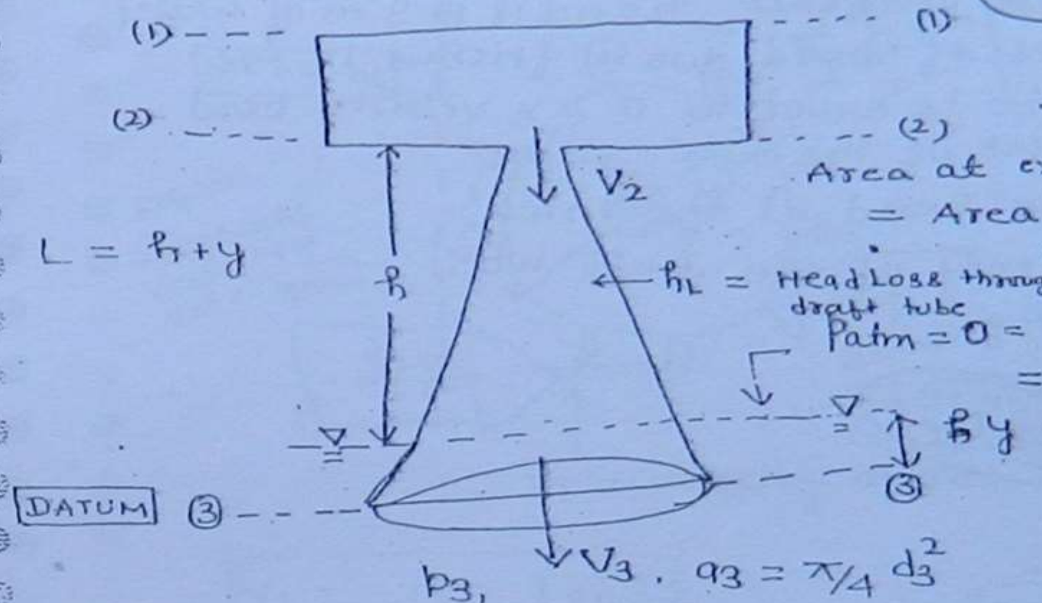
$$\therefore \tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow \theta = 60.8^\circ$$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_1/2} \Rightarrow \phi = 24.9^\circ$$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \Rightarrow \boxed{\alpha = 11.64^\circ}$$

DRAFT TUBE :



Area at exit of Runner = Area at inlet of draft tube $a_2 = \frac{\pi}{4} d_2^2$

$P_{atm} = 0$ = in gauge scale = 10.3 m head of water

p_3, V_3. $a_3 = \pi/4\, d_3^2$

Apply B. Eqn b/w (2) & (3)

$$(h+y) + \frac{V_2^2}{2g} + \frac{P_2}{w} = 0 + \frac{V_3^2}{2g} + \frac{P_3}{w} + h_L$$

$$= \frac{V_3^2}{2g} + \frac{P_{atm}}{w} + y + h_L$$

NOTE: Since P_2 is below atmosphere than it should be such that it should not fall below vapour pressure.

Efficiency of draft-tube :

$$\eta = \frac{\text{Actual converging of K.H. into Pressure head}}{\text{original K.H.}}$$

$$\boxed{\eta = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g} - h_L}{V_2^2/2g}}$$

sheet 21

A conical draft tube having inlet & outlet dia. 1m & 1.5m discharges water at outlet with velocity of 2.5 m/sec. The total length of the draft tube is 6m & 1.20 m Length of the draft tube is immersed in water. If the atmospheric pressure head is 10.3 m of water and Loss of head due to friction in the draft tube is equal to 0.2 × velocity head at outlet of the tube. Find

(i) Pressure head at the inlet.

(ii) Efficiency of the draft tube.

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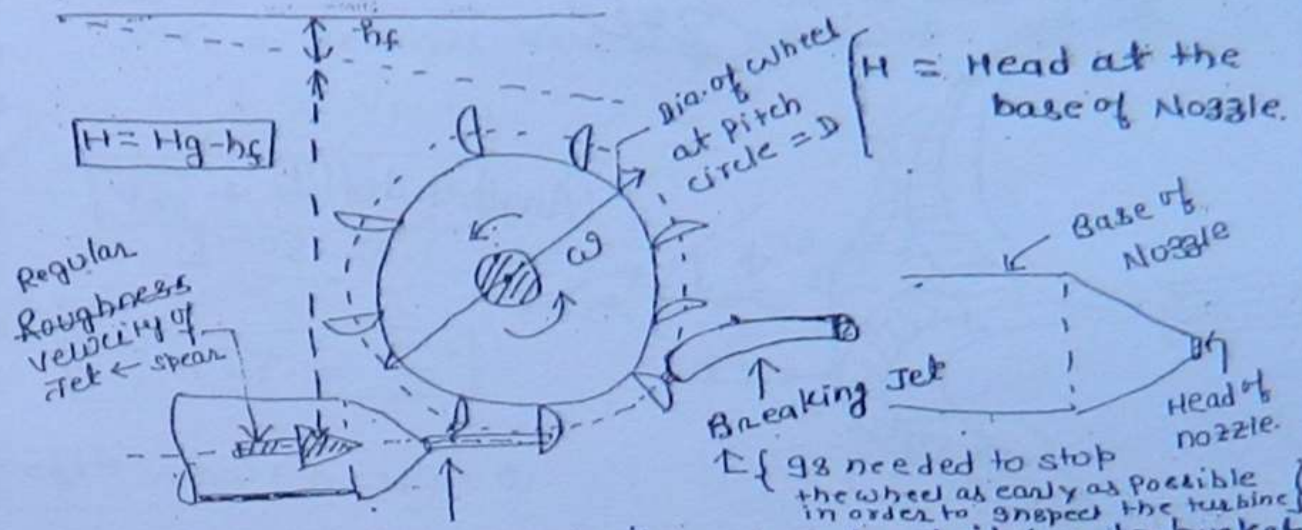
2> PELTON WHEEL :-

Tangential flow Impulse Turbine



V_1 = velocity of Jet striking to bucket

$$\therefore \boxed{V_1 = C_v \cdot \sqrt{2gH}}$$

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$$\boxed{C_v \simeq 0.97 \text{ to } 0.98}$$

→ When wheel is vertical and shaft is horizontal than it is called 'horizontal alignment', whereas 'vertical alignment' is that wheel is horizontal & shaft is vertical.

→ In the Pelton wheel hemispherical bucket are mounted on the pitch circle which may be 15 to 25 in number.

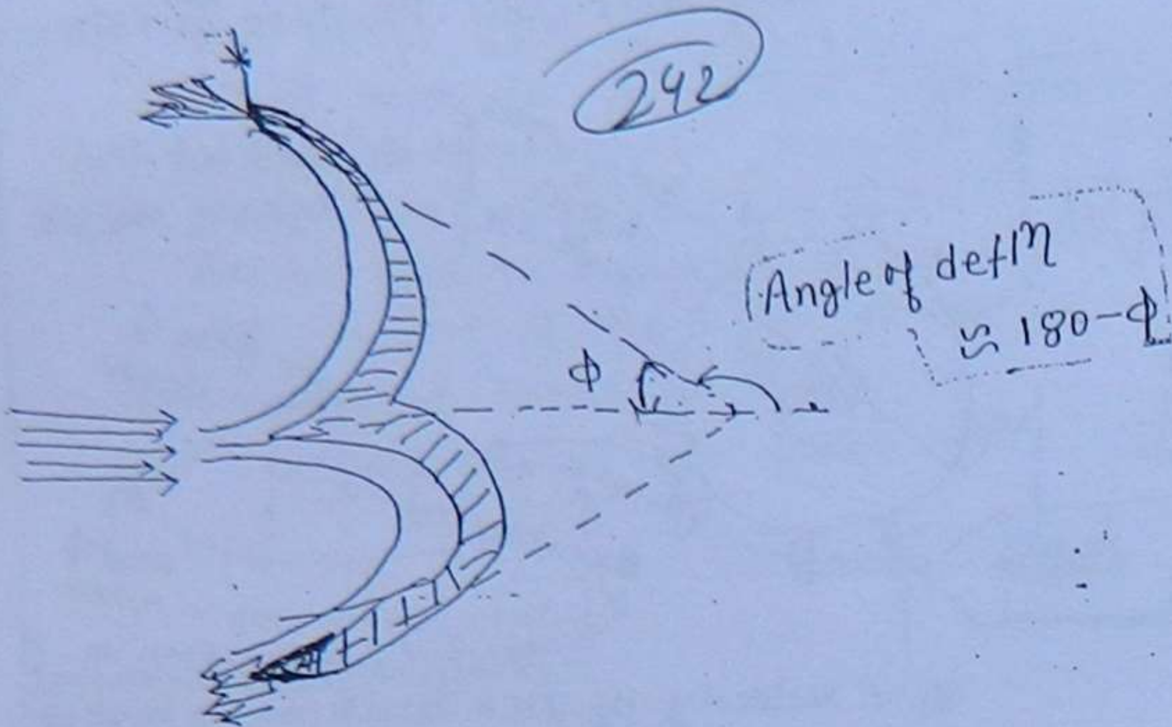
→ As far as possible Less No. of vanes should be provided so as to minimize friction losses but specified minimum no. of vanes should be provided so as to prevent the loss of discharge without hitting the vanes. No. of vanes required depends upon Jet ratio which is defined as dia of pitch circle to dia of Jet

$$m = \text{Jet Ratio} = \frac{\text{Dia of Pitch circle}}{\text{Dia of Jet}} = D/d$$

$$\therefore \boxed{\text{No. of Vanes} = 15 + 0.5\, m}$$

– where m is generally b/w 10 to 15 ≃ 12

Vanes are made [illegible] hemispherical bucket [illegible]



- Theoretically defl^n. angle should be 180° inorder to produce maxm. work-done but practically it causes retardation of coming vanes by hitting on the back of vanes hence appropriate angle of defl^n (160° to 165°) is desirable. In order to minimize friction Losses vanes are polished on inside.

→ Two hemispherical buckets are provided because it will cancell the y-component of force and alignment of bucket can be preserved.

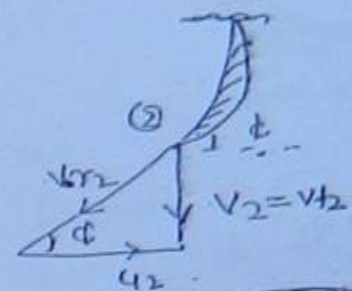
$$\boxed{u_1 = u_2 = \frac{\pi D N}{60}}$$

d = dia of jet.
D = Dia. of Buckets (Mean dia.)

V_{r_1} = Relative velocity at inlet

$= V_1 - u_1$

$$\boxed{\overline{V_{r_1}} + \bar{u}_1 = \bar{V}_1}$$



At exit

$$\boxed{\overrightarrow{V_{r_2}} + \overrightarrow{u_2} = \overrightarrow{V_2}}$$

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$$\boxed{\text{Defl}^{n} \text{ Angle} = 180 - \phi}$$

ϕ = Vane angle at exit

θ = Angle b/w u_1 & V_{r_1}

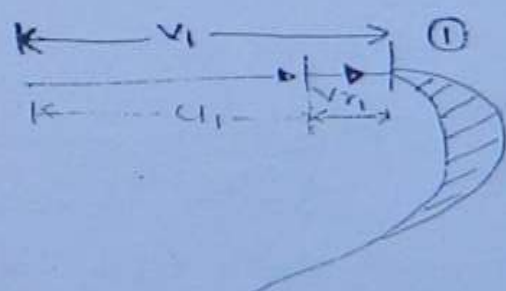
↓

Vane angle at inlet

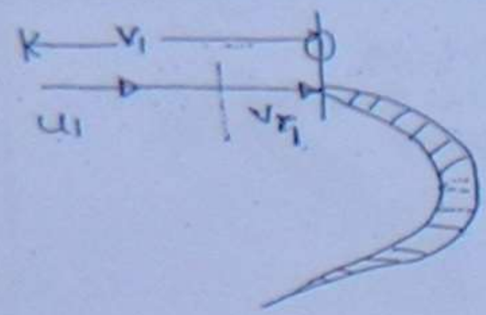
Velocity triangle at inlet:

Since u_1, V_{r_1}, V_1 are in same direction

∴ inlet vel. triangle reduces into a straight line.



Inlet velocity diagram:



$$\boxed{V_{r_1} = V_1 - u_1}$$

$$\boxed{\begin{aligned}\theta &= 0^\circ \\ \alpha &= 0^\circ\end{aligned}}$$

$V_{w_1} = V_1 \cos\alpha = V_1$

$V_{f_1} = 0$ { radial velocity at inlet }

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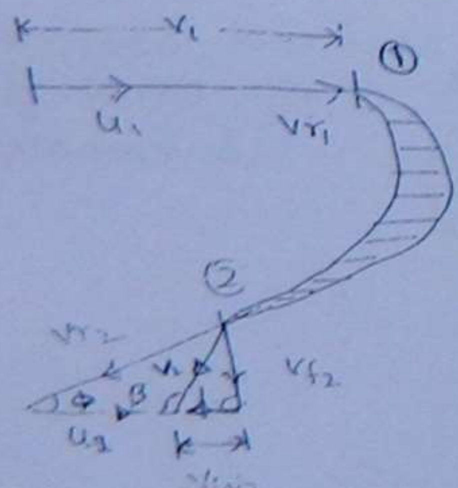
If Loss of friction on the surface of vane is negligible than $\boxed{V_{r_1} = V_{r_2}}$

$\boxed{V_{r_2} = V_{r_1}}$ ---------- when friction Loss is negligible

$\boxed{V_{r_2} = K V_{r_1}}$ ---------- If friction Losses are accounted. $(K < 1)$

Exit velocity diagram:



$$\boxed{\bar{V}_{r_2} + \bar{u}_2 = \bar{V}_2}$$

$$u_2 = u_1 = \frac{\pi D N}{60}$$

$$\boxed{\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}}$$

where $\boxed{V_{w_2} = -ve}$

$$= \frac{\omega Q}{g}\left[V_{w_1} u_1 - [-V_{w_2}] u_2\right]$$

$$= \frac{\omega Q}{g}\left[V_{w_1} u_1 + V_{w_2} u_2\right]$$

Discharge through turbine:

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$$= n \times \left(\frac{\pi d^2}{4}\right) \times V_1$$

[where $n \rightarrow$ No. of vanes
$d \rightarrow$ dia. of Jet
$V_1 \rightarrow$ Velocity of Jet
$= C_V \sqrt{2g.H}$
$H \rightarrow$ Net head at the base of Nozzle.]

Power available at the base of Nozzle:

(1) $\underset{(H.P)}{W.P.} = \omega Q H$ [Power available at the base of Nozzle]

(3) $R.P. = \frac{\omega Q}{g}\left[V_{w_1} u_1 + V_{w_2} u_2\right]$ $[u_1 = u_2]$

(2) Power available at the inlet of Jet / vane

$= K.E.$ of Jet per second

$= \frac{1}{2} \times$ Mass flowing per second $\times V_1^2$

$= \frac{1}{2} \times \rho Q \times V_1^2$

$= \frac{1}{2}\left(\frac{\omega}{g}\right)(aV_1) \times V_1^2$ [$a \rightarrow$ area of Jet]

$= \frac{\omega a V_1^3}{2g}$

Jet velocity

(4) Power available at the shaft

S.P. or Break Power

$= R.P. -$ Losses

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a) Efficiency of Nozzle: ↓

$$\eta_N = \frac{K.E./sec}{\text{Power at the base of Nozzle}} = \frac{\frac{1}{2}\frac{wQ}{g} \times V_1^2}{wQH}$$

$$= \frac{V_1^2}{2gH} = \frac{C_v^2(2gH)}{(2gH)} = C_v^2$$

$\therefore$ $\boxed{\eta_N = C_v^2}$

b) Hydraulic efficiency: ↓

$$\Rightarrow \eta_h = \frac{R.P.}{K.E./sec \text{ of Jet}}$$

$$= \frac{\frac{wQ}{g}\left[V_{w1}u_1 + V_{w2}u_2\right]}{\frac{wQ}{2g} \times V_1^2}$$

NOTE:- If hydraulic efficiency is calculated on the basis of power available at the base of nozzle then

$$\eta_B = \frac{R.P.}{\text{Power available at the base of Nozzle}}$$

$$= \frac{\frac{wQ}{g}\left[V_{w1}u_1 + V_{w2}u_2\right]}{wQH}$$

$$\eta_B = \frac{V_{w1}u_1 + V_{w2}u_2}{gH}$$

It means Energy loss at the base of nozzle is neglected

c) Mechanical efficiency ↓

$$\eta_M = S.P./R.P.$$

d) overall efficiency:- Accounting the Nozzle Losses

$$\boxed{\eta_o = \eta_N \times \eta_h \times \eta_M}$$

Volumetric Efficiency :

If entire volume of Jet is not striking to the vane than volumetric efficiency

$$\boxed{\eta_v = \frac{Q'}{Q}}$$

[Q' = total volume of water striking the Jet per sec.

Q =

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Specification for design of Pleton wheel :

(a) speed ratio $\phi = \frac{u_1}{\sqrt{2gH}}$ [ϕ is also called side clearance angle

$= 0.45$ to 0.47

(b) Angle of defln of Jet through the bucket is

$\simeq 165^\circ$ [If not given]

(c) Jet ratio $= \frac{\text{Dia. of pitch circle}}{\text{Dia. of Jet}} = D/d = m$

$\simeq$ 11 to 15 & commonly it is adopted 12.

(d) No. of buckets on the runner should be as Less as possible in order to minimize friction losses But in-order to Maximize Volumetric Efficiency optimum No. of Buckets are given by

$$\boxed{Z = 15 + \frac{D}{2d} = 15 + \frac{0.5\,m}{2}}$$

[$m = 12$

$\simeq 21$

[Range 18 to 25]

(e) No. of Jets $(n) = \frac{\text{Total discharge}}{\text{Discharge through one Jet}} \not> 6$

(f) width of bucket may be taken as height times dia. of Jet & Vertical depth of bucket may be taken as 1.5 times dia. of Jet

Prob
ES/2001

Pelton wheel

i) Head at the base of Nozzle = 32 M (H)

ii) discharge of the Nozzle = 0.18 m^3/sec (Q)

iii) area of Jet = 7500 mm^2 (a)

iv) Power available at the shaft = 44 KW (S.P.)

v) Mech. efficiency = 94 % ($\eta_M = 0.94$)

Calculate the power Loss

a) In the Nozzle

b) In the Runner

c) In the Mech. friction

$$\eta_M = \frac{S.P.}{R.P.}$$

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$$R.P. = \frac{S.P}{\eta_M} = \frac{44}{0.94} = 46.8 \text{ KW}$$

$$\text{velocity of Jet} = Q/a = \frac{0.18}{7500 \times 10^{-6}} = 24.0 \text{ m/sec}$$

$$\text{K.E per second of the Jet} = \frac{1}{2} \times \left(\frac{wQ}{g}\right) \times V_1^2$$

$$= \frac{1}{2} \times \frac{9.810 \times 0.18}{9.81} \times (24)^2 = 51.84 \text{ KW}$$

Power available at the base of Nozzle

$$= wQH$$

$$= 9.81 \times 0.18 \times 23 = 56.5 \text{ KW}$$

Loss at Nozzle

$$= 56.5 - 51.84 = 4.66 \text{ KW}$$

Loss in Runner

$$= 51.84 - 46.68 = 5.04 \text{ KW}$$

Mechanical Losses = 46.5 - 44 = 2.5 KW

= ? % - losses

?? shaft losses.

A pelton wheel has a mean bucket speed of 12 M/sec is supplied with water at a rate of 750 lit/sec under a head of 35 M. If the buckets deflects the jet by an angle of 160°. Find the horse power & efficiency of the bucket taking $C_v = 0.98$ & Neglecting the friction in the Losses

$u_1 = u_2 = 12$ M/sec

$Q = 750$ l/sec

$= 0.75$ m³/sec

$H = 35$ M

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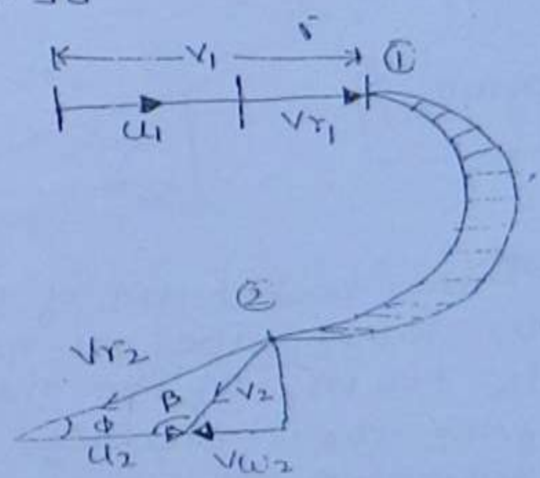
Defl^n Angle $= 180 - \phi$

$= 160°$

$\therefore$ side clearance angle or runner vane angle $(\phi) = 20°$

$C_v = 0.98$



$\boxed{V_{r_1} = V_1 - u_1}$

$= 25.68 - 12$

$= 13.68$ m/sec

$V_1 = C_v\sqrt{2gH}$

$= 0.98\sqrt{2 \times 9.81 \times 35}$

$= 25.68$ m/sec

$V_{r_1} = V_{r_2} = 13.68$ m/sec

$\boxed{|V_{r_2}\cos\phi| = |u_2| + |V_{w_2}|}$

$\Rightarrow 13.68\cos 20 = 12 + V_{w_2}$

$\Rightarrow V_{w_2} = 0.855$ m/sec

$$= \frac{wQ}{g}\left[V_{w_1}u_1 + V_{w_2}u_2\right]$$

$$= \frac{9.81 \times 0.75}{9.81}\left[25.68 + 0.855\right] \times 12$$

$$280$$

Efficiency:

Hydraulic efficiency $\eta_{cy} = \frac{R.P.}{K.E./second}$

$$K.E./second = \frac{1}{2} \times \left(\frac{wQ}{g}\right) \times V_1^2$$

$$= \frac{1}{2} \times \frac{9.81 \times 0.75}{9.81} \times 25.68^2$$

$$= 247.29 \text{ KW}$$

$$\therefore \eta_R = \frac{238.8}{247.29} \times 100\ \%$$

$$= 96.56\ \%$$

Prob3 5/2004

A pelton wheel has mean bucket dia. of 1M and is running at 1000 R.P.M. The net head on the pelton wheel is 700 m. If the side clearance angle is 15° and discharge through the nozzle 0.1 m^3/sec. Find

a) Power available at the nozzle

b) Hydraulic Efficiency of turbine, take $C_v = 1$

$$u_1 = u_2 = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 1000}{60} = 52.35 \text{ m/sec.}$$

Net head on the pelton wheel = 700 m (H)

$\phi = 15°$

$Q = 0.1\ m^3/sec$, $C_v = 1.0$

$$V_1 = C_v\sqrt{2gH}$$

$$= 1.0\sqrt{2 \times 9.81 \times 700}$$

$$= 117.19 \text{ m/sec}$$

$\phi = \frac{u_1}{\sqrt{2gH}}$ = 0.47 to 0.45 speed ratio can not be applied to calculate u_1

Power at Nozzle = $\rho g Q H$ = 9.81 × 700 × [illegible]

$= 6866.7\ kW$

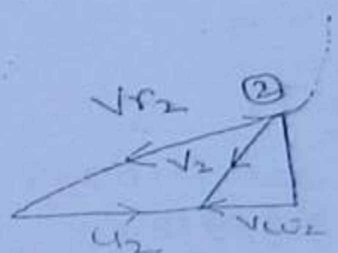
$$\eta_B = \frac{V_{w_1} u_1 + V_{w_2} u_2}{gH}$$ { Losses at Nozzle are negligible

$V_{w_1} = V_1 = 117.19$

$V_{r_1} = V_1 - u_1 = 117.19 - 52.35 = 64.83$

Incoming bucket to be frictionless

$V_{r_2} = V_{r_1} = 64.83\ m/sec.$



$$\boxed{|u_2| + |V_{w_2}| = |V_{r_2}\cos\phi|}$$

$= 64.83 \cos 15$

$\therefore V_{w_2} = 10.261\ m/sec$

$\therefore \eta_B = 97.18\ \%.$

Prob 4

A pipline 1200 m Long supply water to three simple pelton wheel. The head over the nozzle is 360 M. CV = 0.98, friction factor for the pipe = 0.02. The turbine efficiency based on the head of the nozzle is 0.85. If specific speed based on the base of nozzle is 15.3 [Ns is such that n = r.p.m, P = kW, H = meter] If head loss due to friction in the pipe line is 12 M and operating speed of each turbine is 360 r.p.m. determine

(i) Total power developed

(ii) The Dia of pipeline

(iii) The dia of each nozzle.

(iv) Discharge

$$\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$$

$P \rightarrow$ H.P (Power/s ...)

$H \rightarrow$ Head

$L = 1200 m$

no. of turbine = 3

$H = 360 m$

$f = 0.02$

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NOTE:- For Multiple jet pelton wheel the sp. speed is Based on Break Power per jet.

$$\eta = 0.85 = \frac{S.P.}{\text{water available at the base of nozzle}}$$

$$\eta_M = \frac{S.P.}{R.P.}$$

$$0.85 = \frac{55.06}{wQH}.$$

$$\Rightarrow Q = 1.834 \ m^3/sec.$$

Total discharge supplied by pipe = 18.34

$\therefore$ Discharge through each nozzle

$$= \frac{1.834}{3} = 0.611 \ m^3/sec$$

$$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 360}$$

$$\therefore V_1 = 82.36 \ m/sec.$$

~~discharage~~ dia. of each jet/nozzle is

$$\frac{\pi d^2}{4} = \frac{0.611}{82.36} \Rightarrow d = 0.097 \ m = 97 \ mm$$

$$\boxed{h_f = \frac{fLQ^2}{12.1 D^5}} = \frac{0.02 \times 1200 \times 18.34^2}{12.1 \ D^5}$$

$$= 12$$

$$\Rightarrow \boxed{D = 0.88 \ m}$$

Specific speeds for various turbines:-

[M.K.S. Unit]

a) Pelton wheel $P \to H.P.$

→ Single Jet $N_s = 10$ to 30

→ Multi Jet $= 30$ to 60

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b) Francis turbine $N_s \Rightarrow 60$ to 300

c) Kaplan turbine $N_s \Rightarrow 300$ to 1000

Unit quantities:-

A turbine operates most efficiently at its design points at a particular combination of H, Q & N. But in practice these variable do not remain constant therefore concept of unit quantity is important to

(i) Predict the behaviour of turbine working at different condition

(ii) To make the comparison of performance of turbine of the same type but diff-size

(iii) It may be used to compare the performance of turbine of different type

(iv) To correlate the use of experimental data.

a) unit speed: It is the theoritical speed at which a given turbine would operate under a head of 1M.

