



Design of Joint

S K Mondal

Theory at a glance (IES, IAS, GATE & PSU)

Cotters

- In machinery, the general term “**shaft**” refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc., and which is subjected to torsion and to transverse or axial loads acting singly or in combination.
- An “**axle**” is a non-rotating member that supports wheels, pulleys, and carries no torque.
- A “**spindle**” is a short shaft. Terms such as *line-shaft*, *head-shaft*, *stub shaft*, *transmission shaft*, *countershaft*, and *flexible shaft* are names associated with special usage.

A cotter is a flat wedge-shaped piece of steel as shown in figure below. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods.

Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.

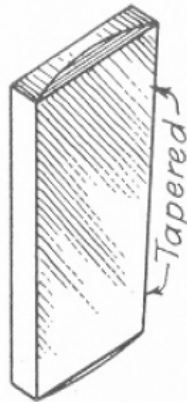


Figure- A typical cotter with a taper on one side only

A typical cotter joint is as shown in figure below. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e. $\alpha_1 + \alpha_2 < \beta_1 + \beta_2$ where α_1, α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and β_1, β_2 are the corresponding angles of friction. This also means that if taper is given on

one side only then $\alpha < \beta_1 + \beta_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.

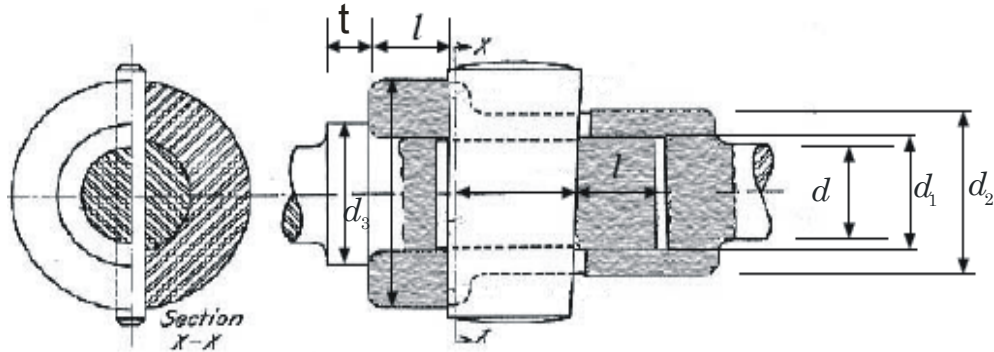


Figure- Cross-sectional views of a typical cotter joint

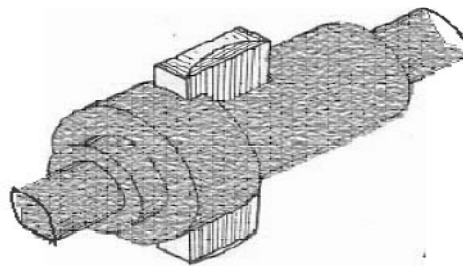


Figure- An isometric view of a typical cotter joint

Design of a cotter joint

If the allowable stresses in tension, compression and shear for the socket, rod and cotter be σ_t , σ_c and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

1. Tension failure of rod at diameter d

$$\frac{\pi}{4} d^2 \sigma_t = P$$

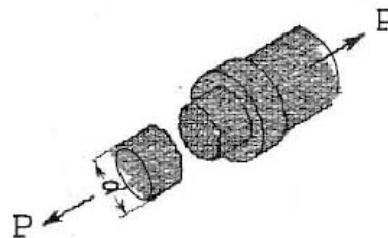


Fig. Tension failure of the rod

2. Tension failure of rod across slot

$$\left(\frac{\pi}{4} d_1^2 - d_1 t \right) \sigma_t = P$$

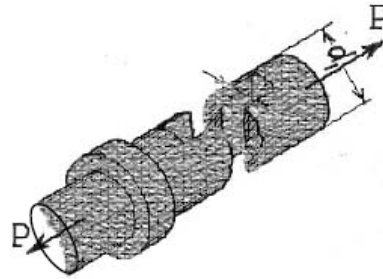


Fig. Tension failure of rod across slot

3. Tensile failure of socket across slot

$$\left(\frac{\pi}{4} (d_2^2 - d_1^2) - (d_2 - d_1) t \right) \sigma_t = P$$

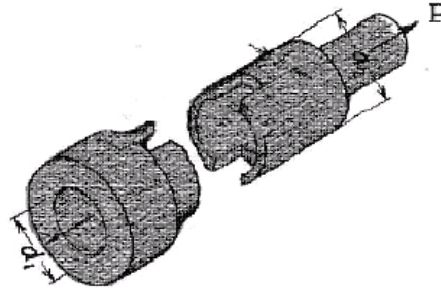


Fig. Tensile failure of socket across slot

4. Shear failure of cotter

$$2bt\tau = P$$

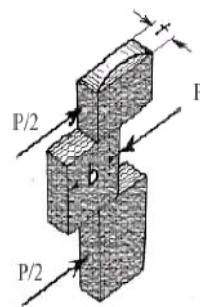


Fig. Shear failure of cotter

5. Shear failure of rod end

$$2l_1 d_1 \tau = P$$

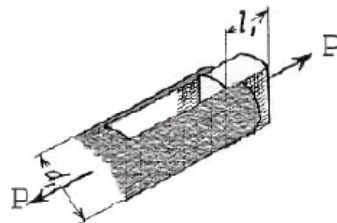


Fig. Shear failure of rod end

6. Shear failure of socket end

$$2l(d_3 - d_1) \tau = P$$

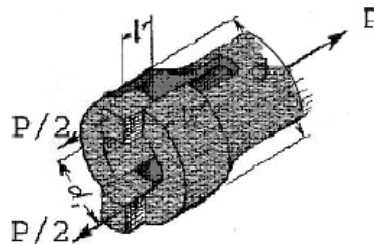


Fig. Shear failure of socket end

7. Crushing failure of rod or cotter

$$d_1 t \sigma_c = P$$

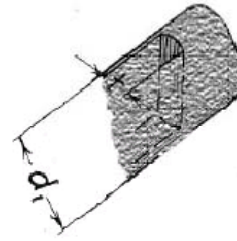


Fig. Crushing failure of rod or cotter

8. Crushing failure of socket or rod

$$(d_3 - d_1) t \sigma_c = P$$

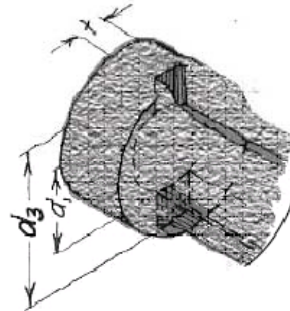


Fig. Crushing failure of socket or rod

9. Crushing failure of collar

$$\frac{\pi}{4} (d_4^2 - d_1^2) \sigma_c = P$$

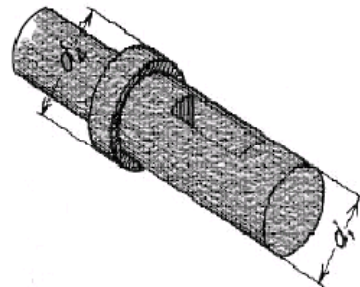


Fig. Crushing failure of collar

10. Shear failure of collar

$$\pi d_1 t_1 \tau = P$$

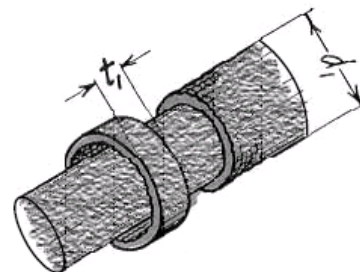
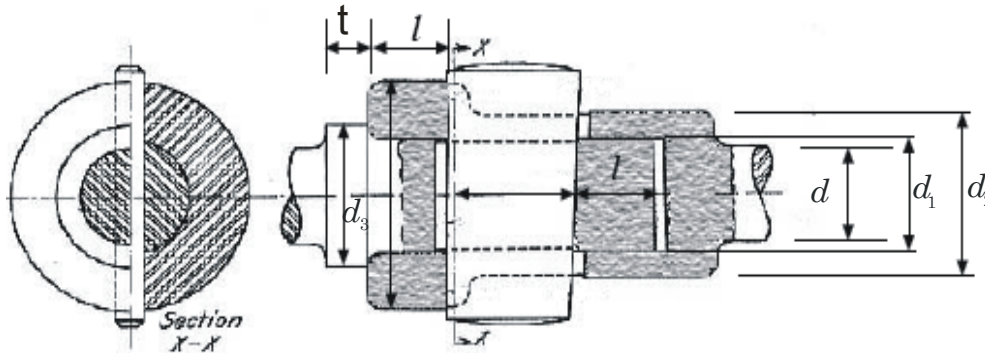


Fig. Shear failure of collar

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in figure below, we may approximate the loading as shown in figure below.



Axial load $(P) = \frac{\pi}{4} d^2 \sigma_y$. On substitution this gives $d=20$ mm. In general

Standard shaft size in mm is

6 mm to 22 mm diameter

25 mm to 60 mm diameter

60 mm to 110 mm diameter

110 mm to 140 mm diameter

140 mm to 160 mm diameter

500 mm to 600 mm diameter

2 mm in increment

5 mm in increment

10 mm in increment

15 mm in increment

20 mm in increment

30 mm in increment

We therefore choose a suitable rod size to be 25 mm.

Refer to figure

For tension failure across slot $\left(\frac{\pi}{4} d^2 - d_1 t \right) \sigma_y = P$. This gives

$d_1 t = 1.58 \times 10^{-4} \text{ m}^2$. From empirical relations we may take $t=0.4d$ i.e. 10 mm and this gives $d_1 = 15.8$ mm. Maintaining the proportion let $d_1 = 1.2 d = 30$ mm.

Refer to figure

The tensile failure of socket across slot $\left\{ \left(\frac{\pi}{4} d_2^2 - d_1^2 \right) - (d_2 - d_1) t \right\} \sigma_y = P$

This gives $d_2 = 37$ mm. Let $d_2 = 40$ mm

Refer to figure above

For shear failure of cotter $2bt\tau = P$. On substitution this gives $b = 22.72$ mm.

Let $b = 25$ mm.

Refer to figure

For shear failure of rod end $2l_1 d_1 \tau = P$ and this gives $l_1 = 7.57$ mm. Let $l_1 = 10$ mm.

Refer to figure

For shear failure of socket end $2l(d_2 - d_1)\tau = P$ and this gives $l = 22.72$ mm. Let $l = 25$ mm.

Refer to figure

For crushing failure of socket or rod $(d_3 - d_1)t\sigma_c = P$. This gives $d_3 = 75.5$ mm. Let $d_3 = 77$ mm.

Refer to figure

For crushing failure of collar $\frac{\pi}{4}(d_4^2 - d_1^2)\sigma_c = P$. On substitution this gives $d_4 = 38.4$ mm. Let $d_4 = 40$ mm.

Refer to figure

For shear failure of collar $\pi d_1 t_1 \tau = P$ which gives $t_1 = 4.8$ mm. Let $t_1 = 5$ mm.

Therefore the final chosen values of dimensions are

$d = 25$ mm; $d_1 = 30$ mm; $d_2 = 40$ mm; $d_3 = 77$ mm; $d_4 = 40$ mm; $t = 10$ mm; $t_1 = 5$ mm; $l = 25$ mm; $l_1 = 10$ mm; $b = 27$ mm.

Knuckle Joint

A knuckle joint (as shown in figure below) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.

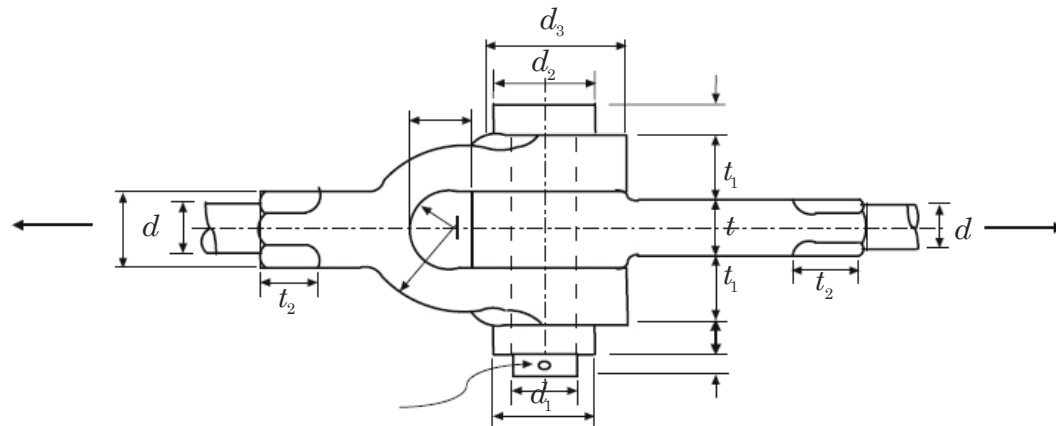


Figure- A typical knuckle joint

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and fork-end eyes and is secured by a collar and a split pin.

Normally, empirical relations are available to find different dimensions of the joint and they are safe from design point of view. The proportions are given in the figure above.

d = diameter of rod

$d_1 = d$	$t = 1.25d$
$d_2 = 2d$	$t_1 = 0.75d$
$d_3 = 1.5.d$	$t_2 = 0.5d$

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses are shown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σ_t , σ_c and τ .

1. Failure of rod in tension:

$$\frac{\pi}{4} d^2 \sigma_t = P$$

2. Failure of knuckle pin in double shear:

$$2 \frac{\pi}{4} d_1^2 \tau = P$$

3. Failure of knuckle pin in bending (if the pin is loose in the fork)

Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in figure below.

Equating the maximum bending stress to tensile or compressive yield stress we have

$$\sigma_t = \frac{16P \left(\frac{t_1}{3} + \frac{t}{4} \right)}{\pi d_1^3}$$

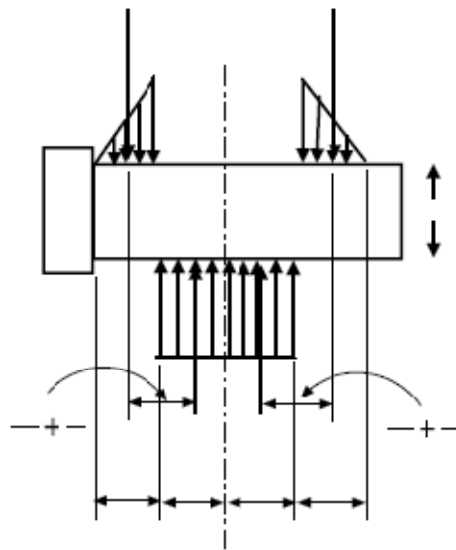


Figure- Bending of a knuckle pin

4. Failure of rod eye in shear:

$$(d_2 - d_1) \tau = P$$

5. Failure of rod eye in crushing:

$$d_1 t \sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1) t \sigma_t = P$$

7. Failure of forked end in shear:

$$2(d_2 - d_1) t_1 \tau = P$$

Design of Joint

S K Mondal's

Chapter 1

8. Failure of forked end in tension:

$$2(d_2 - d_1)t_1\sigma_t = P$$

9. Failure of forked end in crushing:

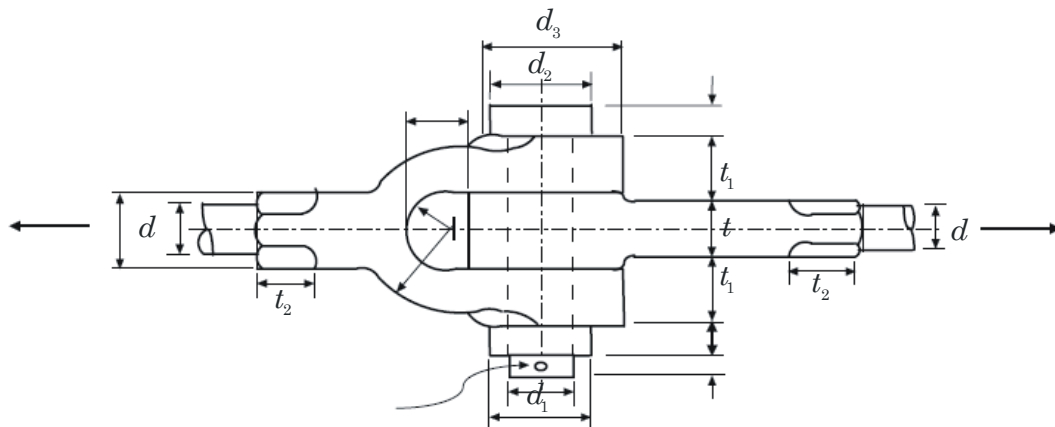
$$2d_1t_1\sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks.

For example using the 2nd equation we have $\tau = \frac{2P}{\pi d_1^2}$. We may now put value of d_1 from

empirical relation and then find **FACTOR OF SAFETY, (F.S.)** = $\frac{\tau_y}{\tau}$ which should be more than one.

Q. Two mild steel rods are connected by a knuckle joint to transmit an axial force of 100 kN. Design the joint completely assuming the working stresses for both the pin and rod materials to be 100 MPa in tension, 65 MPa in shear and 150 MPa in crushing.



Refer to figure above

For failure of rod in tension, $P = \frac{\pi}{4}d^2\sigma_y$. On substituting $P = 100$ kN, $\sigma_y = 100$ MPa we

have $d = 35.6$ mm. Let us choose the rod diameter $d = 40$ mm which is the next standard size.

We may now use the empirical relations to find the necessary dimensions and then check the failure criteria.

$$\begin{aligned} d_1 &= 40 \text{ mm} & t &= 50 \text{ mm} \\ d_2 &= 80 \text{ mm} & t_1 &= 30 \text{ mm;} \\ d_3 &= 60 \text{ mm} & t_2 &= 20 \text{ mm;} \end{aligned}$$

$$\text{Split pin diameter} = 0.25 d_1 = 10 \text{ mm}$$

To check the failure modes:

1. **Failure of knuckle pin in shear:** $P / \left(2 \cdot \frac{\pi}{4} d_1^2 \right) = \tau_y$, which gives $\tau_y = 39.8$

MPa. This is less than the yield shear stress.

2. **For failure of knuckle pin in bending:** $\sigma_y = \frac{16P \left(\frac{t_1}{3} + \frac{t}{4} \right)}{\pi d_1^3}$. On substitution

this gives $\sigma_y = 179$ MPa which is more than the allowable tensile yield stress of 100 MPa. We therefore increase the knuckle pin diameter to 55 mm which gives $\sigma_y = 69$ MPa that is well within the tensile yield stress.

3. **For failure of rod eye in shear:** $(d_2 - d_1)t\tau = P$. On substitution $d_1 = 55$ mm $\tau = 80$ MPa which exceeds the yield shear stress of 65 MPa. So d_2 should be at least 85.8 mm. Let d_2 be 90 mm.
4. **for failure of rod eye in crushing:** $d_1 t \sigma_c = P$ which gives $\sigma_c = 36.36$ MPa that is well within the crushing strength of 150 MPa.
5. **Failure of rod eye in tension:** $(d_2 - d_1)t\sigma_t = P$. Tensile stress developed at the rod eye is then $\sigma_t = 57.14$ MPa which is safe.
6. **Failure of forked end in shear:** $2(d_2 - d_1)t_1\tau = P$. Thus shear stress developed in the forked end is $\tau = 47.61$ MPa which is safe.
7. **Failure of forked end in tension:** $2(d_2 - d_1)t_1\sigma_y = P$. Tensile strength developed in the forked end is then $\sigma_y = 47.61$ MPa which is safe.
8. **Failure of forked end in crushing:** $2d_1t_1\sigma_c = P$ which gives the crushing stress developed in the forked end as $\sigma_c = 42$ MPa. This is well within the crushing strength of 150 MPa.

Therefore the final chosen values of dimensions are:

$d_1 = 55$ mm	$t = 50$ mm
$d_2 = 90$ mm	$t_1 = 30$ mm; and $d = 40$ mm
$d_3 = 60$ mm	$t_2 = 20$ mm;

Keys

In the assembly of pulley, key and shaft, Key is made the **Weakest** so that it is cheap and easy to replace in case of failure.

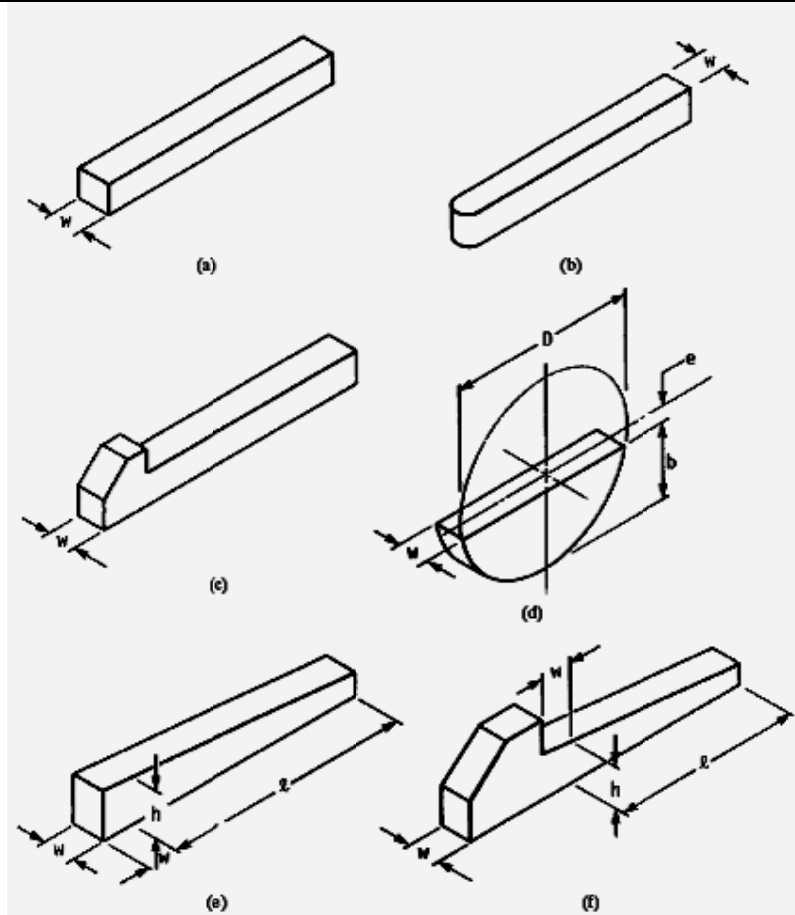


Figure- (a) Square or rectangular key. (b) Square or rectangular key with one end rounded; also available with both ends rounded. (c) Square or rectangular key with gib head. (d) Woodruff key; also available with flattened bottom. (e) Tapered rectangular key; ℓ = hub length, h = height; taper is 1/8 in for 12 in or 1 for 100 for metric sizes. (f) Tapered gib-head key; dimensions and taper same as in (e).

Rectangular and square key

1. **Rectangular sunk key:** A rectangular sunk key is shown in figure below. The usual proportions of this key are:

Width of key, (w)	$= d / 4$
Thickness of key, (t)	$= 2w / 3 = d / 6$

Where d = Diameter of the shaft or diameter of the hole in the hub.
The key has **taper 1 in 100** on the top side only.

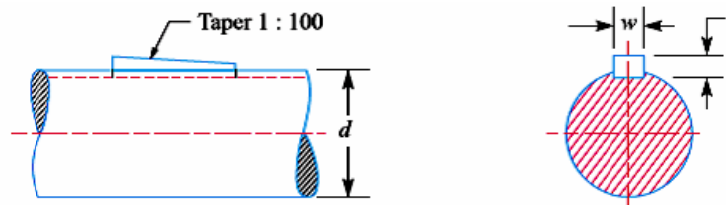


Figure-. Rectangular sunk key.

2. Square sunk key: The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. Gib-head key: It is a rectangular sunk key with a head at one end known as ***gib head***. It is usually provided to facilitate the removal of key. A gib head key is shown in figure below and its use in shown in figure below.

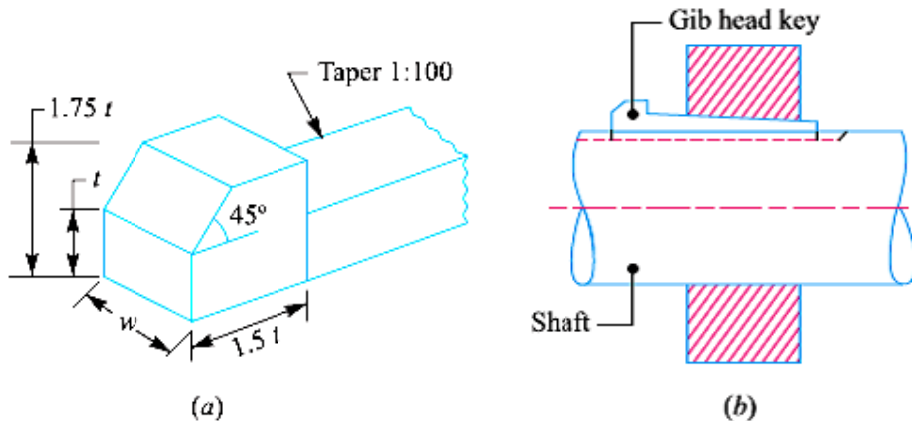


Figure- Gib-head key.

The usual proportions of the gib head key are:

Width, (w)	= $d / 4$
Thickness at large end, (t)	= $2w / 3 = d / 6$

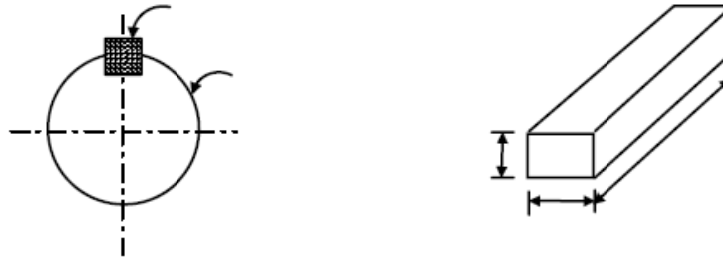
If a square key of sides $d/4$ is used then. In that case, for shear failure we have

$$\left(\frac{d}{4} \times l \right) \tau_x \frac{d}{2} = T$$

$$\text{or } \tau_x = \frac{8T}{ld^2} \quad [\text{Where } \tau_x \text{ the yield is stress in shear and } l \text{ is the key length.}]$$

Q. A heat treated steel shaft of tensile yield strength of 350 MPa has a diameter of 50 mm. The shaft rotates at 1000 rpm and transmits 100 kW through a gear. Select an appropriate key for the gear.

Solution: Consider a rectangular key of width (w), thickness (t) and length (L) as shown in figure below. The key may fail (a) in shear or (b) in crushing.



Figure

Shear failure: The failure criterion is
$$= \tau_y \cdot w \cdot L \cdot \frac{d}{2}$$

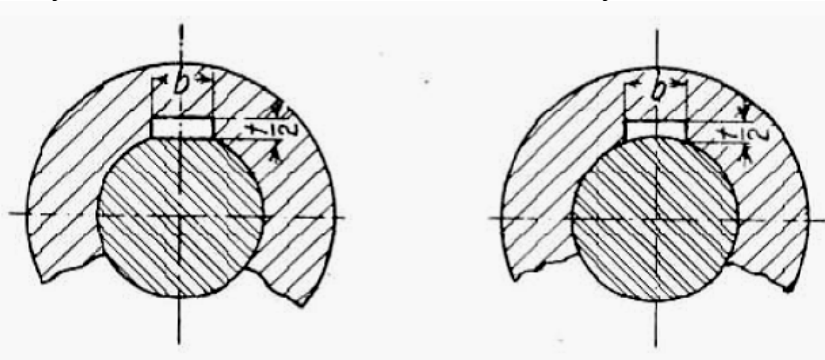
Where torque transmitted is,
$$(T) = \text{Power} / \left(\frac{2\pi N}{60} \right)$$

N being in rpm, w, L and d are the width, length and diameter of the shaft respectively and τ_y is the yield stress in shear of the key material. Taking τ_y to be half of the tensile yield stress and substituting the values in above equations and we have $wL = 2.19 \times 10^{-4} \text{ m}^2$.

Crushing failure
$$= \sigma_c \cdot \frac{t \cdot L}{2} \cdot \frac{d}{2}$$

Flat key

A **flat key**, as shown in figure below is **used for light load** because they depend entirely on friction for the grip. The sides of these keys are parallel but the top is slightly tapered for a tight fit. These keys have about half the thickness of sunk keys.



Flat key

Saddle key

Saddle key

A **saddle key**, shown in figure above, is very similar to a flat key except that the bottom side is concave to fit the shaft surface. These keys also have friction grip and therefore cannot be used for heavy loads. A simple pin can be used as a key to transmit large torques. Very little stress concentration occurs in the shaft in these cases. This is shown in figure above.

Tangent Key

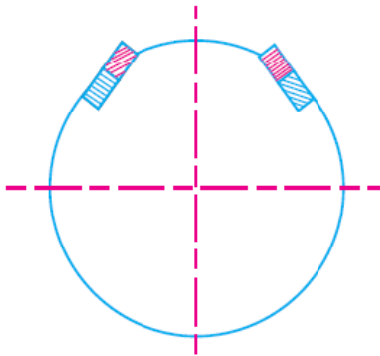


Figure- Tangent Key

Feather key

A feather key is used when one component slides over another. The key may be fastened either to the hub or the shaft and the keyway usually has a sliding fit.

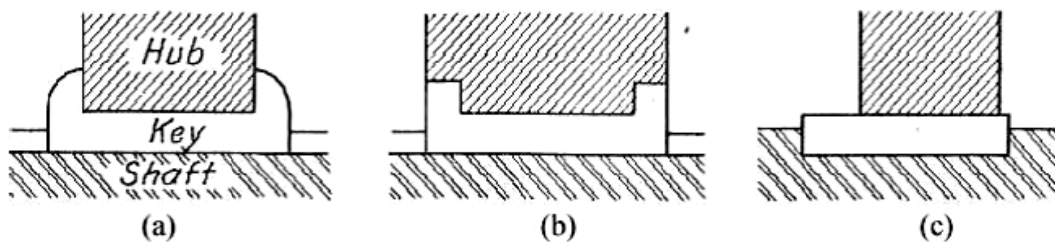


Figure- feather key

Woodruff key

A **woodruff key** is a form of sunk key where the key shape is that of a truncated disc, as shown in figure below. It is usually used for shafts less than about 60 mm diameter and the keyway is cut in the shaft using a milling cutter, as shown in the figure below. It is widely used in machine tools and automobiles due to the extra advantage derived from the extra depth.

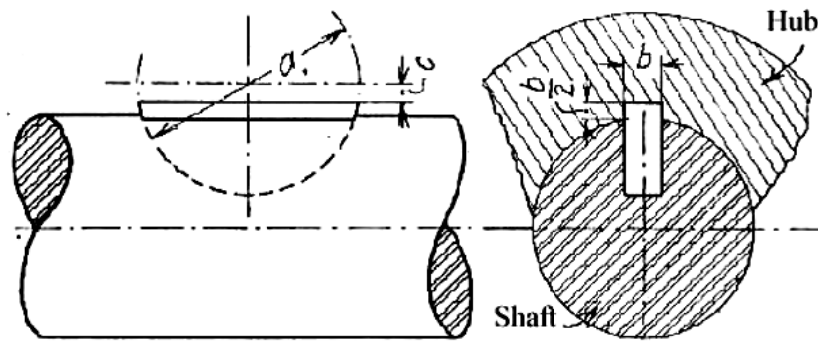


Figure- Woodruff key

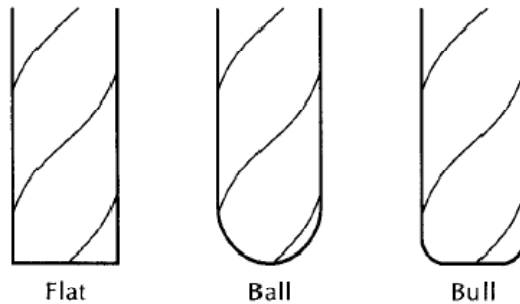
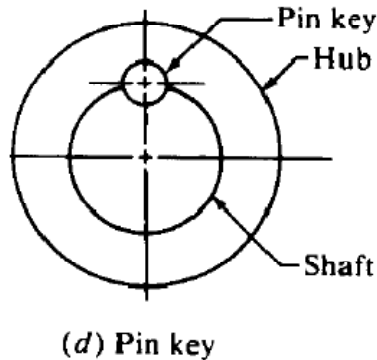
The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The main dis-advantages of a woodruff key are as follows:

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.

Circular (Pin) Keys

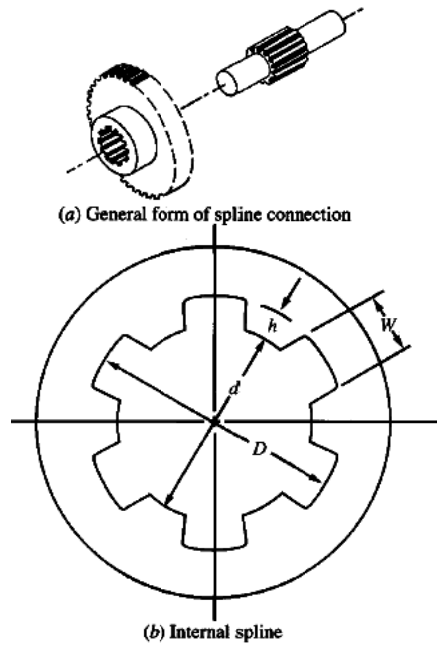


Significantly lower stress concentration factors result from this type of key as compared to parallel or tapered keys. A ball end mill can be used to make the circular key seat.

Splines

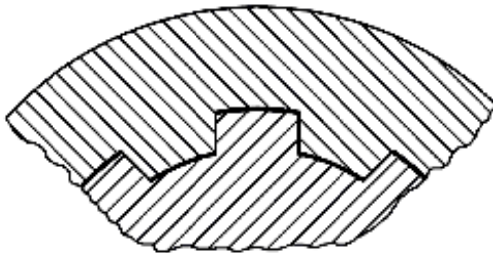
Splines are essentially stub by gear teeth formed on the outside of the shaft and on the inside of the hub of the load-transmitting component. Splines are generally much more expensive to manufacture than keys, and are usually not necessary for simple torque transmission. **They are typically used to transfer high torques.** One feature of a spline is that it can be made with a reasonably loose slip fit to allow for large axial motion between the shaft and component while still transmitting torque. This is useful for connecting two shafts where relative motion between them is common, such as in connecting a power takeoff (PTO) shaft of a tractor to an implement.

Stress concentration factors are greatest where the spline ends and blends into the shaft, but are generally quite moderate.

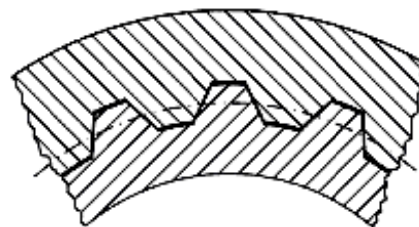


- Splines can be thought of as a series of axial keyways with mating keys machined onto a shaft.
- There are two major types of splines used in industry: 1) straight-sided splines, and 2) involute splines.
- Splines provide a more uniform circumferential transfer of torque to the shaft than a key.

Straight-sided spline

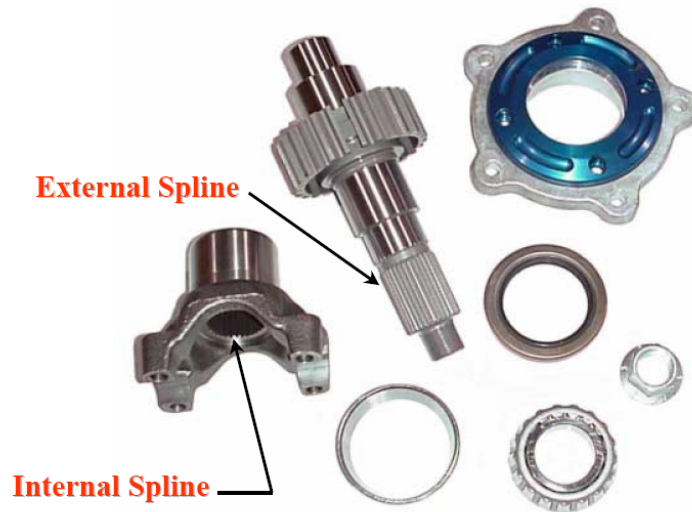


Involute spline



Splines.

Splined Shaft and Hub



Spline Manufacturing Methods

Splines are either “cut” (machined) or rolled. Rolled splines are stronger than cut splines due to the cold working of the metal. Nitriding is common to achieve very hard surfaces which reduce wear.

Welded joints

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium *e.g.* to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Types of welded joints:

Welded joints are primarily of **two kinds**:

(a) Lap or fillet joint

Obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints (see figure below).

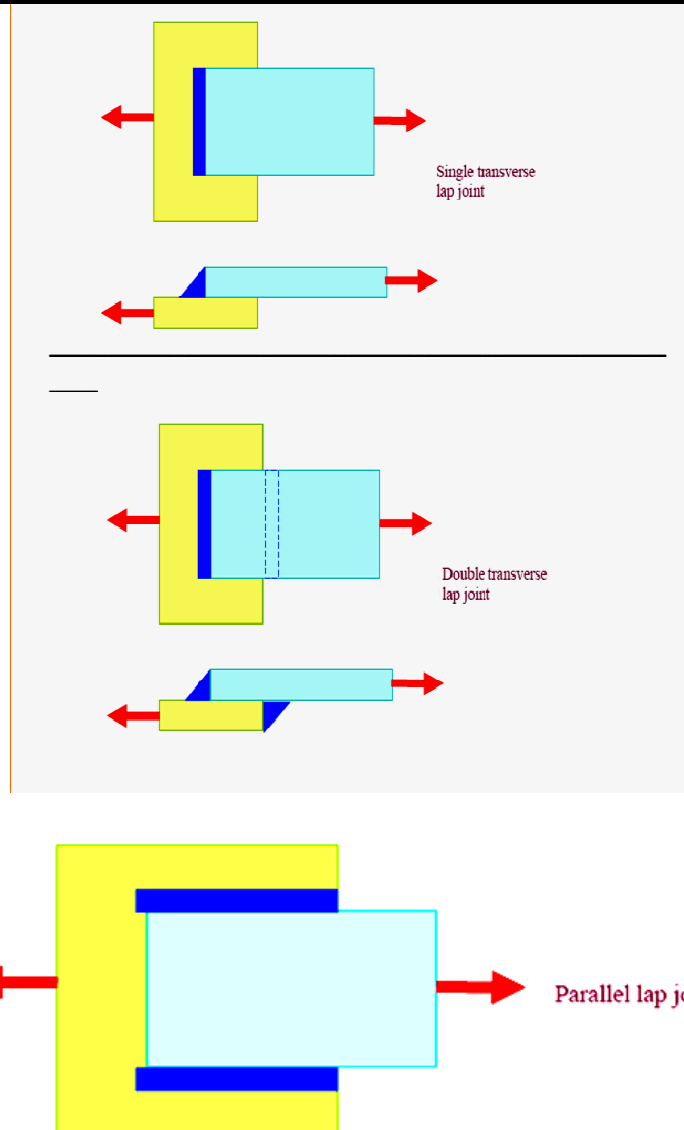


Figure- Different types of lap joints

(b) Butt Joints

Formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g.

- Square butt joint
- Single V-butt joint, double V-butt joint
- Single U-butt joint, double U-butt joint
- Single J-butt joint, double J-butt joint
- Single bevel-butt joint, double bevel butt joint.

These are schematically shown in figure below

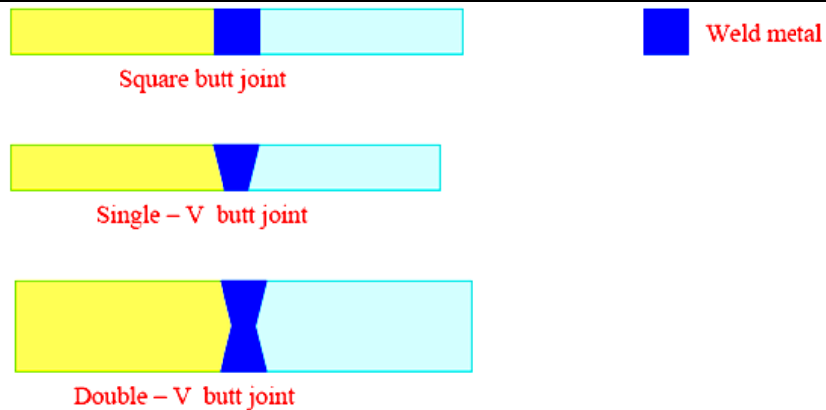
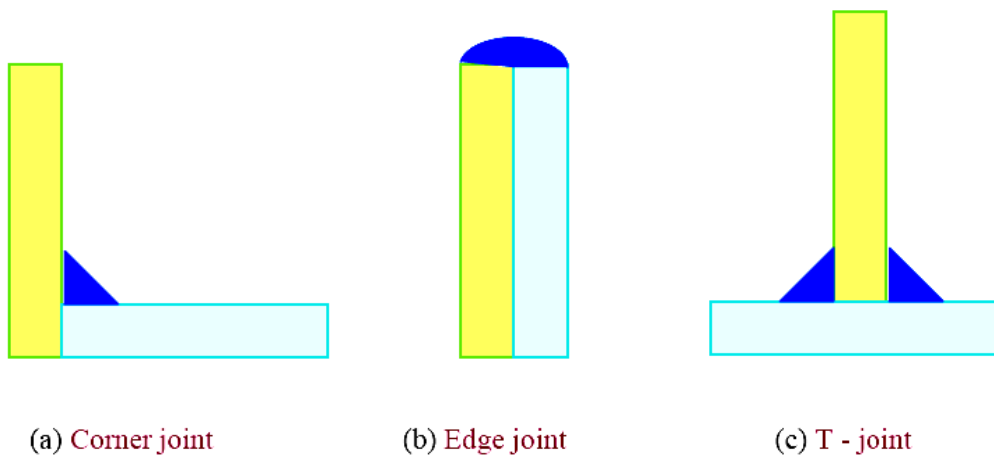


Figure- Different types of butt joints

There are other types of welded joints, for example,

- Corner joint (see figure below)
- Edge or seal joint (see figure below)
- T-joint (see figure below)

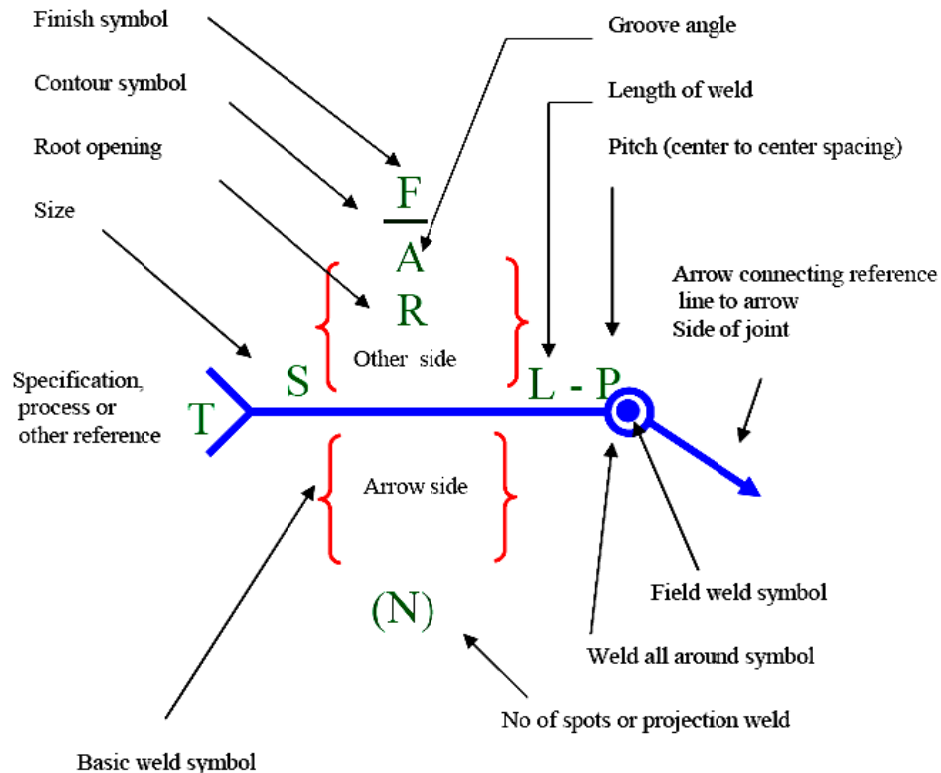


Welding symbol

A welding symbol has following basic elements:

1. Reference line.
2. Arrow.
3. Basic weld symbols (like fillet, butt joints etc.)
4. Dimensions
5. Supplementary symbols.
6. Finish symbols
7. Tail.
8. Specification processes

These welding symbols are placed in standard locations (see figure below)



Example: If the desired weld is a fillet weld of size 10 mm to be done on each side of Tee joint with convex contour, the weld symbol will be as following

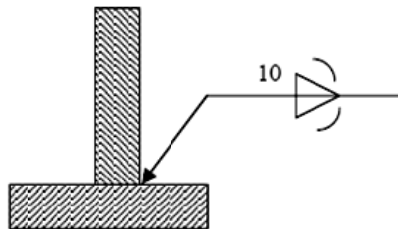


Figure- Tee joint

Design of a butt joint

The main failure mechanism of welded butt joint is tensile failure. Therefore the strength of a butt joint is

$$P = \sigma_t \ell t$$

Where σ_t = allowable tensile strength of the weld material.

t = thickness of the weld

ℓ = length of the weld.

For a square butt joint t is equal to the thickness of the plates. In general, this need not be so (see figure below).

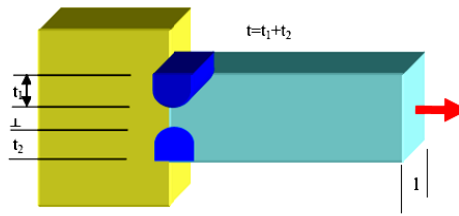


Figure- butt joint

Design of transverse fillet joint

Consider a single transverse joint as shown in figure below. The general stress distribution in the weld metal is very complicated. In design, a simple procedure is used assuming that entire load P acts as shear force on the throat area, which is the smallest area of the cross section in a fillet weld. If the fillet weld has equal base and height, (h , say), then the cross section of the throat is easily seen to be $\frac{hl}{\sqrt{2}}$. With the above consideration the permissible load carried by a transverse fillet weld is

$$P = \tau_s \cdot A_{throat}$$

Where τ_s = allowable shear stress

A_{throat} = throat area.

For a double transverse fillet joint the allowable load is twice that of the single fillet joint.

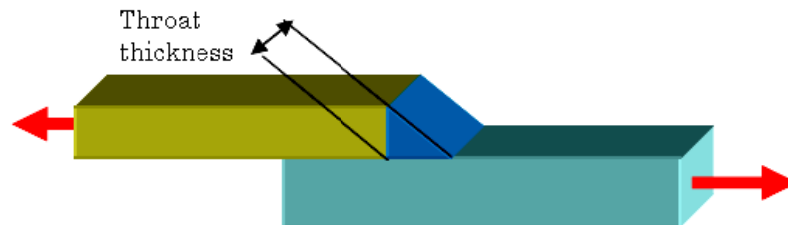
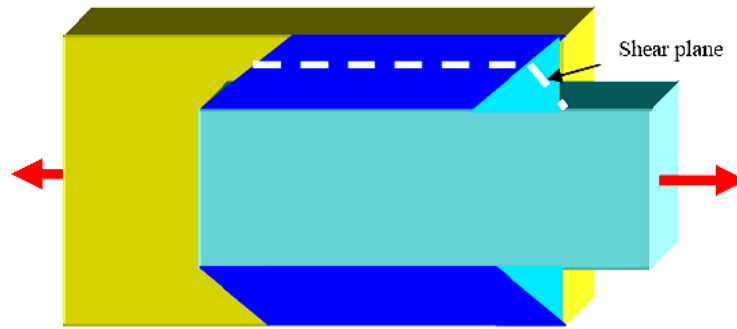


Figure- Enlarge view of fillet welding

Design of parallel fillet joint

Consider a parallel fillet weld as shown in figure below. Each weld carries a load $P/2$. It is easy to see from the strength of material approach that the maximum shear occurs along the throat area (try to prove it). The allowable load carried by each of the joint is $\tau_s A_t$, where the throat area $A_t = \frac{lh}{\sqrt{2}}$. The total allowable load is $P = 2 \tau_s A_t$.



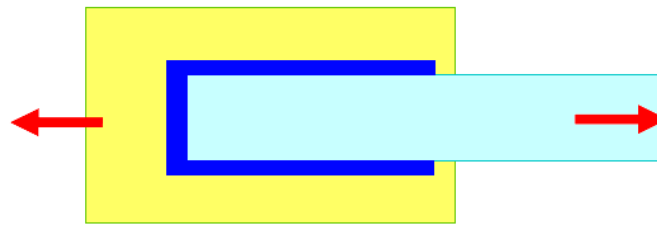
In designing a weld joint the design variables are h and l . They can be selected based on the above design criteria. When a combination of transverse and parallel filled joint required (see figure below) the allowable load is

$$P = 2\tau_s A_t + \tau_s A'_t$$

Where

A_t = throat area along the longitudinal direction.

A'_t = throat area along the transverse direction.



Figure

Design of circular fillet weld subjected to torsion

Consider a circular shaft connected to a plate by means of a fillet joint as shown in figure below. If the shaft is subjected to a torque, shear stress develops in the weld in a similar way as in parallel fillet joint. Assuming that the weld thickness is very small compared to the diameter of the shaft, the maximum shear stress occurs in the throat area. Thus, for a given torque the maximum shear stress in the weld is

$$\tau_{\max} = \frac{T \left(\frac{d}{2} + t_{throat} \right)}{J_p}$$

Where

T = torque applied.

d = outer diameter of the shaft

t_{throat} = throat thickness

J_p = polar moment of area of the throat section.

$$= \frac{\pi}{32} \left[(d + 2t_{throat})^4 - d^4 \right]$$

When $t_{throat} \ll d$, $\tau_{max} = \frac{T \frac{d}{2}}{\frac{\pi}{4} t_{throat} d^3} = \frac{2T}{\pi t_{throat} d^2}$

The throat dimension and hence weld dimension can be selected from the equation

$$\frac{2T}{\pi t_{throat} d^2} = \tau_{max}$$

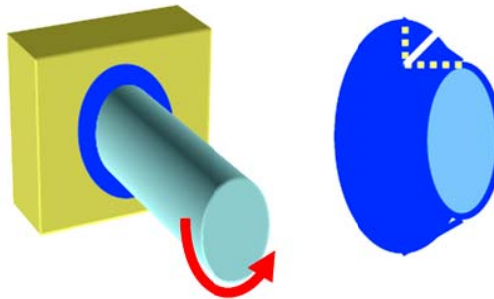


Fig.

- Q.** A plate 50 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld. Assume allowable shear strength to be 56 MPa.

Solution. In a parallel fillet welding two lines of welding are to be provided. Each line shares

a load of $P = \frac{50}{2} \text{ kN} = 25 \text{ kN}$. Maximum shear stress in the parallel fillet weld is $\frac{P}{lt}$, where t = throat length = $\frac{12.5}{\sqrt{2}} \text{ mm}$. Since $\frac{P}{lt} \leq \tau_s = 56 \times 10^6$. Hence the minimum length of the weld is

$\frac{25 \times 10^3 \times \sqrt{2}}{56 \times 12.5 \times 10^3} = 50.5 \text{ mm}$. However some extra length of the weld is to be provided as allowance for starting or stopping of the bead. A usual allowance of 12.5 mm is kept. (Note that the allowance has no connection with the plate thickness)

- Q.** Two plates 200 mm wide and 10 mm thick are to be welded by means of transverse welds at the ends. If the plates are subjected to a load of 70 kN, find the size of the weld assuming the allowable tensile stress 70 MPa.

Solution: According to the design principle of fillet (transverse) joint the weld is designed assuming maximum shear stress occurs along the throat area. Since tensile strength is specified the shear strength may be calculated as half of tensile strength, i.e., $\tau_s = 35 \text{ MPa}$. Assuming there are two welds, each weld carries a load of 35 kN and the size of the weld is calculated from

$$35 \times 10^3 = l \times \left(\frac{10 \times 10^{-3}}{\sqrt{2}} \right) \times 35 \times 10^6$$

Or $l = 141.42 \text{ mm}$.

Adding an allowance of 12.5 mm for stopping and starting of the bead, the length of the weld should be 154 mm.

- Q.** A 50 mm diameter solid shaft is to be welded to a flat plate and is required to carry a torque of 1500 Nm. If fillet joint is used for welding what will be the minimum size of the weld when working shear stress is 56 MPa.

Solution. According to the procedure for calculating strength in the weld joint,

$$\frac{2T}{\pi t_{throat} d^2} = \tau_s$$

Where the symbols have usual significance. For given data, the throat thickness is 6.8 mm. Assuming equal base and height of the fillet the minimum size is 9.6 mm. Therefore a fillet weld of size 10 mm will have to be used.

- Q.** A strap of mild steel is welded to a plate as shown in the following figure. Check whether the weld size is safe or not when the joint is subjected to completely reversed load of 5 kN.

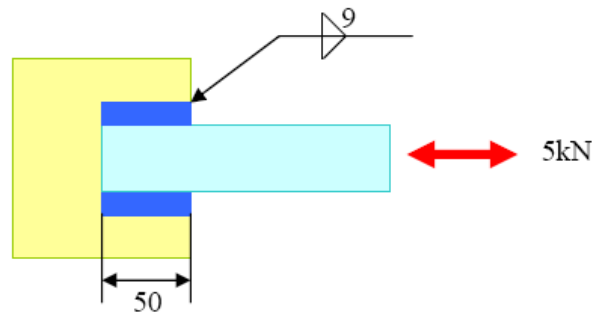


Fig.

Solution. As shown in the figure the joint is a parallel fillet joint with leg size as 9 mm and the welding is done on both sides of the strap. Hence the total weld length is $2(50) = 100$ mm.

In order to calculate the design stress the following data are used

$k_{-1} = 2.7$ (parallel fillet joint, refer table 3) (there is required a table to solve the problem.....)

$w = 0.9$ cm

$K = -1$ for completely reversed loading

The value of the allowable fatigue stress (assuming the weld to be a line) is then

$\sigma_{-1} = \frac{358 \times 0.9}{1.5} = 214.8 \text{ kgf/cm} = 214800 \text{ N/m}$ (approx). The design stress is Therefore

$\sigma_{-1,d} = \frac{214800}{2.7} = 79556 \text{ N/m}^2$. Since the total length of the weld is 0.1 m, the maximum fluctuating load allowable for the joint is 7955.6 N. The joint is therefore safe.

Threaded fasteners

Bolt - Threaded fastener designed to pass through holes in mating members and to be secured by tightening a nut from the end opposite the head of the bolt.

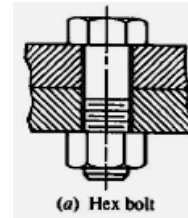
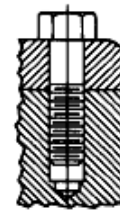


Fig.

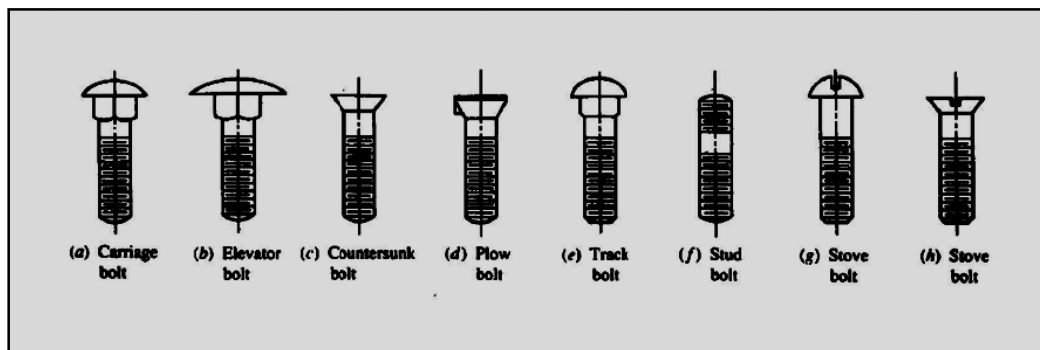
Screw - Threaded fastener designed to be inserted through a hole in one member and into a threaded hole in a mating member.



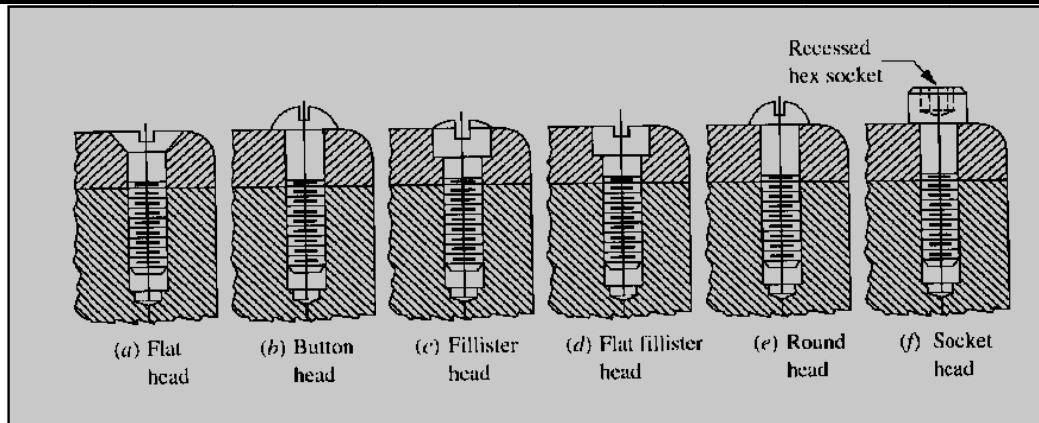
(b) Hex head cap screw

Fig.

Bolts



Machine Screws



Sheet Metal and Lag Screws

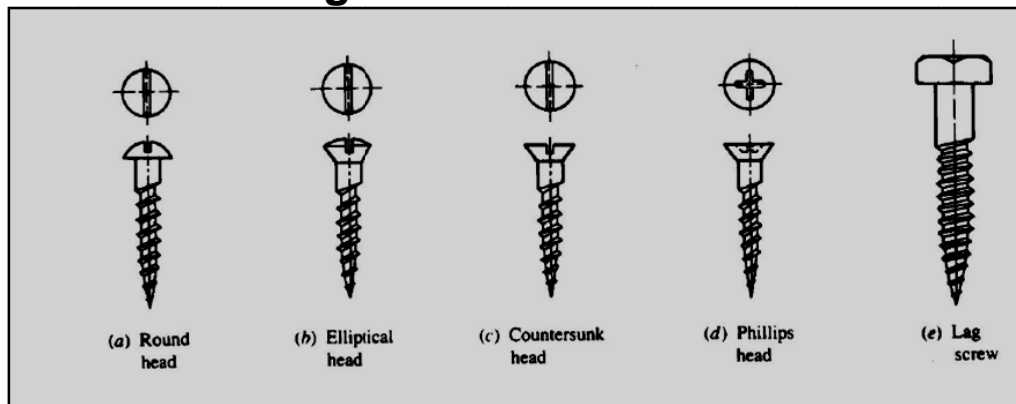
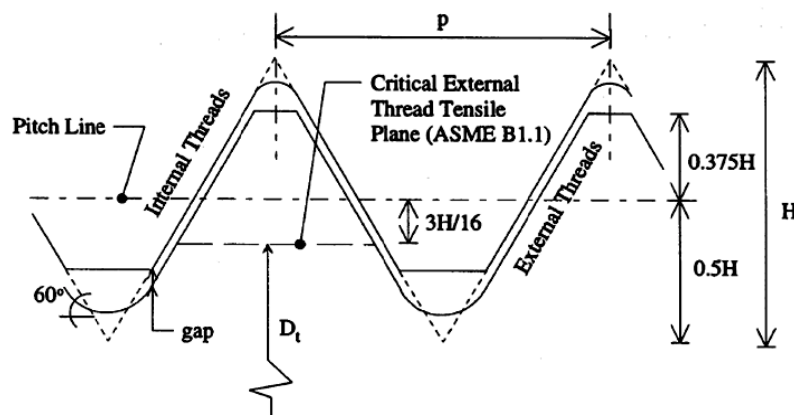


Figure-Sheet metal screws are often self-tapping.

Thread Profiles



The pitch line or diameter is located at $\frac{1}{2}$ the height of the theoretical sharp v-thread profile.

Thread Series

Thread Series - groups of diameter-pitch combinations distinguished from each other by the number of threads per inch applied to a specific diameter.

M-Series

Metric system of diameters, pitches, and tolerance/allowances.

Tensile Stress Area

The average axial stress in a fastener is computed using a “tensile stress area”.

$$\sigma_{ave} = \frac{F}{A_t}$$

$$A_t = \frac{\pi}{4} \left[\frac{D_r + D_p}{2} \right]^2$$

$F \equiv$ Axial Force
 $D_r \equiv$ Root Diameter
 $D_p \equiv$ Pitch Diameter
 $A_t \equiv$ Tensile Stress Area
 $\sigma_{ave} \equiv$ Average axial stress

Length of Engagement (Equal Strength Materials)

If the internal thread and external thread material have the same strength, then

Tensile Strength (External Thread) $\sigma_t = \frac{F_{\max}}{A_t}$	Shear Strength (Internal Thread) $0.5\sigma_t = \frac{F_{\max}}{A_{s,i} \cdot L_e}$
------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------

Where

$$F_{\max} = \sigma_t A_t = 0.5\sigma_t A_{s,i} L_e$$

$$L_e = \frac{2A_t}{A_{s,i}}$$

Bolt/Nut Design Philosophy

ANSI standard bolts and nuts of equal grades are designed to have the bolt fail before the threads in the nut are stripped.

The engineer designing a machine element is responsible for determining how something should fail taking into account the safety of the operators and public. Length of engagement is an important consideration in designing machine elements with machine screws.

Objective Questions (For GATE, IES & IAS)

Design of Joint

S K Mondal's

Chapter 1

Previous 20-Years GATE Questions

Keys

GATE-1. Square key of side " $d/4$ " each and length l is used to transmit torque " T " from the shaft of diameter " d " to the hub of a pulley. Assuming the length of the key to be equal to the thickness of the pulley, the average shear stress developed in the key is given by [GATE-2003]

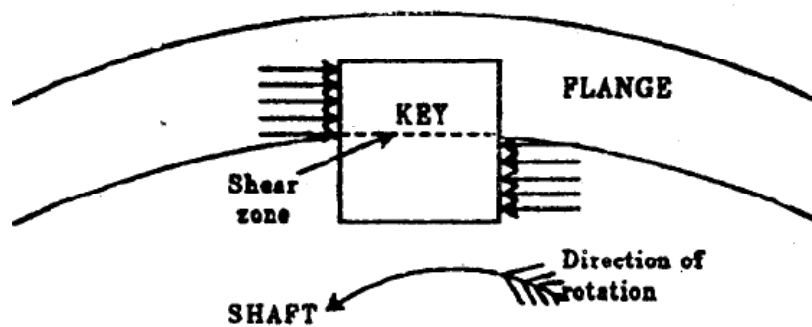
- (a) $\frac{4T}{ld}$ (b) $\frac{16T}{ld^2}$ (c) $\frac{8T}{ld^2}$ (d) $\frac{16T}{\pi d^3}$

GATE-1. Ans. (c) If a square key of sides $d/4$ is used then. In that case, for shear failure we have $\left(\frac{d}{4} \times l\right) \tau_x \frac{d}{2} = T$
or $\tau_x = \frac{8T}{ld^2}$ [Where τ_x is the yield stress in shear and l is the key length.]

GATE-2. A key connecting a flange coupling to a shaft is likely to fail in [GATE-1995]

- (a) Shear (b) tension (c) torsion (d) bending

GATE-2. Ans. (a) Shear is the dominant stress on the key



Welded joints

GATE-3. A 60 mm long and 6 mm thick fillet weld carries a steady load of 15 kN along the weld. The shear strength of the weld material is equal to 200 MPa. The factor of safety is [GATE-2006]

- (a) 2.4 (b) 3.4 (c) 4.8 (d) 6.8

GATE-3. Ans. (b)

$$\begin{aligned} \text{Factor of safety} &= \frac{\text{Strength of material}}{\text{Actual load or strength on material}} \\ &= \frac{200(\text{in MPa})}{15 \times 10^3} \times \frac{200(\text{in MPa})}{58.91(\text{in MPa})} = 3.4 \\ &= \frac{6}{60 \times \frac{6}{\cos 45^\circ} \times 10^{-6}} \times 10^{-6} (\text{in MPa}) \end{aligned}$$

Threaded fasteners

Design of Joint

S K Mondal's

Chapter 1

GATE-4. A threaded nut of M16, ISO metric type, having 2 mm pitch with a pitch diameter of 14.701 mm is to be checked for its pitch diameter using two or three numbers of balls or rollers of the following sizes [GATE-2003]

- (a) Rollers of 2 mm ϕ (b) Rollers of 1.155 mm ϕ
(c) Balls of 2 mm ϕ (d) Balls of 1.155 mm ϕ

GATE-4. Ans. (b)

Previous 20-Years IES Questions

Cotters

IES-1. **Assertion (A):** A cotter joint is used to rigidly connect two coaxial rods carrying tensile load.

Reason (R): Taper in the cotter is provided to facilitate its removal when it fails due to shear. [IES-2008]

- (a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-1. Ans. (b) A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment. The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a locking device is provided. The locking device may be a taper pin or a set screw used on the lower end of the cotter. The cotter is usually made of mild steel or wrought iron. A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjected to axial tensile or compressive forces.

IES-2. Match List I with List II and select the correct answer using the code given below the Lists: [IES 2007]

List I
(Application)

- A. Boiler shell
B. Marine shaft coupling
C. Crosshead and piston rod
D. Automobile gear box
(gears to shaft)

Code:	A	B	C	D
(a)	1	4	2	5
(c)	1	5	2	4

List II
(Joint)

1. Cotter joint
2. Knuckle joint
3. Riveted joint
4. Splines
5. Bolted Joint

	A	B	C	D
(b)	3	5	1	4
(d)	3	4	1	5

IES-2. Ans. (b)

IES-3. Match List-I (Parts to be joined) with List-II (Type of Joint) and select the correct answer using the code given below: [IES-2006]

List-I

- A. Two rods having relative axial motion
B. Strap end of the connecting rod
C. Piston rod and cross head
D. Links of four-bar chain

	A	B	C	D
(a)	1	3	4	2

List -II

1. Pin Joint
2. Knuckle Joint
3. Gib and Cotter Joint
4. Cotter Joint

	A	B	C	D
(b)	2	4	3	1

Design of Joint

S K Mondal's

Chapter 1

(c) 1 4 3 2 (d) 2 3 4 1

IES-3. Ans. (d)

IES-4. Match List I with List II and select the correct answer.

[IES-1994]

List I (Types of joints)

List II (An element of the joint)

A. Riveted joint

1. Pin

B. Welded joint

2. Strap

C. Bolted joint

3. Lock washer

D. Knuckle joint

4. Fillet

Codes: A	B	C	D	A	B	C	D
-----------------	----------	----------	----------	----------	----------	----------	----------

(a) 4	3	2	1	(b) 2	3	4	1
-------	---	---	---	-------	---	---	---

(c) 2	4	3	1	(d) 2	4	1	3
-------	---	---	---	-------	---	---	---

IES-4. Ans. (c)

IES-5. In a gib and cotter joint, the gib and cotter are subjected to

[IES-2006]

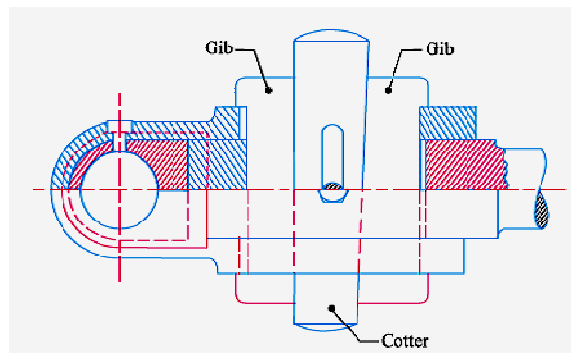
(a) Single shear only

(b) double shear only

(c) Single shear and crushing

(d) double shear and crushing

IES-5. Ans. (d)



IES-6. Match List I (Items in joints) with List II (Type of failure) and select the correct answer using the codes given below the Lists: [IES-2004]

List I

List II

A. Bolts in bolted joints of engine cylinder cover plate

1. Double transverse shear

B. Cotters in cotter joint

2. Torsional shear

C. Rivets in lap joints

3. Single transverse shears

D. Bolts holding two flanges in a flange coupling

4. Tension

A	B	C	D	A	B	C	D
----------	----------	----------	----------	----------	----------	----------	----------

(a) 4	1	3	2	(b) 4	2	3	1
-------	---	---	---	-------	---	---	---

(c) 3	1	4	2	(d) 3	2	4	1
-------	---	---	---	-------	---	---	---

IES-6. Ans. (a)

IES-7. In a cotter joint, the width of the cotter at the centre is 50 mm and its thickness is 12 mm. The load acting on the cotter is 60 kN. What is the shearing stress developed in the cotter? [IES-2004]

(a) 120 N/mm²

(b) 100 N/mm²

(c) 75 N/mm²

(d) 50 N/mm²

IES-7. Ans. (d) It is a case of double shear.

$$\text{Shear stress} = \frac{\text{Load}}{2 \times \text{Area}} = \frac{60 \times 10^3}{2 \times 50 \times 12} = 50 \text{ N/mm}^2$$

Design of Joint

S K Mondal's

Chapter 1

IES-8. The spigot of a cotter joint has a diameter D and carries a slot for cotter. The permissible crushing stress is x times the permissible tensile stress for the material of spigot where $x > 1$. The joint carries an axial load P . Which one of the following equations will give the diameter of the spigot?

[IES-2001]

$$(a) D = 2\sqrt{\frac{P}{\pi\sigma_t} \frac{x-1}{x}} \quad (b) D = 2\sqrt{\frac{P}{\pi\sigma_t} \frac{x+1}{x}} \quad (c) D = \frac{2}{\pi}\sqrt{\frac{P}{\sigma_t} \frac{x+1}{x}} \quad (d) D = \frac{2P}{\pi\sigma_t}\sqrt{x+1}$$

IES-8. Ans. (b)

IES-9. Match List-I (Machine element) with List-II (Cause of failure) and select the correct answer using the codes given below the lists:

[IES-1998]

List-I

A. Axle

B. Cotter

C. Connecting rod

D. Journal bearing

List-II

1. Shear stress

2. Tensile/compressive stress

3. Wear

4. Bending stress

Code:	A	B	C	D	A	B	C	D
(a)	1	4	2	3	(b)	4	1	2
(c)	4	1	3	2	(d)	1	4	3

IES-9. Ans. (b)

- In machinery, the general term “*shaft*” refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc., and which is subjected to torsion and to transverse or axial loads acting singly or in combination.
- An “*axle*” is a non-rotating member that supports wheels, pulleys, and carries no torque.
- A “*spindle*” is a short shaft. Terms such as *line-shaft*, *head-shaft*, *stub shaft*, *transmission shaft*, *countershaft*, and *flexible shaft* are names associated with special usage.

