ACE
Engineering Academy

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ESE-2018 MAINS TEST SERIES
Question Cum Answer Booklet (QCAB)

Mechanical Engineering **Test-2** **Paper-II**

Engineering Mechanics + Design of Machine Elements

Time Allowed: 3 Hours **Maximum Marks: 300**

ACE HALL TICKET No. :

HALL TICKET No. : (Issued by UPSC)

NAME OF THE CANDIDATE :

NAME OF THE CENTRE :

BRANCH : BATCH : ROLL No. : MOBILE No. :

TEST CODE : 302

DATE : 07-04-2018

INSTRUCTIONS TO CANDIDATES:

- This Question-cum- Answer (QCA) Booklet contains **72** pages. Immediately on receipt of booklet, please check that this QCA booklet does not have any misprint or torn or missing pages or items, etc. If so, get it replaced by a fresh QCA booklet.
- Candidates must read the instructions on this page and the following pages carefully before attempting the paper.
- Candidates should attempt all questions strictly in accordance with the specified instructions and in the space prescribed under each question in the booklet. Any answer written outside the space allotted may not be given credit.
- Question Paper in detachable form is available at the end of the QCA booklet and can be removed and taken by the candidates after conclusion of the exam

For filling by Examiners only

Question No.	Page No.	Marks
1	3	
2	12	
3	19	
4	27	
5	35	
6	44	
7	52	
8	60	
Grand Total		

Signature of the Invigilator

Signature of the Student

Marks Secured after Scrutiny

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions:

*There are **EIGHT** questions divided into **TWO** sections.*

*Candidate has to attempt **FIVE** questions in all.*

*Question No.**1** and Question No.**5** are compulsory and out of the remaining, any **THREE** are to be attempted choosing at least **ONE** from each section.*

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-Cum-Answer (QCA) Booklet in the space provided. No marks will be given for answers written in a medium other than the authorized one.

Assume suitable data, if considered necessary and indicated the same clearly.

Unless otherwise mentioned, symbols and notations carry their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-Cum-Answer Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

DONT'S:

1. Do not write your Name or Roll number or Sr. No. of Question-Cum-Answer-Booklet anywhere inside this Booklet.
   Do not sign the "Letter Writing" questions, if set in any paper by name, nor append your roll number to it.
2. Do not write anything other than the actual answers to the questions anywhere inside your Question-Cum-Answer-Booklet.
3. Do not tear off any leaves from your Question-Cum-Answer-Booklet. If you find any page missing, do not fail to notify the Supervisor/invigilator.
4. Do not write anything on the Question Paper available in detachable form or admission certificate and write answers at the specified space only.
5. Do not leave behind your Question-Cum-Answer-Booklet on your table unattended, it should be handed over to the Invigilator after conclusion of the exam.

DO'S :

1. Read the instructions on the cover page and the instructions specific to this Question Paper mentioned on the next page of this Booklet carefully and strictly follow them.
2. Write your Roll number and other particulars, in the space provided on the cover page of the Question-Cum-Answer-Booklet.
3. Write legibly and neatly. Do not write in bad/illegible handwriting.
4. For rough notes or calculations the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be valued.
6. Hand over your Question-Cum-Answer-Booklet personally to the invigilator before leaving the examination hall.
7. **Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.**

SECTION - A

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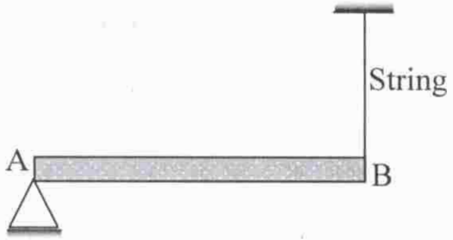
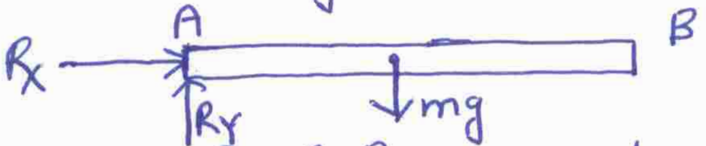
01(a). A uniform rod AB of mass 2 kg is hinged at one end A. The rod is kept in the horizontal position by a massless string tied to point B. Find the reaction of the hinge (in N) on end A of the rod at the instant when string is cut ($g = 10\ m/s^2$).



(12 M)

Soln

F.B.D of rod 'AB' at the instant the string is cut is drawn below



where, R_x & R_y are hinge reactions along x and Y respectively

Translational equation of centre of mass gives, $mg - R_y = m \times a_{c.m}$ ---- (i)

where, $a_{c.m}$ = acceleration of centre of mass

Taking Torque about A

$T_A = mg \times \frac{l}{2} = I_A \times \alpha$ ------ (ii)

where, [l = length of rod,
I_A = moment of inertia of rod about A
α = angular acceleration of rod]

$I_A = \frac{ml^2}{3}$

$a_{c.m} = \frac{l}{2}\alpha$ ---- (iii)

From (i), (ii) and (iii) we get ~~$R_x=0$ &~~ $R_y=5N$

Also, just after the string is cut, $\omega=0$

$\Rightarrow R_x = m\omega^2 \frac{l}{2} = 0$

So, net reaction at the hinge = 5N

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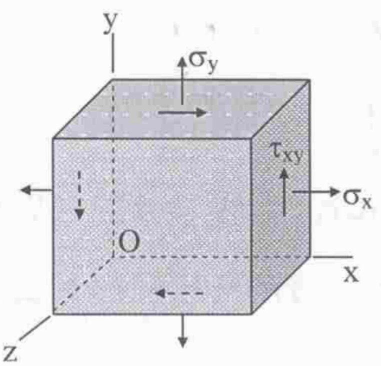
01(b). Assume that the normal strains ε_x and ε_y for an element in plane stress as shown in the figure below are measured with strain gages.



(i) Obtain a formula for the normal strain ε_z in the z-direction in terms of ε_x, ε_y, and Poisson's ratio μ.

(ii) Obtain a formula for the dilatation e in terms of ε_x, ε_y, and Poisson's ratio μ.

(6 + 6 =12 M)

Sol^n Given data:

Strain in x-direction $= \varepsilon_x$

strain in Y-direction $= \varepsilon_y$

poisson's ratio $= \mu$

(i) Normal strain in Z-direction is given by,

$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y)$

where, $\sigma_z = 0$

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)$$

$$\therefore \varepsilon_z = \frac{-\mu}{E}\left[\frac{E}{1-\mu^2}(\varepsilon_x+\mu\varepsilon_y) + \frac{E}{1-\mu^2}(\varepsilon_y+\mu\varepsilon_x)\right]$$

$$= \frac{-\mu}{1-\mu^2}\left[\varepsilon_x + \mu\varepsilon_y + \varepsilon_y + \mu\varepsilon_x\right]$$

$$= \frac{-\mu}{1-\mu^2} \times (\varepsilon_x+\varepsilon_y)(1+\mu)$$

$$\Rightarrow \varepsilon_z = \frac{-\mu}{1-\mu} \times (\varepsilon_x + \varepsilon_y)$$

(ii) ~~Dia~~ Dilatation is given by,

$$e = \frac{\varepsilon_v}{v} = \frac{1-2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{1-2\mu}{E}\left[\frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y) + \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x) + 0\right]$$

$$= \frac{1-2\mu}{1-\mu^2}\left[\varepsilon_x + \mu\varepsilon_y + \varepsilon_y + \mu\varepsilon_x\right]$$

$$= \frac{1-2\mu}{1-\mu^2} \times (1+\mu)(\varepsilon_x+\varepsilon_y)$$

$$\Rightarrow e = \frac{1-2\mu}{1-\mu} \times (\varepsilon_x + \varepsilon_y)$$

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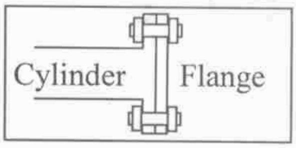
Candidates must not write on this margin

01(c). A steam engine cylinder has an effective diameter of 250 mm. It is subjected to a maximum steam pressure of 1.5 MPa. The cylinder cover is fixed to the cylinder flange by means of 12 studs as shown in figure. The pitch circle diameter of the studs is 400 mm. The permissible tensile stress in the studs is limited to 30 N/mm^2.



(i) Determine the nominal diameter 'd' of the studs if core diameter, $d_c = 0.84d$.

(ii) Calculate the circumferential pitch of the studs. **(8 + 4 = 12 M)**

Sol^n

Given data:

$D = 250$ mm , $P = 1.5$ Mpa ,

$n = 12$, $d_p = 400$ mm , $S_{yt} = 30$ Mpa

(i) The force exerted on the flange is given by,

$$F = P \times A = 1.5 \times \frac{\pi}{4} \times D^2$$

$$= 1.5 \times \frac{\pi}{4} \times (250)^2 = 73631.08 \text{ N}$$

Now, this force is resisted by the flange which is fixed with 12 studs.

Thus, $$\frac{F}{\frac{\pi}{4}(d_c)^2 \times n} = S_{yt}$$

$$\therefore \frac{73631.08}{12 \times \frac{\pi}{4} \times 30} = d_c^2$$

$$\therefore d_c = 16.14 \text{ mm}$$

Given that, $d_c = 0.84\, d$

$$\Rightarrow d = \frac{16.14}{0.84} = 19.21 \text{ mm} \cong 20 \text{ mm}$$

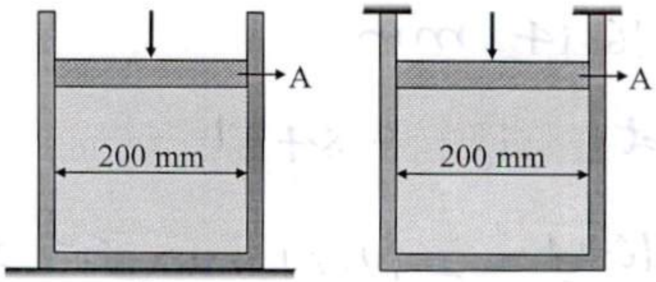
(ii) Circumferential pitch,

$$t = \frac{\pi d_p}{n}$$

$$t = \frac{\pi \times 400}{12} = 104.72 \text{ mm}$$

Candidates must not write on this margin

01(d). The thin-walled cylinder can be supported in one of two ways as shown in the figure below.



(I) (II)

The wall has a thickness of 7 mm, and the inner diameter of the cylinder is 200 mm. Determine the state of stress in the wall of the cylinder for both cases if the piston 'A' causes a uniform internal pressure of 300 kPa. **(12 M)**

Candidates must not write on this margin

SOL^n

Given data :

$P = 300$ Kpa , $d = 200$ mm , $t = 7$ mm

Case (I) :

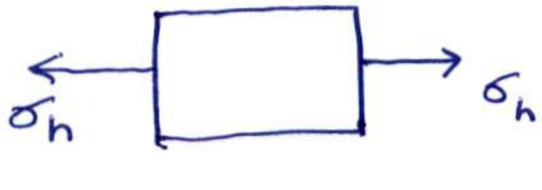
Hoop stress,

$$\sigma_h = \frac{Pd}{2t} = \frac{300 \times 10^3 \times 200}{2 \times 7} = 4.29 \times 10^5 \text{ pa}$$

Longitudnal stress,

$\sigma_L = 0$

As top end of the cylinder is not constrained, there will not be any longitudnal stress in the wall.