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$ r.p.m

for a turbine to run under a head of 1m

$$\boxed{Q_U = \frac{Q}{\sqrt{H}}}$$

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> Unit power: It is the theoretical power which a turbine would produce under a head of 1m.

$$\boxed{P_U = \frac{P}{H^{3/2}}}$$

Model quantities:

When the results obtain from the experiment conducted on the model are applied to the prototype in the field following similarity exists:

a) $\dfrac{H}{N^2 D^2} = \text{costant}$

i.e. $\dfrac{H_M}{N_M D_M^2} = \dfrac{H_P}{N_P D_P^2}$

b) $\dfrac{P}{N^3 D^5} = \text{constant}$

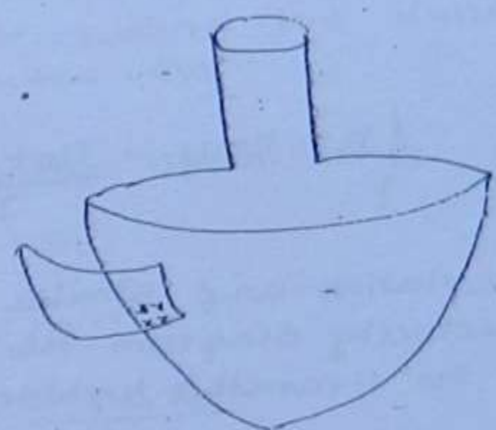
$\Rightarrow \dfrac{P_M}{N_M^3 D_M^5} = \dfrac{P_P}{N_P^3 D_P^5}$

c) $\dfrac{Q}{N D^3} = \text{constant}$

$\Rightarrow \dfrac{Q_M}{N_M D_M^3} = \dfrac{Q_P}{N_P D_P^3}$

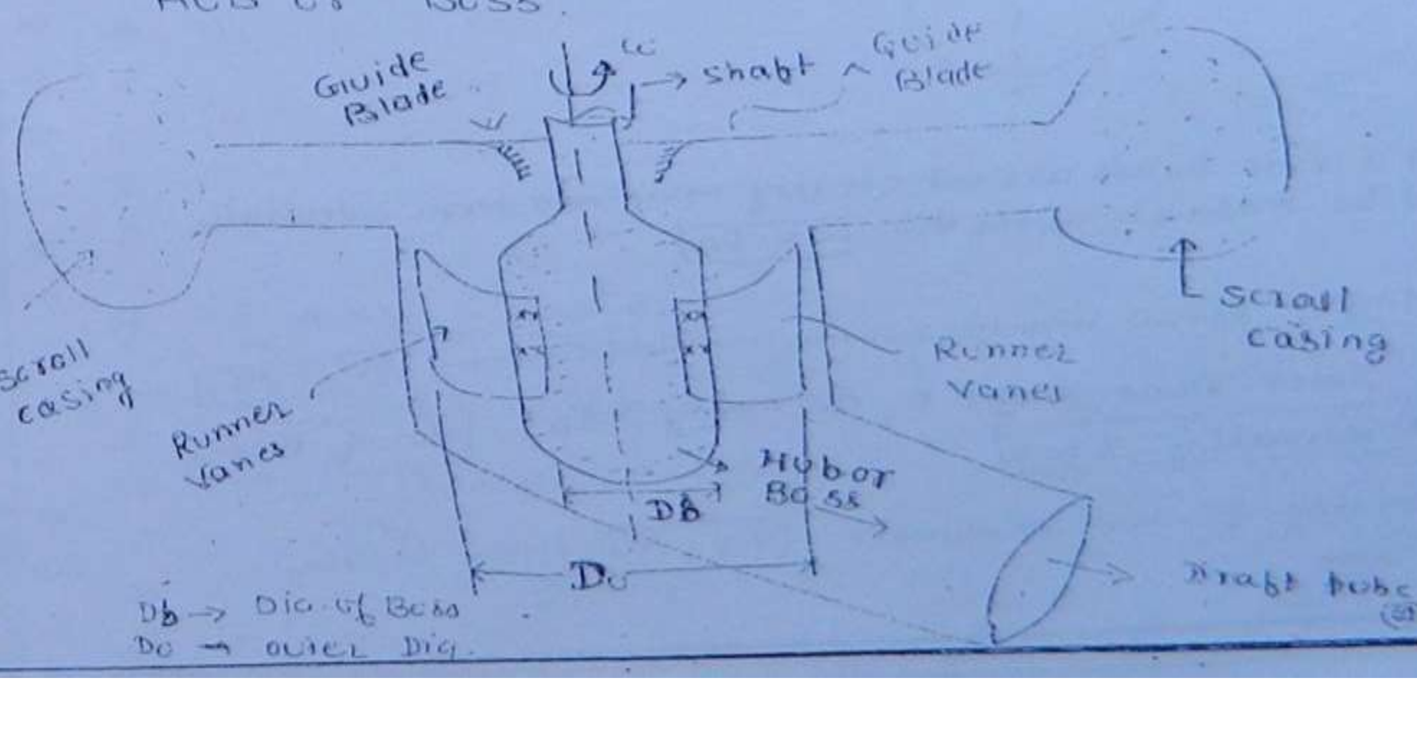
3> Kaplan & Propeller Turbine:]

- Axial flow Rxn turbines
- Propeller turbines & kaplan turbines are similar in principal except that kaplan had improvement over propeller w.r.t. direction of the blades are Flexible i.e. Propeller turbines has fixied vanes connected through rivets permananetly where kapaln has provided bolts & an additional bolt hole.



Important unit of propeller turbines:]

a> For the axial flow reaction turbine the shaft of the turbine is vertical, the lower end of the shaft is made larger which is known as Hub or Boss.



ii> $Q = A_{f_1} \cdot V_{f_1}$
$= A_{f_2} \cdot V_{f_2}$
$\Rightarrow V_{f_1} = V_{f_2}$

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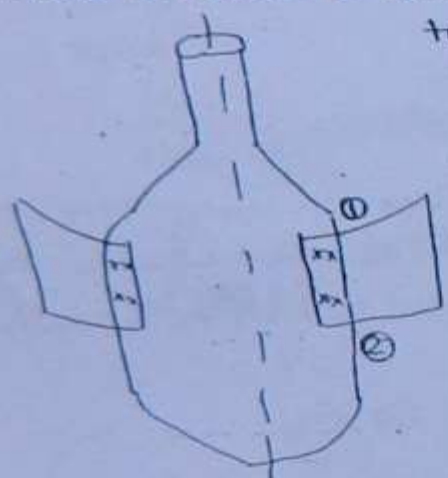
iii> If friction Losses are negligible than
$\boxed{V_{r_1} = V_{r_2}}$

iv> Vane angle from Do to Db May change.

$\therefore \boxed{u_1 = u_2 = \frac{\pi D N}{60}}$

$\left.\begin{array}{l} u_1 \text{ at } D = D_o \\ u_2 \text{ at } D = D_b \end{array}\right\}$ $\left\{ D = D_{Avg} = \frac{D_o + D_b}{2} \right\}$

* Power & Efficiency calculation are similar to Francisis turbine & velocity diagram also similar to Francisis turbine.



$\boxed{H = \frac{V_1^2}{2g} + \frac{V_{w_1} u_1}{g}}$

NOTE:
1> If entry & exit points are not clearly maintain than calculations should be made at outer dia. $\boxed{d = D_o}$

2> velocity of flow through runner remains constant i.e. $\boxed{A_{f_1} = A_{f_2}}$

3) The Inlet Vane Angle θ, normally > 90. $\boxed{\text{No of Blades is Normally 3 to 6}}$

4> Ignoring friction resistance, $\boxed{V_{r_1} = V_{r_2}}$ may be taken

5) Under assumptions B.Eqn may be applied at Inlet and exit point

when $V_{r_2} = C$, friction on the blades is negligible

Probt (ICE III) A propeller runner turbine runner has outer dia 4.5m & dia. of hub is 2M. It is required to develop power 20600 KW when running at 150 r.p.m. under a head of 21 M. Assuming hydraulic efficiency 94% & overall efficiency of 88%. Determine the runner vane angle at inlet & outlet at the mean exit of vane. Assume velocity of whirl at outlet is zero. Also determine the vane angle at outlet & inlet at outer dia.

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Calculation at Mean diameter:

$$D_m = \frac{D_o + D_b}{2}$$

$$= 3.25 \text{ m}$$

$$u_1 = u_2 = \frac{\pi D_m N}{60}$$

$$= \frac{\pi \times 3.25 \times 150}{60}$$

$$= 25.525 \text{ M/sec.}$$

Shaft Power = 20600 KW

$$\eta_o = 0.88$$

$$\eta_o = \frac{S.P}{H.P.}$$

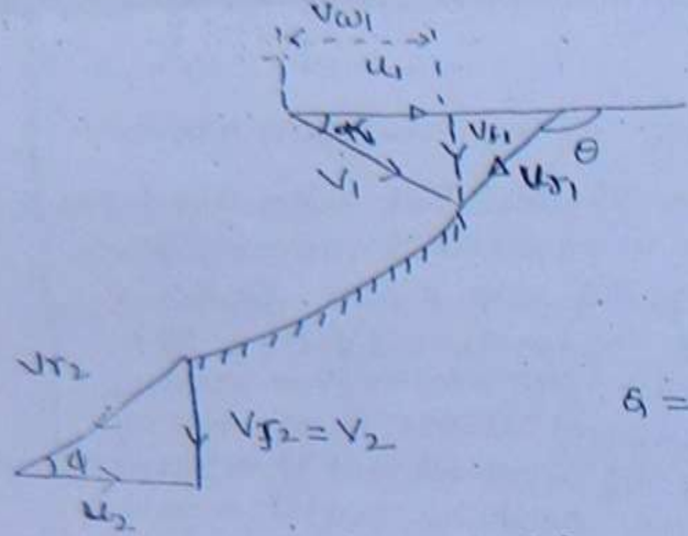
$$\omega Q H = \frac{20,600}{0.88}$$

$$\Rightarrow 9.81 \, Q \times 21 = \frac{20,600}{0.88}$$

$$\Rightarrow Q = 113.63 \text{ m}^3/\text{sec.}$$

$$\text{Hydraulic efficiency } (\eta_h) = \frac{V_{w_1} u_1}{gH} = 0.94$$

$$\Rightarrow V_{w_1} = 7.58 \text{ m/sec.}$$



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$$Q = \pi/4 \left(D_0^2 - D_b^2 \right) \times V_{f1}$$

$$\Rightarrow V_{f1} = 8.90 \text{ M/sec}$$

$$V_{f1} = V_{f2} = V_2 = 8.90 \text{ M/sec.}$$

$$\tan(180-\theta) = \left(\frac{V_{f1}}{u_1 - V_{w1}} \right) = \frac{8.90}{(25.525 - 7.58)}$$

$$\therefore \tan(180-\theta) =$$

$$\Rightarrow \boxed{\theta = 153.6^\circ}$$

From exit velocity diagram

$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{8.95}{25.525}$$

$$\Rightarrow \boxed{\phi = 19.22^\circ}$$

case II^nd. Repeat the calculation for $d = D_0$

NOTE

When it is given that runner dia is d m than it is the outer dia. of the runner & in case of the Pelton wheel pitch circle dia. should be taken

Prob2 A kaplan turbine has a runner dia. of 4 m. And hub dia. 1.2 meter. The discharge through turbine is 70 m3/sec. The η_h & η_m can be taken as 0.9 & 0.93 respectively. Assuming absence of whirl at outlet & discharge is free from friction estimate the Net head available in the turbine & the power developed. Speed Ratio is 2.0. Also estimate specific speed.

$$gH$$

$$V_{f_1} = \frac{Q}{\pi/4\,(D_0^2 - D_b^2)(\in \cdot A f_1)} = 6.12 \text{ m/sec}$$

$$= V_{f_2} = V_2$$

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$$\boxed{H = \frac{V_2^2}{2g} + \frac{V_{\omega_1} u_1}{g}}$$

$$\Rightarrow \quad H = \frac{6.12^2}{2 \times 9.81} + 0.9H$$

$$\Rightarrow \quad H = 19.1\text{ m}$$

$$S.P. = \underline{H.P. \times \eta_0} = \omega Q H \times \eta_h \times \eta_m$$

$$S.P = 10{,}972 \text{ KW}$$

$$\phi = \frac{u_1}{\sqrt{2gH}} \Rightarrow u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 19.1}$$

$$u_1 = 38.71 \text{ m/sec}$$

$$u_1 = \frac{\pi D_0 N}{60} \Rightarrow N = 184.76 \text{ rpm}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{184.76\sqrt{10{,}972 = \underline{\underline{S.P}}}}{(19.1)^{5/4}}$$

N_s (specific speed) = 484.31 (SI unit)

prob3

The hub dia. of a kaplan turbine working under a head of 12 m is 0.35 times dia. of runner, the turbine is running at 150 r.p.m. If the vane angle of extreme edge at outlet is 15 and Flow ratio is 0.26.

a) Find dia. of runner → $D_0 = 6.55$ m

b) Find dia. of boss →

c) discharge through runner → 241.77 m^3/sec

d) Assume velocity of virl at outlet is zero total head of 25M. The centre line of the machine is 3M above the water level, in the tail rest. The abs. velocity of flow leaving the vanes on the runner wheel is in the radial direction. the outlet dia = 0.45 x inletdia. The tangential velocity of runner at exit the rim is 10.8 m/sec & velocity at entry is 3.3 M/sec. And velocity at the exit of the draft tube is 2.2 M/sec. Assuming that flow enters (the runner wheel) without shock.

determine

1) outlet Angle of the guide blades (α)
2) Inlet & outlet angle (θ & ϕ)
3) The pressure head at inlet & outlet of the runner

Assume that loss due to friction in the guide blade, runner blade plate & the draft tube is 4%, 6% & 5% respectively of the available head.

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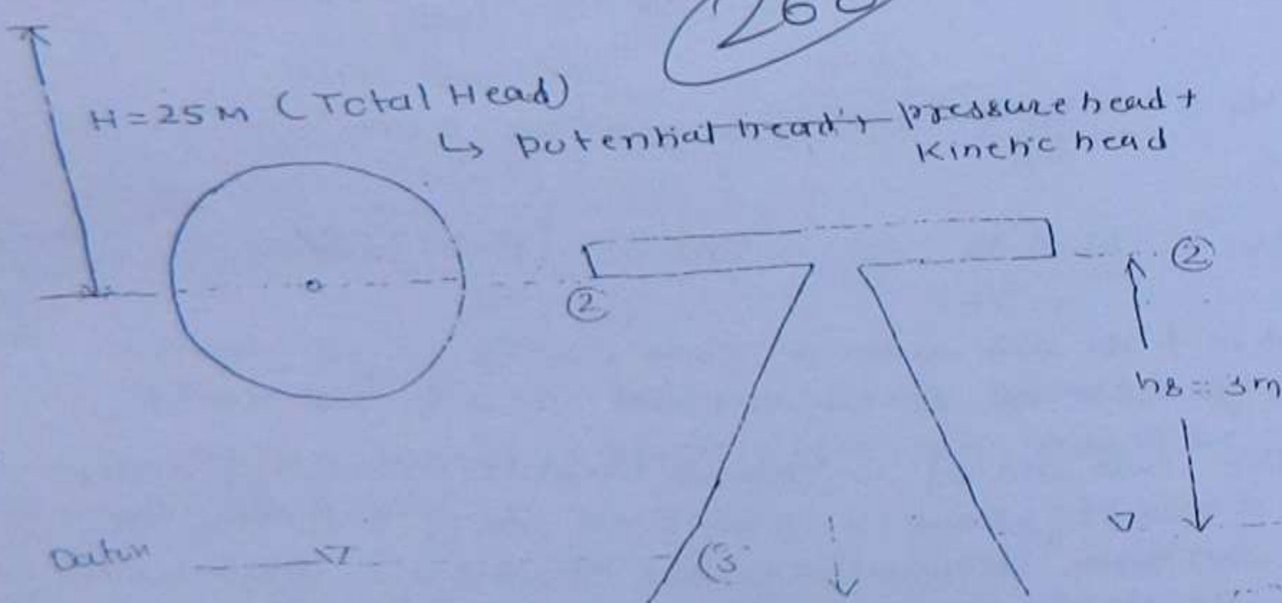

H = 25 M

V_1 = 10.8 m/sec

V_{f1} = 3.3 m/sec

p/w & p/w = ?

V_3 = 2.2 m/sec

Loss in guide blades ...

$$= 0.04\left(\frac{V_1^2}{2g}\right) = 0.04H$$

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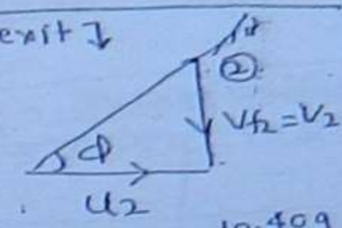
Loss in Runner Vanes

$$= 0.06\left(\frac{V_2^2}{2g}\right) = 0.06\,[0.96H]$$

Loss in Draft tube

$$= 0.05\left(\frac{V_3^2}{2g}\right)$$

At exit



$V_{w1} = 10.409$

$u_1/u_2 = d_1/d_2$

$\Rightarrow u_2 = 4.52$ M/sec

Apply B. Eqn b/w (2) & (3)

$$\left(\frac{V_2^2}{2g} + \frac{P_2}{w} + 3\right) = \frac{p_{atm}}{w} + \frac{V_3^2}{2g} + 0 + h_L + 0.05x$$

$$=$$

$$\Rightarrow 0.95x = \frac{p_{atm}}{w} + \frac{V_3^2}{2g}$$

$$= 10.3 + \frac{2.2^2}{2\times 9.81}$$

$$\Rightarrow x = \frac{10.5}{0.95} = 11.10$$

Total Head at inlet

$$24 = Z + P_1/w + \frac{V_1^2}{2g}$$

$$= 3 + \frac{P_1}{w} + \frac{10.92^2}{2\times 9.81}$$

$$\therefore P_1/w = 14.92\ m$$

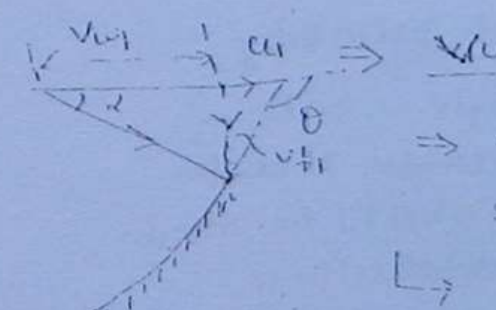
∴ Total head at the exit diameter $= 11.10$

B. Eqn B/w inlet & outlet of Runner

Head at inlet (Y) = Head at outlet + work done on Runner / by unit of water + Losses

$$Y = 11.10 + \frac{\frac{wQ}{g}[V_{w1}u_1]}{wQ} + 0.06Y$$

$$0.94Y = 11.10 + \frac{V_{w1}u_1}{g}$$



$$\Rightarrow \frac{V_{w1}u_1}{g} = 24 - 11.10$$

$$\Rightarrow V_{w1} = 10.409\ M/sec$$

since $V_{w1} < u_1 \Rightarrow \theta > 90$

$$\hookrightarrow \tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{3.3}{10.409}$$

$$\tan(180-\theta) = \frac{V_{f1}}{u_1 - V_{w1}}$$

$$\Rightarrow \boxed{\alpha = 17.59^\circ}$$

$$\boxed{V_1 = \sqrt{V_{f1}^2 + V_{w1}^2} = 10.92\ m}$$

$$\Rightarrow \boxed{\theta = 96.76^\circ}$$

PUMPS :-

* These utilize mechanical power supplied by the shaft or utilize Man-power to convert it into hydraulic power or water power.

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- PUMP
  - Dynamic pressure pump
    - centrifugal Pumps
    - Turbine pumps
    - Jet Pumps
    - Propeller pumps
  - Displacement pump
    - Reciprocating Pump
    - Rotary

centrifugal pump → These works on the principle of forced Vortex - Motion

Reciprocating pump → works on the principle of suction pressure

*1) Centrifugal pump: ↓

1) These have high output & high efficiency ~~& are~~ used for Low head & high discharge.

a) when Head is Less than 15m these are called Low Head pump

b) 15 m < H < 45 m → Medium Head pump

c) $\boxed{H > 45m}$ → High head pump

→ Head here means suction head primarily

NOTE: when $\boxed{\text{head is} > 40\,m}$, single state centrifugal pump is not desirable therefor Multistage pump in series should be used. it is practically observed that when head is $\boxed{\text{b/w } 12 \text{ to } 8 \text{ m}}$ centrifugal pump are most efficient

** The no. of vanes on Rotating unit [illegible]
are 6 to 12.

* centrifugal pumps are exectly inverse of Francis turbine it Means these are outward radial flow pumps.

(263)

Principle: ↓

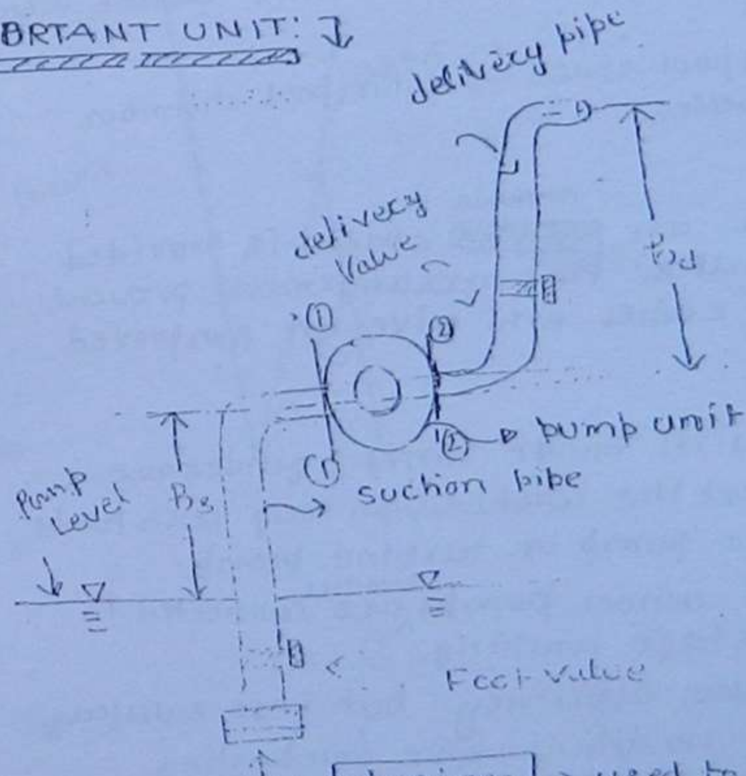
By rotating impeller, pressure head difference is created.

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

[$r_1 \rightarrow$ inner radii
$r_2 \rightarrow$ outer radii]

→ This pressure head created is utilized to Lift the water against Manometric head (it is the head against which pump has to work)

IMPORTANT UNIT: ↓



strainer → used to prevent entry of blockage material

$$\boxed{H_s = h_s + h_d}$$

static head = suction head + delivery head

pressure in suction pipe is always below atmospheric

→ minimum pressure exist at (1)-(1) i.e. entry of pump

→ This pmin should not falls vapour pressure of water [illegible] cavitation phenomenon & water hammer problem. ($p_{min} = p_s$)

Imp. parts of pump: ⤵

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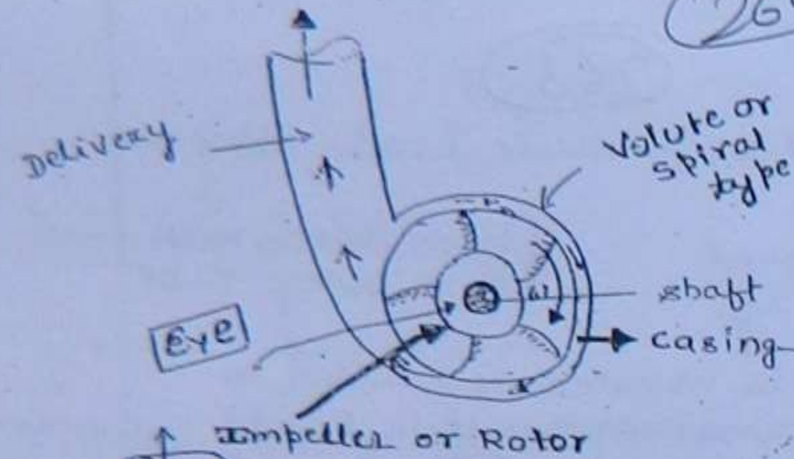
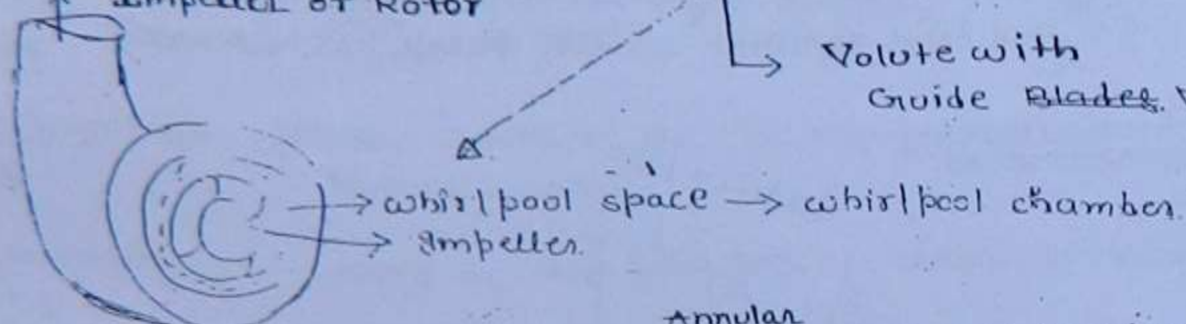


No space is left for expansion b/w Runner vane & casing

casing →
- Volute or spiral ⤵
- Vortex or whirlpool
- Volute with Guide ~~Blades~~ Vanes.



— In whirlpool chamber an Annular spaced is provided b/w casing & impeller, this arrangement prevent the formation of eddies and gives an improved performance.

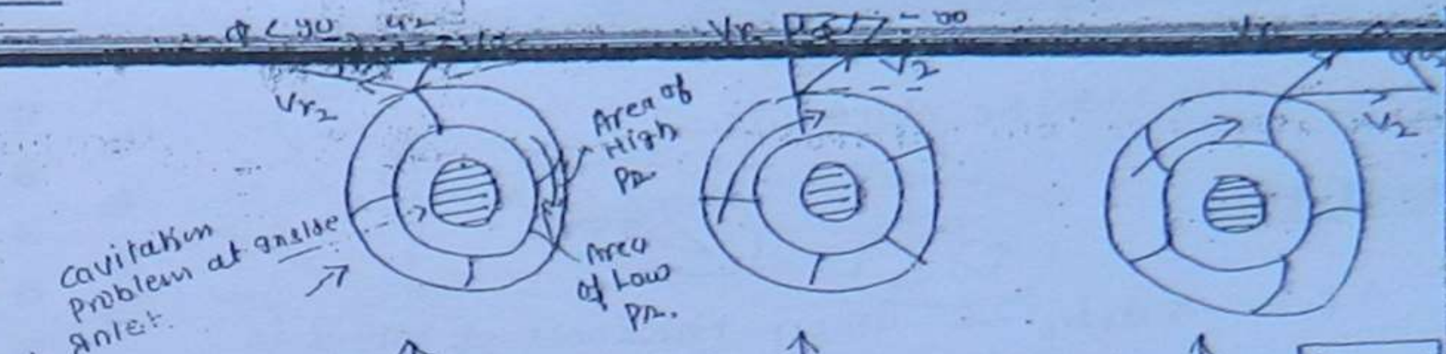
* In case of volute with Guide vanes, guides are provided to divert the water, properly such pumps are called diffuser pump or turbine pump. These are adopted when pumps (impeller) are connected in series for multistage pumping.