IES-10. The piston rod and the crosshead in a steam engine are usually connected by means of

[IES-2003]

- (a) Cotter joint (b) Knuckle joint (c) Ball joint (d) Universal joint

IES-10. Ans. (a)

IES-11. A cotter joint is used when no relative motion is permitted between the rods joined by the cotter. It is capable of transmitting

[IES-2002]

- (a) Twisting moment (b) an axial tensile as well as compressive load
(c) The bending moment (d) only compressive axial load

IES-11. Ans. (b)

IES-12. Match List I with List II and select the correct answer using the codes given below the lists:

[IES-1995]

List I

(Different types of detachable joints)

A. Cotter joint

B. Knuckle joint

C. Suspension link joint

D. Turn buckle (adjustable joint)

List II

(Specific use of these detachable joints)

1. Tie rod of a wall crane

2. Suspension bridges

3. Diagonal stays in boiler

4. Cross-head of a steam engine

Codes:	A	B	C	D	A	B	C	D
(a)	4	2	3	1	(b)	4	3	2
(c)	3	2	1	4	(d)	2	1	4

Design of Joint

S K Mondal's

Chapter 1

IES-12. Ans. (a)

IES-13. Match List I with List II and select the correct answer using the codes given below the lists: [IES-1993]

List I (Type of joint)

- A. Cotter joint
- B. Knuckle joint
- C. Turn buckle
- D. Riveted joint

List II (Mode of jointing members)

- 1. Connects two rods or bars permitting small amount of flexibility
- 2. Rigidly connects two members
- 3. Connects two rods having threaded ends
- 4. Permanent fluid-tight joint between two flat pieces
- 5. Connects two shafts and transmits torque

Codes:	A	B	C	D	A	B	C	D
(a)	5	1	3	2	(b)	2	1	3
(c)	5	3	2	4	(d)	2	3	1

IES-13. Ans. (b) A cotter is a flat wedge-shaped piece of steel. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods. Connection of piston rod to the cross-head of a steam engine, valve rod and its stem etc are examples of cotter joint.

IES-14. Assertion (A): When the coupler of a turn buckle is turned in one direction both the connecting rods either move closer or move away from each other depending upon the direction of rotation of the coupler. [IES-1996]

Reason (R): A turn buckle is used to connect two round rods subjected to tensile loading and requiring subsequent adjustment for tightening or loosening.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-14. Ans. (b)

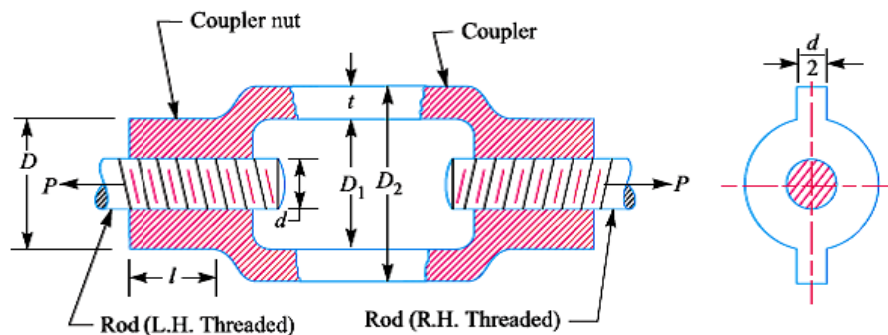


Fig. Turnbuckle

Keys

IES-15. In the assembly of pulley, key and shaft

[IES-1993; 1998]

- (a) pulley is made the weakest
- (b) key is made the weakest

Design of Joint

S K Mondal's

Chapter 1

(c) Key is made the strongest

(d) all the three are designed for equal strength

IES-15. Ans. (b) Key is made the weakest so that it is cheap and easy to replace in case of failure.

IES-16. Match List-I (Type of keys) with List-II (Characteristic) and select the correct answer using the codes given below the Lists: [IES-1997]

List-I

A. Woodruff key

B. Kennedy key

C. Feather key

D. Flat key

List-II

1. Loose fitting, light duty

2. Heavy duty

3. Self-aligning

4. Normal industrial use

Code:	A	B	C	D		A	B	C	D
(a)	2	3	1	4	(b)	3	2	1	4
(c)	2	3	4	1	(d)	3	2	4	1

IES-16. Ans. (b) A feather key is used when one component slides over another. The key may be fastened either to the hub or the shaft and the keyway usually has a sliding fit.

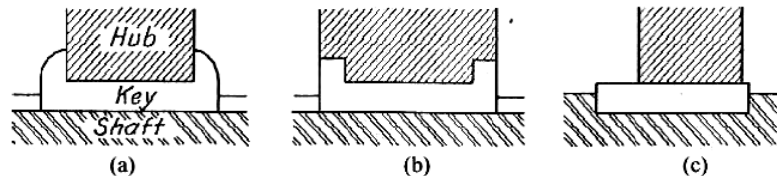


Fig. feather key

IES-17. Match List-I with List-II and select the correct answer using the code given below the lists: [IES-2008]

List-I (Key/splines)

A. Gib head key

B. Woodruff key

C. Parallel key

D. Splines

List-II (Application)

1. Self aligning

2. Facilitates removal

3. Mostly used

4. Axial movement possible

Code:	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	1	2	4	3
(c)	2	1	3	4	(d)	2	1	4	3

IES-17. Ans. (c)

IES-18. A spur gear transmitting power is connected to the shaft with a key of rectangular section. The type (s) of stresses developed in the key is/are.

(a) Shear stress alone

(b) bearing stress alone

[IES-1995]

(c) Both shear and bearing stresses

(d) shearing, bearing and bending stresses.

IES-18. Ans. (c) Key develops both shear and bearing stresses.

IES-19. Assertion (A): The effect of keyways on a shaft is to reduce its load carrying capacity and to increase its torsional rigidity. [IES-1994]

Reason (R): Highly localized stresses occur at or near the corners of keyways.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IES-19. Ans. (d)

Design of Joint

S K Mondal's

Chapter 1

IES-20. Which key is preferred for the condition where a large amount of impact torque is to be transmitted in both direction of rotation? [IES-1992]

- (a) Woodruff key (b) Feather key (c) Gib-head key (d) Tangent key

IES-20. Ans. (d)

IES-21. What is sunk key made in the form of a segment of a circular disc of uniform thickness, known as? [IES-2006]

- (a) Feather key (b) Kennedy key (c) Woodruff key (d) Saddle key

IES-21. Ans. (c)

IES-22. What are the key functions of a master schedule? [IES-2005]

1. To generate material and capacity requirements
2. To maintain valid priorities
3. An effective capacity utilization
4. Planning the quantity and timing of output over the intermediate time horizons

Select the correct answer using the code given below:

- (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

IES-22. Ans. (b)

IES-23. A square key of side $d/4$ is to be fitted on a shaft of diameter d and in the hub of a pulley. If the material of the key and shaft is same and the two are to be equally strong in shear, what is the length of the key? [IES-2005]

- (a) $\frac{\pi d}{2}$ (b) $\frac{2\pi d}{3}$ (c) $\frac{3\pi d}{4}$ (d) $\frac{4\pi d}{5}$

IES-23. Ans. (a)

IES-24. Which one of the following statements is correct? [IES-2004]

While designing a parallel sunk key it is assumed that the distribution of force along the length of the key

- (a) Varies linearly (b) is uniform throughout
(c) varies exponentially, being more at the torque input end
(d) varies exponentially, being less at torque output end

IES-24. Ans. (c) *Parallel sunk key.* The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft. In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

IES-25. Match List-I (Device) with List-II (Component/Accessory) and select the correct answer using the codes given below the Lists: [IES-2003]

List-I

(Device)

- A. Lifting machine
B. Fibre rope drive
C. Differential gear
D. Belt drive

Codes: A B C D

- (a) 4 3 1 2
(c) 4 3 2 1

List-II

(Component/Accessory)

1. Idler of Jockey pulley
2. Sun wheel
3. Sheave
4. Power screw

A B C D

- (b) 3 4 1 2
(d) 3 4 2 1

IES-25. Ans. (c)

IES-26. A pulley is connected to a power transmission shaft of diameter d by means of a rectangular sunk key of width w and length l . The width of the key is taken as $d/4$. For full power transmission, the shearing strength of the key is equal to the torsional shearing strength of the shaft. The ratio of the length of the key to the diameter of the shaft (l/d) is [IES-2003]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{\sqrt{2}}$ (c) $\frac{\pi}{2}$ (d) π

IES-26. Ans. (c)

$$\text{Shearing strength of key: } F = \tau \left(\frac{d}{4} \cdot l \right)$$

$$\text{Torque (T)} = F \cdot \frac{d}{2} = \tau \left(\frac{d}{4} \cdot l \right) \cdot \frac{d}{2}$$

$$\text{Torsional shearing, } \frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{2}$$

$$\text{or } T = \pi d^3 \times \frac{\tau}{16}$$

For same strength

$$\tau \left(\frac{d}{4} \cdot l \right) \cdot \frac{d}{2} = \pi d^3 \times \frac{\tau}{16}$$

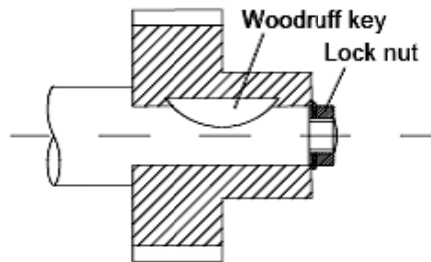
$$\text{or } \frac{l}{d} = \frac{\pi}{2}$$

IES-27. Assertion (A): A Woodruff key is an easily adjustable key.

Reason (R): The Woodruff key accommodates itself to any taper in the hub or boss of the mating piece. [IES-2003]

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-27. Ans. (b)



The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The main dis-advantages of a woodruff key are as follows:

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.

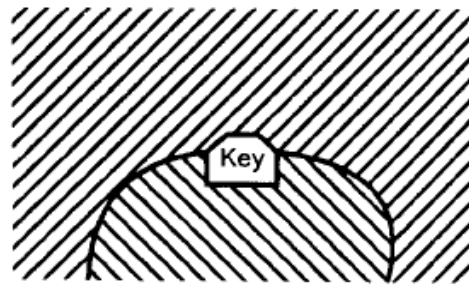
Design of Joint

S K Mondal's

Chapter 1

IES-28. The key shown in the above figure is a

- (a) Barth key
- (b) Kennedy key
- (c) Lewis key
- (d) Woodruff key



[IES-2000]

IES-28. Ans. (a)

IES-29. Match List I (Keys) with List II (Characteristics) and select the correct answer using the codes given below the Lists: [IES-2000]

List I			List II						
A. Saddle key			1. Strong in shear and crushing						
B. Woodruff key			2. Withstands tension in one direction						
C. Tangent key			3. Transmission of power through frictional resistance						
D. Kennedy key			4. Semicircular in shape						
Code:	A	B	C	D	A	B	C	D	
(a)	3	4	1	2	(b)	4	3	2	1
(c)	4	3	1	2	(d)	3	4	2	1

IES-29. Ans. (d)

IES-30. Match List-I with List-II and select the correct answer using the code given below the Lists: [IES-2009]

List-I				List-II					
(Description)				(shape)					
A. Spline				1. Involute					
B. Roll pin				2. Semicircular					
C. Gib-headed key				3. Tapered on one side					
D. Woodruff key				4. Circular					
Code:	A	B	C	D	A	B	C	D	
(a)	1	3	4	2	(b)	2	3	4	1
(c)	1	4	3	2	(d)	2	4	3	1

IES-30. Ans. (c)

IES-31. The shearing area of a key of length 'L', breadth 'b' and depth 'h' is equal to

- (a) $b \times h$
- (b) $L \times h$
- (c) $L \times b$
- (d) $L \times (h/2)$

[IES-1998]

IES-31. Ans. (c)

Splines

IES-32. Consider the following statements:

[IES-1998]

A splined shaft is used for

1. Transmitting power
2. Holding a flywheel rigidly in position
3. Moving axially the gear wheels mounted on it
4. Mounting V-belt pulleys on it.

Of these statements

- (a) 2 and 3 are correct
- (b) 1 and 4 are correct

Design of Joint

S K Mondal's

Chapter 1

(c) 2 and 4 are correct

(d) 1 and 3 are correct

IES-32. Ans. (d)

Welded joints

IES-33. In a fillet welded joint, the weakest area of the weld is [IES-2002]

(a) Toe

(b) root

(c) throat

(d) face

IES-33. Ans. (c)

IES-34. A single parallel fillet weld of total length L and weld size h subjected to a tensile load P , will have what design stress? [IES 2007]

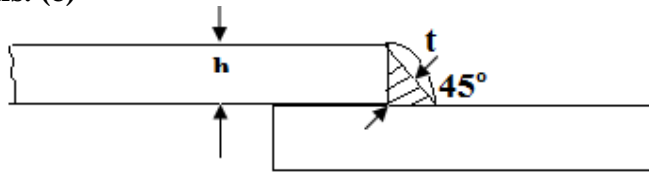
(a) Tensile and equal to $\frac{P}{0.707Lh}$

(b) Tensile and equal to $\frac{P}{Lh}$

(c) Shear and equal to $\frac{P}{0.707Lh}$

(d) Shear and equal to $\frac{P}{Lh}$

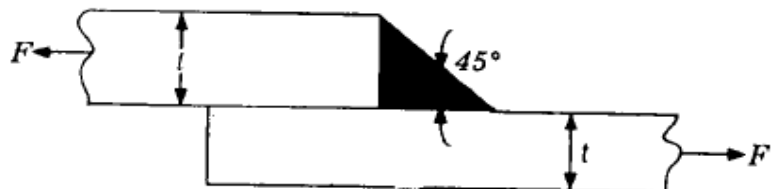
IES-34. Ans. (c)



$$\text{Throat, } t = h \cos 45^\circ = \frac{1}{\sqrt{2}} h = 0.707h$$

$$T = \frac{P}{Lt} = \frac{P}{0.707Lh}$$

IES-35. Two metal plates of thickness 't' and width 'w' are joined by a fillet weld of 45° as shown in given figure.



[IES-1998]

When subjected to a pulling force 'F', the stress induced in the weld will be

(a) $\frac{F}{wt \sin 45^\circ}$

(b) $\frac{F}{wt}$

(c) $\frac{F \sin 45^\circ}{wt}$

(d) $\frac{2F}{wt}$

IES-35. Ans. (a)

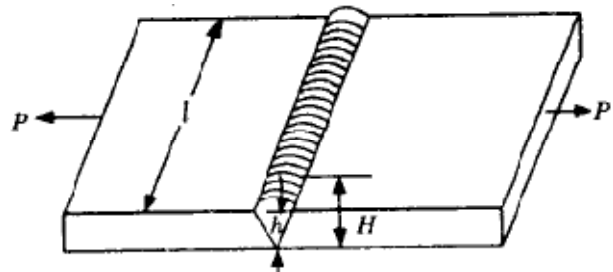
IES-36. A butt welded joint, subjected to tensile force P is shown in the given figure, l = length of the weld (in mm) h = throat of the butt weld (in mm) and H is the total height of weld including reinforcement. The average tensile stress σ_t , in the weld is given by

(a) $\sigma_t = \frac{P}{Hl}$

(b) $\sigma_t = \frac{P}{hl}$

(c) $\sigma_t = \frac{P}{2hl}$

(d) $\sigma_t = \frac{2P}{Hl}$



[IES-1997]

IES-36. Ans. (b)

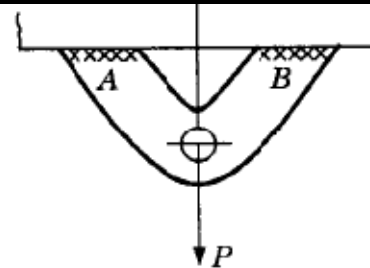
Design of Joint

S K Mondal's

Chapter 1

IES-37. In the welded joint shown in the given figure, if the weld at B has thicker fillets than that at A, then the load carrying capacity P , of the joint will

- (a) increase
- (b) decrease
- (c) remain unaffected
- (d) exactly get doubled



[IES-1997]

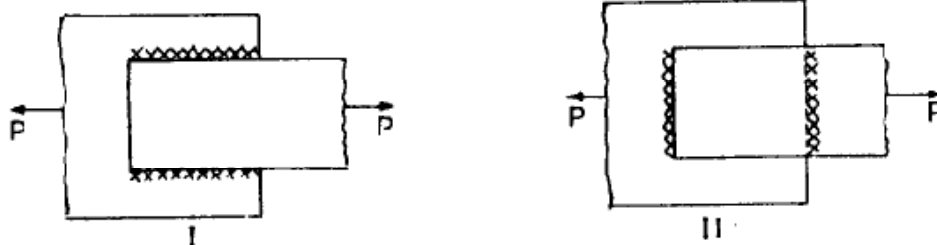
IES-37. Ans. (c)

IES-38. A double fillet welded joint with parallel fillet weld of length L and leg B is subjected to a tensile force P . Assuming uniform stress distribution, the shear stress in the weld is given by [IES-1996]

- (a) $\frac{\sqrt{2}P}{B.L}$
- (b) $\frac{P}{2.B.L}$
- (c) $\frac{P}{\sqrt{2}.B.L}$
- (d) $\frac{2P}{B.L}$

IES-38. Ans. (c)

IES-39. The following two figures show welded joints (x x x x x indicates welds), for the same load and same dimensions of plate and weld. [IES-1994]



The joint shown in

- (a) fig. I is better because the weld is in shear and the principal stress in the weld is not in line with P
- (b) fig. I is better because the load transfer from the tie bar to the plate is not direct
- (c) fig. II is better because the weld is in tension and safe stress of weld in tension is greater than that in shear
- (d) fig. II is better because it has less stress concentration.

IES-39. Ans. (c) Figure II is better because the weld is in tension and safe stress of weld in tension is greater than shear.

IES-40. Assertion (A): In design of double fillet welding of unsymmetrical sections with plates subjected to axial loads lengths of parallel welds are made unequal.

Reason (R): The lengths of parallel welds in fillet welding of an unsymmetrical section with a plate are so proportioned that the sum of the resisting moments of welds about the centre of gravity axis is zero. [IES-2008]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

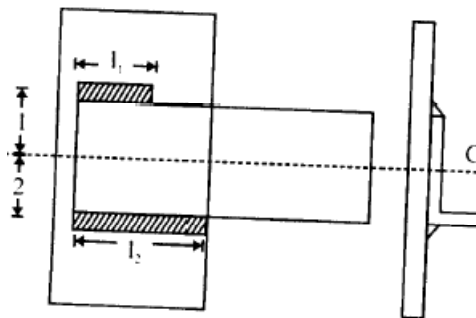
IES-40. Ans. (a) Axially loaded unsymmetrical welded joints

Design of Joint

S K Mondal's

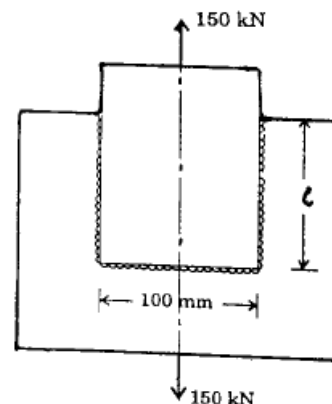
Chapter 1

$$\begin{aligned}\tau &= \frac{P_1}{A_1} \\ P_1 &= \tau A_1 \\ P_1 &= \tau \times t \times I_1 \\ P_2 &= \tau \times t \times I_2 \\ P_1 y_1 &= P_2 y_2 \\ \tau t I_1 y_1 &= \tau t I_2 y_2 \\ I_1 y_1 &= I_2 y_2\end{aligned}$$



IES-41. Two plates are joined together by means of single transverse and double parallel fillet welds as shown in figure given above. If the size of fillet is 5 mm and allowable shear load per mm is 300 N, what is the approximate length of each parallel fillet?

- (a) 150 mm
- (b) 200 mm
- (c) 250 mm
- (d) 300 mm



[IES-2005]

IES-41. Ans. (b) $300 \times (100 + 2l) = 15000$ or $l = 200$

IES-42. A circular rod of diameter d is welded to a flat plate along its circumference by fillet weld of thickness t . Assuming τ_w as the allowable shear stress for the weld material, what is the value of the safe torque that can be transmitted?

[IES-2004]

- (a) $\pi d^2 \cdot t \cdot \tau_w$
- (b) $\frac{\pi d^2}{2} \cdot t \cdot \tau_w$
- (c) $\frac{\pi d^2}{2\sqrt{2}} \cdot t \cdot \tau_w$
- (d) $\frac{\pi d^2}{\sqrt{2}} \cdot t \cdot \tau_w$

IES-42. Ans. (b)

Shear stress = τ_w

Shear force = $\tau_w \times \pi d t$

$$\text{Torque (T)} = \tau_w \times \pi d t \times \frac{d}{2} = \frac{\pi d^2}{2} \cdot t \cdot \tau_w$$

IES-43. A circular solid rod of diameter d welded to a rigid flat plate by a circular fillet weld of throat thickness t is subjected to a twisting moment T . The maximum shear stress induced in the weld is

[IES-2003]

- (a) $\frac{T}{\pi d^2}$
- (b) $\frac{2T}{\pi d^2}$
- (c) $\frac{4T}{\pi d^2}$
- (d) $\frac{2T}{\pi d^3}$

$$\text{IES-43. Ans. (b)} \quad \tau = \frac{T \cdot r}{J} = \frac{T \cdot \left(\frac{d}{2}\right)}{\frac{\pi d^3}{4}} = \frac{2T}{\pi d^2}$$

Design of Joint

S K Mondal's

Chapter 1

IES-44. The permissible stress in a filled weld is 100 N/mm^2 . The fillet weld has equal leg lengths of 15 mm each. The allowable shearing load on weldment per cm length of the weld is [IES-1995]

- (a) 22.5 kN (b) 15.0 kN (c) 10.6 kN (d) 7.5 kN .

IES-44. Ans. (c) Load allowed = $100 \times 0.707 \times 10 \times 15 = 10.6 \text{ kN}$

Threaded fasteners

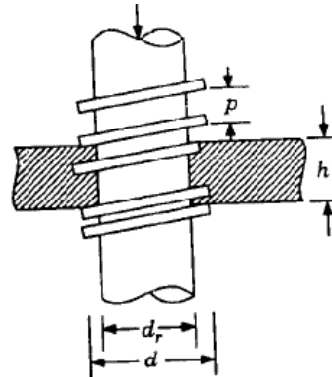
IES-45. A force 'F' is to be transmitted through a square-threaded power screw into a nut. If 't' is the height of the nut and 'd' is the minor diameter, then which one of the following is the average shear stress over the screw thread? [IES 2007]

- (a) $\frac{2f}{\pi dt}$ (b) $\frac{F}{\pi dt}$ (c) $\frac{F}{2\pi dt}$ (d) $\frac{4F}{\pi dt}$

IES-45. Ans. (b)

IES-46. Consider the case of a square-threaded screw loaded by a nut as shown in the given figure. The value of the average shearing stress of the screw is given by (symbols have the usual meaning)

- (a) $\frac{2F}{\pi d_r h}$ (b) $\frac{F}{\pi d_r h}$
(c) $\frac{2F}{\pi dh}$ (d) $\frac{F}{\pi dh}$



[IES-1997]

IES-46. Ans. (b)

IES-47. Assertion (A): Uniform-strength bolts are used for resisting impact loads.

Reason (R): The area of cross-section of the threaded and unthreaded parts is made equal. [IES-1994]

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-47. Ans. (c) A is true and R is false.

IES-48. How can shock absorbing capacity of a bolt be increased? [IES 2007]

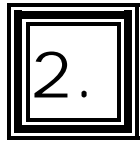
- (a) By tightening it properly
(b) By increasing the shank diameter
(c) By grinding the shank
(d) By making the shank diameter equal to the core diameter of thread

IES-48. Ans. (d)

IES-49. The number of slots in a 25 mm castle nut is [IES-1992]

- (a) 2 (b) 4 (c) 6 (d) 8

IES-49. Ans. (c)



Design of Friction Drives

Theory at a glance (GATE, IES, IAS & PSU)

Couplings

Introduction

Couplings are used to connect two shafts for torque transmission in varied applications. It may be to connect two units such as a motor and a generator or it may be to form a long line shaft by connecting shafts of standard lengths say 6-8m by couplings. Coupling may be rigid or they may provide flexibility and compensate for misalignment. They may also reduce shock loading and vibration. A wide variety of commercial shaft couplings are available ranging from a simple keyed coupling to one which requires a complex design procedure using gears or fluid drives etc.

However there are two main types of couplings:

- **Rigid couplings.**
- **Flexible couplings.**

Rigid couplings are used for shafts having **no misalignment** while the **flexible couplings** can absorb **some amount of misalignment in the shafts to be connected**. In the next section we shall discuss different types of couplings and their uses under these two broad headings.

Types and uses of shaft couplings

Rigid couplings

Since these couplings cannot absorb any misalignment the shafts to be connected by a rigid coupling must have good lateral and angular alignment. The types of misalignments are shown schematically in figure below.

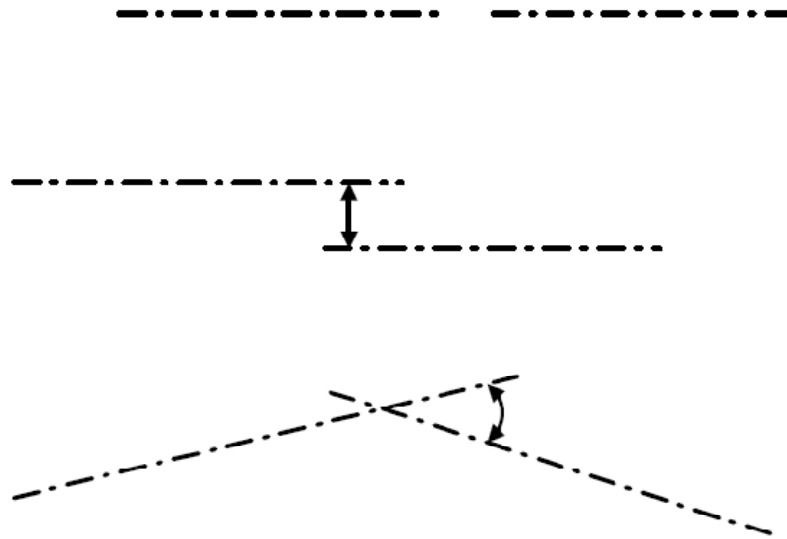


Figure- Types of misalignments in shafts

Sleeve coupling

One of the simple types of rigid coupling is a sleeve coupling which consists of a cylindrical sleeve keyed to the shafts to be connected. A typical sleeve coupling is shown in figure below

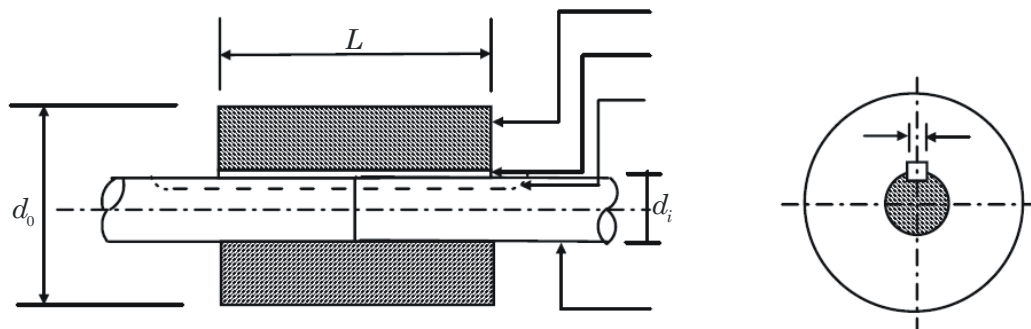


Figure- A typical sleeve coupling

Normally sunk keys are used and in order to transmit the torque safely it is important to design the sleeve and the key properly. The key design is usually based on shear and bearing stresses. If the torque transmitted is T , the shaft radius is r and a rectangular sunk key of dimension b and length L is used then the induced shear stress τ (figure below) in the key is given by

$$\tau = \frac{T}{\left(b \frac{L}{2} r\right)}$$

And for safety

$$(2T / bLr) < \tau_y$$

Where τ_y is the yield stress in shear of the key material. A suitable factor of safety must be used. The induced crushing stress in the key is given as

$$\sigma_{br} = \frac{T}{\left(\frac{b}{2} \frac{L}{2} r\right)}$$

And for a safe design

$$4T / (bLr) < \sigma_c$$

Where σ_c is the crushing strength of the key material.

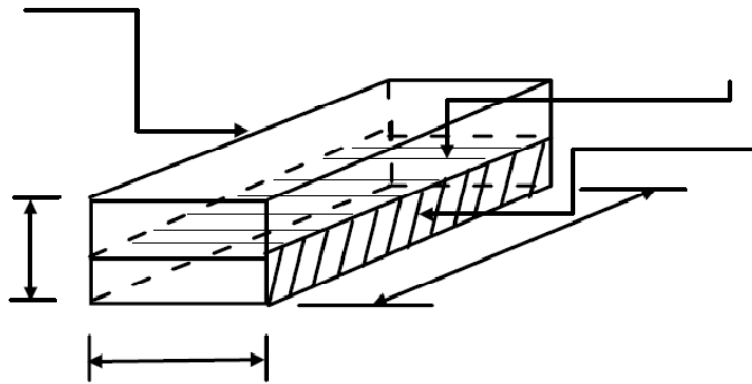


Figure- Shear and crushing planes in the key.

The sleeve transmits the torque from one shaft to the other. Therefore if d_i is the inside diameter of the sleeve which is also close to the shaft diameter d (say) and d_o is outside diameter of the sleeve, the shear stress developed in the sleeve is

$$\tau_{sleeve} = \frac{16Td_o}{\pi(d_o^4 - d_i^4)} \quad \text{and the shear stress in the shaft is given by}$$

$$\tau_{shaft} = \frac{16T}{\pi d_i^3}. \quad \text{Substituting yield shear stresses of the sleeve and shaft materials for}$$

τ_{sleeve} and τ_{shaft} both d_i and d_o may be evaluated.

However from the empirical proportions we have:

$$d_o = 2d_i + 12.5 \text{ mm and } L = 3.5d.$$

These may be used as checks.

Sleeve coupling with taper pins

Torque transmission from one shaft to another may also be done using pins as shown in figure below.

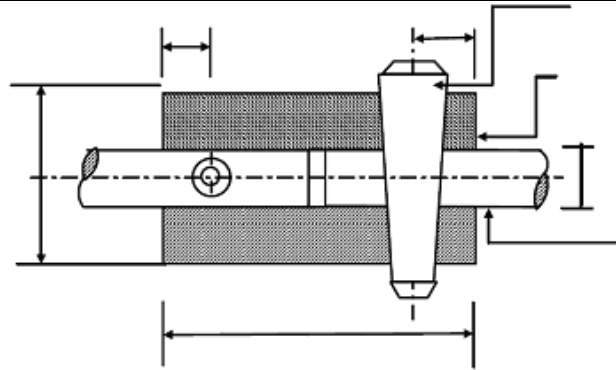


Figure- A representative sleeve coupling with taper pins

The usual proportions in terms of shaft diameter d for these couplings are:
 $d_0 = 1.5d$, $L = 3d$ and $a = 0.75d$.

The mean pin diameter $d_{mean} = 0.2$ to $0.25 d$. For small couplings d_{mean} is taken as $0.25d$ and for large couplings d_{mean} is taken as $0.2d$. Once the dimensions are fixed we may check the pin for shear failure using the relation

$$2 \left(\frac{\pi}{4} d_{mean}^2 \right) \tau \left(\frac{d}{2} \right) = T.$$

Here T is the torque and the shear stress τ must not exceed the shear yield stress of the pin material. A suitable factor of safety may be used for the shear yield stress.

Clamp coupling

A typical clamp coupling is shown in figure below. It essentially consists of two half cylinders which are placed over the ends of the shafts to be coupled and are held together by through bolt.

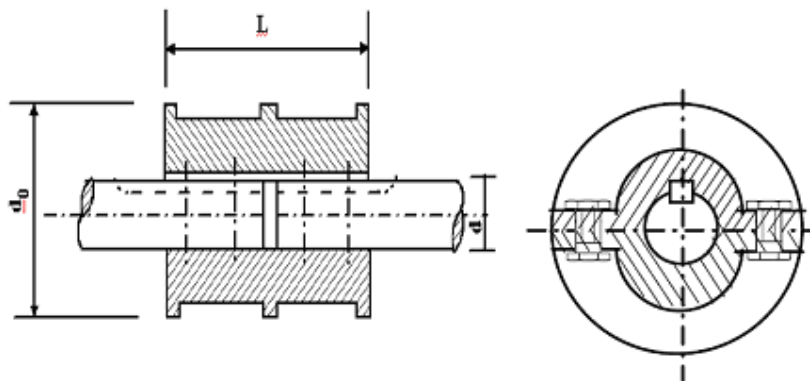


Figure- A representative clamp coupling

The length of these couplings ' L ' usually vary between 3.5 to 5 times the and the outside diameter ' d_0 ' of the coupling sleeve between 2 to 4 times the shaft diameter d . It is assumed that even with a key the torque is transmitted due to the friction grip. If now the number of bolt on each half is n , its core diameter is d_c and the coefficient of friction between the shaft and sleeve material is μ we may find the torque transmitted T as follows.

The **clamping pressure** between the shaft and the sleeve is given by

$$p = \frac{n}{2} \times \frac{\pi}{4} d_c^2 \times \sigma_t / (dL / 2)$$

Where **n** is the total **number of bolts**, the number of effective bolts for each shaft is **n/2** and σ_t is the allowable tensile stress in the bolt. The tangential force per unit area in the shaft periphery is $F = \mu p$. The torque transmitted can therefore be given by

$$T = \frac{\pi d L}{2} \mu p \cdot \frac{d}{2}$$

Ring compression type couplings

The coupling (figure below) consists of two cones which are placed on the shafts to be coupled and a sleeve that fits over the cones. Three bolts are used to draw the cones towards each other and thus wedge them firmly between the shafts and the outer sleeve. The usual proportions for these couplings in terms of shaft diameter d are approximately as follows:

$d_1 = 2d + 15.24 \text{ mm}$	$L_1 = 3d$
$d_2 = 2.45d + 27.94 \text{ mm}$	$L_2 = 3.5d + 12.7 \text{ mm}$
$d_3 = 0.23d + 3.17 \text{ mm}$	$L_3 = 1.5d$

And the taper of the cone is approximately 1 in 4 on diameters.

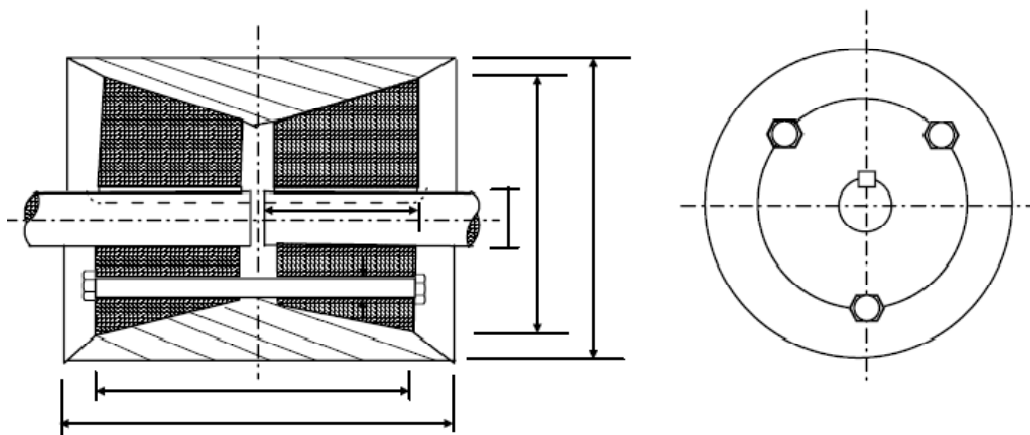


Figure- A representative ring compression type coupling.

Oldham coupling

These couplings can accommodate both **lateral and angular misalignment** to some extent. An Oldham coupling consists of two flanges with slots on the faces and the flanges are keyed or screwed to the shafts. A cylindrical piece, called the disc, has a narrow rectangular raised portion running across each face but at right angle to each other. The disc is placed between the flanges such that the raised portions fit into the slots in the flanges. The disc may be made of flexible materials and this absorbs some misalignment. A schematic representation is shown in figure below.

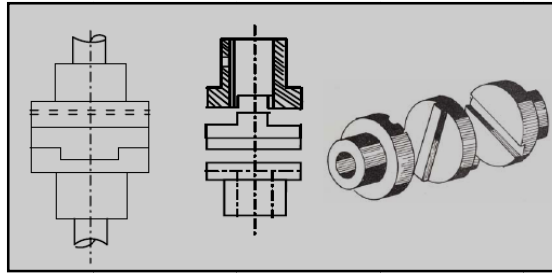


Figure- A schematic diagram of an Oldham coupling

Universal joints (or Hooke's or) coupling

These joints are capable of handling relatively **large angular misalignment** and they are widely used in agricultural machinery, machine tools and automobiles. A typical universal joint is shown in figure below. There are many forms of these couplings, available commercially but they essentially consist of two forks keyed or screwed to the shaft. There is a center piece through which pass two pins with mutually perpendicular axes and they connect the two fork ends such that a large angular misalignment can be accommodated. The coupling, often known as, **Hooke's coupling** has no torsional rigidity nor can it accommodate any parallel offset.

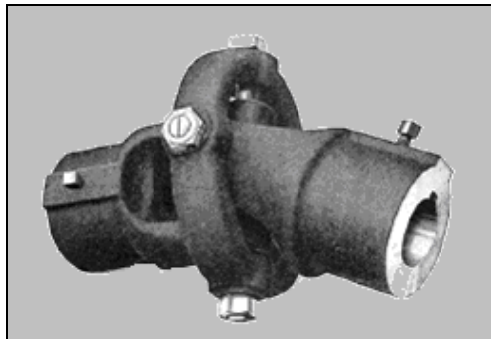


Figure- -Universal joints (or Hooke's or) coupling

Design procedures for rigid and flexible rubber-bushed couplings

Flange coupling

It is a very widely used rigid coupling and consists of two flanges keyed to the shafts and bolted. This is illustrated in figure below.

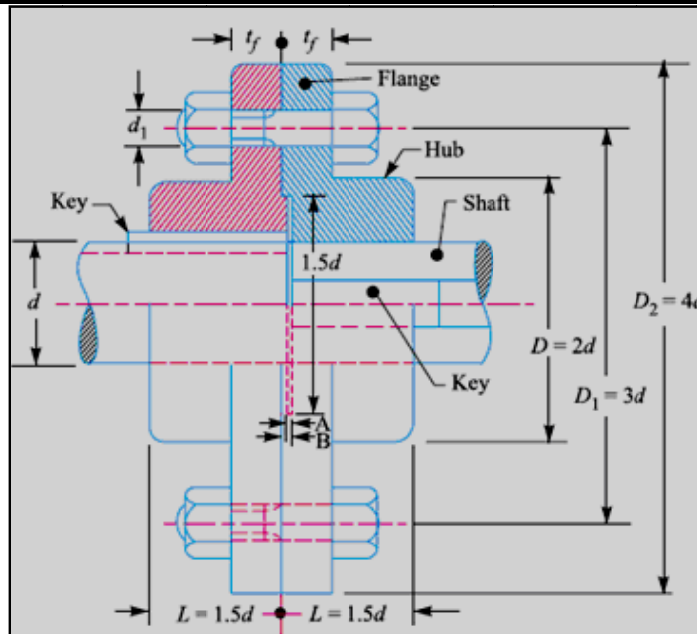


Figure- A typical flange coupling

The main features of the design are essentially

- (a) Design of bolts.
- (b) Design of hub.
- (c) Overall design and dimensions.

Rigid Flange Coupling

A typical rigid flange coupling is shown in Figure above. It essentially consists of two cast iron flanges which are keyed to the shafts to be joined. The flanges are brought together and are bolted in the annular space between the hub and the protecting flange. The protective flange is provided to guard the projecting bolt heads and nuts. The bolts are placed equi-spaced on a bolt circle diameter and the number of bolts depends on the shaft diameter d . A spigot 'A' on one flange and a recess on the opposing face is provided for ease of assembly.

The design procedure is generally based on determining the shaft diameter d for a given torque transmission and then following empirical relations different dimensions of the coupling are obtained. Check for different failure modes can then be carried out. Design procedure is given in the following steps:

- (1) Shaft diameter ' d ' based on torque transmission is given by

$$d = \left(\frac{16T}{\pi \tau_s} \right)^{1/3}$$

Where T is the torque and τ_y is the yield stress in shear.

- (2) Hub diameter, $d_1 = 1.75d + 6.5 \text{ mm}$

Design of Friction Drives

S K Mondal's

Chapter 2

(3) Hub length, $L = 1.5d$

But the hub length also depends on the length of the key. Therefore this length L must be checked while finding the key dimension based on shear and crushing failure modes.

(4) Key dimensions:

If a square key of side's b is used then b is commonly taken as $\frac{d}{4}$. In that case, for shear failure we have

$$\left(\frac{d}{4} \cdot L_k \right) \cdot \tau_y \cdot \frac{d}{2} = T$$

Where τ_y is the yield stress in shear and L_k is the key length.

This gives

$$L_k = \frac{8T}{d^2 \tau_y}$$

If L_k determined here is less than hub length L we may assume the key length to be the same as hub length.

For crushing failure we have

$\left(\frac{d}{8} \cdot L_k \right) \sigma_c \cdot \frac{d}{2} = T$ Where σ_c is crushing stress induced in the key. This gives

$$\sigma_c = \frac{16T}{L_k d^2}$$

And if $\sigma_c < \sigma_{cy}$, the bearing strength of the key material, the key dimensions chosen are in order.

(5) Bolt dimensions:

The bolts are subjected to shear and bearing stresses while transmitting torque.

Considering the **shear failure** mode we have

$$= n \cdot \frac{\pi}{4} d_b^2 \tau_{yb} \frac{d_c}{2}$$

Where n is the number of bolts, d_b nominal bolt diameter, T is the torque transmitted, τ_{yb} is the shear yield strength of the bolt material and d_c is the bolt circle diameter. The bolt

Design of Friction Drives

S K Mondal's

Chapter 2

diameter may now be obtained if n is known. The number of bolts n is often given by the following empirical relation:

$$n = \frac{4}{150}d + 3$$

Where d is the shaft diameter in mm. The bolt circle diameter must be such that it should provide clearance for socket wrench to be used for the bolts. The empirical relation takes care of this.

Considering **crushing failure** we have

$$= n \cdot d_b t_2 \sigma_{cyb} \frac{d_c}{2}$$

Where t_2 is the flange width over which the bolts make contact and σ_{cyb} is the yield crushing strength of the bolt material. This gives t_2 . Clearly the bolt length must be more than $2t_2$ and a suitable standard length for the bolt diameter may be chosen from hand book.

(6) A protecting flange is provided as a guard for bolt heads and nuts. The thickness t_3 is less than $\frac{t_2}{2}$ the corners of the flanges should be rounded.

(7) The spigot depth is usually taken between 2-3mm.

(8) Another check for the shear failure of the hub is to be carried out. For this failure mode we may write

$$= \pi d_1 t_2 \tau_{yf} \frac{d_1}{2}$$

Where d_1 is the hub diameter and τ_{yf} is the shear yield strength of the flange material.

Knowing τ_{yf} we may check if the chosen value of t_2 is satisfactory or not.

Finally, knowing hub diameter d_1 , bolt diameter and protective thickness t_2 We may decide the overall diameter d_3 .

Flexible rubber – bushed couplings

Flexible coupling

Design of Friction Drives

S K Mondal's

Chapter 2

As discussed earlier these couplings can accommodate some misalignment and impact. A large variety of flexible couplings are available commercially and principal features of only a few will be discussed here.

This is simplest type of flexible coupling and a typical coupling of this type is shown in Figure below.

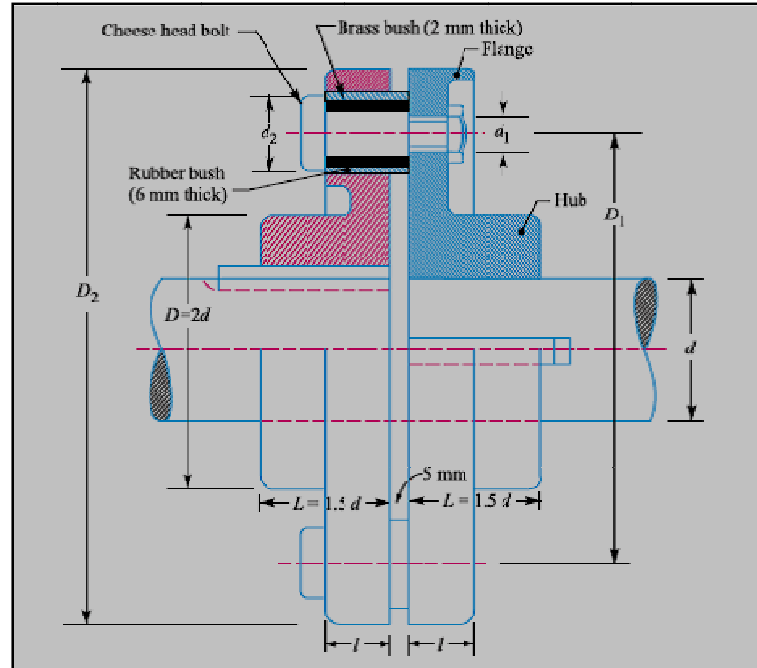


Figure- A typical flexible coupling with rubber bushings.

In a rigid coupling the torque is transmitted from one half of the coupling to the other through the bolts and in this arrangement shafts need be aligned very well.

However in the bushed coupling the rubber bushings over the pins (bolts) (as shown in Figure above) provide flexibility and these coupling can accommodate some misalignment.

Because of the rubber bushing the design for pins should be considered carefully.

(1) Bearing stress

Rubber bushings are available for different inside and out side diameters. However rubber bushes are mostly available in thickness between 6 mm to 7.5 mm for bores upto 25 mm and 9 mm thickness for larger bores. Brass sleeves are made to suit the requirements. However, brass sleeve

Thickness may be taken to be 1.5mm. The outside diameter of rubber bushing d_r is given by

$$d_r = d_b + 2 t_{br} + 2 t_r$$

Where d_b is the diameter of the bolt or pin, t_{br} is the thickness of the brass sleeve and t_r is the thickness of rubber bushing. We may now write

$$n \cdot d_r \cdot t_2 \cdot p_b \cdot \frac{d_c}{2} = T$$

Where d_c is the bolt circle diameter and t_2 the flange thickness over the bush contact area. A suitable bearing pressure for rubber is 0.035 N/mm^2 and the **number of pin** is given by

$$n = \frac{d}{25} + 3 \quad \text{where } d \text{ is in mm.}$$

The d_c here is different from what we had for rigid flange bearings. This must be judged considering the hub diameters, outside diameter of the bush and a suitable clearance. A rough drawing is often useful in this regard.

From the above torque equation we may obtain bearing pressure developed and compare this with the bearing pressure of rubber for safety.

(2) Shear stress

The pins in the coupling are subjected to shear and it is a good practice to ensure that the shear plane avoids the threaded portion of the bolt. Unlike the rigid coupling the shear stress due to torque transmission is given in terms of the tangential force F at the outside diameter of the rubber bush.

Shear stress at the neck area is given by

$$\tau_b = \frac{p_b t_2 d_r}{\frac{\pi}{4} d_{neck}^2}$$

Where d_{neck} is bolt diameter at the neck i.e. at the shear plane.

Bending Stress

The pin loading is shown in Figure below.

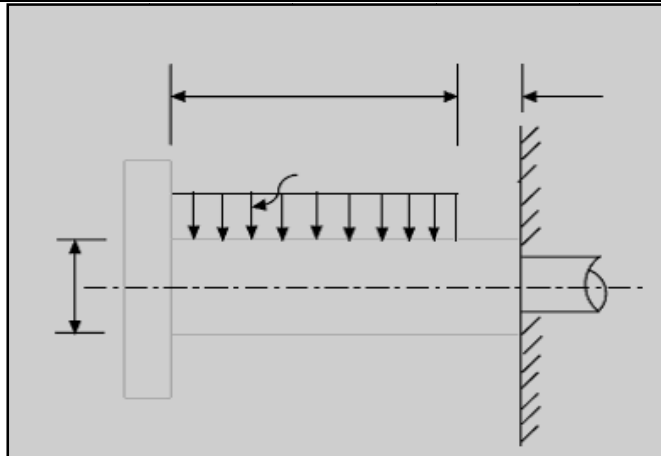


Figure- Loading on a pin supporting the bushings.

Clearly the bearing pressure that acts as distributed load on rubber bush would produce bending of the pin. Considering an equivalent concentrated load $F = pt_2d$ the bending stress is

$$\sigma_b = \frac{32F(t_2 / 2)}{\pi d_{br}^3}$$

Knowing the shear and bending stresses we may check the pin diameter for principal stresses using appropriate theories of failure.

We may also assume the following empirical relations:

Hub diameter = 2d

Hub length = 1.5d

$$\text{Pin diameter at the neck} = \frac{0.5d}{\sqrt{n}}$$

Design of Friction Drives

S K Mondal's

Chapter 2

Problems with Solution

- Q. Design a typical rigid flange coupling for connecting a motor and a centrifugal pump shafts. The coupling needs to transmit 15 KW at 1000 rpm. The allowable shear stresses of the shaft, key and bolt materials are 60 MPa, 50 MPa and 25 MPa respectively. The shear modulus of the shaft material may be taken as 84GPa. The angle of twist of the shaft should be limited to 1 degree in 20 times the shaft diameter.

Solution: The shaft diameter based on strength may be given by

$$d = \sqrt[3]{\frac{16T}{\pi\tau_y}} \quad \text{Where } T \text{ is the torque transmitted and } \tau_y \text{ is the allowable yield stress in shear.}$$

$$\text{Here } T = \text{Power} / \left(\frac{2\pi N}{60} \right) = \frac{15 \times 10^3}{\left(\frac{2\pi \times 1000}{60} \right)} = 143 \text{ Nm}$$

And substituting $\tau_y = 60 \times 10^6 \text{ Pa}$ we have.

Let us consider a shaft of 25 mm which is a standard size.

From the rigidity point of view

$$\frac{T}{J} = \frac{G\theta}{L}$$

Substituting $T = 143 \text{ Nm}$, $J = \frac{\pi}{32}(0.025)^4 = 38.3 \times 10^{-9} \text{ m}^4$, $G = 84 \times 10^9 \text{ Pa}$

$$\begin{aligned} \frac{\theta}{L} &= \frac{143}{38.3 \times 10^{-9} \times 84 \times 10^9} \\ &= 0.044 \text{ radian per meter} \end{aligned}$$

The limiting twist is 1 degree in 20 times the shaft diameter

Which is $\frac{\frac{\pi}{180}}{20 \times 0.025} = 0.035 \text{ radian per meter}$

Therefore, the shaft diameter of 25mm is safe.

We now consider a typical rigid flange coupling as shown in Figure above

Hub

Design of Friction Drives

S K Mondal's

Chapter 2

Using empirical relations

Hub diameter $d_1 = 1.75d + 6.5$ mm. This gives

$$d_1 = 1.75 \times 25 + 6.5 = 50.25 \text{ mm say } d_1 = 51 \text{ mm}$$

Hub length $L = 1.5d$. This gives $L = 1.5 \times 25 = 37.5$ mm, say $L = 38$ mm.

$$\text{Hub thickness, } t_1 = \frac{d_1 - d}{2} = \frac{51 - 25}{2} = 13 \text{ mm}$$

Key

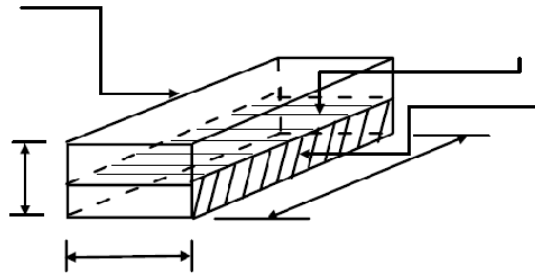
Now to avoid the shear failure of the key (refer to Figure above)

$$\left(\frac{d}{4} L_k \right) \cdot \tau_y \cdot \frac{d}{2} = T, \text{ Where the key width } w = \frac{d}{4} \text{ and the key length is } L_k$$

$$\text{This gives } L_k = \frac{8T}{(\tau_y d^2)} \text{ i.e. } \frac{8 \times 143}{50 \times 10^6 \times (0.025)^2} = 0.0366 \text{ m} = 36.6 \text{ mm}$$

The hub length is 37.5 mm. Therefore we take $L_k = 37.5$ mm.

To avoid crushing failure of the key (Refer to Figure below)



$$= \left(\frac{d}{8} L_k \right) \sigma \cdot \frac{d}{2}, \text{ Where } \sigma \text{ is the crushing stress developed in the key.}$$

$$\text{This gives } \sigma = \frac{16T}{L_k d^2}$$

Substituting $T = 143 \text{ Nm}$, $L_k = 37.5 \times 10^{-3} \text{ m}$ and $d = 0.025 \text{ m}$

$$\sigma = \frac{16 \times 143 \times 10^{-6}}{37.5 \times 10^{-3} \times (0.025)^2} = 97.62 \text{ MPa}$$

Assuming an allowable crushing stress for the key material to be 100 MPa, the key design is safe. Therefore the key size may be taken as: a square key of 6.25 mm size and 37.5 mm long. However keeping in mind that for a shaft of diameter between 22 mm and 30 mm a rectangular key of 8 mm width, 7 mm depth and length between 18 mm and 90 mm is

Design of Friction Drives

S K Mondal's

Chapter 2

recommended. We choose a standard key of 8mm width, 7mm depth and 38mm length which is safe for the present purpose.

Bolts

To avoid shear failure of bolts

$$= n \frac{\pi}{4} d_b^2 \tau_{yb} \frac{d_c}{2}$$

Where **number of bolts n** is given by the empirical relation

$$n = \frac{4}{150} d + 3 \quad \text{Where } d \text{ is the shaft diameter in mm.}$$

Which gives $n=3.66$ and we may take $n=4$ or more.

Here τ_{yb} is the allowable shear stress of the bolt and this is assumed to be 60 MPa.

d_c is the bolt circle diameter and this may be assumed initially based on hub diameter $d_1=51$ mm and later the dimension must be justified Let $d_c=65$ mm.

Substituting the values we have the **bolt diameter** (d_b) as

$$d_b = \left(\frac{8T}{n\pi\tau_{yb}d_c} \right)^{\frac{1}{2}} \text{ i.e. } \left(\frac{8 \times 143}{4\pi \times 25 \times 10^6 \times 65 \times 10^{-3}} \right)^{\frac{1}{2}} = 7.48 \times 10^{-3}$$

Which gives $d_b = 7.48$ mm.

With higher factor of safety we may take $d_b = 10$ mm which is a standard size.

We may now check for crushing failure as

$$nd_b t_2 \sigma_c \frac{d_c}{2}$$

Substituting $n=4$, $d_b=10$ mm, $\sigma_c=100$ MPa, $d_c=65$ mm & $T = 143$ Nm and this gives $t_2 = 2.2$ mm.

$$\text{However empirically we have } t_2 = \frac{1}{2} t_1 + 6.5 = 13 \text{ mm}$$

Therefore we take $t_2=13$ mm which gives higher factor of safety.

Protecting flange thickness

Protecting flange thickness t_3 is usually less than $\frac{1}{2}t_2$ we therefore take $t_3 = 8\text{mm}$ since there is no direct load on this part.

Spigot depth

Spigot depth which is mainly provided for location may be taken as 2mm.

Check for the shear failure of the hub

To avoid shear failure of hub we have

$$T = \pi d_1 t_2 \tau_f \frac{d_1}{2}$$

Substituting $d_1 = 51\text{mm}$, $t_2 = 13\text{mm}$ and $T = 143\text{Nm}$, we have shear stress in flange τ_f as

$$\tau_f = \frac{2T}{(\pi d_1^2 t_2)}$$

And this gives $\tau_f = 2.69\text{ MPa}$ which is much less than the yield shear value of flange material 60MPa.

- Q. Determine the suitable dimensions of a rubber bush for a flexible coupling to connect of a motor and a pump. The motor is of 50 KW and runs at 300rpm. The shaft diameter is 50mm and the pins are on pitch circle diameter of 140mm. The bearing pressure on the bushes may be taken as 0.5MPa and the allowable shear and bearing stress of the pin materials are 25 MPa and 50 MPa respectively. The allowable shear yield strength of the shaft material may be taken as 60MPa.**

Solution: A typical pin in a bushed flexible coupling is as shown in Figure below.

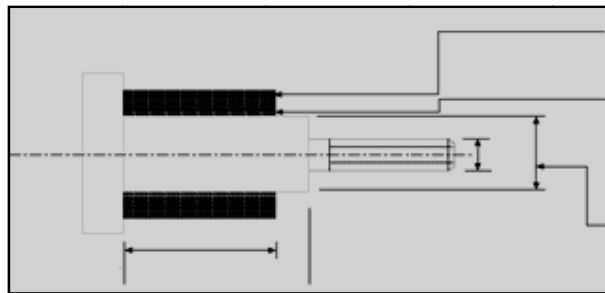


Figure- A typical pin for the bushings

There is an enlarged portion on which a flexible bush is fitted to absorb the misalignment. The threaded portion provided for a nut to tighten on the flange. Considering the whole pin there are three basic stresses developed in the pin in addition to the tightening stresses. There are (a) shear stresses at the unthreaded neck area (b) bending stress over the loaded portion (L) of the enlarged portion of the pin and (c) bearing stress.

However, before we consider the stresses we need to determine the pin diameter and length. Here the torque transmitted

Design of Friction Drives

S K Mondal's

Chapter 2

$$T = \frac{50 \times 10^3}{\frac{2\pi \times 3000}{60}} = 159 \text{ Nm}$$

Based on torsional shear of the shaft diameter, $(d) = \left(\frac{16T}{\pi \tau_y} \right)^{\frac{1}{3}}$

Substituting $T=159\text{Nm}$ and $\tau_y = 60\text{MPa}$, we have $d = 23.8\text{mm}$. Let the shaft diameter be 25mm . From empirical relations we have

Pin diameter at the neck , $(d_{neck}) = \frac{0.5d}{\sqrt{n}}$

Where the number of pins, $(n) = \frac{4d}{150} + 3$

Substituting $d = 25 \text{ mm}$ we have

$$n = 3.67 \text{ (say) } 4$$

$$d_{neck} = 6.25 \text{ (say) } 8\text{mm}$$

On this basis the shear stress at the neck $= \frac{T}{\left[\frac{\pi}{4} d_{neck}^2 n \frac{d_c}{2} \right]}$ which gives 11.29 MPa and this

is much less than yield stress of the pin material.

There is no specific recommendation for the enlarged diameter based on d_{neck} but the enlarged diameters should be enough to provide a neck for tightening. We may choose

$d_{enlarged} = 16\text{mm}$ which is a standard size. Therefore we may determine the inner diameter of the rubber bush as

$$d_{bush} = \text{Enlarged diameter of the pin} + 2 \times \text{brass sleeve thickness.}$$

A brass sleeve of 2mm thickness is sufficient and we have

$$d_{bush} = 20\text{mm}$$

Rubber bush of core diameter up to 25mm are available in thickness of 6mm . Therefore we choose a bush of core diameter 20mm and thickness 6mm .

In order to determine the **bush length** we have

$$T = npLd_{bush} \frac{d_c}{2}$$

Where p is the bearing pressure, (L_{dbush}) is the projected area and d_c is the pitch circle diameter. Substituting $T = 159\text{Nm}$, $p = 0.5\text{MPa}$, $d_{bush} = 0.02\text{m}$ and $d_c = 0.14\text{m}$ we have $L = 56.78\text{ mm}$.

The rubber bush chosen is therefore of 20mm bore size, 6mm wall thickness and 60 mm long.

POWER SCREW

Introduction

A power screw is a drive used in machinery to convert a rotary motion into a linear motion for power transmission. It produces uniform motion and the design of the power screw may be such that

- (a) Either the screw or the nut is held at rest and the other member rotates as it moves axially. A typical example of this is a screw **clamp**.
- (b) Either the screw or the nut rotates but does not move axially. A typical Example for this is a **press**.

Other applications of power screws are **jack screws, lead screws of a lathe, screws for vices, presses etc.**

Power screw normally uses square threads but ACME or Buttress threads may also be used. Power screws should be designed for smooth and noiseless transmission of power with an ability to carry heavy loads with high efficiency. We first consider the different thread forms and their proportions:

Square threads

The thread form is shown in figure below. These threads have high efficiency but they are difficult to manufacture and are expensive. The proportions in terms of pitch are:

$$h_1 = 0.5 p; h_2 = 0.5 p - b; H = 0.5 p + a; e = 0.5 p$$

a and b are different for different series of threads.

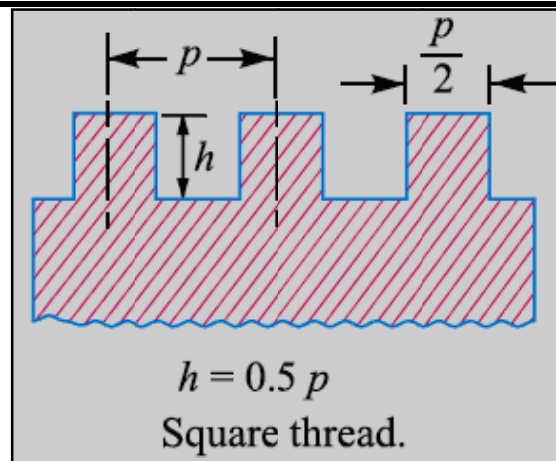


Figure- Some details of square thread form

There are different series of this thread form and some nominal diameters, corresponding pitch and dimensions a and b as per I.S. 4694-1968.

According to IS-4694-1968, a square thread is designated by its nominal diameter and pitch, as for example, SQ 10 x 2 designates a thread form of nominal diameter 10 mm and pitch 2 mm.

Acme or Trapezoidal threads

The Acme thread form is shown in figure below. These threads may be used in applications such as lead screw of a lathe where loss of motion cannot be tolerated. The included angle $2\phi = 29^\circ$ and other proportions are

$$a = \frac{p}{2.7} \text{ And } h = 0.25 p + 0.25 \text{ mm}$$

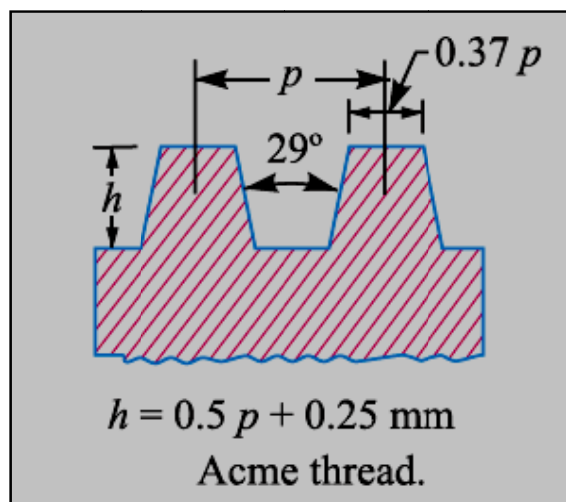


Figure- Some details of **Acme or Trapezoidal** threads forms.

A metric trapezoidal thread form is shown in figure below and different proportions of the thread form in terms of the pitch are as follows:

Design of Friction Drives

S K Mondal's

Chapter 2

Inclined angle = 30° ; $H_1 = 0.5 p$; $z = 0.25 p + H_1/2$; $H_3 = h_3 = H_1 + a_c = 0.5 p + a_c$

a_c is different for different pitch, for example

$a_c = 0.15$ mm for $p = 1.5$ mm ; $a_c = 0.25$ mm for $p = 2$ to 5 mm;

$a_c = 0.5$ mm for $p = 6$ to 12 mm ; $a_c = 1$ mm for $p = 14$ to 44 mm.

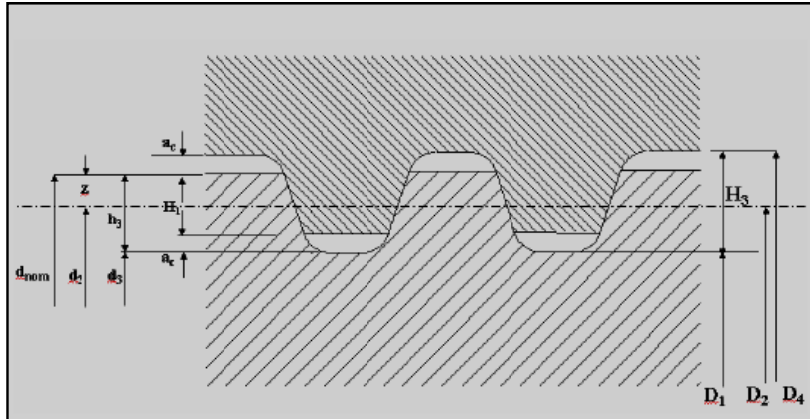


Figure- Some details of a metric Trapezoidal thread form.

Some standard dimensions for a trapezoidal thread form are given in table below as per IS 7008 (Part II and III) - 1973:

Nominal Diameter (mm)	8	10	5	25	50	75	100	150	200	250	300
pitch (mm)	1.5	2	4	5	8	10	12	16	18	22	24

Dimensions of a trapezoidal thread form.

According to IS7008-1973 trapezoidal threads may be designated as, for example, Tr 50 x 8 which indicates a nominal diameter of 50 mm and a pitch of 8 mm.

Buttress thread

This thread form can also be used for power screws but they can transmit power only in one direction. Typical applications are screw jack, vices etc.

A Buttress thread form is shown in figure below, and the proportions are shown in the figure in terms of the pitch.

On the whole the square threads have the highest efficiency as compared to other thread forms but they are less sturdy than the trapezoidal thread forms and the adjustment for wear is difficult for square threads.

When a large linear motion of a power screw is required two or more parallel threads are used. These are called multiple start power drives.

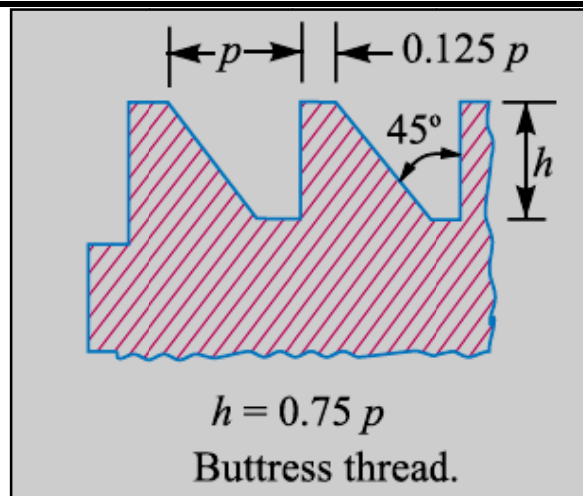


Figure- details of a Buttress thread form

Efficiency of a Power screw

A square thread power screw with a single start is shown in figure below. Here p is the pitch, α the helix angle, d_m the mean diameter of thread and F is the axial load. A developed single thread is shown in figure below **where** $L = n p$ for a multi-start drive, n being the number of starts. In order to analyze the mechanics of the power screw we need to consider two cases:

- (a) Raising the load
- (b) Lowering the load.

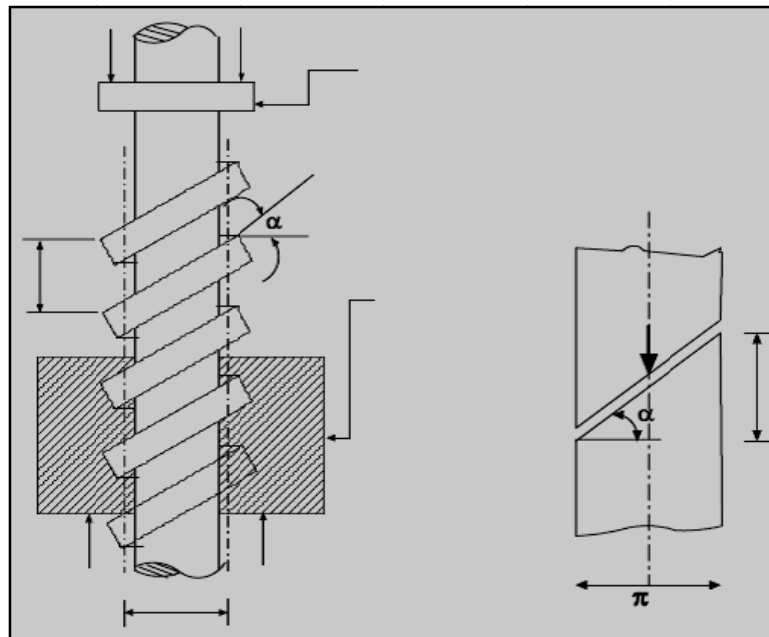


Fig. A square thread power screw Fig. Development of a single thread

Raising the load

This requires an axial force P as shown in figure below. Here N is the normal reaction and μN is the frictional force.

Design of Friction Drives

S K Mondal's

Chapter 2

For equilibrium

$$P - \mu N \cos \alpha - N \sin \alpha = 0$$

$$W + \mu N \sin \alpha - N \cos \alpha = 0$$

This gives

$$N = W / (\cos \alpha - \mu \sin \alpha)$$

$$P = \frac{W(\mu \cos \alpha + \sin \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Where

P= Effort applied at the circumference of the screw to lift the load.

W= load to be lifted.

μ = coefficient of friction ($\tan \phi$),

where ϕ is friction angle

α = helix angle

p= pitch of the screw

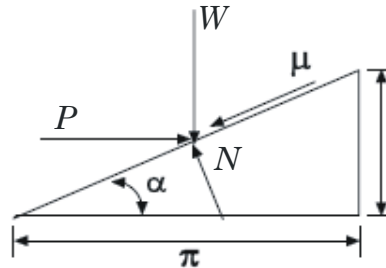


Fig. Forces at the contact surface
For raising the load.

Torque transmitted during raising the load is then given by

$$T_R = P \frac{d_m}{2} = W \frac{d_m}{2} \frac{(\mu \cos \alpha + \sin \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Since, $\tan \alpha = \frac{L}{\pi d_m}$ we have

$$T_R = W \frac{d_m}{2} \frac{(\mu \pi d_m + L)}{(\pi d_m - \mu L)}$$

The force system at the thread during lowering the load is shown in figure below. For equilibrium

$$P - \mu N \cos \alpha + N \sin \alpha = 0$$

$$F - N \cos \alpha - \mu N \sin \alpha = 0$$

This gives

$$N = W / (\cos \alpha + \mu \sin \alpha)$$

$$P = \frac{W(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

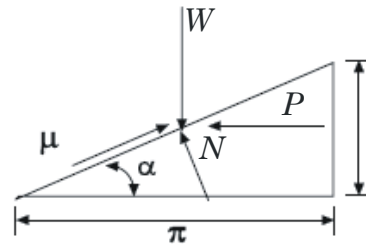


Fig. Forces at the contact
Surface for lowering the load.

Torque required to lower the load is given by

$$T_L = P \frac{d_m}{2} = W \frac{d_m}{2} \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

And again taking $\tan \alpha = \frac{L}{\pi d_m}$ we have

$$T_L = F \frac{d_m}{2} \frac{(\mu \pi d_m - L)}{(\pi d_m + \mu L)}$$

Condition for self locking

The load would lower itself without any external force if

$$\mu \pi d_m < L$$

And some external force is required to lower the load if

$$\mu \pi d_m \geq L$$

This is therefore the condition for self locking.

Efficiency of the power screw is given by

$$\eta = \frac{\text{Work output}}{\text{Work input}}$$

Here work output = $F \cdot L$

Work input = $P \cdot \pi d_m$

This gives

$$\eta = \frac{W}{P} \tan \alpha$$

The above analysis is for square thread and **for trapezoidal thread** some modification is required. Because of the thread angle the force normal to the thread surface is increased as shown in figure below. The torque is therefore given by

$$T = W \frac{d_m}{2} \frac{(\mu \pi d_m \sec \phi + L)}{(\pi d_m - \mu L \sec \phi)}$$

This considers the increased friction due to the wedging action. The trapezoidal threads are not preferred because of high friction but often used due to their ease of machining.

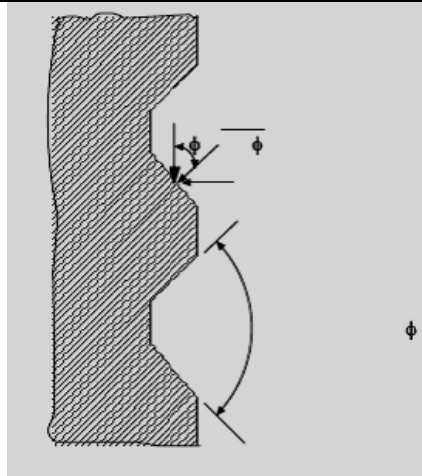


Figure- Normal force on a trapezoidal thread surface

Bursting effect on the nut

Bursting effect on the nut is caused by the horizontal component of the axial load F on the screw and this is given by (figure above)

$$F_x = F \tan \phi$$

For an ISO metric nut $2\phi = 60^\circ$ and $F_x = 0.5777 F$.

