dislike for gambling *as such* but rather refers to his willingness to gamble for the sake of the various possible outcomes.)

As we have seen, in the case of people who have vNM utility functions at all, factor (i) will be completely inoperative, so that their only reason for gambling will be *instrumental*, based on their desire to achieve some specific outcomes.

Yet, when it is claimed that vNM utility functions express people's attitudes toward *gambling* without any qualification, it is natural to assume that their *intrinsic* attitude toward gambling — i.e., their *intrinsic* like or dislike for gambling — is being meant, even though, as we have seen, people's vNM utility functions cannot be affected by this attitude at all.

7. Von Neumann-Morgenstern utilities as *cardinal* utilities

I now propose to argue that vNM utility functions are cardinal utility functions. There are two basic differences between merely *ordinal* and *cardinal* utility functions. One is that the former allow meaningful comparisons only between the relevant individual's *utility levels* but not between his *utility differences*, whereas the latter allow both kinds of comparisons in a meaningful way. Thus, regardless of whether U_i is an ordinal or a cardinal utility function of individual i, the preference statement $A \succ B$ will be represented by the inequality $U_i(A) > U_i(B)$ whereas the indifference statement $A \sim B$ will be represented by the equation $U_i(A) = U_i(B)$.

On the other hand, if U_i is merely an *ordinal* utility function then inequalities and equalities between utility differences such as

$$(10) \qquad \Delta U_i(A,B) = U_i(A) - U_i(B) \text{ and } \Delta U_i(C,D) = U_i(C) - U_i(D)$$

will have no introspective or behavioral meaning. In contrast, if U_i is a *cardinal* utility function then such inequalities and equalities will be meaningful. (As we shall see, in the special case where U_i is a vNM utility function, such inequalities and equalities will tell us something about i's preferences and indifferences between certain lotteries.)

The other difference is that an ordinal utility function U_i tells us only *what* i's preferences are whereas, if U_i is a cardinal utility function, then it will also permit us to *compare* i's different preferences as to their *intensities* or, equivalently, as to their *relative importance* for i.

The relevant mathematical facts will be stated in the form of the following:

LEMMA. *Consider the inequality*

$$(11) \qquad \Delta U_i(A,B) > \Delta U_i(C,D).$$

This inequality will hold if and only if

$$L_1 = \left(A, \frac{1}{2}; D, \frac{1}{2}\right) \succ L_2 = \left(B, \frac{1}{2}; C, \frac{1}{2}\right). \tag{12}$$

Moreover, the Lemma remains true even if in (11) and in (12) the signs $>$ and $\succ$ are replaced by the signs $=$ and $\sim$, respectively.

To verify the first two sentences of the Lemma, note that, in view of (10), inequality (11) can be written also in the form

$$\frac{1}{2}U_i(A) + \frac{1}{2}U_i(D) > \frac{1}{2}U_i(B) + \frac{1}{2}U_i(C). \tag{13}$$

Yet, (13) implies, and is also implied by, statement (12). The last sentence of the Lemma can be verified in a similar way.

The Lemma shows how statements about one utility difference $\Delta U_i(A, B)$ being *larger than*, or being *equal to*, another utility difference $\Delta U_i(C, D)$ can be reduced to statements about i's *preference* for some lottery L_1 over some lottery L_2, or about i's *indifference* between the two lotteries. It also shows how, conversely, statements about i's preferences and indifferences can be reduced to inequalities and equalities between utility differences.

I now propose to show that, in view of our Lemma, if i prefers A to B but prefers C to D, then the utility differences $U_i(A, B)$ and $U_i(C, D)$ can be used to measure the *intensities* of these two preferences by i, or, equivalently, the *relative importance* of these two preferences for him.

Again consider the two lotteries

$$L_1 = \left(A, \frac{1}{2}; D, \frac{1}{2}\right) \quad \text{and} \quad L_2 = \left(B, \frac{1}{2}; C, \frac{1}{2}\right).$$

We can obtain L_1 from L_2 by making two moves: *Move I* will consist in replacing prize B by prize A in lottery L_2 whereas *Move II* will consist in replacing prize C by prize D. Since by assumption we have $A \succ B$ but $C \succ D$, Move I will amount to replacing a given prize by a *preferred* prize while Move II will amount to replacing a given prize by a *less preferred* prize. It is natural to assume that i will prefer lottery L_1 to lottery L_2 if and only if his preference for A over B has *greater intensity* or, equivalently, if it has *greater importance* for him, than his preference for C over D.

Yet, by our Lemma, i will prefer L_1 over L_2 if and only if $\Delta U_i(A, B)$ is *larger* than $AU_i(C, D)$. This means that i's preference for A over B

will have *greater intensity* and will have *greater importance* for him if and only if $\Delta U_i(A, B)$ is *larger* than $\Delta U_i(C, D)$. In other words, the two utility differences $\Delta U_i(A, B) = U_i(A) - U_i(B)$ and $\Delta U_i(C, D) = U_i(C) - U_i(D)$ can be used as *measures* for the *intensities* and for the relative *importance* of i's preference for A over B, and of his preference for C over D. This is of course an intuitively very plausible result: The mere fact that i prefers A to B is indicated by the piece of information that the utility difference $\Delta U_i(A, B)$ is *positive*. Thus, it is not surprising to find that the *magnitude* of this utility difference indicates the *intensity* of this preference and its *importance* for him.

8. Marginal utilities, complementarity, and substitution

Economists use vNM utilities primarily in analyzing choices involving risk and uncertainty. Other things being equal, the *more concave* a person's vNM utility function for money, i.e., the more strongly it displays *decreasing marginal utilities*, the less willing he will be to take risks; and the *more convex* his vNM utility function for money, i.e., the more strongly it displays *increasing marginal utilities*, the more willing he will be to take risks (cf. Friedman and Savage, 1948).

Yet, once vNM utility functions are available, they can be used also in other branches of economic theory. For instance, they can be used to replace the well-known Hicks-Allen definitions for complements and for substitutes (Hicks, 1939) by much simpler definitions. Let A and B denote specific amounts of commodities α and β. Let U_i be i's vNM utility function. Let $U_i(A\&B)$ denote the utility that i derives from consuming A and B *together*, and let $U_i(A)$ and $U_i(B)$ denote the utilities he derives from consuming A and B *separately*.

Then, A and B will be *complements* if

$$U_i(A\&B) > U_i(A) + U_i(B); \tag{14}$$

and they will be *substitutes* if

$$U_i(A\&B) < U_i(A) + U_i(B). \tag{15}$$

Under these definitions, i's vNM utility function for money will display *concavity*, i.e., *decreasing marginal utilities*, in those income ranges where among the commodities consumed by i *substitution* relations predominate. The opposite will be true in those income ranges where among