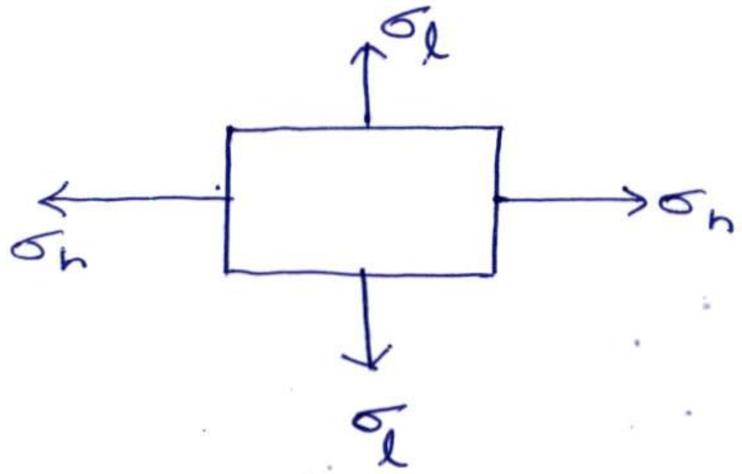
Case(II)

Hoop stress,

$$\sigma_h = \frac{pd}{2t} = 4.29 \times 10^5 \text{ pa}$$

Longitudnal stress,

$$\sigma_L = \frac{pd}{4t} = \frac{\sigma_h}{2} = 2.14 \times 10^5 \text{ pa}$$

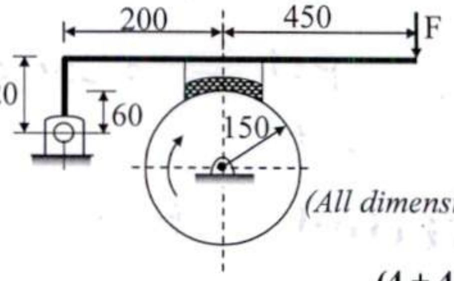


01(e). A single block brake with a torque capacity of 15 N-m is shown in figure. The coefficient of friction is 0.3 and the maximum pressure on the brake lining is 1 N/mm^2. The width of the block is equal to its length. Calculate

(i) the actuating force, F

(ii) the dimensions of the block

(iii) the resultant hinge-pin reaction.



(All dimensions are in mm)

(4 + 4 + 4 = 12 M)

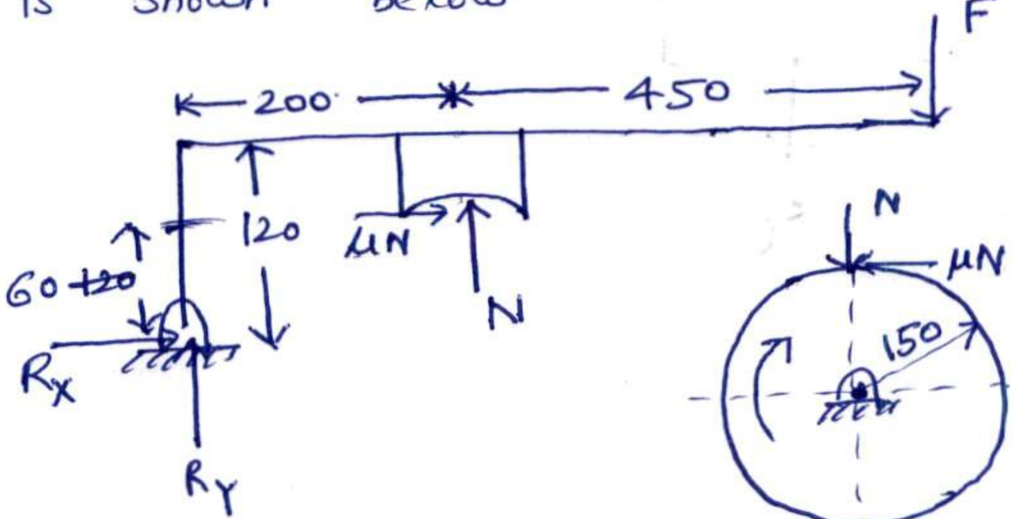
Soln

Given data :

$M_t = 15$ N-m , $\mu = 0.3$, $l = w$, $p = 1\ N/mm^2$

$R = 150$ mm

The free body diagram of the brake is shown below



(i) Braking torque is given by,

$M_t = \mu N R$

$15 \times 10^3 = 0.3 \times N \times 150$

$\therefore N = 333.33$ N

By taking the moment about hinge,

$F \times (200 + 450) - N \times 200 + \mu N \times 60 = 0$

$\therefore$ $F \times 650 - 333.33 \times 200 + 0.3 \times 333.33 \times 60 = 0$

$\therefore$ $F = 93.33\,N$

(ii) Normal reaction,

$N = p \times lw$

Here, p = permissible pressure between block and brake drum

l = length of block

w = width of block

$333.33 = 1 \times l \times l \quad (\because l = w)$

$\therefore$ $l = w = 18.26\,mm$

(iii) For the lever,

$\Sigma F_x = 0$

$\therefore$ $R_x + \mu N = 0$

$\therefore$ $R_x = -0.3 \times 333.33 = -100\,N$

$\Sigma F_y = 0$

$\therefore$ $R_y + N - F = 0$

$\therefore$ $R_y = 93.33 - 333.33 = -240\,N$

Thus, resultant hinge reaction

$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(-100)^2 + (-240)^2} = 260\,N$

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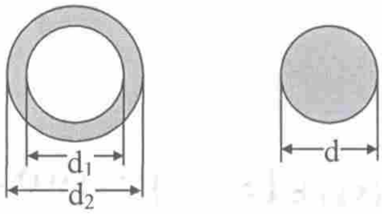
02(a). A hollow aluminum tube used in a roof structure has an outside diameter d_2 = 104 mm and an inside diameter d_1 = 82 mm as shown in the figure below. The tube is 2.75 m long, and the aluminum has shear modulus, G = 28 GPa.



(i) If the tube is twisted in pure torsion by torques acting at the ends, what is the angle of twist (in degrees) when the maximum shear stress is 48 MPa?

(ii) What diameter d is required for a solid shaft as shown in the figure above to resist the same torque with the same maximum stress?

(iii) What is the ratio of the weight of the hollow tube to the weight of the solid shaft?

(8 + 8 + 4 = 20 M)

Sol^n

Given data :

$d_2 = 104$ mm , $d_1 = 82$ mm

$L = 2.75$ m , $G = 28$ Mpa

(i) Angle of twist for a given $\tau_{max} = 48$ Mpa

By using torsion formulae,

$$\theta = \frac{\tau \times L}{G \times r}$$

Here, $\tau = \tau_{max}$

~~Where~~ when $r = r_{max} = \frac{d_2}{2} = \frac{104}{2} = 52$ mm

$$\therefore \theta = \frac{48 \times 2.75 \times 10^3}{28 \times 10^3 \times 52}$$

$$= 0.091 \times \frac{180}{\pi} = 5.19^\circ$$

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(ii) For a solid shaft having diameter 'd' maximum shear stress is given by,

$$\frac{(\tau_{max})_S}{d/2} = \frac{T_S}{\frac{\pi}{32} \times d^4}$$

$$\therefore (\tau_{max})_S = \frac{16 T_S}{\pi d^3} \quad \text{-----} \quad (1)$$

For a hollow shaft,

$$\frac{(\tau_{max})_H}{d_2/2} = \frac{T_H}{\frac{\pi}{32} \times (d_2^4 - d_1^4)}$$

$$\therefore (\tau_{max})_H = \frac{16\, d_2\, T_H}{\pi (d_2^4 - d_1^4)} \quad \text{---} \quad (2)$$

given that, $(\tau_{max})_S = (\tau_{max})_H$ and $T_S = T_H$

$\therefore$ from equations (1) and (2),

$$\frac{1}{d^3} = \frac{d_2}{d_2^4 - d_1^4}$$

$$\therefore d^3 = \frac{d_2^4 - d_1^4}{d_2}$$

$$\therefore d = \left[\frac{(104)^4 - (82)^4}{104}\right]^{1/3} = 88.37 \text{ mm}$$

(iii) Ratio of weight

$$\frac{W_H}{W_S} = \frac{\rho_H A_H L_H}{\rho_S A_S L_S}$$

Candidates must not write on this margin

where, $\rho_H = \rho_S$ and $L_H = L_S$

$$\therefore \frac{W_H}{W_S} = \frac{A_H}{A_S} = \frac{\frac{\pi}{4}(d_2^2 - d_1^2)}{\frac{\pi}{4} d^2}$$

$$\Rightarrow \frac{W_H}{W_S} = \frac{104^2 - 82^2}{88.37^2} = 0.524$$

02(b).

(i). A ball bearing operates on the following work cycle:

Element No.	Radial load (N)	Speed (rpm)	Element time (%)
1	3000	720	30
2	7000	1440	50
3	5000	900	20

The dynamic load capacity of the bearing is 16.6 kN. Calculate the equivalent radial load and the bearing life. **(16 M)**

Soln

Given data : $C = 16.6$ kN

By considering the work cycle of one minute, the revolutions N in element time are calculated and tabulated as:

Element No	Radial load(N), P	Speed (rpm)	Element time (s)	Revolutions N in element time
1	3000	720	$60 \times \frac{30}{100} = 18$	$\frac{720}{60} \times 18 = 216$
2	7000	1440	$60 \times \frac{50}{100} = 30$	$\frac{1440}{60} \times 30 = 720$
3	5000	900	$60 \times \frac{20}{100} = 12$	$\frac{900}{60} \times 12 = 180$
Total			60	1116

Candidates must not write on this margin

Equivalent radial load is given by,

$$P_e = \left[\frac{\Sigma N P^3}{\Sigma N}\right]^{1/3}$$

$$= \left[\frac{N_1 P_1^3 + N_2 P_2^3 + N_3 P_3^3}{N_1 + N_2 + N_3}\right]^{1/3}$$

$$= \left[\frac{216 \times (3000)^3 + 720 \times (7000)^3 + 180 \times (5000)^3}{1116}\right]$$

$$P_e = 6271.57 \text{ N}$$

Rated bearing life

$$L_{10} = \left(\frac{C}{P_e}\right)^n$$

Here, $n = 3$ for ball bearing

$$\therefore L_{10} = \left(\frac{16.6 \times 10^3}{6271.57}\right)^3$$

$$L_{10} = 18.54 \text{ million revolutions}$$

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02(b).

(ii). What is the objective of pre loading balls in rolling contact bearings ? **(4 M)**

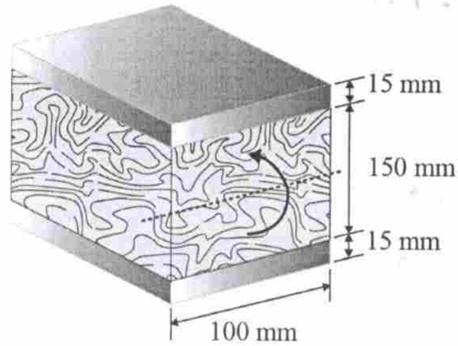
SOL^n The necessary radial internal clearance in an assembled ball bearing may increase noise and rotational vibrations due to movement of the balls inside the bearings. To minimize the relative movement of balls, an axial "preload" should be applied to the bearing. It increases the stiffness of bearing which increases the natural frequeny of the system hence reducing the noise and vibration. Appropriate preload force depends on the size of ball bearing. If insufficient preload ~~is~~ is applied, vibration and fretting wear may occur inside the bearing.

02(c). The wooden section of the beam is reinforced with two steel plates as shown in the figure below. The beam is subjected to an internal moment of M = 30 kN-m.

(i) Determine the maximum bending stresses developed in the steel plates and wood beams.

(ii) Sketch the stress distribution over the combined cross section.