Collar friction

If collar friction μ_c is considered then another term $\mu F d_c/2$ must be added to torque expression. Here d_c is the effective friction diameter of the collar. Therefore we may write the torque required to raise the load as

$$T = W \frac{d_m}{2} \frac{(\mu \pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c W \frac{d_c}{2}$$

Problems with Solution

- Q. The C-clamp shown in figure below uses a 10 mm screw with a pitch of 2 mm. The frictional coefficient is 0.15 for both the threads and the collar. The collar has a frictional diameter of 16 mm. The handle is made of steel with allowable bending stress of 165 MPa. The capacity of the clamp is 700 N.
- Find the torque required to tighten the clamp to full capacity.
 - Specify the length and diameter of the handle such that it will not bend unless the rated capacity of the clamp is exceeded. Use 15 N as the handle force.

Design of Friction Drives

S K Mondal's

Chapter 2

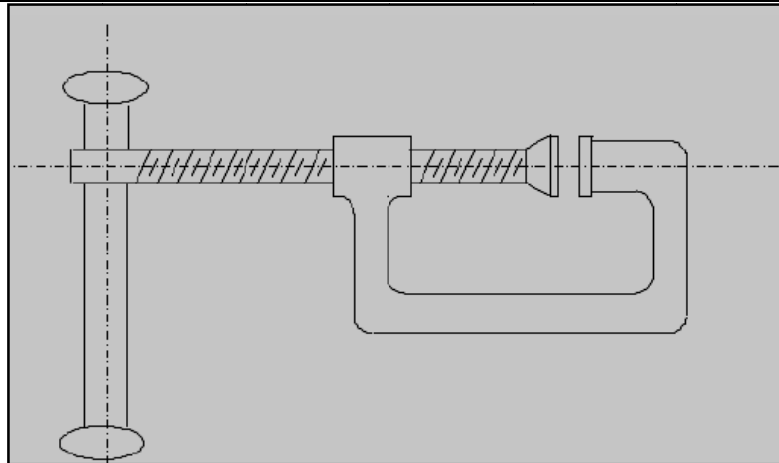


Figure- C- Clamp

Solution: (a) Nominal diameter of the screw, (d) = 10 mm.

Pitch of the screw, (p) = 2 mm.

Choosing a square screw thread we have the following dimensions:

Root diameter, (d₃) = d_{nominal} - 2h₃ = 7.5 mm (since a_c = 0.25 mm and h₃ = 0.5p + a_c)

Pitch diameter, d₂ = d_{nominal} - 2z = 8 mm. (since z = 0.5 p)

Mean diameter, d_m = (7.5+8)/2 = 7.75 mm.

$$\text{Torque, } T = F \frac{d_m}{2} \frac{(\mu \pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F \frac{d_c}{2}$$

Here F = 700 N, $\mu = \mu_c = 0.15$, L = p = 2 mm (assuming a single start screw thread) and d_c = 16 mm. This gives T = 1.48 Nm.

Equating the torque required and the torque applied by the handle of length L we have 1.48 = 15 L since the assumed handle force is 15 N.

This gives L = 0.0986 m. Let the handle length be 100 mm.

The maximum bending stress that may be developed in the handle is

$$\sigma = \frac{M_y}{I} = \frac{32M}{\pi d^3} \text{ Where } d \text{ is the diameter of the handle.}$$

Taking the allowable bending stress as 165 MPa we have

$$d = \left(\frac{32M}{\pi \sigma_y} \right)^{1/3} = \left(\frac{32 \times 1.48}{\pi \times 165 \times 10^6} \right)^{1/3} = 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm}$$

With a higher factor of safety let d = 10 mm.

Q. A single square thread power screw is to raise a load of 50 kN. A screw thread of major diameter of 34 mm and a pitch of 6 mm is used. The coefficient of friction at the thread and collar are 0.15 and 0.1 respectively. If the collar frictional diameter is 100 mm and the screw turns at a speed of 1 rev s⁻¹ find

(a) The power input to the screw.

(b) The combined efficiency of the screw and collar.

Solution: (a) Mean diameter, d_m = d_{major} - p/2 = 34 - 3 = 31 mm.

Design of Friction Drives

S K Mondal's

Chapter 2

$$\text{Torque } T = F \frac{d_m}{2} \frac{(\mu \pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F \frac{d_c}{2}$$

Here $F = 5 \times 10^3 \text{ N}$, $d_m = 31 \text{ mm}$, $\mu = 0.15$, $\mu_c = 0.1$, $L = p = 6 \text{ mm}$ and $d_c = 100 \text{ mm}$

$$\begin{aligned} \text{Therefore } T &= 50 \times 10^3 \times \frac{0.031}{2} \left(\frac{0.15\pi \times 0.031 + 0.006}{\pi \times 0.031 - 0.15 \times 0.006} \right) + 0.1 \times 50 \times 10^3 \times \frac{0.1}{2} \\ &= 416 \text{ Nm} \end{aligned}$$

$$\text{Power input} = T \omega = 416 \times 2\pi \times 1 = 2613.8 \text{ Watts.}$$

(b) The torque to raise the load only (T_0) may be obtained by substituting $\mu = \mu_c = 0$ in the torque equation. This gives

$$T_0 = F \frac{d_m}{2} \left(\frac{L}{\pi d_m} \right) = \frac{FL}{2\pi} = \frac{50 \times 10^3 \times 0.006}{2\pi} = 47.75$$

$$\text{Therefore } \eta = \frac{FL / 2\pi}{T} = \frac{47.75}{416} = 0.1147 \text{ i.e. } 11.47\%$$

Clutches

Introduction of Friction clutches

A *Clutch* is a machine member used to connect the driving shaft to a driven shaft, so that the driven shaft may be started or stopped at will, without stopping the driving shaft. A clutch thus provides an interruptible connection between two rotating shafts.

Clutches allow a high inertia load to be started with a small power.

A popularly known application of clutch is in automotive vehicles where it is used to connect the engine and the gear box. Here the clutch enables to crank and start the engine disengaging the transmission. Disengage the transmission and change the gear to alter the torque on the wheels. Clutches are also used extensively in production machinery of all types.

Mechanical Model

Two inertia's I_1 and I_2 traveling at the respective angular velocities ω_1 and ω_2 , and one of which may be zero, are to be brought to the same speed by engaging. Slippage occurs because the two elements are running at different speeds and energy is dissipated during actuation, resulting in temperature rise.

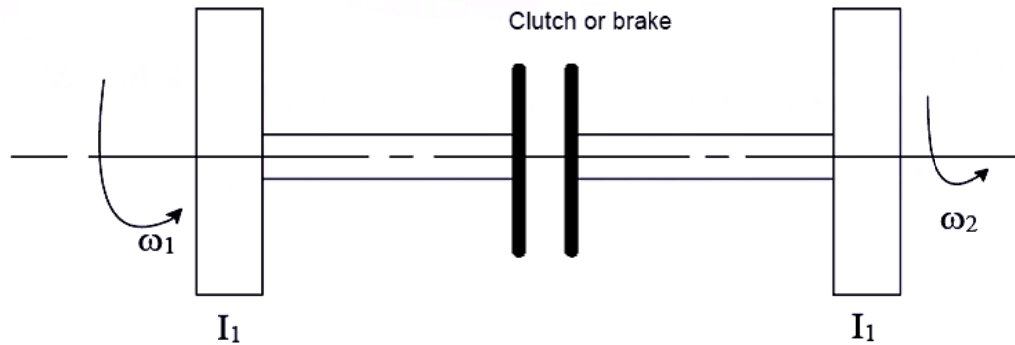


Figure- Dynamic Representation of Clutch or Brake

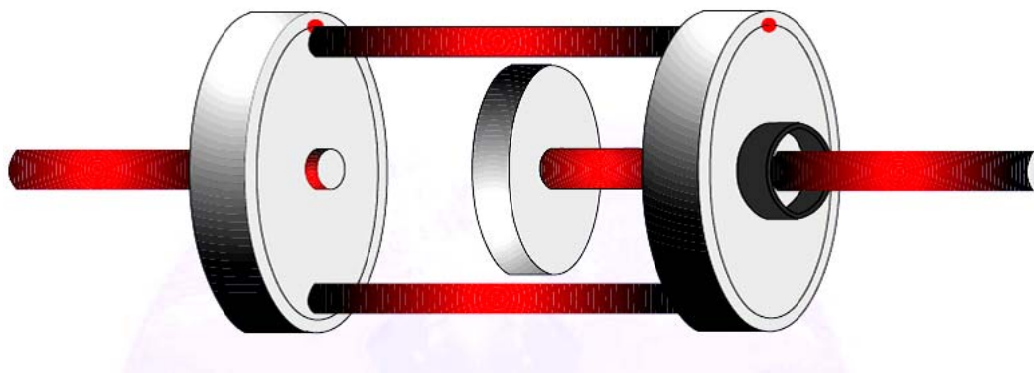


Figure- clutch view

To design analyze the performance of these devices, a knowledge on the Following are required.

1. The torque transmitted
2. The actuating force.
3. The energy loss.
4. The temperature rise.

FRICITION CLUTCHES

As in brakes a wide range of clutches are in use wherein they vary in there are in use their working principle as well the method of actuation and application of normal forces. The discussion here will be limited to mechanical type friction clutches or more specifically to the plate or disc clutches also known as axial clutches

Frictional Contact axial or Disc Clutches

An axial clutch is one in which the mating frictional members are moved in a direction parallel to the shaft. A typical clutch is illustrated in the figure below. It consists of a driving disc connected to the drive shaft and a driven disc connected to the driven shaft. A friction plate is attached to one of the members. Actuating spring keeps both the members in contact and power/motion is transmitted from one member to the other. When the power of motion is to be interrupted the driven disc is moved axially creating a gap between the members as shown in the figure below.

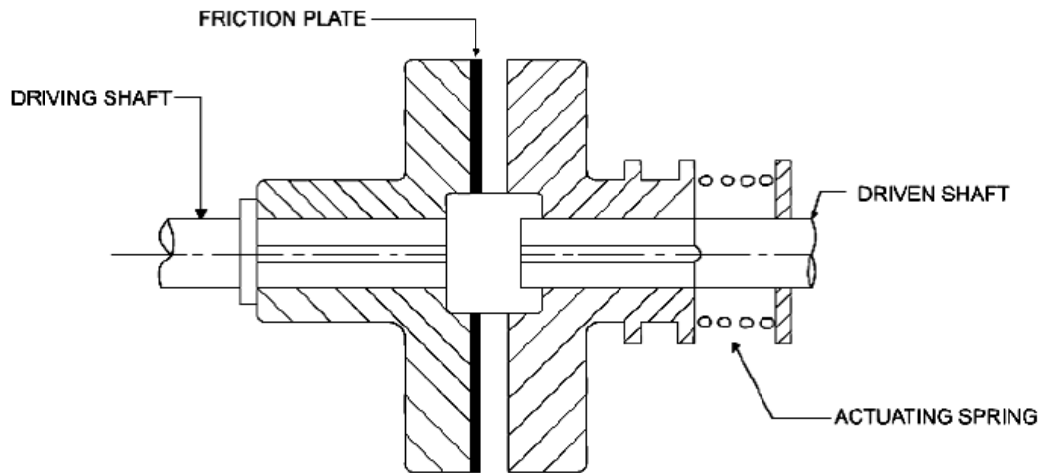


Figure- Disc Clutches

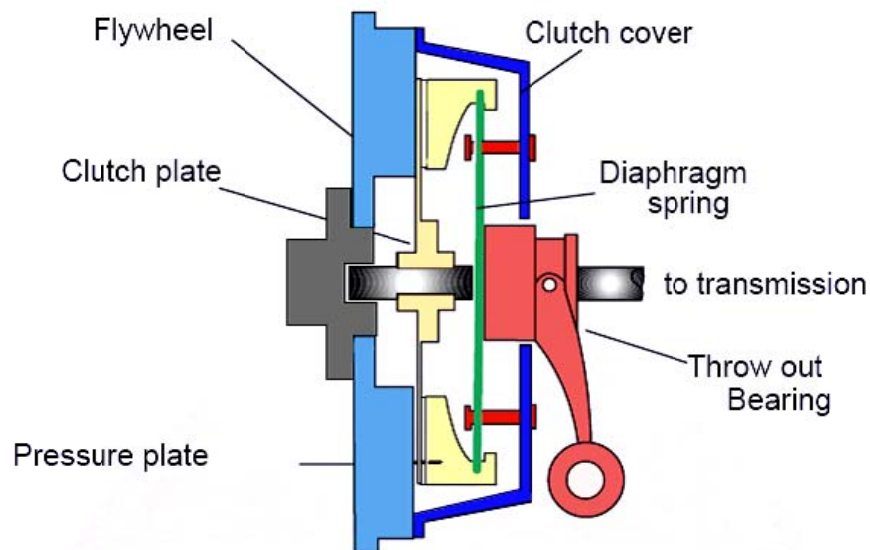


Fig.

METHOD OF ANALYSIS

The torque that can be transmitted by a clutch is a function of its geometry and the magnitude of the actuating force applied as well the condition of contact prevailing between the members. The applied force can keep the members together with a uniform pressure all over its contact area and the consequent analysis is based on uniform pressure condition.

Torque transmitting capacity

Uniform pressure theory

However as the time progresses some wear takes place between the contacting members and this may alter or vary the contact pressure appropriately and uniform pressure condition may no longer prevail. Hence the analysis here is based on uniform wear condition.

Elementary Analysis

Assuming uniform pressure and considering an elemental area dA

$$dA = 2\pi.r \, dr$$

The normal force on this elemental area is

$$dN = 2\pi.r.dr.p$$

The frictional force dF on this area is therefore

$$dF = f.2\pi.r.dr.p \quad (f = \text{co-efficient of friction})$$

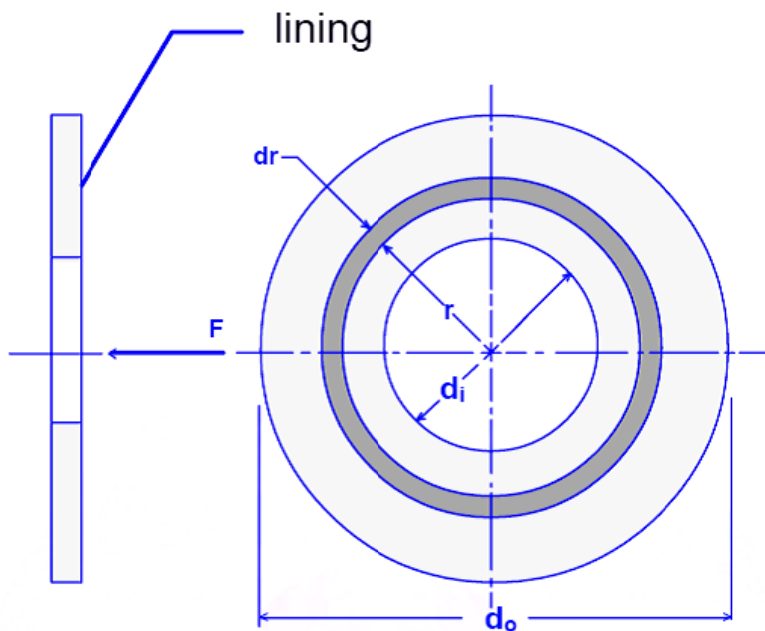


Figure- A single-Surface Axial Disk Clutch

Now the torque that can be transmitted by this elemental are is equal to the frictional force times the moment arm about the axis that is the radius 'r'

$$\begin{aligned} \text{i.e. } T &= dF.r = f.dN.r = f.p.A.r \\ &= f.p.2.\pi.r. \, dr .r \end{aligned}$$

The total torque that could be transmitted is obtained by integrating this equation between the limits of inner radius r_i to the outer radius r_o .

$$T = \int_{r_i}^{r_0} 2\pi p f r^2 dr = \frac{2}{3} \pi p f (r_0^3 - r_i^3)$$

Integrating the normal force between the same limits we get the actuating force that need to be applied to transmit this torque.

$$F_a = \int_{r_i}^{r_0} 2\pi p r dr$$

$$F_a = \pi (r_0^2 - r_i^2) \cdot p$$

Equation 1 and 2 can be combined together to give equation for the torque

$$T = f F_a \cdot \frac{2}{3} \frac{(r_0^3 - r_i^3)}{(r_0^2 - r_i^2)}$$

Uniform wear theory

According to some established theories the wear in a mechanical system is proportional to the 'PV' factor where P refers the contact pressure and V the sliding velocity. Based on this for the case of a plate clutch we can state

The constant-wear rate R_w is assumed to be proportional to the product of pressure p and velocity V .

$$R_w = pV = \text{constant}$$

And the velocity at any point on the face of the clutch is $V = r \cdot \omega$

Combining these equation, assuming a constant angular velocity ω

$$Pr = \text{constant} = K$$

The largest pressure p_{uma} must then occur at the smallest radius r_i ,

$$K = p_{\text{max}} r_i$$

Hence pressure at any point in the contact region

$$p = p_{\text{max}} \frac{r_i}{r}$$

In the previous equations substituting this value for the pressure term p and integrating between the limits as done earlier we get the equation for the torque transmitted and the actuating force to be applied.

I.e. The axial force F_a is found by substituting $p = p_{\text{max}} \frac{r_i}{r}$ for p .

And integrating equation $dN = 2\pi p r dr$

$$F = \int_{r_i}^{r_0} 2\pi p r dr = \int_{r_i}^{r_0} 2\pi \left(p_{\max} \frac{r_i}{r} \right) r dr = 2\pi p_{\max} r_i (r_0 - r_i)$$

Similarly the Torque

$$T = \int_{r_i}^{r_0} f 2\pi p_{\max} r_i r dr = f \pi p_{\max} r_i (r_0^2 - r_i^2)$$

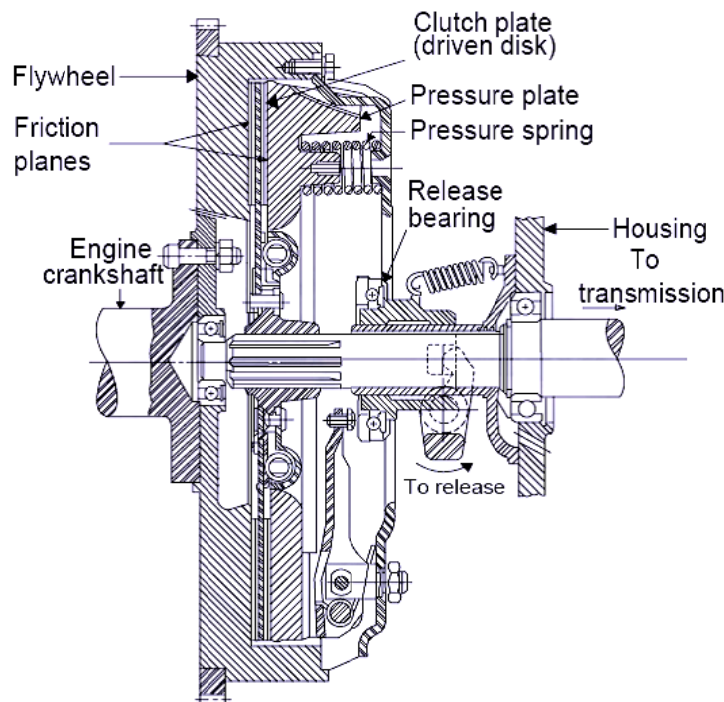
Substituting the values of actuating force F_a

The equation can be given as

$$T = f F_a \cdot \frac{(r_0 + r_i)}{2}$$

Single plates dry Clutch – Automotive application

The clutch used in automotive applications is generally a single plate dry clutch. In this type the clutch plate is interposed between the flywheel surface of the engine and pressure plate.



Figure

Single Clutch and Multiple Disk Clutch

Basically, the clutch needs three parts. These are the engine flywheel, a friction disc called the clutch plate and a pressure plate. When the engine is running and the flywheel is rotating, the pressure plate also rotates as the pressure plate is attached to the flywheel. The friction disc is located between the two. When the driver has pushed down the clutch pedal the clutch is released. This action forces the pressure plate to move away from the friction disc. There are

now air gaps between the flywheel and the friction disc, and between the friction disc and the pressure plate. No power can be transmitted through the clutch.

Multiple Plate Clutches

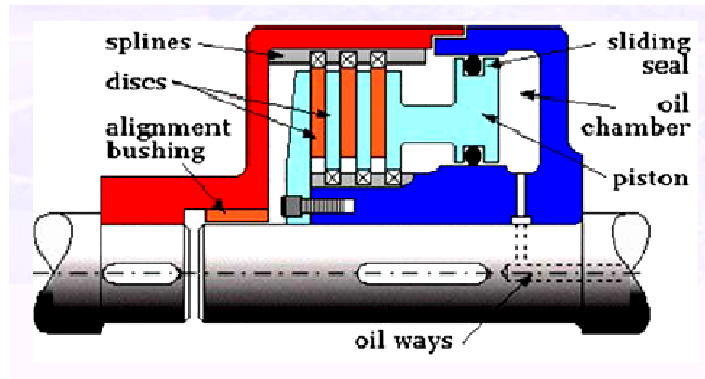


Fig. Multiple Plate Clutches

The properties of the frictional lining are important factors in the design of the clutches

Operation of Clutch

When the driver releases the clutch pedal, power can flow through the clutch. Springs in the clutch force the pressure plate against the friction disc. This action clamps the friction disk tightly between the flywheel and the pressure plate. Now, the pressure plate and friction disc rotate with the flywheel.

As both side surfaces of the clutch plate is used for transmitting the torque, a term 'N' is added to include the number of surfaces used for transmitting the torque.

By rearranging the terms the equations can be modified and a more general form of the equation can be written as

$$T = N \cdot f \cdot F_a \cdot R_m$$

Where

T = the torque (Nm).

N = the number of frictional discs in contact.

f = the coefficient of friction

F_a = the actuating force (N).

R_m = the mean or equivalent radius (m).

Note that $N = n_1 + n_2 - 1$

Where n_1 = number of driving discs.

n_2 = number of driven discs.

Design of Friction Drives

S K Mondal's

Chapter 2

Values of the actuating force F and the mean radius r_m for the two conditions of analysis are summarized.

Clutch Construction

Two basic types of clutch are the coil-spring clutch and the diaphragm-spring clutch. The difference between them is in the type of spring used. The coil spring clutch shown in left figure below uses coil springs as pressure springs (only two pressure springs is shown). The clutch shown in right figure below uses a diaphragm spring.



Fig.

The coil-spring clutch has a series of coil springs set in a circle.

Plate to hub Connection

Secondly the plate and its hub are entirely separate components, the drive being transmitted from one to the other through coil springs interposed between them. These springs are carried within rectangular holes or slots in the hub and plate and arranged with their axes aligned appropriately for transmitting the drive. These dampening springs are heavy coil springs set in a circle around the hub. The hub is driven through these springs. They help to smooth out the torsional vibration (the power pulses from the engine) so that the power flow to the transmission is smooth.

In a simple design all the springs may be identical, but in more sophisticated designs they are arranged in pairs located diametrically opposite, each pair having a different rate and different end clearances so that their role is progressive providing increasing spring rate to cater to wider torsional damping.

The clutch plate is assembled on a splined shaft that carries the rotary motion to the transmission. This shaft is called the clutch shaft, or transmission input shaft.

This shaft is connected to the gear box or forms a part of the gear box.

Friction facing or pads

Design of Friction Drives

S K Mondal's

Chapter 2

It is the friction pads or facing which actually transmit the power from the fly wheel to hub in the clutch plate and from there to the out put shaft. There are grooves in both sides of the friction-disc facings. These grooves prevent the facings from sticking to the flywheel face and pressure plate when the clutch is disengaged. The grooves break any vacuum that might form and cause the facings to stick to the flywheel or pressure plate. The facings on many friction discs are made of cotton and asbestos fibers woven or molded together and impregnated with resins or other binding agents. In many friction discs, copper wires are woven or pressed into the facings to give them added strength. However, asbestos is being replaced with other materials in many clutches. Some friction discs have ceramic-metallic facings.

Such discs are widely used in multiple plate clutches

To minimize the wear problems, all the plates will be enclosed in a covered Chamber and immersed in an oil medium

Such clutches are called wet clutches.

Cone clutches

At high rotational speeds, problems can arise with multi coil spring clutches owing to the effects of centrifugal forces both on the spring themselves and the lever of the release mechanism.

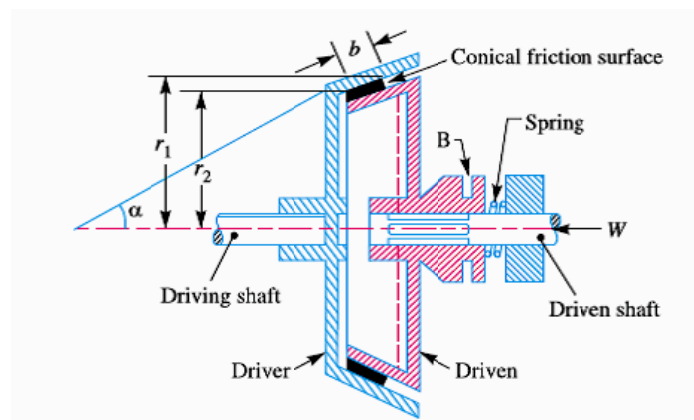


Figure- Cone clutches

These problems are obviated when diaphragm type springs are used, and a number of other advantages are also experienced.

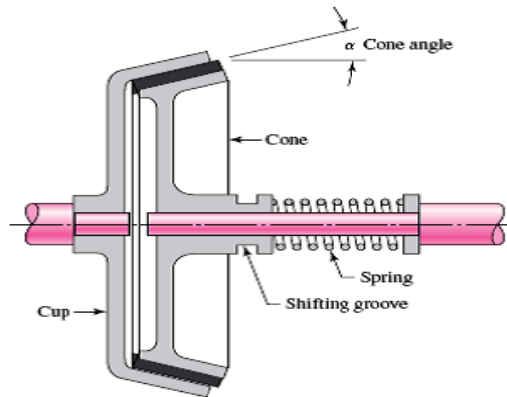


Figure- cross section of cone clutch

Centrifugal clutches

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in figure below. The outer surface of the shoes is covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

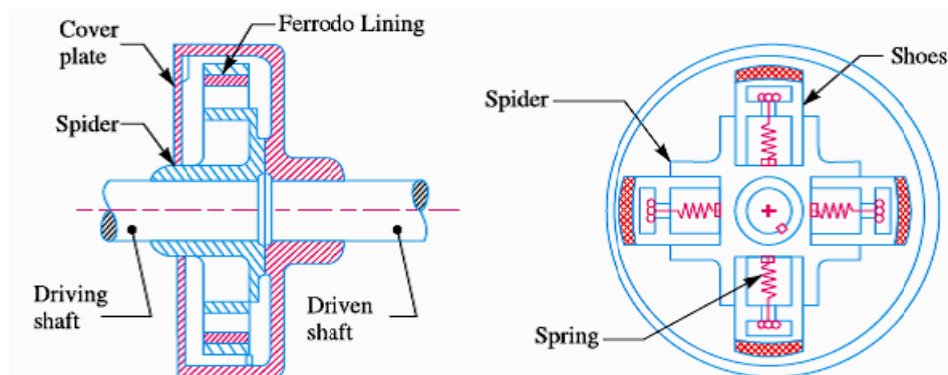


Figure- Centrifugal clutch.

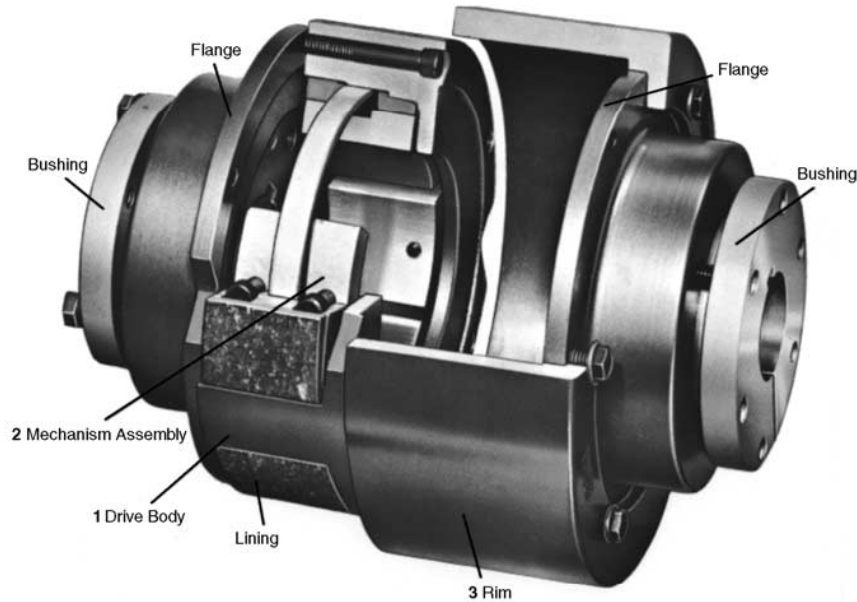


Figure- Centrifugal clutch.

Energy Considerations

Kinetic energy is absorbed during slippage of a clutch and this energy appears as heat. The clutch or brake operation is completed at the instance in which the two angular velocities ω_1 and ω_2 become equal. Let the time required for the entire operation be t_1 , then,

$$t_1 = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T(I_1 + I_2)}$$

This is derived by writing the equations of motion involving inertia i.e.

$$I_1 \ddot{\theta}_1 = -T \quad \dot{\theta}_1 = -\frac{T}{I_1}t + \omega_1$$

$$I_2 \ddot{\theta}_2 = -T \quad \dot{\theta}_2 = -\frac{T}{I_2}t + \omega_2$$

$$\dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2 = -\frac{T}{I_1}t + \omega_1 - \left(-\frac{T}{I_2}t + \omega_2\right)$$

$$= \omega_1 - \omega_2 - T \left(\frac{I_1 + I_2}{I_1 I_2} \right) t$$

From which $t = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T(I_1 + I_2)}$

As at the instance of completion of clutching

Operation $\omega_1 - \omega_2 = 0$ Assuming the torque to be constant, the rate of energy dissipation during the operation is then, The total energy dissipated during the clutching operation or braking cycle is obtained by integrating the above equation from $t=0$ to $t = t_1$. The result can be summed up as,

Design of Friction Drives

S K Mondal's

Chapter 2

$$E = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

$$U = T\theta = T \left[\omega_1 - \omega_2 - T \left(\frac{I_1 + I_2}{I_1 I_2} \right) \cdot t \right]$$

$$E = \int_0^{t_1} u dt = T \int_0^{t_1} \left[\omega_1 - \omega_2 - T \left(\frac{I_1 + I_2}{I_1 I_2} \right) t \right] dt$$

$$E = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

Thus the energy absorbed during clutch slip is a function of the magnitude of the inertia and the angular velocities only. This energy compared to the brake energy may be negligible. Heat dissipation and temperature rise are governed by the same equations presented during brakes. To contain the temperature rise when very frequent clutching operations, wet clutches rather than dry clutches are often use.

Friction materials and their properties

The most important member in a mechanical brake is the friction material. A good friction material is required to possess the following properties:

- High and reproducible coefficient of friction.
- Imperviousness to environmental conditions.
- Ability to withstand high temperature (thermal stability)
- High wear resistance.
- Flexibility and conformability to any surface.

Some common friction materials are woven cotton lining, woven asbestos lining, molded asbestos lining, molded asbestos pad, Sintered metal pads etc.

Typical characteristics of some widely used friction linings are given in the table

Table Properties of common clutch/ Brake lining materials				
Friction Material Against Steel or CI	Dynamic Coefficient of Friction		Maximum Pressure KPa	Maximum Temperature °C
	dry	in oil		
Molded	0.25-0.45	0.06-0.09	1030-2070	204-260
Woven	0.25-0.45	0.08-0.10	345-690	204-260
Sintered metal	0.15-0.45	0.05-0.08	1030-2070	232-677
Cast iron or hard steel	0.15-0.25	0.03-0.06	690-720	260

WORKED OUT EXAMPLE 1

Design an automotive plate clutch to transmit a torque of 550 N-m. The coefficient of friction is 0.25 and the permissible intensity of pressure is 0.5 N/mm². Due to space limitations, the outer diameter of the friction disc is fixed as 250 mm.

Design of Friction Drives

S K Mondal's

Chapter 2

Using the uniform wear theory, calculate:

The inner diameter of the friction disc

The spring force required to keep the clutch in engaged position

Solution: As noted the friction disc of the automotive clutch is fixed between the flywheel on one side and the pressure plate on the other. The friction lining is provided on both sides of the friction disc.

Therefore two pairs of contacting surfaces-one between the fly wheel and the friction disc and the other between the friction disc and the pressure plate. Therefore, the torque transmitted by one pair of contacting surfaces is $(550/2)$ or 275 N-m

$$(T_t)_f = \pi \mu p_a r_i (r_o^2 - r_i^2)$$

$$(275 \times 10^3) = \pi (0.25)(0.5) r_i (125^2 - r_i^2)$$

$$\text{From the Eqn } 8r_i (125^2 - r_i^2) = 5602254$$

Rearranging the terms, we have

The above equation is solved by the trial and error method. It is a cubic equation, with following three roots:

$$(i) \ r_i = 87.08 \text{ mm}$$

$$(ii) \ r_i = 56.145 \text{ mm}$$

$$(iii) \ r_i = -143.23 \text{ mm}$$

Mathematically, all the three answer are correct. The inner radius cannot be negative. As a design engineer, one should select the inner radius as 87.08 mm , which results in a minimum area of friction lining compared with 56.145 . For minimum cost of friction lining.

$$r_i = 87 \text{ mm}$$

Actuating force needed can be determined using the equation

$$F_a = 2\pi p_a r_i (r_o - r_i) = 2\pi (0.5)(87)(125 - 87) = 10390.28 \text{ N}$$

WORKED OUT EXAMPLE 2

A multiple-disc wet clutch is to be designed for transmitting a torque of 85 N.m . Space restriction limit the outside disk diameter to 100 mm . Design values for the molded friction material and steel disks to be used are $f=0.06$ (wet) and $p_{\max}=1400 \text{ kPa}$. Determine appropriate values for the disc inside diameter, the total number of discs, and the clamping force.

Solution

Known: A multiple – disc with outside disc diameter, $d_o \leq 100 \text{ mm}$,

Dynamic friction coefficient, $f=0.06$ (wet)

And maximum disc allowable pressure, $p_{\max}=1400 \text{ kPa}$,

To transmits a torque, $T= 85 \text{ N.m}$

Find: Determine the disc inside diameter d_i , the total number of disks N , and the clamping force F_a .

Design of Friction Drives

S K Mondal's

Chapter 2

Decisions and Assumptions

Use the largest allowable outside disc diameter, $d_o=100$ mm ($r_o=50$ mm).

Select $r_i=29$ mm (based on the optimum d/D ratio of 0.577)

The coefficient of friction f is a constant.

The wear rate is uniform at the interface.

The torque load is shared equally among the disc.

Design Analysis:

Using design equation for torque under constant wear gives

$$N = T / \left[\pi p_{\max} r_i f (r_o^2 - r_i^2) \right] = 6.69$$

Since N must be an even integer, use $N=8$. It is evident that this requires a total of 4+5, or nine discs, remembering that the outer disks have friction surfaces on one side only. 3. With no other changes, this will give a clutch that is over designed by a factor of $8/6.69=1.19$. Possible alternatives include (a) accepting

the 19 percent over design, (b) increasing r_i , (c) decreasing r_o , and (d) leaving both radii unchanged and reducing both p_{\max} and F by a factor of 1.194. With the choice of alternative d, the clamping force is computed to be just sufficient to produce the desired torque:

$$T = Ff \left(\frac{r_o + r_i}{2} \right) N = 85 \text{ N.m} = F(0.06) \left(\frac{0.050 + 0.029}{2} \right) 8,$$

$F = 4483$ N

Rounding up the calculated value of F , we

Find that the final proposed answers are (a) inside diameter= 58 mm, (b)

Clamping force= 4500 N and (C) a total of nine discs.

Belt and Chain drives

The four principal types of belts are shown, with some of their characteristics.

Crowned pulleys are used for flat belts, and *grooved pulleys*, or *sheaves*, for round and V belts. Timing belts require *toothed wheels*, or *sprockets*. In all cases, the pulley axes must be separated by a certain minimum distance, depending upon the belt type and size, to operate properly. Other characteristics of belts are:

- They may be used for long center distances.
- Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shafts is neither constant nor exactly equal to the ratio of the pulley diameters.
- In some cases an idler or tension pulley can be used to avoid adjustments in center Distance that are ordinarily necessitated by age or the installation of new belts.

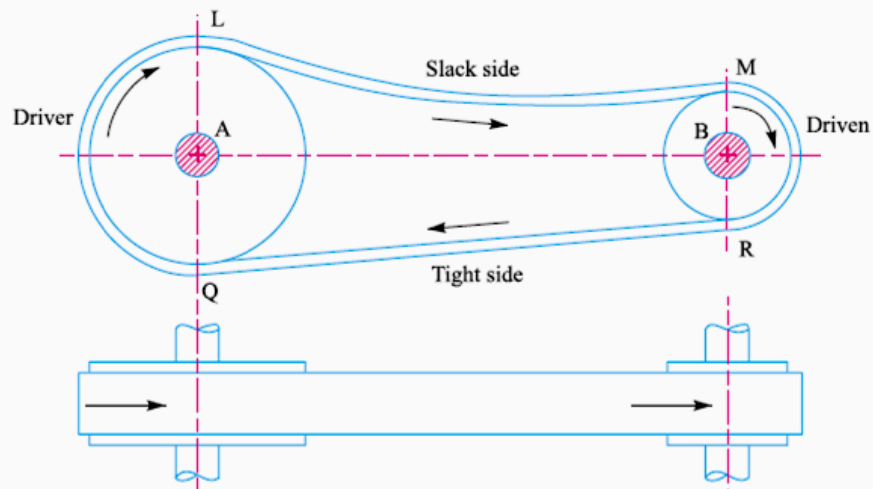


Figure- Open belt drive

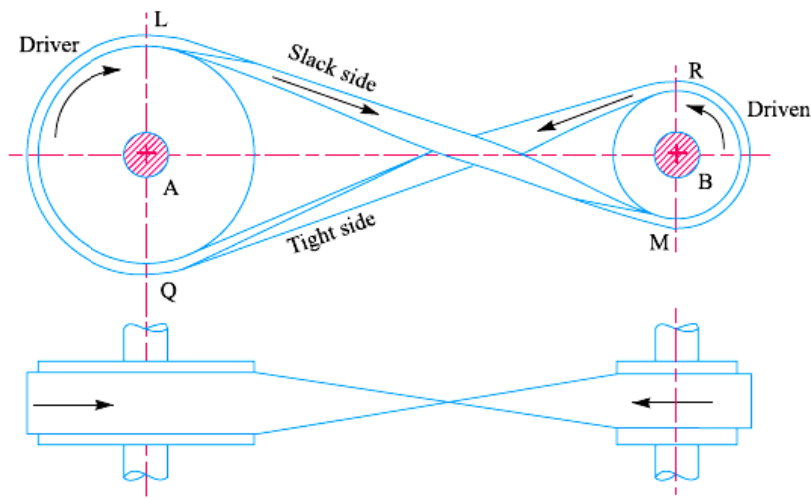


Figure- Crossed or twist belt drive.

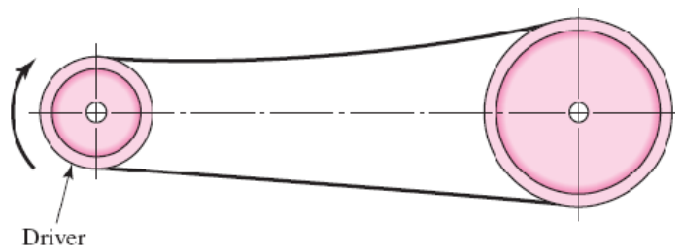


Figure- Non reversing open belt

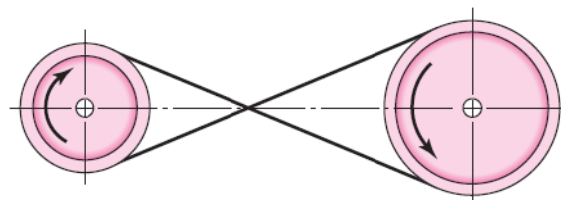


Figure- Reversing crossed belt.
Crossed belts must be separated to prevent Rubbing if high-friction materials are used.

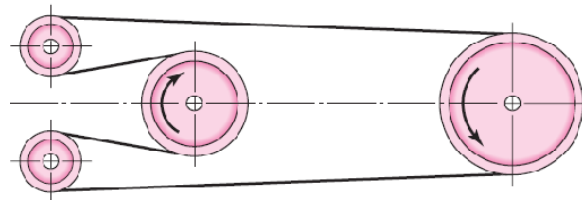


Figure- Reversing open-belt drive.

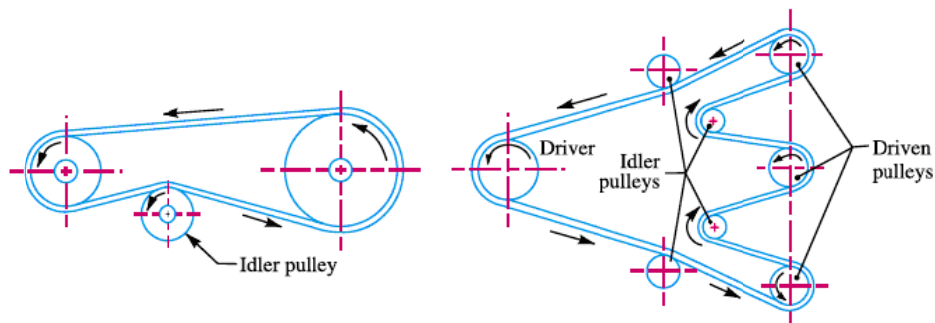


Figure- Belt drives with single idler pulley. Figure- Belt drive with many idler pulleys

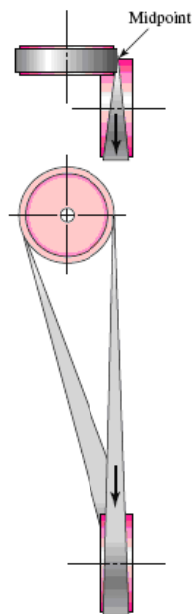
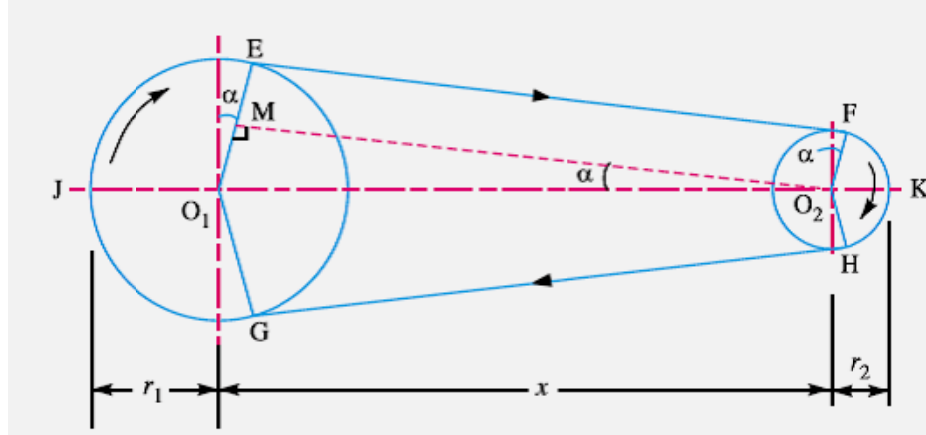


Figure- Quarter-twist belt drive; an idler guide pulley must be Used if motion is to be in both Direction.

Length of the belt



$$\Rightarrow \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \dots(\text{in terms of pulley radius})$$

$$\Rightarrow \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \dots(\text{in terms of pulley diameters})$$

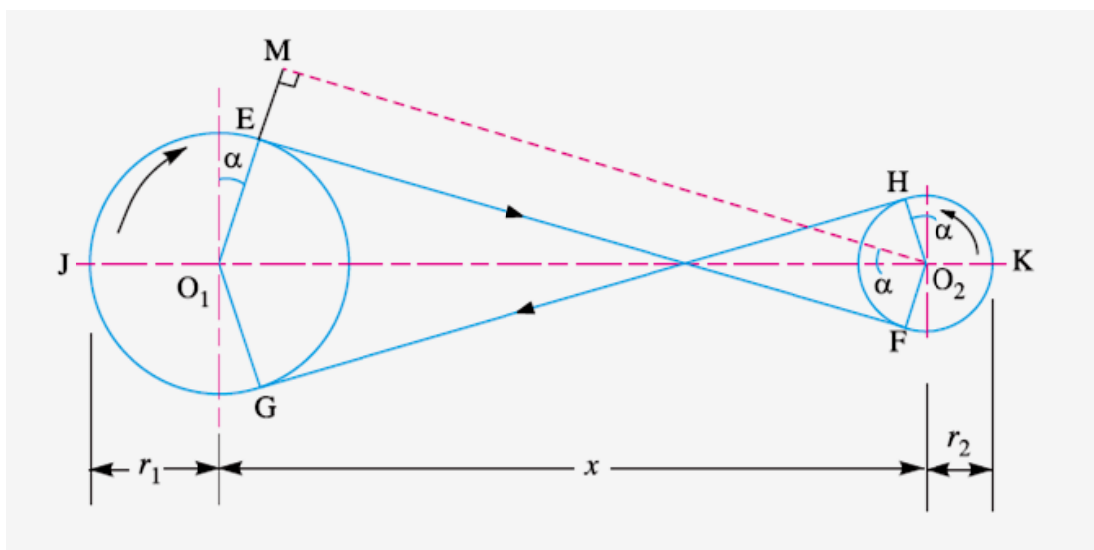


Figure- Crossed belt drive.

- The length of the belt in the case of a **cross-belt drive** is given in terms of centre distance between pulleys (C), diameters of the pulleys D and d as

$$\Rightarrow \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \dots(\text{in terms of pulley radii})$$

$$\Rightarrow \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \dots(\text{in terms of pulley diameters})$$

Belt tension

T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

Design of Friction Drives

S K Mondal's

Chapter 2

θ = Angle of contact in radians (*i.e.* angle subtended by the arc AB, Along which the belt touches the pulley, at the centre).

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta}$$

Notes: 1. While determining the angle of contact, it must be remembered that it is the angle of contact at the smaller pulley, if both the pulleys are of the same material. We know that

$$\sin \alpha = \frac{r_1 - r_2}{x} \quad \dots (\text{for open belt drive})$$

$$= \frac{r_1 + r_2}{x} \quad \dots (\text{for cross-belt drive})$$

\therefore Angle of contact or lap,

$$\theta = (180^\circ - 2\alpha) \frac{\pi}{180} \text{ rad} \quad \dots (\text{for open belt drive})$$

$$= (180^\circ + 2\alpha) \frac{\pi}{180} \text{ rad} \quad \dots (\text{for cross-belt drive})$$

2. When the pulleys are made of different material (*i.e.* when the coefficient of friction of the pulleys or the angle of contact are different), then the design will refer to the pulley for which $\mu \cdot \theta$ is small.

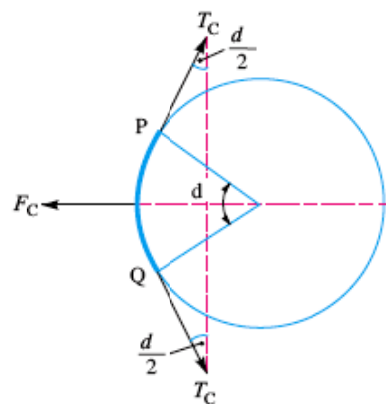
\therefore **Power transmitted,**

$$P = (T_1 - T_2) v$$

Centrifugal tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension. At lower belt speeds (less than 10m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10m /s), its effects is considerable and thus should be taken into account.

Consider a small portion PQ of the belt subtending an angle $d\theta$ at the centre of the pulley, as shown in figure.



T_c = Centrifugal tension

m = Mass of belt per unit length in kg,

v = Linear velocity of belt in m/s,

r = Radius of pulley over which the belt runs in meters.

$$\therefore T_c = m \cdot v^2$$

When centrifugal tension is taken into account, then total tension in the tight side,

Design of Friction Drives

S K Mondal's

Chapter 2

$$T_{t1} = T_1 + T_c$$

And total tension in the slack side,

$$T_{t2} = T_2 + T_c$$

Power transmitted,

$$P = [(T_1 + T_c) - (T_2 + T_c)]v = (T_1 - T_2) v \quad \dots(\text{in watts}) \dots (\text{same as before})$$

Thus we see that the centrifugal tension has no effect on the power transmitted.

The ratio of driving tensions may also be written as

$$\log_e \left(\frac{T_{t1} - T_c}{T_{t2} - T_c} \right) = \mu \cdot \theta$$

Where T_{t1} = Maximum or total tension in the belt.

Condition for maximum power

Condition for maximum power transmission:-

$$P = (T_1 - T_2) v$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow P = (T_{\max} - T_c) \left(1 - \frac{1}{e^{\mu\theta}} \right) v$$

$$\Rightarrow P = (T_{\max} v - mv^3) \left(1 - \frac{1}{e^{\mu\theta}} \right)$$

$$\Rightarrow \frac{dP}{dv} = (T_{\max} - 3mv^2) \left(1 - \frac{1}{e^{\mu\theta}} \right) = 0$$

$$\Rightarrow mv^2 = \frac{T_{\max}}{3}$$

$$\Rightarrow T_c = \frac{T_{\max}}{3}$$

$$T_{\max} = 3T_c$$

Selection of V-belt drive

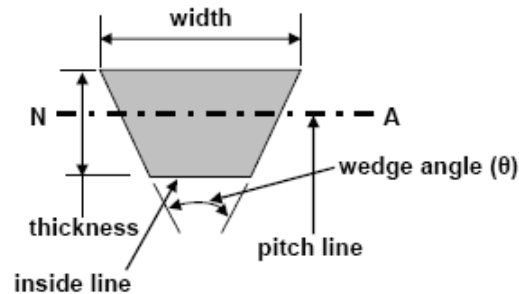


Fig. Nomenclature of V-belt

- For similar materials having the same maximum permissible tension V-belt transmits more power than flat belt with same velocity ratio and centre distance.
- As two sides of V-belt are in contact with side faces of pulley groove, larger contact area gives greater effective frictional force.
- In a multiple V belt drive, when a single belt is damaged, it is preferable to change the complete set to ensure uniform loading.

V-belt designation

B – 2786 – Gr50 → standard size of belt

↓ ↓

Type nominal
Of inside
V belt length

Q.1. How a V-belt section is selected?

Ans. Depending upon the required power transmission, a belt section is chosen. However, the smaller pulley diameter should be less than the pulley diameter as mentioned for the chosen belt section.

Q.2. Why angle of wrap correction factor and belt length correction factor is required to modify power rating of a belt?

Ans. The power rating of V-belts are based on angle of wrap, $\alpha = 180^\circ$. Hence, for any angle of wrap, other than 180° , a correction factor is required. Similarly, if the belt length is different from optimum belt length for which the power rating is given, then belt length correction factor is used, because, amount of flexing in the belt in a given time is different from that in optimum belt length.

Q.3. How a V-belt is designated?

Ans. Let a V-belt of section A has inside length of 3012 mm. Then its designation will be **A 3012/118**. Where, 118 is the corresponding length in inches.

Initial tension in the belt

When a belt is wound round the two pulleys (*i.e.* driver and follower), its two ends are joined together, so that the belt may continuously move over the pulleys, since the motion of the belt

Design of Friction Drives

S K Mondal's

Chapter 2

(from the driver) and the follower (from the belt) is governed by a firm grip due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called **initial tension**.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers to the other side (decreasing tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side

Let T_0 = Initial tension in the belt,
 T_1 = Tension in the tight side of the belt,
 T_2 = Tension in the slack side of the belt, and
 α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side
 $= T_1 - T_0$

And increase in the length of the belt on the tight side
 $= \alpha (T_1 - T_0)$... (i)

Similarly, decrease in tension in the slack side
 $= T_0 - T_2$

And decrease in the length of the belt on the slack side
 $= \alpha (T_0 - T_2)$... (ii)

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations (i) and (ii), we have
 $\alpha (T_1 - T_0) = \alpha (T_0 - T_2)$

Or $T_1 - T_0 = T_0 - T_2$

$\therefore T_0 = \frac{T_1 + T_2}{2}$... (Neglecting centrifugal tension)

$$= \frac{T_1 + T_2 + 2T_C}{2}$$

... (Considering centrifugal tension)

Note: In actual practice, the belt material is not perfectly elastic. Therefore, the sum of the tensions T_1 and T_2 , when the belt is transmitting power, is always greater than twice the initial tension. According to **C.G. Barth**, the relation between T_0 , T_1 and T_2 is given by

$$\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_0}$$

Chain drive

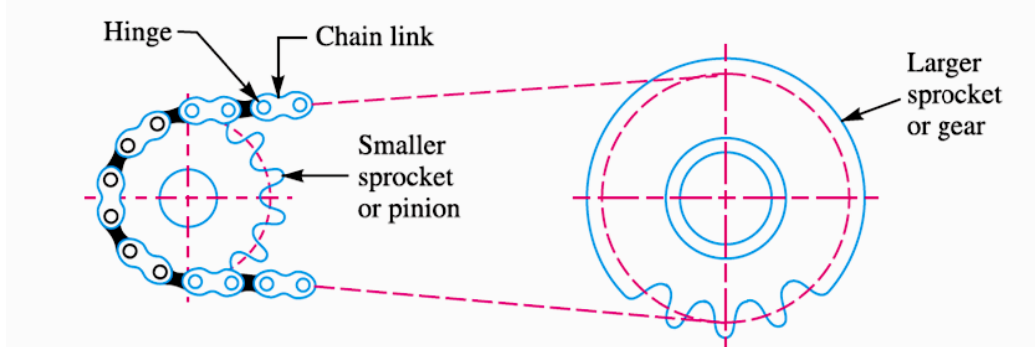


Figure- Sprockets and chain.

The chains are mostly used to transmit motion and power from one shaft to another, when the centre distance between their shafts is short such as in bicycles, motor cycles, agricultural machinery, conveyors, rolling mills, road rollers etc. The chains may also be used for long centre distance of up to 8 metres. The chains are used for velocities up to 25 m /s and for power up to 110kW. In some cases, higher power transmission is also possible.

Advantages and Disadvantages of Chain Drive over Belt or Rope Drive

Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. It may be used for both long as well as short distances.
4. It gives high transmission efficiency (upto 98 percent).
5. It gives less load on the shafts.
6. It has the ability to transmit motion to several shafts by one chain only.
7. It transmits more power than belts.
8. It permits high speed ratio of 8 to 10 in one step.
9. It can be operated under adverse temperature and atmospheric conditions.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance, particularly lubrication and slack adjustment.
3. The chain drive has velocity fluctuations especially when unduly stretched.

Features of chain drive

- Three major types of chain are used for power transmission: roller, engineering steel, and silent.

Design of Friction Drives

S K Mondal's

Chapter 2

- Roller chains are probably the most common and are used in a wide variety of low-speed to high-speed drives.
- Engineering steel chains are used in many low-speeds, high-load drives.
- Silent chains are mostly used in high-speed drives.
- **Silent Chain:** Silent (inverted-tooth) chains are standardized in for pitches of 0.375 to 2.0 in. Silent chain is an assembly of toothed link plates interlaced on common pins. The sprocket engagement side of silent chain looks much like a gear rack. Silent chains are designed to transmit high power at high speeds smoothly and relatively quietly. Silent chains are a good alternative to gear trains where the center distance is too long for one set of gears.
- Chains can span long center distances like belts, and positively transmit speed and torque like gears.
- For a given ratio and power capacity, chain drives are more compact than belt drives, but less compact than gear drives.
- Mounting and alignment of chain drives does not need to be as precise as for gear drives. Chain drives can operate at 98 to 99 percent efficiency under ideal conditions.
- Chain drives are usually less expensive than gear drives and quite competitive with belt drives.

• Number of Sprocket Teeth

Slow speed	12 teeth
Medium speed	17 teeth
High speed	25 teeth

- Given that P = chain pitch, c = centre distance, N and n = number of teeth on large and small sprocket respectively, the length of chain in terms of pitches can be approximated by $\frac{2c}{P} + (N + n) / 2P + [(N - n) / 2P]^2 \frac{P}{c}$
- **Speed Ratio.** The maximum recommended speed ratio for a single-reduction roller-chain drive is **7:1**. Speed ratios up to 10:1 are possible with proper design, but a double reduction is preferred.
- For roller chain drive with sprocket having z teeth, the velocity of the driven shaft with respect to that of drive will be approximately $(V_{\max} - V_{\min}) \propto \left[1 - \cos\left(\frac{180}{z}\right) \right]$
- In order to reduce the variation in chain speed, the number of teeth on the sprocket should be increased. It has been observed that the speed variation is 4% for a sprocket with 11 teeth, 1.6% for a sprocket with 24 teeth.
- For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth. For durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

- **Chain Length:** Required chain length may be estimated from the following approximate equation:

$$L \doteq 2C + \frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{4\pi^2 C}$$

Where

L = Chain length, in chain pitches

N = Number of teeth on small sprocket

N_1 = Number of teeth on small sprocket

N_2 = Number of teeth on large sprocket

C = Centre distance, in chain pitches

- V = Velocity of chain in m/s

$$V_{\max} = \frac{\pi d N}{60} \text{ m/s}$$

Where d = Pitch circle diameter of the smaller or driving sprocket in metres.

The centre of the sprocket and its linear velocity is given by

$$V_{\min} = \frac{\pi d N \cos \theta / 2}{60} \text{ m/s}$$

Rope drive

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance.

It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 meters apart.

If large amounts of power are to be transmitted, by the flat belt, then it would result in excessive belt cross-section.

The ropes drives use the following two types of ropes:

1. Fibre ropes, and
2. Wire ropes.

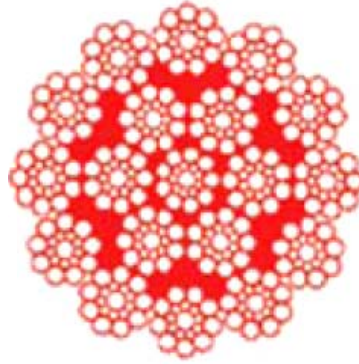
The fibre ropes operate successfully when the pulleys are about 60 meters apart, while the wire ropes are used when the pulleys are upto 150 meters apart.

Advantages of Fiber Rope Drives

The fibre rope drives have the following advantages:

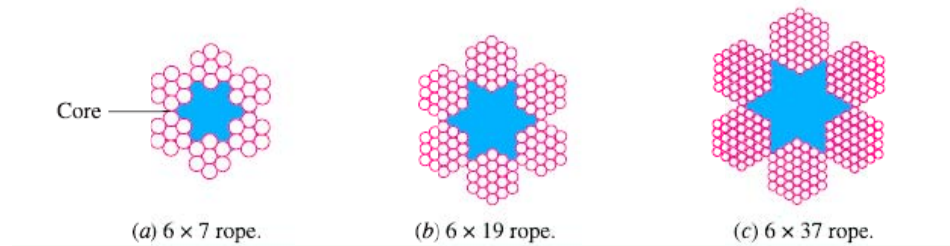
1. They give smooth, steady and quiet service.
2. They are little affected by out door conditions.

3. The shafts may be out of strict alignment.
4. The power may be taken off in any direction and in fractional parts of the whole amount.
5. They give high mechanical efficiency.



Wire strands

Figure- Wire strands



- 6×20 wire rope: 6 indicates number of strands in the wire rope and 20 indicates no of wire in a strand.

Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as **creep**. The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

Where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and
 E = Young's modulus for the material of the belt.

Note: Since the effect of creep is very small, therefore it is generally neglected.

Power screw

- The power screws (also known as *translation screws*) are used to convert rotary motion into translator motion.
- For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material.
- In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.
- In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings.

Power Screw

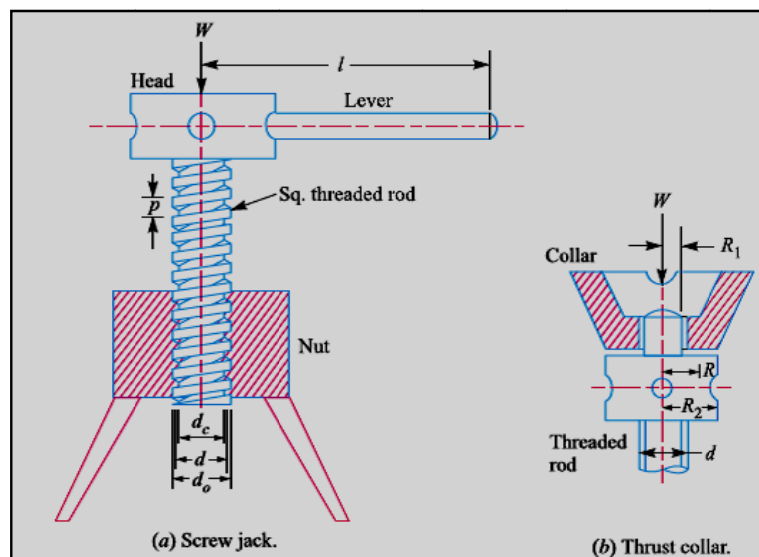


Figure- Types of power screws.

(i) Forms of threads

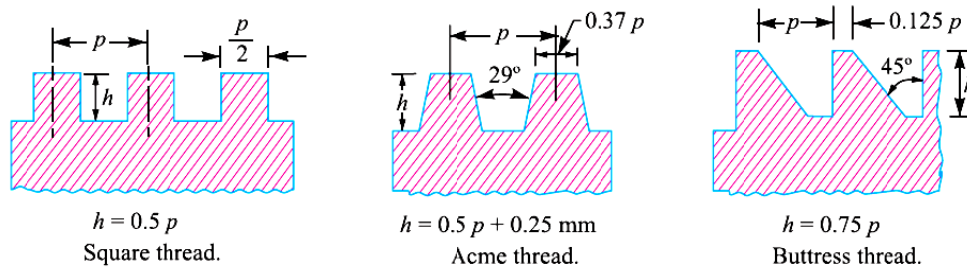


Figure Forms of threads

Design of Friction Drives

S K Mondal's

Chapter 2

$$(ii) \boxed{\tan \alpha = \frac{\ell}{\pi d_m}} \quad | \ell = (\text{number of starts}) \times p$$

$$(iii) d_m = d - 0.5p$$

$$(iv) d_c = d - p$$

$$(v) \text{Lifting Load: } (P) = \frac{W(\mu \cos \alpha + \sin \alpha)}{(\cos \alpha - \mu \sin \alpha)} [\text{Square thread}]$$

$$(vi) \text{efficiency } (\eta) = \frac{\tan \alpha}{\tan (\phi + \alpha)}$$

(vii) efficiency will be maximum if, $\alpha = 45^\circ - \phi / 2$

$$\eta_{\max} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

where ϕ = friction angle

α = Helix angle

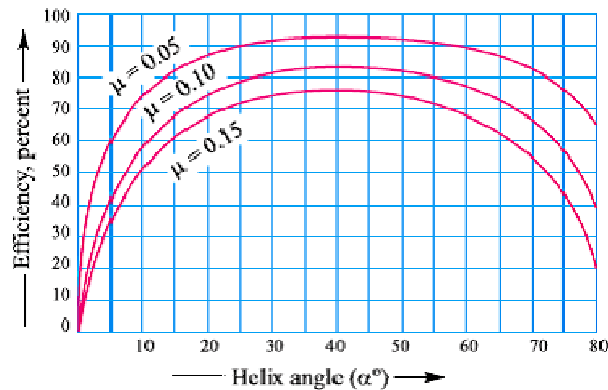
p = pitch of the screw

$$(viii) \text{Lowering Load: } P = \frac{W(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)},$$

if $\phi < \alpha$, self - Locking is not possible *Known as over hauling*

(ix) For self locking screw efficiency is less than 50%. If the efficiency is more than 50% the screw is said to be over hauling.

(x) Efficiency Vs helix angle:



Figure; - Efficiency Vs helix angle

When the helix angle further increases say 70°, the efficiency drops. This is due to the fact that the normal thread force becomes large and thus the force of friction and the work of friction become large as compared with the useful work. This results in low efficiency.

$$(xi) \text{ Collar friction } (M_t)_c = \frac{\mu_c W}{3} \left(\frac{D^3 - d_c^3}{D^2 - d_c^2} \right) \quad \text{Uniform pressure theory.}$$

$$(M_t)_c = \frac{\mu_c W}{4} (D + d_c) \quad \text{Uniform wear theory.}$$

$$(xii) \text{ Total torque } (T_t) = M_t + (M_t)_c$$

$$(xiii) \text{ Overall efficiency } \eta_0 = \frac{Wl}{2\pi T_t}.$$

(xiv) Stresses in screw

Design of Friction Drives

S K Mondal's

Chapter 2

(a) **Direct tensile** or compressive stress due to an axial load

$$\sigma_t \text{ or } \sigma_c = \frac{\frac{W}{\pi d_c^2}}{4} \quad \parallel \sigma_{allowable} = \frac{\sigma_y}{4}$$

N.B. when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions.

Then $P_{cr} = \frac{\pi^2 EI}{l_e^2}$; Length of nut = Lift + $2xd_c$

must be $P_{cr} > w$

(b) **Torsional shear stress:** This is obtained by considering the minimum cross section of the screw.

$$\tau = \frac{16(T_t)}{\pi(d_c)^3} \quad \text{where } T_t = \text{Torque in screw}$$

\therefore Maximum Shear stress on the minor diameter of screw, $\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

(c) **Shear stress due to axial load:** The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to axial load.

$$\begin{aligned} \text{Transverse shear stress for screw, } \tau_{screw} &= \frac{W}{\pi d_c t z} \\ \text{Transverse shear stress for nut, } \tau_{nut} &= \frac{W}{\pi d t z} \end{aligned} \quad \parallel \quad z = \text{number of thread in engagement} = \frac{h}{p}$$

(d) **Bearing pressure:**

$$p_b^* = \frac{W}{\pi / 4 (d^2 - d_c^2) z} = \frac{W}{\pi d_m t z} \quad \parallel \quad \frac{d^2 - d_c^2}{4} = \frac{d + d_c}{2} \times \frac{d - d_c}{2} = d_m \times \frac{P}{2} = d_m \times t$$

N.B. In order to reduce wear of the screw and nut, the bearing pressure on the thread surface must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between screw & nut and the nature of lubrication. In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

Objective Questions (GATE, IES & IAS)

Previous 20-Years GATE Questions

Couplings

GATE-1. The bolts in a rigid flanged coupling connecting two shafts transmitting power are subjected to [GATE-1996]

- (a) Shear force and bending moment (b) axial force.
(c) Torsion and bending moment (d) torsion

Design of Friction Drives

S K Mondal's

Chapter 2

GATE-1. Ans. (a) The bolts are subjected to shear and bearing stresses while transmitting torque.

Uniform pressure theory

GATE-2. A clutch has outer and inner diameters 100 mm and 40 mm respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material 0.4, the torque carrying capacity of the clutch is [GATE-2008]
(a) 148 Nm (b) 196 Nm (c) 372 Nm (d) 490 Nm

GATE-2. Ans. (b) Force(P) = $\frac{\pi p}{4}(D^2 - d^2)$

$$T = \frac{\mu P}{3} \cdot \frac{(D^3 - d^3)}{(D^2 - d^2)}$$
$$= \frac{\mu \pi}{12} \cdot p \cdot (D^3 - d^3) = \frac{0.4 \times \pi \times 2 \times 10^6}{12} (0.1^3 - 0.04^3) = 196 \text{ Nm}$$

GATE-3. A disk clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is [GATE-2006]
(a) 39.4 mm (b) 49.5 mm (c) 97.9 mm (d) 142.9 mm

GATE-3. Ans. (a)

$$\text{Torque, } T = \frac{P \times 60}{2\pi \times N} = 23.87 \text{ N m}$$
$$= \text{Axial thrust, } W = P \times \pi(r_1^2 - r_2^2)$$
$$\text{But } T = \frac{2}{3} \mu \times P \times \pi(r_1^2 - r_2^2) \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} = \mu w r$$
$$\therefore r_2 = 39.4 \text{ mm}$$

Belt and Chain drives

GATE-4. Total slip will Occur in a belt drive when [GATE-1997]

- (a) Angle of rest is zero
- (b) Angle of creep is zero
- (c) Angle of rest is greater than angle of creep
- (d) Angle of creep is greater than angle of rest

GATE-4. Ans. (a)

Belt tension

GATE-5. The ratio of tension on the tight side to that on the slack side in a flat belt drive is [GATE-2000]

- (a) Proportional to the product of coefficient of friction and lap angle
- (b) An exponential function of the product of coefficient of friction and lap angle.
- (c) Proportional to the lap angle
- (d) Proportional to the coefficient of friction

GATE-5. Ans. (b)

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

Design of Friction Drives

S K Mondal's

Chapter 2

GATE-6. The difference between tensions on the tight and slack sides of a belt drive is 3000 N. If the belt speed is 15 m/s, the transmitted power in kW is

- (a) 45 (b) 22.5 (c) 90 (d) 100 [GATE-1998]

GATE-6. Ans. (a)

Given, $T_1 - T_2 = 3000\text{N}$

where T_1, T_2 = tensions on tight and slack side respectively

$$v = \text{belt speed} = 15 \text{ m / sec}$$

$$\begin{aligned}\text{Power} &= (T_1 - T_2)v \\ &= 3000 \times 15 = 45000 \text{ watt} = 45 \text{ kW}\end{aligned}$$

GATE-7. The percentage improvement in power capacity of a flat belt drive, when the wrap angle at the driving pulley is increased from 150° to 210° by an idler arrangement for a friction coefficient of 0.3, is [GATE-1997]

- (a) 25.21 (b) 33.92 (c) 40.17 (d) 67.85

GATE-7. Ans. (d) We know that Power transmitted $(P) = (T_1 - T_2) \cdot v$ W

$$\text{Case-I: } \frac{T_1}{T_2} = e^{\mu\theta} \text{ or } \frac{T_1}{T_2} = e^{0.3 \times \left(\frac{5\pi}{6}\right)} \text{ or } T_1 = 2.193 T_2 \Rightarrow P_1 = 1.193 T_2 v \text{ W}$$

$$\text{Case-II: } \frac{T_1}{T_2} = e^{\mu\theta} \text{ or } \frac{T_1}{T_2} = e^{0.3 \times \left(\frac{7\pi}{6}\right)} \text{ or } T_1 = 3.003 T_2 \Rightarrow P_2 = 2.003 T_2 v \text{ W}$$

$$\text{Therefore improvement in power capacity} = \frac{P_2 - P_1}{P_1} \times 100\% = 67.88\%$$

Centrifugal tension

GATE-8. With regard to belt drives with given pulley diameters, centre distance and coefficient of friction between the pulley and the belt materials, which of the statement below are FALSE? [GATE-1999]

- (a) A crossed flat belt configuration can transmit more power than an open flat belt configuration
(b) A "V" belt has greater power transmission capacity than an open flat belt
(c) Power transmission is greater when belt tension is higher due to centrifugal effects than the same belt drive when centrifugal effects are absent.
(d) Power transmission is the greatest just before the point of slipping is reached

GATE-8. Ans. (c)

Rope drive

GATE-9. In a 6×20 wire rope, No.6 indicates the

[GATE-2003]

- (a) diameter of the wire rope in mm
(b) Number of strands in the wire rope
(c) Number of wires
(d) Gauge number of the wire

GATE-9. Ans. (b) 6×20 wire rope: 6 indicates number of strands in the wire rope and 20 indicates no of wire in a strand.

Self locking screw

GATE-10. What is the efficiency of a self-locking power screw?