→ These have Maximum Efficiency but Less satisfactory when opertating conditions are flactuating (Power or head)

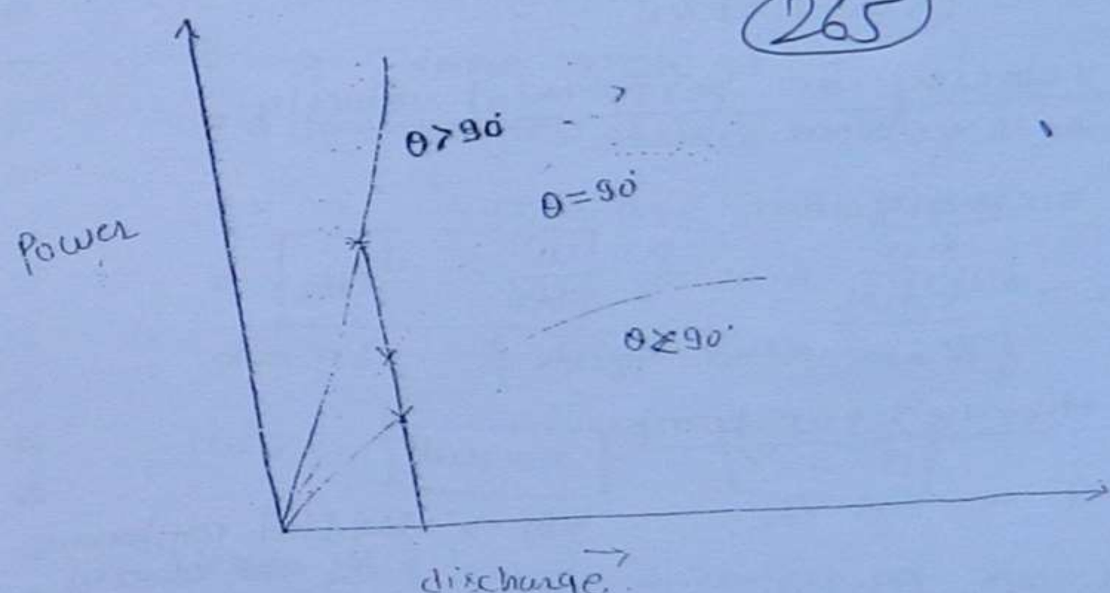
* Impeller: ⤵

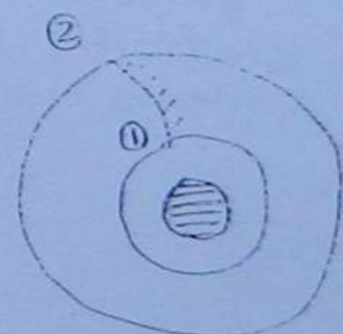
It is the rotating unit of a pump similar to runner unit of turbine

→ Impeller has 6 to 12 curved Vanes.



Backward Vanes	Radial vane	Forward Vanes or Gurrad Vanes
Minimum Power	Medium Power	Max^m Power
* Less efficient		* Most efficient





① → Entry point
② → Exit point
d_1 → Inner dia. (Inlet dia.)
d_2 → Exit dia. (outer dia)

Generally

$\boxed{d_1 \simeq \frac{1}{2} d_2}$

* size of impeller
- Means outer dia $\simeq d_2$

b_2 be the outlet width "

Following points May be Noted: ↓

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(1) Area of flow:

$$A_{f_1} = \pi d_1 b_1 \rightarrow$$ when thickness of vanes is negligible

$$= \cancel{\pi d_1 b_1}$$

$$= (\pi d_1 - nt) b_1 \rightarrow$$ If thickness of each vane is t & there are n vanes.

similarly at exit

$$A_{f_2} = \pi d_2 b_2$$

$$= (\pi d_2 - nt) b_2$$

(2) Tangential velocity or peripherial velocity:

$$u_1 = \frac{\pi d_1 N}{60}$$

$$u_2 = \frac{\pi d_2 N}{60}$$

$$\Rightarrow \boxed{\frac{u_1}{u_2} = d_1/d_2}$$

(3) Discharge through the pump:

$$Q = A_{f_1} \cdot V_{f_1}$$

$$= A_{f_2} \cdot V_{f_2}$$

V_{f_1} & V_{f_2} → Radial components of abs. velocity

velocity diagram :-

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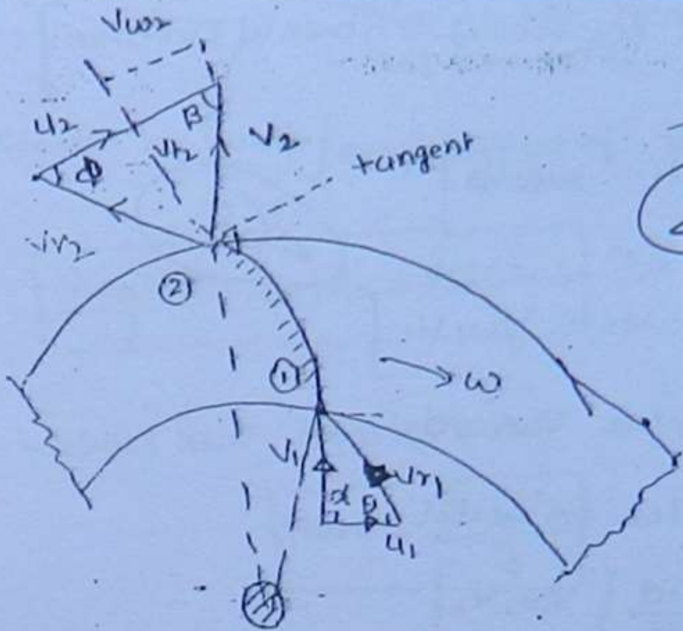

u_1 & u_2 → Tangential or peripherial velocities

Φ → Vane Angle at exit

$\Phi < 90^\circ$, vanes called backward curved

$\Phi = 90^\circ$, vanes are radial at exit

$\Phi > 90^\circ$ vanes forward curved

$\alpha = 90^\circ$ { Angle betn u_1 & V_1 }

Then $\boxed{V_{w1} = 0}$ $\boxed{V_1 = V_{f1}}$

→ Hence discharge is radial at inlet, it Means water enters at inlet without whirl and if there is no impact Loss then discharge is said to be without whirl & shock

$$= -\left[\text{work done per second in case of Turbine}\right]$$

$$= -\frac{wQ}{g}\left[V_{w_1}u_1 - V_{w_2}u_2\right]$$

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$$\therefore \text{work/sec.} = \frac{wQ}{g}\left[V_{w_2}u_2 - \overset{\to 0}{\cancel{V_{w_1}}}u_1\right]$$

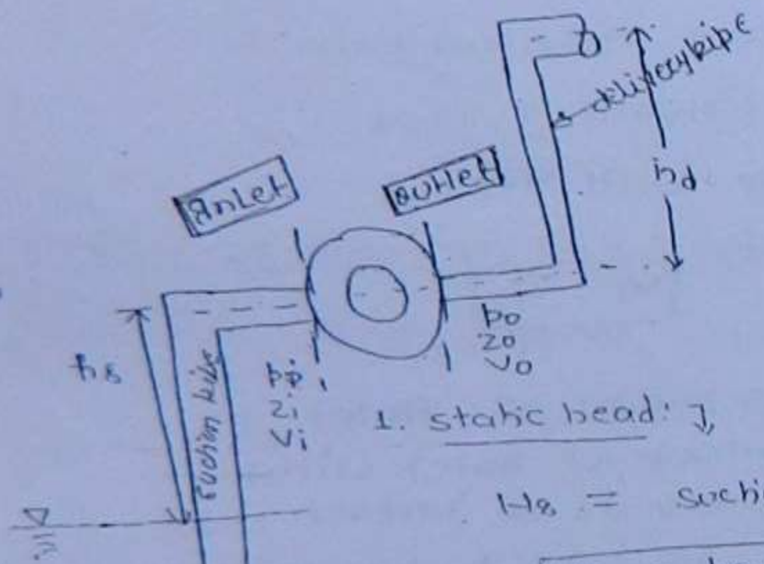
For Maxm Power $V_{w_1} = 0$

work-done by Impeller on water/sec

$$= \frac{wQ}{g}\left[V_{w_2}u_2\right]$$

$$= \text{Impeller power} \,|\, \text{Rotor power}$$

Important Definitions: ↓



1. static head: ↓

$$H_s = \text{suction head} + \text{delivery head}$$

$$\boxed{H_s = h_s + h_d}$$

2. Friction head ↓ (h_f)

$$\boxed{\text{Total friction head} = h_{fs} + h_{fd}}$$

↓ friction head in suction pipe

Manometric head: (Hm)

Imp

— This is the head against which pump has to work.

work-done by pump / sec / unit wt. of water = Manometric head + Losses.

⇒ A) $$\boxed{\frac{V_{w_2} U_2}{g} = H_m + \text{Losses}}$$ Imp

[Assuming $V_{w_1} = 0$]

— If Losses are negligible

$$\boxed{H_m = \frac{V_{w_2} U_2}{g}}$$

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B) H_m = Total energy head at outlet − Total energy head at inlet

$$= \left[\frac{p_o}{w} + z_o + \frac{V_o^2}{2g}\right] - \left[\frac{p_i}{w} + z_i + \frac{V_i^2}{2g}\right]$$

$V_o \rightarrow V_d$ (velocity in delivery pipe)

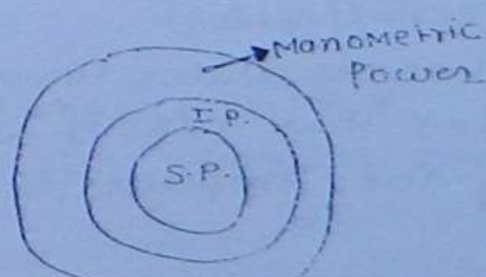
$V_i \rightarrow V_s$ (velocity in suction pipe)

$\boxed{\begin{matrix} z_o \simeq z_i \\ p_o = p_d \end{matrix}}$, $\boxed{p_i \simeq p_s}$

C) $$H_m = (h_s + h_d) + (h_{fs} + h_{fd}) + \underbrace{\left(\frac{V_d^2}{2g}\right)}_{\text{Negligible}}$$

$$\boxed{H_m = \underbrace{(h_s + h_d)}_{\text{Static Head } (H_S)} + \underbrace{(h_{fs} + h_{fd})}_{\text{Friction Head}}}$$ Imp

6) Efficiency & Powers of Pump:



$\boxed{\text{S.P.} > \text{I.P.} > \text{Manometric power (water)}}$ Imp

→ Manometric power $= wQ\,H_m$

1.) Mechanical Efficiency (η_{cy}) :

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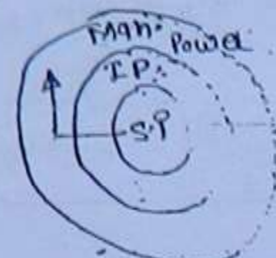

$$\boxed{\eta_{Mech.} = \frac{I.P.}{S.P}}$$

2) Manometric Efficiency :

[Reverse calculation]

$$\boxed{\eta_{Mano.} = \frac{\text{Mano. power}}{\text{Imp. power}}}$$

$$= \frac{wQH_m}{\frac{wQ}{g}\left[V_{w_2}u_2\right]}$$

$$= \frac{gH_m}{V_{w_2}u_2}$$

define
[$\eta_o \rightarrow$ than $\eta_{Mano.}$
$\eta_m \rightarrow$]

3) overall Efficiency :

$$\boxed{(\eta_o) = \frac{\text{Man. power}}{S.P}}$$

$$= \eta_{Mech.} \times \eta_{Man.}$$

NOTE: If Leakage Losses are also accounted than Volumetric Efficiency

$$\eta_v = \frac{\text{Discharge at delivery point}}{\text{Discharge at Inlet}}$$

$$= Q/(Q+\Delta Q)$$

$\Delta Q =$ Loss of water through casing

It may be noted that Loss of water takes place after Leaving the Impeller plates

overall Efficiency

$$\boxed{\eta_o = \eta_{mech.} \times \eta_{man.} \times \eta_{volumetric}}$$

6> Mean speed required to start the pumping of water

Minimum speed should be such that head developed should be greater than Hm.

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$$\therefore \boxed{\frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} \geq Hm}$$ imp

$\omega \rightarrow$ Rad. sec $= \frac{2\pi N}{60}$

$r_1 \rightarrow d_1/2$

$r_2 \rightarrow d_2/2$

especial: For a given value of ω & Hm it is possible to work-out minimum dia. of impeller (outer dia.) which will be required for pumping of water

$$\Rightarrow \frac{\omega^2}{2g}\left[\frac{d_2^2}{4} - \frac{d_1^2}{4}\right] \geq Hm$$

Take $d_2 = 2d_1$

$\Rightarrow \boxed{d_1 = d_2/2}$

$$\therefore \frac{d_2^2}{4}\left[1 - 1/4\right] \geq \frac{2gHm}{\omega^2}$$

$$\Rightarrow \boxed{d_2 = \frac{10.23}{\omega}\sqrt{Hm}}$$ imp

7> Multi-stage centrifugal pump:

(a) To produce high head, impeller should be connected in series. If n no. of impellers are connected in series & each lifts manometric heads equal to (Hm) than

For series connections $\boxed{Q = const.}$

Total head Lifted = n Hm

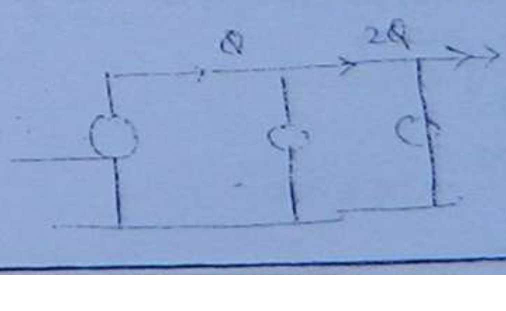
Here total discharge remains constant

(b) Parallel connection: — Impellers or pumps are mounted parallel to increase the discharge

$\boxed{Head = const.}$

Total discharge = nQ

Total head remains constant



(1) $\left(\frac{N\sqrt{Q}}{Hm^{3/4}}\right)_m = \left[\frac{N\sqrt{Q}}{(Hm)^{3/4}}\right]_p$

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(2) $\left(\frac{Hm}{D^2 N^2}\right)_m = \left(\frac{Hm}{D^2 N^2}\right)_p$

(3) $\left(\frac{Q}{N D^3}\right)_m = \left(\frac{Q}{N D^3}\right)_p$

(4) $\left(\frac{P}{N^3 D^5}\right)_m = \left(\frac{P}{N^3 D^5}\right)_m$

9> specific speed of pumps:

output

$$\boxed{N_s = \frac{N\sqrt{Q}}{(Hm)^{3/4}}}$$

Turbine

$$\boxed{N_s = \frac{N\sqrt{P}}{(H)^{5/4}}}$$

10> cavitation & Thoma Number:

In centrifugal impeller the pressure is minimum on the underside of vane at entry where vapour pressure may be form, these vapour pressure carried to a region of high pr. near to exit where bubble collapse causing pitting & sewer demage to metal surface. apperently entry vane tips at exit are the most suspitable for water hammer attack. Therefor harmful effect of cavitation are:

1> pitting & erosion of surface due to continuous hammering
2> sudden drop in head & decrease in the efficiency in the pump
3> Noise & vibration
4> corrosion problem

problems

A centrifugal pump having outer dia equal to two times the inner dia. & running at 1000 rpm works against a total head of 40m. The velocity of flow through impeller is constant & at 2.5 m/sec. The vanes are set-back at an angle of 40° at outlet. If the outer dia of impeller is 150 cm & width of outlet is 5cm than determine

a) vane Angle at inlet

b) work done by impeller/sec on water

c) Manometric efficiency

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$N = 1000$ r.p.m

$H_m = 40m$

$V_{f_1} = V_{f_2} = 2.5$ m/sec

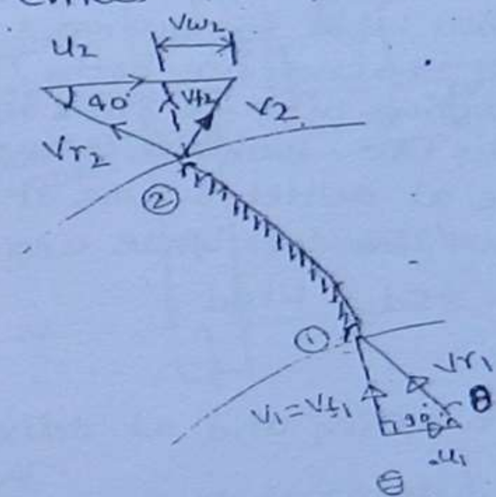
$\phi = 40°$

$d_2 = 0.5m$

$d_1 = 0.25m$

$b_2 = 0.05m$

→ water enters into pump without whirl & shock. $[V_{w_1} = 0]$



$$\tan 40° = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\Rightarrow \boxed{V_{w_2} = 23.19 \text{ m/sec}}$$

B) $\tan\theta = \frac{V_{f_1}}{u_1}$

$$\Rightarrow \boxed{\theta = 10.81}$$

A) $u_2 = \frac{\pi d_2 N}{60}$

$= \frac{\pi \times 0.5 \times 1000}{60}$

$= 26.17$ m/sec

C) Manometric efficiency

$= \frac{g H_m}{V_{w_2} u_2} =$

$= \frac{\text{Manometric efficien power}}{I.P}$

$Q = A_{f_2} \times V_{f_2} = (\pi d_2 b_2) \times V_{f_2}$

$= \pi \times 0.5 \times 0.05 \times 2.5$

$= 0.1966$ m³/sec

work done by imp/sec (I.P)

$= \frac{wQ}{g}\left[V_{w_2} u_2\right] = \frac{9.81 \times 0.1966 \times 23.19 \times 2.617}{9.81}$

$= 119.9$ kW.

2000 — when running at 600 r.p.m. discharges at the rate of 8000 Lit/min against a head of 8.5 m. The water enters the impeller without whirl & shock. The inner dia. is 0.25 m & the vanes are set back at outlet at an angle of 45° & area of flow which is constant from inlet to outlet of the impeller of 0.06 m². determine

(i) Manometric efficiency of pump

(ii) The vane Angle at inlet → 39°

(iii) Minimum speed at which the pump commences to work.

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Prob 9 · A C.P lifts water under a static Lift of 40 m of which 3 m is suction Lift. The suction & delivery pipes are of 30 cm dia. both. The friction Loss in suction pipe is 2 m & in delivery pipe is 6 m. The impeller is 0.5 m dia. & 3 cm. wide at outlet & runs at a speed of 1200 r.p.m. The exit blade angle is 20° and $\eta_{mano.} = 85\%$. Find

a) The discharge

b) Pressure at the suction and at delivery point.

1) $h_m = h_s + h_d + h_{fs} + h_{fd}$

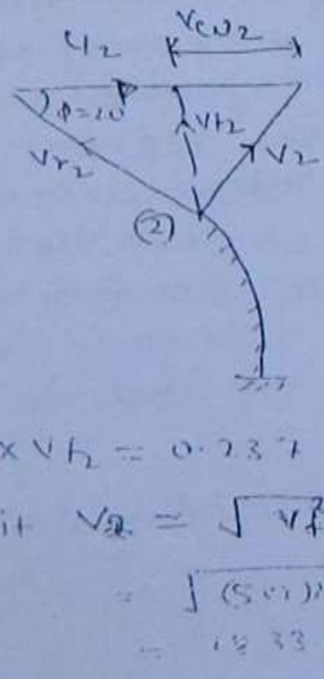
$= 40 + 2 + 6$

$= 48\,m$

2) $\boxed{\eta_{manometric} = \dfrac{gH_m}{V_{w2}u_2}}$

↑ calculate

$V_{w2} = 17.63$



using exit velocity diagram

3) $\boxed{\tan 20° = \dfrac{V_{f2}}{u_2 - V_{w2}}}$

$\Rightarrow V_{f2} = 5.02\ m/sec$

$Q = A_{f2} \times V_{f2} = (\pi d_2 b_2) \times V_{f2} = 0.237\ m^3/sec$

Absolute velocity at exit $V_2 = \sqrt{V_{f2}^2 + V_{w2}^2}$

$= \sqrt{(5.02)^2 + (17.63)^2}$

$= 18.33\ m/sec$

2) Reciprocating pump:

These work on the principle of creating suction head.

Main parts:

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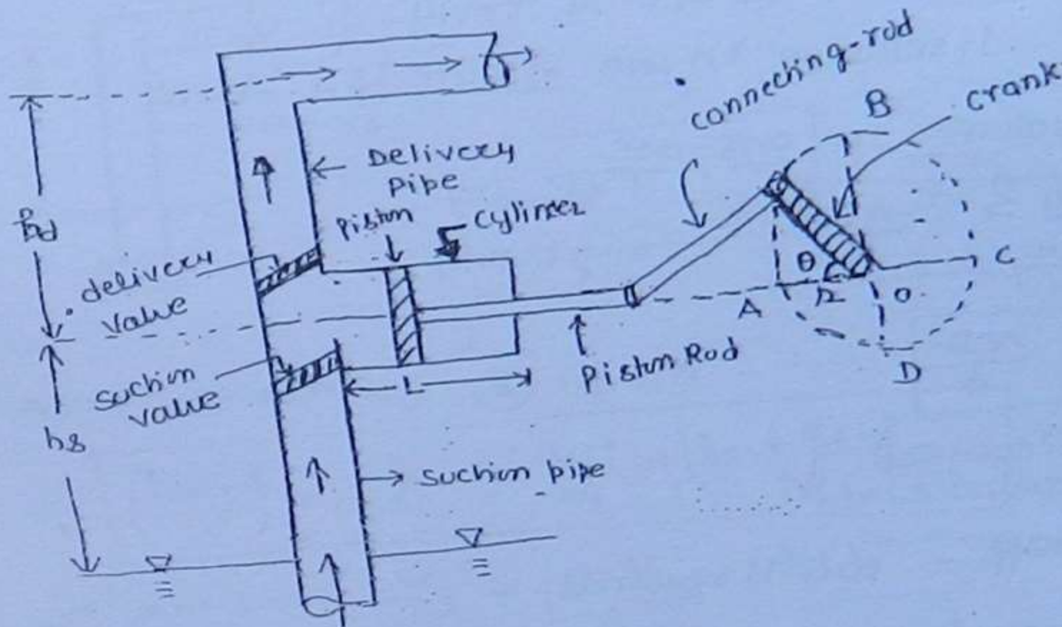

Working:

The crank is rotated by an external source of power. when the crank is at A. suction stokes started and when $\boxed{\theta = 0^\circ - 180^\circ}$ suction stokes complettes. The piston is moving in the cylinder forward & backward.

A → Area of cylinder = Area of piston

L = Length of cylinder chamber

$\boxed{L = 2r}$

when $\theta = 0^\circ$ to 90°, accln takes place and when $\theta = 90^\circ$ to 180°, deaccln. & at $\theta = 90^\circ$ the velocity of piston is Maximum.

for delivery pipe → For $\theta = 180^\circ$ to 270° ⇒ Accln of delivery stoke
For $\theta = 270^\circ$ to 360° ⇒ Deaccln of delivery stoke
at $\theta = 270^\circ$ ⇒ velocity is Max

It may be noted that while suction stoke only suction valve is open & delivery valve is closed & vice versa.

a> Discharge through pump:↓

Let N be the r.p.m. of crank

1 ~~pipe~~ one revoln. of crank

Vol. of water discharged = AL

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If in crank, there are N revln.

than discharge in one Minute is = ALN

∴ discharge in one sec.

Imp $\boxed{Q = \frac{ALN}{60}}$ $N \rightarrow R.P.M$

$\boxed{A = \frac{\pi D^2}{4}}$

b> Power Required:↓

$$\boxed{P = \omega Q \, H_{required}}$$

$$H_{required} = (h_s + h_d) + \underbrace{h_{fs} + h_{fd}}$$

↳ If Neglected

than $H = h_s + h_d$

$$\therefore \boxed{P = \omega Q (h_s + h_d)}$$

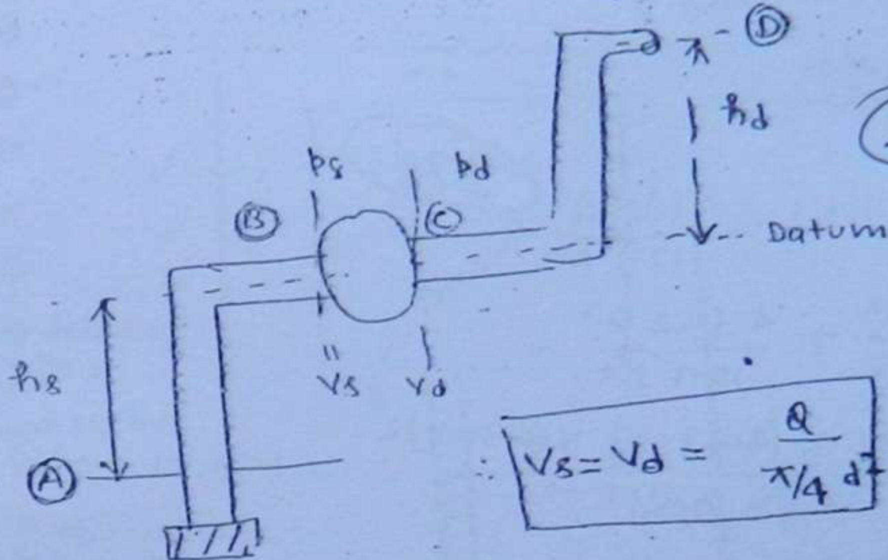
COMMENTS: ↓

1> The discharge through the pump is not continuous in nature therefore power required is fluctating

Imp 2> Reciprocating pumps are suitable for high suction head & Low delivery head & Low discharge.