(Take E_w = 10 GPa and E_{st} = 200 GPa)



(14 + 6 = 20 M)

Soln

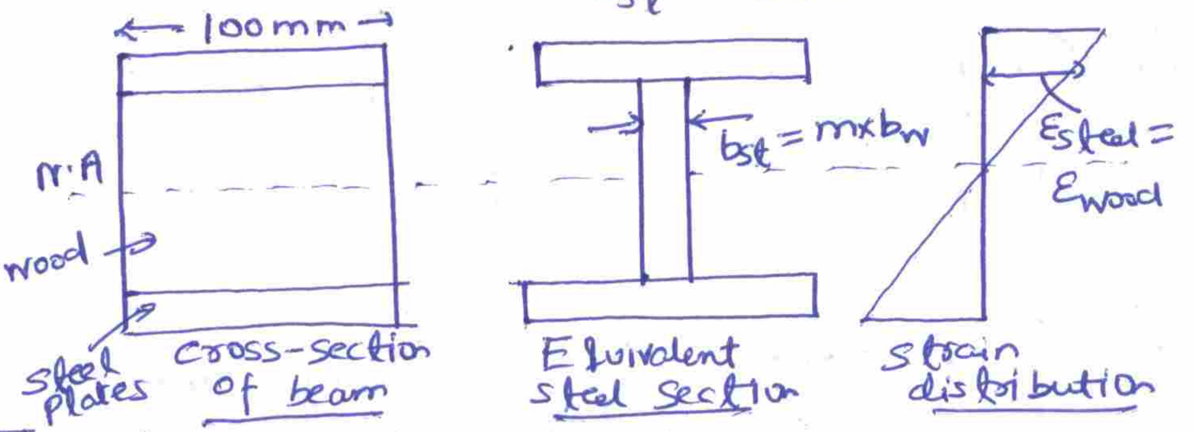
Given data: $M = 30\,kN\text{-}m$

$E_w = 10\,GPa$

$E_{st} = 200\,GPa$

(i) To analyse the composite section, let us first transform it to fully steel section.

Modular ratio, $m = \frac{E_w}{E_{st}} = \frac{10}{200} = 0.05$



Thus, $b_{st} = m \times b_w = 0.05 \times 100 = 5\,mm$

Moment of inertia of I-section is given by,

$$I = \frac{100 \times 180^3}{12} - \frac{(100-5) \times 150^3}{12} = 2.19 \times 10^7\,mm^4$$

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By using flexural formula,

$$(\sigma_b)_{max} = \frac{M}{I} \times y_{max}$$

For steel plate

$$(\sigma_b)_{y=90mm} = \frac{30 \times 10^6}{2.19 \times 10^7} \times 90$$

$$\Rightarrow (\sigma_{st})_{max} = 123.29 \text{ Mpa}$$

$$(\sigma_b)_{y=75mm} = \frac{30 \times 10^6}{2.19 \times 10^7} \times 75 = 102.74 \text{ Mpa}$$

For the wooden section:

We know that strain distribution is linear for the given composite section.

Thus, $(\varepsilon_{st})_{y=75mm} = (\varepsilon_w)_{y=75mm}$

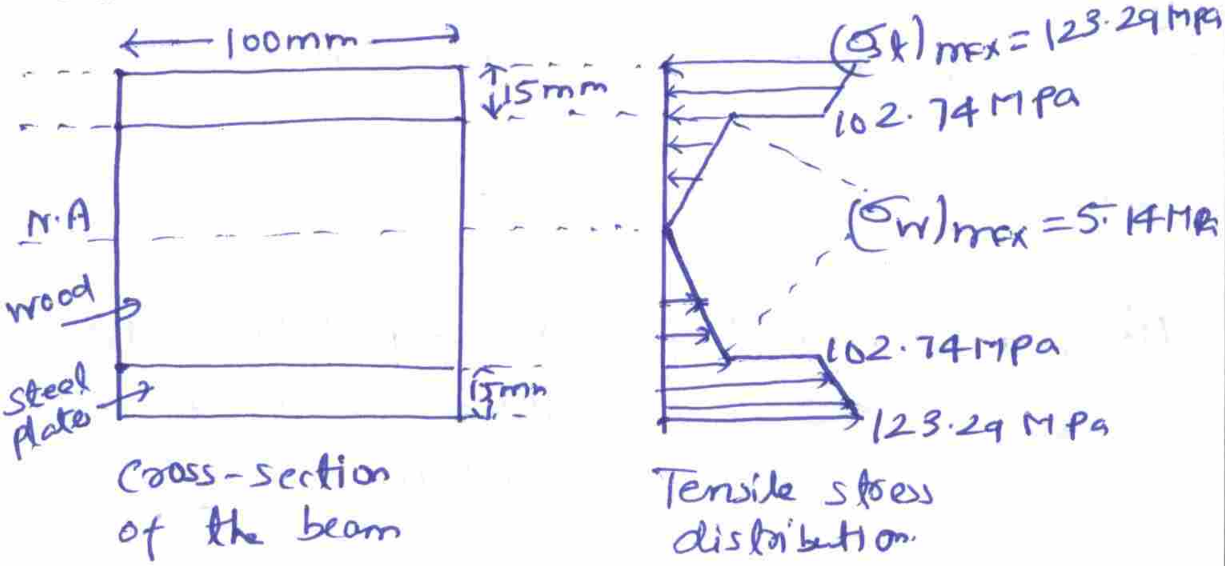
$$\therefore \frac{(\sigma_{st})_{y=75mm}}{E_{st}} = \frac{(\sigma_w)_{y=75mm}}{E_w}$$

$$\therefore (\sigma_w)_{y=75mm} = \frac{E_w}{E_{st}} \times (\sigma_{st})_{y=75mm}$$

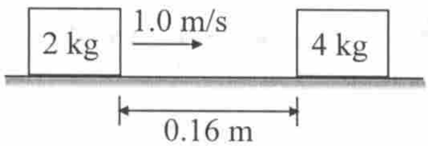
$$= 0.05 \times 102.74$$

$$\Rightarrow (\sigma_w)_{max} = 5.14 \text{ Mpa}$$

(ii) The stress distribution will be symmetric about neutral axis (i.e. Compression above the N.A. and tensile below the N.A) as shown below.



03(a). The friction coefficient between the horizontal surface and each of the blocks shown in figure is 0.20. The collision between the blocks is perfectly elastic. Find the separation between the two blocks when they come to rest. (Take g = 10 m/s^2).



(20 M)

Soln

Just before striking the block of mass 4kg, the velocity of 2Kg block (V_i) can be obtained by using basic kinematic equation

$$V_i^2 = 1^2 - 2 \times \mu \times g \times 0.16 \quad [\because \text{Retardation} = \mu \times g]$$

$$= 1^2 - 2 \times 0.2 \times 10 \times 0.16$$

$$V_i = 0.6 \text{ m/s}$$

Just after collision let the velocity of block 2Kg is 'V_1' towards left and velocity of block 4Kg is 'V_2' towards right

By conservation of linear momentum just before and just after the collision

$$2 \times 0.6 = 4v_2 - 2v_1 \quad \text{— } \underline{\underline{i}}$$

Also, $e = 1 = \dfrac{v_2 + v_1}{0.6}$ [∵ For completely elastic collision coefficient of restitution, $e = 1$]

$$v_2 + v_1 = 0.6 \quad \text{— } \underline{\underline{ii}}$$

From (i) and (ii), $v_1 = 0.2\ m/s$, $v_2 = 0.4\ m/s$

Distance travelled by block of mass 2kg before it comes to rest $= \dfrac{0.2^2}{2 \times \mu \times g}$

$$= \frac{0.2^2}{2 \times 0.2 \times 10} = 0.01\ m$$

Distance travelled by block of mass 4kg before it comes to rest $= \dfrac{0.4^2}{2 \times 0.2 \times 10} = 0.04\ m$

Total seperation between the two blocks when they come to rest $= 0.01 + 0.04 = 0.05\ m$

Candidates must not write on this margin

03(b). A thick cylinder of 120 mm internal diameter and 180 mm external diameter is subjected to an external pressure of 9 MPa.

(i) Determine the maximum value of the internal pressure that can be applied if the maximum allowable circumferential stress is to be 30 MPa.

(ii) Plot the variation of radial and circumferential stresses developed in the material of the cylinder. **(14 + 6 = 20 M)**

SOL^n

Given data:

$d_i = 120\,mm$, $d_o = 180\,mm.$

$P_o = 9\,Mpa$, $\sigma_c = 30\,Mpa$

(i) When the cylinder is subjected to internal and external pressure, the radial and hoop stresses can be written respectively as,

$$P = -A + \frac{B}{d^2} \quad \text{—— (i)}$$

$$\sigma_c = A + \frac{B}{d^2} \quad \text{—— (ii)}$$

Candidates must not write on this margin

Thus, the maximum hoop stress occurs at the inner diameter,

$$\therefore (\sigma_c)_{max} = A + \frac{B}{d_i^2}$$

$$\therefore 30 = A + \frac{B}{120^2} \quad --- \quad (iii)$$

Now, at the outer diameter, $p = p_o$

$$\therefore p_o = -A + \frac{B}{d_o^2}$$

$$\therefore 9 = -A + \frac{B}{180^2} \quad ---- \quad (iv)$$

By adding equations (iii) and (iv),

$$30+9 = A + \frac{B}{120^2} - A + \frac{B}{180^2}$$

$$\therefore 39 = B\left(\frac{1}{120^2} + \frac{1}{180^2}\right)$$

$$\therefore B = 388800$$

Thus from equation (iv),

$$A = \frac{388800}{180^2} - 9 = 3$$

By using equation (i),

$$p = -A + \frac{B}{d^2}$$

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At $d = d_i$, $P = P_i$

$$\Rightarrow P_i = -3 + \frac{388800}{120^2} = 24\ MPa$$

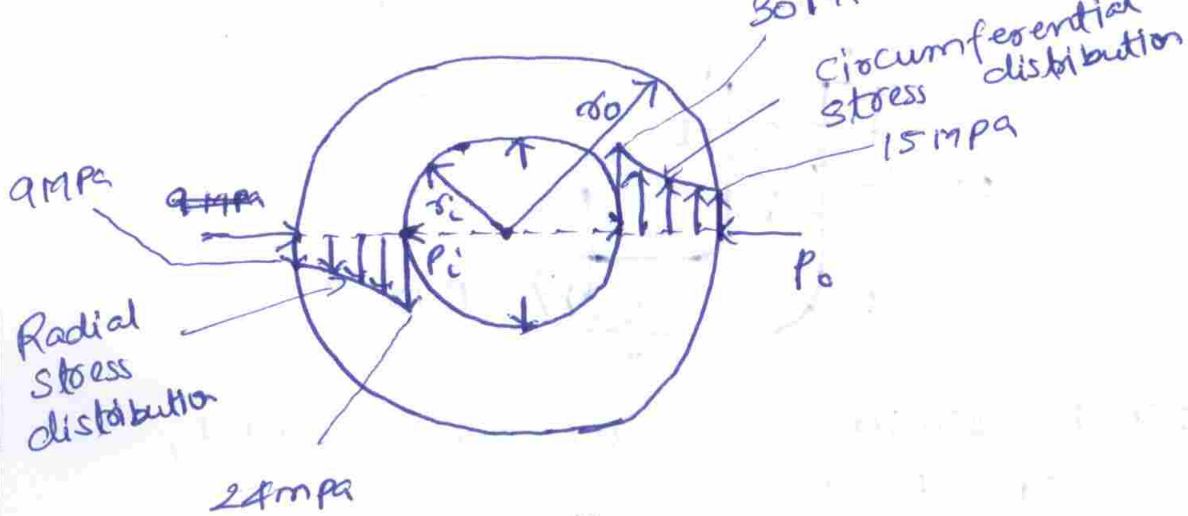
(ii) The value of stresses along the radius of the cylinder are given in the table below:

Radius	Radial stress (σ_r)	Circumferential stress (σ_c)
Inner radius ($r = r_i$)	24 MPa	30 MPa
outer radius ($r = r_o$)	9 MPa	15 MPa

At $r = r_o$

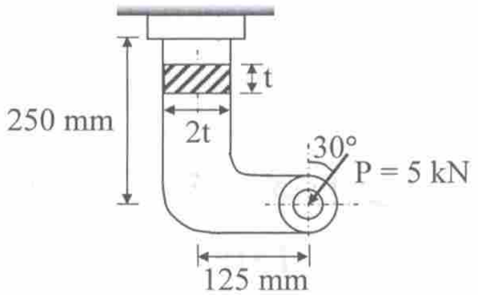
$$\Rightarrow \sigma_c = A + \frac{B}{d_o^2} = 3 + \frac{388800}{180^2} = 15\ MPa$$

Variation of ~~radi~~ the radial and circumferential stress is shown below,



03(c).