[GATE-1994]

Design of Friction Drives

S K Mondal's

Chapter 2

- (a) 70% (b) 60% (c) 55% (d) < 50 %

GATE-10. Ans. (d) We know that the frictional torque for square thread at mean radius while raising load is given by $WR_o \tan(\phi - \alpha)$

Where: (W = load; R_o = Mean Radius; ϕ = Angle of friction; α = Helix angle)
For self locking, angle of friction should be greater than helix angle of screw So that $WR_o \tan(\phi - \alpha)$ will become positive. i.e. we have to give torque to lowering the load.

GATE-11. Self locking in power screw is better achieved by decreasing the helix angle and increasing the coefficient of friction. [GATE-1995]

- (a) True (b) False (c) insufficient logic (d) none of the above

GATE-11. Ans. (a)

Efficiency of screw

GATE-12. Which one of the following is the value of helix angle for maximum efficiency of a square threaded screw? [$\phi = \tan^{-1} \mu$] [GATE-1997]

- (a) $45^\circ + \phi$ (b) $45^\circ - \phi$ (c) $45^\circ - \phi / 2$ (d) $45^\circ + \phi / 2$

GATE-12. Ans. (c)

Previous 20-Years IES Questions

Couplings

IES-1. Consider the following statements in respect of flexible couplings:

1. The flanges of flexible coupling are usually made of grey cast iron FG200. [IES-2006]
2. In the analysis of flexible coupling, it is assumed that the power is transmitted by the shear resistance of the pins.
3. Rubber bushes with brass lining are provided to absorb misalignment between the two shafts.

Which of the statements given above are correct?

- (a) 1, 2 and 3 (b) Only 1 and 2 (c) Only 2 and 3 (d) Only 1 and 3

IES-1. Ans. (d) Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress theory.

IES-2. Which of the following stresses are associated with the design of pins in bushed pin-type flexible coupling? [IES-1998]

1. Bearing stress
2. Bending stress
3. Axial tensile stress
4. Transverse shear stress

Select the correct answer using the codes given below

- (a) 1, 3 and 4 (b) 2, 3 and 4 (c) 1, 2 and 3 (d) 1, 2 and 4

IES-2. Ans. (d)

IES-3. Match List I with List II and select the correct answer using the codes given below the lists: [IES-1995]

- | List I | List II |
|----------------|-----------------------------------------------------------------------------------------------------|
| A. Crank shaft | 1. Supports the revolving parts and transmits torque. |
| B. Wire shaft | 2. Transmits motion between shafts where it is not possible to effect a rigid coupling between them |

Design of Friction Drives

S K Mondal's

Chapter 2

C. Axle					3. Converts linear motion into rotary motion				
D. Plain shaft					4. Supports only the revolving parts.				
Codes:	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	4	2	3	1
(c)	3	2	4	1	(d)	1	4	2	3

IES-3. Ans. (c)

IES-4. The bolts in a rigid flanged coupling connecting two shafts transmitting power are subjected to [IES-2002]

- (a) Shear force and bending moment (b) axial force.
(c) Torsion and bending moment (d) torsion

IES-4. Ans. (a) The bolts are subjected to shear and bearing stresses while transmitting torque.

Introduction Friction clutches

IES-5. Which one of the following is not a friction clutch? [IES-2003]

- (a) Disc or plate clutch (b) Cone clutch
(c) Centrifugal clutch (d) Jaw clutch

IES-5. Ans. (d)

IES-6. Which one of the following pairs of parameters and effects is not correctly matched? [IES-1998]

- (a) Large wheel diameterReduced wheel wear
(b) Large depth of cutIncreased wheel wear
(c) Large work diameterIncreased wheel wear
(d) Large wheel speedReduced wheel wear

IES-6. Ans. (d)

IES-7. Two co-axial rotors having moments of inertia I_1 , I_2 and angular speeds ω_1 and ω_2 respectively are engaged together. The loss of energy during engagement is equal to [IES-1994]

- (a) $\frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$ (b) $\frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 - I_2)}$ (c) $\frac{2I_1 I_2 (\omega_1 - \omega_2)^2}{(I_1 + I_2)}$ (d) $\frac{I_1 \omega_1^2 - I_2 \omega_2^2}{(I_1 + I_2)}$

IES-7. Ans. (c)

IES-8. Which of the following statements hold good for a multi-collar thrust bearing carrying an axial thrust of W units? [IES-1996]

- Friction moment is independent of the number of collars.
- The intensity of pressure is affected by the number of collars.
- Co-efficient of friction of the bearing surface is affected by the number of collars.

- (a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

IES-8. Ans. (a)

IES-9. Which of the following statements regarding laws governing the friction between dry surfaces are correct? [IES-1996]

- The friction force is dependent on the velocity of sliding.
- The friction force is directly proportional to the normal force.
- The friction force is dependent on the materials of the contact surfaces.
- The frictional force is independent of the area of contact

Design of Friction Drives

S K Mondal's

Chapter 2

(a) 2, 3 and 4

(b) 1 and 3

(c) 2 and 4

(d) 1, 2, 3 and 4

IES-9. Ans. (a)

Uniform pressure theory

IES-10. **Assertion (A):** In case of friction clutches, uniform wear theory should be considered for power transmission calculation rather than the uniform pressure theory.

Reason (R): The uniform pressure theory gives a higher friction torque than the uniform wear theory. [IES-2003]

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IES-10. Ans. (b) Uniform pressure theory is applicable only when the clutches are new i.e., the assumption involved is that axial force W is uniformly distributed.

Moreover torque transmitted in uniform pressure is more hence for safety in design uniform wear theory is used.

IES-11. When the intensity of pressure is uniform in a flat pivot bearing of radius r, the friction force is assumed to act at [IES-2001]

(a) r

(b) $r/2$

(c) $2r/3$

(d) $r/3$

IES-11. Ans. (c)

IES-12. In a flat collar pivot bearing, the moment due to friction is proportional to (r_1 and r_2 are the outer and inner radii respectively) [IES-1993]

(a) $\frac{r_1^2 - r_2^2}{r_1 - r_2}$

(b) $\frac{r_1^2 - r_2^2}{r_1 + r_2}$

(c) $\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}$

(d) $\frac{r_1^3 - r_2^3}{r_1 - r_2}$

IES-12. Ans. (c)

Uniform wear theory

IES-13. In designing a plate clutch, assumption of uniform wear conditions is made because [IES-1996]

(a) It is closer to real life situation

(b) it leads to a safer design.

(c) It leads to cost effective design

(d) no other assumption is possible.

IES-13. Ans. (a)

Multi-disk clutches

IES-14. In case of a multiple disc clutch, if n_1 is the number of discs on the driving shaft and n_2 is the number of discs on the driven shaft, then what is the number of pairs of contact surfaces? [IES-2008]

(a) $n_1 + n_2$

(b) $n_1 + n_2 - 1$

(c) $n_1 + n_2 + 1$

(d) $n_1 + 2n_2$

IES-14. Ans. (b)

IES-15. In a multiple disc clutch if n_1 and n_2 are the number of discs on the driving and driven shafts, respectively, the number of pairs of contact surfaces will be [IES-2001; 2003]

(a) $n_1 + n_2$

(b) $n_1 + n_2 - 1$

(c) $n_1 + n_2 + 1$

(d) $\frac{n_1 + n_2}{2}$

Design of Friction Drives

S K Mondal's

Chapter 2

IES-15. Ans. (b)

IES-16. In the multiple disc clutch, If there are 6 discs on the driving shaft and 5 discs on the driven shaft, then the number of pairs of contact surfaces will be equal to [IES-1997]

- (a) 11 (b) 12 (c) 10 (d) 22

IES-16. Ans. (c) No. of active plates = $6 + 5 - 1 = 10$

Cone clutches

IES-17. Which one of the following is the correct expression for the torque transmitted by a conical clutch of outer radius R, Inner radius r and semi-cone angle α assuming uniform pressure? (Where W = total axial load and μ = coefficient of friction) [IES-2004]

- (a) $\frac{\mu W(R+r)}{2 \sin \alpha}$ (b) $\frac{\mu W(R+r)}{3 \sin \alpha}$
 (c) $\frac{2\mu W(R^3 - r^3)}{3 \sin \alpha(R^2 - r^2)}$ (d) $\frac{3\mu W(R^3 - r^3)}{4 \sin \alpha(R^2 - r^2)}$

IES-17. Ans. (c)

Centrifugal clutches

IES-18. On the motors with low starting torque, the type of the clutch to be used is [IES-2003]
 (a) Multiple-plate clutch (b) Cone clutch
 (c) Centrifugal clutch (d) Single-plate clutch with both sides effective

IES-18. Ans. (c)

IES-19. Consider the following statements regarding a centrifugal clutch:
 It need not be unloaded before engagement. [IES-2000]

1. It enables the prime mover to start up under no-load conditions.
 2. It picks up the load gradually with the increase in speed
 3. It will not slip to the point of destruction
 4. It is very useful when the power unit has a low starting torque
- Which of these are the advantages of centrifugal clutch?

- (a) 1, 2 and 4 (b) 1, 3 and 5 (c) 2, 3 and 5 (d) 1, 3, 4 and 5

IES-19. Ans. (c)

IES-20. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1998]

List-I					List-II			
A. Single-plate friction clutch					1. Scooters			
B. Multi-plate friction clutch					2. Rolling mills			
C. Centrifugal clutch					3. Trucks			
D. Jaw clutch					4. Mopeds			
Code:	A	B	C	D	A	B	C	D
(a)	1	3	4	2	(b)	1	3	2
(c)	3	1	2	4	(d)	3	1	4

IES-20. Ans. (d)

Design of Friction Drives

S K Mondal's

Chapter 2

Belt and Chain drives

IES-21. The creep in a belt drive is due to the [IES-2001]

- (a) Material of the pulleys
- (b) Material of the belt
- (c) Unequal size of the pulleys
- (d) Unequal tension on tight and slack sides of the belt

IES-21. Ans. (d)

- When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is to reduce slightly the speed of the driven pulley or follower.
- Here english meaning of 'creep' is 'very slow motion' and not 'When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep.'
- Therefore the belt creep is very slow motion between the belt and the pulley surfaces due to unequal tension on tight and slack sides of the belt.
- Don't confuse with material of the belt because the belt creep depends on both the materials of the pulley and the materials of the belt.

IES-22. Assertion (A): In design of arms of a pulley, in belt drive, the cross-section of the arm is, elliptical with minor axis placed along the plane of rotation. [IES-2001]

Reason (R): Arms of a pulley in belt drive are subjected to complete reversal of stresses and is designed for bending in the plane of rotation.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-22. Ans. (a)

IES-23. Assertion (A): In pulley design of flat belt drive, the cross-sections of arms are made elliptical with major axis lying in the plane of rotation. [IES-1999]

Reason (R): Arms of a pulley in belt drive are subjected to torsional shear stresses and are designed for torsion.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-23. Ans. (c)

IES-24. Which one of the following belts should not be used above 40°C? [IES-1999]

- (a) Balata belt
- (b) Rubber belt
- (c) Fabric belt
- (d) Synthetic belt

IES-24. Ans. (b)

IES-25. In μ is the actual coefficient of friction in a belt moving in grooved pulley, the groove angle being 2α , the virtual coefficient of friction will be

- (a) $\mu / \sin \alpha$
- (b) $\mu / \cos \alpha$
- (c) $\mu \sin \alpha$
- (d) $\mu \cos \alpha$ [IES-1997]

IES-25. Ans. (a)

IES-26. In flat belt drive, if the slip between the driver and the belt is 1%, that between belt and follower is 3% and driver and follower pulley diameters are equal, then the velocity ratio of the drive will be [IES-1996]

- (a) 0.99
- (b) 0.98
- (c) 0.97
- (d) 0.96.

IES-26. Ans. (d)

Design of Friction Drives

S K Mondal's

Chapter 2

IES-27. Assertion (A): Crowning is provided on the surface of a flat pulley to prevent slipping of the belt sideways. **[IES-2006]**

Reason (R): Belt creep, which is the reason for slip of the belt sideways, is fully compensated by providing crowning on the pulley.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-27. Ans. (c) Belt creep has no effect on sideways.

Length of the belt

IES-28. The length of the belt in the case of a cross-belt drive is given in terms of centre distance between pulleys (C), diameters of the pulleys D and d as

- (a) $2C + \frac{\pi}{2}(D + d) + \frac{(D + d)^2}{4C}$
- (b) $2C + \frac{\pi}{2}(D - d) + \frac{(D + d)^2}{4C}$ **[IES-2002]**
- (c) $2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$
- (d) $2C + \frac{\pi}{2}(D - d) + \frac{(D - d)^2}{4C}$

IES-28. Ans. (a)

IES-29. Assertion (A): Two pulleys connected by a crossed belt rotate in opposite directions.

Reason (R): The length of the crossed belt remains constant. **[IES-2008]**

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-29. Ans. (b) Two pulleys connected by open belt rotate in same direction whereas two pulleys connected by crossed belt rotate in opposite direction.
The length of crossed belt is given by

$$L_c = \pi(r_1 + r_2) + 2C + \left(\frac{r_1 + r_2}{C}\right)^2$$

So length of crossed belt is constant. Both the statements are correct but Reason is not the correct explanation of Assertion.

IES-30. Which one of the following statements relating to belt drives is correct?

- (a) The rotational speeds of the pulleys are directly proportional to their diameters
- (b) The length of the crossed belt increases as the sum of the diameters of the pulleys increases
- (c) The crowning of the pulleys is done to make the drive sturdy
- (d) The slip increases the velocity ratio **[IES 2007]**

IES-30 Ans.(b) $L = \pi(r_1 + r_2) + 2C + \frac{(r_1 + r_2)^2}{C}$ where C = centre distance of shafts.

Belt tension

IES-31. Assertion (A): In a short centre open-belt drive, an idler pulley is used to maintain the belt tension and to increase the angle of contact on the smaller pulley.

Design of Friction Drives

S K Mondal's

Chapter 2

Reason (R): An idler pulley is free to rotate on its axis and is put on the slack side of the belt. [IES-1994]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-31Ans. (a) Both A and R are true, and R provides correct explanation for A.

IES-32. In a Belt drive, if the pulley diameter is doubled keeping the tension and belt width constant, then it will be necessary to [IES-1993]

- (a) Increase the key length (b) increase the key depth
- (c) Increase the key width (d) decrease the key length

IES-32Ans. (c) Due to twice increase in diameter of pulley, torque on key is double and has to be resisted by key width. Length can't be increased as belt width is same.

IES-33. The following data refers to an open belt drive: [IES-1993]

	Pulley A	Pulley B
Purpose	Driving	Driven
Diameter.....	450 mm	750 mm
Angle of contact.....	$\theta_A = 150^\circ$	$\theta_A = 210^\circ$
Coefficient of friction between belt and pulley	$f_A = 0.36$	$f_A = 0.22$

The ratio of tensions may be calculated using the relation $(T_1/T_2) = \exp(z)$ where z is

- (a) $f_A \theta_A$ (b) $f_B \theta_B$ (c) $(f_A + f_B)(\theta_A + \theta_B)/4$ (d) $(f_A \theta_A + f_B \theta_B)/2$

IES-33Ans. (a) $\frac{T_1}{T_2} = e^{f_A \theta_A}$ where f and θ are taken for smaller pulley.

Centrifugal tension

IES-34. Centrifugal tension in belts is [IES-1999]

- (a) Useful because it maintains some tension even when no power is transmitted
- (b) Not harmful because it does not take part in power transmission
- (c) Harmful because it increases belt tension and reduces the power transmitted
- (d) A hypothetical phenomenon and does not actually exist in belts

IES-34Ans. (c)

IES-35. In the case of a vertical belt pulley drive with T_c as centrifugal tension and T_o as the initial tension, the belt would tend to hang clear of the tower pulley when [IES-1997]

- (a) $T_c < T_o$ (b) $T_c < T_o/3$ (c) $T_c > T_o$ (d) $T_c < T_o/2$

IES-35Ans. (c)

IES-36. Consider the following statements in case of belt drives: [IES 2007]

1. Centrifugal tension in the belt increases the transmitted power.
2. Centrifugal tension does not affect the driving tension
3. Maximum tension in the belt is always three times the centrifugal tension.

Which of the statements given above is/are correct?

- (a) 1, 2 and 3 (b) 2 and 3 only
- (c) 1 and 3 only (d) 1 only

Design of Friction Drives

S K Mondal's

Chapter 2

IES-36Ans. (b)

IES-37. In case of belt drives, the effect of the centrifugal tension is to: [IES-2006]

- (a) Cause the belt to leave the pulley and increase the power to be transmitted
- (b) Cause the belts to stay on the pulley and increase the power to be transmitted
- (c) Reduce the driving power of the belt
- (d) Stretch the belt in longitudinal direction

IES-37Ans. (d) Centrifugal tension has no effect on the power to be transmitted.

$$T_C = m.v^2$$

When centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

Power transmitted,

$$P = (T_{t1} - T_{t2})v = [(T_1 + T_C) - (T_2 + T_C)]v = (T_1 - T_2)v \quad \dots \text{(same as before)}$$

Condition for maximum power

IES-38. In a flat belt drive the belt can be subjected to a maximum tension T and centrifugal tension T_c . What is the condition for transmission of maximum power? [IES-2008]

- (a) $T = T_c$
- (b) $T = \sqrt{3} T_c$
- (c) $T = 2T_c$
- (d) $T = 3T_c$

IES-38Ans. (d)

Condition for maximum power transmission:-

$$P = (T_1 - T_2)v$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow P = (T_{\max} - T_c) \left(1 - \frac{1}{e^{\mu\theta}} \right) v$$

$$\Rightarrow P = (T_{\max}v - mv^3) \left(1 - \frac{1}{e^{\mu\theta}} \right)$$

$$\Rightarrow \frac{dP}{dv} = (T_{\max} - 3mv^2) = 0$$

$$\Rightarrow mv^2 = \frac{T_{\max}}{3}$$

$$\Rightarrow T_c = \frac{T_{\max}}{3}$$

$$\therefore T_{\max} = 3T_c$$

IES-39. Which one of the following statements with regard to belt drives is NOT correct? [IES-2000]

- (a) Increase in the angle of wrap of the belt enables more power transmission
- (b) Maximum power is transmitted when the centrifugal tension is three times the tight side tension
- (c) Wide and thin belt is preferable for better life than a thick and narrow one
- (d) Crown is provided on the pulley to make the belt run centrally on the pulley

IES-39Ans. (b)

Design of Friction Drives

S K Mondal's

Chapter 2

IES-40. When a belt drive is transmitting maximum power [IES-1996]

- (a) Effective tension is equal to centrifugal tension.
- (b) Effective tension is half of centrifugal tension.
- (c) Driving tension on slack side is equal to the centrifugal tension.
- (d) Driving tension on tight side is twice the centrifugal tension.

IES-40Ans. (d)

IES-41. The power transmitted by a belt is dependent on the centrifugal effect in the belt. The maximum power can be transmitted when the centrifugal tension is [IES-2002]

- (a) $1/3$ of tension (T_1) on the tight side
- (b) $1/3$ of total tension (T_t) on the tight side
- (c) $1/3$ of tension (T_2) on the slack side
- (d) $1/3$ of sum of tensions T_1 and T_2 i.e. $1/3 (T_1 + T_2)$

IES-41Ans. (b)

Selection of V-belt drive

IES-42. Assertion (A): For similar materials having the same maximum permissible tension V-belt transmits more power than flat belt with same velocity ratio and centre distance. **[IES-2001]**

Reason (R): As two sides of V-belt are in contact with side faces of pulley groove, larger contact area gives greater effective frictional force.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-42Ans. (a)

IES-43. In a multiple V belt drive, when a single belt is damaged, it is preferable to change the complete set to [IES-1993]

- (a) Reduce vibration
- (b) reduce slip
- (c) Ensure uniform loading
- (d) ensure proper alignment

IES-43Ans. (c) If a single belt breaks, all belts are replaced to ensure uniform loading.

IES-44. Consider the following:

V-belts are specified by their

[IES-2008]

1. Nominal inside length in mm
2. Nominal pitch length
3. Belt cross section symbol
4. weight/unit length of the belt

Which of the above are correct?

- (a) 1, 2, 3 and 4
- (b) 1 and 2 only
- (c) 1 and 3 only
- (d) 3 and 4 only

IES-44Ans. (a)

V-belt designation

B – 2786 – Gr50 → standard size of belt

↓ ↓

Type nominal

of inside

v belt length

Design of Friction Drives

S K Mondal's

Chapter 2

Initial tension in the belt

IES-45. Given that T_1 and T_2 are the tensions on the tight and slack sides of the belt respectively, the initial tension of the belt taking into account centrifugal tension T_c , is equal to [IES-1997]

- (a) $\frac{T_1 + T_2 + T_c}{3}$ (b) $\frac{T_1 + T_2 + 2T_c}{2}$ (c) $\frac{T_1 + T_2 + 3T_c}{3}$ (d) $\frac{T_1 - T_2 + 3T_c}{3}$

IES-45Ans. (b)

Chain drive

IES-46. Which one of the following drives is used for a constant velocity ratio, positive drive with large centre distance between the driver and driven shafts? [IES-2004]

- (a) Gear drive (b) Flat belt drive (c) Chain drive (d) V-belt drive

IES-46Ans. (c)

IES-47. Assertion (A): Slider-crank chain is an inversion of the four-bar mechanism.

Reason(R): Slider-crank chain often finds applications in most of the reciprocating machinery. [IES-2003]

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-47Ans. (b)

IES-48. Match List I (Applications) with List II (Drive element) and select the correct answer using the codes given below the Lists: [IES-2000]

List I

- A. Automobile differential
B. Bicycle
C. Planing machine
D. Radiator fan of automobile

Code: A B C D

- (a) 4 3 1 2
(c) 4 2 1 3

List II

1. Flat belt
2. V-belt
3. Chain drive
4. Gear drive

A B C D

- (b) 1 3 4 2
(d) 1 2 4 3

IES-48Ans. (a)

IES-49. Sources of power loss in a chain drive are given below: [IES-1995]

1. Friction between chain and sprocket teeth.
2. Overcoming the chain stiffness.
3. Overcoming the friction in shaft bearing.
4. Frictional resistance to the motion of the chain in air or lubricant.
The correct sequence of descending order of power loss due to these sources is

- (a) 1,2,3,4 (b) 1,2,4,3 (c) 2,1,3,4 (d) 2,1,4,3

IES-49Ans. (a) Power loss in descending order takes place as 1, 2 3 and 4.

IES-50. Given that P = chain pitch, c = centre distance, [IES-1994]
 N , n = number of teeth on large and small sprocket respectively
the length of chain in terms of pitches can be approximated by

Design of Friction Drives

S K Mondal's

Chapter 2

(a) $\frac{2c}{P}$

(b) $\frac{2c}{P} + (N + n) / 2$

(c) $\frac{2c}{P} + [(N - n) / 2P]^2 \frac{P}{c}$

(d) $\frac{2c}{P} + (N + n) / 2P + [(N - n) / 2P]^2 \frac{P}{c}$

IES-50Ans. (d)

IES-51. For roller chain drive with sprocket having 10 teeth, the velocity of the driven shaft with respect to that of drive will be approximately [IES-2008]

- (a) same
- (b) 5% above
- (c) 5% below
- (d) 5% above to 5% below

IES-51Ans. (d)

$$(V_{\max} - V_{\min}) \propto \left[1 - \cos\left(\frac{180}{z}\right) \right]$$

In order to reduce the variation in chain speed, the number of teeth on the sprocket should be increased. It has been observed that the speed variation is 4% for a sprocket with 11 teeth, 1.6% for a sprocket with 24 teeth.

For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth. For durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

Rope drive

IES-52. In a 6 × 20 wire rope, No.6 indicates the [IES- 2001; 2003; 2007]

- (a) diameter of the wire rope in mm
- (b) Number of strands in the wire rope
- (c) Number of wires
- (d) Gauge number of the wire

IES-52Ans. (b) 6 × 20 wire rope: 6 indicates number of strands in the wire rope and 20 indicates no of wire in a strand.

IES-53. Consider the following types of stresses in respect of a hoisting rope during acceleration of load: [IES-2000]

1. Direct stress due to weight hoisted and weight of the rope
2. Bending stresses due to bending of rope over the sheave
3. Stresses due to initial tightening.
4. Acceleration stresses

Which of these are the correct types of stresses induced in a hoisting rope during acceleration of load?

- (a) 1, 2 and 3
- (b) 2, 3 and 4
- (c) 1, 2 and 4
- (d) 1, 3 and 4

IES-53Ans. (c)

IES-54. Assertion (A): In lifts, wire ropes are preferred over solid steel rods of same diameter.

Reason (R): Wire ropes are more flexible than steel rods and also provide plenty of time for remedial action before failure. **[IES-1999]**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Design of Friction Drives

S K Mondal's

Chapter 2

IES-54Ans. (a)

IES-55. Given that W = weight of load handled, W_r = weight of rope and f = acceleration, the additional load in ropes of a hoist during starting is given by [IES-1997]

$$(a) F_a = \left(\frac{W - W_r}{g} \right) f \quad (b) F_a = \left(\frac{W + W_r}{g} \right) f \quad (c) F_a = \frac{W}{g} f \quad (d) F_a = \frac{W_r}{g} f$$

IES-55Ans. (b)

IES-56. Effective stress in wire ropes during normal working is equal to the stress due to [IES-1996]

- (a) Axial load plus stress due to bending.
- (b) Acceleration / retardation of masses plus stress due to bending.
- (c) Axial load plus stress due to acceleration / retardation.
- (d) bending plus stress due to acceleration/retardation.

IES-56Ans. (a)

IES-57. When compared to a rod of the same diameter and material, a wire rope [IES-1994]

- (a) Is less flexible
- (b) Has a much smaller load carrying capacity.
- (c) Does not provide much warning before failure.
- (d) Provides much greater time for remedial action before failure.

IES-57Ans. (d) A wire rope provides much greater time for remedial action before failure.

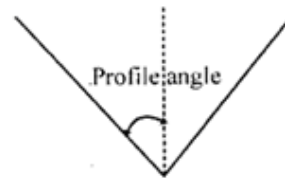
Types of power screw

IES-58. Power screws are used to produce uniform, slow and powerful motion such as required in presses, jacks and other machinery. 'V' threads are usually *not* used for this application due to low efficiency. This is because:

- (a) Profile angle is zero
- (b) Profile angle is moderate [IES-2005]
- (c) Profile angle is large
- (d) There is difficulty in manufacturing the profile

IES-58Ans. (c)

Square thread most efficient.
Profile angle is zero which causes excessive bursting force.



IES-59. Consider the following statements regarding power screws: [IES-1994]

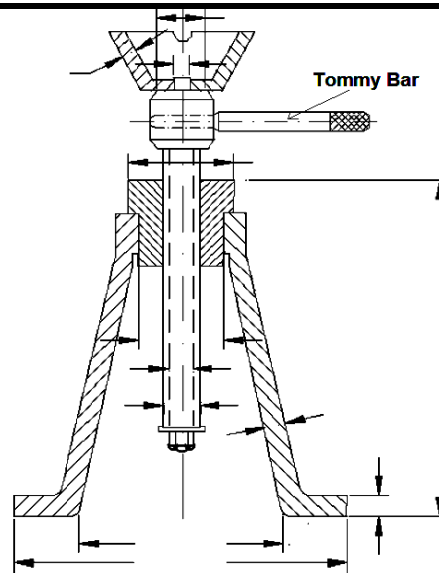
1. The efficiency of a self-locking screw cannot be more than 50%.
2. If the friction angle is less than the helix angle of the screw, then the efficiency will be more than 50%.
3. The efficiency of ACME (trapezoidal thread) is less than that of a square thread.

Of these statements

- (a) 1, 2 and 3 are correct
- (b) 2 and 3 are correct
- (c) 1 and 3 are correct
- (d) 1 and 2 are correct

IES-59Ans. (c)

IES-60. Assertion (A): Buttress thread is a modified square thread profile which is employed on the lead screw of machine tools. [IES-2001]



IES-65. The load cup of a screw jack is made separate from the head of the spindle to [IES-1995]

- (a) Enhance the load carrying capacity of the jack
- (b) Reduce the effort needed for lifting the working load
- (c) Reduce the value of frictional torque required to be countered for lifting the load
- (d) Prevent the rotation of load being lifted.

IES-65Ans. (d)

IES-66. Under service conditions involving jarring, vibration and pulsation of the working load, the bolt of choice would [IES 2007]

- (a) short bolt with high rigidity
- (b) long bolt with increased elasticity
- (c) Bolt with a dished washer
- (d) bolt with castle nut

IES-66Ans. (d)

IES-67. If P is the pitch of a square thread, then the depth of thread d is given by

- (a) $0.5 P$
- (b) P
- (c) $1.5 P$
- (d) $2.0 P$

IES-67Ans. (a)

IES-68. The frictional torque for square thread at mean radius while raising load is given by [IES-1993]

(W = load; R_o = Mean Radius; ϕ = Angle of friction; α = Helix angle)

- (a) $WR_o \tan(\phi - \alpha)$
- (b) $WR_o \tan(\phi + \alpha)$
- (c) $WR_o \tan \alpha$
- (d) $WR_o \tan \phi$

IES-68Ans. (b)

Self locking screw

IES-69. What is the efficiency of a self-locking power screw? [IES-2006; 1997]

- (a) 70%
- (b) 60%
- (c) 55%
- (d) $< 50 \%$

IES-69Ans. (d) We know that the frictional torque for square thread at mean radius while raising load is given by $WR_o \tan(\phi - \alpha)$

Where: (W = load; R_o = Mean Radius; ϕ = Angle of friction; α = Helix angle)

For self locking, angle of friction should be greater than helix angle of screw So that

$WR_o \tan(\phi - \alpha)$ will become positive. i.e. we have to give torque to lowering the load.

Design of Friction Drives

S K Mondal's

Chapter 2

- IES-70. To ensure self-locking in a screw jack it is essential that helix angle is
(a) Larger than friction angle (b) smaller than friction angle. [IES-1996]
(c) Equal to friction angle (d) such as to give maximum efficiency in lifting.

IES-70Ans. (b)

Efficiency of screw

- IES-71. The maximum efficiency of a screw jack having square threads with friction angle ϕ is [IES 2007]

- (a) $\frac{1 - \tan(\phi/2)}{1 + \tan(\phi/2)}$ (b) $\frac{1 - \tan \phi}{1 + \tan \phi}$
(c) $\frac{1 - \sin \phi}{1 + \sin \phi}$ (d) $\frac{1 - \sin(\phi/2)}{1 + \sin(\phi/2)}$

IES-71Ans. (c)

- IES-72. Assertion (A): The maximum efficiency $\left(\eta = \frac{1 - \sin \phi}{1 + \sin \phi} \right)$ of a screw jack is same,

where ϕ is the friction angle, for both motion up and motion down the plane.

Reason (R): The condition for the maximum efficiency for motion up and motion down the plane is same, given by $\alpha = \frac{\pi}{4} - \frac{\phi}{2}$ where α = helix angle. [IES-2003]

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-72Ans. (a)

- IES-73. A screw jack is said to be self-locking if its efficiency is [IES-2002]
(a) Less than 50% (b) equal to 50% (c) more than 50% (d) 100%

IES-73Ans. (a)

- IES-74. Which one of the following is the value of helix angle for maximum efficiency of a square threaded screw? [$\phi = \tan^{-1} \mu$] [IES-2004]
(a) $45^\circ + \phi$ (b) $45^\circ - \phi$ (c) $45^\circ - \phi/2$ (d) $45^\circ + \phi/2$

IES-74Ans. (c)

Collar friction

- IES-75. Stresses in a screw thread are estimated by considering the thread to be:
(a) Long cantilever beam projecting from the pitch cylinder [IES-2006]
(b) Long cantilever beam projecting from the root cylinder
(c) Short cantilever beam projecting from the root cylinder
(d) Short cantilever beam projecting from the pitch cylinder

IES-75Ans. (c)

- Q.20. A power screw of 32 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 12 kN. Permissible thread bearing pressure is 6 MPa; considering bearing action between the threads in engagement, what is the number of threads in engagement with the screw? [IES-2009]
(a) 6 (b) 7 (c) 9 (d) 10

Design of Friction Drives

S K Mondal's

Chapter 2

20. Ans. (c)

Previous 20-Years IAS Questions

Uniform wear theory

IAS-1. The frictional torque transmitted in a flat pivot bearing, assuming uniform wear, is [IAS-2002]

- (a) μWR (b) $\frac{3}{4} \mu WR$ (c) $\frac{2}{3} \mu WR$ (d) $\frac{1}{2} \mu WR$

(Where μ = Coefficient of friction; W = Load over the bearing; R = Radius of bearing)

IAS-1Ans. (d) Use frictional clutch formula.

$$T = \frac{\mu W}{4}(D + d), d = 0 \text{ and } D = 2R \text{ gives } T = \frac{\mu \pi R}{2}$$

Belt and Chain drives

IAS-2. A pulley and belt in a belt drive from a [IAS-2001]

- (a) Cylindrical pair (b) turning pair (c) rolling pair (d) sliding pair

IAS-2Ans. (c)

IAS-3. Crushed ore is dropped on a conveyor belt at the rate of 300 kg/s. The belt moves at speed of 2 m/s. The net force acting on the belt that keeps it moving at the same speed is [IAS-2001]

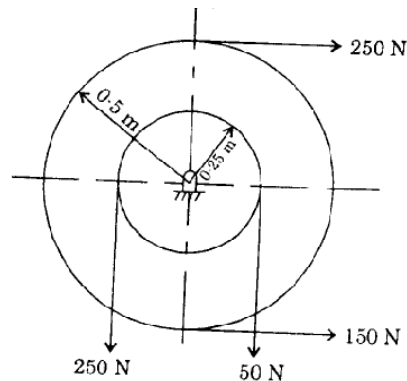
- (a) 30 N (b) 60 N (c) 300 N (d) 600 N

IAS-3Ans. (d) Force = $\frac{d}{dt}(mv) = \frac{dm}{dt} \times v = 300 \times 2 = 600 \text{ N}$

Belt tension

IAS-4. A Differential pulley is subjected to belt tensions as shown in the diagram. The resulting force and moment when transferred to the centre of the pulley are, respectively

- (a) 400 N and 0 Nm
(b) 400 N and 100 Nm
(c) 500 N and 0 Nm
(d) 500 N and 100 Nm



[IAS-2003]

IAS-4Ans. (c)

Design of Friction Drives

S K Mondal's

Chapter 2

$$\begin{aligned}
 &F_y = 300 \text{ N} \\
 &\text{Resultant force at the centre} \quad \uparrow F_x = 400 \text{ N} \\
 &\Rightarrow \sqrt{300^2 + 400^2} = 500 \text{ N} \\
 &\text{Resultant moment due to horizontal force} \\
 &\quad = (250 - 150) \times 0.5 = 50 \text{ N-m (clockwise)} \\
 &\text{and Resultant moment due to vertical force} \\
 &\quad = (250 - 50) \times 0.25 = 50 \text{ N-m (Anticlockwise)} \\
 &\therefore \text{Net moment} = 50 \text{ N} - 50 \text{ N} = 0
 \end{aligned}$$

Selection of V-belt drive

IAS-5. A 50 kW motor using six V belts is used in a pulp mill. If one of the belts breaks after a month of continuous running, then [IAS 1994]

- (a) The broken belt is to be replaced by a similar belt
- (b) All the belt are to be replaced
- (c) The broken belt and two adjacent belts are to be replaced
- (d) The broken belt and one adjacent belt are to be replaced

IAS-5Ans. (b)

Types of power screw

IAS-6. Match List I with List II and select the correct answer using the code given below the Lists: [IAS-2007]

List I (Type of Thread)				List II (Use)				
A. Square thread				1. Used in vice				
B. Acme thread				2. Used in lead screw				
C. Buttress thread				3. Used in screw jack				
D. Trapezoidal thread				4. Used in power transmission devices in machine tool				
Code:	A	B	C	D	A	B	C	D
(a)	2	3	4	1	(b)	2	3	4
(c)	3	2	1	4	(d)	3	2	1

IAS-6Ans. (c)

Design of Friction Drives

S K Mondal's

Chapter 2

Answers with Explanation (Objective)



Design of Power Transmission System

Theory at a glance (GATE, IES, IAS & PSU)

Spur gear

Basic Purpose of Use of Gears

Gears are widely used in various mechanisms and devices to transmit power And motion positively (without slip) between parallel, intersecting (axis) or Non-intersecting non parallel shafts,

- Without change in the direction of rotation
- With change in the direction of rotation
- Without change of speed (of rotation)
- With change in speed at any desired ratio

Often some gearing system (rack – and – pinion) is also used to transform Rotary motion into linear motion and vice-versa.

- A **SPUR GEAR** is cylindrical in shape, with teeth on the outer circumference that are straight and parallel to the axis (hole). There are a number of variations of the basic spur gear, including pinion wire, stem pinions, rack and internal gears.



Fig.

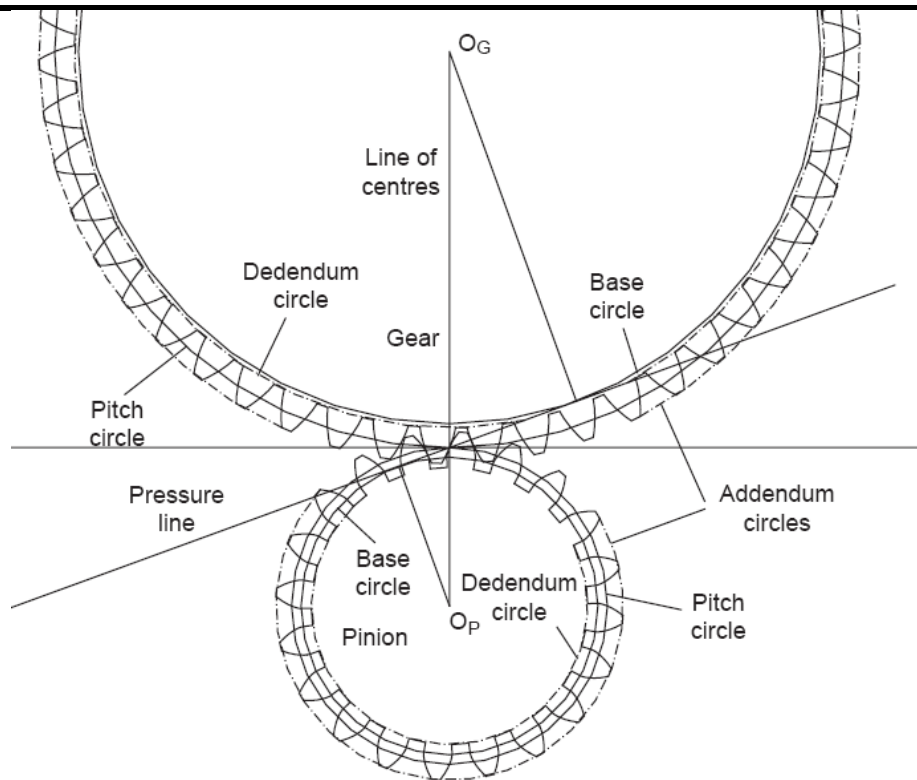


Figure- Layout of a pair of meshing spur gears

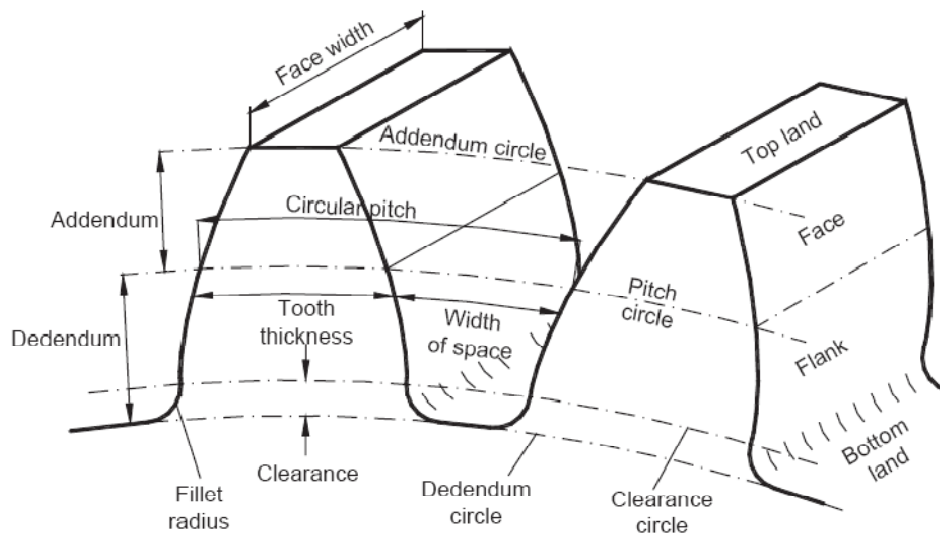


Figure- Spur gear schematic showing principle terminology

For a pair of meshing gears, the smaller gear is called the 'pinion', the larger is called the 'gear wheel' or simply the 'gear'.

Pitch circle

This is a theoretical circle on which calculations are based. Its diameter is called the pitch diameter.

$$d = mT$$

Where d is the pitch diameter (mm); m is the module (mm); and T is the number of teeth. Care must be taken to distinguish the module from the unit symbol for a meter.

Circular pitch

This is the distance from a point on one tooth to the corresponding point on the adjacent tooth measured along the pitch circle.

$$p = \pi m = \frac{\pi d}{T}$$

Where p is the circular pitch (mm); m the module; d the pitch diameter (mm); and T the Number of teeth.

Module.

This is the ratio of the pitch diameter to the number of teeth. The unit of the module should be millimeters (mm). The module is defined by the ratio of pitch diameter and number of teeth. Typically the height of a tooth is about 2.25 times the module. Various modules are illustrated in figure.

$$m = \frac{d}{T}$$

- **Addendum**, (a). This is the radial distance from the pitch circle to the outside of the tooth.
- **Dedendum**, (b). This is the radial distance from the pitch circle to the bottom land.

Clearance (C) is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

Backlash

BACKLASH is the distance (spacing) between two “mating” gears measured at the back of the driver on the pitch circle. Backlash, which is purposely built in, is very important because it helps prevent noise, abnormal wear and excessive heat while providing space for lubrication of the gears.

- The backlash for spur gears depends upon (i) module and (ii) pitch line velocity.
- Factor affected by changing center distance is backlash.

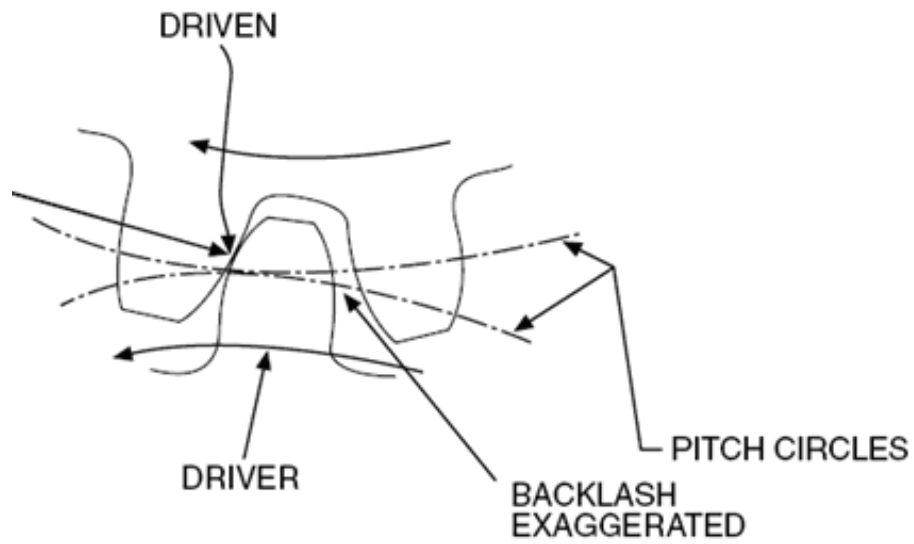


Fig.

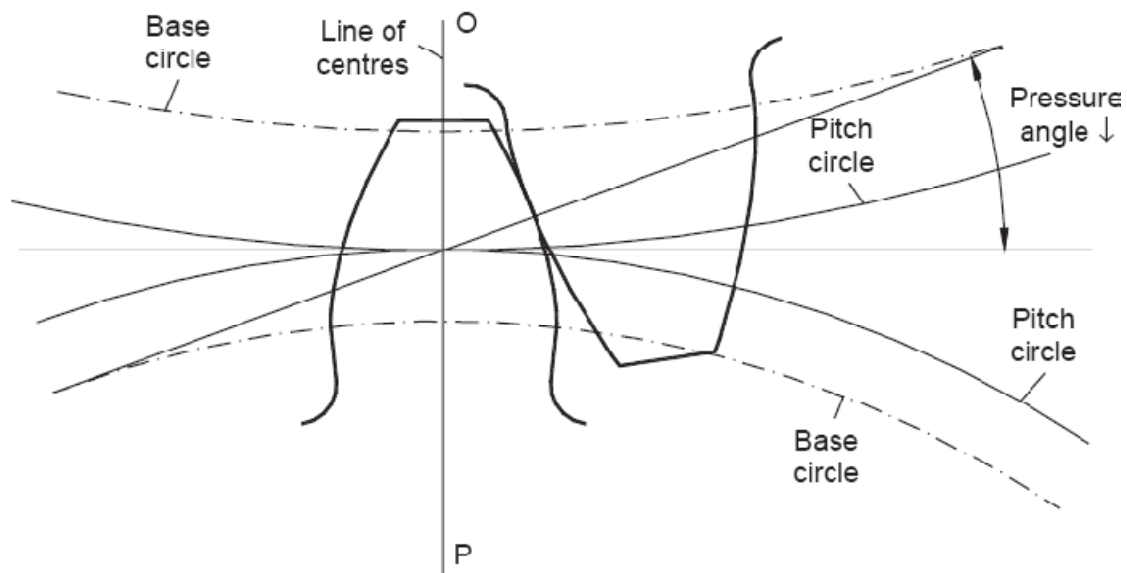


Figure- Schematic showing the pressure line and pressure angle

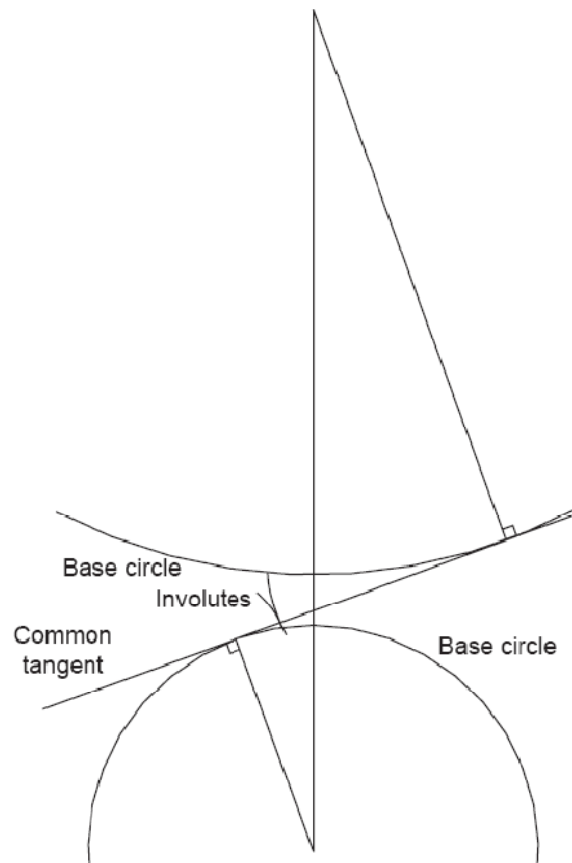


Figure- Schematic of the involute form

Pitch Circle and pitch point

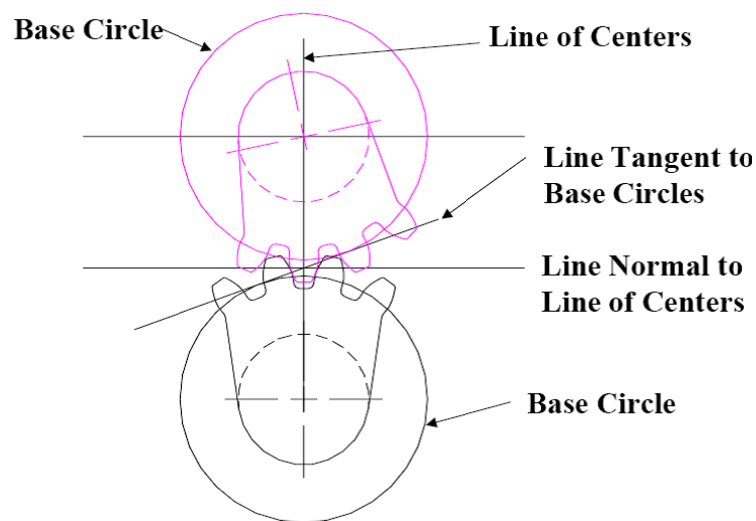


Fig.

Line of Action – Line tangent to both base circles

Pitch Point – Intersection of the line of centers with the line of action

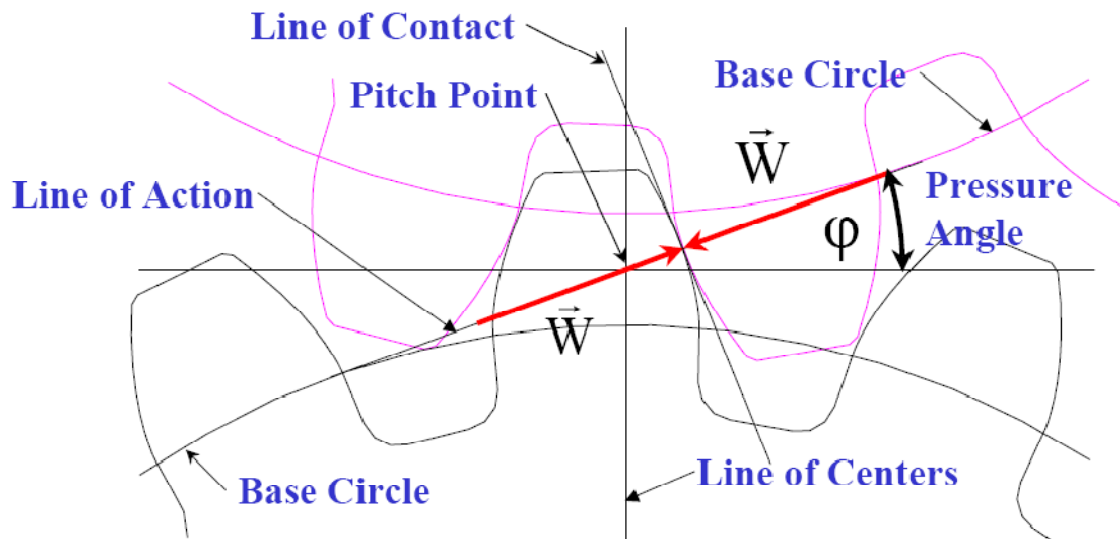


Fig.

Pitch Circle – Circle with origin at the gear center and passing through the pitch point.

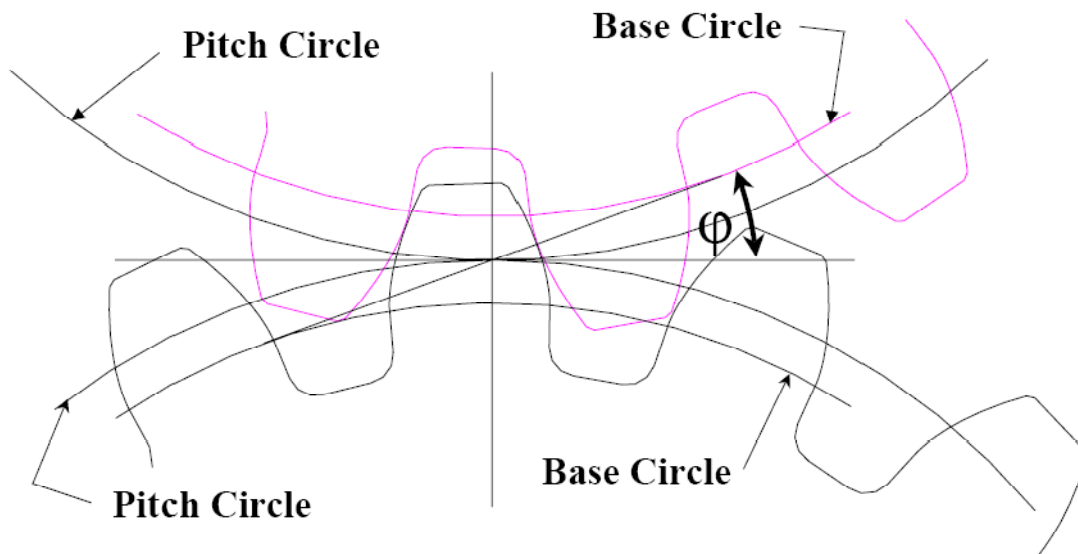


Fig.

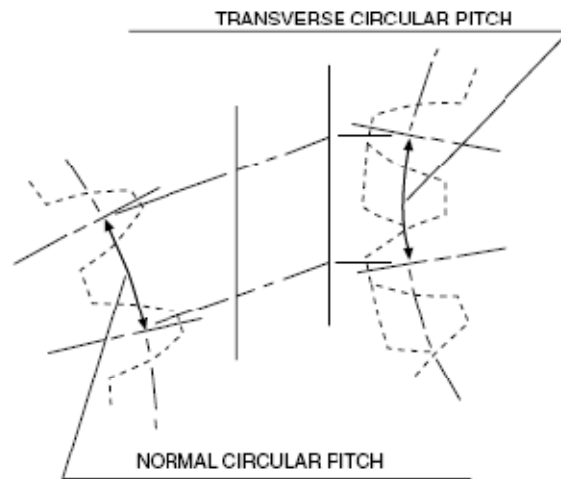


Fig.

Pressure anglee – Angle between the line normal to the line of centers and the line of action.

- The pressure angle of a spur gear normally varies **from 14° to 20°**
- The value of pressure angle generally used for involute gears are **20°**
- Relationship Between Pitch and Base Circles

$$r_b = r \cos \phi$$

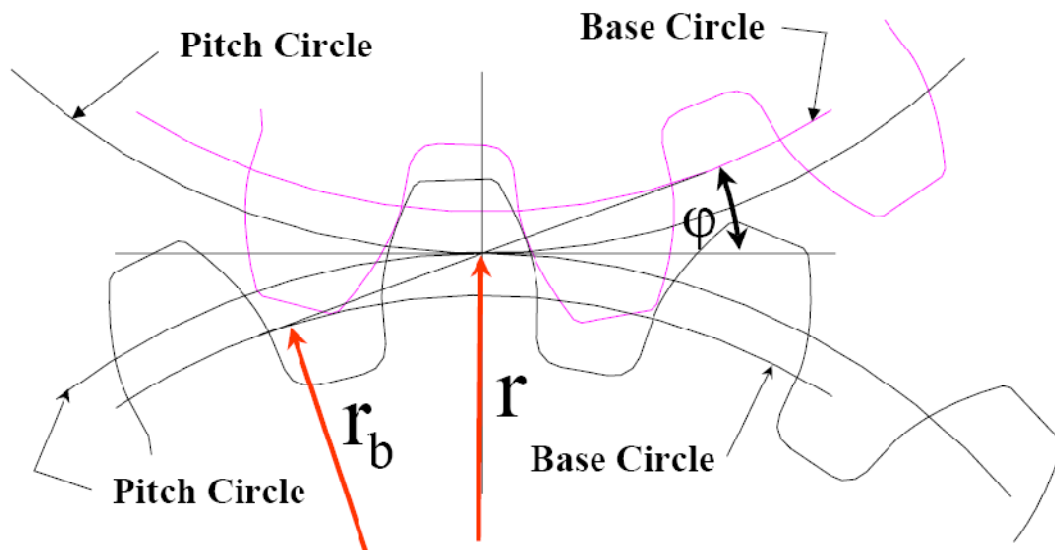


Fig.

The following four systems of gear teeth are commonly used in practice.

1.	$14\frac{1}{2}^0$	Composite system.
2.	$14\frac{1}{2}^0$	Full depth involute system.
3.	20^0	Full depth involute system
4	20^0	Stub involutes system.

The $14\frac{1}{2}^0$ **composite system** is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the $14\frac{1}{2}^0$ **full depth involute system** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the 20^0 **full depth involute system** may be cut by hobs. The increase of the pressure angle from $14\frac{1}{2}^0$ to 20^0 results in a stronger tooth, because the tooth acting as a beam is wider at the base. The 20^0 **stub involute system** has a **strong tooth** to take heavy loads.

Classification of Gears

Gears can be divided into several broad classifications.

1. Parallel axis gears:

- (a) Spur gears
- (b) Helical gears
- (c) Internal gears.

2. Non-parallel, coplanar gears (intersecting axes):

- (a) Bevel gears
- (b) Face gears,
- (c) Conical involute gearing.

3. Non-parallel, non- coplanar gears (nonintersecting axes):

- (a) Crossed axis helical
- (b) Cylindrical worm gearing
- (c) Single enveloping worm gearing,
- (d) Double enveloping worm gearing,
- (e) Hypoid gears,
- (f) Spiroid and helicon gearing,
- (g) Face gears (off centre).

4. Special gear types:

- (a) Square and rectangular gears,
- (b) Elliptical gears.

RACK

RACKS are yet another type of spur gear. Unlike the basic spur gear, racks have their teeth cut into the surface of a straight bar instead of on the surface of a cylindrical blank.

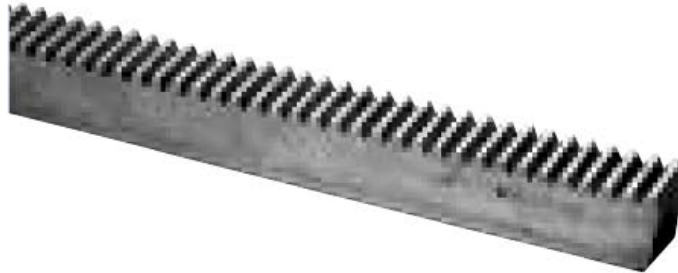


Fig. Rack

Helical gear

The helical gears may be of *single helical type* or *double helical type*. In case of single helical gears there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears (*i.e.* **herringbone gears**) are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are provided on each gear and the resulting axial **thrust is zero**.

Herringbone gears

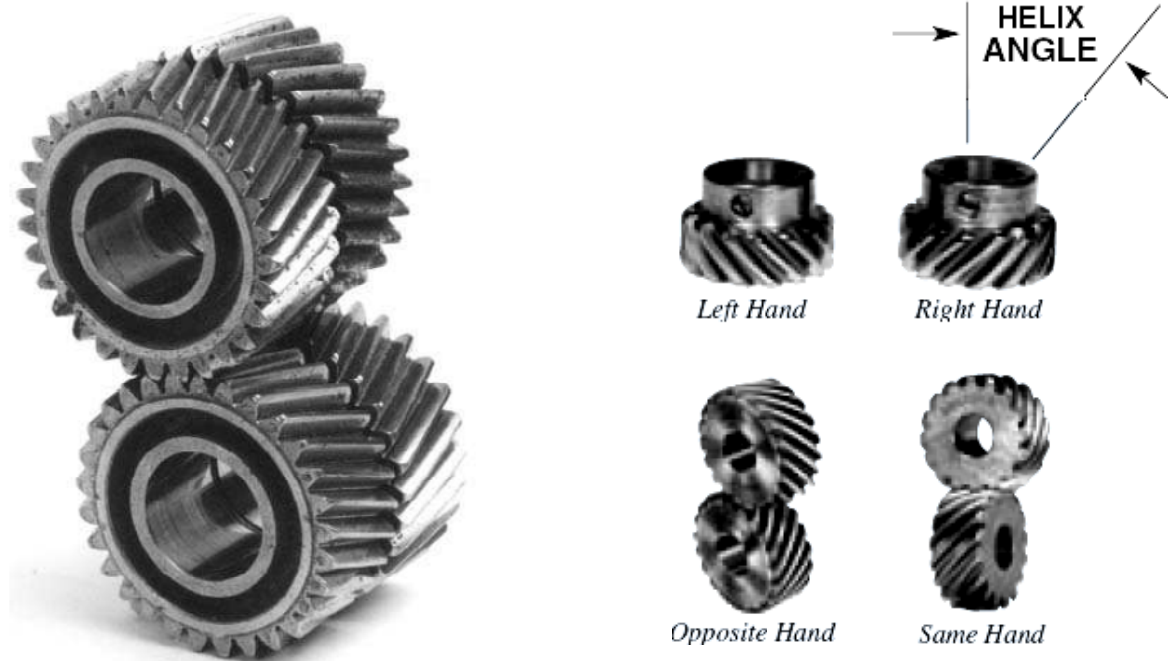


Figure-Herringbone gear

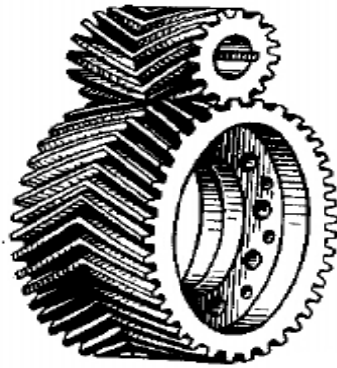


Figure- Herringbone gear



Figure- Crossed axis helical gears

- In spur gears, the contact between meshing teeth occurs along the entire face width of the tooth, resulting in a sudden application of the load which, in turn, results in impact conditions and generates noise.

- In helical gears, the contact between meshing teeth begins with a point on the leading edge of the tooth and gradually extends along the diagonal line **across** the tooth. There is a gradual pick-up of load by the tooth, resulting in smooth engagement and silence operation.

Bevel Gears



Fig.



Fig.

Worm Gear

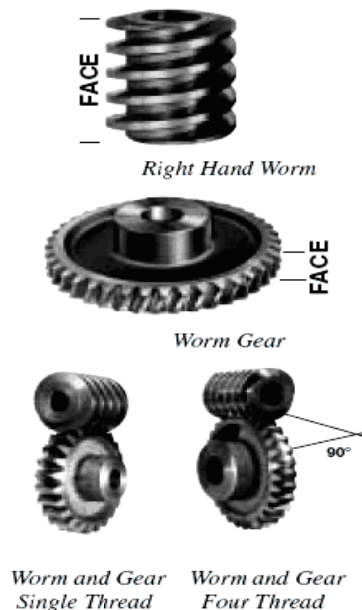


Fig.



Fig.

Hypoid Gears

Hypoid gears resemble bevel gears and spiral bevel gears and are used on crossed-axis shafts. The distance between a hypoid pinion axis and the axis of a hypoid gear is called the *offset*. Hypoid pinions may have as few as five teeth in a high ratio set. Ratios can be obtained with hypoid gears that are not available with bevel gears. High ratios are easy to obtain with the hypoid gear system.

Hypoid gears are matched to run together, just as zero or spiral bevel gear sets are matched. The geometry of hypoid teeth is defined by the various dimensions used to set up the machines to cut the teeth.

- Hypoid gears are similar in appearance to spiral-bevel gears. They differ from spiral-bevel gears in that the axis of the pinion is offset from the axis of the gear.

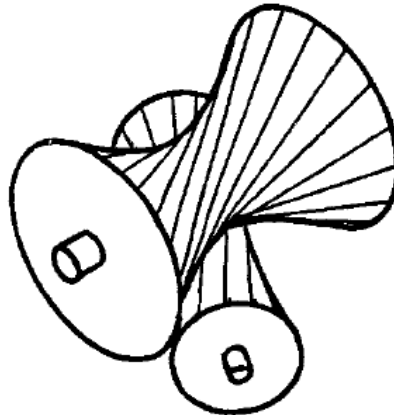


Fig.

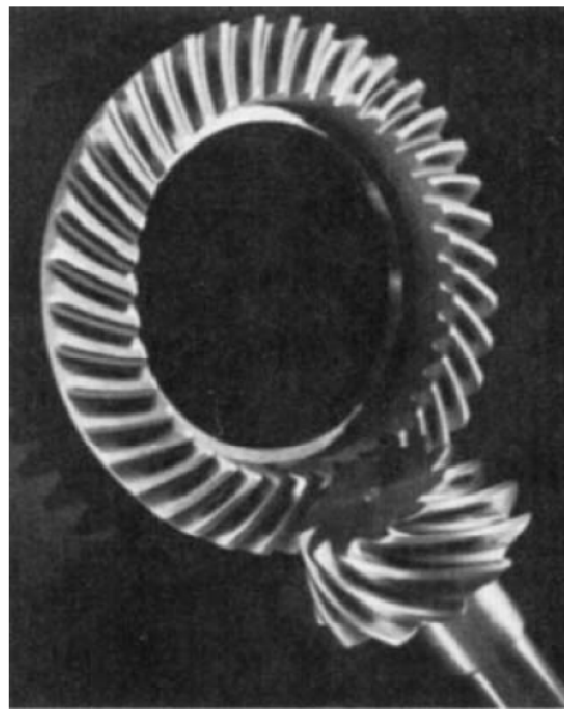


Figure- Hypoid gear

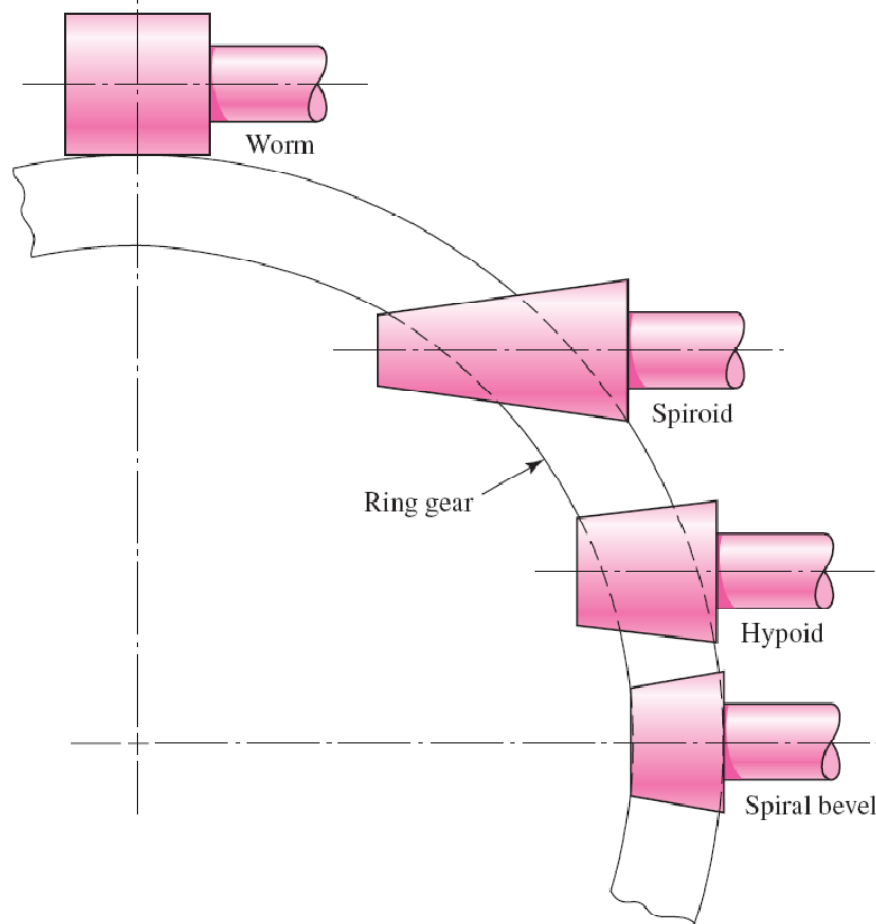


Figure- Comparison of intersecting and offset-shaft bevel-type gearings

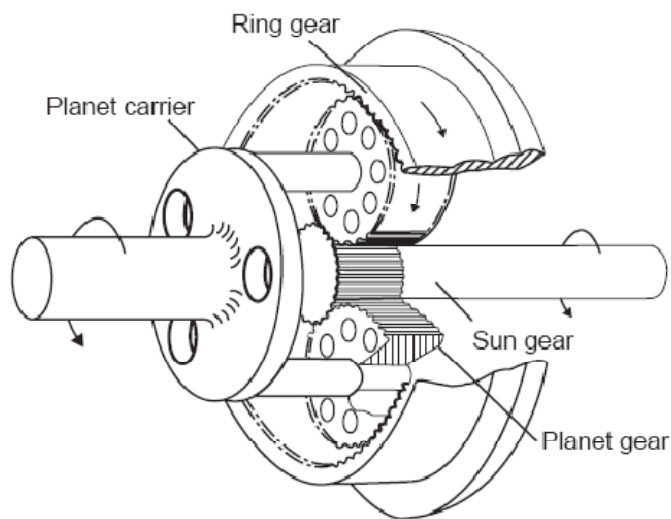


Figure-Epicyclic gears

Mitres gear

Miter gears are identical to bevel gears except that in a miter gear set, both gears always have the same number of teeth. Their ratio, therefore, is **always 1 to 1**. As a result, miter gears are not used when an application calls for a change of speed.

- When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as *mitres*.



Straight Tooth

Spiral Tooth

Figure- Miter gears

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

The number of teeth on the pinion (T_p) in order to avoid interference may be obtained from the following relation:

$$T_p = \frac{2A_w}{G \left[\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1 \right]}$$

Where A_w = Fraction by which the standard addendum for the wheel should be Multiplied, (generally $A_w = 1$)
 G = Gear ratio or velocity ratio = $T_G / T_P = D_G / D_P$,
 ϕ = Pressure angle or angle of obliquity.

- Minimum number of teeth for involute rack and pinion arrangement for pressure angle of 20° is

$$T_{\min} = \frac{2A_R}{\sin^2 \theta} = \frac{2 \times 1}{\sin^2 20^\circ} = 17.1 \quad \text{as } > 17 \quad \text{So, } T_{\min} = 18$$

- The minimum number of teeth on the pinion to operate without interference in standard full height involute teeth gear mechanism with 20° pressure angle is **18**.
- In **full depth** $14\frac{1}{2}^\circ$ degree involute system, the smallest number of teeth in a pinion which meshes with rack with out interference is **32**.

Forms of teeth

Cycloidal teeth

A **cycloid** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as **epicycloid**. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called **hypocycloid**.

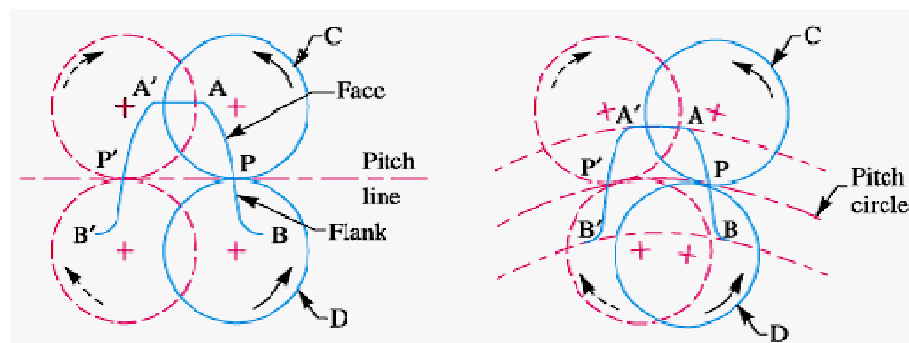


Fig. cycloidal teeth of a gear

Advantages of cycloidal gears

Following are the advantages of cycloidal gears:

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears for the same pitch. Due to this reason, the cycloidal teeth are preferred especially for cast teeth.
2. In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible.
3. In cycloidal gears, **the interference does not occur** at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

Involute teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel as shown in figure below. In connection with toothed wheels, the circle is known as base circle.