3> In order to make the more continuous supply of water double acting pump may be used.

hence $V_{f1} = V_1 = Q/A_{f1}$



$$\therefore \boxed{V_s = V_d = \frac{Q}{\frac{\pi}{4} d^2}} = \frac{0.237}{\frac{\pi}{4}(0.33)^2}$$

$$= 2.46 \text{ m/sec.}$$

Apply B. Eqn b/w (A) and (B)

$$\boxed{\left(\frac{p_{atm}}{\omega}\right) = \frac{p_s}{\omega} + \frac{V_s^2}{2g} + [h_s + \underbrace{h_{fs}}_{\text{Loss in suction pipe}}]}$$

in term of water $\rightarrow$ $10.3 = \frac{p_s}{\omega} + \frac{2.46^2}{2 \times 9.81} + 3 + 2$

$$\Rightarrow \boxed{\frac{p_s}{\omega} = 5} > 2.5 \text{ m of vapour pressure at } 20^\circ c$$

$\boxed{\text{O.K.}}$

Apply B. Eqn b/w (C) and (D)

$$\frac{p_d}{\omega} + \cancel{\frac{V_d^2}{2g}} + 0 = \frac{p_{atm}}{\omega} + \cancel{\frac{V_d^2}{2g}} + (h_d + \underbrace{h_{fd}}_{\text{Loss in delivery pipe}})$$

$$p_d/\omega = 10.3 + 3.7 + 6 = 53.3 \text{ m}$$

prob 4. In a pumping station 18000 m3 water is to be lifted per day from a intake well to a sedimentation tank under a static head of 21 m. Length of suction & delivery pipes are 10 m & 150 m respectively. dia of pipes is constant = 0.50 m. There are two shifts of working each of 8 hrs. If the efficiency of pump & motor combined is 80% and friction coeff is 0.01. Recommends the unit of pumps each having B.H.P of (= 30) ~~Pipe~~ pump

$t = 16 hr$

$\therefore Q = \frac{18000}{16 \times 60 \times 60} = 0.3125 \ m^3/sec$

$H_s = 21 m$

$h_f = h_{fs} + h_{fd}$ (278)

$h_{fs} = \frac{4 f L_s Q^2}{12.1 D^5}, \quad h_{fd} = \frac{4 f L_d Q^2}{12.1 D^5}$

$\therefore h_f = \frac{4 f L_s Q^2}{12.1 D^5} + \frac{4 f L_d Q^2}{12.1 D^5}$

$= \frac{4 \times 0.01 \times (40+150) \times (0.3125)^2}{12.1 \times (0.5)^2} = 1.96 m$

$\therefore H_m = H_s + h_f$

$= 21 + 1.96 = 22.96 \ m$

$\therefore$ Manometric power $= wQH_m \rightarrow KW$

$= \frac{\rho Q H_m}{75} (H.P.)$

$= \frac{1000 \times 0.3125 \times 22.96}{75}$

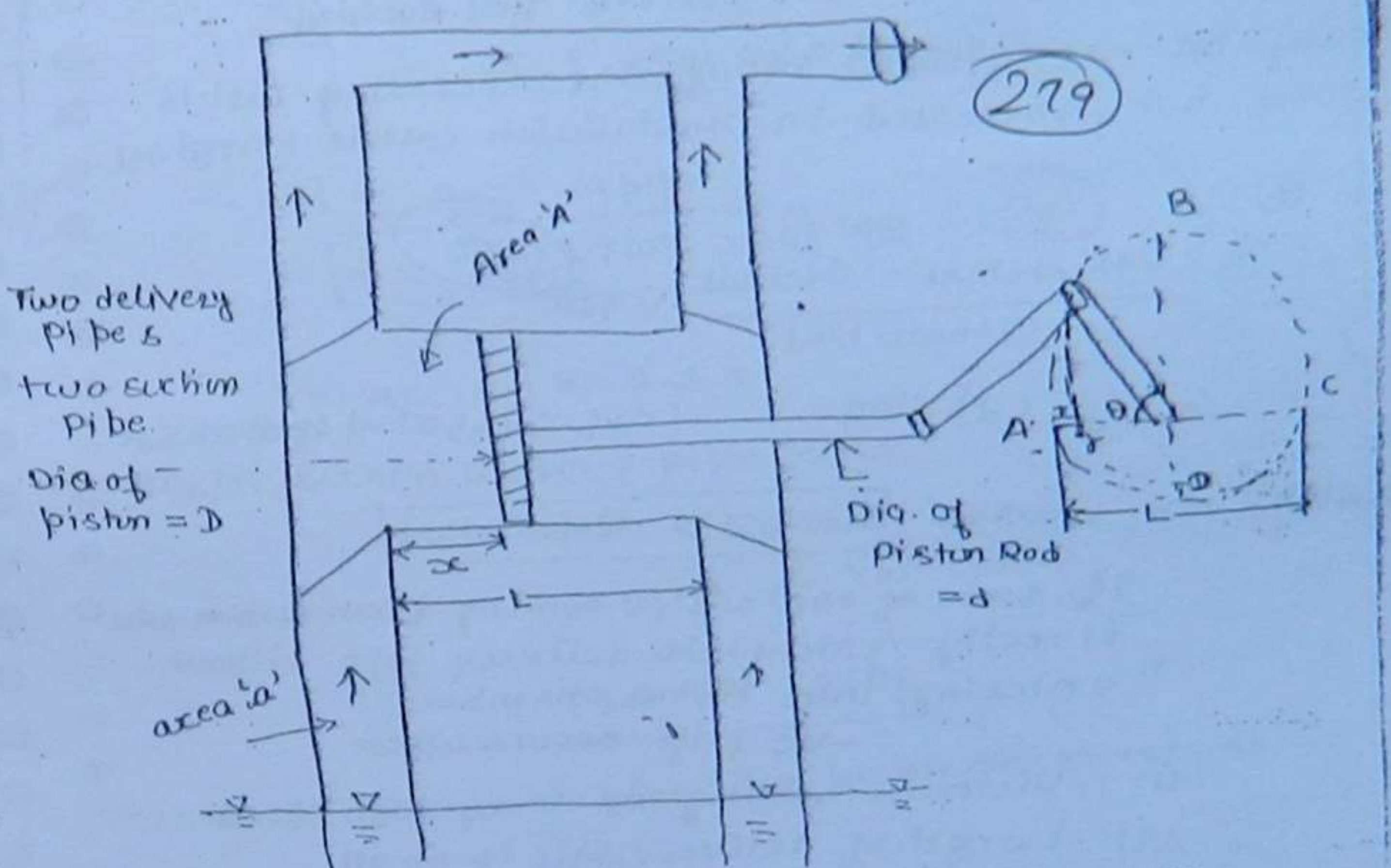
$= 95.67 \ H.P.$

$\eta_o = \frac{\text{Mano. Power (H.P)}}{S.H.P.}$

$S.H.P. = \frac{95.67}{0.80} = 119.58 \ (B.H.P)$

No. of pump required $= \frac{\text{Total B.H.P}}{\text{B.H.P of one pump}}$

$= \frac{119.58}{30} \simeq 4$



Area on the ~~right~~ Left of the piston $= \pi/4\, D^2$

Area ,, ,, ~~Left~~ Right ,, $= \pi/4\,(D^2 - d^2)$

Length $\boxed{L = 2r}$

so, Total volume of water delivered in one revolution of crank

$$V = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L$$

Volume of water delivered in one minute

$$= \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] L \cdot N$$

$$\text{Discharge/sec} = \frac{\frac{\pi}{4}\left[D^2 + (D^2 - d^2)\right] LN}{60}$$

→ If area. $d << D$ then

imp $\boxed{Q = \frac{2 \times \frac{\pi}{4} D^2 \times LN}{60} = \frac{2ALN}{60}}$ → hence discharge get doubled

Power Required: $\boxed{P \propto Q}$

$\therefore$ Power required get doubled

It may be noted that though the operating cost is doubled but increased in installation cost is marginal

Slip: → (%)

$$= \frac{Q_{theoretical} - Q_{actual}}{Q_{theoretical}} \times 100$$

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$$= (1 - C_d) \times 100$$

[C_d: coefficient of discharge

-ve slip: → when $\boxed{Q_{actual} > Q_{theoretical}}$

If some of the water coming from suction pipe directly enters into delivery pipe without entering into piston chamber.

-ve slip occurs when

(i) piston is moving at very high speed
(ii) Length of delivery pipe is small

Effect of Accln of piston on the velocity of suction and delivery pipe: ↓

$$x = r(1 - \cos\theta)$$

velocity of piston

$$V = \frac{dx}{dt} = r\sin\theta\,(d\theta/dt)$$

$$= r\omega\sin\theta$$

$$\boxed{\frac{dx}{dt} = r\omega\sin\theta}$$ Imp.

V_{max} occurs at $\theta = \pi/2$

Let v be the velocity of water in suction / delivery and a be the area of that pipe

$$\boxed{av = AV}$$ Imp.

$V_2 = (A/a)\, r\omega \sin\theta \rightarrow$ velocity of water

$\therefore \frac{dV}{dt}$ = accleration of water in pipe

$= (A/a) \times r\omega \cos\theta \, (d\theta/dt)$

$\therefore \alpha$ = Accln. in pipe

$$\boxed{\alpha = (A/a)\, r\omega^2 \cos\theta} \text{ imp}$$

(281)

α_{Max} at $\theta = 0^\circ$ & π

- Force required in suction water / pipewater

$= \text{Mass} \times \text{accln}$

$= (\rho a l) \times A/a \; r\omega^2 \cos\theta$

$$\boxed{F = \rho l A \, r\omega^2 \cos\theta}$$

- Pressure head in pipe due to piston movement

$= F/a$

$= \rho l \,(A/a)\, r\omega^2 \cos\theta$

$\therefore$ pressure head $= p/\rho g$

$$\boxed{h = (l/g) \times (A/a) \times r\omega^2 \cos\theta} \text{ imp}$$

- Friction head on suction / Delivery pipe

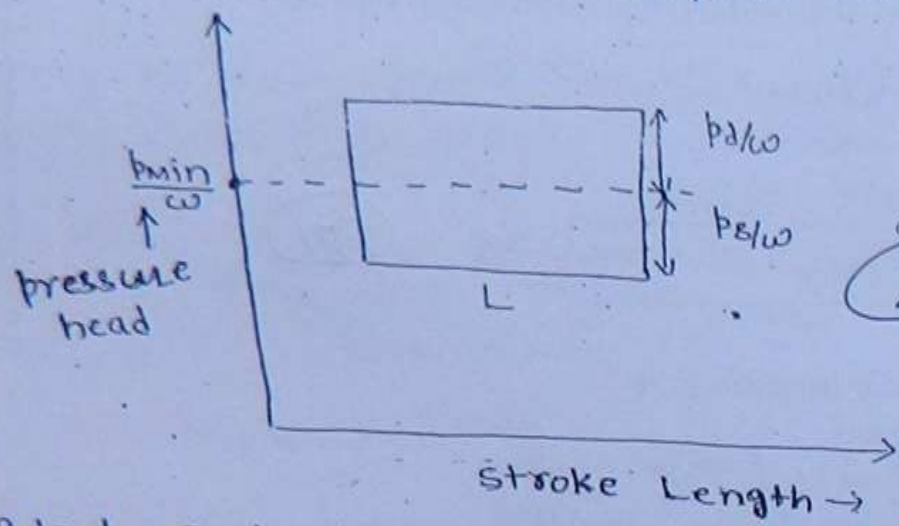
$$\boxed{h_{f_s} = \frac{f\, l_s V^2}{2 g d_s}}$$

[d → dia of suction pipe

$$= \frac{f\, l_s \left[\left(\frac{A}{a}\right) r\omega \sin\theta \right]^2}{2g\, d_s}$$

$$\therefore \boxed{(h_f)_{max} = \frac{f\, l_s \left[(A/a)\, r\omega \right]^2}{2 g\, d_s}}$$

distance travelled by the piston in complete revolution of crank.



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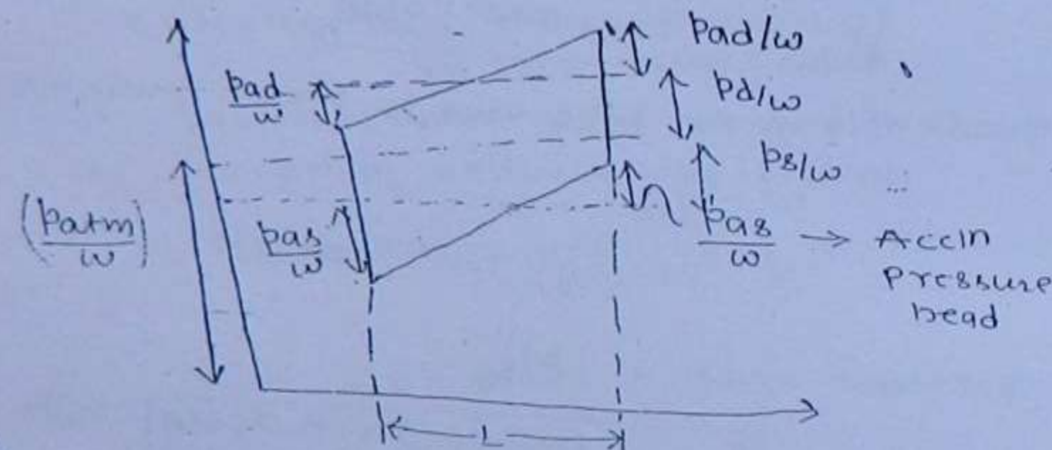
Ideal Indicator diagram when hf is neglected & effect of accln. also neglected.



Significance :-

(i) Area of Indicator diagram is directly proportion to discharge and Q ∝ Power hence If a Indicator diagram of a pump is given than discharge output and power consumed can be compared of two pumps

A C.P. has an impeller of 0.5 m of outer dia, when running at 600 r.p.m. discharges 8000 ltr/min against a head of 8.5 m.

CS/2001
ES/2001

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Prob 2

The cylinder bore dia. of a single acting reciprocating pump is 150 mm and stroke is 300 mm. The pump runs at 50 rpm. and water is lifted to a height of 25 m. The length of delivery pipe is $L_d = 22$ mm and $d_d = 100$ mm.

Find the theoretical discharge & power required to running the pump, if actual discharge = 4.2 l/s. Find the % slip. Also determine accln head at the begining and middle of stroke.

$A = \pi/4 \, D^2$

$D = 150$ mm

$L = 2r = 300$ mm

$N = 50$

$l_s = 25$

$l_d = 22$

$$\boxed{Q_{th} = \frac{ALN}{60}} = \frac{\pi/4 (0.15)^2 \times 0.3 \times 50}{60} \quad (m^3/sec)$$

$$= 4.42 \times 10^{-3} \ m^3/sec$$

$$= 4.42 \ l/sec$$

$$Q_a = 4.2 \ l/sec.$$

$$\therefore C_d = 4.2/4.42 = 0.95$$

$$\therefore \% \text{ slip} = \frac{4.42 - 4.2}{4.42} \times 100 = 4.95\%.$$

Accln head = pressure head due to accln

Accln head in suction pipe :

$$h_{as} = (l_s/g) \times A/a_s \times r\omega^2 \cos\theta$$

$\left[\omega = \frac{2\pi N}{60} \Rightarrow r = L/2 = 150 \text{ mm} \right]$

Accln head in delivery pipe :

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times r\omega^2 \cos\theta$$

$[\theta = 0^\circ, 90^\circ]$

$[\theta = 180^\circ, 270^\circ]$

velocity of flow from inlet to exit remains constant & the turbine discharges radially so that the degree of reaction (ρ) can be expressed as

$$\rho = \frac{1}{2}\left[1 - \frac{\cot\theta}{\cot\alpha - \cot\theta}\right]$$

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where θ is the runner vane angle at inlet
α is the guide blade angle &
$\rho \rightarrow$ degree of rxn. defined as ratio of pressure head droped to the hydraulic work-done in the runner.

Assume that losses in the runner are negligible.

$$\rho = \frac{\text{Pressure head drop b/w inlet \& outlet of Runner}}{\text{work-done by the water on the runner / sec. / unit wt. of water}}$$

$$= \frac{p_1/w - p_2/w}{\left(\frac{V_{w_1}u_1 - V_{w_2}u_2}{g}\right)}$$

Apply the B-eqn b/w inlet & exit of runner

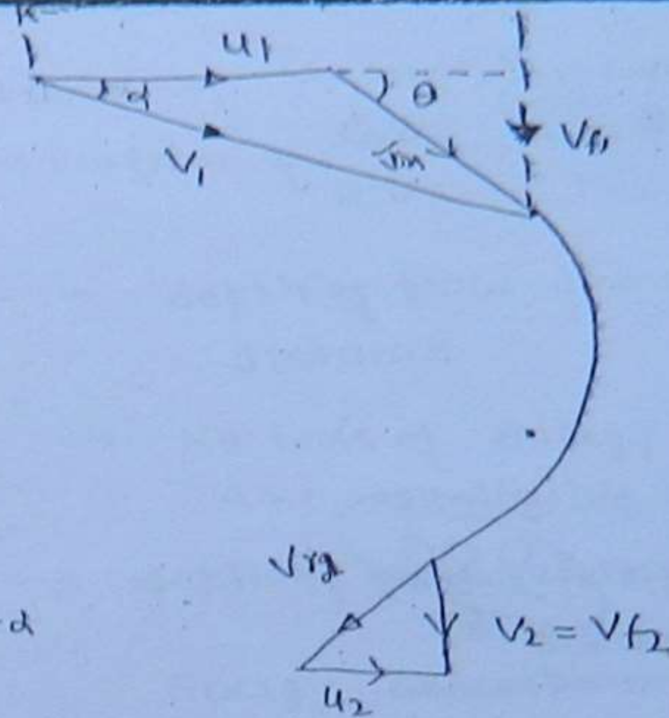
$$p_1/w + z_1 + \frac{V_1^2}{2g} = p_2/w + \frac{V_2^2}{2g} + z_2 + \frac{V_{w_1}u_1}{g}$$

$$(p_1/w - p_2/w) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) + \frac{V_{w_1}u_1}{g}$$

[No losses are considered]

$$\rho = \frac{\left[\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right] + \frac{V_{w_1}u_1}{g}}{\left(\frac{V_{w_1}u_1}{g}\right)}$$

$$\rho = 1 + \left(\frac{V_2^2 - V_1^2}{2V_{w_1}u_1}\right)$$



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$V_{f_1} = V_{f_2} = V_2$

$V_{w_2} = 0$

$$\boxed{\cot\theta = \frac{V_{w_1} - u_1}{V_{f_1}}}$$

$$\boxed{\frac{V_{w_1}}{V_{f_1}} = \cot\alpha}$$

$\therefore u_1 = V_{w_1} - V_{f_1}\cot\theta$

$\therefore V_{w_1} = V_{f_1}\cot\alpha$

$\therefore u_1 = V_{f_1}\left[\cot\alpha - \cot\theta\right]$ —(ii)

$$\boxed{V_2 = V_{f_1}}$$

$\therefore V_1^2 = V_{f_1}^2 + V_{w_1}^2$

$\therefore V_1 = \sqrt{1 + \cot^2\alpha} \cdot V_{f_1}$

$$\therefore \boxed{V_1 = V_{f_1}\sqrt{1 + \cot^2\alpha} = V_{f_1}\operatorname{cosec}\alpha} \text{ —(i)}$$

$$\therefore R = 1 + \frac{V_{f_1}^2 - V_{f_1}^2\operatorname{cosec}^2\alpha}{2V_{f_1}\cot\alpha \times V_{f_1}(\cot\alpha - \cot\theta)}$$

$$R = 1 + \frac{V_{f_1}^2(1 - \operatorname{cosec}^2\alpha)}{2V_{f_1}^2\cot\alpha(\cot\alpha - \cot\theta)}$$

$$= 1 - \frac{\cot^2\alpha}{2\cot\alpha(\cot\alpha - \cot\theta)}$$

$$= 1 - \frac{\cot\alpha}{2(\cot\alpha - \cot\theta)}$$

$$\Downarrow$$

$$\boxed{R = \frac{1}{2}\left[1 - \frac{\cot\theta}{(\cot\alpha - \cot\theta)}\right]}$$

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OPEN CHANNEL FLOW

O.C.F
- Uniform → V = const, d (depth) = const, A = const [channel is prismatic], Q = const
- Non-uniform
  - GVF
  - RVF

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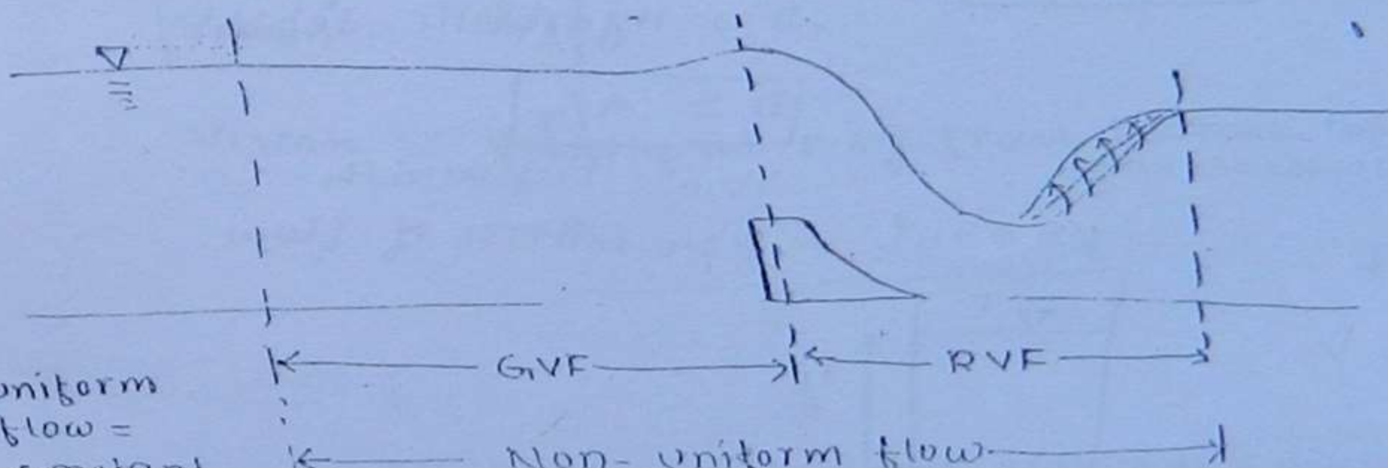
GVF → depth of flow changes over a long distance

→ No loss of energy if friction losses are negligible.

RVF → depth of flow changes suddenly & Energy ~~description~~ dissipation takes place at the point of Jump formation.



uniform flow = constant depth.

* For Laminar Flow in open channel

$\boxed{Re \le 500}$ → Laminar

$\boxed{Re > 2000}$ → Turbulent

$$Re = \frac{\rho V R}{\mu} = \frac{VR}{\nu}$$

R → Hydraulic Radius $= A/P$ → wetted perimeter

→ Hydraulic Mean depth

Subcritical Flow: ↓

If $F_R < 1$

Subcritical / Tranquil / Streaming / stable flow
(velocity Low, depth of flow high)

critical Flow: ↓

If $F_R = 1 \Rightarrow$ critical Flow

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supercritical Flow: ↓

If $F_R > 1 \Rightarrow$ supercritical Flow / shooting / Rapid / unstable flow.
($V\uparrow$, depth of flow Low)

$$\boxed{F_R = \frac{V}{\sqrt{gD}}}$$

$V \rightarrow$ Mean velocity $= Q/A$

$D \rightarrow$ Hydraulic depth

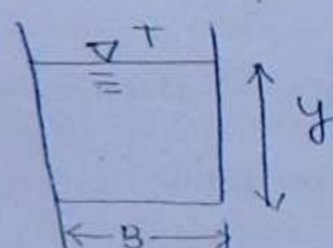
$$\boxed{D = A/T}$$

T = Top width

A = Area of flow

For eg: ↓

1)



$\therefore$ Hyd. depth $= A/T$

$= By/B = y$

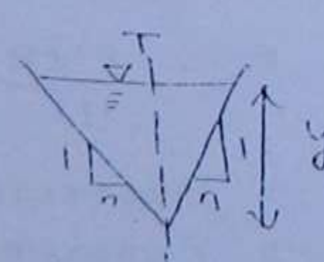
$=$ depth of flow {In case of Rect. channel}

2) $T = 2ny$

$A = \frac{1}{2} \times y \times 2ny = ny^2$

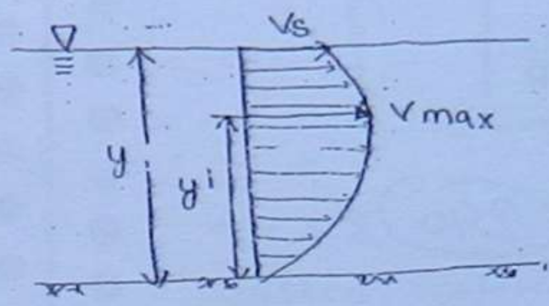
$\therefore$ Hydraulic depth $= A/T$

$= y/2$



[side slope = nH:1V]

Velocity distribution in open channel :-



(289)

y' = distance of V_{max} from bottom $\simeq 0.8y$ to $0.95y$ *gate*

$\boxed{V_s \simeq 0.91\, V_{max}}$ $\xrightarrow{\text{gate}}$ (May be b/w 0.85 to $0.95\, V_{max}$).

↳ velocity at surface

$\boxed{V_{Mean} = Q/A}$

$V_{Mean} \simeq$ velocity at $0.6y$ from ~~bottom~~ top (Freesurface)

$$\simeq \frac{V_{at\, 0.2y} + V_{at\, 0.8y}}{2}$$

UNIFORM FLOW :-

Methods to determine velocity & discharge

(a) Chezy's Eqn :-

$$V = C\sqrt{RS}$$

$$\boxed{Q = AC\sqrt{RS}}$$

S = slope of the channel bottom
A = Area of flow
R = Hydraulic Radius [Effective length Parameter]
C = Chezy's constant

$\boxed{C = L^{1/2} T^{-1}}$ gate

Chezy's constant can be calculated by the Kutter's or Bazin's eqn and C depends upon the surface roughness

$$\boxed{C = \frac{23 + \frac{0.00155}{S} + \frac{1}{n}}{\left\{1 + \left[\left(23 + \frac{0.00155}{S}\right)\right]\left[\frac{n}{\sqrt{R}}\right]\right\}}}$$

n = Kutter's rugosity coeff. = Roughness of channel surface

$$\boxed{C = \frac{23 + \frac{0.00155}{S} + 1/n}{\left[1 + \left[\left(23 + \frac{0.00155}{S}\right)\right] \frac{n}{\sqrt{R}}\right]}}$$

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Bazin's eqn: ↓

$$\boxed{C = \frac{157.6}{1.81 + K/\sqrt{R}}}$$

K = Bazin's coeff.

$$\boxed{C = \frac{187}{1 + M/R}}$$

$$\boxed{C = \frac{m/sec}{\sqrt{m}} = L^{1/2} T^{-1/2}}$$

Average or Mean velocity is findout

Manning's Equation: ↓

$$\boxed{V = 1/N \, R^{2/3} S^{1/2}}$$

N = Manning's Rugosity constant

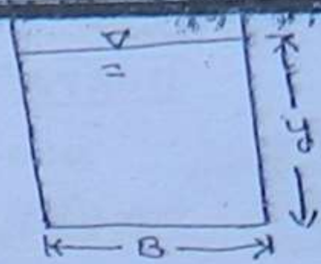
$$\boxed{N = L^{-1/3} T^{1}}$$

$$\boxed{C = 1/N \, R^{1/6}}$$

v. Imp.

$$\boxed{C = \sqrt{8g/f} = 1/N \, R^{1/6}}$$

A) Rectangular section:



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$A = By$

Wetted perimeter $= B + 2y$

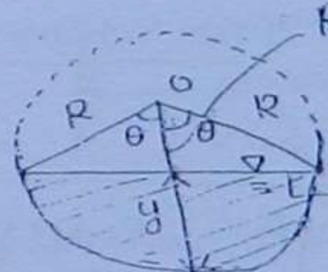
$R = A/P = \dfrac{By}{B+2y}$

$D = $ Hy. depth $= A/T = y$

gate $\boxed{\text{Section Factor} = \sqrt{A^3/T}}$

$= \left[\dfrac{(By)^3}{B}\right]^{1/2} = B \cdot y^{3/2}$

B) circular section:



• Area of Flow

$= \theta R^2 - 2\sqrt{R^2(R-y)^2} \times \frac{1}{2} \times (R-y)$

[$\theta \rightarrow$ Radian

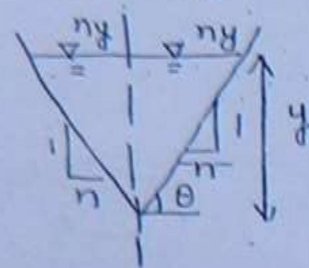
$= R^2\left[\theta - \frac{1}{2}\sin\theta\cos\theta\right]$

$\boxed{A = R^2\left[\theta - \dfrac{\sin 2\theta}{2}\right]}$

gate

- Top width $T = 2R\sin\theta$ ✓
- Wetted Perimeter $P = 2\theta R$

$T = 2ny$



side slopes

$\boxed{nH : 1V}$

$E_c = 5/4\ y_c$

$y_c = \left(\frac{2Q^2}{g m^2}\right)^{1/5}$

$F = \sqrt{2}\cdot\frac{V}{\sqrt{gy}}$

$\tan\theta = 1/n$

$A = ny^2$

$T = 2ny$

$R = A/P = \frac{ny}{2\sqrt{1+n^2}}$

$P = 2y\sqrt{1+n^2}$

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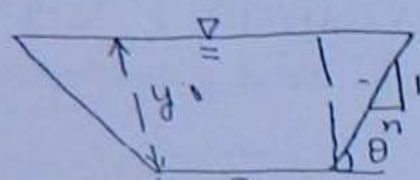
d) Trapezoidal section:



$T = B + 2ny$

$A = \left(\frac{B+T}{2}\right)y = \left(\frac{B+B+2ny}{2}\right)y$

$\boxed{A = (B+ny)\,y}$ ✓

Wetted perimeter $\boxed{P = B + 2y\sqrt{1+n^2}}$

e) Parabolic channel:

$T = B$

Hydraulic depth

$\boxed{D = A/T = 2/3\, y}$

• Section Factor

$Z = 2/9\sqrt{6}\ B y^{3/2}$

$A = 2/3\ Ty = 2/3\ By$

Wetted perimeter $P = B + 8/3\ \frac{y^2}{B}$

$= \frac{3B^2 + 8y^2}{3B}$

Prob: Find the discharge for the channel section shown in figure whose bed slope is .0001 and Manning's $N = 0.018$

$S = 0.0001$

$N = 0.018$ → $\boxed{0.012 \text{ to } 0.025}$

For smooth Lined channels. — For very Rough Earthen channel

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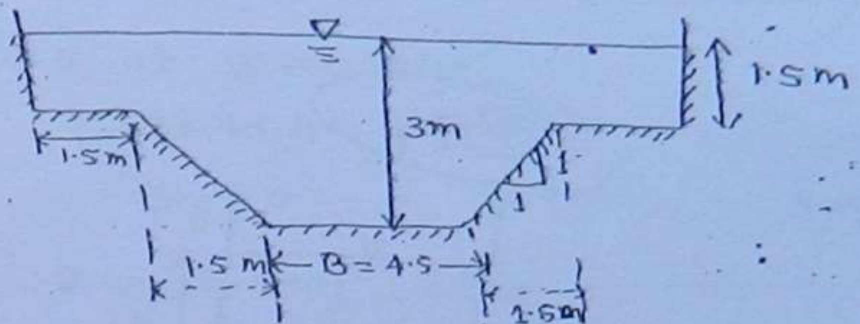

open channel
- steady
  - uniform → Flow in a prismatic channel with constant velocity.
  - GVF → (i) Backing up of water in a stream due to a dam. (ii) dropping of water surface due to a sudden drop in canal bed
  - RVF → A hydraulic jump occurring below a spillway / sluice gate.
  - Spatially varied → Flow over a bottom rack
- unsteady
  - Gradually varied → Passage of flood wave in a river
  - Rapidly varied → A surge moving up a canal & Bore travelling up a river
  - Spatially varied. → surface Runoff due to Rainfall.