(i). A bracket, made of steel FeE 200 (S_{yt} = 200 N/mm^2) and subjected to a force of 5 kN acting at an angle of 30° to the vertical, is shown in figure. The factor of safety is 4. Determine the dimensions of the cross-section of the bracket by using maximum normal stress theory.

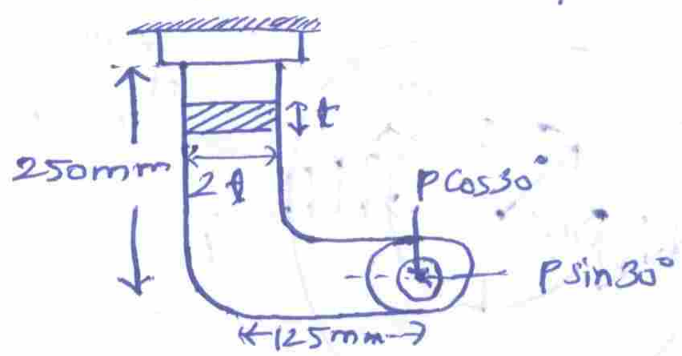


(15 M)

Soln

Given data :

$S_{yt} = 200$ GPa , $P = 5$ KN , FS = 4



Direct tensile stress in the bracket,

$\sigma_d = \frac{P\cos 60}{A}$ $\frac{P\cos 30}{A}$

$$= \frac{5\times10^3 \times \cos 30}{2t \times t} = \frac{2165.06}{t^2} \text{ Mpa}$$

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Bending moment in the bracket at fixed end,

$$M = (P\cos 30 \times 125) + (P\sin 30 \times 250)$$

$$= (5\times10^3 \times \cos 30 \times 125) + (5\times10^3 \times \sin 30 \times 250)$$

$$= 1.17\times10^6 \text{ N-mm}$$

By using flexural formula,

$$(\sigma_b)_{max} = \frac{M}{I} \times y_{max}$$

$$= \frac{1.17\times10^6}{\frac{t\times(2t)^3}{12}} \times \frac{2t}{2}$$

$$= \frac{1.75\times10^6}{t^3} \text{ Mpa}$$

Now, maximum principal stress,

$$\sigma_1 = (\sigma_b)_{max} + \sigma_d$$

$$\therefore \frac{S_{yt}}{F.S} = \frac{1.75\times10^6}{t^3} + \frac{2165.06}{t^2}$$

$$\text{or, } \frac{200}{4} = \frac{1.75\times10^6}{t^3} + \frac{2165.06}{t^2}$$

or, $50t^3 - 2165.06t - 1.75\times10^6 = 0$

$\Rightarrow t = 33.15\,mm \approx 34\,mm$

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03(c).

(ii). Write brief notes on maximum principle stress theory and maximum shear stress theory. Also state that, which theory is applicable to which type of materials. **(5 M)**

Soln Maximum Principal stress theory

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal stress in a biaxial stress system reaches the limiting strength of the material in a simple tension test. The limiting strength for ductile materials is yield point stress and for brittle materials the limiting strength is ultimate stress.

This theory is mostly used for designing of brittle materials.

Maximum shear stress theory:

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test.

This theory is mostly used for ductile materials.

04(a). An angle is welded to a frame by two fillet welds having 10 mm throat thickness, as shown in the figure below. A load of 16 kN is applied normal to the gravity axis at a distance of 300 mm from the centre of gravity of welds. Find maximum shear stress in the welds, assuming each weld to be 100 mm long and parallel to the axis of the angle.

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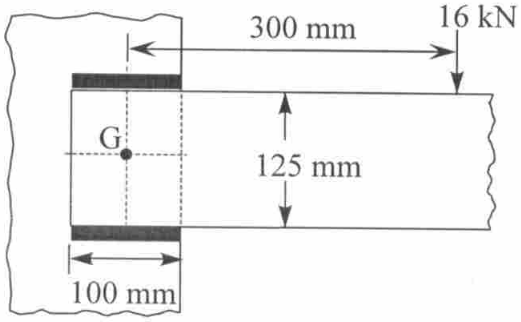


(20 M)

Soln

Given data:

$t = 10$ mm , $P = 16$ kN,

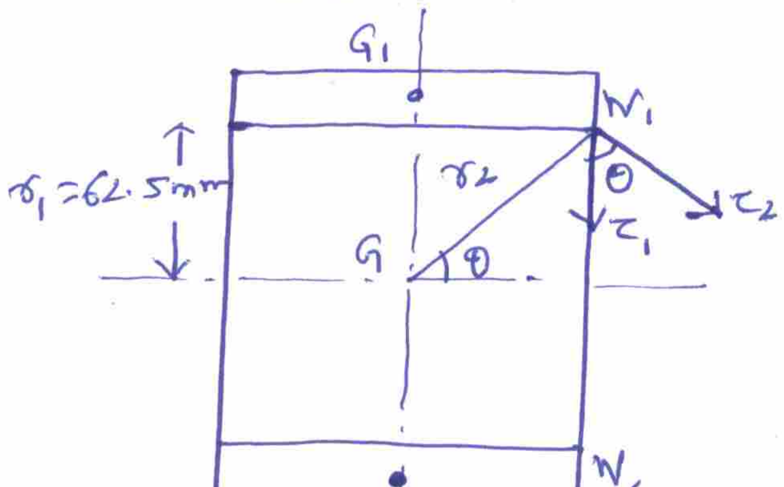
$e = 300$ mm, $l = 100$ mm

We know that, total throat area,

$$A = 2 \times t \times l = 2 \times 10 \times 100 = 2000 \text{ mm}^2$$

Direct primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{16 \times 10^3}{2000} = 8 \text{ MPa}$$



From the above figure, polar moment of inertia J_1 of weld W_1 about G is given by

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$$J_1 = J_{G_1} + A r_1^2$$

$$= A\left(\frac{l^2}{12} + r_1^2\right)$$

$$= 2000 \times \left(\frac{100^2}{12} + 62.5^2\right)$$

$$= 47.4 \times 10^5 \ mm^4$$

Due to symmetry, polar ~~more~~ moment of inertia of two welds,

$$J = J_1 + J_2 = 2J$$

$$= 2 \times 47.4 \times 10^5 = 94.8 \times 10^5 \ mm^4$$

From the figure shown above,

$$r_2 = \sqrt{62.5^2 + 50^2} = 80.04 \ mm$$

Shear stress due to the turning moment i.e., Secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J}$$

$$= \frac{16 \times 10^3 \times 300 \times 80.04}{94.8 \times 10^5} = 40.53 \ MPa$$

Now, resultant shear stress,

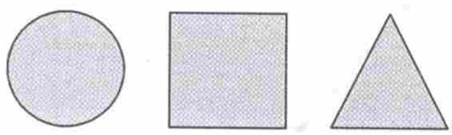
$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta}$$

here, $\cos\theta = \dfrac{50}{80.04}$

$$\Rightarrow \tau = \sqrt{8^2 + 40.53^2 + 2 \times 8 \times 40.53 \times \frac{50}{\cancel{8.0}\,80.04}}$$

$$= 45.95 \text{ Mpa}$$

04(b). Three pinned-end columns of the same material but different shapes have the same length and the same cross-sectional area as shown in the figure below. The columns are free to buckle in any direction. The columns have cross section as follows: (1) a circle, (2) a square, and (3) an equilateral triangle. Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.



(1) (2) (3)

(20 M)

Sol^n

Given data :

- Material of each section is same
- Length of each section is same
- Cross-sectional area of each section is same.

Critical buckling load is given by,

$$P = \frac{\pi^2 E I}{L_e^2}$$

Here, E and L_e are constant.

$\therefore$ $P \propto I$

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(1) circle:

$$I_1 = \frac{\pi}{64} d^4 \quad \text{and} \quad A = \frac{\pi}{4} d^2$$

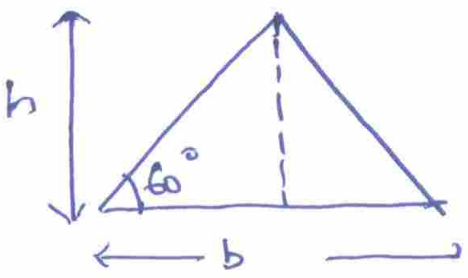
$$\therefore \; I_1 = \frac{A^2}{4\pi}$$

(2) Square (~~let~~ let side = a):

$$I_2 = \frac{a^4}{12} \quad \text{and} \quad A = a^2$$

$$\therefore \; I_2 = \frac{A^2}{12}$$

(3) Equilateral triangle:



$$I_3 = \frac{bh^3}{36} \quad \text{and} \quad A = \frac{bh}{2}$$

From geometry,

$$\sin 60 = \frac{h}{b}$$

$$\therefore \; h = b\sin 60 = \frac{\sqrt{3}\,b}{2}$$

$$\therefore \; A = \frac{\sqrt{3}}{4} b^2$$

$$\therefore \; b^2 = \frac{4A}{\sqrt{3}}$$

Now, $I_3 = \frac{b\left(\frac{\sqrt{3}\,b}{2}\right)^3}{36} = \frac{3\sqrt{3}}{8\times 36} \times b^4$

$= \frac{3\sqrt{3}}{8\times 36} \times \left(\frac{4A}{\sqrt{3}}\right)^2$

$\therefore \quad I_3 = \frac{\sqrt{3}}{18} A^2$

Thus, $P_1 : P_2 : P_3 = I_1 : I_2 : I_3$

$= \frac{A^2}{4\pi} : \frac{A^2}{12} : \frac{\sqrt{3}}{18} A^2 =$

$= 0.08 : 0.083 : 0.096$

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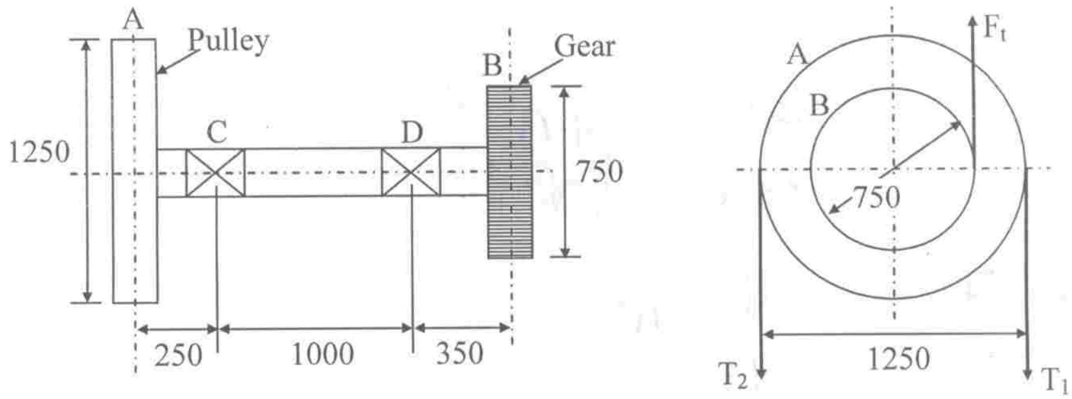
04(c). Figure shows a shaft carrying a pulley A and a gear B and supported by two bearings C and D. The shaft transmits 20 kW at 150 r.p.m. The tangential force F_t on the gear B acts vertically upwards as shown.