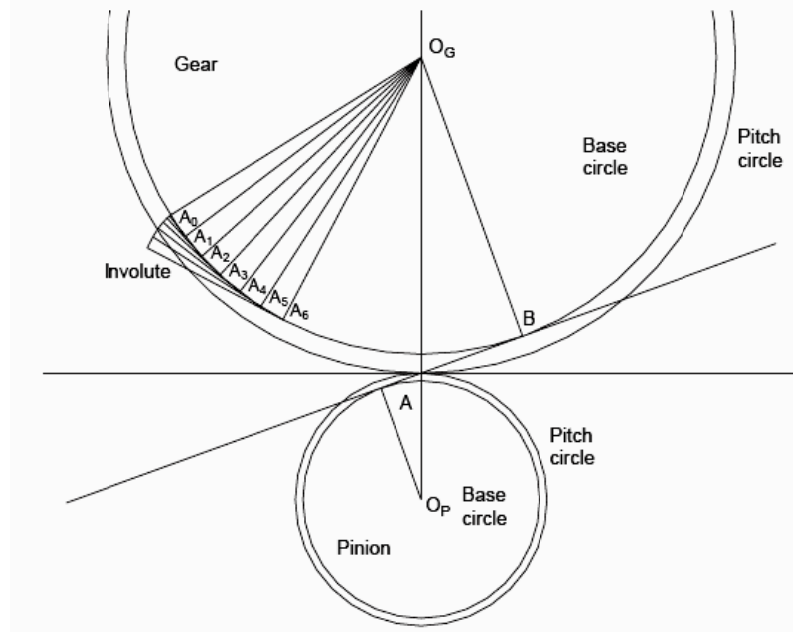


Fig.

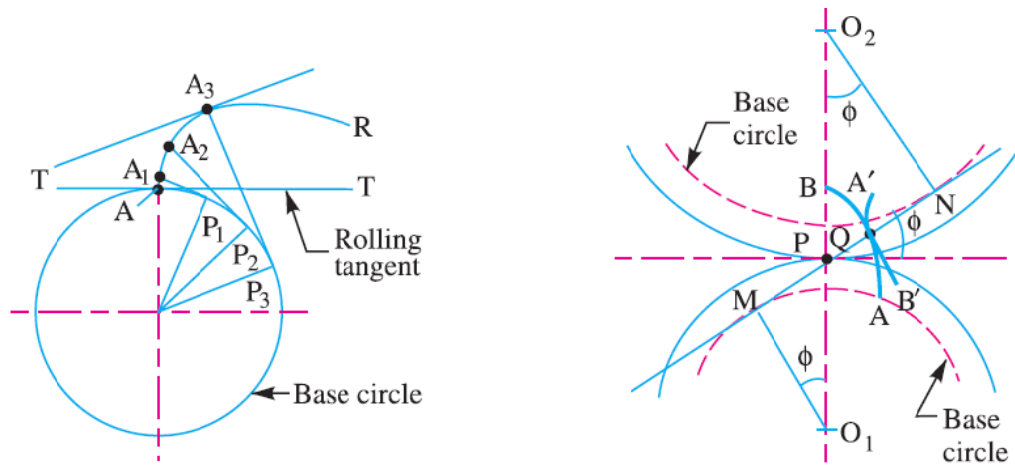


Figure-involute teeth

- The tooth profile most commonly used in gear drives for power transmission is an involute. It is due to easy manufacturing.

Advantages of involute gears

Following are the advantages of involute gears:

1. The most important advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without changing the velocity ratio. This is not true for cycloidal gears which require exact centre distance to be maintained.

2. In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains **constant**. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts increasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve whereas in cycloidal gears, double curves (*i.e.* epicycloids and hypocycloid) are required for the face and flank respectively.

Thus the involute teeth are easy to manufacture than cycloidal teeth. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Note: The only disadvantage of the involute teeth is that **the interference occurs** with pinions having smaller number of teeth. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.

Contact ratio

Note: The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** *i.e.* number of pairs of teeth in contact.

$$\begin{aligned}\text{Contact ratio} &= \frac{\text{length of arc of contact}}{\text{circular pitch}} \\ &= \frac{\sqrt{R_A^2 - R^2 \cos^2 \phi} + \sqrt{r_A^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi}{P_c (\cos \phi)}\end{aligned}$$

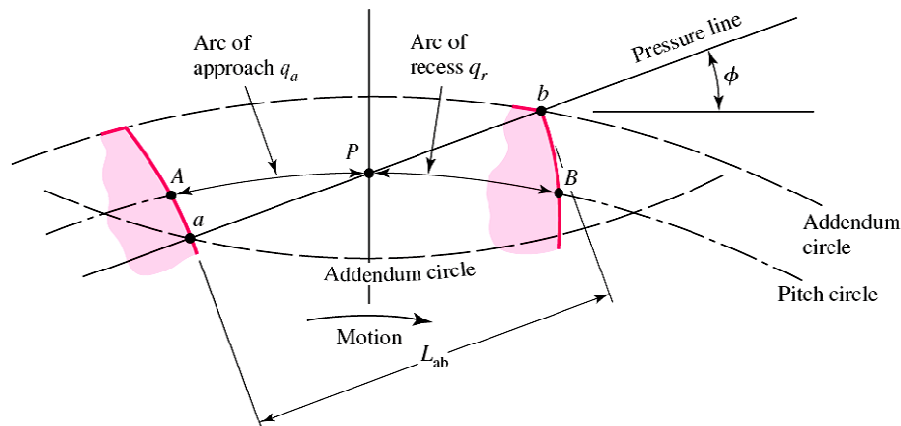


Fig.

The zone of action of meshing gear teeth is shown in figure above. We recall that tooth Contact begins and ends at the intersections of the two addendum circles with the pressure line. In figure above initial contact occurs at *a* and final contact at *b*. Tooth profiles drawn through these points intersect the pitch circle at *A* and *B*, respectively. As shown, the distance *AP* is called **the arc of approach** (q_a), and the distance *P B*, **the arc of recess** (q_r). The sum of these is the **arc of action** (q_v).

- The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** *i.e.* number of pairs of teeth in contact. The contact ratio for gears is greater than one. **Contact ratio should be at least 1.25.** For maximum smoothness and quietness, the contact ratio should be between 1.50 and 2.00. High-speed applications should be designed with a face-contact ratio of 2.00 or higher for best results.

Interference

- The contact of portions of tooth profiles that are not conjugate is called *interference*.
- Contact begins when the tip of the driven tooth contacts the flank of the driving tooth. In this case the flank of the driving tooth first makes contact with the driven tooth at point A, and this occurs *before* the involute portion of the driving tooth comes within range. In other words, contact is occurring below the base circle of gear 2 on the *noninvolute* portion of the flank. The actual effect is that the involute tip or face of the driven gear tends to dig out the noninvolute flank of the driver.

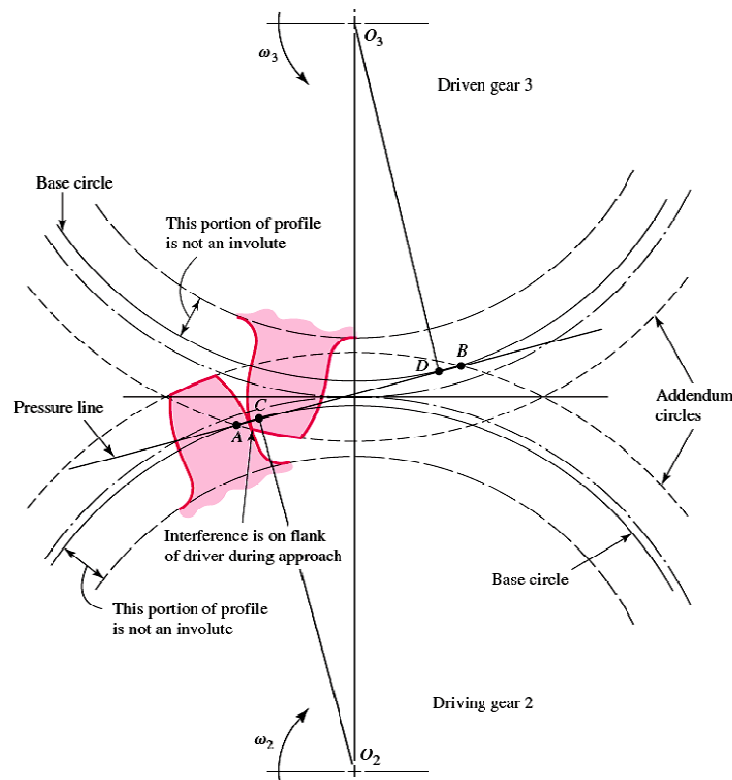


Fig.

- Interference can be eliminated by using more teeth on the pinion. However, if the pinion is to transmit a given amount of power, more teeth can be used only by increasing the pitch diameter.
- Interference can also be reduced by using a larger pressure angle. This results in a smaller base circle, so that more of the tooth profile becomes involute.
- The demand for smaller pinions with fewer teeth thus favors the use of a 25° pressure angle even though the frictional forces and bearing loads are increased and the contact ratio decreased.
- **There are several ways to avoid interfering:**
 - Increase number of gear teeth
 - Modified involutes
 - Modified addendum
 - Increased centre distance.

Face Width

Face width. It is the width of the gear tooth measured parallel to its axis.

Face width.

We know that **face width**,
 $b = 10 m$

Where, **m** is module.

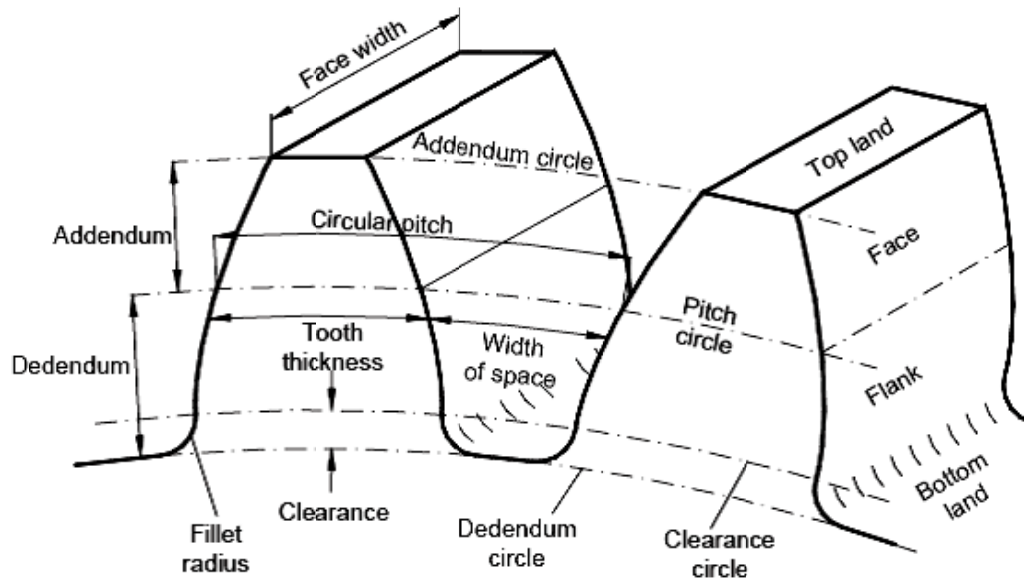


Fig.

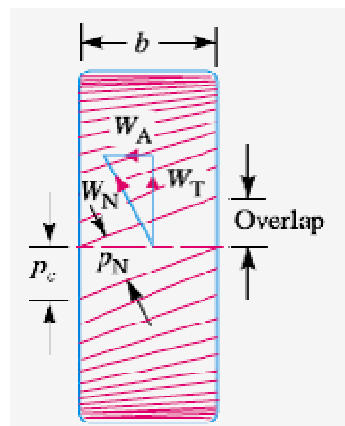


Fig. Face width of helical gear.

Beam Strength of Gear Tooth

The beam strength of gear teeth is determined from an equation (**known as Lewis equation**) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to

assume that the load is distributed among several teeth, considering each tooth as a cantilever beam.

Notes: (i) The **Lewis equation** is applied only to the weaker of the two wheels (*i.e.* pinion or gear).

(ii) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(iii) When the pinion and the gear are made of different materials, then the product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.

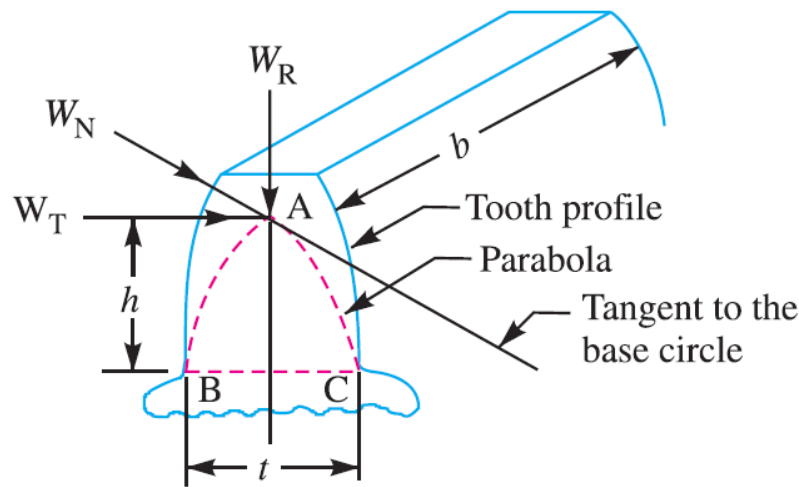


Figure- Tooth of a gear

The maximum value of the bending stress (or the permissible working stress):

$$\sigma_w = \frac{(W_T \times h)t / 2}{b.t^3 / 12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

Where

M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b.t^3/12$,

b = Width of gear face.

Lewis form factor or tooth form factor

$$W_T = \sigma_w \cdot b \cdot p_c \cdot y = \sigma_w \cdot b \cdot \pi m \cdot y$$

The quantity y is known as **Lewis form factor** or **tooth form factor** and W_T (which is the tangential load acting at the tooth) is called the **beam strength of the tooth**.

Lewis form factor or tooth form factor

$$\begin{aligned}
 y &= 0.124 - \frac{0.684}{T}, \text{ for } 14\frac{1}{2}^{\circ} \text{ composite and full depth involute system.} \\
 &= 0.154 - \frac{0.912}{T}, \text{ for } 20^{\circ} \text{ full depth involute system.} \\
 &= 0.175 - \frac{0.841}{T}, \text{ for } 20^{\circ} \text{ stub system.}
 \end{aligned}$$

Example: A spur gear transmits 10 kW at a pitch line velocity of 10 m/s; driving gear has a diameter of 1.0 m. find the tangential force between the driver and the follower, and the transmitted torque respectively.

Solution: Power transmitted = Force × Velocity

$$\Rightarrow 10 \times 10^3 = \text{Force} \times 10$$

$$\Rightarrow \text{Force} = \frac{10 \times 10^3}{10} = 1000 \text{ N/m}$$

$$\text{Torque Transmitted} = \text{Force} \times \frac{\text{diameter}}{2}$$

$$= 1000 \times \frac{1}{2} = 1000 \times 0.5$$

$$= 500 \text{ N-m} = 0.5 \text{ kN-m}$$

Wear Strength of Gear Tooth

Wear strength (σ_w) = $bQdpK$,

$$\text{Where, } Q = \frac{2T_g}{T_g + T_p} \quad \text{for external gear}$$

$$= \frac{2T_g}{T_g - T_p} \quad \text{for internal gear}$$

$$\begin{aligned}
 \text{load - stress factor (k)} &= \frac{\sigma_c^2 \sin \phi \cos \phi}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \\
 &= 0.16 \left(\frac{\text{BHN}}{100} \right)^2
 \end{aligned}$$

Gear Lubrication

All the major oil companies and lubrication specialty companies provide lubricants for gearing and other applications to meet a very broad range of operating conditions. General gear lubrication consists of high-quality machine oil when there are no temperature extremes or other adverse ambient conditions. Many of the automotive greases and oils are suitable for a broad range of gearing applications.

For adverse temperatures, environmental extremes, and high-pressure applications, consult the lubrication specialty companies or the major oil companies to meet your particular requirements or specifications.

The following points refer especially to spiral and hypoid bevel gears:

- (a) Both spiral and hypoid bevel gears have combined rolling and sliding motion between the teeth, the rolling action being beneficial in maintaining a film of oil between the tooth mating surfaces.
- (b) Due to the increased sliding velocity between the hypoid gear pair, a more complicated lubrication system may be necessary.

Simple Gear train

A gear train is one or more pairs of gears operating together to transmit power. When two gears are in mesh, their pitch circles roll on each other without slippage.

If r_1 is pitch radius of gear 1; r_2 is pitch radius of gear 2; ω_1 is angular velocity of gear 1; and ω_2 is angular velocity of gear 2 then the pitch line velocity is given by

$$V = |r_1 \omega_1| = |r_2 \omega_2|$$

The velocity ratio is

$$\frac{|\omega_1|}{|\omega_2|} = \frac{r_2}{r_1}$$

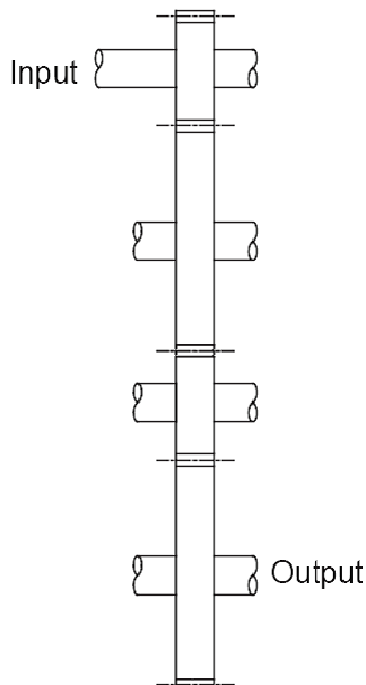


Figure- Simple gear train

Compound gear train

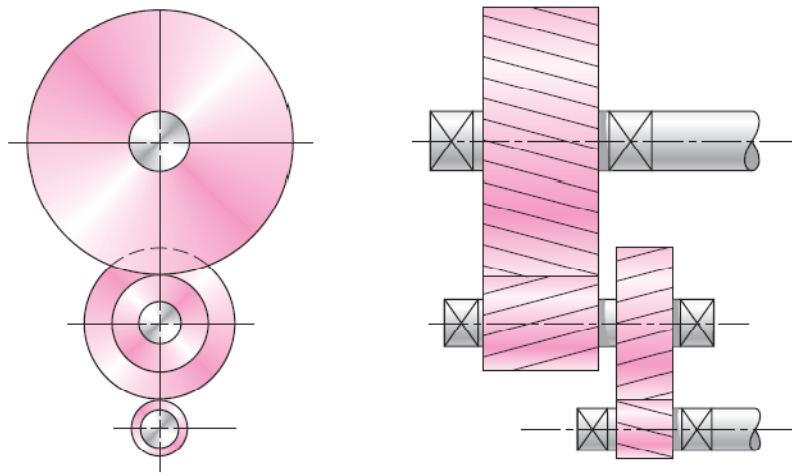


Figure -Compound gear train

The velocity ratio in the case of the compound train of wheels is equal to

$$= \frac{\text{Product of teeth on the followers}}{\text{Product of teeth on the drivers}}$$

The velocity ratio of the following gear train is

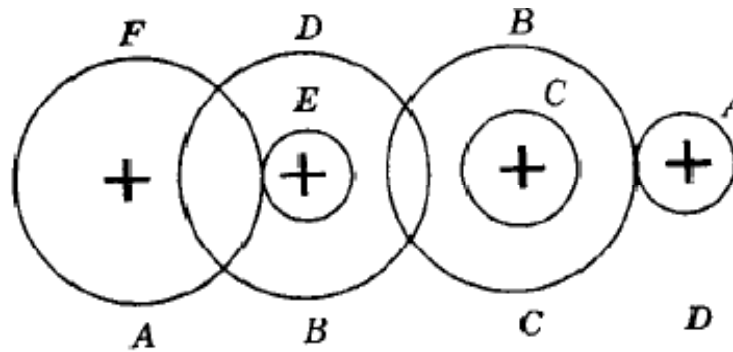


Figure- velocity ratio

$$\frac{N_F}{N_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F}$$

Reverted gear train

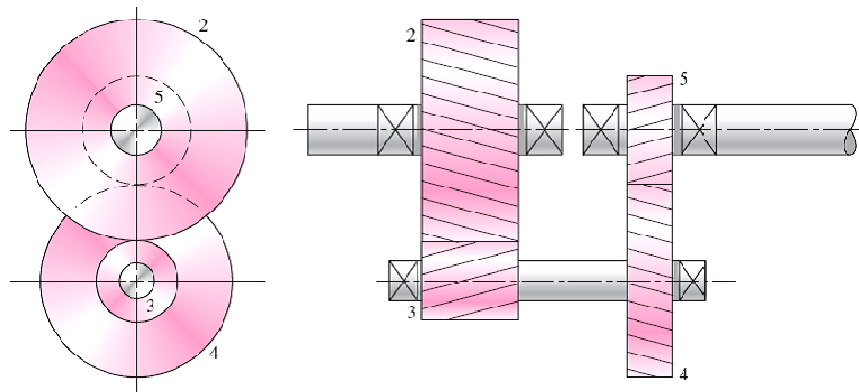


Figure- reverted gear train or a compound reverted gear train

It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Fig above. This configuration is called a *compound reverted gear train*. This requires the distances between the shafts to be the same for both stages of the train.

The distance constraint is

$$\frac{d_2}{2} + \frac{d_3}{2} = \frac{d_4}{2} + \frac{d_5}{2}$$

The diametric pitch relates the diameters and the numbers of teeth, $P = T/d$. Replacing All the diameters give

$$T_2 / (2P) + T_3 / (2P) = T_4 / (2P) + T_5 / (2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition Stated in terms of numbers of teeth:

$$T_2 + T_3 = T_4 + T_5$$

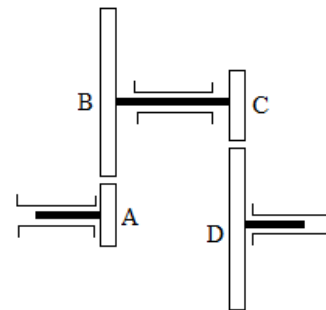
This condition must be exactly satisfied, in addition to the previous ratio equations, to Provide for the in-line condition on the input and output shafts.

In the compound gear train shown in the figure, gears A and C have equal numbers of teeth and gears B and D have equal numbers of teeth.

From the figure $r_A + r_B = r_C + r_D$ or $T_A + T_B = T_C + T_D$ and as $N_B + N_C$ it must be $T_B = T_D$ & $T_A = T_C$

$$\text{Or } \frac{N_B}{N_A} = \frac{N_D}{N_C} \text{ or } N_C = \sqrt{N_A N_D}$$

[where $N_B = N_C$]



Epicyclic gear train

Consider the following Epicyclic gear train

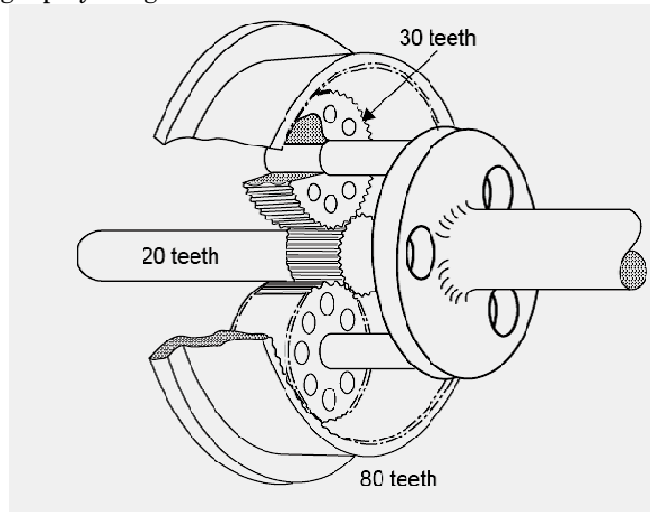


Figure- Epicyclic gear train

For the epicyclic gearbox illustrated in figure, determine the speed and direction of the final drive and also the speed and direction of the planetary gears. The teeth numbers of the sun, planets and ring gear are 20, 30 and 80, respectively. The speed and direction of the sun gear is 1000 rpm clockwise and the ring gear is held stationary.

Solution

$$n_{arm} = \frac{n_{sun}}{(80/20) + 1} = \frac{-1000}{5} = -200 \text{ rpm}$$

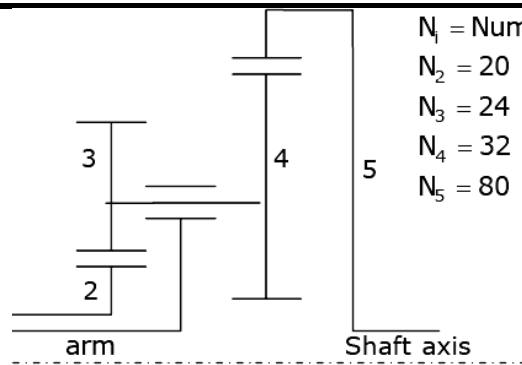
The speed of the final drive is 200 rpm clockwise. The reduction ratio for the gearbox is Given by $n_{sun}/n_{arm} = 1000/200 = 5$. To determine the speed of the planets use

The planets and sun are in mesh, so

$$\begin{aligned} \frac{n_{planet} / n_{arm}}{n_{sum} / n_{arm}} &= -\frac{N_S}{N_P} \\ \frac{n_{planet} - n_{arm}}{n_{sum} - n_{arm}} &= -\frac{N_S}{N_P} \\ \frac{n_{planet} - (-200)}{-1000 - (-200)} &= -\frac{20}{30} \\ n_{planet} &= -\frac{20}{30} \times (-800) - 200 = 333 \text{ rpm} \end{aligned}$$

The speed of rotation of the planetary gears is 333 rpm counter-clockwise.

Now make a table for **the epicyclic gear arrangement shown in the figure below.**



N_i = Number of teeth for gear i
 $N_2 = 20$
 $N_3 = 24$
 $N_4 = 32$
 $N_5 = 80$

	Arm	2	3	4	5
1.	0	+x	$\frac{-N_2}{N_3}x$	$\frac{-N_2}{N_3}x$	$\frac{-N_4}{N_5} \times \frac{N_2}{N_3}x$
2.	y	y	y	y	y
	y	x + y	$y - \frac{N_2}{N_3}x$		$y - \frac{N_4}{N_5} \times \frac{N_2}{N_3}x$

Formula List for Gears:

(a) Spur Gear

Name Speed ratio

1. Spur & Helical 6:1 to 10:1 for high speed helical
For high speed spur.
2. Bevel 1:1 to 3:1
3. Worm 10:1 to 100:1 provided $\angle 100$ KW

SPUR GEAR

(i) Circular pitch (p) = $\frac{\pi d}{T}$

(ii) Diametral pitch (P) = $\frac{T}{d}$

(iii) $pP = \pi$

(iv) Module (m) = $\frac{d}{T} = \frac{1}{P}$ or $d = mT$

(v) Speed ratio (G) = $\frac{\omega_p}{\omega_g} = \frac{T_g}{T_p}$

(vi) centre-to-centre distance = $\frac{1}{2}(d_g + d_p)$
 $= \frac{1}{2}m(T_g + T_p)$

(vii) Addendum (h_a) = 1m

(h_f) = 1.25 m

Clearance (C) = 0.25 m

(viii) $P_t = \frac{2T}{d}$

$$P_r = P_t \tan \alpha$$

$$P_N = \frac{P_t}{\cos \alpha}$$

(ix) Minimum number of teeth to avoid interference

$$T_{\min} = \frac{2A_w}{\sin^2 \phi} \quad \left\| \text{For } 20^\circ \text{ full depth } T = 18 \text{ to } 20\right.$$

(x)

$$T_{\min, \text{pinion}} = \frac{2 \times A_w}{G \left[\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1 \right]}$$

$$G = \text{Gear ratio} = \frac{Z_g}{Z_p} = \frac{\omega_p}{\omega_g}$$

$$A_p = \text{fraction of addendum to module} = \frac{h_a}{m} \text{ for pinion}$$

$$A_w = \text{fraction of addendum to module} = \frac{h_f}{m} \text{ for gear (generally 1)}$$

(xi) Face width $8m < b < 12m$; usually $b = 10m$

(xii) Beam strength $\sigma_b = mb\sigma_b Y \rightarrow \text{Lewis equation}$

$$\text{Where } \sigma_b = \frac{\sigma_{\text{ult}}}{3}$$

Lewis form factor, $Y = \left(0.154 - \frac{0.912}{z} \right)$ for 20° full depth gear.

(xiii) Wear strength (σ_w) = $bQdpK$

$$\text{Where } Q = \frac{2T_g}{T_g + T_p} \quad \text{for external gear}$$

$$= \frac{2T_g}{T_g - T_p} \quad \text{for internal gear}$$

$$\begin{aligned} \text{load-stress factor (k)} &= \frac{\sigma_c^2 \sin \phi \cos \phi}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \\ &= 0.16 \left(\frac{\text{BHN}}{100} \right)^2 \end{aligned}$$

(xiv)

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t \quad \text{where } C_v = \frac{3}{3 + V}, \quad \text{for ordinary cut gear } v < 10 \text{ m/s}$$

$$= \frac{6}{6 + v}, \quad \text{for hobbed generated } v > 20 \text{ m/s}$$

$$= \frac{5.6}{5.6 + \sqrt{v}}, \quad \text{for precision gear } v > 20 \text{ m/s}$$

(xv)**Spott's equation,** $(P_{\text{eff}}) = C_s P_t + P_d$

$$\text{where } P_d = \frac{e n_p T_p b r_1 r_2}{2530 \sqrt{r_1^2 + r_2^2}} \quad \text{for steel pinion and steel gear}$$

$$e = 16.00 + 1.25\phi \quad \text{for grade 8}$$

$$\phi = m + 0.29\sqrt{d} \quad \text{for all.}$$

(b) Helical Gear

$$(1) p_n = p \cos \alpha$$

where p = transverse circular pitch.

p_n = normal circular pitch.

$$(2) P_n = \frac{P}{\cos \alpha}$$

where P_n = normal diametral pitch.

P = transverse diametral pitch.

α = helix angle.

$$(3) pP = \pi$$

$$(4) m_n = m \cos \alpha \left[P = \frac{1}{m} \right]$$

m = transverse module.

m_n = normal module.

$$(5) p_a = \frac{p}{\tan \alpha} = \frac{\pi m}{\tan \alpha};$$

p_a = axial pitch.

$$(6) \cos \alpha = \frac{\tan \phi_n}{\tan \phi}$$

ϕ_n = normal pressure angle (usually 20°).

ϕ = transverse pressure angle.

$$(7) d = \frac{TP}{\pi} = zm = \frac{Tm_n}{\cos \alpha};$$

d = pitch circle diameter.

$$(8) a = \frac{m_n}{2 \cos \alpha} \{T_1 + T_2\} \rightarrow \text{centre - to - centre distance.}$$

$$(9) T' = \frac{T}{\cos^3 \alpha}; d' = \frac{d}{\cos^2 \alpha}$$

(10) An imaginary spur gear is considered with a pitch circle diameter of d' and module m_n is called 'formative' or 'virtual' spur gear.

(11) Helix angle α varies from **15 to 25°**.

(12) Preference value of normal modulus (m_n) = 1, 1.25, 1.5, 2, 3, 4, 5, 6, 8

(13) Addendum (h_a) = m_n ; dedendum (h_f) = $1.25 m_n$, clearance = $0.25 m_n$

(14)

$$P_t = \frac{2M_t}{d} = P \cos \phi_n \cos \alpha; M_t = \frac{60 \times 10^6 \times (KW)}{2\pi N} \text{ N - mm}$$

$$P_r = P_t \left(\frac{\tan \phi_n}{\cos \alpha} \right) = P \sin \alpha \phi_n$$

$$P_a = P_t \tan \alpha = P \cos \phi_n \sin \alpha$$

(15) Beam strength $S_b = m_n b \sigma_b Y'$

(16) Wear strength $S_w = \frac{b Q d_p K}{\cos^2 \alpha}$

(17)

$$P_{\text{eff}} = \frac{C_s P_t}{C_v} ; C_v = \frac{5.6}{5.6 + \sqrt{v}} ; P_d = \frac{e n_p T_p b r_1 r_2}{2530 \sqrt{r_1^2 + r_2^2}}$$

$$P_{\text{eff}} = C_s P_t + P_d \cos \alpha_n \cos \psi$$

(c) Worm Gear

(i) Specified and designated by $T_1 / T_2 / q / m$

$$\text{Where: } q \text{ is diametric quotient} = \frac{d_1}{m}$$

(ii) The threads of the worm have an involute helicoids profile.

(iii) Axial pitch (p_x) = distance between two consecutive teeth-measured along the axis of the worm.

(iv) The lead (l) = when the worm is rotated one revolution, a distance that a point on the helical profile will move.

$$(v) \quad l = p_x \times T_1; \quad d_z = m T_2$$

(vi) Axial pitch of the worm = circular pitch of the worm wheel

$$P_x = \frac{\pi d_2}{T_2} = \pi m \quad [\text{ICS - 04}]$$

$$l = P_x T_1 = \pi m T_1$$

$$(vii) \text{ Lead angle } (\delta) = \tan^{-1} \left(\frac{T_1}{q} \right) = \tan^{-1} \left(\frac{1}{\pi d_1} \right)$$

$$(viii) \text{ centre-to-centre distance } (a) = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}m(T_1 + T_2)$$

(ix) Preferred values of q : 8, 10, 12.5, 16, 20, 25

(x) Number of starts T_1 usually taken 1, 2, or 4

(xi)

addendum (h_{a_1}) = m	$h_{a_2} = m(2 \cos \delta - 1)$ $h_{f_2} = m(1 + 0.2 \cos \delta)$ $c = 0.2 m \cos \delta$
dedendum (h_{f_1}) = $(2.2 \cos \delta - 1)m$	
clearance (c) = $0.2m \cos \delta$	

(xii) $F = 2m \sqrt{(q+1)}$ effective face width of the root of the worm wheel.

(xiii)

$$\delta = \sin^{-1} \left(\frac{F}{d_{a_1} + 2C} \right); l_r = (d_{a_1} + 2C) \sin^{-1} \left(\frac{F}{d_{a_1} + 2C} \right) = \text{length of the root of the worm wheel teeth}$$

$$(xiv) (P_1)_t = \frac{2m_t}{d_1}; (P_1)_a = (P_1)_t \frac{\cos\alpha\cos\delta - \mu\sin\delta}{\cos\alpha\sin\delta + \mu\cos\delta}; (P_1)_r = (P_1)_t \frac{\sin\alpha}{\cos\alpha\sin\delta + \mu\cos\delta}$$

$$(xv) \text{ Efficiency } (\eta) = \frac{\text{Power output}}{\text{Power input}} = \frac{\cos\alpha - \mu\tan\delta}{\cos\alpha + \mu\tan\delta}$$

$$(xvi) \text{ Rubbing velocity } (V_s) = \frac{\pi d_1 n_1}{60000 \cos\delta} \text{ m / s (remaining 4 cheak)}$$

$$(xvii) \text{ Thermal consideration } H_g = 1000(1 - \eta) \times (\text{KW})$$

$$H_d = K(t - t_0)A$$

$$\text{KW} = \frac{K(t - t_0)A}{1000(1 - \eta)}$$

Previous 20-Years GATE Questions

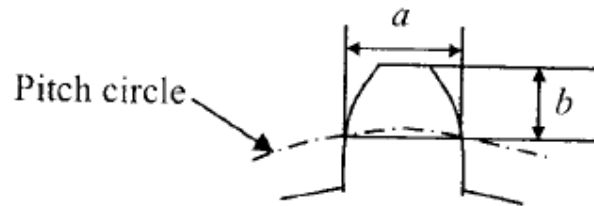
Spur gear

GATE-1. Match the type of gears with their most appropriate description. [GATE-2008]

Type of gear	Description
P Helical	1. Axes non parallel and intersecting
Q Spiral	2. Axes parallel and teeth are inclined to the axis
R Hypoid	3. Axes parallel and teeth are parallel to the axis
S Rack and pinion	4. Axes are perpendicular and intersecting, and teeth are inclined to the axis
	5. Axes are perpendicular and used for large speed reduction
	6. Axes parallel and one of the gears has infinite radius
(a) P-2, Q- 4, R- 1, S- 6	(c) P-2, Q- 6, R- 4, S- 2
(b) P-1, Q- 4, R- 5, S- 6	(d) P-6, Q- 3, R- 1, S- 5

GATE-1Ans. (a)

GATE-2. One tooth of a gear having 4 module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions 'a' and 'b', respectively, are closest to [GATE-2008]



- | | |
|---------------------|---------------------|
| (a) 6.08 mm, 4 mm | (b) 6.48 mm, 4.2 mm |
| (c) 6.28 mm, 4.3 mm | (d) 6.28 mm, 4.1 |

GATE-2Ans. (a)

Classification of Gears

GATE-3. Match the following [GATE-2004]

Type of gears	Arrangement of shafts
P. Bevel gears	1. Non-parallel off-set shafts
Q. Worm gears	2. Non-parallel intersecting shafts
R. Herringbone gears	3. Non-parallel non-intersecting shafts
S. Hypoid gears	4. Parallel shafts
(a) P-4 Q-2 R-1 S-3	(b) P-2 Q-3 R-4 S-1

(c) P-3 Q-2 R-1 S-4

(d) P-1 Q-3 R-4 S-2

GATE-3Ans. (b)

Pitch point

GATE-4. In spur gears, the circle on which the involute is generated is called the

(a) Pitch circle

(b) clearance circle

[GATE-1996]

(c) Base circle

(d) addendum circle

GATE-4Ans. (a)

Minimum Number of Teeth

GATE-5. The minimum number of teeth on the pinion to operate without interference in standard full height involute teeth gear mechanism with 20° pressure angle is [GATE-2002]

(a) 14

(b) 12

(c) 18

(d) 32

GATE-5Ans. (c)

Interference

GATE-6 Tooth interference in an external involute spur gear pair can be reduced by [GATE-2010]

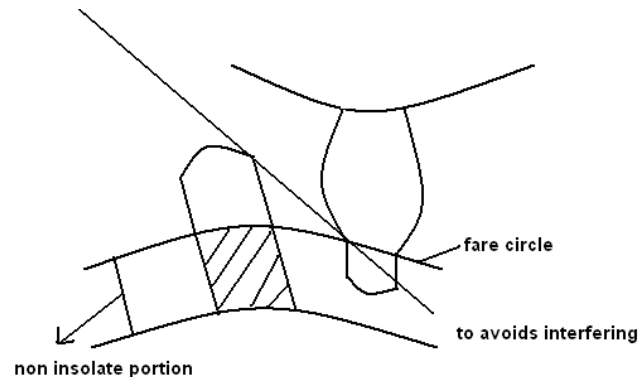
(a) Decreasing center distance between gear pair

(b) Decreasing module

(c) Decreasing pressure angle

(d) Increasing number of gear teeth

GATE-6Ans. (d)



There are several ways to avoid interfering:

- i. Increase number of gear teeth
- ii. Modified involutes
- iii. Modified addendum
- iv. Increased centre distance

GATE-7. Interference in a pair of gears is avoided, if the addendum circles of both the gears intersect common tangent to the base circles within the points of tangency. [GATE-1995]

(a) True

(b) False

(c) Insufficient data

(d) None of the above

GATE-7Ans. (a)

GATE-8. Twenty degree full depth involute profiled 19-tooth pinion and 37-tooth gear are in mesh. If the module is 5 mm, the centre distance between the gear pair will be [GATE-2006]

- (a) 140 mm (b) 150 mm
(c) 280 mm (d) 300 mm

GATE-8Ans. (a)

$$\text{Centre distance} = \frac{D_1 + D_2}{2} = \frac{mT_1 + mT_2}{2} = \frac{5(19 + 37)}{2} = 140\text{mm}$$

Beam Strength of Gear Tooth

GATE-9. A spur gear has a module of 3 mm, number of teeth 16, a face width of 36 mm and a pressure angle of 20°. It is transmitting a power of 3 kW at 20 rev/s. Taking a velocity factor of 1.5, and a form factor of 0.3, the stress in the gear tooth is about [GATE-2008]

- (a) 32 MPa (b) 46 MPa
(c) 58 MPa (d) 70 MPa

GATE-9Ans. (c)

Statement for Linked Answer GATE-10 and GATE-11:

A 20° full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. Its face width is 25 mm.

GATE-10. The tangential force transmitted (in N) is [GATE -2009]
(a) 3552 (b) 2611 (c) 1776 (d) 1305

GATE-10Ans. (a)

GATE-11. Given that the tooth geometry factor is 0.32 and the combined effect of dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is [GATE -2009]

- (a) 242.0 (b) 166.5 (c) 121.0 (d) 74.0

GATE-11Ans. (b)

Simple Gear train

Note: - Common Data for GATE-12 & GATE-13.

A gear set has a pinion with 20 teeth and a gear with 40 teeth. The pinion runs at 0 rev/s and transmits a power of 20 kW. The teeth are on the 20° full –depth system and have module of 5 mm. The length of the line of action is 19 mm.

GATE-12. The center distance for the above gear set in mm is [GATE-2007]
(a) 140 (b) 150 (c) 160 (d) 170.

GATE-12Ans. (b)

GATE-13 The contact ratio of the contacting tooth [GATE-2007]
(a) 1.21 (b) 1.25 (c) 1.29 (d) 1.33

GATE-13Ans. (c)

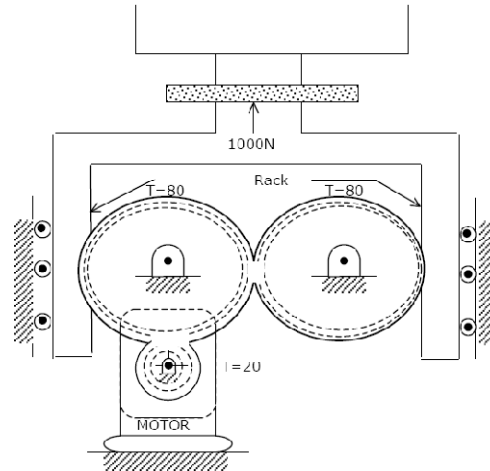
GATE-14. The resultant force on the contacting gear tooth in N is: [GATE-2007]
(a) 77.23 (b) 212.20 (c) 225.80 (d) 289.43

GATE-14Ans. (c)

Compound gear train

Data for GATE-15 & GATE-16 are given below. Solve the problems and choose correct answers.

A compacting machine shown in the figure below is used to create a desired thrust force by using a rack and pinion arrangement. The input gear is mounted on the motor shaft. The gears have involute teeth of 2 mm module.

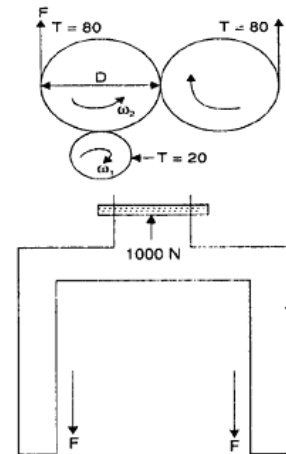


GATE-15. If the drive efficiency is 80%, then torque required on the input shaft to create 1000 N output thrust is [GATE-2004]

- (a) 20 Nm (b) 25 Nm (c) 32 Nm (d) 50 Nm

GATE-15Ans. (b)

Given : Module $m = 2$, $\frac{D}{T} = 2$
 $\therefore D = 80 \times 2 = 160 \text{ mm}$
 $2F = 1000$, or $F = 500 \text{ N}$
 Let T_1 be the torque applied by motor.
 T_2 be the torque applied by gear.
 \therefore Power transmission = 80%
 Now, $T_1 \omega_1 = \frac{2T_2 \times \omega_1}{0.8}$
 or $T_1 = \frac{2 \times F \times (D/2)}{0.8} \times \frac{\omega_1}{\omega_2}$
 $= 2 \times 500 \times \frac{0.16}{2} \times \frac{1}{0.8} \times \frac{1}{4}$
 $= 25 \text{ N-m.}$



GATE-16. If the pressure angle of the rack is 20° , then force acting along the line of action between the rack and the gear teeth is [GATE-2004]

- (a) 250 N (b) 342 N (c) 532 N (d) 600 N

GATE-16Ans. (c)

$$P \cos \phi = F$$

\therefore Force acting along the line of action,

$$\begin{aligned}
 P &= \frac{F}{\cos \phi} \\
 &= \frac{500}{\cos 20^\circ} \\
 &= 532\text{N}
 \end{aligned}$$

Reverted gear train

Data for GATE-17 & GATE-18 are given below. Solve the problems and choose correct answers.

The overall gear ratio in a 2 stage speed reduction gear box (with all spur gears) is 12. The input and output shafts of the gear box are collinear. The countershaft which is parallel to the input and output shafts has a gear (Z_2 teeth) and pinion ($Z_3 = 15$ teeth) to mesh with pinion ($Z_1 = 16$ teeth) on the input shaft and gear (Z_4 teeth) on the output shaft respectively. It was decided to use a gear ratio of 4 with 3 module in the first stage and 4 module in the second stage.

GATE-17. Z_2 and Z_4 are

[GATE-2003]

- (a) 64 and 45 (b) 45 and 64 (c) 48 and 60 (d) 60 and 48

GATE-17Ans. (a)

$$\text{Given, } \frac{N_1}{N_2} = 12, \frac{N_1}{N_2} = 4 = \frac{D_2}{D_1}$$

$$m_1 = 3, m_2 = 4$$

$$\text{Now, } \frac{D_1}{Z_1} = \frac{D_2}{Z_2}$$

$$\Rightarrow \frac{Z_1}{Z_2} = \frac{D_1}{D_2} = \frac{N_2}{N_1} = \frac{1}{4}$$

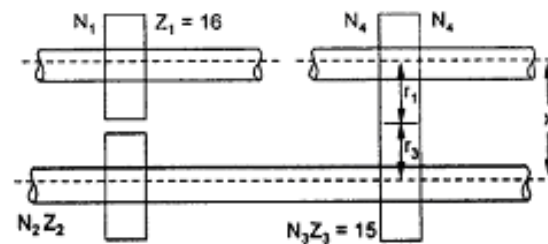
$$\Rightarrow Z_2 = Z_1 \times 4 = 64$$

$$\Rightarrow 12 = \frac{D_4}{D_3}$$

$$\Rightarrow \frac{D_4}{D_3} = 3$$

$$\text{Also, } \frac{Z_3}{Z_4} = \frac{D_3}{D_4}$$

$$\begin{aligned}
 \Rightarrow Z_4 &= Z_3 \frac{D_4}{D_3} = Z_3 \times 3 = 15 \times 3 \\
 &= 45
 \end{aligned}$$



GATE-18. The centre distance in the second stage is

[GATE-2003]

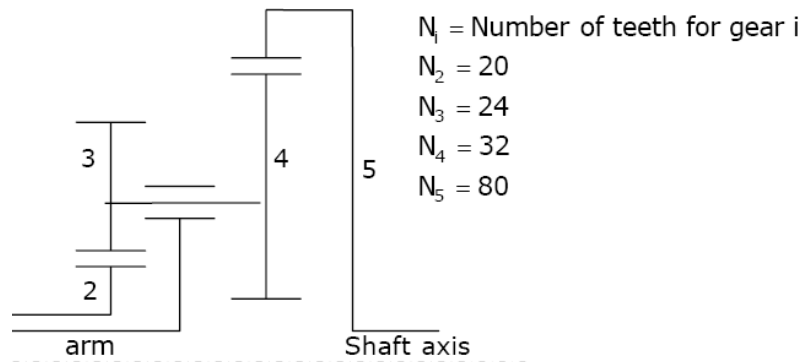
- (a) 90 mm (b) 120 mm (c) 160 mm (d) 240mm

GATE-18Ans. (b)

$$\begin{aligned} \text{Now,} \quad x &= r_4 + r_3 = \frac{D_4 + D_3}{2} \\ \text{But} \quad \frac{D_4}{Z_4} &= \frac{D_3}{Z_3} = 4 \\ \Rightarrow \quad D_4 &= 180, D_3 = 60 \\ \therefore \quad x &= \frac{180 + 60}{2} = 120 \text{ mm} \end{aligned}$$

Epicyclic gear train

GATE-19. For the epicyclic gear arrangement shown in the figure, $\omega_2 = 100$ rad/s clockwise (CW) and $\omega_{arm} = 80$ rad/s counter clockwise (CCW). The angular velocity ω_5 , (in rad/s) is [GATE-2010]



(a) 0

(b) 70 CW

(c) 140 CCW

(d) 140 CW

GATE-19Ans. (c)

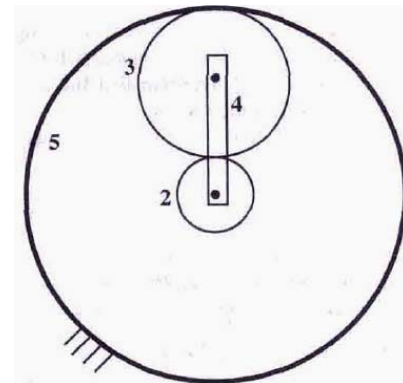
	Arm	2	3	4	5
1.	0	$+x$	$\frac{-N_2}{N_3}x$	$\frac{-N_2}{N_3}x$	$\frac{-N_4}{N_5} \times \frac{N_2}{N_3}x$
2.	y	y	y	y	y
	y	$x + y$	$y - \frac{N_2}{N_3}x$		$y - \frac{N_4}{N_5} \times \frac{N_2}{N_3}x$

$$x + y = 100 \text{ (cw)}$$

$$y = -80 \text{ (ccw)}$$

$$\text{Speed of Gear } (W_5) = -80 - \frac{32}{80} \times \frac{20}{24} \times 180 = -140 = 140 \text{ (ccw)}$$

GATE-20. An epicyclic gear train is shown schematically in the adjacent figure. The sun gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm (ccw = counter-clockwise and cw = clockwise).



[GATE -2009]

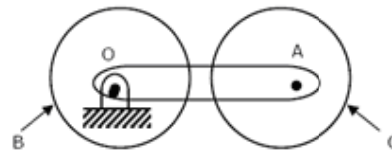
The arm 4 attached to the output shaft will rotate at

- (a) 10 rpm ccw
- (b) 10 rpm cw
- (c) 12 rpm cw
- (d) 12 rpm ccw

GATE-20Ans. (a)

GATE-21 The arm OA of an epicyclic gear train shown in figure revolves counter clockwise about O with an angular velocity of 4 rad/s. Both gears are of same size. Tire angular velocity of gear C, if the sun gear B is fixed, is [GATE-1995]

- (a) 4 rad / sec
- (b) 8 rad / sec
- (c) 10 rad / sec
- (d) 12 rad / sec



GATE-21Ans. (b)

Explanation

	Arm A	B	C
Fix arm A			
Give one rotation to B	0	1	-1
Multiply by x	0	+x	-x
Add y	y	X+ y	y-x

B is fixed, therefore

$$x + y = 0$$

$$y = \text{rad/sec (ccw)}$$

\Rightarrow

$$x = -4 \text{ rad/sec (cw)}$$

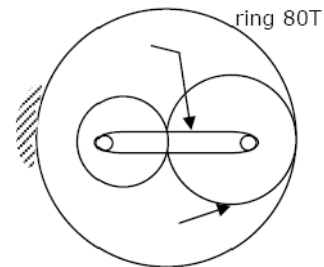
Angular velocity of gear

$$C = y - x = 4 - (-4) = 8 \text{ rad/s}$$

GATE-22. The sun gear in the figure is driven clockwise at 100 rpm. The ring gear is held stationary.

For the number of teeth shown on the gears, the arm rotates at

- (a) 0 rpm (b) 20 rpm
(c) 33.33 rpm (d) 66.67 rpm



[GATE-2001]

GATE-22Ans. (b)

Arm	Sun	Planet	Ring
+1	+1	+1	+1
0	$\frac{80}{30} \times \frac{30}{20}$	$-\frac{80}{30}$	-1
1	5	$-\frac{5}{3}$	0

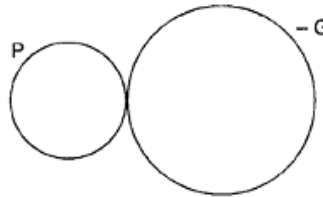
For 5 Revolutions Of Sun, Arm rotates by 1

\therefore for 100 revolutions of Sun, Arm rotates by $\frac{100}{5} = 20$

GATE-23. Two mating spur gears have 40 and 120 teeth respectively. The pinion rotates at 1200 rpm and transmits a torque of 20 Nm. The torque transmitted by the gear is [GATE-2004]

- (a) 6.6 Nm (b) 20 Nm (c) 40 Nm (d) 60 Nm

GATE-23Ans. (d)



We know $\frac{N_P}{N_G} = \frac{T_G}{T_P}$

where, N_P = speed of pinion, N_G = speed of gear wheel

T_G = number of teeth of gear,

T_P = number of teeth of pinion

$\therefore \frac{1200}{N_G} = \frac{120}{40}$

or $N_G = 400 \text{ r.p.m.}$

Since power transmitted by both gear will be equal

i.e. $T_P \omega_P = T_G \omega_G$

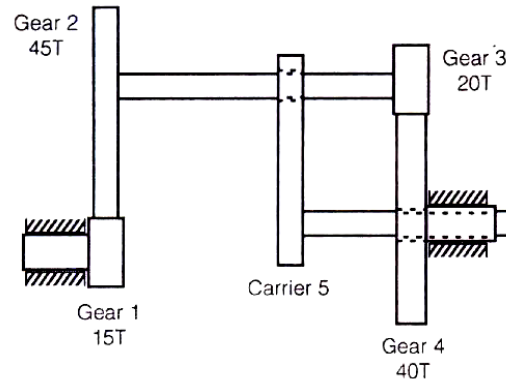
where, T_P = torque transmitted by pinion, T_G = torque transmitted by gear wheel

$\therefore \frac{20 \times 2\pi \times 1200}{60} = \frac{T_G \times 2\pi \times 400}{60}$

\therefore torque transmitted by gear, $T_G = 60 \text{ N.m.}$

Common Data for GATE-24, GATE-25:

A planetary gear train has four gears and one carrier. Angular velocities of the gears are ω_1 , ω_2 , ω_3 , and ω_4 respectively. The carrier rotates with angular velocity ω_5 ,



GATE-24. What is the relation between the angular velocities of Gear 1 and Gear 4?
[GATE-2006]

GATE-24Ans. (a)

$$\frac{\omega_1 - \omega_5}{\omega_2 - \omega_5} = 3 \quad (\text{with respect to arm 5 or carrier 5})$$

$$\frac{\omega_3 - \omega_5}{\omega_4 - \omega_5} = 2 \quad (\text{with respect to carrier 5})$$

As, $\omega_3 = \omega_2$

$$\therefore \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$$

GATE-25. For ($\omega_1 = 60$ rpm clockwise (cw) when looked from the left, what is the angular velocity of the carrier and its direction so that Gear 4 rotates in counter clockwise (ccw) direction at twice the angular velocity of Gear 1 when looked from the left?
[GATE-2006]

- (a) 130 rpm, cw (b) 223 rpm, ccw
(c) 256 rpm, cw (d) 156 rpm, ccw

GATE-25Ans. (d)

$\omega_1 = 60$ rpm (Clockwise)

$\omega_4 = 120$ rpm (Counter clock wise)

$$\frac{60 - \omega_5}{-120 - \omega_5} = 6$$

$\therefore \omega_5 = -156$ i.e. counter clockwise

Worm Gears

GATE-26. Large speed reductions (greater than 20) in one stage of a gear train are possible through
[GATE-2002]

- (a) Spur gearing (b) Worm gearing (c) Bevel gearing (d) Helical gearing

GATE-26Ans. (b)

GATE-27. A 1.5 kW motor is running at 1440 rev/min. It is to be connected to a stirrer running at 36 rev /min. The gearing arrangement suitable for this application is
[GATE-2000]

- | | |
|-----------------------|------------------|
| (a) Differential gear | (b) helical gear |
| (c) Spur gear | (d) worm gear |

GATE-27Ans. (d) speed reduction = $1440/36 = 40$

GATE-28. To make a worm drive reversible, it is necessary to increase [GATE-1997]

- | | |
|----------------------|--------------------------|
| (a) centre distance | (b) worm diameter factor |
| (c) Number of starts | (d) reduction ratio |

GATE-28Ans. (c)

Previous 20-Years IES Questions

Spur gear

IES-1. The velocity ratio between pinion and gear in a gear drive is 2.3, the module of teeth is 2.0 mm and sum of number of teeth on pinion and gear is 99. What is the centre distance between pinion and the gear? [IES 2007]

- | | | | |
|-------------|-----------|--------------|------------|
| (a) 49.5 mm | (b) 99 mm | (c) 148.5 mm | (d) 198 mm |
|-------------|-----------|--------------|------------|

IES-1. Ans. (b) Centre distance = $\frac{D_1 + D_2}{2} = \frac{mT_1 + mT_2}{2} = \frac{m}{2} (T_1 + T_2) = \frac{2}{2} \times 99 = 99\text{mm}$

IES-2. Consider the following statements: [IES-2001]

When two gears are meshing, the clearance is given by the

1. Difference between dedendum of one gear and addendum of the mating gear.
2. Difference between total and the working depth of a gear tooth.
3. Distance between the bottom land of one gear and the top land of the mating gear.
4. Difference between the radii of the base circle and the dedendum circle.

Which of these statements are correct?

- | | | | |
|----------------|----------------|----------------|----------------|
| (a) 1, 2 and 3 | (b) 2, 3 and 4 | (c) 1, 3 and 4 | (d) 1, 2 and 4 |
|----------------|----------------|----------------|----------------|

IES-2. Ans. (a)

IES-3. The working surface above the pitch surface of the gear tooth is termed as [IES-1998]

- | | | | |
|--------------|--------------|-----------|----------|
| (a) Addendum | (b) dedendum | (c) flank | (d) face |
|--------------|--------------|-----------|----------|

IES-3. Ans. (d)

IES-4. Match the following $14\frac{1}{2}^\circ$ composite system gears [IES-1992]

List I

A. Dedendum

B. Clearance

C. Working depth

List II

1. $\frac{2}{pd}$

2. $\frac{0.157}{pd}$

3. $\frac{1.157}{pd}$

D. Addendum				4. $\frac{1}{pd}$				
Code:	A	B	C	D	A	B	C	D
(a)	1	2	3	4	(b)	4	3	2
(c)	3	2	1	4	(d)	3	1	2

IES-4. Ans. (c)

IES-5. Match List I with List II and select the correct answer using the codes given below the lists: [IES-1993]

List I (Standard tooth/arms)					List II (Advantages or disadvantages)				
A. 20° and 25° systems					1. Results in lower loads on bearing				
B. 14.5° stub-tooth system					2. Broadest at the base and strongest in bending				
C. 25° Full depth system					3. Obsolete				
D. 20° Full depth system					4. Standards for new applications				
Code:	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	1	2	4
(c)	3	2	1	4	(d)	4	2	3	1

IES-5. Ans. (a)

IES-6. **Assertion (A):** When one body drives another by direct contact, their contact points must have equal components of velocity normal to the surfaces at the point of contact.

Reason (R): Two points in the same body must have the same component of velocity relative to the third body, in the direction of the line joining the two points. [IES-1993]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-6Ans. (a)

Classification of Gears

IES-7. Match List I with List II and select the correct answer [IES-1996]

List I				List II				
A. Helical gears				1. Non-interchangeable				
B. Herring bone gears				2. Zero axial thrust				
C. Worm gears				3. Quiet motion				
D. Hypoid Gears				4. Extreme speed reduction				
Codes:	A	B	C	D	A	B	C	D
(a)	1	2	3	4	(b)	3	2	1
(c)	3	1	4	2	(d)	3	2	4

IES-7Ans. (d)

IES-8. Match List-I (Type of Gears) with List-II (Characteristics) and select the correct answer using the code given below the Lists: [IES-2006]

List-I		List-II	
A. Helical gearing		1. Zero axial thrust	
B. Herringbone gearing		2. Non-inter-changeable	
C. Worm gearing		3. Skew shafts	
D. Hypoid gearing		4. Parallel shafts	

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	3	2	4	1
(c)	4	2	3	1	(d)	3	1	4	2

IES-8Ans. (a)

IES-9. Match List I with List II and select the correct answer using the code given below the Lists: [IES 2007]

List I				List II			
A. Worm gear				1. Imposes no thrust load on the shaft			
B. Spur gear				2. To transmit power between two non-intersecting shafts which are perpendicular to each other			
C. Herringbone gear				3. To transmit power when the shafts are parallel			
D. Spring level gear				4. To transmit power when the shafts are at right angles to one another			
Code: A	B	C	D	A	B	C	D
(a)	1	2	3	(b)	2	3	1
(c)	1	2	4	(d)	2	3	4

IES-9Ans. (b)

IES-10. Match List I (Type of Gear/Gear Train) with List II (Different Usage and Drive) and select the correct answer using the code given below the Lists: [IES-2005]

List I				List II			
A. Epicyclic gear train				1. Reduces end thrust			
B. Bevel Gear				2. Low gear ratio			
C. Worm-worm Gear				3. Drives non-parallel nonintersecting shafts			
D. Herringbone Gear				4. Drives non-parallel intersecting shafts			
				5. High gear ratio			
A	B	C	D	A	B	C	D
(a)	5	4	3	(b)	2	3	4
(c)	5	3	4	(d)	2	4	3

IES-10Ans. (a)

IES-11. Which type of gear is used for shaft axes having an offset? [IES-2004]

- | | |
|------------------|------------------------|
| (a) Mitre gears | (b) Spiral bevel gears |
| (c) Hypoid gears | (d) Zerol gears |

IES-11Ans. (c)

IES-12. The gears employed for connecting two non-intersecting and non-parallel, i.e., non-coplanar shafts are [IES-2003; 2005]

- | | | | |
|-----------------|------------------|-------------------|-----------------|
| (a) Bevel gears | (b) Spiral gears | (c) Helical gears | (d) Mitre gears |
|-----------------|------------------|-------------------|-----------------|

IES-12Ans. (b)

IES-13. When two shafts are neither parallel nor intersecting, power can be transmitted by using [IES-1998]

- | | |
|--------------------------|-----------------------------|
| (a) A pair of spur gears | (b) a pair of helical gears |
| (c) An Oldham's coupling | (d) a pair of spiral gears |

IES-13Ans. (d)

IES-14. In a single reduction, a large velocity ratio is required. The best transmission is [IES-1999]

- (a) Spur gear drive (b) helical gear drive
(c) Bevel gear drive (d) worm gear drive

IES-14Ans. (a)

IES-15. Which one of the following pairs is not correctly matched? [IES-1995]

- (a) Positive drive Belt drive
(b) High velocity ratio Worm gearing
(c) To connect non-parallel and non-intersecting shafts Spiral gearing.
(d) Diminished noise and smooth operation Helical gears.

IES-15Ans. (a)

Mitres gear

IES-16. Mitre gears [IES-1992]

- (a) spur-gears with gear ratio 1: 1
(b) Skew gears connecting non-parallel and nonintersecting shafts
(c) Bevel gears transmitting power at more than or less than 90°
(d) Bevel gears in which the angle between the axes is 90° and the speed ratio of the gears is 1: 1

IES-16Ans. (d)

IES-17. Match List-I (Gears) with List-II (Configurations) and select the correct answer using the codes given below the Lists: [IES-2003]

List-I (Gears)	List-II (Configurations)
A Spur	1. Connecting two non-parallel or intersecting but coplanar shafts
B. Bevel	2. Connecting two parallel and coplanar shafts with teeth parallel to the axis of the gear wheel
C. Helical	3. Connecting two parallel and coplanar shafts with teeth inclined to the axis of the gear wheel
D. Mitre	4. Connecting two shafts whose axes are mutually perpendicular to each other

Codes:	A	B	C	D	A	B	C	D	
(a)	2	4	3	1	(b)	3	1	2	4
(c)	2	1	3	4	(d)	3	4	2	1

IES-17Ans. (c)

Pitch point

IES-18. Gearing contact is which one of the following? [IES 2007]

- (a) Sliding contact (b) Sliding contact, only rolling at pitch point
(c) Rolling contact (d) Rolling and sliding at each point of contact

IES-18Ans. (b)

When pair of teeth touch at the pitch point, they have for the instant pure rolling action. At any other position they have the sliding action.

IES-19. When two spur gears having involute profiles on, their teeth engage, the line of action is tangential to the [IES-2003]

(a) Pitch circles

(b) Dedendum circles

(c) Addendum circles

(d) Base circles

IES-19Ans. (d)

Pressure angle

IES-20. What is the value of pressure angle generally used for involute gears?

[IES-2006]

(a) 35° (b) 30° (c) 25° (d) 20°

IES-20Ans. (d)

IES-21. Consider the following, modifications regarding avoiding the interference between gears: [IES-2003]

1. The centre distance between meshing gears be increased

2. Addendum of the gear be modified

3. Teeth should be undercut slightly at the root

4. Pressure angle should be increased

5. Circular pitch be increased

Which of these are effective in avoiding interference?

(a) 1, 2 and 3

(b) 2, 3, 4 and 5

(c) 1, 4 and 5

(d) 3, 4 and 5

IES-21Ans. (b)

IES-22. An external gear with 60 teeth meshes with a pinion of 20 teeth, module being 6 mm. What is the centre distance in mm? [IES-2009]

(a) 120

(b) 180

(c) 240

(d) 300

IES-22Ans. (c)

$$\begin{aligned}\text{Centre distance in mm} &= \frac{m}{2}(T_1 + T_2) \\ &= \frac{6}{2}(60 + 20) \\ &= 240 \text{ mm}\end{aligned}$$

IES-23. Assertion (A): An involute rack with 20° pressure angle meshes with a pinion of 14.5° pressure angle. [IES-2002]

Reason (R): Such a matching is impossible.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IES-23Ans. (d)

IES-24. Compared to gears with 20° pressure angle involute full depth teeth, those with 20° pressure angle and stub teeth have [IES 2007]

1. Smaller addendum.

2. Smaller dedendum.

3. Smaller tooth thickness.

4. Greater bending strength.

Which of the statements given above are correct?

(a) 1, 2 and 3

(b) 1, 2 and 4

(c) 1, 3 and 4

(d) 2, 3 and 4

IES-24Ans. (b)

IES-25. Consider the following statements:

[IES-1999]

A pinion of $14\frac{1}{2}^\circ$ pressure angle and 48 involute teeth has a pitch circle diameter of 28.8 cm. It has

- | | |
|---------------------|----------------------------------------|
| 1. Module of 6 mm | 2. Circular pitch of 18 mm |
| 3. Addendum of 6 mm | 4. Diametral pitch of $\frac{11}{113}$ |

Which of these statements are correct?

- (a) 2 and 3 (b) 1 and 3 (c) 1 and 4 (d) 2 and 4

IES-25Ans. (b) Module = $\frac{d}{T} = \frac{288}{48} = 6\text{mm}$

Circular pitch = $\frac{\pi d}{T} = \pi \times 6 = 18.84 \text{ mm}$; addendum = 1 module = 6 mm

diametral pitch = $\frac{T}{d} = \frac{1}{6}$

Circular pitch = $\pi \times 6 = 18.84 \text{ mm}$

IES-26. Which of the following statements are correct? [IES-1996]

1. For constant velocity ratio transmission between two gears, the common normal at the point of contact must always pass through a fixed point on the line joining the centres of rotation of the gears.
2. For involute gears the pressure angle changes with change in centre distance between gears.
3. The velocity ratio of compound gear train depends upon the number of teeth of the input and output gears only.
4. Epicyclic gear trains involve rotation of at least one gear axis about some other gear axis.

- (a) 1, 2 and 3 (b) 1, 3 and 4 (c) 1, 2 and 4 (d) 2, 3 and 4

IES-26Ans. (c)

IES-27. Which one of the following is true for involute gears? [IES-1995]

- (a) Interference is inherently absent
- (b) Variation in centre distance of shafts increases radial force
- (c) A convex flank is always in contact with concave flank
- (d) Pressure angle is constant throughout the teeth engagement.

IES-27Ans. (d) For involute gears, the pressure angle is constant throughout the teeth engagement.

IES-28. In involute gears the pressure angle is [IES-1993]

- | | |
|------------------------------------|------------------------------------|
| (a) Dependent on the size of teeth | (b) dependent on the size of gears |
| (c) Always constant | (d) always variable |

IES-28Ans. (c) The pressure angle is always constant in involute gears.

Minimum Number of Teeth

IES-29. Which one of the following statements is correct? [IES-2004]

Certain minimum number of teeth on the involute pinion is necessary in order to

- | | |
|----------------------------------|-------------------------------------------|
| (a) Provide an economical design | (b) avoid Interference |
| (c) Reduce noise in operation | (d) overcome fatigue failure of the teeth |

IES-29Ans. (b)

- IES-30.** A certain minimum number of teeth is to be kept for a gear wheel
 (a) So that the gear is of a good size [IES-1999]
 (b) For better durability
 (c) To avoid interference and undercutting
 (d) For better strength

IES-30Ans. (c)

- IES-31.** In full depth $14\frac{1}{2}^\circ$ degree involute system, the smallest number of teeth in a pinion which meshes with rack with out interference is [IES-1992]
 (a) 12 (b) 16 (c) 25 (d) 32

IES-32Ans. (d)

- IES-33.** Match List I with List II and select the correct answer using the codes given below the lists:

List I (Terminology)

List II (Relevant terms)

[IES-1995]

A. Interference

1. Arc of approach, arc of recess, circular pitch

B. Dynamic load on tooth

2. Lewis equation

C. Static load

3. Minimum number of teeth on pinion

D. Contract ratio

4. Inaccuracies in tooth profile

Codes: A B C D A B C D

(a) 3 4 1 2 (b) 1 2 3 4

(c) 4 3 2 1 (d) 3 4 2 1

IES-33Ans. (d)

- IES-34 Assertion (A):** When a pair of spur gears of the same material is in mesh, the design is based on pinion. [IES-2002; 1993]

Reason (R): For a pair of gears of the same material in mesh, the 'strength factor' of the pinion is less than that of the gear.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

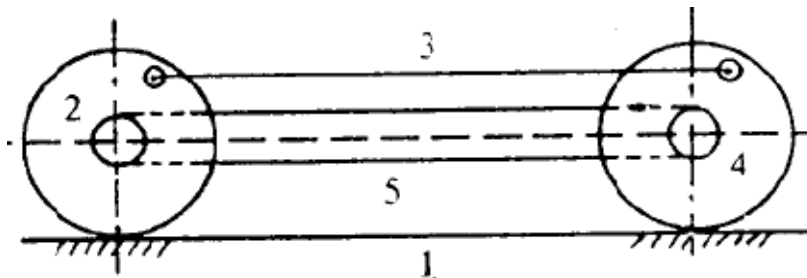
IES-34Ans. (a)

Cycloidal teeth

- IES-35.** The curve traced by a point on the circumference of a circle which rolls along the inside of affixed circle, is known as [IES-1992]
 (a) Epicycloid (b) hypocycloid
 (c) Cardioid (d) involute

IES-35Ans. (b)

IES-36



In the mechanism shown above, link 3 has [IES-2004]

- (a) Curvilinear translation and all points in it trace out identical cycloids
- (b) Curvilinear translation and all points in it trace out identical involutes
- (c) Linear translation & all points in it trace out identical helices
- (d) Linear translation & all points in it trace out identical ellipses

IES-36Ans. (a)

IES-37. A thin circular disc is rolling with a uniform linear speed, along a straight path on a plane surface. [IES-1994]

Consider the following statements in this regard:

- 1. All points on the disc have the same velocity.
- 2. The centre of the disc has zero acceleration.
- 3. The centre of the disc has centrifugal acceleration.
- 4. The point on the disc making contact with the plane surface has zero acceleration of these statements

- (a) 1 and 4 are correct
- (b) 3 and 4 are correct
- (c) 3 alone is correct
- (d) 2 alone is correct.

IES-37Ans. (d)

Involute teeth

IES-38. In the case of an involute toothed gear, involute starts from [IES-1997]

- (a) Addendum circle
- (b) dedendum circle
- (c) Pitch circle
- (d) base circle

IES-38Ans. (b)

IES-39. Consider the following statements: [IES-2006]

- 1. A stub tooth has a working depth larger than that of a full-depth tooth.
- 2. The path of contact for involute gears is an arc of a circle.

Which of the statements given above is/are correct?

- (a) Only 1
- (b) Only 2
- (c) Both 1 and 2
- (d) Neither 1 nor 2

IES-39Ans. (d) 1. A stub tooth has a working depth lower than that of a full-depth tooth.