* In GVF, frictional resistance plays an important Role.

* In GVF, RVF No flow is externally added or taken out of the system.

→ spatially varied flow — either some flow is added or abstracted from the system.

→ Specific Force is sum of the Pressure Force + Momentum Flux per unit wt. of the fluid at a section.

- Sp. Force is constant in a horizontal, Frictionless channel.

- critical flow condition is governed by the channel Geometry and discharge. Other channel properties such as the Bed slope and Roughness do not influence the critical flow condition for any given discharge.

Economical section: ↓

A section of a channel is said to be economical when its cost of construction is Least or for a given discharge and given area.

For a given sectional Area, dimension of section design in such a way that discharge carrying capacity is Maximum.

Rectangular section: ↓

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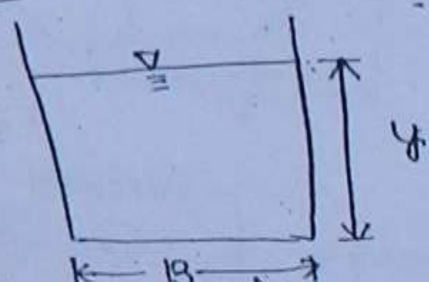
$A = By =$ constant



For Maxm Q

we know that

$Q \propto V \propto R^{1/2} \rightarrow$ chezy's

$\propto R^{2/3} \rightarrow$ Manning

$q_{Max} \rightarrow R$ should be Maxm

For a given area $R = A/P$

For $R_{max} \rightarrow P$ should be ~~Maximum~~ Minimum

wetted perimeter $P = B + 2y$

$P = A/y + 2y$

$\therefore$ For P_{min} $dP/dy = 0$

$-A/y^2 + 2 = 0$

$\Rightarrow \boxed{y = B/2}$ gaip

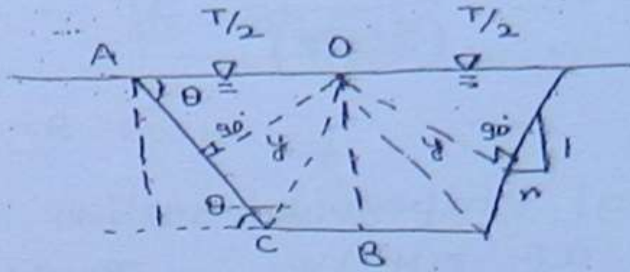
$\therefore$ $\boxed{\text{depth of flow} = \tfrac{1}{2}\ \text{width}}$ gaip

Hydraulic Radius $R = A/P = \dfrac{By}{B+2y} = \dfrac{B \cdot B/2}{B + 2 \cdot B/2}$

$\boxed{R = B/4 = y/2}$ gaip

~~Trapezoidal section~~

case I: → side slopes are constant



$$A = (B+ny)y$$
$$P = B + 2y\sqrt{1+n^2}$$
$$T = B + 2ny$$

For Maxm Q at given area

P should be Minimum

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$\therefore \; dP/dy = 0$

$$P = B + 2y\sqrt{n^2+1}$$
$$= A/y - ny + 2y\sqrt{n^2+1}$$
$$\therefore \; dP/dy = -A/y^2 - n + 2\sqrt{n^2+1}$$
$$= -\left(\frac{(B+ny)y}{y^2}\right) - n + 2\sqrt{n^2+1} = 0$$
$$= -(B+ny) - ny + 2y\sqrt{n^2+1} = 0$$

$$\therefore \; \boxed{y\sqrt{n^2+1} = \frac{B+2ny}{2}}$$

$$T/2 = \frac{B+2ny}{2}$$
$$\text{side} = y\sqrt{n^2+1}$$

∴ For Most Economical channel

$\boxed{\text{½ top width = one of sloping side Length}}$

$$\therefore \; R = A/P = \frac{(B+ny)y}{B+2y\sqrt{n^2+1}}$$
$$= \frac{(B+ny)y}{B+2\left(\frac{B+2ny}{2}\right)} \Rightarrow \boxed{R = y/2}$$
$$= y/2$$

$$\frac{(B+2ny/2)}{} = y/AC = \frac{y}{y\sqrt{n^2+1}} = \frac{OB}{\left(\frac{B+2ny}{2}\right)}$$

$$\therefore \boxed{OB = y}$$

Condition: For an economical trapezoidal section side slope will be at nH: 1V

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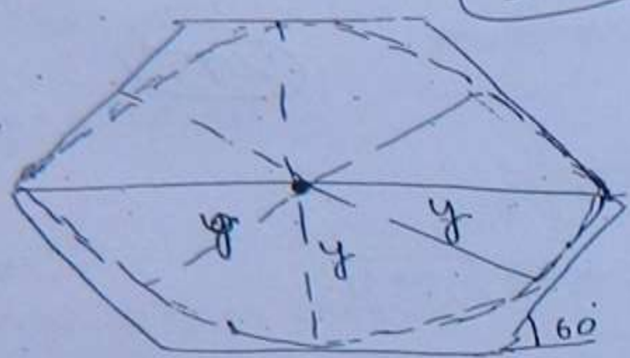


$\boxed{\theta = 60°}$ ↳ channel is Most Economical

$\boxed{1H:3V}$

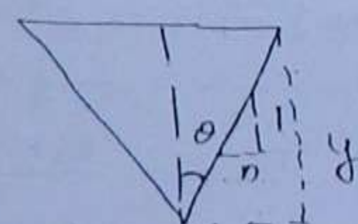
Trapezoidal section is a part of Hexagon whose centre at Middle of top width.

Triangular section: ↴

$A = ny^2$

$P = 2y\sqrt{1+n^2}$

$R = A/P$



∴ For Maxm R, P Min

∴ $\partial P/\partial n = 0$

$P = 2 \times \frac{\sqrt{A}}{\sqrt{n}} \sqrt{n^2+1}$

$P = 2\sqrt{A}\sqrt{1+1/n} \Rightarrow P^2 = 4A\,(n+1/n)$

$2P\left(\frac{dP}{dn}\right) = 4A\,(1-1/n^2)$

$\boxed{\frac{dP}{dn} = 2\sqrt{A}}$

$\frac{dP}{dn} = 0, \quad n = 1 \Rightarrow \boxed{\theta = 45°}$

i. Triangular section to be most economical $\theta = 45$ and $\boxed{R = \frac{y}{2\sqrt{2}}}$

$$\therefore \boxed{R = y/2\sqrt{2}}$$

CIRCULAR SECTION:

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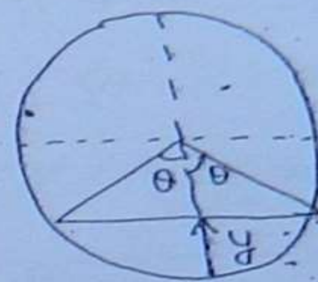
1) For Maxm. velocity condition

2) For Maxm. discharge condition



$V \propto (R = A/P)^{1/2}$
$\propto R^{2/3}$

$Q \propto A \cdot R^{2/3}$ ↳ Manning's eqn

$\propto A \cdot R^{1/2}$ chezy's Eqn.

$\therefore$ For V_{max} – R should be Maxm.

$$\therefore A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$P = 2\theta R$$

$$\frac{dA}{d\theta} = R^2[1-\cos 2\theta]$$

$$dP/d\theta = 2R$$

$$\therefore dR/d\theta = 0 \Rightarrow \partial/d\theta\,(A/P) = 0$$

$$= \frac{P\cdot\left(\frac{dA}{d\theta}\right) - A\cdot\frac{dP}{d\theta}}{P^2} = 0$$

$$\Rightarrow \frac{PR^2[1-\cos 2\theta] - A(2R)}{P^2} = 0$$

$$\Rightarrow 2R\theta[1-\cos 2\theta]R^2 - R^2\left[\theta - \frac{\sin 2\theta}{2}\right] \times 2R = 0$$

$$\Rightarrow \boxed{2\theta = \tan 2\theta}$$

$$\boxed{\begin{aligned}\tan\alpha &= \alpha \\ \alpha &= 4.5\ \text{rad}\end{aligned}} \longrightarrow \text{Trial \& Hit}$$

V. Imp

$$\boxed{2\theta = 4.5\ \text{rad} = 257^\circ\cdot 30'}$$

mp

$$\boxed{\begin{aligned} R_{Max} &= 0.60R \\ &= 0.80D \end{aligned}}$$

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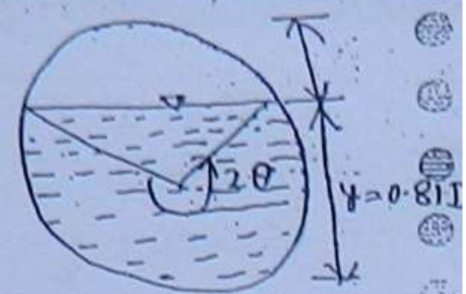
For V_{Max}:

$$\boxed{\begin{aligned} 2\theta &= 257^{\circ}\,30' \\ y &= 0.81D \\ R_{Max} &= 0.3D \end{aligned}}$$



Condition For Max^m discharge:

A) using chezy's eqn:

$$Q = CA\,R^{1/2}\cdot S^{1/2}$$

For $Q_{Max} \Rightarrow (A\,R)^{1/2}{}_{max}$

$$\therefore \frac{d}{d\theta}\left[A R^{1/2}\right] = 0$$

$$\Rightarrow \frac{d}{d\theta}\left[\frac{A^{3/2}}{P^{1/2}}\right] = 0$$

$$\Rightarrow \boxed{\begin{aligned} 2\theta &= 308^{\circ} \\ y &= 0.95D \\ R &= 0.29D \end{aligned}}$$

B) using Manning eqn:

$$Q = \frac{1}{N}\cdot A\,R^{2/3}\,S^{1/2}$$

$$Q_{Max} = (A R^{2/3})_{max}$$

$$\therefore \boxed{\begin{aligned} 2\theta &= 302^{\circ}\,20' \\ y &= 0.938D \\ R &= 0.29D \end{aligned}}$$

COMMENT: → From techanical consideration Manning's result are More relalistic because Mannings N based on surface great roughness which can be computing directly whereas chezy's C is given arbitary.

G.V.F. :

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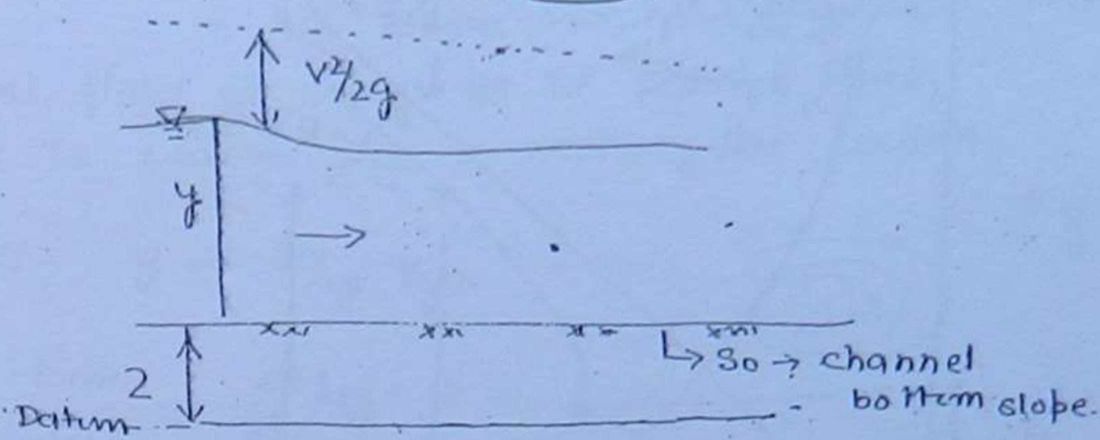


Total Energy head = E = $Z + y + \frac{V^2}{2g}$

Assuming uniform velocity of section $\alpha = 1$

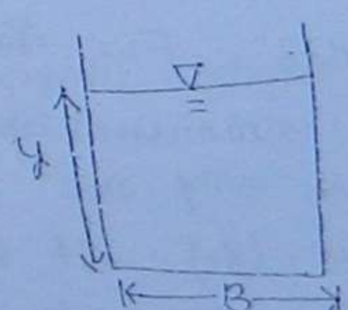
Specific Energy: If the ~~vertical~~ channel bottom is taken as datum than total energy per unit wt. is called specific Energy.

∴ For G.V.F. specific energy is constant

$$E = y + V^2/2g$$

Case: 1st: For Rectangular section

Specific Energy



$$\boxed{E = E_p + E_k}$$

$$E = y + V^2/2g$$

$$= y + \frac{Q^2}{2gA^2}$$

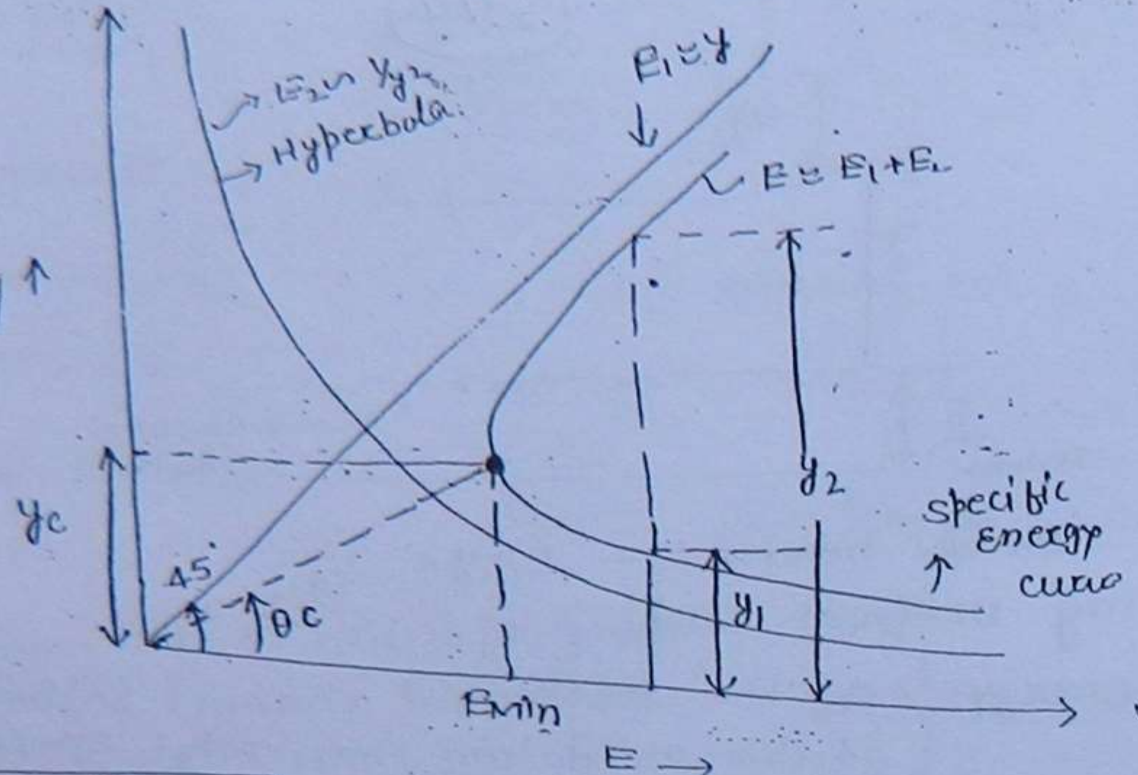
∴ Let discharge per unit width $q = Q/B$

$$\therefore E = y + q^2/2gy^2 = y + (q^2/2g) \times 1/y^2$$

$$= E_1 + E_2$$

$E_1 = E_p = y$

$E_2 = E_K = q^2/2g * 1/y^2$ 300



$$\boxed{\tan\theta = y_c/E_{Min}}$$
$$= \frac{2/3\ E_{min}}{E_{min}}$$

$\Rightarrow \boxed{\theta_c = 33.7°}$ For Rectangular
$= 38.65°$ For Triangular
$= 36.8°$ For Parabolic

imp

For G.V.F. Minimum specific Energy occurs at which there is only one depth of flow called critical depth of flow (y_c). It Means at critical flow specific energy is Minimum.

For other sp. energy there will be two depth of flow y_1 & y_2 known as alternate depths. For Rectangular section y_c is critical depth $2/3$ of Emin.

$y_c = \frac{2}{3} E_{min}$ For Rectangular

$= \frac{3}{4} E_{min}$ For Parabolic.

$= \frac{4}{5} E_{min}$ For Triangular

→ At critical flow it may ~~be~~ be proved that discharge is maxm. For a Rectangular section.

$$\therefore \quad E = y + \frac{q^2}{2g} \cdot \frac{1}{y^2}$$

301

$\therefore$ For E_{min}, $dE/dy = 0$

$$\Rightarrow 1 + \frac{q^2}{2g}\left(-\frac{2}{y^3}\right) = 0$$

$$\Rightarrow y^3 = \left(\frac{q^2}{g}\right)$$

$\therefore$ At E_{min}, $y = y_c$

$$\therefore \boxed{y_c^3 = \left(\frac{q^2}{g}\right)} \longrightarrow$$ Valid For Rectangular section.

If $y = y_c$, $E = E_{min}$

$\therefore$ at $y = y_c$, $E_{min} = y_c + \frac{y_c^3}{2} \times \frac{1}{y_c^2}$

$$\therefore \boxed{E_{min} = \frac{3}{2} y_c}$$

At critical flow, for Rectangular section kinetic head is half of potential head.

* For parabolic section: ↓

$$\boxed{\begin{aligned} E_{min} &= \frac{4}{3} y_c \\ &= y_c + \underbrace{y_c/3}_{\text{kinetic Head}} \end{aligned}}$$

Triangular section: ↓

$$\boxed{E_{min} = \frac{5}{4} y_c = (y_c) + \underbrace{y_c/4}_{\text{kinetic Head.}}}$$

$\therefore \quad \frac{V}{\sqrt{gD}} = 1$

$\therefore$ FOR Rect. section $D = A/T = \frac{B \cdot y}{B} = y$

$$\therefore \boxed{V_c = \sqrt{g y_c}}$$

3.02

For Triangular section

$$D = A/T = y/2$$

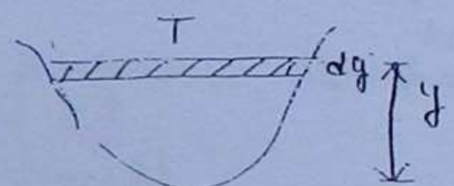
$$\therefore \boxed{V_c = \sqrt{g y_c/2}}$$

For Parabolic section

$$D = A/T = 2/3\, y$$

$$\therefore \boxed{V_c = \sqrt{\frac{2 g y_c}{3}}}$$

critical depth for Non-uniform channel:



$dA = T\, dy$

$\therefore dA/dy = T$

$E = y + \frac{Q^2}{2gA^2}$

For ~~critical~~ critical Flow $dE/dy = 0 = 1 + \frac{Q^2}{2g}\left(-2/A^3\right)\frac{dA}{dy}$

$\therefore \frac{Q^2}{g} = \frac{A^3}{(dA/dy)} = A^3/T$

$\boxed{\frac{Q^2}{g} = A^3/T} \longrightarrow$ For critical Flow
Applicable for Rectangular or Non-uniform section.

grIP

Dynamic Eqn. for Gradually Varied Flow:-

(303)

Assumptions:-

1> Chezy's Formula & Manning Formula is used (with S_0 as Energy slope.)

2> Bottom slope of the channel is very small

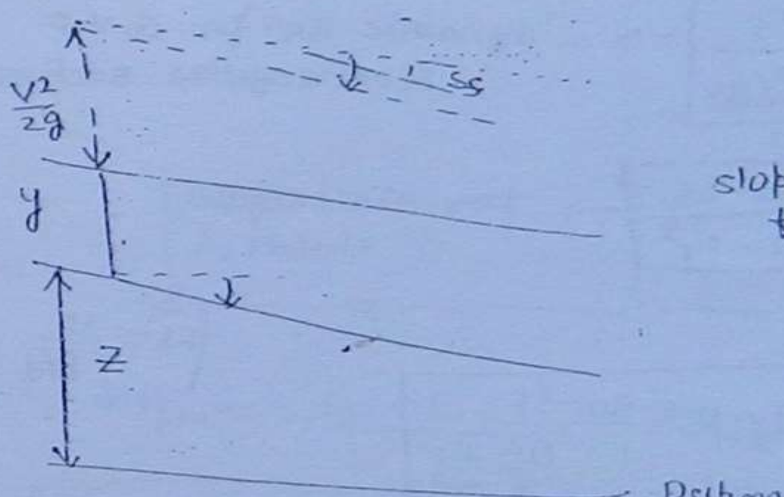
3> channel is prismatic.

4> Energy correction factor is 1

5> Pressure distribution is only hydrostatic

6> Discharge is constant, flow is steady.

7> Roughness coeff. of channel is independent of the depth of flow and taken constant through the Length of channel.



slope is falling in flow direction.

channel slope $\boxed{dZ/dx = -S_0}$

If total Energy is E

$$dE/dx = \text{Energy slope} - S_f$$

$$E = z + y + V^2/2g$$

$$dE/dx = dz/dx + dy/dx + d/dx\left[\frac{V^2}{2g}\right]$$

$$\therefore \quad d/dx\left(\frac{V^2}{2g}\right) = d/dx\left[\frac{Q^2}{2gA^2}\right]$$

$$\frac{d}{dx}\left[\frac{Q^2}{2gB^2y^2}\right] = \frac{Q^2}{2gB^2} \times -\frac{2}{y^3}\frac{dy}{dx} \qquad [A = By$$

$$= \frac{Q^2}{gB^2y^2} \times \bar{x}_y \, \frac{dy}{dx}$$

304

$$= -\frac{V^2}{gy} \cdot \frac{dy}{dx}$$

$$\therefore \frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} - \frac{V^2}{gy} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[1 - \frac{V^2}{gy}\right] = \frac{dE}{dx} - \frac{dz}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{-S_f - (-S_o)}{1 - V^2/gy}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{S_o - S_f}{1 - V^2/gy}} \rightarrow$$ dynamic eqn for G.V.F Rectangular section.

$$\boxed{\frac{dy}{dx} = \frac{S_o - S_f}{1 - F_r^2}} \rightarrow$$ For Rectangular channel

For All sections:

$$\left[F_r = \frac{V}{\sqrt{gy}}\right.$$

$$\boxed{\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2}{g}\frac{T}{A^3}}}$$

$$\boxed{S_o - S_f = \frac{dE}{dx}}$$

↑ differential energy eqn of G.V.F

Energy slope

$$\boxed{S_f = h_f/L}$$

According to chezy's eqn

$$\boxed{\frac{dy}{dx} = S_o\left[\frac{1 - (y_n/y)^3}{1 - (y_c/y)^3}\right]}$$ V.Imp (obj)

$y_c \rightarrow$ critical depth of flow

$y_n \rightarrow$ Normal " "

According to Manning's Eqn: 7.

$$\boxed{dy/dx = S_0\left[\frac{1-(y_n/y)^{10/3}}{1-(y_c/y)^{10/3}}\right]}$$

309

$y \rightarrow$ Actual depth of flow

$y_n \rightarrow$ It is that depth of flow at which flow is uniform

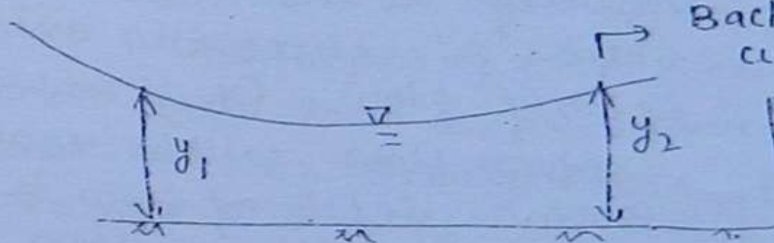
$dy/dx \rightarrow$ Rate of change of depth of flow w.r.t. channel bottom

If $\frac{dy}{dx} > 0 \rightarrow$ depth of flow increasing in direction of flow
↳ Backwater flow curve

$dy/dx = 0 \rightarrow$ depth of flow is constant.

$dy/dx < 0 \rightarrow$ depth of flow is decreasing
↳ drawdown curve.

1)



$\boxed{dy/dx > 0}$

$$dy/dx = \frac{S_0 - S_f}{1 - \frac{V^2}{gy}} \quad ; \quad dy/dx > 0 \text{ if}$$

$$\left[\begin{array}{c} S_0 > S_f \\ \& \\ 1 > \frac{V^2}{gy} \end{array}\right]$$

→ $\boxed{\text{channel slope} > \text{Energy slope}}$

→ Flow is subcritical.

2)

$\boxed{S_0 < S_f}$

$1 < \frac{V^2}{gy}$

$\boxed{dy/dx < 0}$

↑ supercritical flow

[drawdown curve]
↳ convex shape.

$$\therefore \frac{V^2}{gy} \neq 1$$

306

iv) If $\frac{V^2}{gy} = 1$, $F_r = 1$

$\boxed{\frac{dy}{dx} = \infty}$ → Since in G.V.F [dy/dx] is small depth of flow change over a larger length hence this condition beyond the assumption of G.V.F.

For a given discharge normal depth of flow can be calculated as follows:-

Normal depth of flow:-

[depth of flow at which a given discharge flows as uniform flow in a given channel.]

For given values of Mannings N and Chezy C and for given value of Q and channel bottom slope 'S_0' there will exist one depth of flow (y_n) at which the uniform flow will be maintained such a depth of flow is called Normal depth of flow.

- Choking:- (1) U/s, water surface elevation is not affected by the conditions at section (2) till a critical stage is just achieved.

(2) In case of Hump. for all $\boxed{\Delta z \leq \Delta z_{max}}$ — U/s water depth is constant

For all $\boxed{\Delta z > \Delta z_{max}}$ → y_1 increases in subcritical flow
→ y_1 decreases in super critical flow.

(3) In case of width contraction:-

$\boxed{B_2 \geq B_{2m}}$ → U/s depth y_1 is constant

while For $\boxed{B_2 < B_{2m}}$ → U/s depth goes under a change

→ onset of critical condition at (2) is prerequisite to choking.

→ All cases $\boxed{\Delta z > \Delta z_{max}}$, $\boxed{B_2 < B_{2m}}$ ⟹ known as choked condition.