(All dimensions are in mm)

The pulley delivers the power through a belt to another pulley of equal diameter vertically below the pulley A. The ratio of tensions T_1/T_2 is equal to 2.5. The gear and the pulley weigh 900 N and 2700 N respectively. The permissible shear stress for the material of the shaft may be taken as 63 MPa. Assuming the weight of the shaft to be negligible in comparison with the other loads, determine its diameter. Take shock and fatigue factors for bending and torsion as 2 and 1.5 respectively. Neglect the radial component of contact force at gear tooth. **(20 M)**

Soln Given data:

$P = 20\,KW$ $\qquad N = 150$ r.p.m

$\frac{T_1}{T_2} = 2.5$ $\qquad W_B = 900\,N$

$W_A = 2700\,N$ $\qquad \tau = 63\,Mpa$

$K_m = 2$ $\qquad K_t = 1.5$

$D_B = 750\,mm$ $\qquad D_A = 1250\,mm$

We know that, power,

$$P = \frac{2\pi N T}{60}$$

$$\therefore 20\times10^3 = \frac{2\pi\times150\times T}{60} \Rightarrow T = 1273\ N\text{-}m$$

Also, $(T_1 - T_2) R_A = T$

$\therefore (T_1 - T_2) = \frac{1273 \times 10^3}{625} = 2037 \text{ N} \text{ ---- } (1)$

Given that, $\frac{T_1}{T_2} = 2.5$

$\Rightarrow T_1 = 2.5 T_2$ ---- (2)

From (1) ~~&(2)~~ and (2)

$T_1 = 3395 \text{ N}$

$T_2 = 1358 \text{ N}$

$\therefore$ Total vertical load acting downward on the shaft at A,

$= T_1 + T_2 + W_A$

$= 3395 + 1358 + 2700 = 7453 \text{N}$

Assuming that the torque on gear B is same as that of the shaft, therefore the tangential force acting vertically upward on the gear B,

$F_t = \frac{T}{R_B} = \frac{1273 \times 10^3}{375} = 3395 \text{N}$

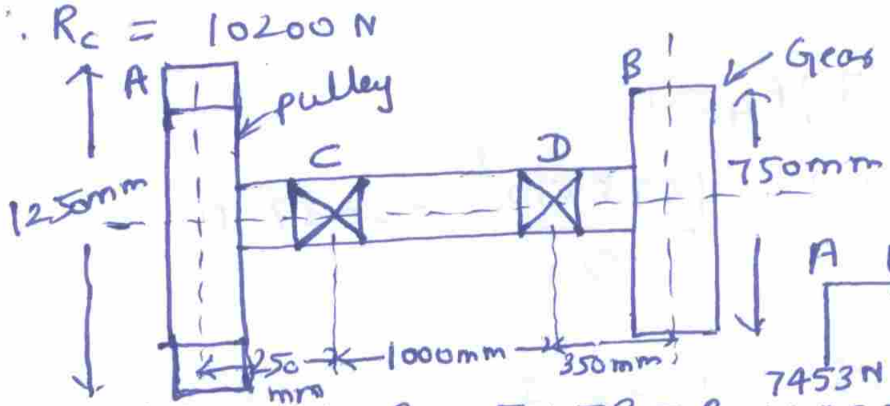
The total vertical load acting upward on the shaft at B,

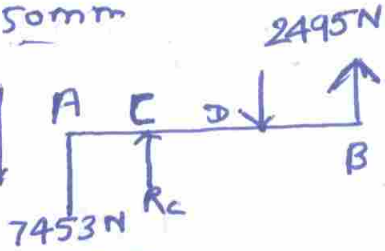
$= F_t - W_B = 3395 - 900 = 2495 \text{ N}$

Now, to find the reactions at the bearings C and D, take moment about D

$R_c \times 1000 = 7453 \times 1250 + 2495 \times 350 = 10.2 \times 10^6$

$\therefore R_c = 10200 \text{ N}$





Also, $\sum F_y = 0 \Rightarrow R_D + 7453 = R_c + 2495$

$\Rightarrow R_D = 5242 \text{ N}$

Bending moment:

$M_A = M_B = 0$

$M_C = 7453 \times 250 = 1863 \times 10^3 \text{ N-mm}$

$M_D = 2495 \times 350 = 873 \times 10^3 \text{ N-mm}$

From above, $M_{max} = M_C = 1863 \times 10^3 \text{ N-mm}$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M_{max})^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1863 \times 10^3)^2 + (1.5 \times 1273 \times 10^3)^2}$$

$$= 4187 \times 10^3 \text{ N-mm}$$

Also, $T_e = \dfrac{\pi}{16} \times \tau \times d^3$

$\therefore 4187 \times 10^3 = \dfrac{\pi}{16} \times 63 \times d^3$

$\Rightarrow d = 69.6 \approx 70 \text{mm}$

SECTION - B

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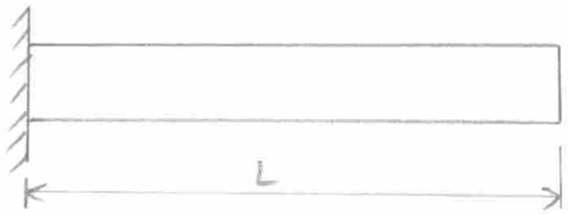
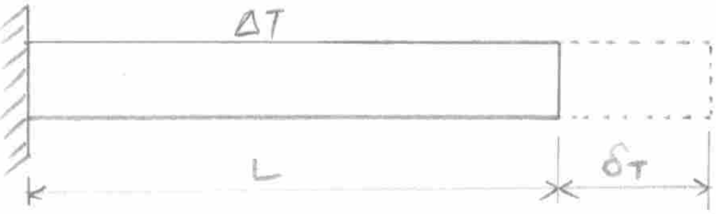
05(a).

(i). A steel rod is fixed at one end and free at the other end. The coefficient of thermal expansion of the steel is α, and modulus of elasticity is E. If the temperature of the rod is increased by ΔT then determine the stress and strain developed in the rod. **(6 M)**

A steel rod, which is fixed at one end and free at the other end is shown below:



When the temperature of the rod is increased by ΔT, the rod will be expanded as shown below:



Here, δ_T = Expansion of the rod

Also, $\delta_T \propto (\Delta T) \times L$

$\therefore \delta_T = \alpha (\Delta T) L$

where, α = Coefficient of thermal expansion

Now, strain, $\varepsilon_T = \dfrac{\delta_T}{L}$

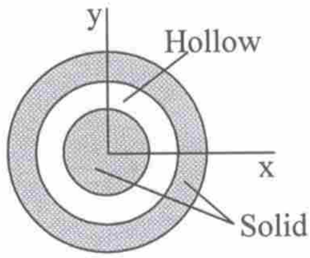
$$\boxed{\therefore \varepsilon_T = \alpha \times (\Delta T)}$$

It is given that the rod is free to expand. Thus, there will not be any stress induced in the rod.

$$\boxed{\therefore \sigma_T = 0}$$

05(a).

(ii). Calculate the moment of inertia I_x for the hollow composite circular area shown in the figure. The origin of the axes is at the center of the concentric circles, and the three diameters are 20, 40, and 60 mm.



(6 M)

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Given data:

$d_1 = 20$ mm,

$d_2 = 40$ mm,

$d_3 = 60$ mm,

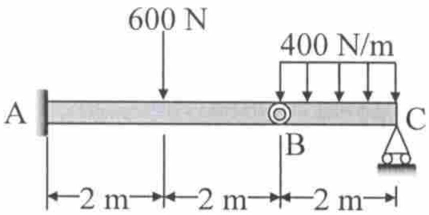
Moment of inertia of a given composite section is calculated by;

$$I_x = \frac{\pi}{64}\left[(d_3^4 - d_2^4) + d_1^4\right]$$

$$= \frac{\pi}{64}\left[(60)^4 - (40)^4 + (20)^4\right]$$

$$\boxed{\therefore I_x = 5.18 \times 10^5 \text{ mm}^4}$$

05(b). The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear force diagram for the beam.



(12 M)

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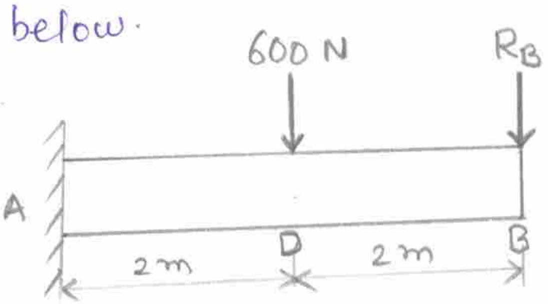
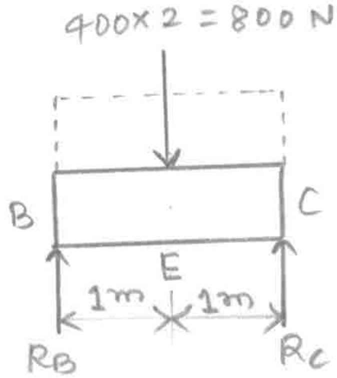
Free body diagrams of each beam are shown below.





Beam BC:

Due to symmetry of the beam, reactions R_B and R_C can be written as,

$$R_B = R_C = \frac{800}{2}$$

$$= 400 \text{ N}$$

Shear force:

$$(S.F.)_C = -R_C = -400 \text{ N}$$

$$(S.F.)_B = -R_C + (400 \times 2)$$

$$= -400 + 800$$

$$= 400 \text{ N}$$

Beam AB:

$$(S.F.)_B = 400\,N$$

$$(S.F.)_{D\,Right} = (S.F.)_B$$

$$= 400\,N$$

$$(S.F.)_{D\,Left} = (S.F.)_{D\,Right} + 600$$

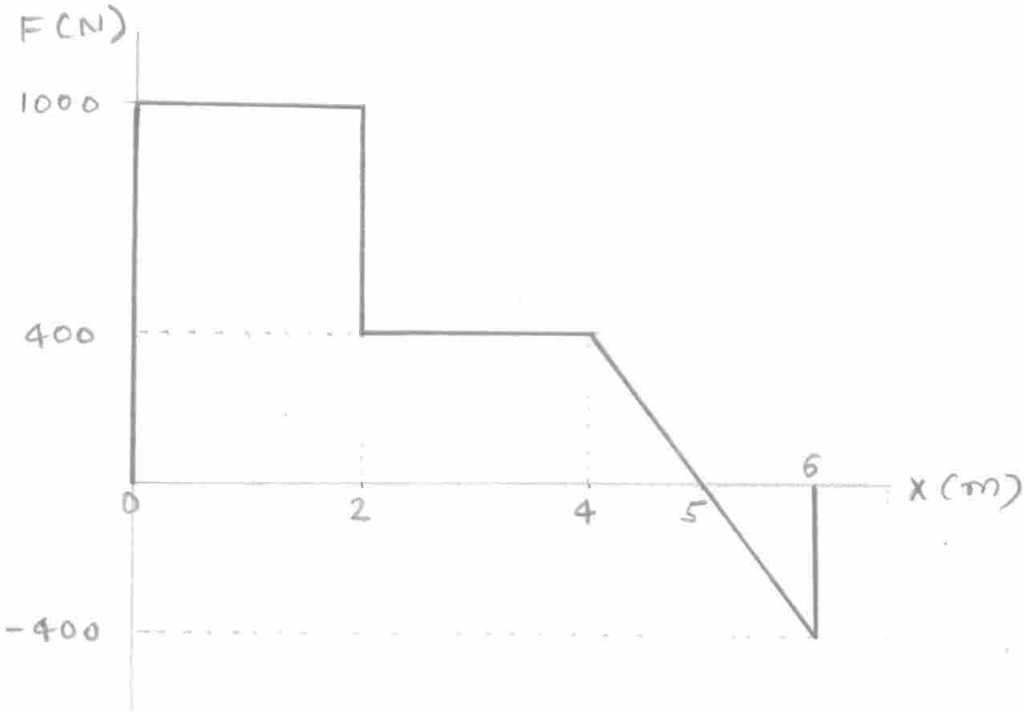
$$= 400 + 600$$

$$= 1000\,N$$

$$(S.F.)_A = (S.F.)_{D\,left}$$

$$= 1000\,N$$

From the shear force values calculated as above, shear force diagram can be drawn as shown below:



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05(c). Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows:

Endurance limit stress = 225 MPa

Yield point stress = 300 MPa

The factor of safety based on yield point may be taken as 1.5. **(12 M)**

Candidates must not write on this margin

Given data:

$b = 120\ mm$, $F.S. = 1.5$,

$W_{max} = 250\ kN$, $W_{min} = 100\ kN$,

$\sigma_e = 225\ MPa$, $\sigma_y = 300\ MPa$,

Let, t = thickness of the plate in mm.