- 2. The path of contact for involute gears is a line.

IES-40. Consider the following statements regarding the choice of conjugate teeth for the profile of mating gears: [IES-1999]

- 1. They will transmit the desired motion
- 2. They are difficult to manufacture.
- 3. Standardisation is not possible
- 4. The cost of production is low.

Which of these statements are correct?

- (a) 1, 2 and 3
- (b) 1, 2 and 4
- (c) 2, 3 and 4
- (d) 1, 3 and 4

IES-40Ans. (a) Cost of production of conjugate teeth, being difficult to manufacture is high.

IES-41. Which one of the following is correct? [IES-2008]

When two teeth profiles of gears are conjugate, the sliding velocity between them

- (a) Is always zero, all through the path of contact?
- (b) Is zero, at certain points along the path of contact?
- (c) Is never zero anywhere on the path of contact?
- (d) Can be made zero by proper selection of profiles

IES-41Ans. (a)

Contact ratio

IES-42. Which one of the following is the correct statement? [IES 2007]

In meshing gears with involute gears teeth, the contact begins at the intersection of the

- (a) Line of action and the addendum circle of the driven gear
- (b) Line of action and the pitch circle of the driven gear
- (c) Dedendum circle of the driver gear and the addendum circle of the driven gear
- (d) Addendum circle of the driver gear and the pitch circle of the driven gear

IES-42Ans. (a)

IES-43. Common contact ratio of a pair of spur pinion and gear is [IES-2008]

- (a) Less than 1.0
- (b) equal to 1
- (c) Between 2 and 3
- (d) greater than 3

IES-43Ans. (c) The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** i.e. number of pairs of teeth in contact. The contact ratio for gears is greater than one. Contact ratio should be at least 1.25. For maximum smoothness and quietness, the contact ratio should be between 1.50 and 2.00. High-speed applications should be designed with a face-contact ratio of 2.00 or higher for best results.

Interference

IES-44. Interference between an involute gear and a pinion can be reduced by which of the following? [IES-2008]

1. Increasing the pressure angle of the teeth in the pair, the number of teeth remaining the same.
2. Decreasing the addendum of the gear teeth and increasing the same for the pinion teeth by the corresponding amount.

Select the correct answer using the code given below:

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

IES-44Ans. (c)

IES-45. In gears, interference takes place when [IES-1993]

- (a) The tip of a tooth of a mating gear digs into the portion between base and root circles
- (b) Gears do not move smoothly in the absence of lubrication
- (c) Pitch of the gear is not same
- (d) gear teeth are undercut

IES-45Ans. (a) In gears, interference takes place when the tip of a tooth of a mating gear digs into the portion between base and root circle.

IES-46. An involute pinion and gear are in mesh. If both have the same size of addendum, then there will be an interference between the [IES-1996]

- (a) Tip of the gear tooth and flank of pinion.
- (b) Tip of the pinion and flank of gear.
- (c) Flanks of both gear and pinion.
- (d) Tips of both gear and pinion.

IES-46Ans. (a)

IES-47. Interference between the teeth of two meshing involute gears can be reduced or eliminated by [IES 2007]

1. Increasing the addendum of the gear teeth and correspondingly reducing the addendum of the pinion.
2. Reducing the pressure angle of the teeth of the meshing gears.
3. Increasing the centre distance

Which of the statements given above is/are correct?

- (a) 1 and 2 (b) 2 and 3
(c) 1 only (d) 3 only

IES-47Ans. (d)

IES-48. Consider the following statements: [IES-2002]

A 20° stub tooth system is generally preferred in spur gears as it results in

1. Stronger teeth
2. Lesser number of teeth on the pinion
3. Lesser changes of surface fatigue failure
4. Reduction of interference

Which of the above statements are correct?

- (a) 1, 2 and 4 (b) 3 and 4 (c) 1 and 3 (d) 1, 2, 3 and 4

IES-48Ans. (a)

IES-49. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-2001]

List-I

- A. Undercutting
B. Addendum
C. Lewis equation
D. Worm and wheel

List-II

1. Beam strength
2. Interference
3. Large speed reduction
4. Intersecting axes
5. Module

Codes:	A	B	C	D	A	B	C	D
(a)	2	5	1	3	(b)	1	5	4
(c)	1	3	4	5	(d)	2	3	1

IES-49Ans. (a)

IES-50. Which one of the following pairs is correctly matched? [IES-1999]

- (a) Governors ... Interference
(b) GearsHunting
(c) Klein's construction.... Acceleration of piston
(d) CamPinion

IES-50Ans. (c)

IES-51. Consider the following characteristics: [IES-1998]

1. Small interference
2. Strong tooth.
3. Low production cost
4. Gear with small number of teeth.

Those characteristics which are applicable to stub 20° involute system would include

- (a) 1 alone (b) 2, 3 and 4 (c) 1, 2 and 3 (d) 1, 2, 3 and 4

IES-51Ans. (b) Involute system is very interference prone.

IES-52. The motion transmitted between the teeth of two spur gears in mesh is generally [IES-1999]

- (a) Sliding (b) rolling
(c) Rotary (d) party sliding and partly rolling

IES-52Ans. (b)

Beam Strength of Gear Tooth

IES-53. In heavy-duty gear drives, proper heat treatment of gears is necessary in order to: [IES-2006]

- (a) Avoid interference
- (b) Prevent noisy operation
- (c) Minimize wear of gear teeth
- (d) Provide resistance against impact loading on gear teeth

IES-53Ans. (c)

IES-54. Consider the following statements pertaining to the basic Lewis equation for the strength design of spur gear teeth: [IES-2005]

1. Single pair of teeth participates in power transmission at any instant.
2. The tooth is considered as a cantilever beam of uniform strength.
3. Loading on the teeth is static in nature.
4. Lewis equation takes into account the inaccuracies of the tooth profile.
5. Meshing teeth come in contact suddenly.

Which of the statements given above are correct?

- (a) 1, 3, 4 and 5 (b) 1, 2, 3 and 4 (c) 1, 2 and 3 (d) 2, 4 and 5

IES-54Ans. (c)

IES-55. **Assertion (A):** The Lewis equation for design of gear tooth predicts the static load capacity of a cantilever beam of uniform strength.

Reason (R): According to law of gears interchangeability is possible only when gears have same pressure angle and same module. [IES-2008]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-55Ans. (b) The beam strength of gear teeth is determined from an equation (known as Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth.

Notes: (i) The Lewis equation is applied only to the weaker of the two wheels (*i.e.* pinion or gear).

(ii) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(iii) When the pinion and the gear are made of different materials, then the product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.

IES-56. In the formulation of Lewis equation for toothed gearing, it is assumed that tangential tooth load F_t , acts on the [IES-1998]

- (a) Pitch point
- (b) tip of the tooth
- (c) Root of the tooth
- (d) whole face of the tooth

IES-56Ans. (b)

IES-57. **Assertion (A):** The Lewis equation for gear tooth with involute profile predicts the static load capacity of cantilever beam of uniform strength. [IES-1994]

Reason (R): For a pair of meshing gears with involute tooth profile, the pressure angle and module must be the same to satisfy the condition of inter-changeability.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-57Ans. (c) For a pair of meshing gears with involute tooth profile, the pressure angle and module must be the same to satisfy the condition of inter-changeability it is not correct. Due to law of gearing.

IES-58. The dynamic load on a gear is due to [IES-2002]

1. Inaccuracies of tooth spacing
 2. Irregularities in tooth profile
 3. Deflection of the teeth under load
 4. Type of service (i.e. intermittent, one shift per day, continuous per day).
- Which of the above statements are correct?

- (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

IES-58Ans. (a)

IES-59. Consider the following statements:

The form factor of a spur gear tooth depends upon the [IES-1996]

1. Number of teeth
2. Pressure angle
3. Addendum modification coefficient
4. Circular pitch

Of these correct statements are

- (a) 1 and 3 (b) 2 and 4 (c) 1, 2 and 3 (d) 1 and 4

IES-59Ans. (c)

IES-60. Assertion (A): If the helix angle of a helical gear is increased, the load carrying capacity of the tooth increases. [IES-1996]

Reason (R): The form factor of a helical gear increases with the increasing in the helix angle.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-61Ans. (a)

IES-62. Match List I with List II and select the correct answer using the codes given below the Lists:

List I

- A. Unwin's formula
- B. Wahl factor
- C. Reynolds's equation
- D. Lewis form factor

Code:	A	B	C	D
(a)	1	4	2	3
(c)	1	3	2	4

List II

1. Bearings
2. Rivets
3. Gears
4. Springs

	A	B	C	D
(b)	2	3	1	4
(d)	2	4	1	3

[IES-2000]

IES-62Ans. (d)

IES-62. A spur gear transmits 10 kW at a pitch line velocity of 10 m/s; driving gear has a diameter of 1.0 m. Find the tangential force between the driver and the follower, and the transmitted torque respectively. [IES-2009]

- (a) 1 kN and 0.5 kN-m (b) 10 kN and 5 kN-m

- (c) 0.5 kN and 0.25 kN-m (d) 1 kN and 1 kN-m
- IES-62Ans. (a) Power transmitted = Force \times Velocity
 $\Rightarrow 10 \times 10^3 = \text{Force} \times 10$
 $\Rightarrow \text{Force} = \frac{10 \times 10^3}{10} = 1000 \text{ N / m}$
 Torque Transmitted = Force $\times \frac{\text{diameter}}{2}$
 $= 1000 \times \frac{1}{2} = 1000 \times 0.5$
 $= 500 \text{ N - m} = 0.5 \text{ kN - m}$

Wear Strength of Gear Tooth

- IES-63. The limiting wear load of spur gear is proportional to (where E_p = Young's modulus of pinion material; E_g = Young's modulus of gear material) [IES-1997]

(a) $(E_p + E_g)^{-1}$ (b) $\left(\frac{E_p + E_g}{E_p E_g}\right)$ (c) $\left(1 + \frac{E_p}{E_g}\right)$ (d) $\left(1 + \frac{E_g}{E_p}\right)$

IES-63Ans. (b)

Gear Lubrication

- IES-64. Match List I (Types of gear failure) with List II (Reasons) and select the correct answer using the codes given below the Lists [IES-2004]

List I

A. Scoring

B. Pitting

C. Scuffing

D. Plastic flow

List II

1. Oil film breakage

2. Yielding of surface under heavy loads

3. Cyclic loads causing high surface stress

4. Insufficient lubrication

A	B	C	D	A	B	C	D
(a) 2	1	3	4	(b) 2	3	4	1
(c) 4	3	1	2	(d) 4	1	3	2

IES-64Ans. (b)

Simple Gear train

- IES-65. In a simple gear train, if the number of idler gears is odd, then the direction or motion of driven gear will [IES-2001]
- (a) Be same as that of the driving gear
 (b) Be opposite to that of the driving gear
 (c) Depend upon the number of teeth on the driving gear
 (d) Depend upon the total number of teeth on all gears of the train

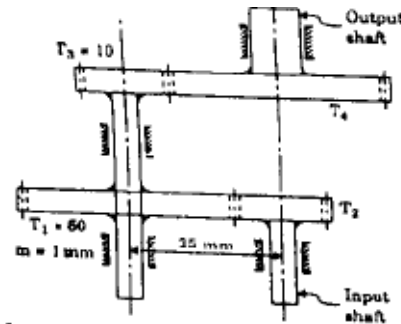
IES-65Ans. (a)

- IES-66. The gear train usually employed in clocks is a [IES-1995]
- (a) Reverted gear train (b) simple gear train
 (c) Sun and planet gear (d) differential gear.

IES-66Ans. (a)

IES-67. In the figure shown above, if the speed of the input shaft of the spur gear train is 2400 rpm and the speed of the output shaft is 100 rpm, what is the module of the gear 4?

- (a) 1.2 (b) 1.4
(c) 2 (d) 2.5



[IES-2005]

IES-67Ans. (b)

$$\frac{mT_2 + mT_1}{2} = 35$$

$$\text{or } T_2 = 10$$

$$N_1 = -N_i \times \frac{T_2}{T_1} = N_3$$

$$N_4 = \frac{-N_3 T_3}{T_4} = +N_i \times \frac{T_2}{T_1} \times \frac{T_3}{T_4} \text{ or } 100 = 2400 \times \frac{10}{60} \times \frac{10}{T_4} \text{ or } T_4 = 40$$

$$\frac{m'T_3 + m'T_4}{2} = 35 \text{ or } m' = \frac{70}{(40 + 10)} = 1.4$$

IES-68 In a machine tool gear box, the smallest and largest spindles are 100 rpm and 1120 rpm respectively. If there are 8 speeds in all, the fourth speed will be [IES-2002]

- (a) 400 rpm (b) 280 rpm (c) 800 rpm (d) 535 rpm

IES-68Ans. (b)

IES-69. A fixed gear having 200 teeth is in mesh with another gear having 50 teeth. The two gears are connected by an arm. The number of turns made by the smaller gear for one revolution of arm about the centre of the bigger gear is [IES-1996]

- (a) $\frac{2}{4}$ (b) 3 (c) 4 (d) 5

IES-69Ans. (d) $1 + 200/50 = 1 + 4 = 5$

Compound gear train

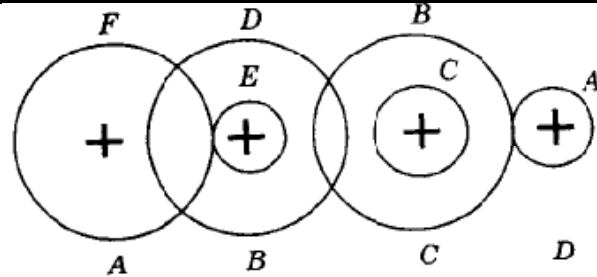
IES-70 The velocity ratio in the case of the compound train of wheels is equal to [IES-2000]

- (a) $\frac{\text{No. of teeth on first driver}}{\text{No. of teeth on last follower}}$ (b) $\frac{\text{No. of teeth on last follower}}{\text{No. of teeth on first driver}}$
(c) $\frac{\text{Product of teeth on the drivers}}{\text{Product of teeth on the followers}}$ (d) $\frac{\text{Product of teeth on the followers}}{\text{Product of teeth on the drivers}}$

IES-70Ans. (d)

IES-71. Consider the gear train shown in the given figure and table of gears and their number of teeth.

Gear	:A	B	C	D	E	F
No of teeth:	20	50	25	75	26	65



[IES-1999]

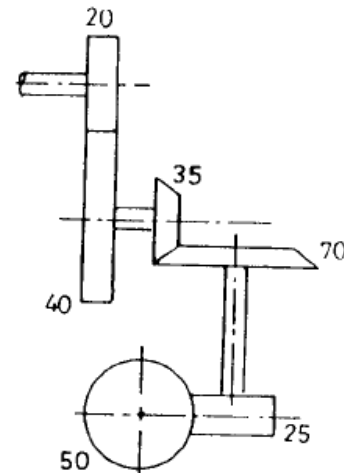
Gears BC and DE are moulded on parallel shaft rotating together. If the speed of A is 975 r.p.m., the speed of F will be

IES-71Ans. (b)

$$\text{Speed ratio } \frac{N_F}{N_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F} = \frac{20 \times 25 \times 26}{50 \times 75 \times 65} = \frac{4}{75} \quad \text{or } N_F = 975 \times \frac{4}{75} = 52 \text{ rpm}$$

IES-72. A compound train consisting of spur, bevel and spiral gears are shown in the given figure along with the teeth numbers marked against the wheels. Over-all speed ratio of the train is

- (a) 8
- (b) 2
- (c) $\frac{1}{2}$
- (d) $\frac{1}{8}$



[IES-1996]

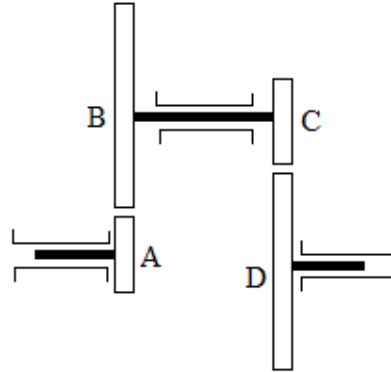
IES-72Ans. (a)

Elements of higher pair like follower in cam is under the action of gravity or spring force .

$$\text{Train value} = \frac{\text{speed of lost driven or follower}}{\text{speed of the first gear}}$$

$$\text{Train value} = \frac{\text{product of no.of teeth no the drives}}{\text{product of no.of teeth on the drives}} \times \frac{\text{speed of the first drive}}{\text{speed of the last driven or follower}}$$

- IES-73.** In the compound gear train shown in the above figure, gears A and C have equal numbers of teeth and gears B and D have equal numbers of teeth. When A rotates at 800 rpm, D rotates at 200 rpm. The rotational speed of compound gears BC would then be
- 300 rpm
 - 400rpm
 - 500 rpm
 - 600rpm



[IES 2007]

IES-73Ans. (b) From the figure $r_A + r_B = r_C + r_D$ or $T_A + T_B = T_C + T_D$ and as $N_B + N_C$ it must be $T_B = T_D$ & $T_A = T_C$

$$\text{Or } \frac{N_B}{N_A} = \frac{N_D}{N_C} \text{ or } N_C = \sqrt{N_A N_D} = \sqrt{800 \times 200} = 400 \text{ rpm } [\because N_B = N_C]$$

Reverted gear train

IES-74. Consider the following statements in case of reverted gear train:[IES-2002]

- The direction of rotation of the first and the last gear is the same.
- The direction of rotation of the first and the last gear is opposite.
- The first and the last gears are on the same shaft.
- The first and the last gears are on separate but co-axial shafts.

Which of these statements is/are correct?

- 1 and 3
- 2 and 3
- 2 and 4
- 1 and 4

IES-74Ans. (d)

IES-75. A reverted gear train is one in which the output shaft and input shaft

- Rotate in opposite directions
- are co-axial
- Are at right angles to each other
- are at an angle to each other

[IES-1997]

IES-75Ans. (b)

IES-76. In a reverted gear train, two gears P and Q are meshing, Q - R is a compound gear, and R and S are meshing. The modules of P and R are 4 mm and 5 mm respectively. The numbers of teeth in P, Q and R are 20, 40 and 25 respectively. The number of teeth in S is

[IES-2003]

- 23
- 35
- 50
- 53

IES-76Ans. (a)

Summation of radius will be constant.

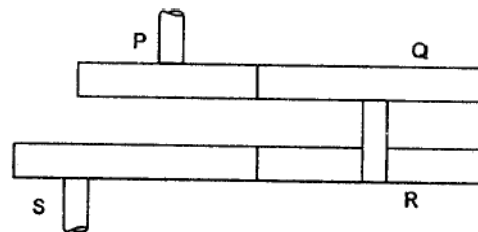
$$R_P + R_Q = R_R + R_S$$

$$\text{or } D_P + D_Q = D_R + D_S$$

$$\text{or } m_1(T_P + T_Q) = m_2(T_R + T_S)$$

$$\text{or } 4(20 + 40) = 5(25 + T_S)$$

$$\text{or } T_S = 23$$



IES-77. Two shafts A and B, in the same straight line are geared together through an intermediate parallel shaft. The parameters relating to the gears and pinions are given in the table:

[IES-2003]

Item	Speed	Teeth	PCD	Module
Driving wheel A	N_A	T_A	D_A	m
Driven wheel B	N_B	T_B	D_B	m
Driven wheel C on the intermediate shaft	N_C	T_C	D_C	m
Driving wheel D on the intermediate shaft, in mesh with B	N_D	T_D	D_D	m

$$(a) \frac{N_A}{N_B} = \frac{T_C}{T_A} \times \frac{T_B}{T_D}$$

$$(b) \frac{N_A}{N_B} = \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$(c) D_A + D_C = D_B + D_D$$

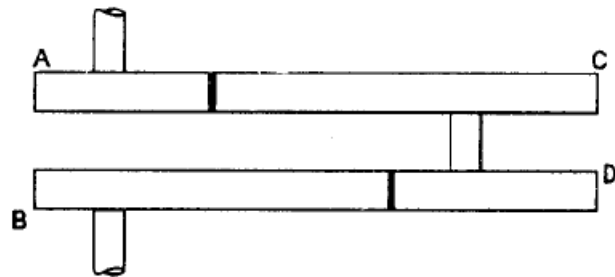
$$(d) T_A + T_C = T_B + T_D$$

IES-77Ans. (b)

$$(i) D_A + D_C = D_B + D_D$$

$$(ii) mT_A + mT_C = mT_B + mT_D$$

$$(iii) \frac{N_A}{N_B} = \frac{N_A}{N_C} \times \frac{N_C}{N_B} = \frac{T_C}{T_A} \times \frac{T_B}{T_D}$$



IES-78. A gear having 100 teeth is fixed and another gear having 25 teeth revolves around it, centre lines of both the gears being jointed by an arm. How many revolutions will be made by the gear of 25 teeth for one revolution of arm? [IES-2009]

(a) 3

(b) 4

(c) 5

(d) 6

IES-78Ans. (c)

Arm	N_A	N_B
0	+1	$\frac{-100}{25}$
Multiplying through out by x		
0	+x	$\frac{-100}{25}x$
y	y + x	y - 4x
Given that y + x = 0 \therefore x = -y = -1		
(\because y = 1)		
\therefore	$N_B = y - 4x = 5$	

Epicyclic gear train

IES-79. If the annular wheel of an epicyclic gear train has 100 teeth and the planet wheel has 20 teeth, the number of teeth on the sun wheel is [IES-2003]

(a) 80

(b) 60

(c) 40

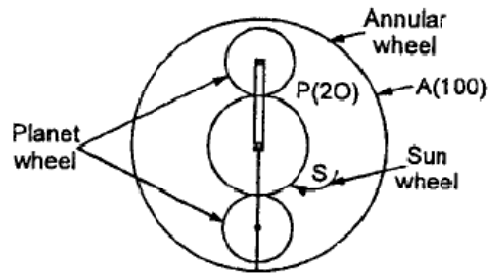
(d) 20

IES-79Ans. (b) From geometry

$$2d_p + d_s = d_A$$

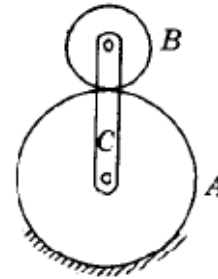
$$\text{or } 2T_p + T_s = T_A$$

$$\text{or } T_s = T_A - 2T_p = 100 - 2 \times 20 = 60$$



IES-80. In the epicyclic gear train shown in the given figure, A is fixed. A has 100 teeth and B has 20 teeth. If the arm C makes three revolutions, the number of revolutions made by B will be

- (a) 12
- (b) 15
- (c) 18
- (d) 24



[IES-1997]

IES-80Ans. (c) For 1 revolution of C,

$$N_B = 1 + \frac{T_A}{T_B} = 1 + \frac{100}{20} = 6 \quad \therefore \text{for 3 revolution, } N_D = 6 \times 3 = 18$$

IES-81. An epicyclic gear train has 3 shafts A, B and C, A is an input shaft running at 100 rpm clockwise. B is an output shaft running at 250 rpm clockwise. Torque on A is 50 kNm (clockwise). C is a fixed shaft. The torque to fix C

- (a) Is 20 kNm anticlockwise
- (b) is 30 kNm anticlockwise
- (c) Is 30 kNm clockwise
- (d) Cannot be determined as the data is insufficient

[IES-2002]

IES-81Ans. (b)

$$\text{Now } \omega_1 M_1 - \omega_2 M_2 = 0$$

$$\therefore M_2 = \frac{100 \times 50}{250} = 20$$

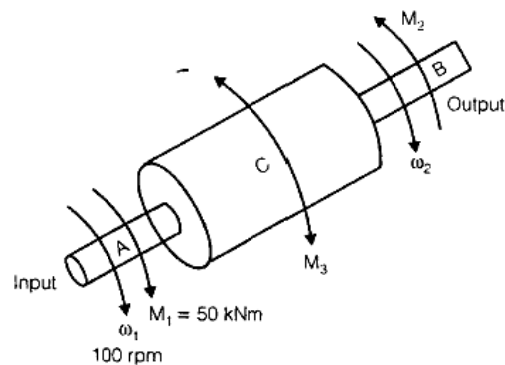
KNm(anticlockwise)

$$\text{and } \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 0$$

$$50 - 20 + \vec{M}_3 = 0$$

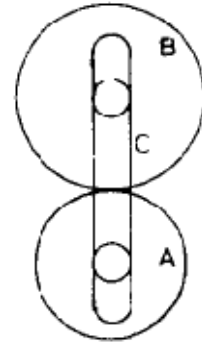
$$\therefore \vec{M}_3 = -30 \text{ kNm (clockwise)}$$

$$= 30 \text{ kNm (anticlockwise)}$$



IES-82. A single epicyclic gear train is shown in the given figure. Wheel A is stationary. If the number of teeth on A and B are 120 and 45 respectively, then when B rotates about its own axis at 100 rpm, the speed of C would be

- (a) 20 rpm (b) $27\frac{3}{11}$ rpm
(c) $19\frac{7}{11}$ rpm (d) 100 rpm



[IES-1994]

IES-82Ans. (c)

Terminology of Helical Gears

IES-83. If α = helix angle, and p_c = circular pitch; then which one of the following correctly expresses the axial pitch of a helical gear? [IES 2007]

- (a) $p_c \cos \alpha$ (b) $\frac{p_c}{\cos \alpha}$ (c) $\frac{p_c}{\tan \alpha}$ (d) $p_c \sin \alpha$

IES-83Ans. (c)

IES-84A helical gear has the active face width equal to b , pitch p and helix angle α . What should be the minimum value of b in order that contact is maintained across the entire active face of the gear? [IES-2004]

- (a) $p \cos \alpha$ (b) $p \sec \alpha$ (c) $p \tan \alpha$ (d) $p \cot \alpha$

IES-84Ans. (d) $b \geq \frac{P}{\tan \alpha}$

IES-85. **Assertion (A):** Helical gears are used for transmitting motion and power between intersecting shafts, whereas straight bevel gears are used for transmitting motion and power between two shafts intersecting each other at 90° . [IES-2000]

Reason (R): In helical gears teeth are inclined to axis of the shaft and are in the form of a helix.

Where as in bevel gears, teeth are tapered both in thickness and height from one end to the other.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-85Ans. (d)

IES-86. **Assertion (A):** Shafts supporting helical gears must have only deep groove ball-bearings. [IES-1999]

Reason (R): Helical gears produce axial thrusts.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-86Ans. (a)

IES-87. Assertion (A): Crossed helical gears for skew shafts are not used to transmit heavy loads. [IES-1995]

Reason (R) The gears have a point contact, and hence are not considered strong.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-87Ans. (b)

Bevel Gears

IES-88. In a differential mechanism, two equal sized bevel wheels A and B are keyed to the two halves of the rear axle of a motor car. The car follows a curved path. Which one of the following statements is correct? [IES-2004]

The wheels A and B will revolve at different speeds and the casing will revolve at a speed which is equal to the

- (a) Difference of speeds of A and B
- (b) Arithmetic mean of speeds of A and B
- (c) Geometric mean of speeds of A and B
- (d) Harmonic mean of speeds of A and B

IES-88Ans. (d)

Worm Gears

IES-89. Assertion (A): Tapered roller bearings must be used in heavy duty worm gear speed reducers. [IES-2005]

Reason (R): Tapered roller bearings are suitable for large radial as well as axial loads.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-89Ans. (a)

IES-90. Consider the following statements in respect of worm gears: [IES-2005]

1. They are used for very high speed reductions.
 2. The velocity ratio does not depend on the helix angle of the worm.
 3. The axes of worm and gear are generally perpendicular and non-intersecting.
- Which of the statements given above are correct?

- (a) 1 and 2
- (b) 1 and 3
- (c) 2 and 3
- (d) 1, 2 and 3

IES-90Ans. (d)

IES-91. For a speed ratio of 100 smallest gear box is obtained by using which of the following? [IES-2008]

- (a) A pair of spur gears
- (b) A pair of bevel and a pair of spur gears in compound gear train
- (c) A pair of helical and a pair of spur gears in compound gear train
- (d) A pair of helical and a pair of worm gears in compound gear train

IES-91Ans. (d)

IES-92. Consider the following statements regarding improvement of efficiency of worm gear drive: [IES-2004]

1. Efficiency can be improved by increasing the spiral angle of worm thread to 45° or more
2. Efficiency can be improved by adopting proper lubrication
3. Efficiency can be improved by adopting worm diameter as small as practicable to reduce sliding between worm-threads and wheel teeth
4. Efficiency can be improved by adopting convex tooth profile both for worm and wheel

Which of the statements given above are correct?

- (a) 1, 2 and 3 (b) 1, 2 and 4 (c) 2, 3 and 4 (d) 1, 3 and 4

IES-92Ans. (a)

$$\text{Gear } \eta_{\text{wormgear}} = \frac{\tan \lambda}{\tan(\phi_v + \lambda)}$$

$$\tan \phi_v = \pi_v$$

$$\tan \lambda = \frac{z_w \cdot m}{d_w}$$

The face of worm gear is made concave to envelope the worm.

IES-93. The lead angle of a worm is 22.5 deg. Its helix angle will be [IES-1994]

- (a) 22.5 deg. (b) 45 deg. (c) 67.5 deg. (d) 90°C.

IES-93Ans. (c) α = Pressure angle \cong lead angle; $\alpha + \beta = 90^\circ$; β = helix angle = $90^\circ - 22.5^\circ = 67.5^\circ$

Previous 20-Years IAS Questions

Spur gear

IAS-1. Match List I (Terms) with List II (Definition) and select the correct answer using the codes given below the lists: [IAS-2001]

List I

List II

A. Module

1. Radial distance of a tooth from the pitch circle to the top of the tooth

B. Addendum

2. Radial distance of a tooth from the pitch circle to the bottom of the tooth

C. Circular pitch

3. Distance on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth

4. Ratio of a pitch circle diameter in mm to the number of teeth

Codes:	A	B	C		A	B	C
(a)	4	1	3	(b)	4	2	3
(c)	3	1	2	(d)	3	2	4

IAS-1Ans. (a)

IAS-2 Consider the following specifications of gears A, B, C and D: [IAS-2001]

Gears	A	B	C	D
Number of teeth	20	60	20	60
Pressure angle	$14\frac{1}{2}^\circ$	$14\frac{1}{2}^\circ$	20°	$14\frac{1}{2}^\circ$
Module	1	3	3	1

- | | | | | | |
|--|----------|-------|-------|-------|-------|
| | Material | Steel | Brass | Brass | Steel |
|--|----------|-------|-------|-------|-------|
- Which of these gears form the pair of spur gears to achieve a gear ratio of 3?**
- (a) A and B (b) A and D (c) B and C (d) C and D
- IAS-2Ans. (b)**
 For a gear pair i) module must be same
 (ii) Pressure angle must be same.
- IAS-3. If the number of teeth on the wheel rotating at 300 r.p.m. is 90, then the number of teeth on the mating pinion rotating at 1500 r.p. m. is [IAS-2000]**
- (a) 15 (b) 18 (c) 20 (d) 60
- IAS-3Ans. (b)**
 Peripheral velocity (πDN) = constant. $\pi D_1 N_1 = \pi D_2 N_2$ and $D = mT$
 or $\pi m T_1 N_1 = \pi m T_2 N_2$ or $T_2 = T_1 \times \frac{N_1}{N_2} = 90 \times \frac{300}{1500} = 18$
 Or you may say speed ratio, $\frac{N_1}{N_2} = \frac{T_2}{T_1}$
- IAS-4. A rack is a gear of [IAS-1998]**
 (a) Infinite diameter (b) infinite module
 (c) zero pressure angle (d) large pitch
- IAS-4Ans. (a)**

Classification of Gears

- IAS-5. Assertion (A):** While transmitting power between two parallel shafts, the noise generated by a pair of helical gears is less than that of an equivalent pair of spur gears. **[IAS-2000]**
Reason(R): A pair of helical gears has fewer teeth in contact as compared to an equivalent pair of spur gears.
- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true
- IAS-5Ans. (c)** In spur gears, the contact between meshing teeth occurs along the entire face width of the tooth, resulting in a sudden application of the load which, in turn, results in impact conditions and generates noise.
 In helical gears, the contact between meshing teeth begins with a point on the leading edge of the tooth and gradually extends along the diagonal line across the tooth. There is a gradual pick-up of load by the tooth, resulting in smooth engagement and silence operation.

Pitch point

- IAS-6. An imaginary circle which by pure rolling action, gives the same motion as the actual gear, and is called [IAS-2000]**
- (a) Addendum circle (b) pitch circle
 (c) Dedendum circle (d) base circle

IAS-6Ans. (b)

Pressure angle

IAS-7. The pressure angle of a spur gear normally varies from [IAS-2000]
 (a) 14° to 20° (b) 20° to 25° (c) 30° to 36° (d) 40° to 50°

IAS-7Ans. (a)

Minimum Number of Teeth

IAS-8. Minimum number of teeth for involute rack and pinion arrangement for pressure angle of 20° is [IAS-2001]
 (a) 18 (b) 20 (c) 30 (d) 34

IAS-8Ans. (a) $T_{\min} = \frac{2h_f}{\sin^2 \theta} = \frac{2 \times 1}{\sin^2 20^\circ} = 17.1$ as > 17 So $T_{\min} = 18$

Cycloidal teeth

IAS-9. The tooth profile most commonly used in gear drives for power transmission is [IAS-1996]
 (a) A cycloid (b) An involute (c) An ellipse (d) A parabola

IAS-9Ans. (b) It is due to easy manufacturing.

Contact ratio

IAS-10. Which one of the following statements is correct? [IAS-2007]
 (a) Increasing the addendum results in a larger value of contact ratio
 (b) Decreasing the addendum results in a larger value of contact ratio
 (c) Addendum has no effect on contact ratio
 (d) Both addendum and base circle diameter have effect on contact ratio

IAS-10Ans. (d) contact ratio = $\frac{\text{length of arc of contact}}{\text{circular pitch}}$

$$= \frac{\sqrt{R_{A^2} - R^2 \cos^2 \theta} + \sqrt{r_{A^2} - r^2 \cos^2 \theta} - (R + r) \sin \theta}{P_c (\cos \theta)}$$

IAS-11. The velocity of sliding of meshing gear teeth is [IAS-2002]
 (a) $(\omega_1 \times \omega_2) x$ (b) $\frac{\omega_1}{\omega_2} x$ (c) $(\omega_1 + \omega_2) x$ (d) $\frac{(\omega_1 + \omega_2)}{x}$

(Where ω_1 and ω_2 = angular velocities of meshing gears
 x = distance between point of contact and the pitch point)

IAS-11Ans. (c)

Interference

IAS-12. For spur with gear ratio greater than one, the interference is most likely to occur near the [IAS-1997]

- | | |
|-----------------------------|-----------------------------------|
| (a) Pitch point | (b) point of beginning of contact |
| (c) Point of end of contact | (d) root of the tooth |

IAS-12Ans. (d)

IAS-13. How can interference in involute gears be avoided? [IAS-2007]

- (a) Varying the centre distance by changing the pressure angle only
- (b) Using modified involute or composite system only
- (c) Increasing the addendum of small wheel and reducing it for the larger wheel only
- (d) Any of the above

IAS-13Ans. (d)

IAS-14. Which one of the following statements in respect of involute profiles for gear teeth is not correct? [IAS-2003]

- (a) Interference occurs in involute profiles,
- (b) Involute tooth form is sensitive to change in centre distance between the base circles.
- (c) Basic rack for involute profile has straight line form
- (d) Pitch circle diameters of two mating involute gears are directly proportional to the base circle diameter

IAS-14Ans. (b)

IAS-15. **Assertion (A):** In the case of spur gears, the mating teeth execute pure rolling motion with respect to each other from the commencement of engagement to its termination. [IAS-2003]

Reason (R): The involute profiles of the mating teeth are conjugate profiles which obey the law of gearing.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-15Ans. (a)

IAS-16. **Assertion (A):** Gears with involute tooth profile transmit constant velocity ratios between shafts connected by them. [IAS-1997]

Reason (R): For involute gears, the common normal at the point of contact between pairs of teeth always passes through the pitch point.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-16Ans. (a)

Compound gear train

IAS-17. There are six gears A, B, C, D, E, F in a compound train. The numbers of teeth in the gears are 20, 60, 30, 80, 25 and 75 respectively. The ratio of the angular speeds of the driven (F) to the driver (A) of the drive is

- (a) $\frac{1}{24}$ (b) $\frac{1}{8}$ (c) $\frac{4}{15}$ (d) 12 [IAS-1995]

IAS-17Ans. (a)

$$\text{The ratio of angular speeds of F to A} = \frac{T_A \cdot T_C \cdot T_E}{T_B \cdot T_D \cdot T_F} = \frac{20 \times 30 \times 25}{60 \times 80 \times 75} = \frac{1}{24}$$

Epicyclic gear train

IAS-18. A fixed gear having 100 teeth meshes with another gear having 25 teeth, the centre lines of both the gears being joined by an arm so as to form an epicyclic gear train. The number of rotations made by the smaller gear for one rotation of the arm is [IAS-1995]

- (a) 3 (b) 4 (c) 5 (d) 6

IAS-18Ans. (c)

$$\text{Revolutions of 25 teeth gear} = 1 + \frac{T_{100}}{T_{25}} (\text{for one rotation of arm}) = 1 + \frac{100}{25} = 5$$

IAS-19. For an epicyclic gear train, the input torque = 100 Nm. RPM of the input gear is 1000 (clockwise), while that of the output gear is 50 RPM (anticlockwise). What is the magnitude of the holding torque for the gear train? [IAS-2007]

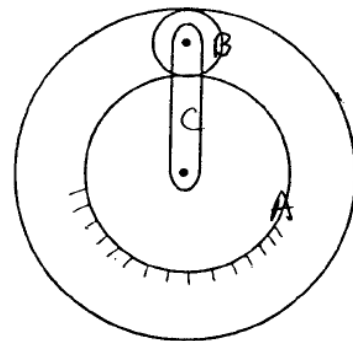
- (a) Zero (b) 500 Nm (c) 2100 Nm (d) None of the above

IAS-19Ans. (c) $T_i + T_o + T_{\text{arm}} = 0$ and $T_i \omega_i + T_o \omega_o + T_{\text{arm}} \omega_{\text{arm}} = 0$

$$\text{Gives, } T_{\text{arm}} = T_i \left(\frac{\omega_i}{\omega_o} - 1 \right) = T_i \left(\frac{N_i}{N_o} - 1 \right) = 100 \times \left(\frac{-1000}{50} - 1 \right) = -2100 \text{ Nm}$$

IAS-20. In the figure shown, the sun wheel has 48 teeth and the planet has 24 teeth. If the sun wheel is fixed, what is the angular velocity ratio between the internal wheel and arm?

- (a) 3.0
(b) 1.5
(c) 2.0
(d) 4.0



[IAS-2004]

IAS-20Ans. (a) $\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \because N_A = 0$

$$\frac{N_B - N_C}{-N_C} = -\frac{48}{24} \quad \text{or} \quad -\frac{N_B}{N_C} + 1 = -2 \quad \text{or} \quad \frac{N_B}{N_C} = 2 + 1 = 3$$

IAS-21. 100 kW power is supplied to the machine through a gear box which uses an epicyclic gear train. The power is supplied at 100 rad/s. The speed of the

output shaft of the gear box is 10 rad/s in a sense opposite to the input speed. What is the holding torque on the fixed gear of the train? [IAS-2004]

- (a) 8 kNm (b) 9 kNm (c) 10 kNm (d) 11 kNm

IAS-21Ans. (b) $T_1 + T_2 + T_3 = 0$

$$T_1 W_1 + T_2 W_2 + T_3 W_3 = 0$$

$$W_3 = 0$$

$$T_1 W_1 = 100 \text{ kW}, W_1 = 100 \text{ rad/s}$$

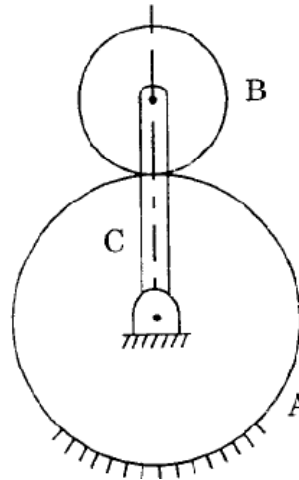
$$\therefore T_1 = 1 \text{ kNm}$$

$$\text{Or } T_2 = -\frac{T_1 W_1}{W_2} = \frac{-100}{(10)} = -10 \text{ kNm}$$

$$T_3 = -T_2 - T_1 = -(-10) - 1 = 9 \text{ kNm}$$

IAS-22. In the epicyclic gear train shown in the figure, $T_A = 40$, $T_B = 20$. For three revolutions of the arm, the gear B will rotate through

- (a) 6 revolutions
(b) 2.5 revolutions
(c) 3 revolutions
(d) 9 revolutions



[IAS-2003]

IAS-22Ans. (d)

Bevel Gears

IAS-23. Assertion (A): Spiral bevel gears designed to be used with an offset in their shafts are called 'hypoid gears' [IAS-2004]

Reason (R): The pitch surfaces of such gears are hyperboloids of revolution.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-23Ans. (a)

Worm Gears

IAS-24. If reduction ratio of about 50 is required in a gear drive, then the most appropriate gearing would be [IAS-1999]

- (a) spur gears (b) bevel gears
(c) Double helical gears (d) worm and worm wheel

IAS-24Ans. (d)

- IAS-25.** Speed reduction in a gear box is achieved using a worm and worm wheel. The worm wheel has 30 teeth and a pitch diameter of 210 mm. If the pressure angle of the worm is 20° , what is the axial pitch of the worm?
 (a) 7 mm (b) 22 mm (c) 14 mm (d) 63 mm [IAS-2004]

IAS-25Ans. (b) $m = \frac{210}{30} = 7$ and $P_x = \pi m = \frac{22}{7} \times 7 = 22 \text{ mm}$

Axial pitch = circular pitch of the worm wheel = πm

- IAS-24.** A speed reducer unit consists of a double-threaded worm of pitch = 11 mm and a worm wheel of pitch diameter = 84 mm. The ratio of the output torque to the input torque is [IAS-2002]
 (a) 7.6 (b) 12 (c) 24 (d) 42

IAS-24Ans. (a) $\frac{\text{Output torque}}{\text{Input torque}} = \frac{\text{pitch diameter of worm wheel}}{\text{pitch of worm}} = \frac{84}{11} = 7.6$

- IAS-25.** The maximum efficiency for spiral gears in mesh is given by (Where θ = shaft angle and ϕ , = friction angle) [IAS-1998]

(a) $\frac{1 + \cos(\theta - \phi)}{1 + \cos(\theta + \phi)}$ (b) $\frac{1 + \cos(\theta + \phi)}{1 + \cos(\theta - \phi)}$
 (c) $\frac{1 - \cos(\theta - \phi)}{1 + \cos(\theta + \phi)}$ (d) $\frac{1 - \cos(\theta + \phi)}{1 + \cos(\theta - \phi)}$

IAS-Ans. (b)

- IAS-26.** Assertion (A): A pair of gears forms a rolling pair. [IAS-1996]

Reason (R): The gear drive is a positive drive.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IAS-26Ans. (d) In rolling pair one link rolls over another fixed link.

Answers with Explanation (Objective)



Design of Bearings

Theory at a glance (GATE, IES, IAS & PSU)

Rolling Contact Bearings

- Rolling contact bearings are also called **anti-friction** bearing due to its low friction characteristics. These bearings are used for radial load, thrust load and combination of thrust and radial load. These bearings are extensively used due to its relatively lower price, being almost maintenance free and for its operational ease. However, friction increases at high speeds for rolling contact bearings and it may be noisy while running.
- In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings.
- We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction.
- It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction.
- Due to this low friction offered by rolling contact bearings, these are called **antifriction bearings**.
- **Why Rolling Contact Bearings?**
Rolling contact bearings are used to minimize the friction associated with relative motion performed under load.

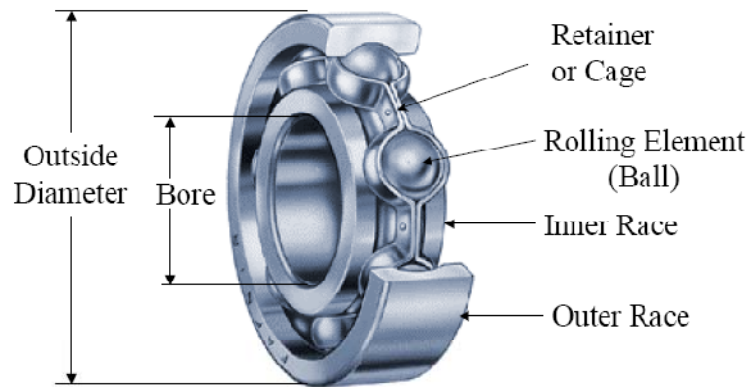
The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.



Inner and outer races are typically pressed onto the shaft or hub with a slight interference fit to make them move with the shaft (inner race) or remain stationary (outer race).

Fig. Bearing Nomenclature

If Average values of effective coefficients of friction for bearings are described below:

- | | |
|-----------------------------------|--------------------------------------------------------|
| 1. Spherical ball bearing - f_1 | 2. Cylindrical roller bearing - f_2 |
| 3. Taper roller bearing - f_3 | 4. Stable (thick film) Sliding contact bearing - f_4 |

Correct sequence is $f_1 < f_2 < f_3 < f_4$

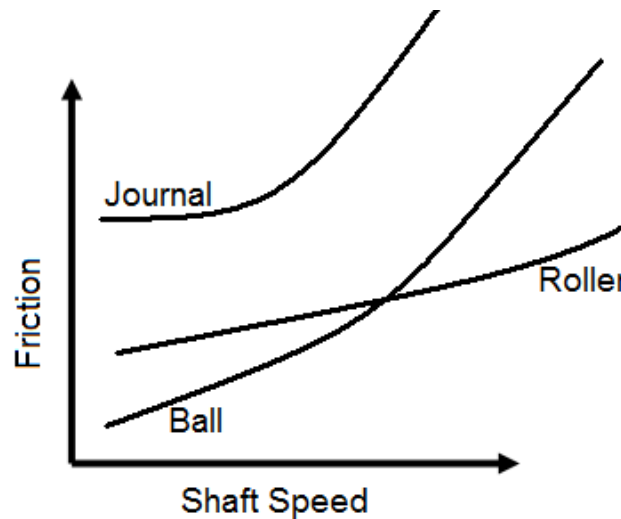


Fig.

Types of Rolling Contact Bearings

1.	Self realigning ball bearing	for hinged condition
----	------------------------------	----------------------

2.	Taper roller bearing	For axial and radial load.
3.	Deep groove ball bearing	for pure radial load
4.	Thrust ball bearing	For pure axial load.

Ball Bearings

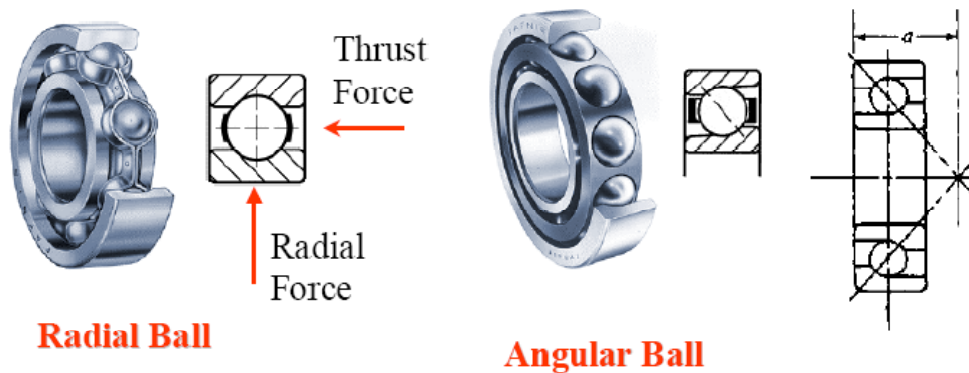


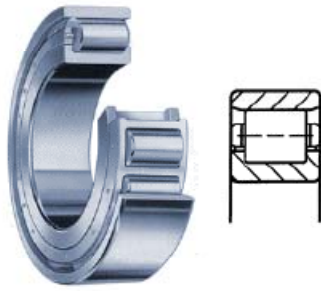
Fig. Ball Bearings

Angular ball bearings have higher thrust load capacity in one direction than due radial ball bearings

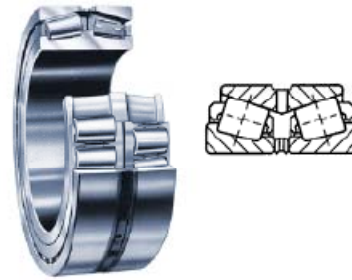


Fig. Double Row Angular Contact Bearing

Double Row Angular Contact Bearing, shown in Fig. above has two rows of balls. Axial displacement of the shaft can be kept very small even for axial loads of varying magnitude.



Radial Cylindrical



Radial Tapered

Roller bearings have higher load capacity than ball bearings.



Thrust

Figure- Roller Bearings

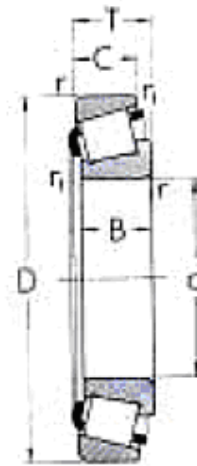


Figure- Taper Roller Bearing

A taper roller bearing is shown in Fig. above. It is generally used for simultaneous heavy radial load and heavy axial load. Roller bearings has more contact area than a ball bearing, therefore, they are generally used for heavier loads than the ball bearings.

Spherical Roller Bearing

A spherical roller bearing, shown in the Fig. below has self aligning property. It is mainly used for heavy axial loads. However, considerable amount of loads in either direction can also be applied.



Figure- Spherical Roller Bearing

Needle Bearings

Needle bearings have very high load ratings and require less space

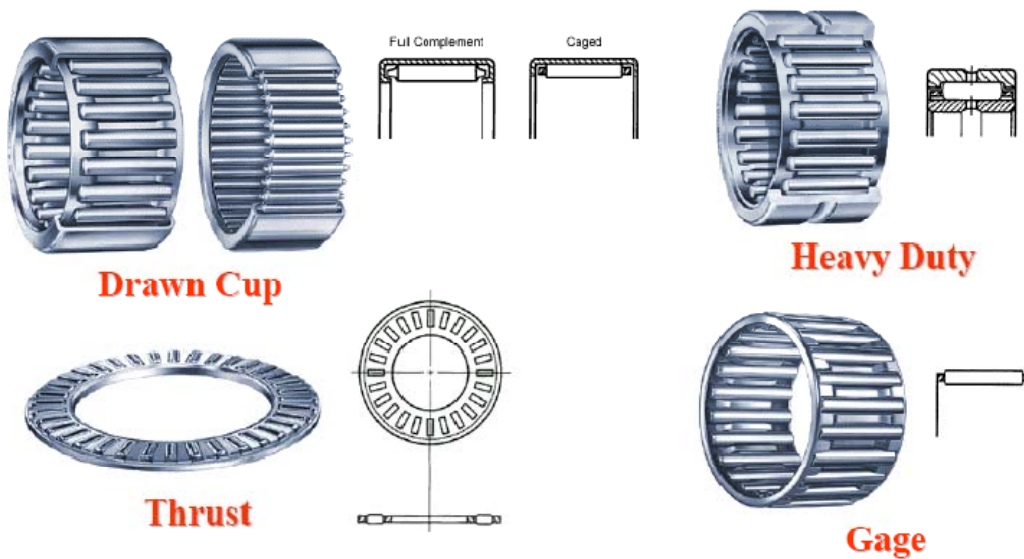


Figure- Spherical Bearings

- If three ball bearing identified as SKF 2015, 3115 and 4215

According to ISO plan for dimension series bearings are provided with two digit numbers. The first number indicates the width series 8, 0, 1, 2, 3, 4, 5 and 6 in order of increasing width. The second number indicate diameter series 7, 8, 9, 0, 1, 2, 3, and 4 in order of ascending outer diameter of bearing. Thus bearing number SKF 2015, 3115 and 4215 shows bearings belonging to different series with 75 mm bore diameter but width is increasing. SKF 2015, 3115 and 4215 shows width is increasing ascending outer diameter of bearing same bore diameter 75 mm. (i.e. 15×5)

Dynamic Load Carrying Capacity

Static Load Capacity

- The static load rating is the load at which permanent deformation of a race or ball will occur.
- The bearing is not rotating when this measurement is made.

Bearing load

If two groups of identical bearings are tested under loads P_1 and P_2 for respective lives of L_1 and L_2 , then,

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1} \right)^a$$

Where,

L = life in millions of revolution or life in hours

a = constant which is **3 for ball bearings and 10/3 for roller bearings**

Basic load rating

It is that load which a group of apparently identical bearings can withstand for a Rating life of one million revolutions.

Hence, in, if say, L_1 is taken as one million then the corresponding load is

$$C = P \left(L \right)^{\frac{1}{a}}$$

Where, C is the basic or dynamic load rating

Therefore, for a given load and a given life the value of C represents the load carrying capacity of the bearing for one million revolutions. This value of C , for the purpose of bearing selection, should be lower than that given in the manufacturer's catalogue. Normally the basic or the dynamic load rating as prescribed in the manufacturer's catalogue is a conservative value, therefore the chances of failure of bearing is very less.

Equivalent Bearing Load and Equivalent radial load

The load rating of a bearing is given for radial loads only. Therefore, if a bearing is subjected to both axial and radial load, then an equivalent radial load is estimated as,

$$P_e = VP_r \text{ or}$$

$$P_e = XVP_r + YP_a$$

Where,

P_e : Equivalent radial load

P_r : Given radial load

P_a : Given axial load

V: Rotation factor (1.0, inner race rotating; 1.2, outer race rotating)

X: A radial factor

Y: An axial factor

The values of X and Y are found from the chart whose typical format and few Representative values are given below.

$\frac{P_a}{C_o}$	e	$\frac{P_a}{P_r} \leq e$		$\frac{P_a}{P_r} \geq e$	
		X	Y	X	Y
0.021	0.21	1.0	0.0	0.56	2.15
0.110	0.30	1.0	0.0	0.56	1.45
0.560	0.44	1.0	0.0	0.56	1.00

The factor, C_o is obtained from the bearing catalogue

Load-life Relationship

Rating Life

Rating life is defined as the life of a group of apparently identical ball or roller bearings, in number of revolutions or hours, rotating at a given speed, so that 90% of the bearings will complete or exceed before any indication of failure occur.

Suppose we consider 100 apparently identical bearings. All the 100 bearings are put onto a shaft rotating at a given speed while it is also acted upon by a load. After some time, one after another, failure of bearings will be observed. When in this process, the tenth bearing fails, then the number of revolutions or hours lapsed is recorded. These figures recorded give the rating life of the bearings or simply L_{10} life (10 % failure). Similarly, L_{50} means, 50 % of the bearings are operational. It is known as median life. Figure below defines the life of rolling contact bearings.

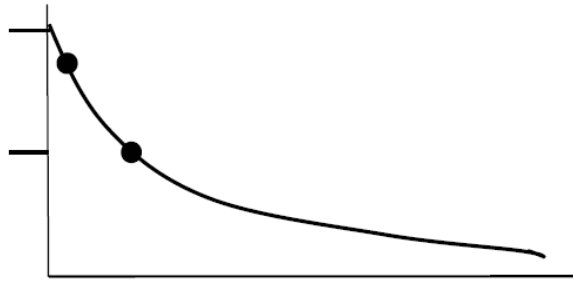


Fig.

Rating Life

$$(i) L = \left(\frac{d}{R} \right)^P$$

$$(ii) L = \frac{60NL_h}{10^6}$$

L = bearing life in (Mrev)
 d = dynamic load capacity
 R = Equivalent bearing load.
 N = speed of rotation
 L_h = bearing life in (hours)
 $P = 3$ for ball bearing
 $= \frac{10}{3}$ for roller bearing

(i) Ball bearing is usually made from chrome nickel steel.

Note: d = dynamic load carrying capacity

$$R = XF_r + YF_a$$

F_r = Radial load, F_a = axial load

The rated life of a ball bearing,

$$L = \left(\frac{d}{R} \right)^P$$

Where,

d = dynamic load capacity

R = Equivalent bearing load

$P = 3$ for ball bearing

$P = \frac{10}{3}$ for roller bearing.

Reliability of a Bearing

We have already discussed that the rating life is the life that 90 per cent of a group of identical bearings will complete or exceed before the first evidence of fatigue develops. The reliability

(R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. According to Weibull, the relation between the bearing life and the reliability is given as

$$\log_e \left(\frac{1}{R} \right) = \left(\frac{L}{a} \right)^b \quad \text{or} \quad \frac{L}{a} = \left[\log_e \left(\frac{1}{R} \right) \right]^{1/b} \quad \dots(i)$$

Where L is the life of the bearing corresponding to the desired reliability R and a and b are constants whose values are

$$a = 6.84, \text{ and } b = 1.17$$

If L_{90} is the life of a bearing corresponding to a reliability of 90% (*i.e.* R_{90}), then

$$\frac{L_{90}}{a} = \left[\log_e \left(\frac{1}{R_{90}} \right) \right]^{1/b} \quad \dots(ii)$$

Dividing equation (i) by equation (ii), we have

$$\frac{L}{L_{90}} = \left[\frac{\log_e (1/R)}{\log_e (1/R_{90})} \right]^{1/b} = * 6.85 \left[\log_e (1/R) \right]^{1/1.17} \quad \dots(\because b=1.17)$$

This expression is used for selecting the bearing when the reliability is other than 90%.

Note: If there is n number of bearings in the system each having the same reliability R , and then the reliability of the complete system will be

$$R_s = R_p$$

Where R_s indicates the probability of one out of p number of bearings failing during its life time.

$$\begin{aligned} \left[\log_e (1/R_{90}) \right]^{1/b} &= \left[\log_e (1/0.90) \right]^{1/1.17} = (0.10536)^{0.8547} = 0.146 \\ \therefore \frac{L}{L_{90}} &= \frac{\left[\log_e (1/R) \right]^{1/b}}{0.146} = 6.85 \left[\log_e (1/R) \right]^{1/1.17}. \end{aligned}$$

Questions and answers

Q. What is rating life of a rolling contact bearing?

Ans. Rating life is defined as the life of a group of apparently identical ball or roller bearings, in number of revolutions or hours, rotating at a given speed, so that 90% of the bearings will complete or exceed before any indication of failure occur.

Q. What is basic load rating of a rolling contact bearing?

Ans. It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

$$C = P \left(L \right)^{\frac{1}{a}}$$

Where, C is the basic load rating and P and L are bearing operating load and life respectively and a is a constant **which is 3 for ball bearings and 10/3 for roller bearings.**

Q. Why determination of equivalent radial load is necessary?

Ans. The load rating of a bearing is given for radial loads only. Therefore, if a bearing is subjected to both axial and radial loads, then equivalent radial load estimation is required.

Example-1:

A certain application requires a bearing to last for 1800 hr with a reliability of 90 percent. What should be the rated life of the bearing?

Given: Bearing must last for 1800 hr with a reliability of 90 percent.

Find: Rated life of bearing.

Solution: The L_{10} life has 90% reliability. Therefore the L_{10} rated life must be 1800 hr.

Example-2:

A ball bearing is to be selected to withstand a radial load of 4 kN and have an L_{10} life of 1200 h at a speed of 600 rev/min. The bearing maker's catalog rating sheets are based on an L_{10} life 3800 h at 500 rev/min. What load should be used to enter the catalog?

Given: Ball bearing must withstand 4 kN for 1200 hr. with a reliability of 90% at a speed of 600 rev/min. The catalog rating sheets are based on an L_{10} life of 3800 hr at 500 rev/min.

Find: Load to be used with catalog data to select the bearing

Solution: $L_1 = (600 \text{ rev/min}) (1200 \text{ hr}) (60 \text{ min/hr}) \quad F_1 = 4 \text{ kN}$
 $= 43.2 \times 10^6 \text{ rev.}$

$L_2 = (500 \text{ rev/min}) (3800 \text{ hr}) (60 \text{ min/hr}) \quad F_2 = ?$
 $= 114 \times 10^6 \text{ rev.}$

$$\frac{L_2}{L_1} = \left(\frac{F_1}{F_2} \right)^k \Rightarrow F_2 = F_1 \left(\frac{L_2}{L_1} \right)^{\frac{1}{k}}$$

For a ball bearing, $k = 3$

$$\Rightarrow F_2 = 4 \text{ kN} * \left(\frac{114 \times 10^6}{43.2 \times 10^6} \right)^{1/3}$$

$$\boxed{F_2 = 5.53 \text{ kN}}$$

Sample problem

A simply supported shaft, diameter 50mm, on bearing supports carries a load of 10kN at its center. The axial load on the bearings is 3kN. The shaft speed is 1440 rpm. Select a bearing for 1000 hours of operation.

Solution

The radial load $P_r = 5$ kN and axial load $P_a = 3$ kN. Hence, a single row deep groove ball bearing may be chosen as radial load is predominant. This choice has wide scope, considering need, cost, future changes etc.

Millions of revolution for the bearing, $L_{10} = \frac{60 \times 1440 \times 1000}{10^6} = 86.4$

For the selection of bearing, a manufacturer's catalogue has been consulted. The equivalent radial load on the bearing is given by,

$$P_e = XVP_r + YP_a$$

Here, $V=1.0$ (assuming inner race rotating)

From the catalogue, $C_0 = 19.6$ kN for 50mm inner diameter.

$$\therefore \frac{P_a}{C_0} = \frac{3.0}{19.6} = 0.153,$$

Therefore, value of e from the table (sample table is given in the text above) and By linear interpolation = 0.327.

Here, $\frac{P_a}{P_r} = \frac{3}{5} = 0.6 > e$. Hence, X and Y values are taken from fourth column of the

Sample table. Here, $X = 0.56$ and $Y = 1.356$

Therefore, $P_e = XVP_r + YP_a = 0.56 \times 1.0 \times 5.0 + 1.356 \times 3.0 = 6.867$ kN

\therefore basic load rating, $C = P(L)^{\frac{1}{3}} = 6.867 \times (86.4)^{\frac{1}{3}} = 30.36$ kN

Now, the table for single row deep groove ball bearing of series- 02 shows that for a 50mm inner diameter, the value of $C = 35.1$ kN. Therefore, this bearing may be selected safely for the given requirement without augmenting the shaft size. A possible bearing could be SKF 6210.

Sliding Contact Bearings

Depending upon the nature of contact. The bearings under this group are classified as:

- (a) Sliding contact bearings, and
- (b) Rolling contact bearings.

In **sliding contact bearings**, as shown in Fig. the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as **plain bearings**.

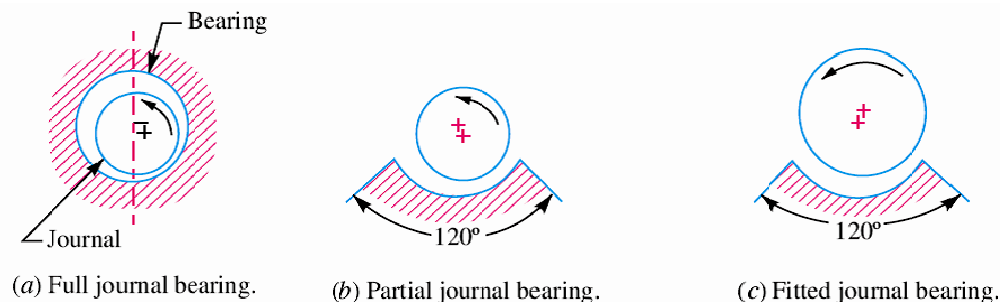


Fig.

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows:

1. Thick film bearings: The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings is also called as *hydrodynamic lubricated bearings*.

2. Thin film bearings: The thin film bearings are those in which, although lubricant is present; the working surfaces partially contact each other at least part of the time. Such type of bearings is also called *boundary lubricated bearings*.

3. Zero film bearings: The zero film bearings are those which operate without any lubricant present.

4. Hydrostatic or externally pressurized lubricated bearing: The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

Conformability: It is the ability of the bearing material to accommodate shaft deflections and bearing inaccuracies by plastic deformation (or creep) without excessive wear and heating.

Embeddability: It is the ability of bearing material to accommodate (or embed) small particles of dust, grit etc., without scoring the material of the journal.

Basic Modes of Lubrication

The load supporting pressure in hydrodynamic bearings arises from either

1. The flow of a viscous fluid in a converging channel (known as *wedge film lubrication*), or
2. The resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as *squeeze film lubrication*).

Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, i.e. the side leakage is neglected.

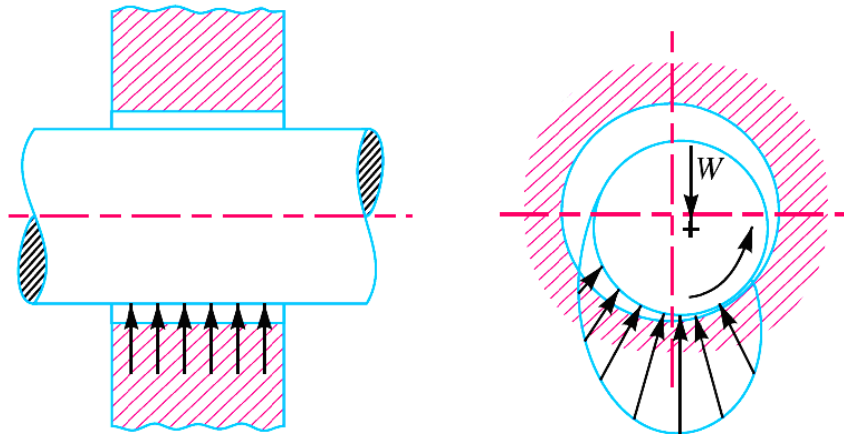


Fig. Variation of pressure in the converging film.

Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in figure below, in which O is the centre of the journal and O' is the centre of the bearing.

Let D = Diameter of the bearing,

d = Diameter of the journal,

and

l = Length of the bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. Diametral clearance. It is the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

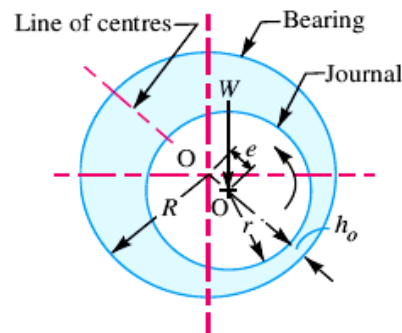


Fig. Hydrodynamic journal bearing.

Note: The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is 0.025 mm

4. Eccentricity: It is the radial distance between the centre (O) of the bearing and the displaced centre (O') of the bearing under load. It is denoted by e .

5. Minimum oil film thickness: It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h_o and occurs at the line of centers as shown in Figure above. Its value may be assumed as $c / 4$.

6. Attitude or eccentricity ratio: It is the ratio of the eccentricity to the radial clearance.

Mathematically, attitude or eccentricity ratio,

$$\epsilon = \frac{e}{c_1} = \frac{c_1 - h_0}{c_1} = 1 - \frac{h_0}{c_1} = 1 - \frac{2h_0}{c} \quad \dots (\because c_1 = c / 2)$$

7. Short and long bearing: If the ratio of the length to the diameter of the journal (*i.e.* l / d) is less than 1, then the bearing is said to be short bearing. On the other hand, if l / d is greater than 1, then the bearing is known as long bearing.

Notes: 1. when the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called square bearing.

2. Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large l / d ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of l / d may be taken as 1 to 2 for general industrial machinery. In crank shaft bearings, the l / d ratio is frequently less than 1.

Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{\mu N}{P} \left(\frac{d}{c} \right)^2$$

For design purposes, its value is taken as follows:

$$\frac{\mu N}{p} \left(\frac{d}{c} \right)^2 = 14.3 \times 10^6 \quad \dots \text{ (When } \mu \text{ is in kg / m-s and } p \text{ is in N / mm}^2 \text{)}$$

*Coefficient of friction,

$$f = \frac{33}{10^8} \left(\frac{\mu N}{p} \right) \left(\frac{d}{c} \right) + k \quad \dots \text{ (when } \mu \text{ is in kg / m-s and } p \text{ is in N / mm}^2 \text{)}$$

From Figure below, we see that the minimum amount of friction occurs at A and at this point the value of $\mu N / p$ is known as bearing modulus which is denoted by K . The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of $\mu N / p$ at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of $\mu N / p = 15 K$ may be used. From above, it is concluded that when the value of $\mu N / p$ is greater than K , then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of $\mu N / p$ is less than K , then the oil film will rupture and there is a metal to metal contact.

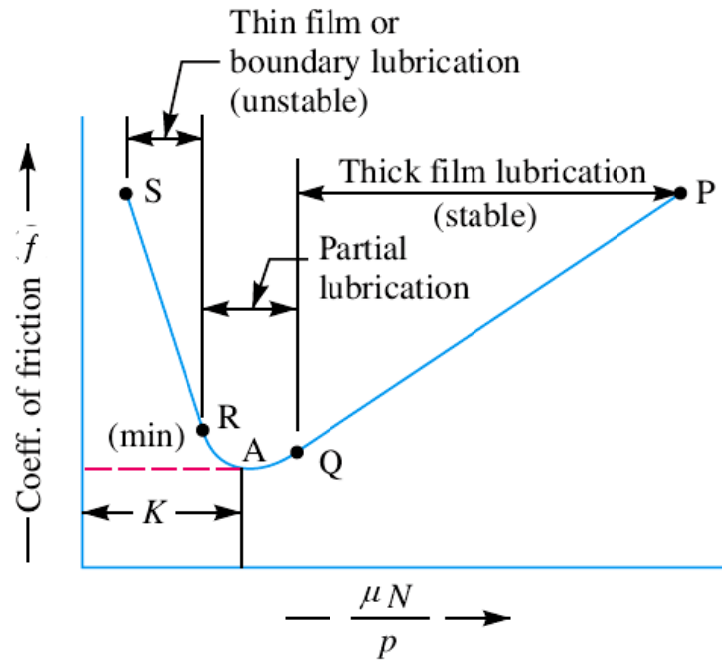


Fig. Variation of coefficient of friction with $\mu N/p$.

Temperature Rise

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu \cdot W \cdot V \text{ N-m/s or J/s or watts}$$

Where

μ = Coefficient of friction,

W = Load on the bearing in N, p = Pressure on the bearing in $\text{N/mm}^2 \times$ Projected area of the bearing in $\text{mm}^2 = p (l \times d)$,

V = Rubbing velocity in $\text{m/s} = \frac{\pi d \cdot N}{60}$, d is in meters, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$$Q_d = C \cdot A (t_b - t_a) \text{ J/s or W}$$

... ($\because 1 \text{ J/s} = 1 \text{ W}$)

Where C = Heat dissipation coefficient in $\text{W/m}^2/^\circ\text{C}$,
 A = Projected area of the bearing in $\text{m}^2 = l \times d$,
 t_b = Temperature of the bearing surface in $^\circ\text{C}$, and
 t_a = Temperature of the surrounding air in $^\circ\text{C}$.

The value of C has been determined experimentally by O. Lasche. The values depend upon the type of bearing, its ventilation and the temperature difference. The average values of C (in $\text{W/m}^2/^\circ\text{C}$), for journal bearings may be taken as follows:

For unventilated bearings (Still air)
 $= 140 \text{ to } 420 \text{ W/m}^2/^\circ\text{C}$

For well ventilated bearings
 $= 490 \text{ to } 1400 \text{ W/m}^2/^\circ\text{C}$

It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t_o) and the temperature of the outside air (t_a). In other words,

$$t_b - t_a = \frac{1}{2}(t_o - t_a)$$

Notes: 1. for well designed bearing, the temperature of the oil film should not be more than 60°C , otherwise the viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temperature of the oil film is often called as the operating temperature of the bearing.

2. In case the temperature of the oil film is higher, then the bearing is cooled by circulating water through coils built in the bearing.

3. The mass of the oil to remove the heat generated at the bearing may be obtained by equating the heat generated to the heat taken away by the oil. We know that the heat taken away by the oil,

$$Q_t = m.S.t \text{ J/s or watts}$$

Where m = Mass of the oil in kg / s ,
 S = Specific heat of the oil. Its value may be taken as $1840 \text{ to } 2100 \text{ J / kg / } ^\circ\text{C}$,
 t = Difference between outlet and inlet temperature of the oil in $^\circ\text{C}$.