→ { In subcritical Flow, water surface will drop due to decrease in sp. energy
{ In supercritical Flow, depth of flow increases due to Reduction in sp. energy.

CRITICAL DEPTH

of constant discharge situation:

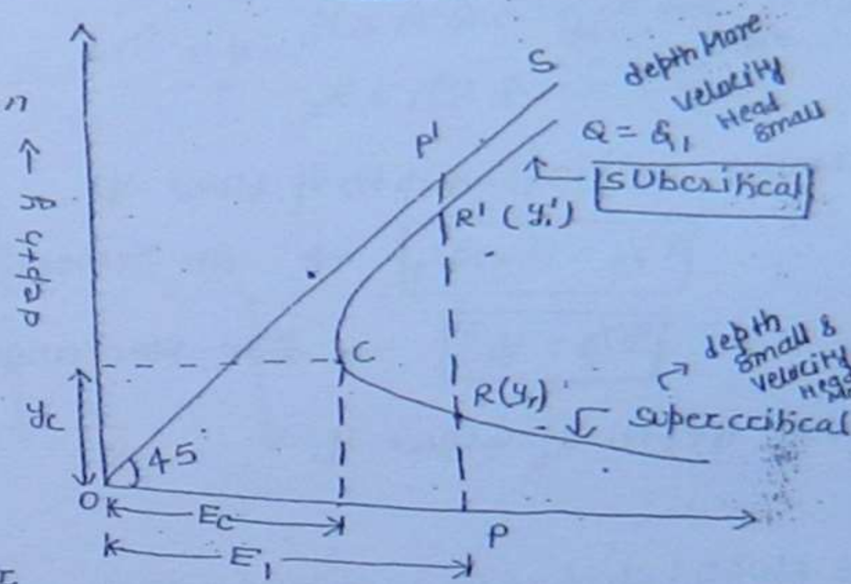
(307)

$$E = y + \frac{Q^2}{2gA^2}$$

* For a channel of known geometry

$E = f(y, Q)$

Keeping Q = constant $Q = Q_1$ the variation of E with y is represented by a cubic parabola.



That at any particular discharge Q_1 can be passed in a given channel at two depths & still maintain the same sp. Energy E.

. sp. Energy $E = y + \frac{v^2}{2g}$ →

The depth of flow can be either $PR = y_1$ or $PR' = y_1'$. These two have same sp. Energy. The intercept $P'R'$ or $P'R$ represents the velocity head. depth $(PR = y_1)$ is smaller and has a Large velocity head while other $(PR' = y_1')$ has a Larger depth and consequently a smaller velocity head.

→ For a given Q, as the sp. energy is increased the difference b/w the two alternate depth increases.

→ If E is decreased, the difference $(y_1' - y_1)$ will decrease and at a certain value $E = E_c$

→ At the Lower Limb CR of the sp. energy curve the depth $\boxed{y_1 < y_c}$ As such $\boxed{V_1 > V_c}$ and $\boxed{F_1 > 1.0}$

↳ supercritical Flow Region

→ In the upper Limb CR', $\boxed{y_1' > y_c}$ as such $\boxed{V_1' < V_c}$ and $\boxed{F_1' < 1.0}$ ⇒ subcritical flow Region.

1. Determine normal depth of flow by y_n.

(308)

$$Q = \frac{1}{N} \cdot A R^{2/3} S^{1/2}$$

OR

$$Q = C \cdot A \cdot R^{1/2} S^{1/2}$$

2. determine critical depth of flow y_c.

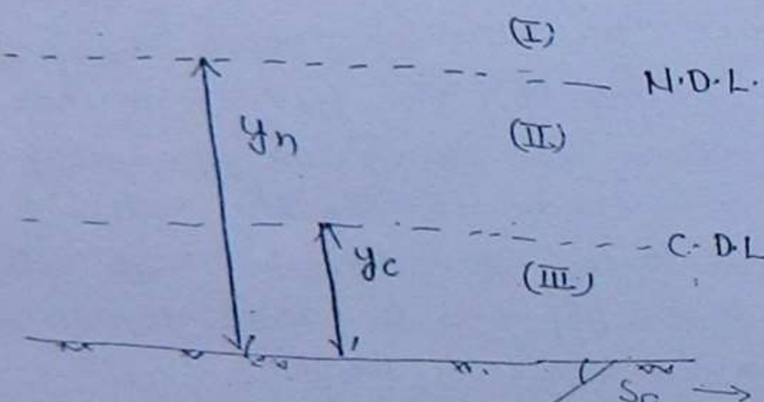
$\boxed{Q^2/g = A^3/T}$ → In General

$\boxed{q^2/g = y_c^3}$ → For Rectangular channel

3. Actual depth of flow y.

(a) Mild slope :

when $\boxed{y_n > y_c}$



→ Normal Mild slope

यहाँ $\boxed{S_0 < S_c}$

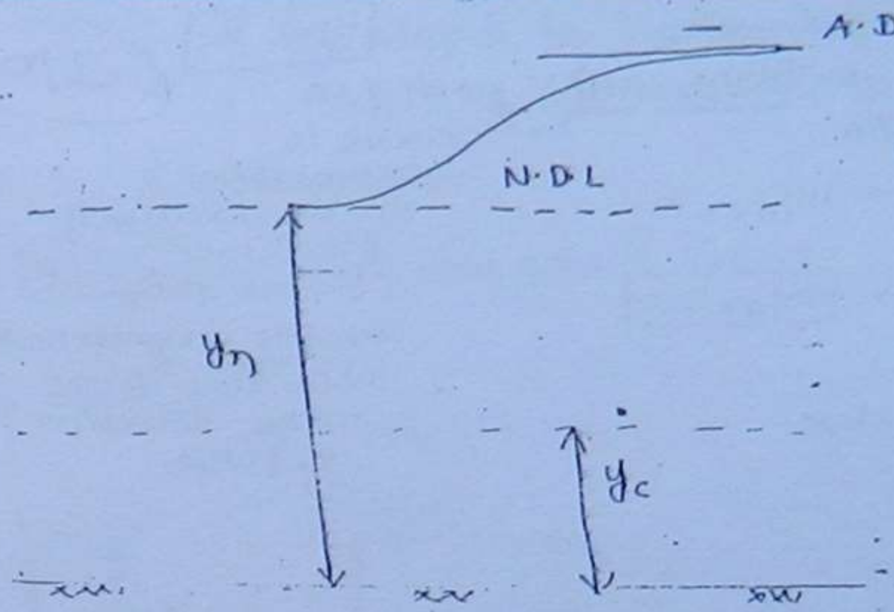
If actual depth of flow y is such that

$\cancel{\boxed{y_c < y < y_n}}$ $\boxed{y > y_n > y_c}$

↳ Than surface profile is M_1 type

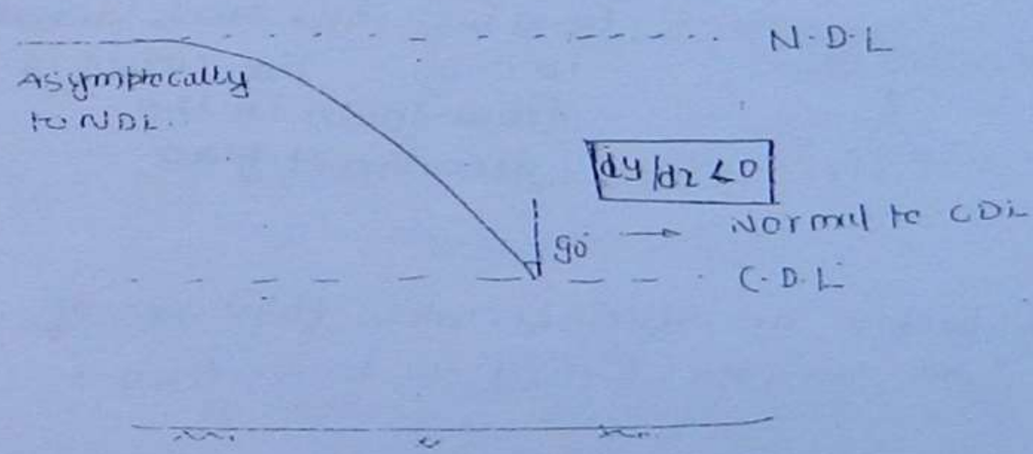
M_1 profile means normal depth line is asympotically and tend to become horizontal in d/s towards actual depth line. The curve water is backwater and rising hence

$$\boxed{dy/dx > 0}$$



If. $\boxed{y_c \le y \le y_n}$ → M_2 profile will be formed

↳ Means asymptocally to normal depth line and Normally to critical depth in the direction of flow. The curve is drawdown.

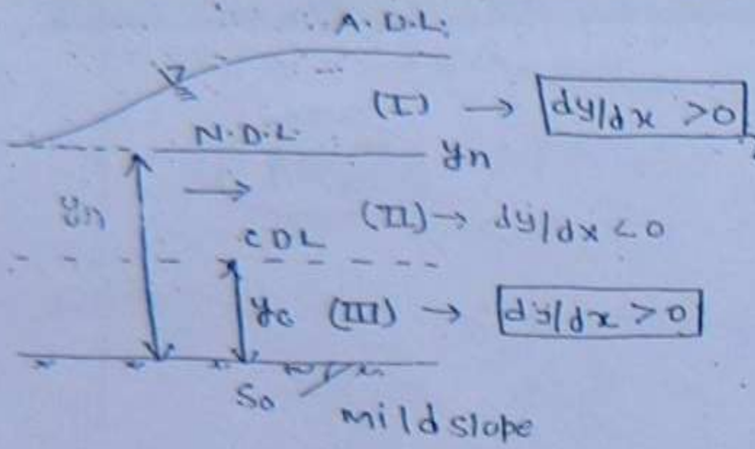


** M_1 & M_2 curve profile are formed when $\boxed{Fr < 1}$

In Mild slope

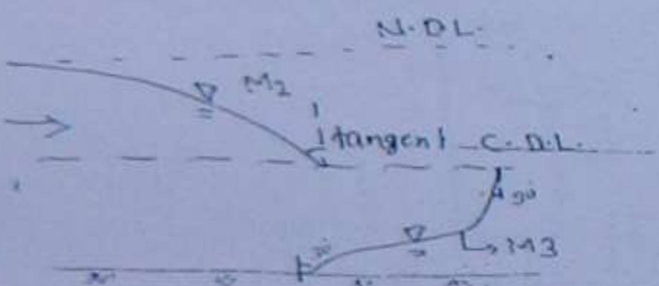
$\boxed{S_0 < S_c}$

3/0



curve is Backwater in the direction of flow. i.e. M_1 profile Meets Asymptotically to NDL and Rises in the direction of flow.

When $\boxed{y_c \leq y \leq y_n}$, than M_2 profile is formed



M_2 profile Meets asmtotatically to N.D.L. and Meets normally to C.D.L. The curve is draw down in the direction of flow.

NOTE. M_1 & M_2 profile are formed when flow are of stable type having Fraud No. Less than 1

When $0 < y \leq y_c$ → M_3 profile is formed

→ M_3 profile is backwater curve which is normal to channel bottom ~~slope~~ slope & C.D.L.

→ M_3 curve is formed when flow is supercritical

2) Horizontal slope:

If slope is horizontal than normal depth of flow tends to infinity.

$Q \propto y_N \; A R^{2/3} S^{1/2} \to 0$

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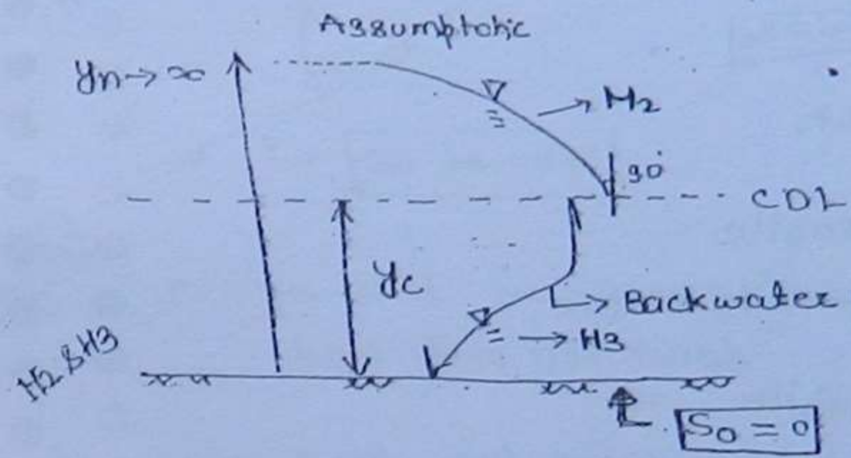
$\Rightarrow$ $\boxed{y_n \to \infty}$ (For horizontal slope)



H1 profiles are not formed and H1 zone does not exist

If $\boxed{y_c \le y \le y_n}$ → than H2 profile is formed
→ drawdown curve.

If $\boxed{y < y_c}$ → than H3 profile is formed.
→ Backwater curve.

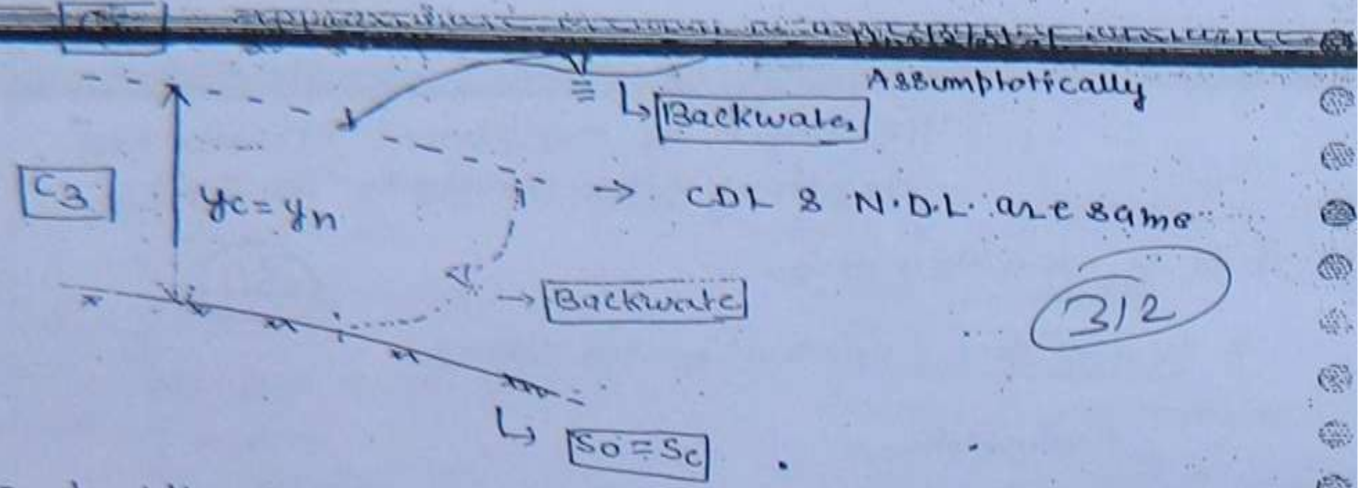
3) critical slope:

Imp

(C1 & C3)

1) when channel bottom slope $\boxed{S_0 = S_c}$ at this stage flow is critical

2) y_n will be Eqaual to y_c. $\boxed{y_n = y_c}$



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C_2 profile does not exist.

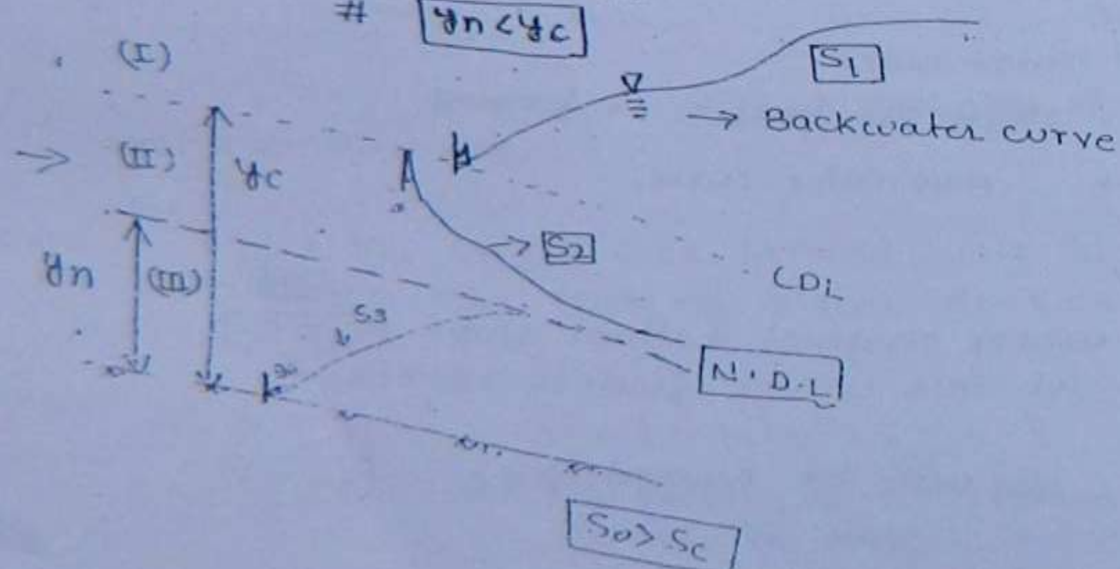
If $y > y_n \Rightarrow C_1$ profile
$(= y_c) \Rightarrow$ Backwater curve

If $y < y_n \Rightarrow C_3$ profile
$(= y_c) \Rightarrow$ ~~drawdown~~ curve.
Backwater

steep slope :

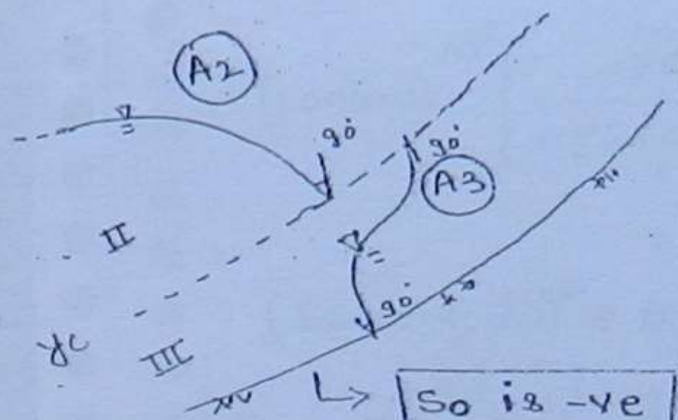
≠ $\boxed{S_0 > S_c}$ imp.

$\boxed{y_n < y_c}$



S_1 profile $\Rightarrow$ when $y > y_c$
S_2 profile $\Rightarrow$ when $y_n \le y \le y_c$
S_3 profile $\Rightarrow$ when $0 < y < y_n$

Adverse slope :



$\boxed{S_0 \text{ is } -ve}$

A2 & A3

* yn will be imaginary so normal depth Line does not exist.

* A1 zone will not exist and only A2 & A3 will exist.

Prob 1 A Rectangular channel 10m wide, carry a discharge of 30 M³/sec. It is Laid at a slope of 0.0001. If at a section in this channel the depth of flow is 1.6m. Find whether upstream or downstream from this section, the depth of flow is 2M. Also determine the surface profile type. Assume N = 0.015. Also determine the distance b/w two depths along the chanel slope. ***

Find Normal depth of flow :

$$Q = 30 \text{ M}^3/\text{sec}$$

$$B = 10 \text{ m}$$

Section 1

$$y = 1.6\text{m}$$

At section 2

$$y = 2\text{m}$$

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$$\therefore A = 10\, y_n$$

$$Q = \frac{1}{N} A R^{2/3} S_0^{1/2}$$

$$30 = \frac{1}{0.015} (10 y_n) \left[\frac{10 y_n}{10 + 2 y_n} \right]^{2/3} (0.0001)^{1/2}$$

$$\therefore y_n^{5/2} = 1.909\,(y_n + 5)$$

$$\Rightarrow y_n = 2.97\ m \quad \text{(By trial \& hit Method)}$$

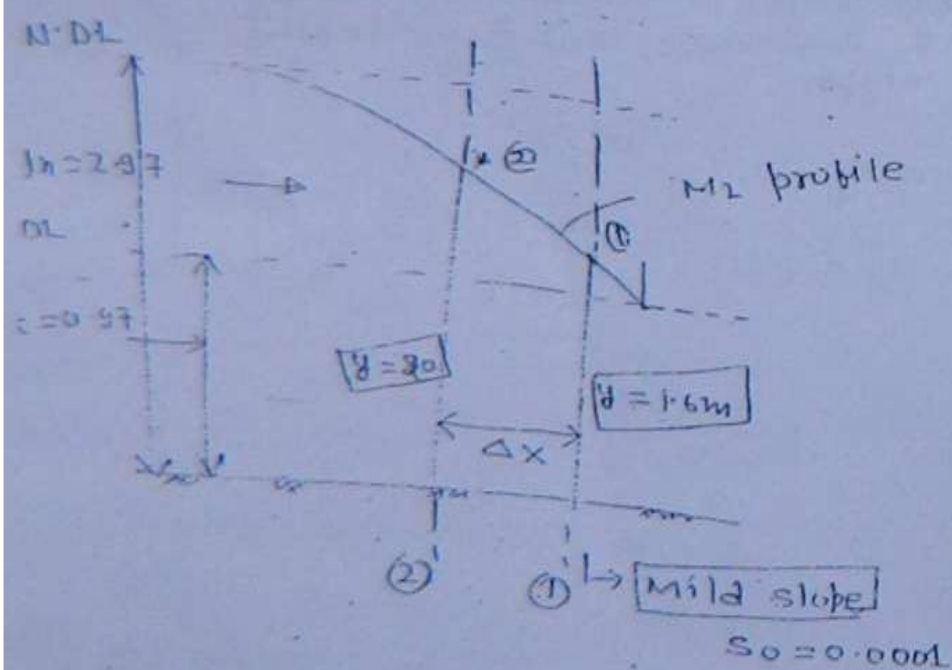
2) Find critical depth

$$q = Q/B = 30/10 = 3\ m^3/sec/m$$

$$\therefore y_c^3 = (q^2/g) \longrightarrow \text{For Rectangular}$$

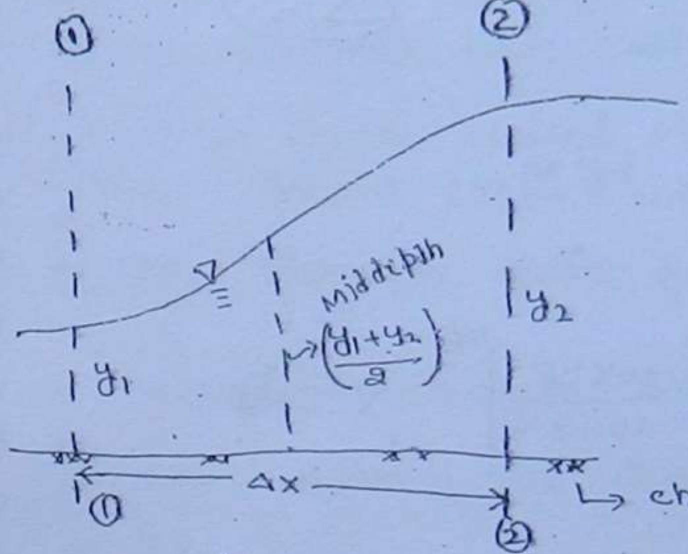
$$\therefore y_c = \left(\frac{3^2}{9.81}\right)^{1/3} = 0.97\ m$$

since $y_n > y_c$ the given slope is Mild slope and since actual depth of flow are b/w y_c & y_n therefor surface profile will be M_2.



Approximate Method to determine distance b/w two depths in G.V.F. ↓

(315)



↳ channel slope = S_0

$E_1 = y_1 + \frac{V_1^2}{2g}$

E_2 = sp. Energy at section ②

$= y_2 + \frac{V_2^2}{2g}$

$$\boxed{\Delta x = \frac{E_2 - E_1}{S_0 - S_e}}$$

slope of Energy Line at mid section

At mid section $y = y_1 + y_2/2$

∴ S_e is given by

$Q = \frac{1}{N} A R^{2/3} S_e^{1/2}$

$A = B \cdot y$ → depth at mid section

$P = B + 2y$, $R = A/P$.

Ref. Ex:

$E_2 = y_2 + V_2^2/2g$

$= 2.0 + \frac{1.5^2}{2 \times 9.81}$

$= 2.11$ m

$\left[V_2 = Q/A_2 = \frac{30}{10 \times 2.0} = 1.5 \right.$

1.6×10

$$= 1.6 + \frac{1.875^2}{2 \times 9.81}$$

$$= 1.779$$

316.

At Mid depth

$$y = \frac{1.6 + 2.0}{2} = 1.8 \text{ M}$$

$$Q = \frac{1}{N} \cdot A R^{2/3} S_e^{1/2}$$

$$30 = \frac{1}{[0.015]} \times [10 \times 1.8] \left[\frac{10 \times 1.8}{10 + 3.6}\right]^{2/3} \times S_e^{1/2}$$

$$\Rightarrow S_e = 0.00043$$

$$\therefore \Delta x = \frac{E_2 - E_1}{S_0 - S_e} = \frac{2.11 - 1.779}{0.0001 - 0.00043} = (-) 1003$$

$$\therefore \Delta x = 1.003 \text{ KM} \rightarrow \text{U/S to section (1)}$$

ob2

$\boxed{\text{choking}}$:

In the case of a channel with a bump and also in the case of a width contraction, it is observed that the U/S water-surface elevation is not affected by the conditions at section (2) till a critical stage is first achieved.

So in the case of a bump for all $\boxed{\Delta Z \leq \Delta Z_{max}}$ the U/S water depth is constant and for all $\Delta Z > \Delta Z_{max}$ the U/S depth is different from y_1. Similarly in the case of the width constriction $\boxed{\text{for } B_2 \geq B_{2min}}$ the U/S depth y_1 is constant; while for all $\boxed{B_2 < B_{2min}}$ the U/S depth undergoes a change.

This creation of critical condition at section (2) is [illegible] choking.

Thus for all cases with $\boxed{\Delta Z > \Delta Z_m}$ & $\boxed{B_2 < B_{2m}}$ are known as choked conditions.

Applications of sp. Energy :

1) Analysis of flow through the channels when one section is transformed into another section such channels are called transition channel.

2) Flow over raised channel bottom slope.
Ex: Broad crested weir

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3) Flow through sluice gate opening.