Area,

$$A = b \times t$$

$$= 120t\ mm^2$$

Mean load is given by,

$$W_m = \frac{W_{max} + W_{min}}{2}$$

$$= \frac{250 + 100}{2}$$

$$= 175\ kN$$

Mean stress,

$$\sigma_m = \frac{W_m}{A}$$

$$= \frac{175}{120t}\ kN/mm^2$$

Variable load is given by,

$$W_v = \frac{W_{max} - W_{min}}{2}$$

$$= 75 \text{ kN}$$

Mean variable stress,

$$\sigma_v = \frac{W_v}{A}$$

$$= \frac{75}{120t} \text{ kN/mm}^2$$

According to Soderberg's formula,

$$\frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{F.S.}$$

$$\therefore \frac{175 \times 10^3}{120t \times 300} + \frac{75 \times 10^3}{120t \times 225} = \frac{1}{1.5}$$

$$\boxed{\therefore t = 11.46 \text{ mm}}$$

Candidates must not write on this margin

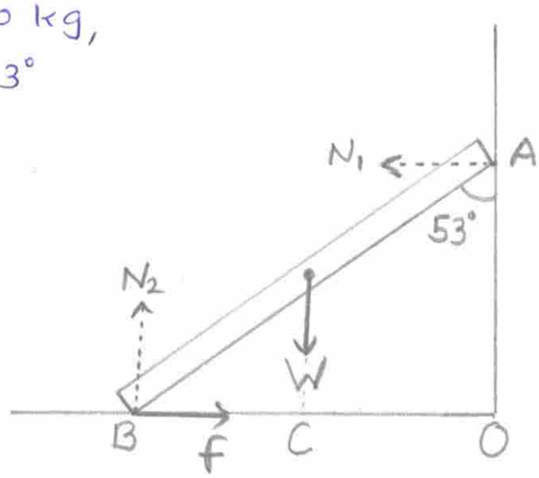
05(d). A uniform ladder of mass 10 kg leans against a smooth vertical wall making an angle of 53° with it. The other end rests on a rough horizontal floor. Find the normal force and the frictional force that the floor exerts on the ladder. **(12 M)**

Candidates must not write on this margin

Given data :

$m = 10$ kg,

$\theta = 53°$



The forces acting on the ladder are shown in the figure above. They are,

(a) Its weight, W

(b) Normal force N_1 by the vertical wall

(c) Normal force N_2 by the floor and

(d) Frictional force f by the floor.

Taking horizontal and vertical components,

$$N_1 = f \quad \text{——} \quad (I)$$

and $$N_2 = W \quad \text{——} \quad (II)$$

Taking torque about B,

$$N_1 \times OA = W \times CB$$

$$\therefore \quad N_1 \times AB \cos 53° = W \times \frac{AB}{2} \times \sin 53°$$

$$\therefore \quad N_1 = \frac{W}{2} \tan 53° \quad \text{——} \quad (III)$$

From equation (II), the normal force by the floor is,

$$N_2 = W$$
$$= mg$$
$$= 10 \times 9.81$$

$$\boxed{\therefore N_2 = 98.1 \text{ N}}$$

From, equations (I) and (III), the frictional force,

$$f = N_1$$
$$= \frac{W}{2} \tan 53^\circ$$
$$= \frac{98.1}{2} \tan 53^\circ$$

$$\boxed{\therefore f = 65.1 \text{ N}}$$

Candidates must not write on this margin

05(e). An antifriction bearing used in a gear box has a specification 7205. What do the numbers signify in the given specification ? **(12 M)**

Candidates must not write on this margin

Specification 7205:

Significance of 7:

- The number represents the type of anti-friction bearing.
- Thus, 7 represents angular contact ball bearing.

Significance of 2:

- The next number signifies the series of bearing.
- Thus, 2 indicates light series.

Significance of 05:

- The number is multiplied with 5 to give the bore diameter of the bearing.

  Thus, 05 x 5 = 25 mm

- Therefore, diameter of bore is 25 mm.

06(a). At a point on the surface of a machine the material is in biaxial stress with $\sigma_x = 32$ MPa, and $\sigma_y = -50$ MPa as shown in figure (a). Figure (b) shows an inclined plane *aa* cut through the same point in the material but oriented at an angle θ.

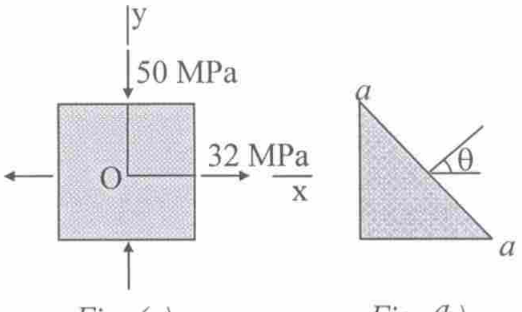


Fig. (a) *Fig. (b)*

(i) Determine the value of the angle θ between zero and 90° such that no normal stress acts on plane *aa*.

(ii) Sketch a stress element having plane *aa* as one of its sides and show all stresses acting on the element. **(10 + 10 = 20 M)**

Given data:

$\sigma_x = 32$ MPa,

$\sigma_y = -50$ MPa,

$\tau_{xy} = 0$

(i) Normal stress at any angle θ is given by,

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Given that, $\sigma_\theta = 0$

$$\therefore 0 = \frac{32 - 50}{2} + \frac{32 + 50}{2} \cos 2\theta + 0$$

$$\therefore \cos 2\theta = \frac{9}{41}$$

$$\boxed{\therefore \theta = 38.66^\circ} \quad (\because 0^\circ < \theta < 90^\circ)$$

(ii) For the stress element which is at an angle $\theta = 38.66^\circ$, shear stress is given by,

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{32 + 50}{2} \sin (2 \times 38.66^\circ) - 0$$

$$= 40 \text{ MPa}$$

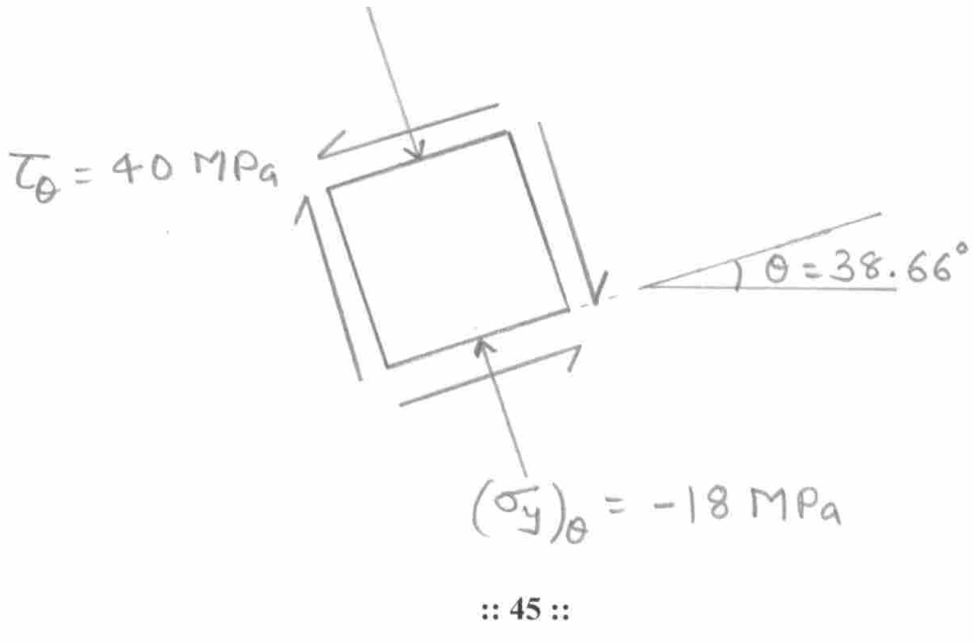
Also, stress invarient,

$$I_1 = \sigma_x + \sigma_y = (\sigma_x)_\theta + (\sigma_y)_\theta$$

$$\therefore 32 - 50 = 0 + (\sigma_y)_\theta$$

$$\therefore (\sigma_y)_\theta = -18 \text{ MPa}$$

The stress element having plane aa as one of its side is shown below:



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06(b).

(i). A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 r.p.m. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the fluctuation of speed is not to exceed 1 percent and the maximum centrifugal stress in the flywheel is to be 5.5 MPa, determine, the mean diameter and the cross-sectional area of the rim.

(The material of the rim has a density of 7200 kg/m³) **(16 M)**

Given data:

$P = 75 \text{ kW}$, $N = 360 \text{ rpm}$,

$\Delta E = 0.9E$, $C_s = 1\%$,

$\sigma_t = 5.5 \text{ MPa}$, $\rho = 7200 \text{ kg/m}^3$

Mean angular speed,

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 360}{60}$$

$$= 37.7 \text{ rad/s}$$

Now, the centrifugal stress,

$$\sigma_t = \rho R^2 \omega^2$$

$$\therefore 5.5 \times 10^6 = 7200 \times R^2 \times (37.7)^2$$

$$\therefore R = 0.733 \, m$$

$$\boxed{\therefore D = 1.466 \, m}$$

Workdone by the flywheel per cycle,

$$E = \frac{P \times 60}{n}$$

$$= \frac{75 \times 10^3 \times 60}{\left(\frac{360}{2}\right)} \quad \left(\because n = \frac{N}{2} \text{ for four stock cycle}\right)$$

$$= 25 \text{ kN.m}$$

Given that, $\Delta E = 0.9 \, E$

$$= 22.5 \text{ kN.m}$$

Also, $\Delta E = m R^2 \omega^2 C_s$

$$\therefore 22.5 \times 10^3 = m \times (0.733)^2 \times (37.7)^2 \times 0.01$$

$$\therefore m = 2946.4 \text{ kg}$$

We know that,

$$m = A \times \pi D \times \rho$$

$$\therefore 2946.4 = A \times \pi \times 1.466 \times 7200$$

$$\boxed{\therefore A = 0.09 \, m^2}$$

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06(b).

(ii). What is cooling stress and how does it induce in flywheel? **(4 M)**

- The stresses developed in casting process due to unequal rate of cooling is called cooling stress.
- In large flywheel there is heavy concentration of mass at rim and hub. It results in unequal cooling rates at rim, the hub and the arms.
- Large stresses are induced in the arms due to the compressive force generated by rim and hub. This resulting stress in arms is due to cooling stress.

06(c).

(i). An engine developing 22 kW at 1000 r.p.m. is fitted with a cone clutch having mean diameter of 300 mm. The cone has a face angle of 12°. If the normal pressure on the clutch face is not to exceed 0.07 N/mm^2 and the coefficient of friction is 0.2, determine the face width of the clutch, and the axial spring force necessary to engage the clutch.