Bearing Materials

The materials commonly used for sliding contact bearings are discussed below:

1. **Babbitt metal:** The tin base and lead base babbitts are widely used as a bearing material, because they satisfy most requirements for general applications. The babbitts are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2 . When applied in automobiles, the babbitt is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an Insert or steel shell. The composition of the babbitt metals is as follows:

Tin base babbitts: Tin 90%; Copper 4.5%; Antimony 5%; Lead 0.5%.

Lead base babbitts: Lead 84%; Tin 6%; Antimony 9.5%; Copper 0.5%.

2. **Bronzes:**

3. **Cast iron:**

4. **Silver:**

5. **Non-metallic bearings:**

6. Soft rubber

(i) Radial clearance $C = R - r$

(ii) $R = e + r + h_0$

(iii) $\varepsilon = \frac{e}{C} = 1 - \frac{h_0}{C}$

(iv) Sommerfeld number

$$\frac{\mu N}{P} \left(\frac{d}{c} \right)^2$$

(v) Petroff's Law

$$f = 2\pi^2 \frac{\mu n_s}{P} \left(\frac{r}{C} \right) + K \rightarrow 0.002$$

(vi) Radius of friction circle
 $= f \times r$

(vii) $C = (0.001)r$

(viii) $h_0 = (0.0002)r$

(ix) $1C_p = \frac{1}{100} P = \frac{1}{1000} \frac{N-S}{m^2} = 10^{-9} N-S/mm^2$

(x) Frictional power (KW)_f = $\frac{2\pi n_s f w r}{10^6}$

R = Radius of bearing

r = Radius of journal

e = eccentricity

h_0 = minimum film thickness.

ε = eccentricity ratio.

μ = viscosity $(N-S/mm^2)$

P = unit bearing pressure Load per unit projection area. $(W/ld)(N/mm^2)$

n_s = journal speed

f = co-efficient of friction

μw = Force

$\mu w \times r$ = Torque

$\mu w r (\omega) = p$

$= \mu w r 2\pi n_s$.

(xi) Flow variable (FV) = $\frac{Q}{n_s C r l}$

(xii) co-efficient of friction variable: CFV = $f \cdot \left(\frac{r}{c} \right)$

(xiii) The angle of eccentricity or attitude angle locates the position of minimum film thickness with respect to the direction of load.

(xiv) $\frac{\mu N}{P}$ is called bearing characteristic Number

μ = absolute viscosity of the lubricant

N = speed, P = Bearing pressure

(xv) When bearing is subjected to large fluctuation of load and heavy impacts. The $\frac{\mu N}{P}$ should be 15 times the bearing modulus

Types of Rolling Contact Bearings

GATE-1. Spherical roller bearings are normally used **[GATE-1992]**

- (a) For increased radial load (b) for increased thrust load
(c) When there is less radial space (d) to compensate for angular misalignment

GATE-1Ans. (d) It is also true for (a) but (d) is more appropriate.

Load-life Relationship

GATE-2. The rated life of a ball bearing varies inversely as which one of the following? **[GATE-1993; IES-2004]**

- (a) Load (b) (load)² (c) (load)³ (d) (load)^{3.33}

GATE-2Ans. (c) $L = \left(\frac{d}{R}\right)^p$, d = dynamic load capacity

R = Equivalent bearing load

p = 3 for ball bearing

= $\frac{10}{3}$ for roller bearing.

GATE-3. The life of a ball bearing at a load of 10 kN is 8000 hours. Its life in hours, if the load is increased to 20 kN, keeping all other conditions the same, is

- (a) 4000 (b) 2000 (c) 1000 (d) 500 **[GATE-2000]**

GATE-3 Ans. (c)

$$Life \propto \left(\frac{1}{P}\right)^3$$

$$\Rightarrow L_2 = L_1 \left(\frac{P_1}{P_2}\right)^3 = 8000 \left(\frac{10}{20}\right)^3 = 1000 \text{ hrs.}$$

GATE-4. The dynamic load capacity of 6306 bearing is 22 kN. The maximum radial load it can sustain to operate at 600 rev/min, for 2000 hours is **[GATE-1997]**

- (a) 4.16 kN (b) 3.60 kN (c) 6.2S kN (d) 5.29 kN

GATE-4 Ans. (d)

$$\begin{aligned} \text{Number of revolutions in life} &= 2000 \times 60 \times 600 \\ &= 72 \times 10^5 \text{ revolutions} \end{aligned}$$

$$L = 72$$

$$\text{Maximum radial load} = \frac{22}{\sqrt[3]{L}} = \frac{22}{\sqrt[3]{72}} = 5.29 \text{ kN}$$

- GATE-5. The basic load rating of a ball bearing is [GATE-1998]**
- (a) The maximum static radial load that can be applied without causing any plastic deformation of bearing components.
 - (b) The radial load at which 90% of the group of apparently identical bearings run for one million revolutions before the first evidence of failure.
 - (c) The maximum radial load that can be applied during operation without any plastic deformation of bearing components.
 - (d) A combination of radial and axial loads that can be applied without any plastic deformation.

GATE-5 Ans. (b)

Basic Modes of Lubrication

- GATE-6. Which one of the following is a criterion in the design of hydrodynamic journal bearings? [GATE-2005]**
- (a) Sommerfeld number
 - (b) rating life
 - (c) Specific dynamic capacity
 - (d) Rotation factor

GATE-6 Ans. (a) Sommerfeld Number, also Known as bearing Characteristic Number,

$$s = \frac{z_n}{P} \cdot \left(\frac{D}{C_d} \right)^2$$

- GATE-7. A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/second carries a load of 2.0 kN. The lubricant used has a viscosity of 20 mPas. The radial clearance is 50 μm. The Sommerfeld number for the bearing is [GATE-2007]**
- (a) 0.062
 - (b) 0.125
 - (c) 0.250
 - (d) 0.785

GATE-7Ans. (b) Sommerfeld number $S = \left(\frac{r}{c} \right)^2 \times \frac{\mu N}{P}$

Where, r is radius of journal

μ is viscosity of lubricant

N is number of revolution per second

P is bearing pressure on projected Area

C is radial clearance

Therefore,

$$P = \frac{P}{d \times l} = \frac{2000}{50 \times 50} = 0.8 \text{ N/mm}^2$$

$$S = \left(\frac{25}{50 \times 10^{-3}} \right)^2 \times \frac{20 \times 20 \times 10^{-3}}{0.8 \times 10^6}$$

$$= 0.125$$

- GATE-8. To restore stable operating condition in a hydrodynamic journal bearing, when it encounters higher magnitude loads, [GATE-1997]**
- (a) Oil viscosity is to be decreased
 - (b) oil viscosity is to be increased
 - (c) Oil viscosity index is to be increased
 - (d) oil viscosity index is to be decreased

GATE-8Ans. (b)

GATE-9. List I

- (A) Automobile wheel mounting on axle
- (B) High speed grinding spindle
- (C) I.C. Engine connecting rod
- (D) Leaf spring eye mounting

List II

[GATE-1997]

- 1. Magneto bearing
- 2. Angular contact bearing
- 3. Taper roller bearing
- 4. Hydrodynamic journal bearing

5. Sintered metal bearing

6. Teflon/Nylon bush.

GATE-9Ans. (A) -3, (B) -1, (C)-4, (D)-6

GATE-10. In thick film hydrodynamic journal bearings, the coefficient of friction

- | | |
|--------------------------------------|----------------------------------------------------|
| (a) Increases with increases in load | (b) is independent of load [GATE-1996] |
| (c) Decreases with increase in load | (d) may increase or decrease with increase in load |

GATE-10Ans. (c)

Hydrostatic Step Bearing 464

GATE-11. Starting friction is low in

[GATE-1992]

- | | |
|---------------------------------------|------------------------------|
| (a) Hydrostatic lubrication | (b) Hydrodynamic lubrication |
| (c) Mixed (or semi-fluid) lubrication | (d) Boundary lubrication |

GATE-11Ans. (a)

Previous 20-Years IES Questions

IES-1. Consider the following statements about antifriction bearings: [IES-2008]

1. Their location influences the lateral critical speed of a rotor.
2. Roller bearings are antifriction bearings.

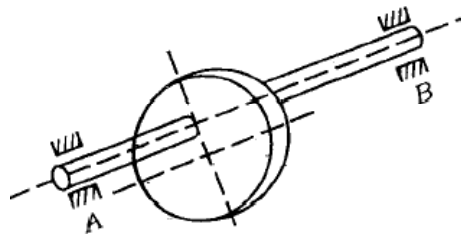
Which of the statements given above is/are correct?

- | | |
|------------------|---------------------|
| (a) 1 only | (b) 2 only |
| (c) Both 1 and 2 | (d) Neither 1 nor 2 |

IES-1Ans. (c)

IES-2. A circular disc having a mass of 30 kg is mounted asymmetrically between two bearings A and B as shown above in the figure. It is used as an eccentric cam with an eccentricity of 0.01 m. If the shaking force on each of the bearings is not to exceed 1500 N, the speed of rotation of the cam should not exceed

- | | |
|----------------|---------------|
| (a) 10 rad/s | (b) 100 rad/s |
| (c) 70.7 rad/s | (d) 140 rad/s |



[IES-2003]

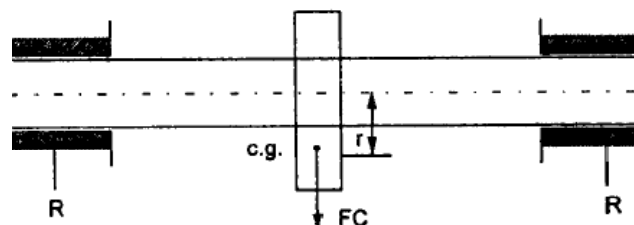
IES-2 Ans. (b)

$$R_{\max} = 1500 \text{ N}$$

$$F_c = 2 \times R = 2 \times 1500 \text{ N}$$

$$m\omega^2 r = 2 \times 1500$$

$$\text{or } \omega = \left(\frac{2 \times 1500}{30 \times 0.01} \right)^{1/2} = 100 \text{ rad / sec}$$



Types of Rolling Contact Bearings

IES-3. In three ball bearing identified as [IES-2008]
SKF 2015, 3115 and 4215

- (a) Bore is common but width is increasing
- (b) Outer diameter is common but bore is increasing
- (c) Width is common but outer diameter is decreasing
- (d) Bore is common but outer diameter is decreasing

IES-3Ans. (a) According to ISO plan for dimension series bearings are provided with two digit numbers. The first number indicates the width series 8, 0, 1, 2, 3, 4, 5 and 6 in order of increasing width. The second number indicate diameter series 7, 8, 9, 0, 1, 2, 3, and 4 in order of ascending outer diameter of bearing. Thus bearing number SKF 2015, 3115 and 4215 shows bearings belonging to different series with 75 mm bore diameter but width is increasing.

SKF 2015, 3115 and 4215 shows width is increasing ascending outer diameter of bearing same bore diameter 75 mm. (i.e. 15×5)

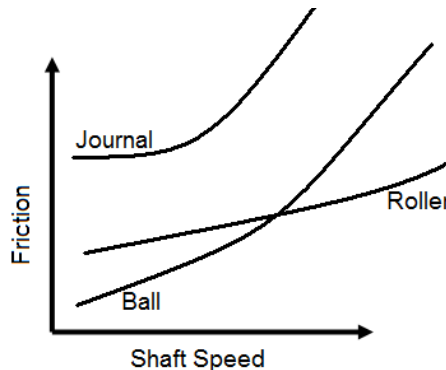
IES-4. Average values of effective coefficients of friction for bearings are described below: [IES 2007]

- | | |
|-----------------------------------|--------------------------------------------------------|
| 1. Spherical ball bearing - f_1 | 2. Cylindrical roller bearing - f_2 |
| 3. Taper roller bearing - f_3 | 4. Stable (thick film) Sliding contact bearing - f_4 |

Which one of the following sequences is correct?

- (a) $f_1 < f_2 < f_3 < f_4$
- (b) $f_1 < f_2 < f_4 < f_3$
- (c) $f_2 < f_1 < f_3 < f_4$
- (d) $f_1 < f_4 < f_2 < f_3$

IES-4Ans. (a)



IES-5. The rating life of a group of apparently identical ball bearings is defined as the number of revolutions or exceeded before the first evidence of fatigue crack by: [IES-2005]

- (a) 100% of the bearings of the group
- (b) 95% of the bearings of the group
- (c) 90% of the bearings of the group
- (d) 66.66% of the bearings of the group

IES-5Ans. (c)

IES-6. Match List I (Type of Bearings) with List II (Type of Load) and select the correct answer using the code given below the Lists: [IES-2005]

- | List I | List II |
|---------------------------|-----------------------------------------------|
| A Deep groove bearing | 1. Radial load |
| B. Tapered roller bearing | 2. Radial and axial load |
| C. Self aligning being | 3. Mainly radial load with shaft misalignment |
| D. Thrust bearing | 4. Mainly axial load |

Design of Bearings

S K Mondal's

Chapter 4

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	3	4	1	2
(c)	1	4	3	2	(d)	3	2	1	4

IES-6Ans. (a)

IES-7. Which one of the following statements is correct?

[IES-2004]

Antifriction bearings are

- | | |
|------------------------------|-----------------------------|
| (a) Sleeve bearings | (b) gas lubricated bearings |
| (c) Ball and roller bearings | (d) journal bearings |

IES-7Ans. (c)

IES-8. The rolling element bearings are

[IES-2003]

- | | |
|---------------------------|---------------------------------|
| (a) Hydrostatic bearings | (b) Squeeze film bearings |
| (c) Antifriction bearings | (d) Grease lubrication bearings |

IES-8Ans. (c)

IES-9. A ball-bearing is characterized by basic static capacity = 11000 N and dynamic capacity = 18000 N. This bearing is subjected to equivalent static load = 5500 N. The bearing loading ratio and life in million revolutions respectively are

[IES-2001]

- (a) 3.27 and 52.0 (b) 3.27 and 35.0 (c) 2.00 and 10.1 (d) 1.60 and 4.1

IES-9Ans. (b) $\text{Loading ratio} = \frac{C}{P} = \frac{18000}{5500} = 3.27$

Life (million revolutions)

$$= \left(\frac{C}{P} \right)^3 = \left(\frac{18000}{5500} \right)^3 = 35$$

IES-10. On what does the basic static capacity of a ball bearing depends?

- (a) Directly proportional to number of balls in a row and diameter of ball **[IES-2009]**
 (b) Directly proportional to square of ball diameter and inverse of number of rows of balls
 (c) Directly proportional to number of balls in a row and square of diameter of ball
 (d) Inversely proportional to square of diameter of ball and directly proportional to number of balls in a row

IES-10Ans. (c)

IES-11. Ball bearings are provided with a cage

[IES-1992]

- (a) To reduce friction
 (b) To maintain the balls at a fixed distance apart
 (c) To prevent the lubricant from flowing out
 (d) To facilitate slipping of balls

IES-11Ans. (b)

IES-12. In a single row deep groove ball-bearing, cages are needed to

[IES-1999]

- (a) Separate the two races
 (b) Separate the balls from the inner race
 (c) Separate the outer race from the balls
 (d) Ensure that the balls do not cluster at one point and maintain proper relative angular positions.

IES-12Ans. (d)

IES-13. Which one of the following statements is NOT true of rolling contact bearing? [IES-1997]

- (a) The bearing characteristic number is given by ZN/p where Z is the absolute viscosity of the lubricant, N is the shaft speed and p is the bearing pressure.
- (b) Inner race of a radial ball bearing has an interference fit with the shaft and rotates along with it
- (c) Outer race of the bearing has an interference fit with bearing housing and does not rotate
- (d) In some cases, the inner race is stationary and outer race rotates

IES-13Ans. (d)

IES-14. Assertion (A): It is desirable to increase the length of arc over which the oil film has to be maintained in a journal bearing. **[IES-1996]**

Reason (R): The oil pressure becomes negative in the divergent part and the partial vacuum created will cause air to leak in from the ends of bearing.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-14Ans. (a)

IES-15. Consider the following statements about anti-friction bearings: [IES-1994]

1. They have low starting and low running friction at moderate speeds.
2. They have high resistance to shock loading.
3. They can carry both radial and thrust loads.
4. Their initial cost is high.
5. They can accommodate some amount of misalignments of shaft.

Of these statements

- (a) 1, 2, 3 and 4 are correct
- (b) 1, 3 and 4 are correct
- (c) 1, 4 and 5 are correct
- (d) 1, 2, 3 and 5 are correct.

IES-15Ans. (a) Self aligning bearing can accommodate some amount of misalignments of shaft.

IES-16. Removal of metal particles from the raceway of a rolling contact bearing is a kind of failure of bearing known as [IES-1995]

- (a) Pitting
- (b) wearing
- (c) spalling
- (d) scuffing

IES-16Ans. (a)

Load-life Relationship

IES-17. The rated life of a ball bearing varies inversely as which one of the following? [GATE-1993; IES-2004]

- (a) Load
- (b) (load)²
- (c) (load)³
- (d) (load)^{3.33}

IES-17 Ans. (c) $L = \left(\frac{d}{R}\right)^p$, d = dynamic load capacity

R = Equivalent bearing load

$p = 3$ for ball bearing

$= \frac{10}{3}$ for roller bearing.

IES-18. If the load on a ball bearing is halved, its life: [IES-2005]

- (a) Remains unchanged
- (b) Increases two times

(c) Increases four times

(d) Increases eight times

IES-18Ans. (d) $L = \left(\frac{d}{R}\right)^3$ d is dynamic load carrying capacity. R is actual load applied if R halved L will increased by $2^3 = 8$ times

Selection of Taper Roller Bearings

IES-19. Assertion (A): Tapered roller bearings are sensitive to the tightening between inner and outer races. **[IES-2002]**

Reason (R): Tapered roller bearings are always provided with adjusting nut for tightening.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-19Ans. (b)

IES-20. Which bearing is preferred for oscillating conditions? **[IES-1992]**

- (a) Double row roller bearing
- (b) Angular contact single row ball bearing
- (c) Taper roller bearing
- (d) Needle roller bearing

IES-20Ans. (d)

IES-21. Match List-I (Bearings) with List-II (Applications) and select the correct answer using the codes given below the lists: **[IES-2001]**

List I

- A. Cylindrical roller
- B. Ball-bearing
- C. Taper rolling bearing
- D. Angular contact ball-bearing

List II

- 1. Radial loads
- 2. Machine needs frequent dismantling and assembling
- 3. Radial loads with lesser thrust
- 4. Shock loads
- 5. Axial expansion of shaft due to rise in temperature

	A	B	C	D
(a)	4	3	1	5
(c)	4	1	2	3

	A	B	C	D
(b)	1	3	2	5
(d)	5	4	1	3

IES-21Ans. (c)

IES-22. Match List-I with List-II and select the correct answer using the codes given below the lists: **[IES-1998]**

List I

- A. End thrust
- B. No cage
- C. More accurate centering
- D. Can be overloaded

List II

- 1. Plain bearing
- 2. Ball bearing
- 3. Needle bearing
- 4. Tapered roller bearing

Code:	A	B	C	D
(a)	3	4	2	1
(c)	3	4	1	2

	A	B	C	D
(b)	4	3	1	2
(d)	4	3	2	1

IES-22 Ans. (d)

IES-23. Match List-I with List-II and select the correct answer using the codes given below the Lists: **[IES-1997]**

List-I

List-II

Design of Bearings

S K Mondal's

Chapter 4

(Bearing)					(Purpose)				
A. Ball bearing					1. Heavy loads with oscillatory motion				
B. Tapered Roller bearings					2. Light loads				
C. Spherical Roller bearings					3. Carrying both radial and thrust loads				
D. Needle Roller bearings					4. Self-aligning property				
Codes:	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	2	1	4	3
(c)	2	3	1	4	(d)	2	3	4	1

IES-23 Ans. (d)

IES-24. Tapered roller bearings can take [IES-1996]

- (a) Radial load only
- (b) Axial load only
- (c) Both radial and axial loads and the ratio of these being less than unity.
- (d) Both radial and axial loads and the ratio of these being greater than unity.

IES-24Ans. (d)

IES-25. In a collar thrust bearing, the number of collars has been doubled while maintaining coefficient of friction and axial thrust same. It will result in

- (a) Same friction torque and same bearing pressure [IES-2002]
- (b) Double friction torque and half bearing pressure
- (c) Double friction torque and same bearing pressure
- (d) Same friction torque and half bearing pressure

IES-25 Ans. (d)

Sliding Contact Bearings

IES-26. Which of the following are included in the finishing operations for porous bearing? [IES-2005]

- 1. Infiltration 2. Sizing 3. Heat treatment 4. Coining**

Select the correct answer using the code given below:

- (a) 1 and 2 (b) 2 and 3 (c) 2 and 4 (d) 1 and 4

IES-26Ans. (a)

Basic Modes of Lubrication

IES-27. In sliding contact bearings, a positive pressure can be built up and a load supported by a fluid only by the use of a: [IES-2005]

- (a) Diverging film
- (b) Converging-diverging film
- (c) Converging film
- (d) Flat film

IES-27Ans. (c)

IES-28. Which one of the following is correct? [IES-2008]

A hydrodynamic slider bearing develops load bearing capacity mainly because of

- (a) Slider velocity
- (b) wedge shaped oil film
- (c) Oil compressibility
- (d) oil viscosity

IES-28Ans. (b) A hydrodynamic slider bearing develops load bearing capacity mainly because of wedge shaped oil film.

IES-29. Assertion (A): In steady rotating condition the journal inside a hydrodynamic journal bearing remains floating on the oil film. [IES-2008]

Reason (R): The hydrodynamic pressure developed in steady rotating conditions in journal bearings balances the load on the journal.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-29Ans. (a) The film pressure created by the moving surface itself pulling the lubricant into a wedge shaped zone at a velocity sufficiently high to create the necessary pressure required to separate the surface against the load on the bearings. Hydrodynamic lubrication is also called as full film lubrication or fluid lubrication. So Assertion and Reason both are correct and A is the correct explanation of R.

IES-30. Increase in values of which of the following results in an increase of the coefficient of friction in a hydrodynamic bearing? [IES 2007]

1. Viscosity of the oil.
2. Clearance between shaft and bearing.
3. Shaft speed.

Select the correct answer using the code given below:

- (a) 1 and 2 only
- (b) 1 and 3 only
- (c) 2 and 3 only
- (d) 1, 2 and 3

IES-30Ans. (b) 2 is false Petroff's law says

$$\text{Co efficient of friction (f)} = 2\pi \frac{\mu N_s}{P} \times \left(\frac{r}{c}\right)$$

f ↑ if (i) μ ↑ ; (ii) c ↓ ; (iii) N_s ↑

IES-31. A journal bearing with hydrodynamic lubrication is running steadily with a certain amount of minimum film thickness. When the load and speed are doubled, how does the minimum film thickness vary? [IES-2008]

- (a) Remains unchanged
- (b) Gets doubled
- (c) Gets reduced to one-fourth of original value
- (d) Gets reduced to half of original value

IES-32Ans. (a) When the load and speed is doubled, the minimum film thickness remains

unchanged. Since, $S = \left(\frac{\mu N}{p}\right) \left(\frac{r}{c}\right)^2$

Since S remains the same even after doubling the speed as well as load and film Thickness depends on the Sommerfeld number.

IES-33. What is the main advantage of hydrodynamic bearing over roller bearing? [IES-2005]

- (a) Easy to assemble
- (b) Relatively low price
- (c) Superior load carrying capacity at higher speeds
- (d) Less frictional resistance

IES-33Ans. (c)

IES-34. Consider the following statements: [IES-1993; 2002; 2006]
Radius of friction circle for a journal bearing depends upon

1. Coefficient of friction
2. Radius of the journal
3. Angular speed of rotation of the shaft

Which of the statements given above are correct?

- (a) 1, 2 and 3
- (b) Only 1 and 2
- (c) Only 2 and 3
- (d) Only 1 and 3

Design of Bearings

S K Mondal's

Chapter 4

IES-34Ans. (b) radius of friction circle = $f \times r$

IES-35. In a journal bearings, the radius of the friction circle increases with the increase in [IES-1997]

- (a) Load
- (b) Radius of the journal
- (c) Speed of the journal
- (d) Viscosity of the lubricant

IES-35Ans. (b)

IES-36. Consider the following statements: [IES 2007]

For a journal rotating in a bearing under film lubrication conditions, the frictional resistance is

1. Proportional to the area of contact
2. Proportional to the viscosity of lubricant
3. Proportional to the speed of rotation
4. Independent of the pressure

Which of the statements given above are correct?

- (a) 1, 2, 3 and 4
- (b) 1 and 4 only
- (c) 2, 3 and 4 only
- (d) 2 and 3 only

IES-36 Ans. (a) Viscous resistance (F) = $T \times \text{Area} = \frac{\mu \pi D N}{60t} \times \pi D L = \frac{\mu \pi^2 D^2 N L}{60t}$

IES-37. The bearing characteristic number in a hydrodynamic bearing depends on

- (a) Length, width and load
- (b) length, width and speed. [IES-1996]
- (c) Viscosity, speed and load
- (d) viscosity, speed and bearing pressure.

IES-37Ans. (d)

IES-38. It is seen from the curve that there is a minimum value of the coefficient of friction (μ) for a particular value of the Bearing Characteristic Number denoted by α . What is this value of the Bearing Characteristic Number called? [IES-2004]

- (a) McKee Number
- (b) Reynolds Number
- (c) Bearing Modulus
- (d) Somerfield Number

IES-38Ans. (c)

IES-39. **Assertion (A):** In equilibrium position, the journal inside a journal bearing remains floating on the oil film. [IES-1995]

Reason (R): In a journal bearing, the load on the bearing is perpendicular to the axis of the journal.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-39Ans. (b) Both A and R are true but R is not correct explanation for A.

IES-40.A full journal bearing having clearance to radius ratio of 1/100, using a lubricant with $\mu = 28 \times 10^{-3}$ Pas supports the shaft journal running at $N = 2400$ r.p.m. If bearing pressure is 1.4 MPa, the Somerfield number is [IES-2001]

- (a) 8×10^{-3}
- (b) 8×10^{-5}
- (c) 0.48
- (d) 0.48×10

IES-40Ans. (a) $s = \frac{\mu N_s}{p} \left(\frac{r}{c} \right)^2$

Design of Bearings

S K Mondal's

Chapter 4

IES-41. A sliding contact bearing is operating under stable condition. The pressure developed in oil film is p when the journal rotates at N r.p.m. The dynamic viscosity of lubricant is μ and effective coefficient of friction between bearing and journal of diameter D is f . Which one of the following statements is correct for the bearing? [IES-2001]

- (a) f is directly proportional to μ and p
- (b) f is directly proportional to μ and N
- (c) f is inversely proportional to p and f
- (d) f is directly proportional to μ and inversely proportional to N

IES-41Ans. (b) Petroff's law $f = 2\pi^2 \frac{\mu n_s}{P} \left(\frac{r}{c} \right) + k \rightarrow 0.002$

IES-42. Which one of the following sets of parameters should be monitored for determining safe operation of journal bearing? [IES-2000]

- (a) Oil pressure, bearing metal temperature and bearing vibration
- (b) Bearing vibration, oil pressure and speed of shaft
- (c) Bearing metal temperature and oil pressure
- (d) Oil pressure and bearing vibration

IES-42Ans. (a)

IES-43. Consider the following pairs of types of bearings and applications:

- 1. Partial Journal bearing..... Rail wagon axles [IES-2000]
- 2. Full journal bearing Diesel engine crank-shaft
- 3. Radial bearing Combined radial and axial loads

Which of these pairs is/are correctly matched?

- (a) 1 alone
- (b) 1 and 2
- (c) 2 and 3
- (d) 1, 2 and 3

IES-43Ans. (b)

IES-44. Match List I with List II and select the correct answer using the code given below the lists: [IES-1995]

List I (Requirement)

- A. High temperature service
- B. High load
- C. No lubrication
- D. Bushings

List II (Type)

- 1. Teflon bearing.
- 2. Carbon bearing
- 3. Hydrodynamic bearing
- 4. Sleeve bearing

Codes: A	B	C	D	A	B	C	D
(a) 1	2	3	4	(b) 4	1	2	3
(c) 2	1	3	4	(d) 2	3	1	4

IES-44Ans. (d)

IES-45. **Assertion (A):** In anti-friction bearings, the frictional resistance is very low as the shaft held by it remains in floating condition by the hydrodynamic pressure developed by the lubricant. [IES-2006]

Reason (R): In hydrodynamic journal bearings, hydrodynamic pressure is developed because of flow of lubricant in a converging-diverging channel

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-45Ans. (d)

IES-46. Satisfactory hydrodynamic film in a journal bearing is formed when

Design of Bearings

S K Mondal's

Chapter 4

- (a) Journal speed is low, unit pressure on the bearing is high and viscosity of lubricant used is low [IES-2006]
 (b) Journal speed is low, unit pressure on the bearing is low and viscosity of lubricant used is low
 (c) Journal speed is high, unit pressure on the bearing is high and viscosity of lubricant used is high
 (d) Appropriate combination of journal speed, unit pressure on bearing and lubricant viscosity exists resulting in low coefficient of friction

IES-46Ans. (c)

IES-47. In an oil-lubricated journal bearing, coefficient of friction between the journal and the bearing. [IES-1995]

- (a) Remains constant at all speeds.
 (b) is minimum at zero speed and increases monotonically with increase in speed.
 (c) is maximum at zero speed and decreases monotonically with increase in speed.
 (d) becomes minimum at an optimum speed and then increases with further increase in speed.

IES-47Ans. (d)

IES-48. Match List I with List II and select the correct answer: [IES-2002]

List I (Bearings)

A. Hydrodynamic Journal bearing

B. Rectangular Hydrostatic bearing

C. Taper Roller bearing

D. Angular contact ball bearing thrust combined

List II (Load type)

1. High radial and thrust load combined

2. Radial load only

3. Thrust load only

4. Medium to low radial and

	A	B	C	D
(a)	2	3	1	4
(c)	2	1	3	4

	A	B	C	D
(b)	4	1	3	2
(d)	4	3	1	2

IES-48 Ans. (a)

IES-49. Assertion (A): Oil as a cutting fluid result in a lower coefficient of friction.

Reason (R): Oil forms a thin liquid film between the tool face and chip, and it provides 'hydrodynamic lubrication'. [IES-2000]

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-49Ans. (c) Oil forms a thin liquid film between the tool face and chip, and it provides 'boundary lubrication'

IES-50. Which one of the following pair is correctly matched? [IES-2000]

- (a) Beauchamp towerFirst experiments on journal bearings
 (b) Osborne ReynoldsAntifriction bearings
 (c) Somerfield number.....Pivot and Collar bearings
 (d) Ball bearings.....Hydrodynamic lubrication

IES-50Ans. (a)

IES-51. Match List-I (Type of Anti-friction bearing) with List-II (Specific Use) and select the correct answer using the code given below the Lists: [IES-2006]

List-I

List-II

A. Self-aligning ball bearing

1. For pure axial load

- B. Taper roller bearing**
C. Deep groove ball bearing
D. Thrust ball bearing

A	B	C	D
(a) 2	1	3	4
(c) 2	4	3	1

- 2. For hinged condition**
3. For pure radial load
4. For axial and radial load

A	B	C	D
(b) 3	4	2	1
(d) 3	1	2	4

IES-51Ans. (c)

IES-52. Which one of the following types of bearings is employed in shafts of gearboxes of automobiles? [IES-1999]

- (a) Hydrodynamic journal bearings (b) Multi-lobed journal bearings
 (c) Antifriction bearings (d) Hybrid journal bearings

IES-52Ans. (c)

IES-53. Assertion (A): In hydrodynamic journal bearings, the rotating journal is held in floating condition by the hydrodynamic pressure developed in the lubricant.

Reason (R): Lubricant flows in a converging-diverging channel. [IES-1994]

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-53Ans. (a) Both A and R are true and R provides correct explanation for A

Hydrostatic Step Bearing 464

IES-54. Assertion (A): Hydrostatic lubrication is more advantageous when compared to hydrodynamic lubrication during starting and stopping the journal in its bearing.

Reason (R): In hydrodynamic lubrication, the fluid film pressure is generated by the rotation of the journal. [IES-1998]

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-54Ans. (b)

Previous 20-Years IAS Questions

Types of Rolling Contact Bearings

IAS-1. Deep groove ball bearings are used for

[IAS-1995]

- (a) Heavy thrust load only
 (b) Small angular displacement of shafts
 (c) Radial load at high speed
 (d) Combined thrust and radial loads at high speed.

IAS-1Ans. (d) Deep groove ball bearings are primarily designed to support radial loads at high speeds. However, this type of construction permits the bearing also to support relatively high thrust loads in either direction.

Load-life Relationship

IAS-2. If $k = 3$ for ball bearings and $k = 3.33$ for roller bearings, which one of the following correctly states the load (P) - Life (L) relationship for rolling contact bearings? [IAS-2004]

- $$(a) \frac{L_1}{L_2} = \left(\frac{P_1}{P_2} \right)^k$$

$$(c) \frac{L_2}{L_1} = \left(\frac{P_1}{P_2} \right)^k$$

$$(b) \frac{L_2}{L_1} = \left(\frac{P_1}{P_2} \right)^{\frac{1}{(k-1)}}$$

$$(d) \frac{L_2}{L_1} = \left(\frac{P_1}{P_2} \right)^{k-1}$$

IAS-2Ans. (c)

$$L = \left(\frac{d}{R} \right)^K \quad [d = \text{dynamic load carrying capacity and } R = \text{Equivalent load}]$$

$$\therefore L \propto \frac{1}{R^K} \quad \therefore \frac{L_2}{L_1} = \left(\frac{R_1}{R_2} \right)^K$$

Basic Modes of Lubrication

IAS-3. In a journal bearing P = average bearing pressure, Z = absolute viscosity of the lubricant, N = rotational speed of the journal. The bearing characteristic number is given by [IAS-1997]

- (a) ZN/p (b) p/ZN (c) Z/pN (d) N/Zp

IAS-3Ans. (a)

IAS-4. Match List-I (Applications) with List-II (Choice of Bearings) and select the correct answer using the codes given below the lists: [IAS-2004]

List - I

(Applications)

A. Granite table of a coordinate

B. Headstock spindle of a lathe

C. Crank shaft of a diesel engine

D. Armature of 0.5 kW induction motor

Codes:

(a) 1 4 3 2

(c) 1 2 3 4

List - II

(Choice of Bearings)

1. Hydrodynamic bearing measuring machine

2. Deep groove ball bearing

3. Hydrostatic bearing

4. Taper roller bearing

A B C D

(b) 3 2 1 4

(d) 3 4 1 2

IAS-4Ans. (a)

IAS-5. In a hydrodynamic journal bearing, there is [IAS-2001]

- (a) A very thin film of lubricant between the journal and the bearing such that there is contact between the journal and the bearing
- (b) A thick film of lubricant between the journal and the bearing
- (c) No lubricant between the journal and the bearing
- (d) A forced lubricant between the journal and the bearing

IAS-5 Ans. (b)

IAS-6. Which one of the following is the lubricator regime during normal operation of a rolling element bearing? [IAS-2000]

- (a) Hydrodynamic lubrication (b) Hydrostatic lubrication

Design of Bearings

S K Mondal's

Chapter 4

(c) Elasto-hydrodynamic lubrication

(d) Boundary lubrication

IAS-6 Ans. (c) There is elastic deformation of the contacting surfaces as surfaces are not sufficiently rigid. Here fluid film pressure is also high.

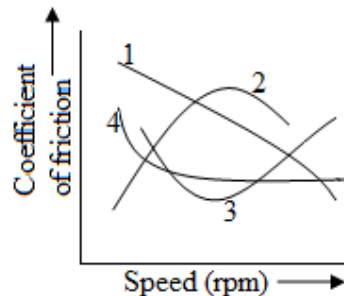
IAS-7. A journal bearing of diameter 25 cm and length 40 cm carries a load of 150 kN. The average bearing pressure is [IAS-1997]

- (a) 1.5 kN/cm² (b) 15 kN/cm² (c) 150 kN/cm² (d) none of the above

IAS-7Ans. (d) The average bearing pressure = $\frac{\text{load}}{\text{projected area}} = \frac{150}{25 \times 40} = 0.15 \text{ kN/cm}^2$

IAS-8. Which one of the curves shown below represents the characteristic of a hydrodynamically lubricated journal bearing?

- (a) 1
(b) 2
(c) 3
(d) 4



[IAS-1998]

IAS-8Ans. (c)

IAS-9. Consider the following statements: [IAS-1996]

For a proper hydrodynamic lubrication for a given journal bearing.

1. the higher the viscosity, the lower the rotating speed needed to float the journal at a given load.
2. The higher the rotating speed, the higher the bearing load needed to float the journal at a given viscosity.
3. the higher the bearing load, the higher the viscosity needed to float the journal at a given speed.

Of these statements:

- (a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 and 3 are correct (d) 1 and 3 are correct

IAS-9Ans. (a)

IAS-10. Assertion (A): An important feature of film lubrication is that once a lubricant film is formed on the mating surfaces by running the bearing with a lubricant having a high degree of oiliness, it is possible to change to a lubricant with a much lower oiliness. [IAS-1999]

Reason (R) Lubricants of high oiliness are liable to decompose or oxidize and hence are not suitable for general lubrication purposes.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-10Ans. (a)

IAS-11. Thrust bearings of the sliding type are often provided with multiple sector-shaped bearing pads of the tilting type instead of a continuous angular bearing surface in order to [IAS 1994]

- (a) Distribute the thrust load more non-uniformly
(b) Provide limited adjustments to shaft misalignments

- (c) Enable the formation of a wedge shaped oil film
 (d) Enable lubricating oil to come into contact with the total bearing area

IAS-11 Ans. (c)

Hydrostatic Step Bearing 464

- IAS-12. The most suitable bearing for carrying very heavy loads with slow speed is
 (a) Hydrodynamic bearing (b) ball bearing [IAS 1994]
 (c) Roller bearing (d) hydrostatic bearing

IAS-12 Ans. (d)

Comparison of Rolling and Sliding Contact bearings

- IAS-13. Match List -I (Bearings) with List-II (Applications) and select the correct answer using the codes given below the lists: [IAS-1998]

List -I

- A. Journal bearing**
B. Thrust bearing
C. Conical pivot bearing
D. Ball bearing

Codes:	A	B	C	D
(a)	3	4	1	2
(c)	3	4	2	1

List-II

- 1. Electric motors**
2. Watches
3. Marine engines
4. Swivelling chairs

	A	B	C	D
(b)	4	2	1	3
(d)	4	2	3	1

IAS-13Ans. (c)



5. Fluctuating Load

Consideration for Design

Objective Questions (IES, IAS, GATE)

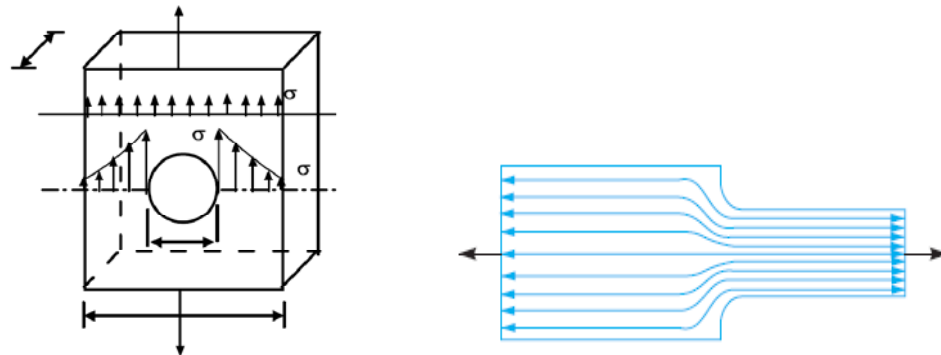
Stress Concentration

Origin of stress concentration

Machine members often have regions in which the state of stress is significantly greater than theoretical predictions as a result of:

1. Geometric discontinuities or stress raisers such as holes, notches, and fillets;
2. Internal microscopic irregularities (non-homogeneities) of the material created by such manufacturing processes as casting and molding;
3. Surface irregularities such as cracks and marks created by machining operations.

These stress concentrations are highly localized effects which are functions of geometry and loading. In this tutorial, we will examine the standard method of accounting for stress concentrations caused by geometric features. Specifically, we will discuss the application of a theoretical or geometric stress-concentration factor for determination of the true state of stress in the vicinity of stress raisers.



Stress concentration due to a central hole in a plate subjected to a uni-axial loading.

Since the designer, in general, is more interested in knowing the maximum stress rather than the actual stress distribution, a simple relationship between the σ_{\max} and σ_{ave} in terms of geometric parameters will be of practical importance.

Many experiments were conducted on samples with various discontinuities and the relationship between the *stress concentration factor* and the geometrical parameters are established, where

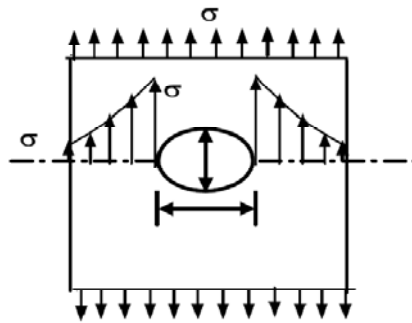
$$\text{Stress concentration factor, } K_t = \frac{\sigma_{\max}}{\sigma_{ave}}$$

It is possible to predict the stress concentration factors for certain geometric shapes using theory of elasticity approach. For example, for an elliptical hole in an infinite plate, subjected to a uniform tensile stress σ_1 (**figure below**), stress distribution around the discontinuity is disturbed and at points remote from the discontinuity the effect is insignificant. According to such an analysis

$$\sigma_3 = \sigma_1 \left(1 + \frac{2b}{a} \right)$$

If $a = b$ the hole reduces to a circular one and therefore $\sigma_3 = 3\sigma_1$ which gives $k_t = 3$.

If, however 'b' is large compared to 'a' then the stress at the edge of transverse crack is very large and consequently k is also very large. If 'b' is small compared to a then the stress at the edge of a longitudinal crack does not rise and $k_t = 1$.



Stress concentration due to a central elliptical hole in a plate subjected to a uni-axial loading.

In design under fatigue loading, stress concentration factor is used in modifying the values of endurance limit while in design under static loading it simply acts as stress modifier. This means Actual stress = $k_t \times$ calculated stress.

For ductile materials under static loading effect of stress concentration is not very serious but for brittle materials even for static loading it is important.

Application to Ductile and Brittle Materials for Static Loading

Ductile Materials

While stress concentration must be considered for fatigue and impact loading of most materials, stress-concentration factors are seldom applied to ductile materials under *static* loading. This design practice is justified by four points:

1. Areas of high stress caused by stress concentrations are highly localized and will not dictate the performance of the part. Rather, it is assumed that the stress state in the cross section as a whole is below the general yield condition;
2. If the magnitude of the loading is large enough to cause yielding due to the stress concentration, the localized area will plastically deform immediately upon loading;

Fluctuating Load Consideration for Design

S K Mondal's
Chapter 5

3. Ductile materials typically work-harden (strain-strengthen) on yielding, resulting in a localized increase in material strength;
4. The static load is never cycled.

It is important to note, that even though the stress-concentration factor is not usually applied to estimate the stresses at a stress raiser in a ductile material, the higher state of stress does in fact exist.

Ductile Material Practice: $\sigma_{\max} = \sigma_0$

Brittle Materials

Stress-concentration factors are always required for brittle materials, regardless of the loading conditions, since brittle failure results in fracture. This type of failure is characteristic of brittle materials which do not exhibit a yielding or plastic range. As a consequence of brittle fracture, the part breaks into two or more pieces having no load carrying capability. To avoid such catastrophic failure, the design practice is to always use a stress-concentration factor for brittle materials to ensure that the state of stress is accurately represented.

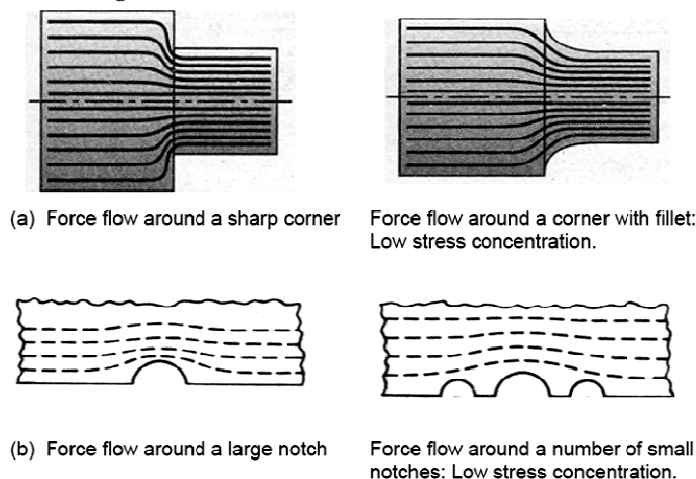
Brittle Material Practice: $\sigma_{\max} = K_t \sigma_0$

Methods of reducing stress concentration

A number of methods are available to reduce stress concentration in machine parts. Some of them are as follows:

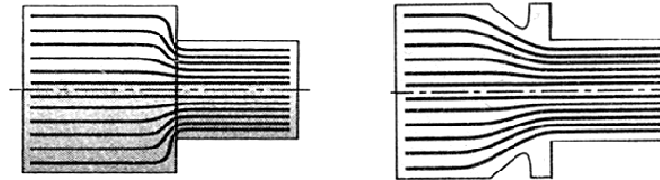
1. Provide a fillet radius so that the cross-section may change gradually.
2. Sometimes an elliptical fillet is also used.
3. If a notch is unavoidable it is better to provide a number of small notches rather than a long one. This reduces the stress concentration to a large extent.
4. If a projection is unavoidable from design considerations it is preferable to provide a narrow notch than a wide notch.
5. Stress relieving groove are sometimes provided.

These are demonstrated in figure below.





(c) Force flow around a wide projection Force flow around a narrow projection:
Low stress concentration.



(d) Force flow around a sudden change in diameter in a shaft Force flow around a stress relieving groove.

Fluctuating Stresses

Types of fluctuating stress

1. The stresses which vary from a minimum value to a maximum value of the same nature, (i.e. tensile or compressive) are called fluctuating stresses.
2. The stresses which vary from zero to a certain maximum value are called repeated stresses.
3. The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called alternating stresses.

Cyclic Stressing

As the name implies, the induced stresses vary in some pattern with time. This can be due to variation in the applied load itself or because of the conditions of use as seen earlier. Let us assume that the pattern of such a variation is sinusoidal. Then the following are the basic terminology associated with variable stresses. The definitions included here are elementary. They are introduced for clarity and convenience.

Maximum stress: (σ_{\max})

The largest or highest algebraic value of a stress in a stress cycle. Positive for tension

Minimum stress: (σ_{\min})

The smallest or lowest algebraic value of a stress in a stress cycle. Positive for tension.

Nominal stress: (σ_{nom})

As obtained or calculated from simple theory in tension, bending and torsion neglecting geometric discontinuities

Fluctuating Load Consideration for Design

S K Mondal's
Chapter 5

$$\sigma_{nom} = F / A \text{ or } M / Z \text{ or } T.r / J$$

$$\text{Hence } \sigma_{max} = F_{max} / A \text{ or } M_{max} / Z \text{ or } T_{max}r / J_p$$

$$\sigma_{min} = F_{min} / A$$

Mean stress (Mid range stress): (σ_m) The algebraic mean or average of the Maximum and minimum stress in one cycle.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress range: (σ_r) the algebraic difference between the maximum and minimum stress in one cycle.

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

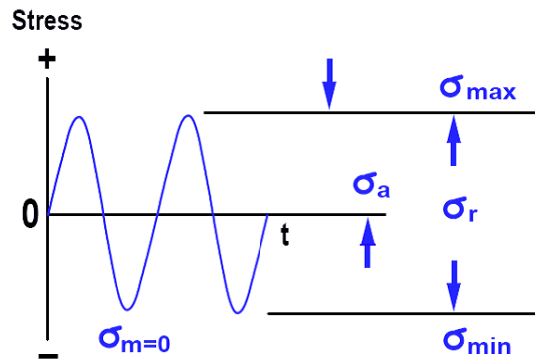
Stress Amplitude: (σ_a) Half the value of the algebraic difference between the maximum and minimum stress in one cycle or half the value of the stress range.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_r}{2}$$

Types of Variations

(a) (Completely) reversible stressing:

Stress variation is such that the mean stress is zero; same magnitude of maximum and minimum stress, one in tension and the other in compression. Now for Completely reversible loading $\sigma_m = \sigma_{max} = \sigma_{min}$; $R = -1$ and $A = 0$



(a) Fully reversed

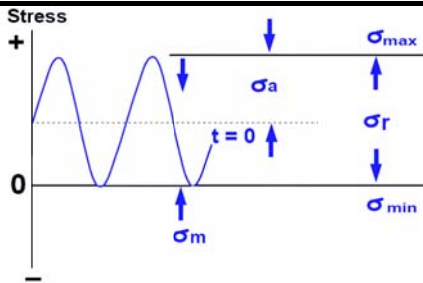
(b) Repeated stressing:

Stress variation is such that the minimum stress is zero. Mean and amplitude stress have the same value for repeated loading

$$\sigma_{min} = 0$$

$$\sigma = \sigma_a = \sigma_{max}/2$$

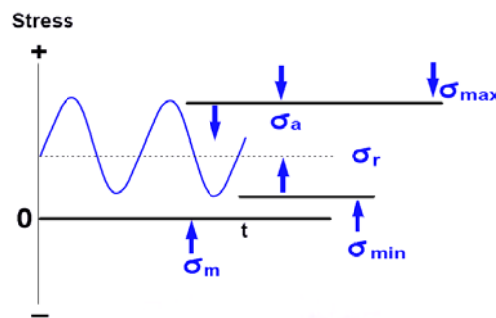
$$R = 0 \text{ and } A = 1$$



(b) Repeated

(c) Fluctuating stressing:

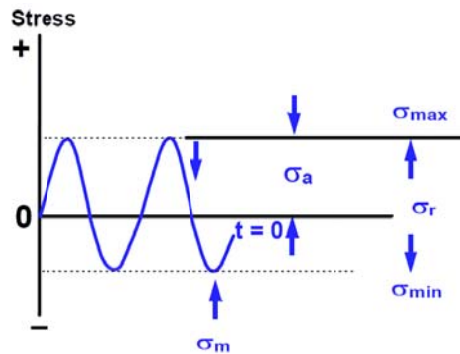
Both minimum and maximum stresses are positive and mean stress also being positive (tensile)



(b) Flutuating

(d) Alternating stressing:

Positive maximum stress and negative minimum stress; mean stress is generally positive but can also be negative.



Endurance Limit

Endurance or Fatigue Limit

In the case of the steels, a knee (flattening or saturation) occurs in the graph, and beyond this knee failure will not occur, no matter how large the numbers of cycles are. The strength (stress amplitude value) corresponding to the knee is called the endurance limit (S_e) or the fatigue limit. However the graph never does become horizontal for non-ferrous metals and alloys, hence these materials do not have an endurance limit.

Endurance or Fatigue limit - definition

Fluctuating Load Consideration for Design

S K Mondal's Chapter 5

Endurance or fatigue limit can be defined as the magnitude of stress amplitude value at or below which no fatigue failure will occur, no matter how large the number of stress reversals are, in other words leading to an infinite life to the component or part being stressed. For most ferrous materials Endurance limit (S_e) is set as the cyclic stress level that the material can sustain for 10 million cycles.

In general, steel alloys which are subjected to a cyclic stress level below the EL (properly adjusted for the specifics of the application) will not fail in fatigue. That property is commonly known as "infinite life". Most steel alloys exhibit the infinite life property, but it is interesting to note that most aluminum alloys as well as steels which have been casehardened by carburizing, do not exhibit an infinite-life cyclic stress level (Endurance Limit).

Factors Influencing Fatigue

(i) Loading

Nature and type of loading: - Axial tension, bending, torsion and combined loading-Mean and Variable components in case of Repeated, Fluctuating and Alternating loading and Frequency of loading and rest periods

(ii) Geometry

Size effects and stress concentration

(iii) Material

Composition, structure, directional properties and notch sensitivity

(iv) Manufacturing

Surface finish, heat treatment, residual stresses

(V) Environment

Corrosion, high temperature, radiation

Material

As noted earlier there are two classes of materials as far as the fatigue behavior is concerned, those material which exhibit well defined endurance limit and those without do not show endurance limit. Most ferrous materials and basic steels fall under the first category and some heat treated alloys of steel, aluminum etc. fall under the second category.

Composition and strength of the material are interrelated and detail discussion on strength follows later. Strength is also related to micro structure and in this respect it is interesting to note that soft structure like ferrite resist fatigue better than hard structure like cementite. However because of the higher strength that can be achieved from the same material by altering the micro structure, such structures are preferred in spite of their poor resistance

Why is the surface so important?

Fatigue failures almost always begin at the surface of a material. The reasons are that (a) the most highly-stresses fibers are located at the surface (bending fatigue) and (b) the intergranular flaws which precipitate tension failure are more frequently found at the surface.

Suppose that a particular specimen is being fatigue tested (as described above). Now suppose the fatigue test is halted after 20 to 25% of the expected life of the specimen and a small thickness of material is machined off the outer surface of the specimen, and the surface condition is restored to its original state. Now the fatigue test is resumed at the same stress level as before. The life of the part will be considerably longer than expected. If that process is repeated several times, the life of the part may be extended by several hundred percent, limited only by the available cross section of the specimen. That proves fatigue failures originate at the surface of a component.

Fluctuating Load Consideration for Design

S K Mondal's
Chapter 5

Frequency: ν or f in units of Hz. For rotating machinery at 3000 rpm, $f = 50$ Hz. In general only influences fatigue if there are environmental effects present, such as humidity or elevated temperatures

Waveform: Is the stress history a sine wave, square wave, or some other wave form? As with frequency, generally only influences fatigue if there are environmental effects.

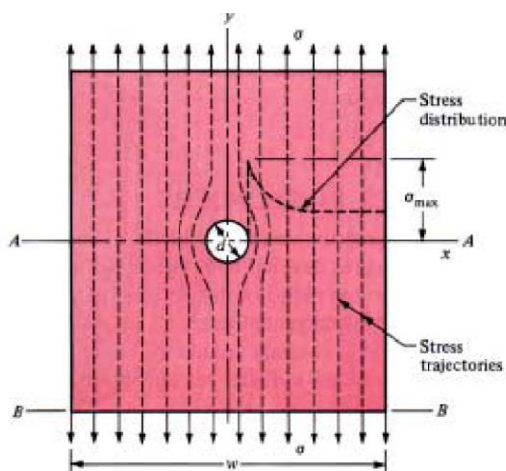
Endurance Limit Multiplying Factors (Marin Factors)

$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times S'_e$ $S_e \equiv$ Endurance limit of part $S'_e \equiv$ Endurance limit of test specimen $k_a \equiv$ Surface factor $k_b \equiv$ Size factor $k_c \equiv$ Load factor $k_d \equiv$ Temperature factor $k_e \equiv$ Miscellaneous – effects factor	<div style="border: 1px solid black; padding: 10px;"> <p>There are several factors that are known to result in differences between the endurance limits in test specimens and those found in machine elements.</p> </div>
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Notch Sensitivity

Geometric Stress Concentration

Factors



$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}}$$

$$\sigma_{\text{nom}} = \frac{F}{A_0}$$

$$A_0 = (w - d)t$$

Geometric stress concentration factors can be used to estimate the stress amplification in the vicinity of a geometric discontinuity.

Geometric Stress Concentration

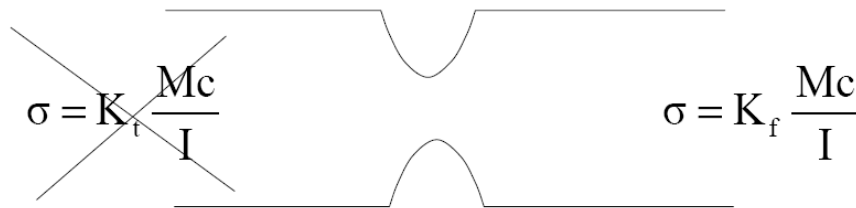
Factors

(Summary)

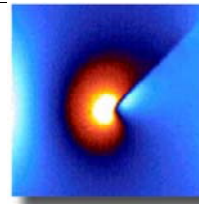
K_t is used to relate the maximum stress at the discontinuity to the nominal stress.
 K_t is used for normal stresses
 K_{ts} is used for shear stresses
 K_t is based on the geometry of the discontinuity
 σ_{nom} is usually computed using the minimum cross section

Un-notched and Notched Fatigue

Specimens



Comparisons of fatigue test results for notched and un-notched specimens revealed that a reduced K_t was warranted for calculating the fatigue life for many materials.



Fatigue Stress Concentration Factors

$$K_f = \frac{\text{Maximum stress in notched specimen}}{\text{Stress in notch free specimen}}$$

or

$$K_f = \frac{\text{Endurance limit of a notch free specimen.}}{\text{Endurance limit of a notched specimen.}}$$

Fatigue Stress Concentration Factors

- K_f is normally used in fatigue calculations but is sometimes used with static stresses.
- Convenient to think of K_f as a stress concentration factor reduced from K_t because of lessened sensitivity to notches.
- If notch sensitivity data is not available, it is conservative to use K_t in fatigue calculations.

Notch Sensitivity Factor

The notch sensitivity of a material is a measure of how sensitive a material is to notches or geometric discontinuities.

$$q = \frac{K_f - 1}{K_t - 1} \quad 0 \leq q \leq 1$$

$$K_f = 1 + q(K_t - 1) \quad 1 \leq K_f \leq K_t$$

It is found that some materials are not very sensitive to the existence of notches or discontinuity. In such cases it is not necessary to use the full value of k_t and instead a reduced value is needed. This is given by a factor known as fatigue strength reduction factor k_f and this is defined as

$$K_f = \frac{\text{Endurance limit of notch free specimens}}{\text{Endurance limit of notched specimens}}$$

Another term called Notch sensitivity factor, q is often used in design and this is defined as

$$q = \frac{K_f - 1}{K_t - 1}$$

The value of 'q' usually lies between 0 and 1. If $q = 0$, $k_f = 1$ and this indicates no notch sensitivity. If however $q = 1$, then $k_f = k_t$ and this indicates full notch sensitivity. Design charts for 'q' can be found in design hand-books and knowing k_t , k_f may be obtained. A typical set of notch sensitivity curves for steel is

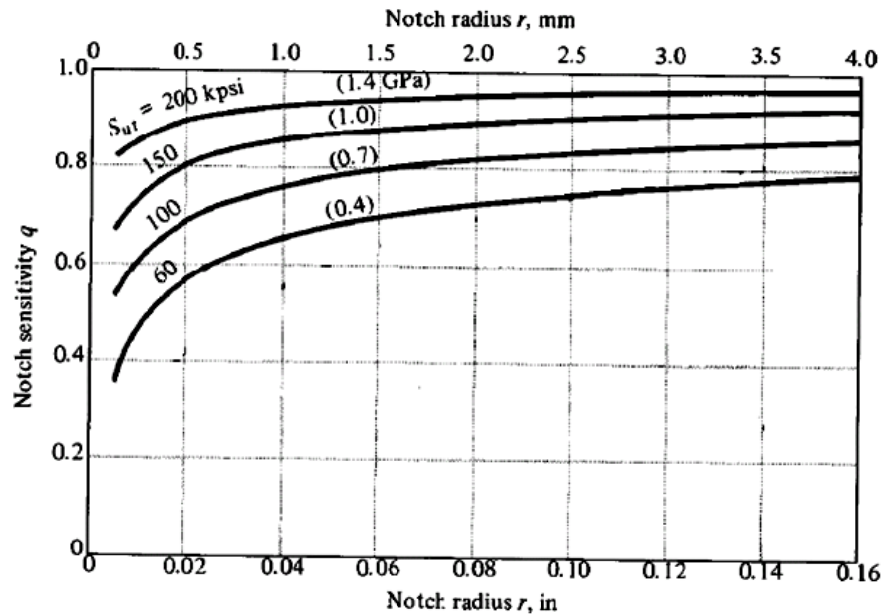


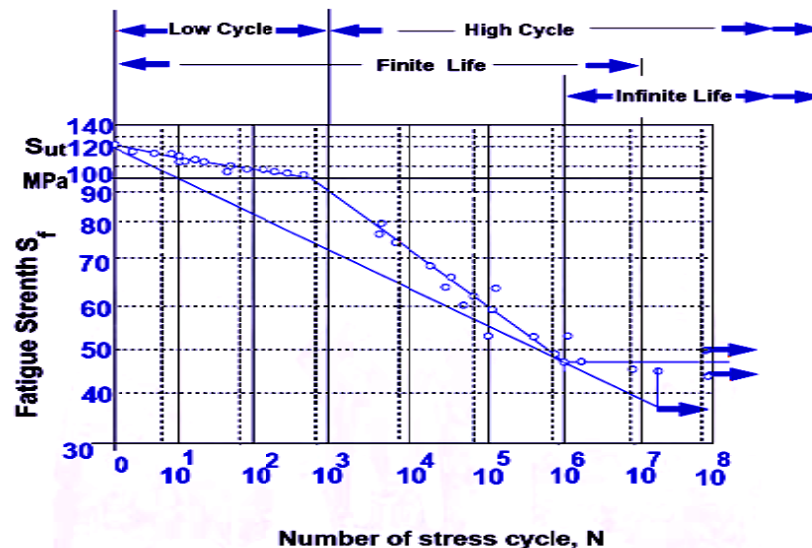
Fig. Variation of notch sensitivity with notch radius for steels of different ultimate tensile strength

Notch sensitivity index q can also be defined as

$$q = \frac{1}{1 + \left(\frac{a}{r}\right)^{1/2}}$$

Where, \sqrt{a} is called the Nubert's constant that depends on materials and their Heat treatments.

Low & High Cycle Fatigue; Finite and Infinite Life problem



Low Cycle Fatigue

The body of knowledge available on fatigue failure from $N = 1$ to $N = 1000$ cycles is generally classified as low-cycle fatigue.

High Cycle Fatigue

High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than 10^3 cycles. (Note that a stress cycle ($N=1$) constitutes a single application and removal of a load and then another application and removal of load in the opposite direction. Thus $N = \frac{1}{2}$ means that the load is applied once and then removed, which is the case with the simple tensile test.)

Finite and Infinite Life

We also distinguish a finite-life and an infinite-life region. Finite life region covers life in terms of number of stress reversals upto the knee point.(in case of steels) beyond which is the infinite-life region. The boundary between these regions cannot be clearly defined except for specific materials; but it lies somewhere between 10^6 and 10^7 cycles, for materials exhibiting fatigue limit.

Fatigue with finite life

This applies to most commonly used machine parts and this can be analyzed by idealizing the S-N curve for, say, steel,

The line between 10^3 and 10^6 cycles is taken to represent high cycle fatigue with finite life and this can be given by

$$\log S = b \log N + c$$

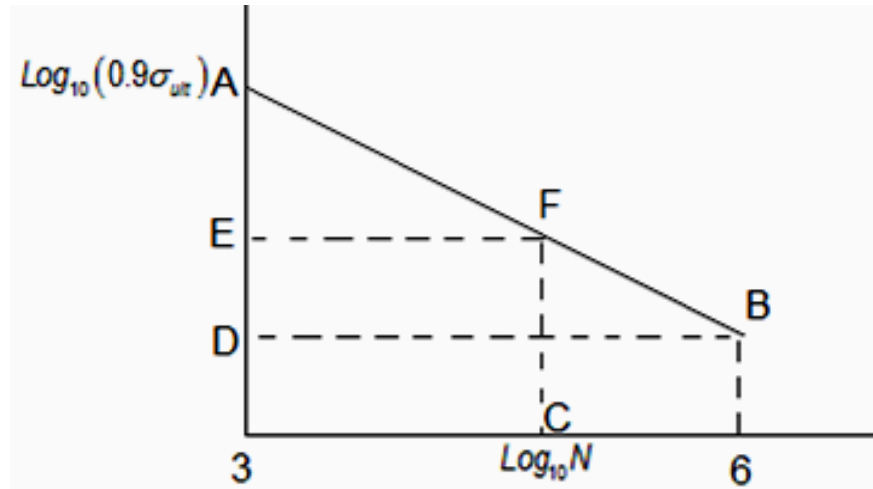
Where S is the reversed stress and b and c are constants.

At point A $\log(0.8\sigma_u) = b \log 10^3 + c$ where σ_u is the ultimate tensile stress and at point B, $\log \sigma_e = b \log 10^6 + c$ where σ_e is the endurance limit.

This gives
$$b = -\frac{1}{3} \log \frac{0.8\sigma_u}{\sigma_e} \text{ and } c = \log \frac{(0.8\sigma_u)^2}{\sigma_e}$$

Example: A cylindrical shaft is subjected to an alternating stress of 100 MPa. Fatigue strength to sustain 1000 cycles is 490 MPa. If the corrected endurance strength is 70 MPa, estimated shaft life will be [GATE-2006]

Solutions:



It is a finite life problem. The line AB is the failure line. Where

$A\{3, \log_{10}(0.9\sigma_{ult})\}$ But here it will be $A\{3, \log_{10}(490)\}$ and

$B\{6, \log_{10}(\sigma_e)\}$ Here it is $B\{6, \log_{10}(70)\}$

Therefore $F\{\log_{10} N, \log_{10}(100)\}$ we have to find N

$$\frac{EF}{AE} = \frac{DB}{AD}$$

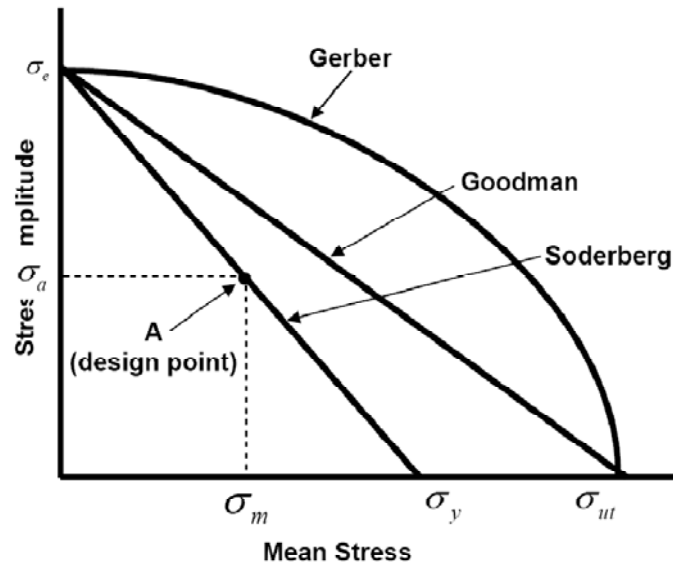
$$\text{Or } \frac{\log_{10} N - 3}{\log_{10} 490 - \log_{10} 100} = \frac{6 - 3}{\log_{10} 490 - \log_{10} 70}$$

or $N = 281914$ cycles.

Soderberg and Goodman Diagrams

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view:

1. Gerber method
2. Goodman method and
3. Soderberg method.



1.	Goodman criterion:	$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ut}} = \frac{1}{FS}$
2.	Soderberg criterion:	$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{FS}$
3.	Gerber criterion:	$\frac{FS \times \sigma_a}{\sigma_e} + \left(\frac{FS \times \sigma_m}{\sigma_{ut}} \right)^2 = 1$

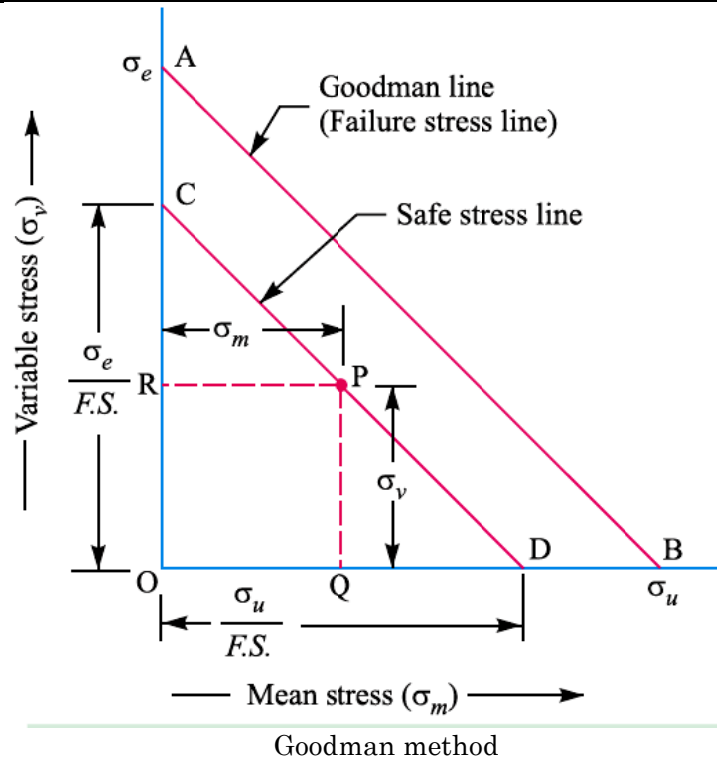
Where,

σ_a = Stress amplitude; σ_e = Endurance limit; σ_m = Mean stress; σ_y = Yield point;

σ_{ut} = Ultimate stress and FS = factor of safety.

ASME-elliptic
$$\left(\sigma_a / S_e \right)^2 + \left(\sigma_m / S_{ut} \right)^2 = 1 / n^2$$

Goodman Diagram



$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

Where

F.S. = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading,

Above expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

For high stress concentration the fatigue stress concentration factor (K_f) is used to multiply the variable stress.

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Where

F.S. = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

K_f = Fatigue stress concentration factor.

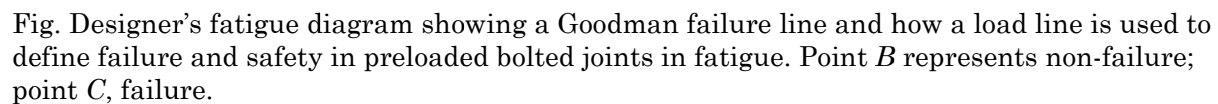


Diagram illustrating the Soderberg line (Failure stress line) and the Safe stress line in a stress analysis plot.

The vertical axis represents Variable stress (σ_v) and the horizontal axis represents Mean stress (σ_m).

Key points and lines:

- A**: Point on the Soderberg line at σ_e on the vertical axis.
- B**: Point on the Soderberg line at σ_y on the horizontal axis.
- C**: Point on the Safe stress line at $\sigma_e / F.S.$ on the vertical axis.
- D**: Point on the Safe stress line at σ_y on the horizontal axis.
- P**: Point on the Soderberg line, representing the failure state. Its coordinates are σ_m (mean stress) and σ_v (variable stress).
- R**: Point on the Safe stress line, representing the safe state. Its coordinates are $\sigma_m / F.S.$ and $\sigma_v / F.S.$.
- Q**: Point on the horizontal axis at σ_m .
- O**: Origin.

The Factor of Safety (F.S.) is defined as the ratio of the distance from the origin to the safe stress line (O-C-R) to the distance from the origin to the failure stress line (O-A-P).

The relationship between the Factor of Safety (F.S.), Mean stress (σ_m), Variable stress (σ_v), Yield stress (σ_y), and Endurance limit (σ_e) is given by:

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

Page 230 of 263

$$\frac{1}{\text{F.S.}} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Example: A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y) and corrected endurance (S_e) strength of the steel material are 600 MPa, 420 MPa and 240 MPa respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is

Solution:

$$\begin{aligned}\sigma_{\max} &= \frac{160 \times 10^3 \text{ N}}{\frac{\pi \times 30^2}{4} \text{ mm}^2} = 226 \text{ MPa} \\ \sigma_{\min} &= \frac{-40 \times 10^3 \text{ N}}{\frac{\pi \times 30^2}{4} \text{ mm}^2} = -56.6 \text{ MPa} \\ \sigma_{\text{mean}} &= \frac{(\sigma_{\max} + \sigma_{\min})}{2} = 84.7 \text{ MPa} \\ \sigma_{\text{min}} &= \frac{(\sigma_{\max} - \sigma_{\min})}{2} = 141.3 \text{ MPa} \\ \text{Therefore } \frac{1}{\text{FOS}} &= \frac{\sigma_{\text{mean}}}{\sigma_y} + \frac{\sigma_v}{\sigma_e} \\ \text{or } \frac{1}{\text{FOS}} &= \frac{84.7}{420} + \frac{141.3}{240} \\ \text{or FOS} &= 1.26\end{aligned}$$

Q. What modification in Soderberg diagram is required when it is used for design of helical springs?

Ans. In the earlier Soderberg diagram, we have used in the design for varying loads on the machine member, had only stress amplitude in the endurance limit representation, since, endurance limit value was for complete reversed loading.