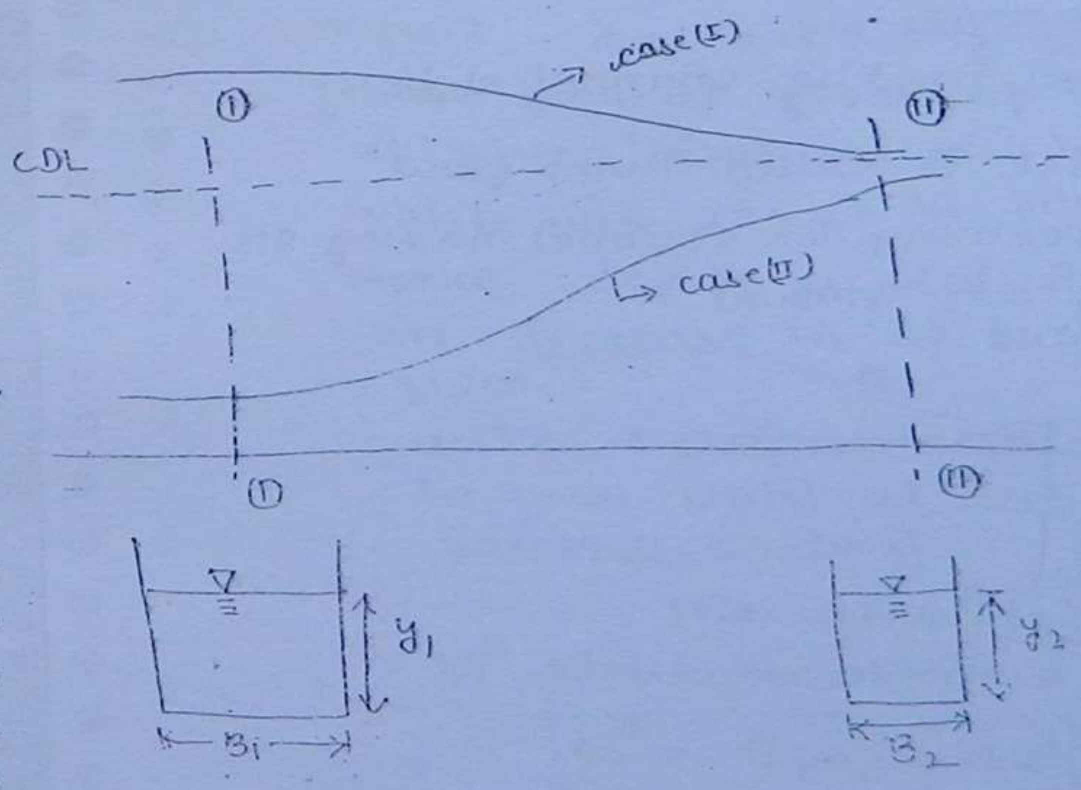
Flow through Rectangular channel Transition :

when the width of the channel is reduced, it can be done by

a) sudden contraction

b) Gradual contraction

In suddenly contraction Loss of Energy is much more than Gradual contraction therefor all practical purpose Gradual contraction is preferred.



Case I:- when at section (1)-(1) depth ... V_1 &

flow is subcritical. ($Fr<1$)

If flow is transform such that V_2 decreases than Flow will tend to become critical and depth will tend to become critical hence Lowering of water surface and type-2 curve will be formed

Case II:- If flow is supercritical at section (1)-(1). than at section (2), flow tend to become critical hence flow will rise will be observed. in actual curve

If flow is through smooth transition and there is no change in bed level than $E_2 = E_1$

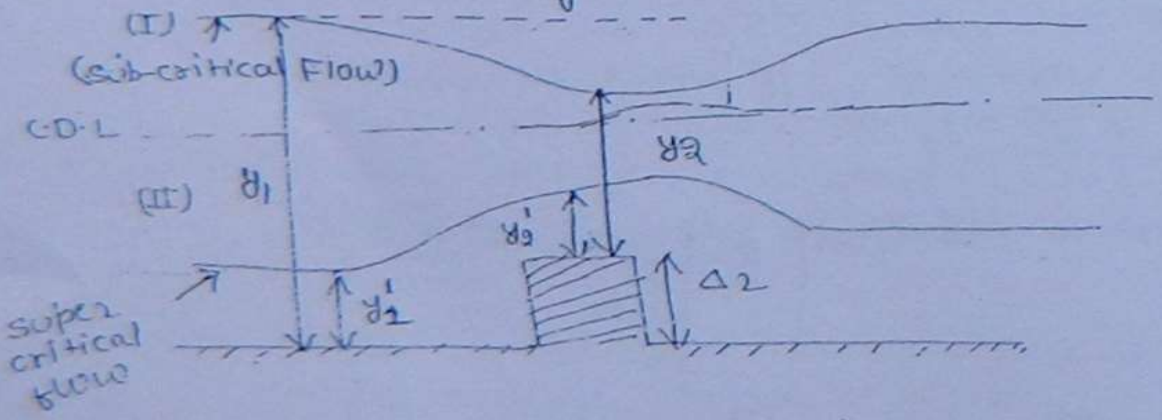
NOTE:- If B_2 is further contracted even after E_2 has reached 'E_{min} (corresponding to critical stage) than piling of water in U/s will occure and flow will not be possible at original depth y_2.

Imp

Flow over Local rise in the channel slope:

If Flow over Local rise is called 'hump flow'.

Let's consider a rectangular channel having its bottom raised by an amount Δ.



(319) $(y_2 < y_1)$

$\boxed{E_1 = \Delta Z + E_2}$

OR

$\boxed{E_1' = \Delta Z + E_2'}$

→ If flow $Fr > 1$ → $\{y_1' < y_c\}$

$(y_2' > y_1')$

→ The Max^m height of hump. (ΔZ = Max) will be obtain when point (2) in the sp. energy curve or when depth of section (2) coincides with critical depth at that section. $(y_2 = y_c)$
Thus flow over raised section will be critical.

Than

$$E_1 = (\Delta Z)_{Max} + E_{\underline{Min}} \qquad \boxed{E_2 = E_{Min}}$$

$$\therefore \boxed{(\Delta Z)_{Max} = E_1 - E_{Min}} \text{ get}$$

NOTE: If the height of hump is further ~~reduced~~ increased beyond ΔZ, Let's say $\Delta Z' > (\Delta Z_{max})$ than flow at given sp. energy E_1 & at given depth y_1 will not be possible which will result in piling of water.
Hence depth y_1 will increase, causing an increase in sp. energy of approaching flow.
Water Level at section (1) will continue to rise until at section (2) flow becomes critical.

The increased sp. energy E_1'' at (1) is obtained as

$$\boxed{E_1'' = E_{Min} + \Delta Z'}$$

$$\boxed{= \tfrac{3}{2} y_c + \Delta Z'} \rightarrow \text{For Rect. channel}$$

A trapezoidal channel of base width 6m and

side slope of 2H:1V carries a flow of $60\ m^3/sec$ at a depth of 2.5 m. There is a smooth transition to a rect. section 6m wide accompined by a gradual lowering of channel bed by 0.6 m

1) Find the depth of water in the rect. section & change in water surface level.

2) In case the drop of water surface level is to be restricated to 0.30 m what is the amount of bed must be lowered

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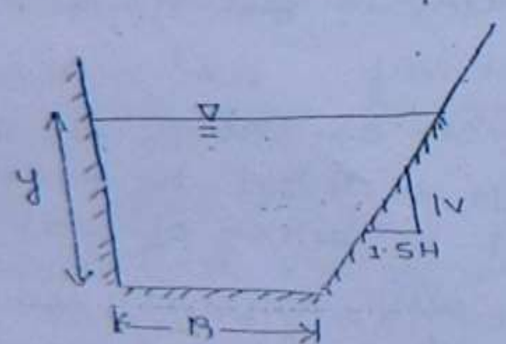
Prob2 [Uniform Flow Problem]

. A lined channel ($N = 0.014$) is of trapezoidal section with one ~~sech~~ side vertical and other side on slope (1.5H: 1V) If the channel is to be deliver $9\ M^3/sec$ when laid on a slope of 0.0002. calculate the dimensions of efficient section which requires minimum lining. Also calculate corresponding mean velocity.



$\left[\begin{array}{l} nH:1V \\ n = 1.5 \end{array}\right.$

For minimum lining, P should be minimum

$$P = y + B + y\sqrt{n^2+1}$$

$$= B + y\left[1 + \sqrt{3.25}\right]$$

$$8 =$$

$$A = \frac{B + (B+ny)}{2} \times y$$

$$\therefore A = \frac{(2B + b, 1.5y)}{2} \times y$$

$$\therefore A = (B + 0.75y) \times y$$

For Given Area perimeter should be Minimum.

$$\therefore B = A/y - 0.75y$$

$$\therefore P = A/y - 0.75y + 2.8y$$

$$\therefore dP/dy = -A/y^2 - 0.75 + 2.8 = 0$$

$$\Rightarrow y^2 = \frac{A}{2.05} \Rightarrow \boxed{A = 2.05y^2}$$

$$\therefore \boxed{B = 1.3y}$$

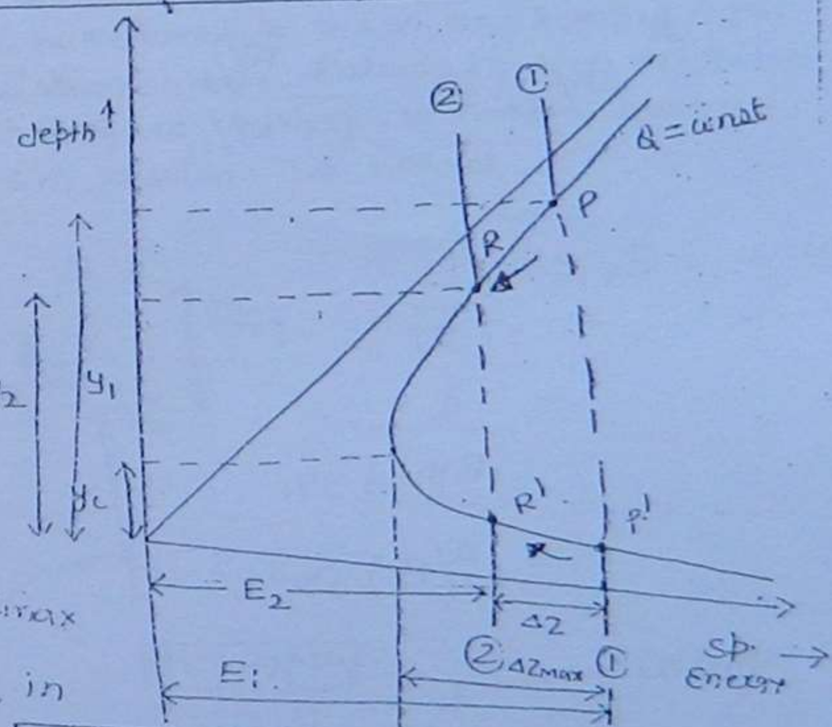
By applying Manning's Equation

$$Q = \frac{1}{N} \cdot A \cdot R^{2/3} S^{1/2} \Rightarrow \text{Get } (y) = ?$$

channel with a hump:

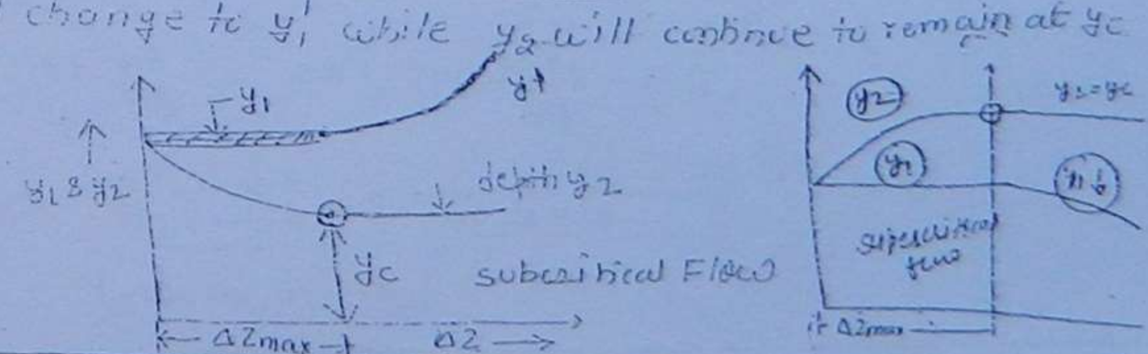
a) subcritical Flow:

when $\boxed{0 < \Delta Z < \Delta Z_{max}}$ the U/S water Level remains stationary at y_1 while the depth of flow at section 2 decreases with ΔZ reaching at Minimum Value of y_c at $\Delta Z = \Delta Z_{max}$



with further increase in the value of ΔZ i.e. $\boxed{\Delta Z > \Delta Z_{max}}$

y_1 will change to y_1' while y_2 will continue to remain at y_c



Q. In a flow through a rect. channel for a certain discharge the Fraud No. corresponding to two alternate depths are F_1 & F_2 than prove that

$$\left(F_2/F_1\right)^{2/3} = \frac{2+F_2^2}{2+F_1^2}$$

(322)

$$F_r = \frac{V}{\sqrt{gY}} = \frac{V}{\sqrt{gD}}$$

$$E_1 = E_2$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow y_1\left[1+\frac{V_1^2}{2gy_1}\right] = Y_2\left[1+\frac{V_2^2}{2gy_2}\right]$$

$$\therefore \quad y_1/y_2 = \frac{1+F_2^2/2}{1+F_1^2/2} = \frac{2+F_2^2}{2+F_1^2}$$

Now $F_1 = \dfrac{V_1}{\sqrt{gY}}$

$$= \frac{q}{By_1\sqrt{gy_1}}$$

$$= q/B\sqrt{gy_1^3}$$

Similarly $F_2 = \dfrac{q}{B\sqrt{g\,y_2^3}}$

$$y_1^3 = \frac{q^2}{B^2 g F_1^2}$$

$$y_2^3 = \frac{q^2}{B^2 g F_2^2}$$

$$\boxed{(F_2/F_1)^{2/3} = \frac{2+F_2^2}{2+F_1^2}}$$

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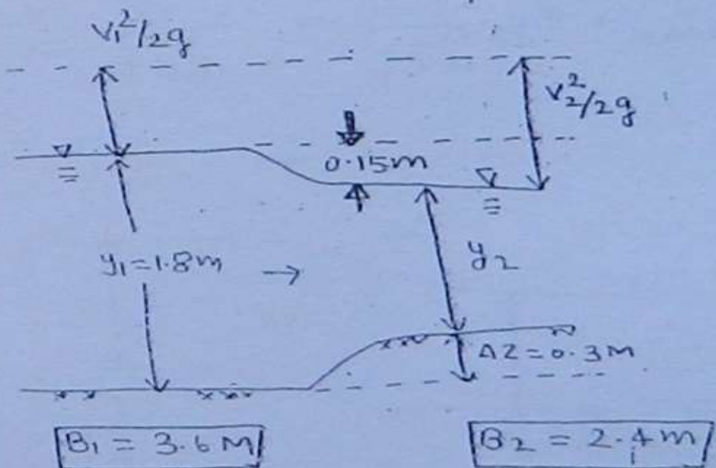
$$\left[F_1 = \frac{V_1}{\sqrt{g y_1}} , \quad F_2 = \frac{V_2}{\sqrt{g y_2}} \right.$$

$$\therefore \quad y_1^3 = \frac{Q^2}{B^2 g F_1^2}$$

$$y_2^3 = \frac{Q^2}{B^2 g F_2^2} \qquad \therefore \quad y_1/y_2 = (F_2/F_1)^{2/3}$$

$$\therefore \boxed{(F_2/F_1)^{2/3} = \frac{2+F_2^2}{2+F_1^2}}$$

Prob2 A 3.6 M wide rect. wide channel carries a water at a depth of 1.8M. In order to measure the discharge the channel width is reduced to 2.4 m and hump of 0.3 m is provided in the bottom. calculate the discharge If the water surface in the contracted surface drop by 0.15 m assume no Losses.



$$E_1 = \Delta Z + E_2$$

$$y_1 + \frac{V_1^2}{2g} = \Delta Z + y_2 + \frac{V_2^2}{2g} \quad \text{——— (1)}$$

Imp $\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 0.15\text{ m}$ ——— (2)

From (2) in Eqn (1)

$\Rightarrow \ y_2 = 1.8 - 0.3 - 0.15 = 1.35\ m$

By using eqn (ii) ...

3.24

$$\frac{Q^2}{2gA_2} - \frac{Q^2}{2gA_1} = 0.15$$

$$\Rightarrow \frac{Q^2}{2g}\left[\frac{1}{A_2} - \frac{1}{A_1}\right] = 0.15$$

$$\Rightarrow Q = 6.418\ m^3/sec \qquad \left[\begin{array}{l} A_1 = 3.6 \times 1.8 \\ A_2 = 2.4 \times 1.35 \end{array}\right.$$

20b8 S/2005 ***

A wide rect. channel carries a flow of 2.76 m^3/sec/per m width. The depth of flow is 1.524 m.

a) calculate the min. rise of flow at a section required to produce critical flow conditions

b) what is corresponding fall in the water Level

Given

$q = 2.76\ m^3/sec/m$

$y_1 = 1.524\ m$

$\Delta Z_{max} = ?$ (For critical flow condition)

At critical Flow condition at section ②

$$E_1 = \Delta Z_{Max} + E_{Min}$$

$$v_1 = q/y_1 = \frac{2.76}{1.524} = 1.81\ m/sec$$

$$E_1 = y_1 + \frac{v_1^2}{2g} = 1.524 + \frac{1.81^2}{2 \times 9.81} = 1.69\ m$$

$$E_{min} = \frac{3}{2}(y_c) \qquad \left[\ y_c = (q^2/g)^{1/3} = 0.919\ m\right.$$

$$\Delta Z_{Max} = E_1 - E_{min} = 1.69 - 1.5 \times 0.955 = 0.312\ m$$

b) $\Delta y = y_1 - \Delta z - y_2$ [illegible]

$= 1.524 - 0.312 - 0.914$

$= 0.294\ m$

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Prob 4

A rect. channel 3.5 m wide laid at a slope of 0.0036, uniform flow occurs at a depth of 2 m. Find how high a bump can be raised on the channel bed without causing a change in U/s depth. If the U/s depth is to be raised to 2.4 m what should be the height of bump. Assume Manning's N = 0.015.

$A_1 = 3.5 \times 2.0 = 7\ m^2$

$P_1 = 3.5 + 2 \times 2 = 7.5\ m$

$R_1 = A_1/P_1 = 0.938$

$V_1 = \frac{1}{N} R_1^{2/3} S^{1/2}$

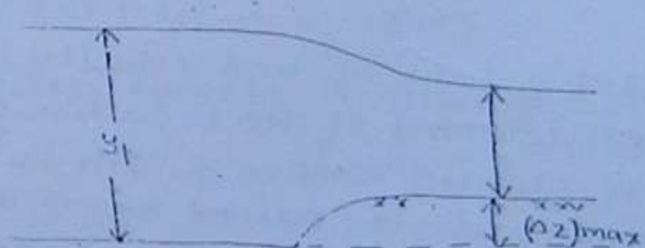
$= 3.82\ m/sec$

check for Froud Number

$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.82}{\sqrt{9.81 \times 2}} = 0.86 < 1$

↳ subcritical flow

TEL



a) If U/s depth is not to be changed than at equilibrium

$$\boxed{E_1 = (\Delta z)_{max} + E_{min}}$$

$(\Delta z)_{max} = E_1 - E_{min}$

$Q = V_1 \times A_1 = 3.82 \times 7$

$Q/B_1 = q = 3.82 \times 7 / 3.5 = 7.64\ m^3/sec/m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\quad}{9.81}\right)$$

$\therefore \Delta Z_{max} = E_1 - 3/2 \times 1.81$

$E_1 = y_1 + \frac{v_1^2}{2g}$

$= 2.0 + \frac{3.82^2}{2\times 9.81}$

$= 2.74$ m

$\therefore \Delta Z_{max} = 2.74 - 1.5\times 1.81$

$= 0.026$ m.

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b) If Now $y_1' = 2.4$ m (VL would change)

Discharge would remain same, y_1 would change.

$\therefore y_1' + \frac{v_1'^2}{2g} = \Delta Z' + E_{min}$ $[\Delta Z > \Delta Z_{max}$

$2.4 + \frac{v_1'^2}{2g} = \Delta Z' + 3/2 \times 1.812$

$\Rightarrow 2.4 + \frac{(3.18)^2}{2\times 9.81} = \Delta Z' + 1.812\times 1.5$

$$\left[v_1' = Q/A_1' = \frac{26.74}{3.5\times 2.4} = 3.18 \text{ m/sec.} \right.$$

$\Delta Z' = 0.198$ m.

Probs

Water flows at a depth of 1.6 m and velocity of 1.1 m/sec in a open channel of Rect. section of width 4m. At a certain section width is reduced to 3.5 m and the bed is raised by 0.35 m through a smooth flat hump. Calculate water surface elevation at the contracted sections as well as the U/S section. Neglect losses.

At section (2) depth $(y_2) = 1.158$ m $[y_2 > y_c$

$y_1 = 1.6$ m

At section (1) elevation is = raised

Rapid Varied Flow (RVF) :-

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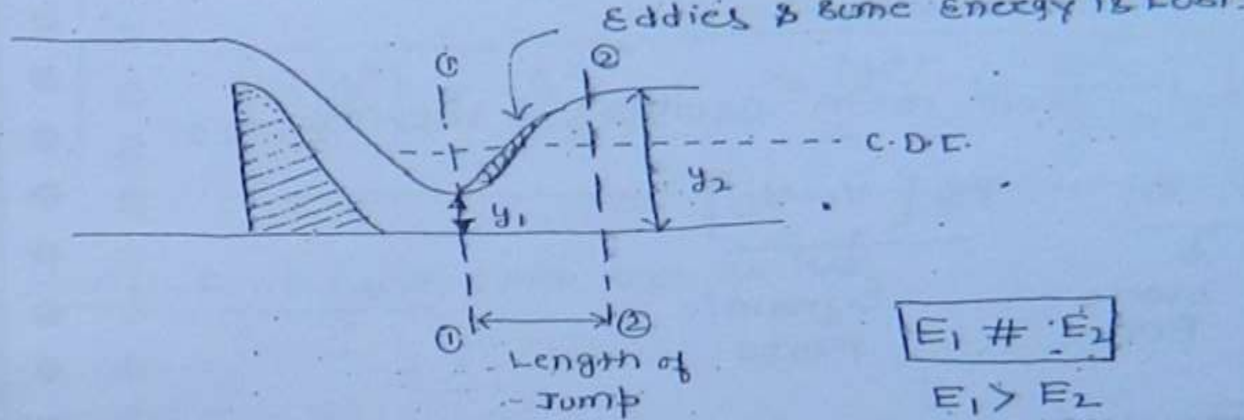


$\boxed{E_1 \neq E_2}$

$E_1 > E_2$

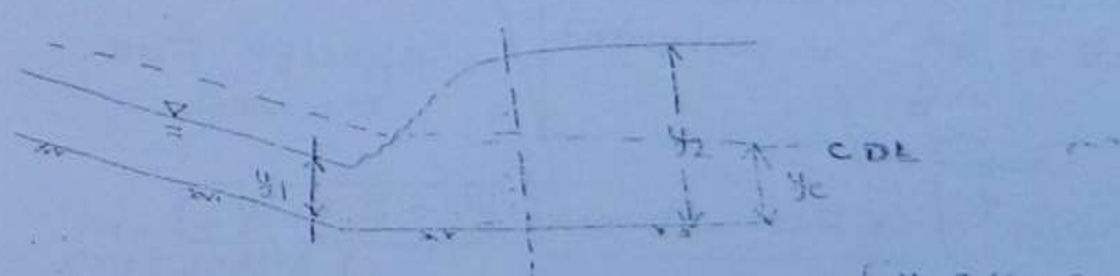
$\boxed{E_1 - E_2 = \Delta E}$ (Loss of Energy)

An essential & necessary condition for hydraulic jump to occur Flow must change supercritical to subcritical & this change is over a small Length hence if flowing fluid having Fraud No. greater than 1, jump May be created

Hydraulic jump is defined as sudden & turbulent Passes of water supercritical to subcritical it is also called shooting, rapid, Tranquill, unstable type.

→ There is considerable dissipation of energy during the formation of Jmp.

Sp. Force concept is always applied with hydraulic jump i.e. Flow changing from supercritical to subcritical

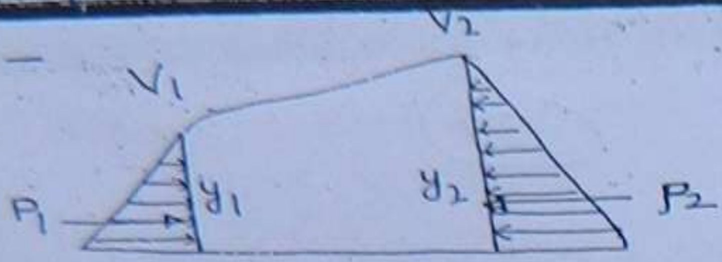


$\boxed{y_1 < y_c < y_2}$

y_1 & y_2 are subsequent depth or conjugate depth

$\boxed{E_1 \neq E_2}$

- But $\boxed{F_1 = F_2}$ (sp. force)



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If Resultant Force is in Equilibrium than $\Sigma F = 0$

$$\Rightarrow \underbrace{(P_1 - P_2)}_{\text{Static Force}} + \underbrace{\rho Q [V_1 - V_2]}_{\text{Dynamic Force}} = 0$$

$\boxed{P_1 = \omega A_1 \bar{x}_1}$ [$\bar{x}_1$ = Position of C.G. From Top surface

$\boxed{P_2 = \omega A_2 \bar{x}_2}$

$$\therefore \omega [A_1 \bar{x}_1 - A_2 \bar{x}_2] + \rho Q [V_1 - V_2] = 0 \quad \text{imp}$$

$$\Rightarrow \omega [A_1 \bar{x}_1 - A_2 \bar{x}_2] = \rho Q \left[Q/A_2 - Q/A_1 \right]$$

$$\Rightarrow \cancel{\rho} g [A_1 \bar{x}_1 - A_2 \bar{x}_2] = \cancel{\rho} Q \left[Q/A_2 - Q/A_1 \right]$$

$$\Rightarrow g [A_1 \bar{x}_1 - A_2 \bar{x}_2] = \frac{Q^2}{A_2} - \frac{Q^2}{A_1}$$

$$\Rightarrow A_1 \bar{x}_1 + \frac{Q^2}{A_1 g} = A_2 \bar{x}_2 + \frac{Q^2}{A_2 g}$$

$$\Rightarrow \text{imp} \quad \boxed{A\bar{x} + \frac{Q^2}{A g} = \text{constant}} \rightarrow \text{specific Force}$$

$F = \text{const.}$

Generally For Rect. channel $\boxed{\bar{x} = \bar{z} = y/2}$

[Either from top or bottom]

~~At minimum Sp. force there is only one critical depth~~

For other values of F there are two depth of flow. y_1 & y_2, called conjugate depth.

$$\boxed{y_1 < y_c < y_2} \text{ imp}$$

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For critical Flow, sp. force is Minimum

for Rect. channel

$$\boxed{F = \frac{q^2 B}{g} \times \frac{1}{y} + \frac{B y^2}{2}}$$

For F(min), $\frac{dF}{dy} = 0$

$$\Rightarrow \frac{q^2 B}{g}\left(-\frac{1}{y^2}\right) + \frac{2By}{2} = 0$$

$$\Rightarrow \boxed{y_c = \left(\frac{q^2}{g}\right)^{1/3}} \quad \text{--- (1)}$$

NOTE: For Non-Rectangular channel

$$\left[\frac{Q^2}{g} = \frac{A^3}{T}\right] \quad \text{----(ii) at critical Flow.}$$

$$F_1 = F_2$$

$$\Rightarrow \frac{Q^2}{A_1 g} + A_1 \bar{z}_1 = \frac{Q^2}{A_2 g} + A_2 \bar{z}_2$$

$[Q = q \cdot B$ For Rect. channel

$$\therefore \frac{q^2 B^2}{B y_1 g} + B y_1 \left(\frac{y_1}{2}\right) = \frac{q^2 B^2}{y_2 B g} + B y_2 \left(\frac{y_2}{2}\right)$$

⇓ on solving

$$2 q^2/g = y_1 y_2 (y_1 + y_2) \quad \text{--- (iii)}$$

$$\Rightarrow \boxed{y_c^3 = \frac{y_1 y_2 (y_1 + y_2)}{2}} \quad \text{imp --- (IV)}$$

**

For alternate depth → $E_1 = E_2$ → $\boxed{y_c^3 = \frac{2 y_1^2 y_2^2}{(y_1 + y_2)}}$ ✓

using eqn (iii) - we can find y_1 & y_2

Let's Find y_2

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$\therefore \quad y_2^2 \cdot y_1 + y_1^2 \cdot y_2 - \frac{2q^2}{g} = 0$

$\Rightarrow \quad x^2 a + b \cdot x + c = 0$

$\Rightarrow \quad y_2 = \frac{-y_1^2 \pm \sqrt{y_1^4 + 4y_2(2q^2/g)}}{2 \times y_1}$

$y_2 = y_1/2 \left[-1 \pm \sqrt{1 + \frac{8q^2}{g y_1^3}} \right]$

$\therefore \quad \frac{y_2}{y_1} = 1/2 \left[-1 \pm \sqrt{1 + \frac{8 V_1^2}{g Y_1}} \right]$

$\therefore$ *** $\boxed{y_2/y_1 = 1/2 \left[-1 + \sqrt{1 + 8 F_{r_1}^2} \right]}$ imp

OR $\quad [F_{r_1} > 1] \quad$ > only For Rect. Section

$\boxed{y_1/y_2 = 1/2 \left[-1 + \sqrt{1 + 8 F_{r_2}^2} \right]}$

Loss of Energy in Jump :-

$\Delta E = E_1 - E_2$

$= \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$

$\boxed{\Delta E = \frac{[y_2 - y_1]^3}{4 y_1 y_2}}$ imp

OR

$\boxed{\Delta E = \frac{[V_1 - V_2]^3}{2g(V_1 + V_2)}}$

$= \omega Q(\Delta E)$

\# Height of Jump $= y_2 - y_1$

\# Length of Jump $= 5$ to 7 times $(y_2 - y_1)$

Fr_1	Type of Jump
1) 1–1.7	Undular Jump
2) 1.7 to 2.5	weak Jump
c) 2.5 to 4.5 goll	oscilating Jump
d) 4.5 to 9.0 goll	steady Jump
e) > 9.0	strong Jump

Prob 1

A Trapezoidal section having bottom width of 8 m & side slope is 1:1, carries a discharge of 30 m³/sec. Find the depth of conjugate to initial depth of 0.75 m before the Jump. Also determine the Loss of energy in the Jump.