(16 M)

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Given data:

$P = 22$ kW, $N = 1000$ rpm

$D_m = 300$ mm, $\alpha = 12^\circ$

$P_{max} = 0.07$ N/mm^2, $\mu = 0.2$

We know that, Power,

$$P = \frac{2\pi N T}{60}$$

$$\therefore T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 22 \times 10^3}{2\pi \times 1000}$$

$$= 210.08 \text{ N.m}$$

By assuming uniform wear theory,

$$T = \frac{\mu W R_m}{\sin\alpha}$$

Here, $W = 2\pi C (R_2 - R_1)$

where, $C = P_{max} \times R_1$

$$\therefore T = \frac{\mu \times 2\pi \times P_{max} \times R_1 \times R_m \times (R_2 - R_1)}{\sin\alpha}$$

$$\therefore \frac{210.08}{R_2 - R_1} = \frac{0.2 \times 2\pi \times 0.07 \times 10^6 \times R_1 \times 0.15}{\sin 12^\circ}$$

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$$\therefore R_1 (R_2 - R_1) = 3.31 \times 10^{-3}$$

$$\therefore R_1 (0.3 - 2R_1) = 3.31 \times 10^{-3}$$

$$\therefore 2R_1^2 - 0.3 R_1 + 3.31 \times 10^{-3} = 0$$

$$\therefore R_1 = 0.138 \text{ m}$$

$$\therefore R_2 = 0.3 - 0.138$$

$$= 0.162 \text{ m}$$

Now, face width,

$$b = \frac{R_2 - R_1}{\sin \alpha}$$

$$= \frac{0.162 - 0.138}{\sin 12^\circ}$$

$$\boxed{\therefore b = 0.115 \text{ m}}$$

Axial spring force,

$$W = 2\pi c (R_2 - R_1)$$

$$= 2\pi P_{max} \times R_1 (R_2 - R_1)$$

$$= 2\pi \times 0.07 \times 10^6 \times 0.138 \times 0.024$$

$$\boxed{\therefore W = 1456.69 \text{ N}}$$

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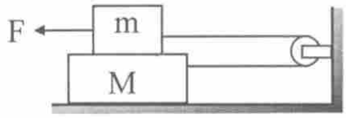
06(c).

(ii). Why are clutches usually designed on the basis of uniform wear? **(4 M)**

- Uniform pressure theory is applicable to only new clutches. When the clutch is new, it has more friction surface which has higher torque capacity.
- Uniform wear assumption gives a lower torque capacity clutch than uniform pressure. Hence, uniform wear theory is used so that clutch will not slip when it becoms old.

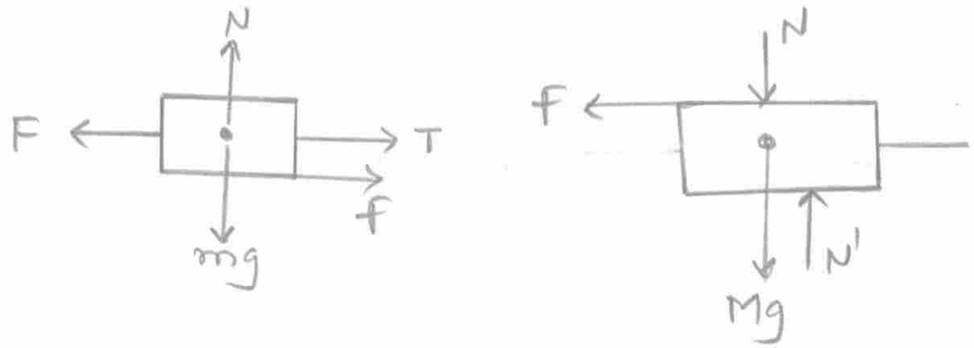
07(a).

(i). The friction coefficient between the two blocks shown in figure is μ but the floor is smooth. What maximum horizontal force F can be applied without disturbing the equilibrium of the system ?



(10 M)

Free body diagrams of blocks of masses 'M' and 'm' are drawn below:



For maximum value of 'F', frictional force 'f' is maximum. So, $f = \mu N$

For equilibrium of upper block of mass 'm',

$$T + f = F \quad \text{—— (I)}$$

$$N = mg \quad \text{—— (II)}$$

Similarly for equilibrium of lower block of mass 'M',

$$T = f \quad \text{—— (III)}$$

$$N' = N + Mg = mg + Mg = (m+M)g \quad \text{—— (IV)}$$

$$f = \mu N \quad \text{—— (V)}$$

From equations (I), (II), (III), (IV) and (V),

$$\boxed{F = 2\mu mg}$$

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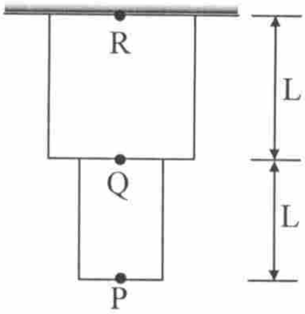
07(a).

(ii). A composite bar made up of steel (Modulus of elasticity = E) is hanging freely under its own weight as shown in the figure below. Self weight and cross sectional area of bar RQ are 2W and 2A respectively while those of bar PQ, are W and A respectively. Considering self weights of the bars, determine the displacement of point P.



(10 M)

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Given data:

$W_1 = W,$ $\quad W_2 = 2W,$

$A_1 = A,$ $\quad A_2 = 2A,$

$E_1 = E,$ $\quad E_2 = E$

Displacement of point P is given by,

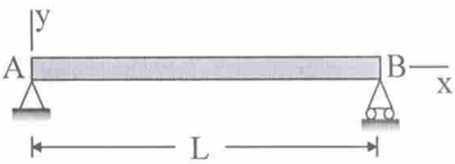
$$\delta_P = (\delta_{P_1})_{\text{Due to self weight of QP}} + (\delta_{P_2})_{\text{Due to self weight of RQ}} + (\delta_{P_3})_{\text{Due to external load (weight of bar PQ)}}$$

$$= \frac{W_1 L_1}{2A_1 E_1} + \frac{W_2 L_2}{2A_2 E_2} + \frac{W_1 L_2}{A_2 E_2}$$

$$= \frac{WL}{2AE} + \frac{(2W)L}{2(2A)E} + \frac{WL}{(2A)E}$$

$$\boxed{\therefore \delta_P = \frac{3WL}{2AE}}$$

07(b). The deflection curve for a simple beam AB as shown in the figure below, is given by the following equation: $v = -\frac{w_o L^4}{\pi^4 EI} \sin \frac{\pi x}{L}$.



(i) Describe the load acting on the beam.

(ii) Determine the reactions at the supports.

(iii) Determine the maximum bending moment M_{max}. **(8 + 6 + 6 = 20 M)**

Given data:

$$v = -\frac{w_o L^4}{\pi^4 EI} \sin\left(\frac{\pi x}{L}\right) \quad \text{——— (I)}$$

(i) The load intensity acting on the beam is given by,

$$w = -EI\, v''''$$

By using equation (I),

$$v' = -\frac{w_o L^4}{\pi^4 EI} \cos\left(\frac{\pi x}{L}\right) \times \frac{\pi}{L}$$

$$\therefore v'' = \frac{w_o L^4}{\pi^4 EI} \sin\left(\frac{\pi x}{L}\right) \times \frac{\pi^2}{L^2} \quad \text{——— (II)}$$

$$\therefore v''' = \frac{w_o L^4}{\pi^4 EI} \cos\left(\frac{\pi x}{L}\right) \times \frac{\pi^3}{L^3} \quad \text{——— (III)}$$

$$\therefore v'''' = -\frac{w_o L^4}{\pi^4 EI} \sin\left(\frac{\pi x}{L}\right) \times \frac{\pi^4}{L^4}$$

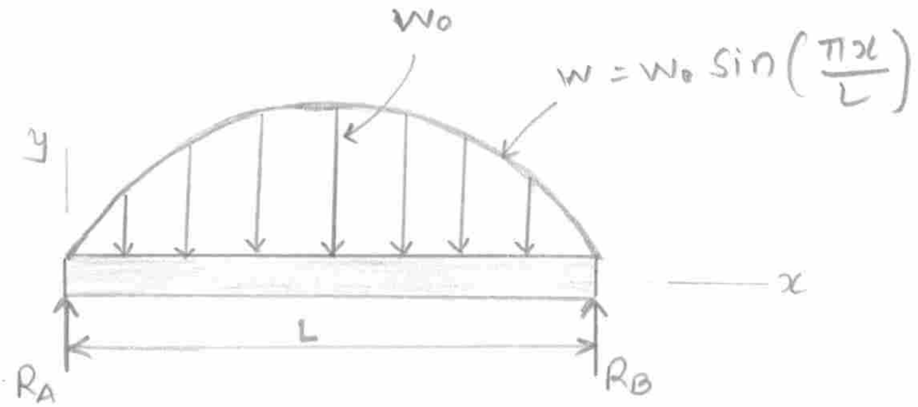
$$= -\frac{w_o}{EI} \sin\left(\frac{\pi x}{L}\right) \quad \text{——— (IV)}$$

Now, the load intensity is given by,

$$w = -EIv''''$$

$$\boxed{\therefore w = w_0 \sin\left(\frac{\pi x}{L}\right)} \quad \text{——— (V)}$$

Thus, load intensity is in the form of sinusoidal curve as shown below:



(ii) By using equation (III), shear force at any section can be given by,

$$F = EIv'''$$

$$= \frac{w_0 L}{\pi} \cos\left(\frac{\pi x}{L}\right)$$

At $x = 0$, $F = R_A$

$$\boxed{\therefore R_A = \frac{w_0 L}{\pi}}$$

At $x = L$, $F = -R_B$

$$\boxed{\therefore R_B = \frac{w_0 L}{\pi}}$$

(iii) By using equation (II), bending moment at any section is given by,

$$M = EIv''$$

$$= \frac{W_0 L^2}{\pi^2} \sin\left(\frac{\pi x}{L}\right)$$

At $x = \frac{L}{2}$, $M = M_{max}$

$$\therefore M_{max} = \frac{W_0 L^2}{\pi^2} \sin\left(\frac{\pi (L/2)}{L}\right)$$

$$\boxed{\therefore M_{max} = \frac{W_0 L^2}{\pi^2}}$$

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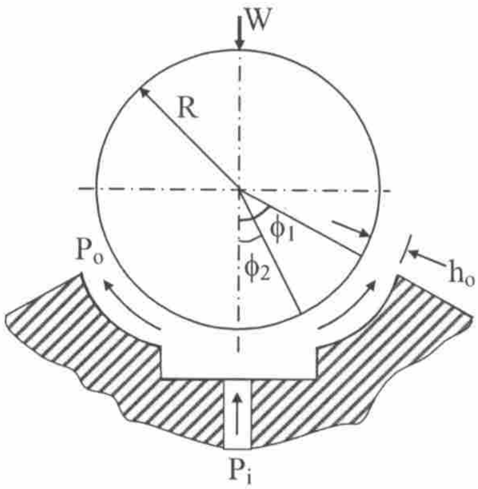
07(c). A hydrostatic spherical step bearing is shown in figure.



Show that the flow requirement is given by :

$$Q = \frac{\pi P_i h_o^3}{6\mu \ln\left[\dfrac{\tan\left(\dfrac{\phi_1}{2}\right)}{\tan\left(\dfrac{\phi_2}{2}\right)}\right]}$$

(20 M)

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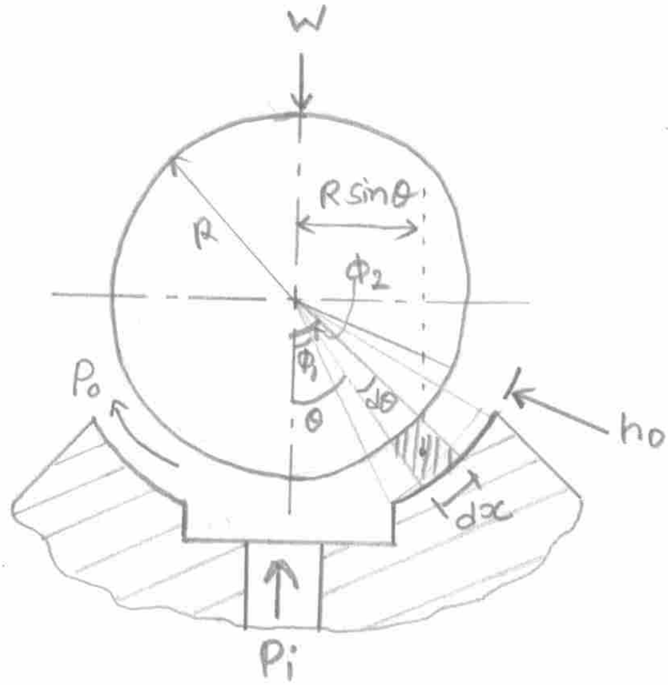
Consider an element ring of length dx and thickness ho at an angle θ as shown below.