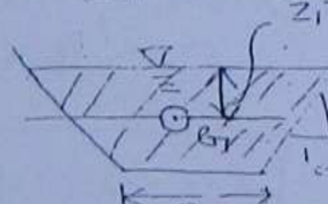
Sp Force

$$F_1 = F_2$$

$$\frac{Q^2}{A_1 g} + A_1 \bar{Z}_1 = \frac{Q^2}{A_2 g} + A_2 \bar{Z}_2 \quad \text{--- (1)}$$

$\rightarrow B + 2ny = 9.5\ m$

$$\bar{Z}_1 = \frac{9.5 + 2 \times 8}{9.5 + 8} \times 0.75/3 = 0.364\ m$$



$\left[\frac{a+2b}{a+b}\right] h/3$

$nH : 1V$

$\Rightarrow n = 1$

$$A_1 = (B + n y_1) y_1 = (8 + 0.75)\, 0.75 = 6.56\ m$$

$Z_1 = 0.364\ m$

$$A_2 = (B + y_2) y_2$$

$$\bar{Z}_2 = \frac{(8 + 2y_2) + 2 \times 8}{8 + 2y_2 + 8} \times y_2/3$$

Solving for y_2 from Eqn (1)

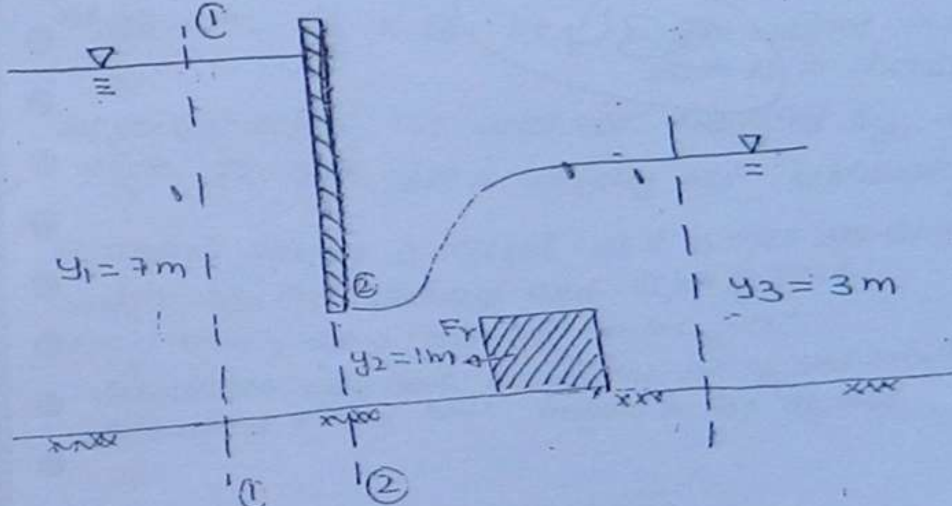
$$y_2 = 4.167\ m$$

$\therefore$ Loss of energy $= E_1 - E_2$

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

prob2
ES/1992

A sluice across a channel is 6m wide, discharges a stream 1m deep. what is the flow rate when u/s depth of sluice is 7m. on the d/s side a concrete block have been placed, to create the condition of hydraulic jump. Determine the force on the block if d/s. depth is 3m.



$$q = (y_1 \times 6) V_1 = (y_2 \times 6) V_2$$
$$= (y_3 \times 6) V_3$$

$$\Rightarrow 7V_1 = V_2 = 3V_3 \text{ ---- } (1)$$

There is No Loss of Energy b/w (1)-(1) & (2)-(2)

so $\boxed{E_1 = E_2}$

$$\Rightarrow y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \text{ --- } (2)$$

By eqn (1) & (2)

$\Rightarrow V_1 = 1.565$ m/sec. $\quad V_3 = 3.65$ m/sec

$V_2 = 10.\text{--}$ m/sec

$$Fr = (P_2 - P_3) + \rho Q [V_2 - V_3]$$

$$= \rho g [A_2 \bar{x}_2 - A_3 \bar{x}_3] + \rho Q [V_2 - V_3]$$

$$= 1000 \times 9.81 \left[(1 \times 6) \times Y_2 - (3 \times 6) \times \frac{1.5}{2} \right] +$$

$$1000 \times 65.77 [10.95 - 3.65] \; N$$

$$\therefore \boxed{Fr = 243.83 \, KN}$$

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(i) Mild slope:- $y_0 > y_c$ → subcritical flow at Normal depth
(ii) Steep slope:- $y_0 < y_c$ → supercritical flow at
(1,3) (iii) critical slope:- $y_0 = y_c$ → critical flow at Normal depth.
2,3 (iv) Horizontal slope:- $S_0 = 0$ → cannot sustain uniform flow
2,3 (v) Adverse slope:- $S_0 < 0$ → " "

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$\frac{dy}{dx} > 0$ if (a) $y > y_0$ and $y > y_c$ or
(b) $y < y_0$ and $y < y_c$

$\frac{dy}{dx} < 0$ if (i) $y_c > y > y_0$ or
(ii) $y_0 > y > y_c$

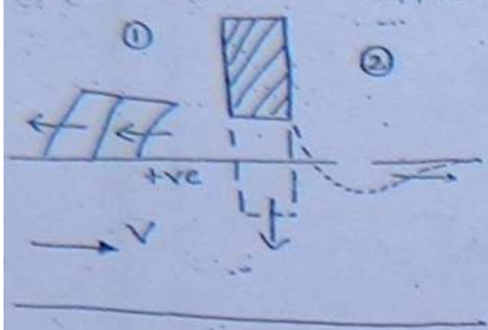
(i) As $y \to y_0 \Rightarrow \frac{dy}{dx} \to 0$ i.e. The water surface approaches the Normal depth line Asymptotically.

(2) As, $y \to y_c \Rightarrow \frac{dy}{dx} \to \infty$ i.e. water surface meets the critical depth line vertically.

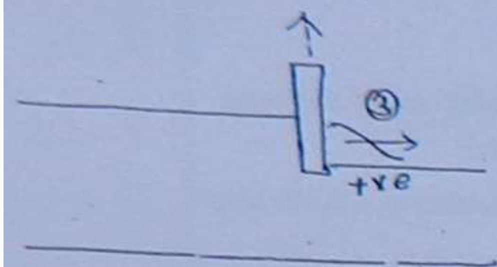
(3) As, $y \to \infty$, $\frac{dy}{dx} \to S_0$ i.e. water surface meets a very Large depth as a horizontal Asymptote.

At critical depth the curves are indicated by dashed line to remind that the GVF Eqn is strictly not applicable in that neighbourhood.

→ A control section is defined as a section in which a fixed Relationship exists b/w the discharge and depth of flow.
- critical depth is also a critical control point.
- subcritical Flows have controls at the D/s end while supercritical Flows have controls in the U/s end.



→ If the flow in a channel is increased suddenly by means of opening gate a wave is formed which travels D/s.

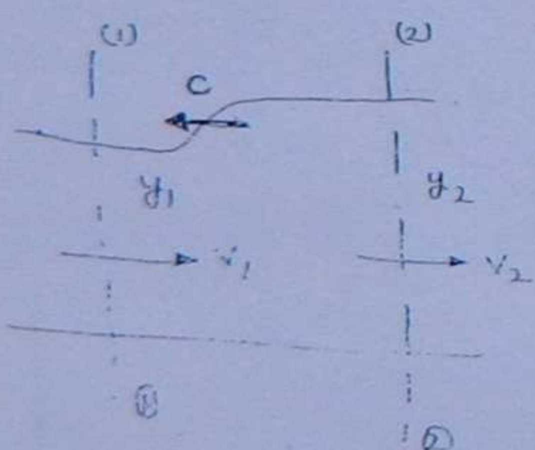


→ similarily if a gate is suddenly closed and a flow is partially reduced, a wave formed travels such wave is called surge wave.

-ve surge wave:- Also known as 'elevation surge wave'
→ If depth of water increases in the direction of motion of wave is called surge wave.

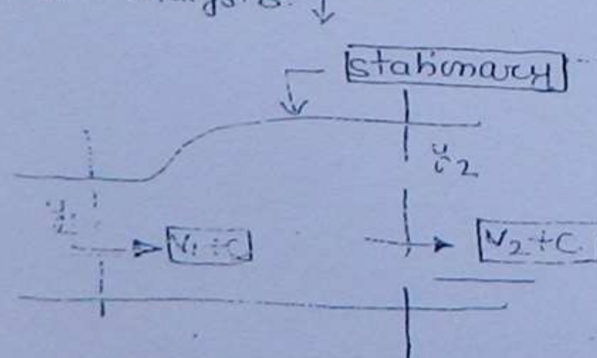
(1) & (3) are +ve surge wave & (2) is -ve surge wave.

Analysis:↓



Let vel. of wave is C.
{celerity of wave}

For Analysis:↓



For prismatic Rectangular channel:

$$Q = by_1(V_1+C) = by_2(V_2+C) \Rightarrow y_1(V_1+C) = y_2(V_2+C)$$

$$\therefore \boxed{C = \frac{V_1y_1 - V_2y_2}{y_2 - y_1}} \quad \text{---(1)}$$

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If there is no resistance force b/w (1) & (2) then Total Force = 0.

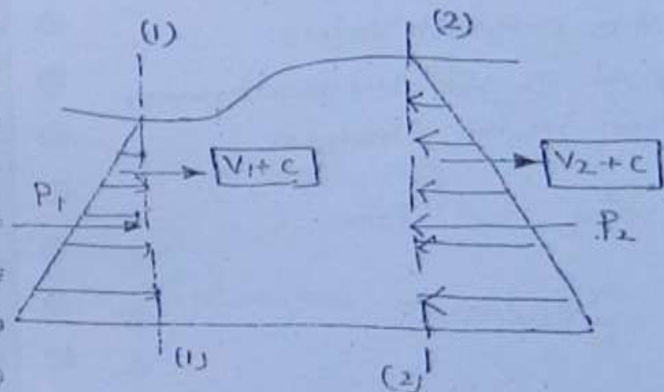
$$= \text{static force} + \text{dynamic force}$$

$$= (P_1 - P_2) + \rho Q\{[V_1+C] - [V_2+C]\} = 0$$

$$\Rightarrow P_1 - P_2 = \rho Q[V_2 - V_1]$$

$$\therefore \omega b \cdot y_1 * y_1/2 - \omega b\, y_2 * y_2/2 = \rho Q[V_2 - V_1]$$

$$\therefore \frac{\rho g b}{2}[y_1^2 - y_2^2] = \rho Q[V_2 - V_1] \quad \text{---(2)}$$



From eqn (1) & (2)

$$\boxed{V_1 + C = \sqrt{\frac{gy_2}{2y_1}(y_1+y_2)}}$$

$[V_1+C] \rightarrow$ vel. of surge relative to water {+ve surge}

$\boxed{\text{For -ve surge}}$: $\boxed{C > V_1}$

$$\therefore \text{Rel. vel.} = C - V_1 = \sqrt{\frac{gy_2}{2y_1}(y_1+y_2)}$$

Special case ↓

If surge is very small and depth of flow is Large than $y_1 \simeq y_2 \simeq y$ then

$$\boxed{V_1 + C = \sqrt{gy}}$$

For Rectangular channel: ↓

$$\frac{V_1}{\sqrt{gy}} = F_r$$

For critical Flow $V_1 = \sqrt{gy_c}$

For subcritical Flow $V_1 < \sqrt{gy}$

For supercritical Flow $V_1 > \sqrt{gy}$

Note: In an open channel if some external creates a small surge is created and if this surge ~~this~~ has zero celerity then $V_1 = \sqrt{gy} \Rightarrow V/\sqrt{gy} = F_r = 1$. Similarly if celerity is +ve means it causes upwards flow than $\boxed{V_1 < \sqrt{gy}}$. Therefore $\boxed{F_r < 1}$ Hence Flow is subcritical..

similarly at celerity −ve wave wave travels d/s.

Ex: 1 A trapezoidal channel with base width of 6m & side slopes with 2H:1V conveys water at the rate of 17 m³/sec with a depth of flow of 1.5 m. If this flow situation is subcritical or supercritical.

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$$F_r = \frac{V}{\sqrt{gD}}, \quad D = A/T = \frac{(b+ny)y}{b+2ny} \qquad [n = 2$$

Ex: 2 A rectangular horizontal channel of 3m width and 2m depth conveys water at 18 m³/sec. If the flow rate is suddenly reduced to 2/3 of its original value. Compute the magnitude of solidity and speed of U/s surge w.r.t water. Assume that there is no friction in the channel.

$$V_1 = Q/A_1 = 18/b \cdot y_1 = \frac{18}{3 \times 2} = 3 \text{ m/sec}$$

$$V_2 = Q_2/A_2 = \frac{2/3 \times 18}{3 \times y_2} = 4/y_2$$

At stable condition

$$b y_1 (V_1 + c) = b y_2 (V_2 + c) \quad \text{—— (1)}$$

$$P_1 - P_2 = \rho Q [V_2 - V_1] \quad \text{—— (2)}$$

$$\boxed{c = \frac{V_1 y_1 - V_2 y_2}{y_2 - y_1}} \quad \text{Imp.}$$

$$= \frac{3 \times 2 - 4/y_2 \times y_2}{y_2 - 2}$$

$$\boxed{c = \frac{2}{y_2 - 2}}$$

$$\boxed{V_1 + c = \sqrt{\frac{g y_2}{2 y_1}(y_1+y_2)}}$$

$$3 + \frac{2}{y_2 - 2} = \sqrt{\frac{9.81 y_2}{2\times 2}(2+y_2)}$$

$\Rightarrow \; y_2 = ?!$

→ clearity: – The velocity of the surge relative to the initial flow velocity in a canal is called clearity.

$C_s = V_\omega - V_1$ — D/S

$C_s = V_\omega + V_1$ — U/S

$$\boxed{c_s = \sqrt{\frac{g}{2}\, y_2/y_1 [y_1+y_2]}}$$

MODELS

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Similirahes: ↓

1> Geometric similarities: – It includes physical parameters such as Length, width, height volume, area. etc.

eg

Area ratio (A_r) = $\frac{\text{Area of prototype}}{\text{Area of Model}}$

$A_r = L_r^2 = (L_P/L_M)^2 = \left(\frac{\text{Length of prototype}}{\text{Length of Model}}\right)^2$

$V_r = L_r^3$

$[L_r = 100:1$ or $1:100$

2> Kinematic similarity: Those parameters which involves effect of time such as vel. ~~acc~~ acc.n, discharge etc.

a> $V_r = V_P/V_m = L_r/T_r$

b> Acc.n ratio $= a_r = \frac{V_r}{T_r} = L_r/T_r^2$

c> discharge ratio $= Q_r = L_r^3/T_r$

3> Dynamic similarity: – It exists if the ratio of all the forces at homologus point Model & prototype are similar.

a) Gravity Force = F_g = m·g

$= \rho L^3 \cdot (L/T^2 \rightarrow$ same) for Model & prototype.

$= \rho L^3 \cdot g$

b) Inertial Force ↳ Effect of Mass and it always acts.

$F_i = M \times \alpha.$

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$= \rho L^3 \times L/T^2$

$= \rho L^2 (L^2/T^2) = \rho L^2 V^2$

c) Viscosity Force:

$F_v = \tau \times A$

$= \mu (V/L) \times L^2 = \mu V L$

d) Pressure Force:

$= p \times A$

$= p \times L^2$

e) Surface Tension Force:

$F_s = \sigma \cdot L$

f) Compressibility Force:

↳ Bulk Modulus

$F_C = K \times A = K \times L^2$

Reynolds Number: ↓

For situation, viscosity forces are very predominant with inertia forces but other forces are less significant than Reynolds Number is defined as:

$$Re = \frac{F_i}{F_v}$$

$$= \frac{\rho L^2 V^2}{\mu V L}$$

$$\therefore \boxed{Re = \rho V L / \mu}$$

Eff.

L → Length parameter

ρ → Mass density

μ → dyn. coeff. of viscosity

Reynolds Model Law:

(341)

In the flow conditions where viscosity forces are very predominant than other forces than this Law is applied.

example

a) Flow in pipes under Laminar conditions

b) Flow of submarine & air plane but submarine are fully emerged.

c) Flow around submerged structure.

d) Flow through Low speed turbo Machines.

→ For such conditions Re for Model will be equal to Re for prototype.

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$\Rightarrow \boxed{\frac{\rho_r V_r L_r}{\mu_r} = 1} \rightarrow \text{Reynolds Law}$$

$$\mu/\rho = \nu, \quad \boxed{\frac{V_r L_r}{\nu_r} = 1}$$

Ex: 1) $T_r = ?$

$$\rho_r \frac{L_r}{T_r} \times \frac{L_r}{\mu_r} = 1$$

$$\Rightarrow \boxed{T_r = L_r^2 \frac{\rho_r}{\mu_r}}$$

2) Accln Ratio $a_r = \frac{V_r}{T_r} = L_r / T_r^2$

$$a_r = \frac{L_r \ \mu_r^2}{L_r^4 \cdot \rho_r^2}$$

$$\therefore \boxed{a_r = \frac{\mu_r^2}{L_r^3 \ \rho_r^2}}$$

Discharge Ratio ... $= \dfrac{\;}{L_r^2 * \rho_r} = \dfrac{\;}{\rho_r}$

> Force Ratio:

$$F_r = M_r \cdot a_r$$

$$= \rho_r \times L_r^3 \times \frac{\mu_r^2}{\rho_r^2 \times L_r^3}$$

$$\therefore \boxed{F_r = \frac{\mu_r^2}{\rho_r}}$$

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5> Power Ratio :

$$P_r = F_r \times V_r$$

$$\therefore P_r = \frac{\mu_r^2}{\rho_r} * \frac{\mu_r}{L_r \cdot \rho_r}$$

$$\therefore \boxed{P_r = \frac{\mu_r^3}{L_r \rho_r^2}}$$

:> Fraud Number: When gravity force is important apart from Inertia force but other forces are Less significant than Fraud Number is defined as

$$F_r = \sqrt{F_i / F_g} = \sqrt{\frac{\rho L_a^2 v^2}{\rho L^3 \cdot g}}$$

$$\therefore \boxed{F_r = \frac{V}{\sqrt{Lg}}}$$

L: Length Parameter

: A/D [For open channel

Fraud Model Law:-

When gravity force is very Important than this Law is applicable.

Ex: i> Flow through open channels (waves & Jumps)

1) Flow over spillway of a dam

3) Flow of Liquid Jet of orifice

4) Flow over weir & Notches.

5) Motion of ship in Rough and turbulent

According to this Law

$$\boxed{(Fr)_p = (Fr)_m}$$

$$\therefore \left(\frac{V}{\sqrt{gL}}\right)_p = \left(\frac{V}{\sqrt{gL}}\right)_m$$

$$\Rightarrow \boxed{\frac{V_r}{g_r^{1/2}\, L_r^{1/2}} = 1}$$

$\boxed{\text{However } g_r = 1}$

a) Time Ratio $T_r = ?$

$$L_r/T_r = V_r = g_r^{1/2} \cdot L_r^{1/2}$$

$$\therefore T_r = \frac{L_r^{1/2}}{g_r^{1/2}}$$

$$\therefore \boxed{T_r = L_r^{1/2}\, g_r^{-1/2}}$$

$$\boxed{T_r = L_r^{1/2}} \quad \text{if } g_r = 1$$

b) Accln Ratio:

$$a_r = g_r = 1$$

c) Force Ratio:

$$F_r = \rho_r L_r^3 \times g_r$$

$$= \rho_r L_r^3$$

d) Discharge Ratio:

$$Q_r = \frac{L_r^3}{T_r} = \frac{L_r^3}{L_r^{1/2}\, g_r^{-1/2}}$$

$$\therefore \boxed{Q_r = L_r^{5/2}\, g_r^{1/2}}$$

$= \rho_r L_r^{3} \cdot g_r \times L_r^{1/2} g_r^{1/2}$

$= \rho_r L_r^{7/2} g_r^{3/2}$

344 Mach number: when compressibility force is significant apart from inertia force than Mack number is important and it is defined as

$$\boxed{M_a = \sqrt{F_i/F_c}}$$

$$\therefore M_a = \sqrt{\frac{\rho L^2 V^2}{K \times L^2}} = \frac{V}{\sqrt{K/\rho}}$$

$$\boxed{M_a = V/c}$$

[$\sqrt{K/\rho} \rightarrow$ velocity of sound (c)]

Mack number is significant when velocity is comparable with sound velocity

If (i) $M_a > 1 \rightarrow$ Flow is supersonic

(ii) $M_a = 1 \rightarrow$ " " sonic

(iii) $M_a >> 1 \rightarrow$ Flow is hypersonic

(iv) $M_a < 1 \rightarrow$ Flow is sub sonic / ultrasonic

(v) $M_a < 0.3 \Rightarrow$ Effect of compressibility is neglected.

$\boxed{M_a < 0.4}$

NOTE: Square of Mack Number is called cauchy's No.

$M_a^2 =$ cauchy's No.

$\frac{V_a^2}{c^2} =$ "

$= \frac{F_i}{F_c}$.

Mach Law: — mainly in compressible fluid flowing high speed, which are comparable with sound.

ex: 1> flow of gases having high speed.
2> water hammer problem
3> ~~Aero~~ dynamic testing
4> Testing of torpido's
5> Motion of airplane with high speed
6> Launching & Projectile of Missiles.

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According to this Law

$$\boxed{(M_a)_M = (M_a)_P}$$

$$\boxed{V_r / C_r = 1}$$

4> Weber Model:

When surface tension force apart from inertia force is important but other forces are less significant than Weber No. is defined as

$$\boxed{W_e = \sqrt{F_i / F_s}}$$

$$\therefore W_e = \sqrt{\frac{\rho L^2 V^2}{\sigma L}}$$

$$\therefore \boxed{W_e = \frac{V}{\sqrt{\sigma / \rho L}}}$$

Weber Model Law:

Application
a> capillary movement of water in soil
b> flow of bloods in veins & ~~arteries~~ arteries
c> thin sheet flow
d> Liquid atomization
e> capillary tube flow.

According to this Law

$$\boxed{(W_e)_P = (W_e)_M}$$

$$\therefore \sqrt{\frac{\sigma_r}{L_r \cdot \rho_r}} = 1$$

2) Euler Number:

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When the pressure force is very important apart from inertia force as compared to other forces than Euler No. is defined as

$$E_u = \sqrt{F_i / F_p}$$

$$= \sqrt{\frac{\rho L^2 V^2}{p \times L^2}} = \frac{V}{\sqrt{p/\rho}}$$

⇒ $1/E_u$ is called Newton Number.

⇒ Newton number square is called pressure coefficient.

$$1/E_u^2 = \text{Newton Number} = \frac{F_p}{F_i} = \frac{p}{\rho V^2},$$

Euler Model Law:

ex:

a) Flow through pipes under pressure
b) Flow over submerged bodies when pressure is important
c) Pressure rise due to sudden closure of value.
d) Discharge through weir & Mouthpieces, under Large head

According to this Law

$$\boxed{(E_u)_m = (E_u)_p}$$

NOTE

1) In some of the cases where viscosity force & gravity force both are important than Reynolds Law & Froud Law both should be applicable.

For Ex:

a) Resistance to ship is generally caused by viscosity & eddies formed by wave here both law should be satisfied

The combination of these two laws is given by

$$\boxed{L_r^{3/2} = \nu_r \quad \text{OR} \quad L_r^{3/2} = \mu_r/\rho_r}$$

Above law satisfies both laws of Reynolds & Frouds (~~viscosity~~)

2) In case of the rivers normally scale adopted for depth is different from scale for width & length therefore slope of bed is exagrated and according to Manning's law

$$V_r = \frac{R_r^{2/3} \, S_r^{1/2}}{n_r}$$

where $S_r = \frac{D_r}{L_r}$ → Depth Ratio or Height Ratio; L_r → Length Ratio / width Ratio

For wide Rivers $A_r = L_r \times D_r$

(Perimeter) $P_r = L_r$

$\therefore R_r = (A_r/P_r) = D_r$

$$\therefore V_r = \frac{D_r^{2/3}}{n_r}\left(\frac{D_r^{1/2}}{L_r^{1/2}}\right)$$

$$\therefore \boxed{V_r = \frac{D_r^{7/6}}{n_r L_r^{1/2}}} \quad \text{—— 1)}$$

→ In case of the rivers in open channel, Froud Model Law is applicable.

$$V_r = D_r^{1/2} \cdot g_r^{1/2} \quad \text{— 2)}$$

By eqn (i) & (ii)

[put $g_r = 1$

$$\boxed{n_r = D_r^{4/6} L_r^{-1/2}} \rightarrow \text{For distorted model.}$$

$$\boxed{n_r = D_r^{2/3} L_r^{-1/2}}$$

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$\rightarrow \sin^2\theta + \cos^2\theta = 1$

$\rightarrow 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

cos θ, tan θ	cos θ, tan θ
$3\pi/2$	0
sin θ, cos θ, tan θ	sin θ, cos θ, tan θ
2π	

$\rightarrow \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\left.\begin{aligned}
&\rightarrow \sin C + \sin D = 2\sin\frac{C+D}{2}\cdot\cos\left(\frac{C-D}{2}\right)\\
&\rightarrow \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)\\
&\rightarrow \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\\
&\rightarrow \cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)
\end{aligned}\right\}$$

$$\left.\begin{aligned}
2\sin A\sin B &= \cos(A-B) - \cos(A+B)\\
2\cos A\cos B &= \cos(A+B) + \cos(A-B)
\end{aligned}\right\}$$

$$\left.\begin{aligned}
2\sin A\cos B &= \sin(A+B) + \sin(A-B)\\
2\cos A\sin B &= \sin(A+B) - \sin(A-B)
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\sin 2A &= 2\sin A\cos A\\
\cos 2A &= \cos^2 A - \sin^2 A\\
&= 2\cos^2 A - 1\\
&= 1 - 2\sin^2 A\\
\tan 2A &= \frac{2\tan A}{1-\tan^2 A}
\end{aligned}\right\}$$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$c^2 = a^2 + b^2 - 2ab\cos C$

similarly

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$