For this element, the flow of lubricant through this ring is given by,

$$Q = \frac{\Delta P b h^3}{12 \mu L}$$

Where, $L = dx = R \cdot d\theta$,

$b = 2\pi R_y = 2\pi R \sin\theta$,

$h = h_o$

$$\therefore Q = \frac{-dP \times 2\pi R \sin\theta \times h_0^3}{12 \times \mu \times R d\theta}$$

The negative sign is introduced in equation, because pressure decreases as θ increases.

$$\therefore dp = \frac{-6Q\mu \, d\theta}{\pi h_0^3 \sin\theta}$$

By integrating the above equation,

$$P = -\frac{6Q\mu}{\pi h_0^3} \int \operatorname{cosec}\theta \, d\theta$$

here, $\int \operatorname{cosec}\theta \, d\theta = -\ln(\operatorname{cosec}\theta + \cot\theta)$

$$= -\ln\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$= -\ln\left(\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= -\ln\left(\cot\frac{\theta}{2}\right)$$

$$= \ln\left(\tan\frac{\theta}{2}\right)$$

$$\therefore P = -\frac{6Q\mu}{\pi h_0^3} \ln\left(\tan\left(\frac{\theta}{2}\right)\right) + C \quad \text{——(I)}$$

At $\theta = \phi_1$, $P = 0$

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$$\therefore \quad C = \frac{6Q\mu}{\pi h_0^3} \ln\left(\tan\left(\frac{\phi_1}{2}\right)\right) \quad \text{— (II)}$$

Also, at $\theta = \phi_2$, $P = P_i$

$$\therefore P_i = -\frac{6Q\mu}{\pi h_0^3} \ln\left(\tan\left(\frac{\phi_2}{2}\right)\right) + C \quad \text{— (III)}$$

By using equations (II) and (III),

$$P_i = -\frac{6Q\mu}{\pi h_0^3} \ln\left(\tan\left(\frac{\phi_2}{2}\right)\right) + \frac{6Q\mu}{\pi h_0^3} \ln\left(\tan\left(\frac{\phi_1}{2}\right)\right)$$

$$\therefore P_i = \frac{6Q\mu}{\pi h_0^3} \ln\left[\frac{\tan\left(\frac{\phi_1}{2}\right)}{\tan\left(\frac{\phi_2}{2}\right)}\right]$$

$$\boxed{\therefore Q = \frac{\pi P_i h_0^3}{6\mu \ln\left[\frac{\tan\left(\frac{\phi_1}{2}\right)}{\tan\left(\frac{\phi_2}{2}\right)}\right]}}$$

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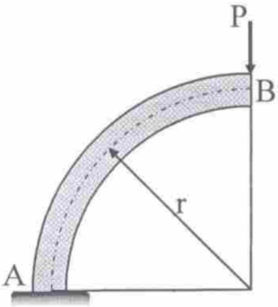
08(a). The curved rod AB has a diameter d. Determine the vertical displacement of end B of the rod. The rod is made of material having a modulus of elasticity of E.
(*Consider the displacement due to bending only*)



(20 M)

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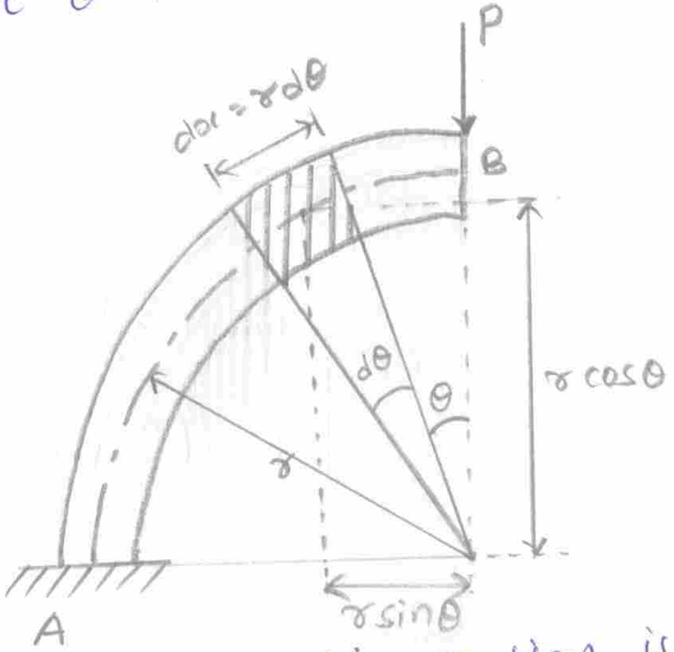
Consider the section of the beam which is at an angle θ as shown below:



Bending moment at this section is given by,

$$M_\theta = P \times r\sin\theta$$

strain energy stored in the beam is given by,

$$U = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \frac{M_\theta^2 \, dx}{2EI}$$

$$= \int_0^{\pi/2} \frac{(Pr\sin\theta)^2 \, r \, d\theta}{2EI}$$

$$= \frac{P^2 r^3}{2EI} \int_0^{\pi/2} \sin^2\theta \, d\theta$$

here, $\int_0^{\pi/2} \sin^2\theta \, d\theta = \int_0^{\pi/2} \left(\frac{1-\cos 2\theta}{2} \right) d\theta$

$$= \frac{1}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{2} - 0 \right) - (0-0) \right]$$

$$= \frac{\pi}{4}$$

$$\therefore U = \frac{P^2 r^3}{2EI} \times \frac{\pi}{4}$$

$$= \frac{\pi P^2 r^3}{8EI}$$

$$= \frac{8 P^2 r^3}{Ed^4} \qquad \left(\because I = \frac{\pi}{64} d^4 \right)$$

By using Castigliano's theorem, displacement of point B is given by,

$$\delta_B = \frac{\partial U}{\partial P}$$

$$\boxed{\therefore \delta_B = \frac{16 P r^3}{Ed^4}}$$

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Candidates must not write on this margin

08(b). Derive an expression for the distortion energy per unit volume for a body subjected to a uniform stress state, given by the principal stresses σ_1 and σ_2 with the principal stress σ_3 being zero. **(20 M)**

Given data:

$\sigma_3 = 0$

We know that, total strain energy can be considered as the sum of two parts, one part representing the energy needed to cause a volume change of the element with no change in shape and the other part representing the energy needed to distort the element.

$$U_{total} = U_v + U_d \quad \text{———} \quad (I)$$

here, $U_{total} = \frac{1}{2}\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3$

$$= \frac{1}{2}\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 \quad [\because \sigma_3 = 0]$$

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We know that,

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1\right]$$

$$\varepsilon_3 = \frac{1}{E}\left[-\mu(\sigma_1+\sigma_2)\right]$$

$$\therefore U_{total} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \quad \text{—— (II)}$$

Now, $U_v = \frac{1}{2} \times$ Average stress $\times$ Volumetric strain

$$= \frac{1}{2} \times \left(\frac{\sigma_1+\sigma_2+0}{3}\right) \times \frac{(1-2\mu)(\sigma_1+\sigma_2)}{E}$$

$$\therefore U_v = \frac{(1-2\mu)(\sigma_1+\sigma_2)^2}{6E} \quad \text{—— (III)}$$

By using equations (I), (II) and (III),

$$U_d = \frac{1}{2E}(\sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2) - \frac{(1-2\mu)}{6E}(\sigma_1+\sigma_2)^2$$

$$= \frac{1}{6E}(3\sigma_1^2+3\sigma_2^2-6\mu\sigma_1\sigma_2) - \frac{(1-2\mu)}{6E}(\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2)$$

$$= \frac{1}{6E}\left[2(\sigma_1^2+\sigma_2^2) - 2\sigma_1\sigma_2 + 2\mu(\sigma_1^2+\sigma_2^2) - 2\mu\sigma_1\sigma_2\right]$$

$$= \frac{1}{3E}(1+\mu)\left[(\sigma_1^2+\sigma_2^2) - 2\sigma_1\sigma_2\right]$$

$$= \frac{1+\mu}{6E}\left[2(\sigma_1^2+\sigma_2^2) - 2\sigma_1\sigma_2\right]$$

$$\boxed{\therefore U_d = \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2)^2+(\sigma_1)^2\right]}$$

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08(c).

(i). A steel pinion with 20° full depth involute teeth is transmitting 7.5 kW power at 1000 rpm from an electric motor. The starting torque of the motor is twice the rated torque. The number of teeth on the pinion is 25, while the module is 4 mm. The face width is 45 mm. Assuming that velocity factor accounts for the dynamic load, calculate the effective load on the gear tooth and the bending stresses in the gear tooth.

(Take value of service factor as 2 and use the table given below for Lewis form factor value)

Number of teeth	Lewis form factor
25	0.340
45	0.399

(16 M)

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Given data:

$P = 7.5 \text{ kW}$, $N_p = 1000 \text{ rpm}$,

$Z_p = 25$, $m = 4 \text{ mm}$,

$b = 45 \text{ mm}$, $C_s = 2$

Power, $P = \dfrac{2\pi N T}{60}$

$$\therefore 7.5 \times 10^6 = \frac{2\pi \times 1000 \times T}{60}$$

$$\therefore T = 71619.72 \text{ N.mm}$$

Torque transmitted by pinion,

$$T = P_t \times \frac{d_P}{2}$$

$$\therefore 71619.72 = P_t \times \frac{4 \times 25}{2}$$

$(\because d_P = m \times Z_P)$

$\therefore P_t = 1432.39$ N

Now, pitch line velocity,

$$v = \frac{\pi d_p N_p}{60}$$

$$= \frac{\pi \times 4 \times 25 \times 1000}{60}$$

$$= 5235.99 \text{ mm/sec}$$

here, v < 10 m/s. so, velocity factor,

$$C_v = \frac{3}{3+v}$$

$$= \frac{3}{3+5.24}$$

$$= 0.364$$

Now, effective load on gear tooth,

$$P_{eff} = \frac{C_s \times P_t}{C_v}$$

$$= \frac{2 \times 1432.39}{0.364}$$

$$\boxed{\therefore P_{eff} = 7870.27 \text{ N}}$$

Beam strength of gear tooth is given by,

$$S_b = P_{eff} = m b \sigma_b Y$$

$$\therefore 7870.27 = 4 \times 45 \times \sigma_b \times 0.340$$

$$\boxed{\therefore \sigma_b = 128.6 \text{ MPa}}$$

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08(c).

(ii). Define the terms 'Pitting' and 'Scoring' in gear system? **(4 M)**

Pitting :

- It is a surface fatigue Failure, characterized by small pits on surface of a body.
- In gears, it occurs when the contact stresses between two meshing teeth exceed the surface endurance strength of the material.

Scoring :

- Scratch and tear of the material due to external load is called scoring.
- In gear system, due to inadequate lubrication, there is a metal to metal contact. As there is no lubrication, high is heat is generated. Later on, welding and tearing action resulting from metallic contact removes the metal.

SPACE FOR ROUGH WORK

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