Here, in spring design, we use endurance limit value for repeated loads only.

Hence, we have both stress amplitude and mean stress value of equal magnitude, $\frac{\tau_e}{2}$.

Therefore, the endurance limit representation in Soderberg diagram changes to $\frac{\tau_e}{2}, \frac{\tau_e}{2}$.

CUMULATIVE FATIGUE DAMAGE

Instead of a single reversed stress σ for n cycles, suppose a part is subjected to σ_1 for n_1 cycles σ_1 for n_2 cycles. etc. Under these conditions our problem is to estimate the fatigue life of a part subjected to these reverse stresses, or to estimate the factor of safety if the part has an infinite life. A search of the literature reveals that this problem has not been solved completely.

Cumulative Fatigue Damage

Thus the theory which is in greatest use at the present time to explain cumulative fatigue damage, i.e the Palmgren-Minor cycle-ratio summation theory also known as Minor's rule can mathematically, stated as

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_i}{N_i} = C$$

Where n is the number of cycles of stress σ applied to the specimen and N is the fatigue life corresponding to σ . The constant C is determined by experiment and is usually found in the range $0.7 \leq C \leq 2.2$. Many authorities recommend $C = 1$ and the short form of the theory can state as:

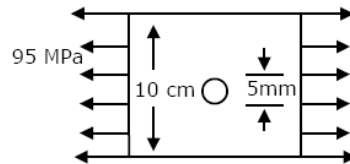
$$\sum \frac{n}{N} = 1$$

<h1 style="margin: 0;">Fluctuating Load Consideration for Design</h1> <h2 style="margin: 0;">S K Mondal's</h2> <h2 style="margin: 0; text-align: right;">Chapter 5</h2> <h3 style="margin: 0; text-align: center;">Objective Questions (IES, IAS, GATE)</h3>

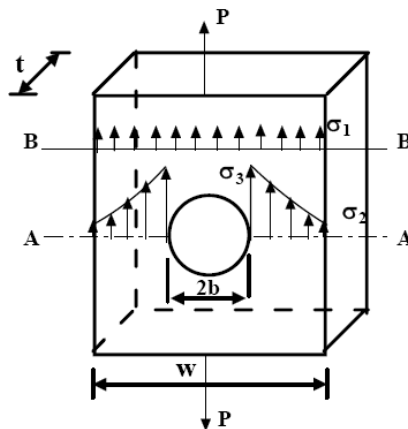
Previous 20-Yrs GATE Questions

GATE 1. A large uniform plate containing a rivet-hole is subjected to uniform uniaxial tension of 95 MPa. The maximum stress in the plate is [GATE-1992]

- (a) 100 MPa
- (b) 285 MPa
- (c) 190 MPa
- (d) Indeterminate



GATE 1. Ans. (b)



Stress concentration due to a central hole in a plate subjected to an uni-axial loading.

$$\sigma_3 = \sigma_1 \left(1 + \frac{2b}{a} \right) \text{ If } a=b \text{ the hole reduces to a circular one and therefore } \sigma_3 = 3\sigma_1 \text{ which gives } k_t = 3.$$

GATE 2. Match 4 correct pairs between list I and List II for the questions

List I

- (a) Strain rosette
- (b) Beams
- (c) Section modulus
- (d) Wahl's stress factor
- (e) Fatigue
- (f) Somerfield number

List II

- 1. Critical speed
- 2. Mohr's circle
- 3. Coil springs
- 4. Flexural rigidity
- 5. Endurance limit
- 6. Core section

[GATE-1994]

GATE 2. Ans. (a) – 2, (c) – 4, (d) – 3, (e) - 5

GATE 3. In terms of theoretical stress concentration factor (K_t) and fatigue stress concentration factor (K_f), then notch sensitivity 'q' is expressed as

[GATE-2004]

- (a) $\frac{(K_f - 1)}{(K_t - 1)}$
- (b) $\frac{(K_f - 1)}{(K_t + 1)}$
- (c) $\frac{(K_t - 1)}{(K_f - 1)}$
- (d) $\frac{(K_f + 1)}{(K_t + 1)}$

GATE 3. Ans. (a)

Fluctuating Load Consideration for Design

S K Mondal's

Chapter 5

GATE4. A thin supercritical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying from 4 to 8 MPa. Assume that the yield, ultimate, and endurance strength of material are 600, 800 and 400 MPa respectively. The factor of safety as per Goodman's relation is [GATE-2007]

- (a) 2.0 (b) 1.6 (c) 1.4 (d) 1.2

GATE 4. Ans. (b) Stress induced $\sigma_1 = \sigma_2 = \frac{pr}{2t}$

$$\sigma_{1\max} = \frac{8 \times 100}{2 \times 1} = 400 \text{ MPa}$$

$$\sigma_{1\min} = \frac{4 \times 100}{2 \times 1} = 200$$

$$\sigma_{2\max} = 400 \text{ MPa} \quad \sigma_{2\min} = 200 \text{ MPa}$$

$$\sigma_{1m} = 300 \text{ MPa} \quad \sigma_{1a} = 100 \text{ MPa}$$

$$\sigma_{2m} = 300 \text{ MPa} \quad \sigma_{2a} = 100 \text{ MPa}$$

Equivalent Stresses

$$\begin{aligned} \sigma_{1me} &= \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}} \\ &= \sqrt{300^2 + 300^2 - 300 \times 300} \\ &= 300 \text{ MPa} \end{aligned}$$

Similarly,

$$\begin{aligned} \Rightarrow \frac{100}{400} + \frac{300}{800} &= \frac{1}{n} \\ n &= 1.6 \end{aligned}$$

GATE 5. A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y), and corrected endurance (S_e) strength of the steel material are 600 MPa, 420 MPa and 240 MPa respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is [GATE -2009]

- (a) 1.26 (b) 1.37 (c) 1.45 (d) 2.00

GATE 5. Ans. (a) $\sigma_{\max} = \frac{160 \times 10^3 \text{ N}}{\frac{\pi \times 30^2}{4} \text{ mm}^2} = 226 \text{ MPa}$

$$\sigma_{\min} = \frac{-40 \times 10^3 \text{ N}}{\frac{\pi \times 30^2}{4} \text{ mm}^2} = -56.6 \text{ MPa}$$

$$\sigma_{\text{mean}} = \frac{(\sigma_{\max} + \sigma_{\min})}{2} = 84.7 \text{ MPa}$$

$$\sigma_{\text{min}} = \frac{(\sigma_{\max} - \sigma_{\min})}{2} = 141.3 \text{ MPa}$$

<h2 style="margin: 0;">Fluctuating Load Consideration for Design</h2> <div style="display: flex; justify-content: space-between; font-weight: bold; font-size: 1.2em;"> S K Mondal's Chapter 5 </div>

Therefore
$$\frac{1}{\text{FOS}} = \frac{\sigma_{\text{mean}}}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

or
$$\frac{1}{\text{FOS}} = \frac{84.7}{420} + \frac{141.3}{240}$$

or $\text{FOS} = 1.26$

GATE 6. The yield strength of a steel shaft is twice its endurance limit. Which of the following torque fluctuation represent the most critical situation according to Soderberg criterion? [GATE-1993]

- (a) -T to +T (b) -T/2 to +T (c) 0 to +T (d) +T/2 to +T

GATE 6. Ans. (a)

GATE 7. An aeroplane makes a half circle towards left. The engine runs clockwise when viewed from the rear. Gyroscopic effect on the aeroplane causes the nose to [GATE-1995]

- (a) Lift (b) dip (c) both Lift and dip (d) None of the above

GATE 7. Ans. (a)

GATE 8. For a disk of moment of inertia I the spin and precession angular velocities are ω and ω_p respectively. The magnitude of gyroscopic couple is..... [GATE-1994]

- (a) $I\omega\omega_p$ (b) $I\omega\omega_p / 2$ (c) $2I\omega\omega_p$ (d) $4I\omega\omega_p$

GATE 8. Ans. (a)

GATE 9. The S-N curve for steel becomes asymptotic nearly at [GATE-2004]

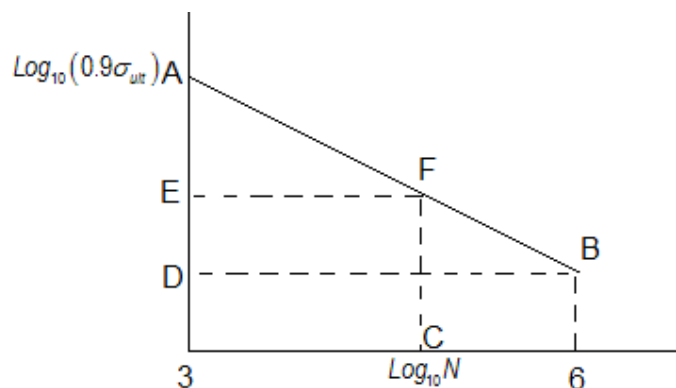
- (a) 10^3 cycles (b) 10^4 cycles (c) 10^6 cycles (d) 10^9 cycles

GATE 9. Ans. (c)

GATE10. A cylindrical shaft is subjected to an alternating stress of 100 MPa. Fatigue strength to sustain 1000 cycles is 490 MPa. If the corrected endurance strength is 70 MPa, estimated shaft life will be [GATE-2006]

- (a) 1071 cycles (b) 15000 cycles (c) 281914 cycles (d) 928643 cycles

GATE 10. Ans. (c)



It is a finite life problem. The line AB is the failure line. Where $A\{3, \log_{10}(0.9\sigma_{ult})\}$

but here it will be $A\{3, \log_{10}(490)\}$ and $B\{6, \log_{10}(\sigma_e)\}$ here it is $B\{6, \log_{10}(70)\}$

Therefore $F\{\log_{10} N, \log_{10}(100)\}$ we have to find N

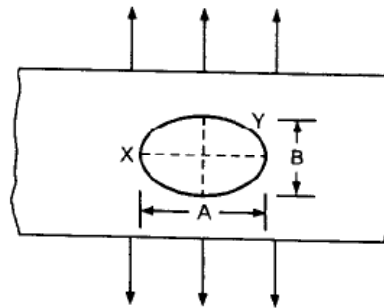
$$\frac{EF}{AE} = \frac{DB}{AD}$$

Or $\frac{\log_{10} N - 3}{\log_{10} 490 - \log_{10} 100} = \frac{6 - 3}{\log_{10} 490 - \log_{10} 70}$
 or $N = 281914$ cycles.

Previous 20-Yrs IES Questions

Stress Concentration

- IES 1. A loaded semi-infinite flat plate is having an elliptical hole ($A/B = 2$) in the middle as shown in the figure. The stress concentration factor at points either X or Y is
- 1
 - 3
 - 5
 - 7



[IES-2000]

IES 1. Ans. (c)

Fluctuating Stresses

- IES 2. In designing a shaft for variable loads, the S.N. diagram can be drawn by
- Joining the S_{ut} at 0 cycles and S_e at 10^6 cycles by a straight line on an S.N. graph
 - Joining the $0.9 S_{ut}$ at 1000 cycles and S_e at 10^6 cycles by a straight line on a log S- log N graph
 - Joining the $0.9 S_{ut}$ at 1000 cycles and S_e at 10^6 cycles by a straight line on an S-N graph
 - Joining the S_{ut} at 1000 cycles and $0.9 S_e$ at 10^6 cycles by a straight line on a log S- log N graph
- (S_{ut} stands for ultimate tensile strength and S_e for the endurance limit)

[IES 2007]

IES 2. Ans. (b)

- IES 3. Consider the following statements: [IES-2005]
- Endurance strength of a component is not affected by its surface finish and notch sensitivity of the material.
 - For ferrous materials like steel, S-N curve becomes asymptotic at 10^6 cycles.

Which of the statements given above is/are correct?

- 1 only
- 2 only
- Both 1 and 2
- Neither 1 nor 2

IES 3. Ans. (b) 1 is false: affected

Endurance Limit

<h2 style="margin: 0;">Fluctuating Load Consideration for Design</h2> <h3 style="margin: 0;">S K Mondal's</h3> <h3 style="margin: 0; text-align: right;">Chapter 5</h3>

IES 4. Match List I with List II and select the correct answer using the codes given below the lists: [IES-1993]

List I (Material properties) List II (Tests to determine material properties)

- | | |
|------------------------------|------------------|
| A. Ductility | 1. Impact test |
| B. Toughness | 2. Fatigue test |
| C. Endurance limit | 3. Tension test |
| D. Resistance to penetration | 4. Hardness test |

Code:	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	4	2	1	3
(c)	3	1	2	4	(d)	4	1	2	3

IES 4. Ans. (c)

IES 5. When σ and Young's Modulus of Elasticity E remain constant, the energy-absorbing capacity of part subject to dynamic forces, is a function of its [IES-1992]

- (a) Length (b) cross-section (c) volume (d) none of the above

IES 5. Ans. (c) Strain energy is given by,

$$U = A.L. \left(\frac{\sigma^2}{2E} \right)$$

Where σ and E remaining constant,

\therefore U is proportional to (A.L.) which is volume.

Also, since U is a function of σ^2 , that portion of the part which is prone to high localised will absorb a high amount of energy, making it vulnerable to failure. Such a part, therefore, is designed to have such a contour that, when it is subjected to time-varying or impact loads or others types of dynamic forces, the part absorbs or less uniform stress distribution along the whole length of the part is ensured.

IES 6. Fatigue strength of a rod subjected to cyclic axial force is less than that of a rotating beam of the same dimensions subjected to steady lateral force. What is the reason? [IES-2009]

- (a) Axial stiffness is less than bending stiffness
 (b) Absence of centrifugal effects in the rod
 (c) The number of discontinuities vulnerable to fatigue is more in the rod
 (d) At a particular time, the rod has only one type of stress whereas the beam has both tensile and compressive stresses

IES 6. Ans. (d)

Soderberg and Goodman Diagrams

IES 7. Assertion (A): Soderberg relation is used for design against fatigue. [IES-1996]

Reason (R): Soderberg relation is based on yield strength of the material whereas all other failure relations for dynamic loading are based on ultimate strength of the material.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is **not** the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES 7. Ans. (a)

IES 8. The design calculations for members subject to fluctuating loads with the same factor of safety yield the most conservative estimates when using

Fluctuating Load Consideration for Design

S K Mondal's Chapter 5

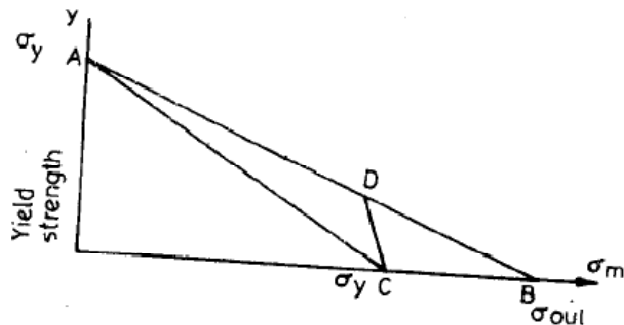
- (a) Gerber relation
(c) Goodman relation

- (b) Soderberg relation [IES-1995]
(d) none of the above.

IES 8. Ans. (b)

IES 9. In the figure shown, it the line AB represents Goodman criterion of failure, then soderberg criterion could be represented by line

- (a) AD
(b) D
(c) DC
(d) AC



[IES-1992]

IES 9. Ans. (d)

Gyroscopic motion

IES 10. Consider the following statements:

[IES-2005]

1. The effect of gyroscopic couple on a car while negotiating a curve is that its outer wheels tend to get lifted from the ground.
2. If spin vector is rotated about the precession vector axis in a direction opposite to that of precession through 90° , the new position of the spin vector indicates the direction of the torque vector.

Which of the following statements given above is/are correct?

- (a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

IES 10. Ans. (d)

IES 11. **Assertion (A):** The precession of the axis of rotation of a shaft causes a gyroscopic reaction couple to act on the frame to which the bearings are fixed.

Reason (R): The reaction of the shaft on each bearing is equal and opposite to the action of the bearing on the shaft.

[IES-2002]

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES 11. Ans. (b)

IES 12. **Assertion (A):** There is a danger of locomotive wheels being lifted above rails at certain speeds.

[IES-2001]

Reason (R): Lifting of the locomotive wheel above rails at certain speed is due to gyroscopic action.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES 12. Ans. (c)

<h1 style="margin: 0;">Fluctuating Load Consideration for Design</h1> <h2 style="margin: 0;">S K Mondal's</h2> <h2 style="margin: 0; text-align: right;">Chapter 5</h2> <h3 style="margin: 0; background-color: #cccccc; padding: 5px;">Previous 20-Yrs IAS Questions</h3>

- IAS 1. A flywheel has a mass of 300 kg and a radius of gyration of 1m. It is given a spin of 100 r.p.m about its horizontal axis. The whole assembly rotates about a vertical axis at 6 rad/sec. The gyroscopic couple experienced will be [IAS-1996]
- (a) 3π kNm (b) 6π kNm (c) 180π kNm (d) 360π kNm

IAS 1. Ans. (b)

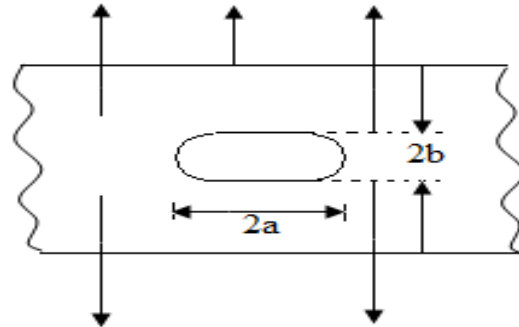
$$\text{Gyroscopic couple} = I\omega\omega_p = mk^2\omega\omega_p = 300 \times 1^2 \times \left(\frac{2\pi \times 100}{60} \right) \times 6 \text{ Nm} = 6\pi \text{ kNm}$$

- IAS 2. A bicycle remains stable in running through a bend because of [IAS 1994]
- (a) Gyroscopic action (b) Coriolis's acceleration
(c) centrifugal action (d) radius of curved path

IAS 2. Ans. (a)

- IAS 3. In a semi-infinite flat plate shown in the figure, the theoretical stress concentration factor k_t for an elliptical hole of major axis $2a$ and minor axis $2b$ is given by

- (a) $K_t = \frac{a}{b}$
(b) $K_t = 1 + \frac{a}{b}$
(c) $K_t = 1 + \frac{2b}{a}$
(d) $K_t = 1 + \frac{2a}{b}$



[IAS-1998]

IAS 3. Ans. (d)

- IAS 4. **Assertion (A):** Endurance limits for all materials are always less than the ultimate strength of the corresponding materials. [IAS 1994]

Reason (R): Stress concentration in a machine part due to any dislocation is very damaging when the part is subjected to variable loading.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS 4. Ans. (b)

- IAS 5. Match List I (Mechanical Property) with List II (Measured in Terms of) and select the correct answer using the codes given below the lists:

List-I				List-II			
(Mechanical Property)				(Measured in Terms of)			
(A) Strength (Fluctuating load)				1. Percentage elongation			
(B) Toughness				2. Modulus of elasticity			
(C) Stiffness				3. Endurance limit			
(D) Ductility				4. Impact strength			
Codes:	A	B	C	D	A	B	C
(a)	2	1	3	4	(b)	3	4
						2	1

[IAS-2003]

(c) 2 4 3 1 (d) 3 1 2 4

IAS 5. Ans. (b)

IAS 6. Match List I with List II and select the correct answer: [IAS-2000]

List I

- A. Proof stress
- B. Endurance limit
- C. Leaf Spring
- D. Modulus of rigidity

List II

- 1. Torsion test
- 2. Tensile test
- 3. Fatigue test
- 4. Beam of uniform strength

A B C D

(a) 2 3 4 1

(c) 3 2 4 1

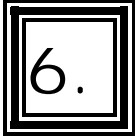
A B C D

(b) 2 3 1 4

(d) 3 2 1 4

IAS 6. Ans. (a)

Answer with Explanation



Miscellaneous

Theory at a glance (GATE, IES, IAS & PSU)

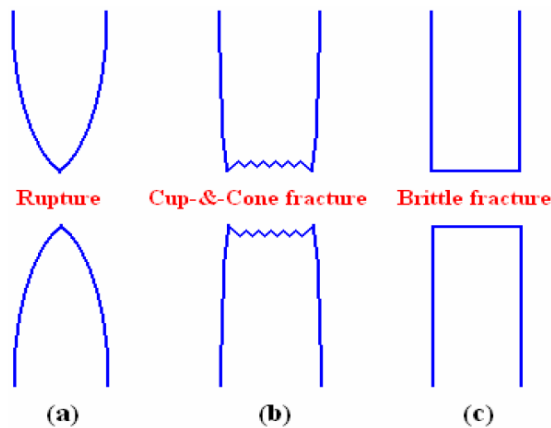
Fracture

- Fracture *defined* as the separation or fragmentation of a solid body into two or more parts under the action of stress.
- Fracture is classified based on several characteristic features:

<i>characteristic</i>	<i>terms used</i>	
Strain to fracture	<i>Ductile</i>	<i>Brittle</i>
Crystallographic mode	Shear	Cleavage
Appearance	Fibrous and gray	Granular and bright
Crack propagation	Along grain boundaries	Through grains

Fracture modes

- Ductile and Brittle are relative terms.
- Most of the fractures belong to one of the following modes:
(a) Rupture, (b) cup-&-cone and (c) brittle.



Ductile fracture Vs Brittle fracture

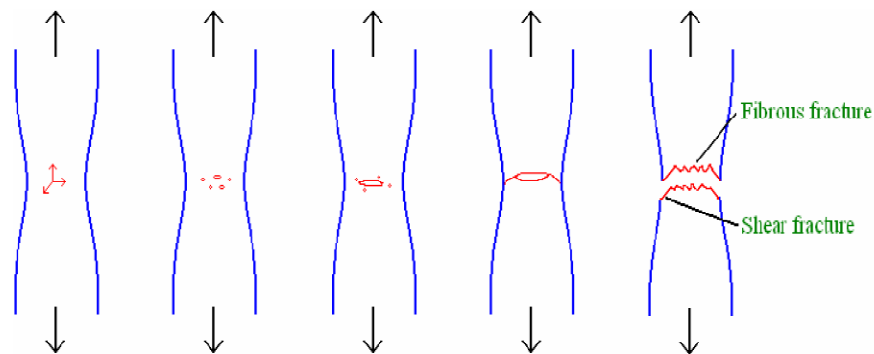
<i>Parameter</i>	<i>Ductile fracture</i>	<i>Brittle fracture</i>
Strain energy required	Higher	Lower
Stress, during cracking	Increasing	Constant
Crack propagation	Slow	Fast
Warning sign	Plastic deformation	None
Deformation	Extensive	Little
Necking	Yes	No
Fractured surface	Rough and dull	Smooth and bright

Ductile fracture

- Ductile fracture in tension occurs after appreciable plastic deformation.
- It is usually preceded by necking.
- It exhibits three stages - (1) formation of cavities (2) growth of cavities (3) final failure involving rapid crack propagation at about 45° to the tensile axis.
- Fractography of ductile fracture reveals numerous spherical dimples separated by thin walls on the fractured surface.
- McClintock's strain to ductile fracture, ϵ_f ,

$$\epsilon_f = \frac{(1-n)\ln(l_0/2b_0)}{\sinh\left[(1-n)(\sigma_a + \sigma_b)/(2\bar{\sigma}/\sqrt{3})\right]}$$

- Stages of void nucleation, void growth, crack initiation and eventual fracture under ductile fracture mode:



Brittle fracture

- Brittle fracture takes place with little or no preceding plastic deformation.
- It occurs, often at unpredictable levels of stress, by rapid crack propagation.
- Crack propagates nearly perpendicular to the direction of applied tensile stress, and hence called cleavage fracture.
- Most often brittle fracture occurs through grains i.e. transgranular.
- Three stages of brittle fracture - (1) plastic deformation that causes dislocation pile-ups at obstacles, (2) micro-crack nucleation as a result of build-up of shear stresses, (3) eventual crack propagation under applied stress aided by stored elastic energy.

Brittle fracture – Griffith Theory

- Nominal fracture stress that causes brittle fracture in presence of cracks (length of interior crack = $2c$), the stress raisers,

$$\sigma_f \approx \left(\frac{E\gamma}{4c} \right)^{1/2}$$

- **Griffith's criteria:** a crack will propagate when the decrease in elastic energy is at least equal to the energy required to create the new crack surface. Thus for thin plates:

$$\sigma = \left(\frac{2E\gamma}{c\pi} \right)^{1/2}$$

- For thick plates: $\sigma = \left(\frac{2E\gamma}{(1-\nu^2)c\pi} \right)^{1/2}$
- When plastic energy is also taken into account (Orowan's modification):

$$\sigma = \left(\frac{2E(\gamma + p)}{c\pi} \right)^{1/2} \approx \left(\frac{E_p}{c} \right)^{1/2}$$

Fracture mechanics

- Relatively new field of mechanics, that deals with possibility whether a crack of given length in a material with known toughness is dangerous at a given stress level or not!
- Fracture resistance of a material in the presence of cracks, known as fracture toughness, is expressed in two forms.

(1) Strain-energy release rate, (G): $G = \frac{\pi\sigma^2 c}{E}$

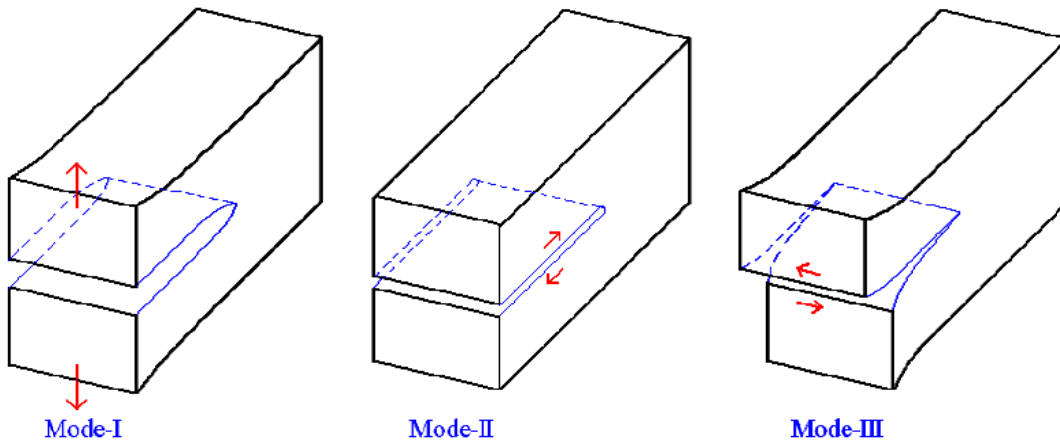
(2) Stress concentration factor, (K): $K = \alpha\sigma\sqrt{c\pi}$

- Both parameters are related as:

For plane stress conditions i.e. thin plates: $K^2 = GE$

For plane strain conditions i.e. thick plates: $K^2 = GE/(1-\nu^2)$

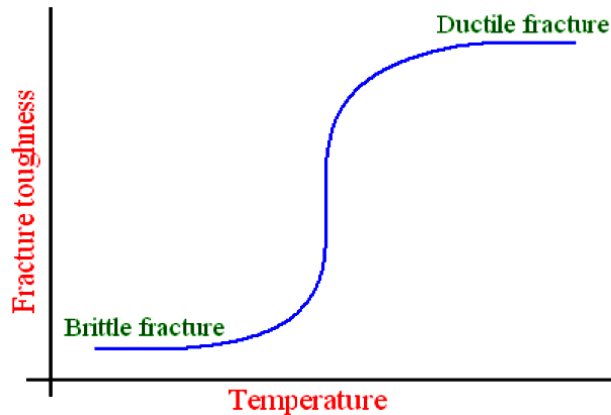
- K depends on many factors, the most influential of which are temperature, strain rate, microstructure and orientation of fracture. The value of K decreases with increasing strain rate, grain size and/or decreasing temperature.
- Depending on the orientation of fracture, three modes of fracture are identified as shown in the figure:



↑ Displacement of crack surfaces

Ductile-to-Brittle transition

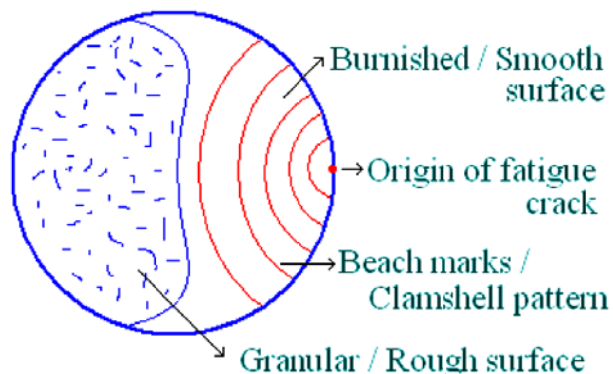
- Energy absorbed during the notch-impact is plotted as a function of temperature to know at what temperature range (DBTT) material fracture in a particular mode.



In metals DBTT is around $0.1-0.2 T_m$ while in ceramics it is about $0.5-0.7 T_m$, where T_m represents absolute melting temperature.

Fatigue failure

- Failure that occurs under fluctuating/cyclic loads – Fatigue.
- Fatigue occurs at stresses that considerable smaller than yield/tensile stress of the material.
- These failures are dangerous because they occur without any warning. Typical machine components subjected to fatigue are automobile crank-shaft, bridges, aircraft landing gear, etc.
- Fatigue failures occur in both metallic and non-metallic materials, and are responsible for a large number fraction of identifiable service failures of metals.
- Fatigue fracture surface is perpendicular to the direction of an applied stress.
- Fatigue failure can be recognized from the appearance of the fracture surface:



- Any point with stress concentration such as sharp corner or notch or metallurgical inclusion can act as point of initiation of fatigue crack.
- Three basic requisites for occurrence of fatigue fracture are: (a) a maximum tensile stress of sufficiently high value (b) a large enough variation or fluctuation in the applied stress and (c) a sufficiently large number of cycles of applied stress.

- Stress cycles that can cause fatigue failure are characterized using the following parameters:
 Range of stress, $\sigma_r = \sigma_{\max} - \sigma_{\min}$
 Alternating stress, $\sigma_a = \sigma_r / 2 = (\sigma_{\max} - \sigma_{\min}) / 2$
 Mean stress, $\sigma_m = (\sigma_{\max} + \sigma_{\min}) / 2$
 Stress ratio, $R = \sigma_{\min} / \sigma_{\max}$
 Amplitude ratio, $A = \sigma_a / \sigma_m = (1-R)/(1+R)$
- Material fails under fatigue mode at higher number of stress cycles if stress applied is lower.
- After a limiting stress, ferrous materials won't fail for any number of stress cycles. This limiting stress is called – *fatigue limit / endurance limit*.
- For non-ferrous materials, there is no particular limiting stress i.e. as stress reduces, number of cycles to failure keep increasing. Hence stress corresponding to 10^7 cycles is considered as characteristic of material, and known as *fatigue strength*. Number of cycles is called *fatigue life*.
- Endurance ratio* – ratio of fatigue stress to tensile stress of a material. For most materials it is in the range of 0.4-0.5.

Fatigue – Crack initiation & propagation

- Fatigue failure consists of four stages: (a) crack initiation –includes the early development of fatigue damage that can be removed by suitable thermal anneal (b) slip-band crack growth – involves the deepening of initial crack on planes of high shear stress (stage-I crack growth) (c) crack growth on planes of high tensile stress – involves growth of crack in direction normal to maximum tensile stress (stage-II crack growth) (d) final ductile failure – occurs when the crack reaches a size so that the remaining cross-section cannot support the applied load.
- Stage-I is secondary to stage-II crack growth in importance because very low crack propagation rates involved during the stage.

Static load Vs Cyclic load

<i>Feature</i>	<i>Static load</i>	<i>Cyclic load</i>
Slip (<i>nm</i>)	1000	1-10
Deformation feature	Contour	Extrusions & Intrusions
Grains involved	All grains	Some grains
Vacancy concentration	Less	Very high
Necessity of diffusion	Required	Not necessary

Fatigue crack growth: Stage-I Vs Stage-II

<i>Parameter</i>	<i>Stage-I</i>	<i>Stage-II</i>
Stresses involved	Shear	Tensile
Crystallographic orientation	Yes	No
Crack propagation rate	Low (nm/cycle)	High (μm/cycle)
Slip on	Single slip plane	Multiple slip planes
Feature	Feature less	Striations

Fatigue crack propagation rate

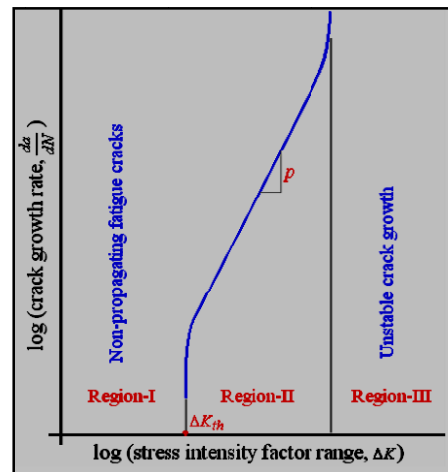
- Studies of fatigue crack propagation rate attained much importance because it can be used as fail-safe design consideration.

$$\frac{da}{dN} = \text{fn}(\sigma, a) = C\sigma_m^a a^n$$

- Paris law:**

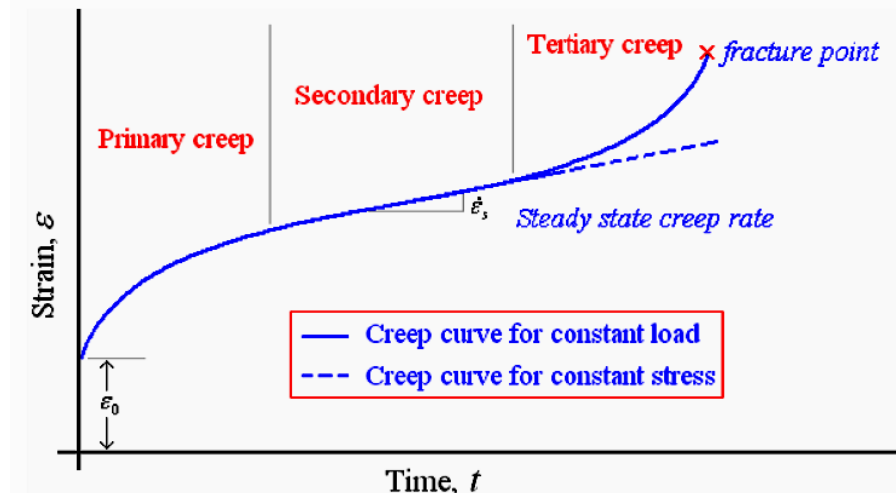
$$\frac{da}{dN} = A(\Delta K)^p$$

$p = 3$ for steels, 3-4 for Al alloys



Creep failure

- Deformation that occurs under constant load/stress and elevated temperatures which is time-dependent is known as *creep*.
- Creep deformation (constant stress) is possible at all temperatures above absolute zero. However, it is extremely sensitive to temperature.
- Hence, creep is usually considered important at elevated temperatures (temperatures greater than $0.4 T_m$, T_m is absolute melting temperature).
- Creep test data is presented as a plot between time and strain known as creep curve.
- The slope of the creep curve is designated as creep rate.

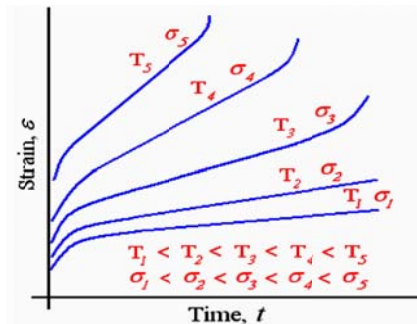
Creep curve

- Creep curve is considered to be consists of three portions.
- After initial rapid elongation, ε_0 , the creep rate decreases continuously with time, and is known as *primary* or *transient creep*.
- Primary creep is followed by *secondary* or *steady-state* or *viscous creep*, which is characterized by constant creep rate. This stage of creep is often the longest duration of the three modes.
- Finally, a third stage of creep known as, *tertiary creep* occurs that is characterized by increase in creep rate.
- Andrade creep equation: $\varepsilon = \varepsilon_0 (1 + \beta t^{1/3}) e^{kt}$
- Garofalo creep equation: $\varepsilon = \varepsilon_0 + \varepsilon_t (1 - e^{-rt}) + \dot{\varepsilon}_s t$
- First stage creep is associated with strain hardening of the sample.
- Constant creep rate during secondary creep is believed to be due to balance between the competing processes of strain hardening and recovery. Creep rate during the secondary creep is called the minimum creep rate.
- Third stage creep occurs in constant load tests at high stresses at high temperatures. This stage is greatly delayed in constant stress tests. Tertiary creep is believed to occur because of either reduction in cross-sectional area due to necking or internal void formation. Third stage is often associated with metallurgical changes such as coarsening of precipitate particles, recrystallization, or diffusion changes in the phases that are present.

Creep rate – Stress & Temperature effects

- Two most important parameter that influence creep rate are: stress and temperature.
- With increase in either stress or temperature (a) instantaneous elastic strain increases (b) steady state creep rate increases and (c) rupture lifetime decreases.

$$\dot{\epsilon}_s = K_2 \sigma^n e^{\frac{Q_c}{RT}}$$



Dynamometers

For consistently accurate and reliable measurement, the following requirements are considered during design and construction of any tool force dynamometers:

- **Sensitivity** : the dynamometer should be reasonably sensitive for precision measurement
- **Rigidity** : the dynamometer need to be quite rigid to withstand the forces without causing much deflection which may affect the machining condition
- **Cross Sensitivity**: the dynamometer should be free from cross sensitivity such that one force (say P_z) does not affect measurement of the other forces (say P_x and P_y)
- Stability against humidity and temperature
- Quick time response
- High frequency response such that the readings are not affected by vibration within a reasonably high range of frequency
- Consistency, i.e. the dynamometer should work desirably over a long period.

Construction and working principle of some common tool – force dynamometers.

The dynamometers being commonly used now-a-days for measuring machining forces desirably accurately and precisely (both static and dynamic characteristics) are

- Either • strain gauge type
Or • piezoelectric type

Strain gauge type dynamometers are inexpensive but less accurate and consistent, whereas, the piezoelectric type are highly accurate, reliable and consistent but very expensive for high material cost and stringent construction.

Measuring Horsepower

Shows the prony brake or dynamometer method of measuring motor horsepower. This method is valid for all types of motors including internal combustion engines, turbines, and all electric motors.

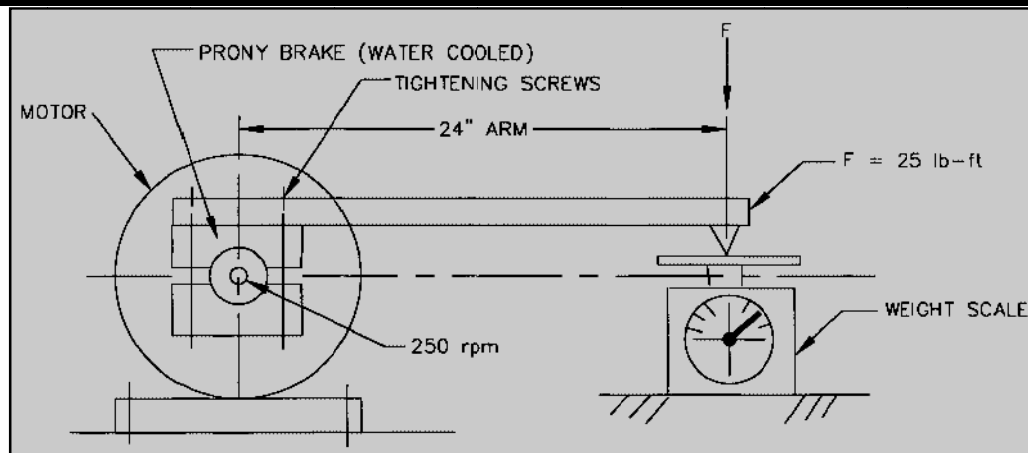


Fig. Prony-brake dynamometer.

$$T_Q = 24 \times 25 = 600 \text{ lb} \cdot \text{in} \text{ or } 50 \text{ lb} \cdot \text{ft}$$

From this

$$T_R = \frac{5250(\text{hp})}{N_r} \quad \text{hp} = \frac{T_R \times N_r}{5250} = \frac{50(250)}{5250} = 2.38 \text{ hp}$$

where hp = horsepower (1 hp = 33,000 ft · lb / min = 550 ft · lb / sec).

T_R = running torque, lb · ft

N_r = running speed, rpm

Brake

1. Types of brakes

Brakes are devices that dissipate kinetic energy of the moving parts of a machine. In mechanical brakes the dissipation is achieved through sliding friction between a stationary object and a rotating part. Depending upon the direction of application of braking force, the mechanical brakes are primarily of three types

- Shoe or block brakes – braking force applied radially
- Band brakes – braking force applied tangentially.
- Disc brake – braking force applied axially.

2. Shoe or block brake

In a shoe brake the rotating drum is brought in contact with the shoe by suitable force. The contacting surface of the shoe is coated with friction material. Different types of shoe brakes are used, viz., single shoe brake, double shoe brake, internal expanding brake, external expanding brake. These are sketched in figure below

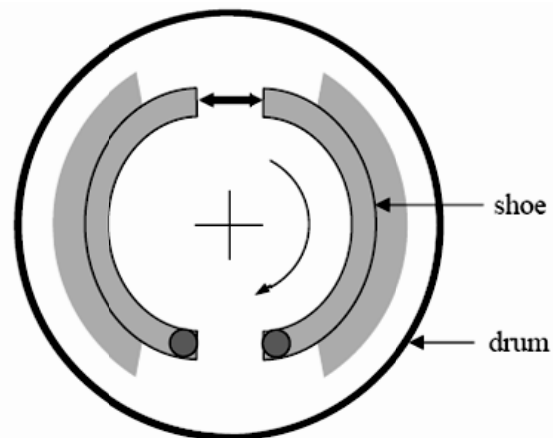
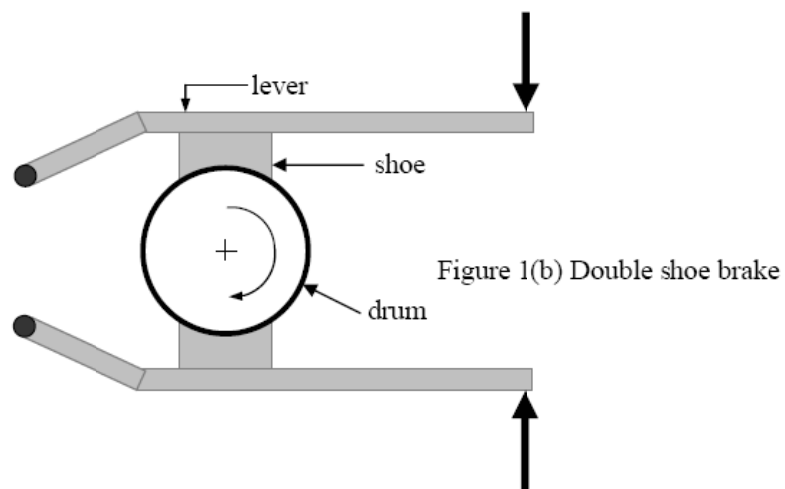
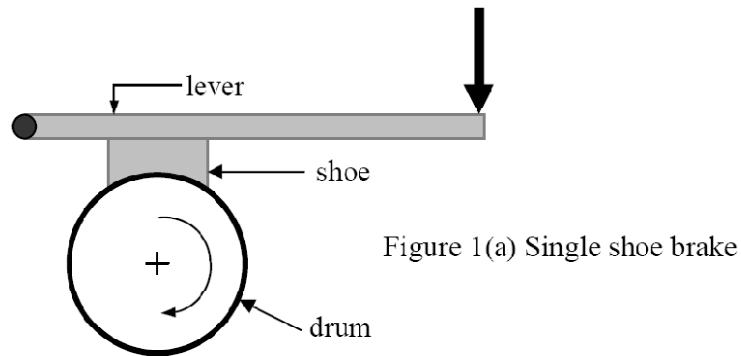


Figure : Internal expanding shoe brake

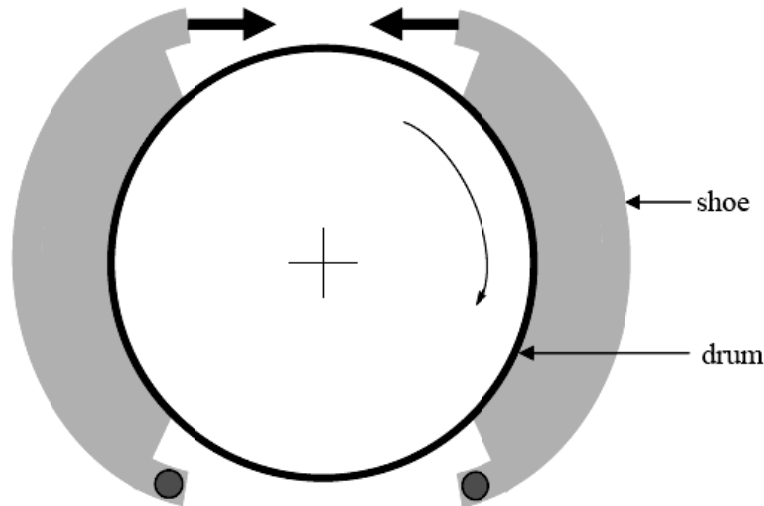


Figure : External expanding shoe brake

Figure : Different shoe brakes

Single Shoe brake

The force needed to secure contact is supplied by a lever. When a force F is applied to the shoe (see figure below) frictional force proportional to the applied force $F_{fr} = \mu' F$ develops, where μ' depends of friction material and the geometry of the shoe. A simplified analysis is done as discussed below.

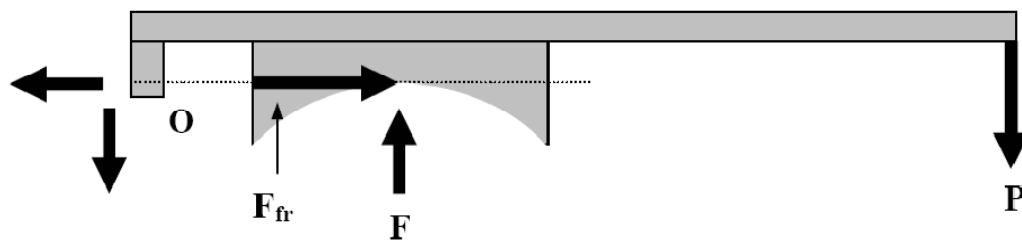


Figure Free body diagram of a brake shoe

Though the exact nature of the contact pressure distribution is unknown, an approximation (based on wear considerations) is made as

$$p(\theta) = p_0 \cos \theta$$

Where the angle is measured from the centerline of the shoe. If Coulomb's law of friction is assumed to hold good, then

$$f_{fr}(\theta) = \mu p_0 \cos \theta$$

Since the net normal force of the drum is F , one has

$$Rb \int_{-\theta_0}^{\theta_0} p(\theta) \cos \theta d\theta = F,$$

Where R and b are the radius of the brake drum and width of the shoe respectively.

The total frictional torque is

$$T = b \int_{-\theta_0}^{\theta_0} f_{fr}(\theta) R^2 d\theta$$

If the total frictional force is assumed to be a concentrated one, then the

Equivalent force becomes $f_{fr} = \frac{T}{R}$. A simple calculation yields,

$$\mu' = \frac{4\mu \sin \theta_0}{2\theta_0 + \sin 2\theta_0}$$

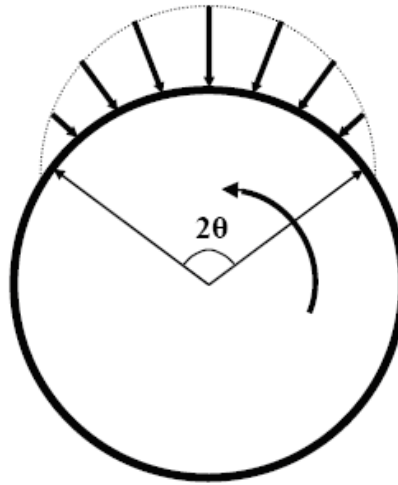


Fig. Pressure distribution on brake

It may be seen that for very small value of θ_0 , $\mu = \mu'$. Even when $\theta_0 = 30^\circ$, $\mu' = 1.0453\mu$. Usually if the contact angle is below 60° , the two values of friction coefficient are taken to be equal.

Consider now single shoe brakes as shown in figures below. Suppose a force P is applied at the end of a lever arm with length l . The shoe placed at a distance x from the hinge experiences a normal force N and a friction force F , whose direction depends upon the sense of rotation of the drum. Drawing free body diagram of the lever and taking moment about the hinge one gets

(a) For clockwise rotation of the brake wheel,

$$Nx + Fa = Pl$$

(b) For anticlockwise rotation of the brake wheel,

$$Nx - Fa = Pl.$$

Where a is the distance between the hinge and the line of action of F and is measured positive when F acts below point O as shown in the figure. Using Coulomb's law of friction the following results are obtained,

$$(a) \text{ For clockwise rotation } F = \frac{\mu Pl}{x + \mu a},$$

$$(b) \text{ For anticlockwise rotation } F = \frac{\mu Pl}{x - \mu a},$$

It may be noted that for anticlockwise rotating brake, if $\mu > \frac{x}{a}$, then the force P has negative value implying that a force is to applied in the opposite direction to bring the lever to

equilibrium. Without any force the shoe will, in this case, draw the lever closer to the drum by itself. This kind of brake is known as 'self-locking, brake. Two points deserve attention.

(1) If $a < 0$, the drum brake with clockwise rotation becomes self-energizing and if friction is large, may be self locking.

(2) If the brake is self locking for one direction, it is never self locking for the opposite direction. This makes the self locking brakes useful for 'back stops of the rotors.

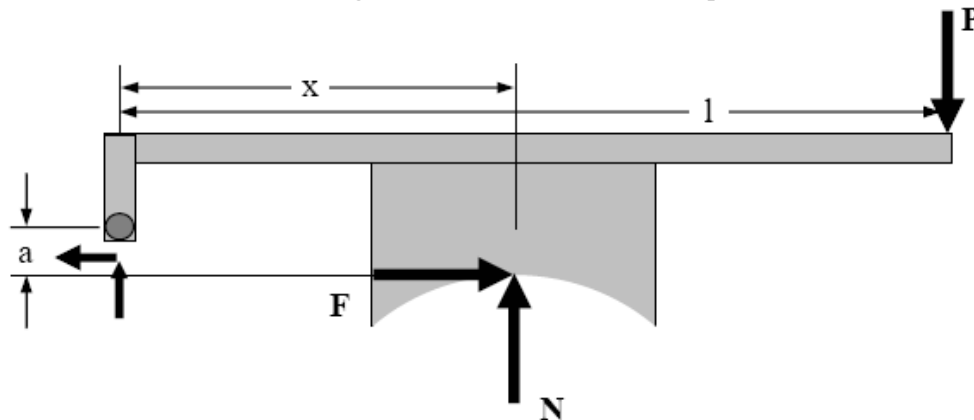


Figure: FBD of shoe (CW drum rotation)

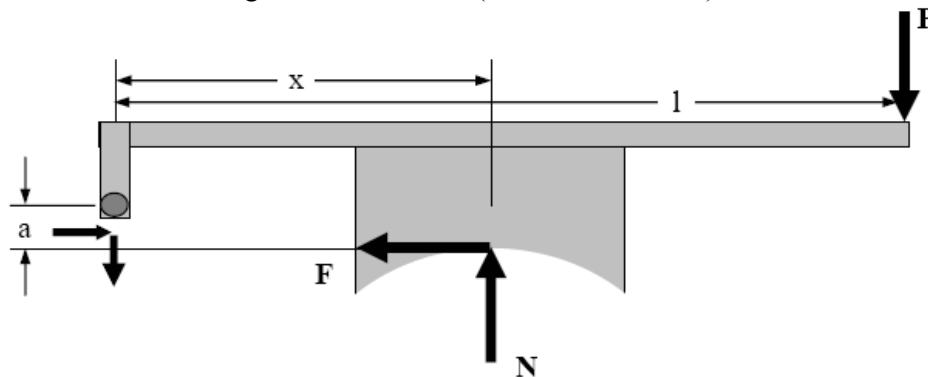


Figure: FBD of shoe (CCW drum rotation)

Double shoe brake

Since in a single shoe brake normal force introduces transverse loading on the shaft on which the brake drum is mounted two shoes are often used to provide braking torque. The opposite forces on two shoes minimize the transverse loading. The analysis of the double shoe brake is very similar to the single shoe brake.

External expanding shoe brake

An external expanding shoe brake consists of two symmetrically placed shoes having inner surfaces coated with frictional lining. Each shoe can rotate about respective fulcrum (say, O_1 and O_2). A schematic diagram with only one shoe is presented (figure below) When the shoes are engaged, non-uniform pressure develops between the friction lining and the drum. The pressure is assumed to be proportional to wear which is in turn proportional to the perpendicular distance from pivoting point (O_1N in figure below). A simple geometrical consideration reveals that this distance is proportional to sine of the angle between the line

joining the pivot and the center of the drum and the line joining the center and the chosen point. This means

$$p(\theta) = p_0 \sin \theta,$$

Where the angle is measured from line OO_1 and is limited as $\theta_1 \leq \theta \leq \theta_2$.

Drawing the free body diagram of one of the shoes (left shoe, for example) and writing the moment equilibrium equation about O_1 (say) the following equation is resulted for clockwise rotation of the drum :

$$F_1 l = M_p - M_f,$$

Where F_1 is the force applied at the end of the shoe, and

$$M_p = \frac{1}{2} p_0 b R \delta \left[(\theta_2 - \theta_1) + \frac{1}{2} (\sin 2\theta_1 - \sin 2\theta_2) \right],$$

$$M_f = \frac{1}{2} \mu_0 b R \delta \left[R (\cos \theta_1 - \cos \theta_2) - \frac{\delta}{4} (\cos 2\theta_1 - \cos 2\theta_2) \right],$$

Where δ is the distance between the center and the pivot (OO_1 in figure below) and ℓ is the distance from the pivot to the line of action of the force F_1 (O_1C in the figure). In a similar manner the force to be applied at the other shoe can be obtained from the equation

$$F_2 l = M_p + M_f$$

The net braking torque in this case is

$$T = \mu p_0 b R^2 (\cos \theta_1 - \cos \theta_2).$$

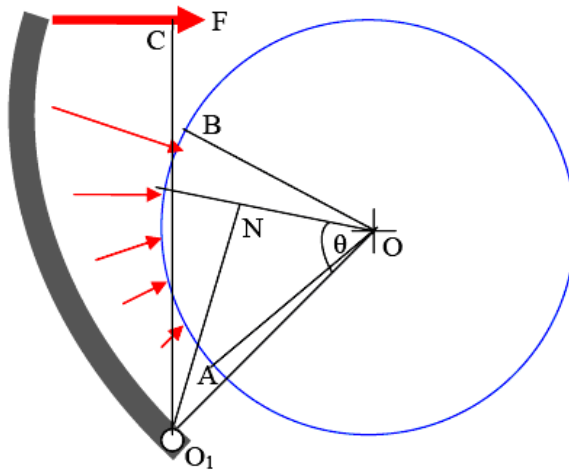


Figure: Force distribution in externally expanding brake.

Internal expanding shoe brake

Here the brake shoes are engaged with the internal surface of the drum. The analysis runs in the similar fashion as that of an external shoe brake.

The forces required are

$$F_1 = M_p + M_f/l$$

And

$$F_2 = (M_p - M_f)/l,$$

Respectively

One of the important members of the expanding shoe brakes is the anchor pin. The size of the pin is to be properly selected depending upon the face acting on it during brake engagement.

1. Band brakes

The operating principle of this type of brake is the following. A flexible band of leather or rope or steel with friction lining is wound round a drum. Frictional torque is generated when tension is applied to the band. It is known (see any text book on engineering mechanics) that the tensions in the two ends of the band are unequal because of friction and bear the following relationship:

$$\frac{T_1}{T_2} = e^{\mu\beta},$$

Where T_1 = tension in the taut side,

T_2 = tension in the slack side,

μ = coefficient of kinetic friction and

β = angle of wrap.

If the band is wound around a drum of radius R , then the braking torque is

$$T_{br} = (T_1 - T_2)R = T_1(1 - e^{-\mu\beta})R$$

Depending upon the connection of the band to the lever arm, the member responsible for application of the tensions, the band brakes are of two types,

(a) Simple band brake:

In simple band brake one end of the band is attached to the fulcrum of the lever arm (see figures below). The required force to be applied to the lever is:

$$P = T_1 \frac{b}{l} = \text{for clockwise rotation of the brake drum and}$$

$$P = T_2 \frac{b}{l} = \text{for anticlockwise rotation of the brake drum,}$$

Where l = length of the lever arm and

b = perpendicular distance from the fulcrum to the point of attachment of other end of the band.

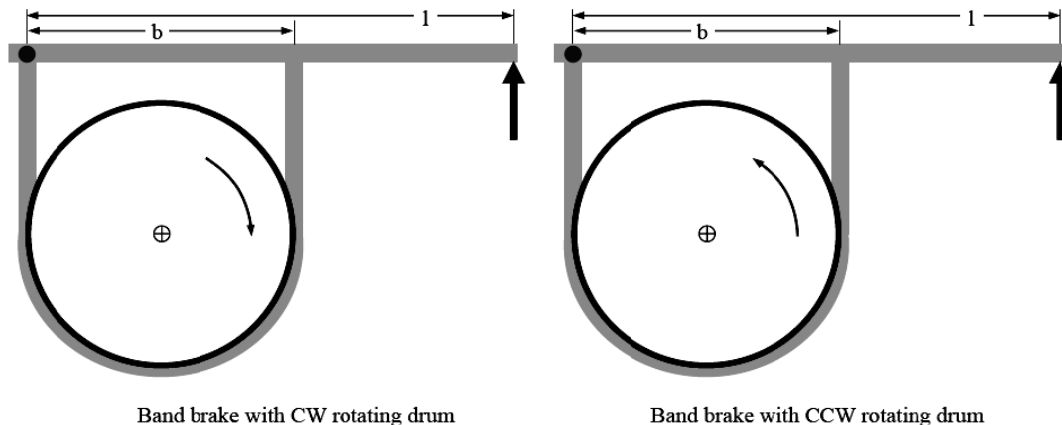


Figure Band brakes

(b) Differential band brake:

In this type of band brake, two ends of the band are attached to two points on the lever arm other than fulcrum (see figures above). Drawing the free body diagram of the lever arm and taking moment about the fulcrum it is found that

$$P = T_2 \frac{a}{l} - T_1 \frac{a}{l}, \text{ for clockwise rotation of the brake drum and}$$

$$P = T_1 \frac{a}{l} - T_2 \frac{a}{l}, \text{ For anticlockwise rotation of the brake drum.}$$

Hence, P is negative if

$$e^{\mu\beta} = \frac{T_1}{T_2} > \frac{a}{b} \text{ For clockwise rotation of the brake drum}$$

And $e^{\mu\beta} = \frac{T_1}{T_2} < \frac{a}{b}$ for counterclockwise rotation of the brake drum. In these cases the force is

to be applied on the lever arm in opposite direction to maintain equilibrium. The brakes are then self locking.

The important design variables of a band brake are the thickness and width of the band. Since the band is likely to fail in tension, the following:

Relationship is to be satisfied for safe operation.

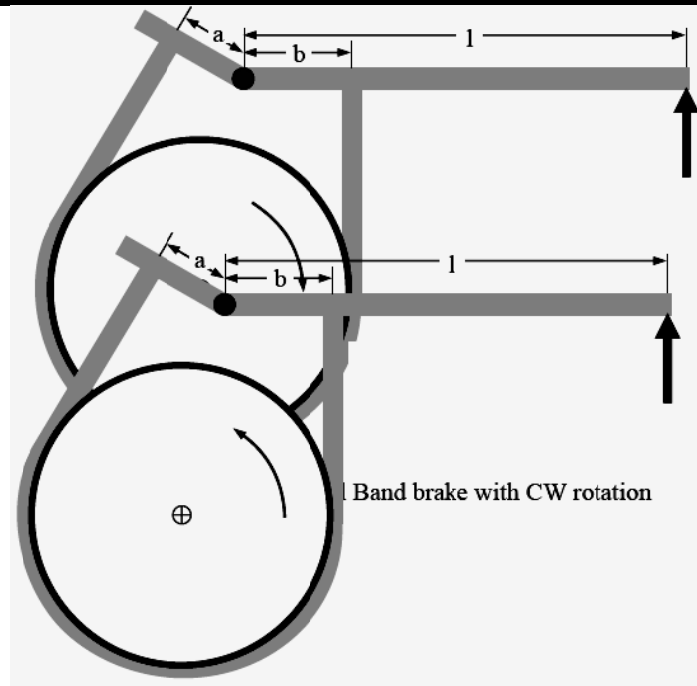
$$T_1 = wts_T$$

Where w = width of the band,

t = thickness of the band and

s_T = allowable tensile stress of the band material. The steel bands of the following dimensions are normally used

w	25-40 mm	40-60 mm	80 mm	100 mm	140-200 mm
t	3 mm	3-4 mm	4-6 mm	4-7 mm	6-10 mm



2. Band and block brakes

Sometimes instead of applying continuous friction lining along the band, blocks of wood or other frictional materials are inserted between the band and the drum. In this case the tensions within the band at both sides of a block bear the relation

$$\frac{T_1}{T_1'} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta},$$

Where T_1 = tension at the taut side of any block

T_1' = tension at the slack side of the same block

2θ = angle subtended by each block at center.

If n number of blocks are used then the ratio between the tensions at taut side to slack side becomes

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right)^n$$

The braking torque is $T_{br} = (T_1 - T_2)R$

3. Disc brake

In this type of brake two friction pads are pressed axially against a rotating disc to dissipate kinetic energy. The working principle is very similar to friction clutch. When the pads are new the pressure distribution at pad-disc interface is uniform, i.e.

$$P = \text{constant.}$$

If F is the total axial force applied then $p = \frac{F}{A}$, where A is the area of the pad.

The frictional torque is given by

$$T_{\text{braking}} = \frac{\mu F}{A} \oint_A r dA$$

where μ = coefficient of kinetic friction and r is the radial distance of an infinitesimal element of pad. After some time the pad gradually wears away. The wear becomes uniform after sufficiently long time, when

$$pr = \text{constant} = c \text{ (say)}$$

where $F = \oint p dA = c \oint \frac{dA}{r}$. The braking torque is

$$T_{\text{braking}}' = \mu \oint p r dA = \mu A c = \frac{\mu A F}{\oint \frac{dA}{r}}$$

It is clear that the total braking torque depends on the geometry of the pad. If the annular pad is used then

$$T_{\text{br}} = \frac{2}{3} \mu F \left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2} \right)$$

$$T_{\text{br}}' = \mu F \left(\frac{R_1 + R_2}{2} \right)$$

Where R_1 and R_2 are the inner and outer radius of the pad.

4. Friction materials and their properties

The most important member in a mechanical brake is the friction material. A Good friction material is required to possess the following properties:

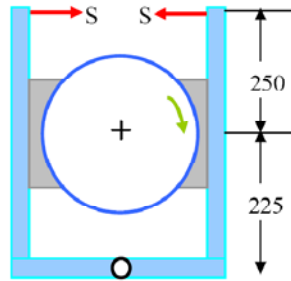
- High and reproducible coefficient of friction.
- Imperviousness to environmental conditions.
- Ability to withstand high temperature (thermal stability)
- High wear resistance.
- Flexibility and conformability to any surface.

Some common friction materials are woven cotton lining, woven asbestos lining, molded asbestos lining, molded asbestos pad, Sintered metal pads etc.

Review questions and answers

Q1. A double shoe brake has diameter of brake drum 300mm and contact angle of each shoe 90 degrees, as shown in figure below. If the coefficient of friction for the

brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 Nm. Also determine the width of the brake shoe if the braking pressure on the lining material is not to exceed 0.28 MPa.



Ans. The friction force required to produce the given torque is

$$F_1 + F_2 = \frac{30}{0.150} = 200(\text{N})$$

The normal forces on the shoes are $N_1 = \frac{F_1}{\mu'}$, $N_2 = \frac{F_2}{\mu'}$, where $\mu' = \frac{4\mu \sin \theta_0}{2\theta_0 + \sin 2\theta_0}$ ($\theta_0 = \frac{\pi}{4}$) = 0.44.

writing the moment equilibrium equations about the pivot points of individual shoes (draw correct FBDs and verify)

$$-Sl + N_1x + F_1a = 0 \Rightarrow F_1 = \frac{Sl}{a + \frac{x}{\mu'}} = 0.718S, \text{ and}$$

$$Sl + N_2x + F_2a = 0 \Rightarrow F_2 = \frac{Sl}{\frac{x}{\mu'} - a} = 1.1314S$$

This yields $S = 98.4(\text{N})$.

ANSWER The design of belt is to be carried out when the braking torque is maximum i.e. $T_{br} = 1000 \text{ N-m}$. According to the principle of band brake

$$T_{br} = T_1(1 - e^{-\mu\beta})R = T_1\left(1 - e^{-0.3 \times \frac{4\pi}{3}}\right) \times 0.25$$

Which yield $T_1 = 5587\text{N}$, $T_2 = e^{-\mu\beta}T_1 = 1587\text{N}$. In order to find the pressure on the band, consider an infinitesimal element. The force balance along the radial direction yields $N = T\Delta\theta$

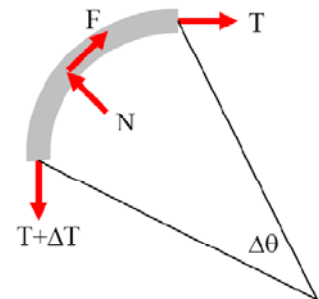
Since $N = pbR\Delta\theta$ so $p = \frac{T}{bR}$.

The maximum pressure is $p_{\max} = \frac{T_1}{bR}$.

Hence $b = \frac{5587}{0.25 \times 0.2 \times 10^6} = 0.112\text{m}$ (approx.)

The thickness t of the band is calculated from the relation $S_t bt = T_1$

Which yields $t = \frac{5587}{70 \times 10^6 \times 0.1117} = 0.0007145 \text{ m}$ or 1 mm (approx.).



Objective Questions (For GATE, IES & IAS)

Previous 20-Yrs GATE Questions

GATE-1. In a 2-D CAD package, clockwise circular arc of radius 5, specified from $P_1(15, 10)$ to $P_2(10, 15)$ will have its center at [GATE-2004]

- (a) (10, 10) (b) (15, 10) (c) (15, 15) (d) (10, 15)

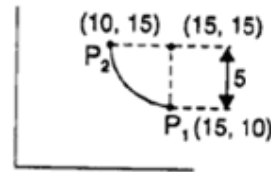
GATE-1. Ans. (c)

Given: $P_1(15, 10)$

$P_2(10, 15)$

Clearly from figure,

Centre of arc having radius
= 5 is (15, 15)



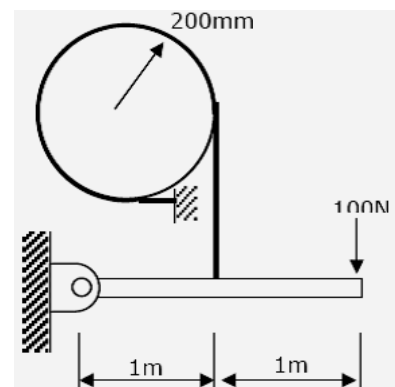
GATE-2. A band brake having band-width of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270 degrees is required to exert a friction torque of 1000 N m. The maximum tension (in kN) developed in the band is [GATE-2010]

- (a) 1.88 (b) 3.56 (c) 6.12 (d) 11.56

GATE-2. Ans. (d)

Statement for Linked Answer GATE- 3 and GATE-4:

A band brake consists of a lever attached to one end of the band. The other end of the band is fixed to the ground. The wheel has a radius of 200 mm and the wrap angle of the band is 270°. The braking force applied to the lever is limited to 100 N, and the coefficient of friction between the band and the wheel is 0.5. No other information is given.



GATE-3. The maximum tension that can be generated in the band during braking is [GATE-2005]

- (a) 1200 N (b) 2110 N (c) 3224 N (d) 4420 N

GATE-3. Ans. (b)

Taking moment about hinge

$$T_2 \times 1 = 100 \times 2$$

$$\frac{T_1}{T_2} = e^{\mu\theta}, \quad \text{where} \quad \theta = \frac{3\pi}{2}$$

GATE-4. The maximum wheel torque that can be completely braked is [GATE-2005]

- (a) 200 N.m (b) 382 N.m (c) 604 N.m (d) 844 N.m

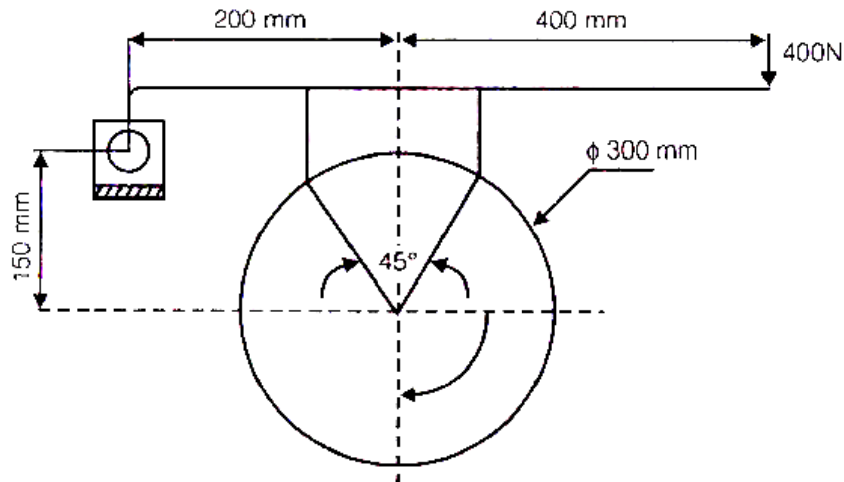
GATE-4. Ans. (b)

GATE-5. In a band brake the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is 180° the coefficient of friction required between drum and the band is [GATE-2003]

- (a) 0.20 (b) 0.25 (c) 0.30 (d) 0.35

GATE-5. Ans. (d)

GATE-6. A block-brake shown below has a face width of 300 mm and a mean coefficient of friction of 0.25. For an activating force of 400 N, the braking torque in Nm is [GATE-2007]



- (a) 30 (b) 40 (c) 45 (d) 60

GATE-6. Ans. (c)

Previous 20-Yrs IES Questions

IES-1. What is the correct sequence of the following steps in engine analysis?

1. Vibration analysis
2. Inertia force analysis. [IES-1997]
3. Balancing analysis
4. Velocity and Acceleration analysis.

Select the correct answer using the codes given below:

- (a) 2, 4, 1, 3 (b) 2, 4, 3, 1 (c) 4, 2, 1, 3 (d) 4, 2, 3, 1

IES-1. Ans. (c)

IES-2. A device for lifting or lowering objects suspended from a hook at the end of a retractable chain or cable is called [IES-1994; 1995]

- (a) Hoist (b) jib crane (c) chain conveyor (d) elevator

IES-2. Ans. (a)

IES-3. Consider the following design considerations: [IES-1995]

1. Tensile failure
2. Creep failure
3. Bearing failure
4. Shearing failure
5. Bending failure

The design of the pin of a rocker arm of an I.C. engine is based on

- (a) 1, 2 and 4 (b) 1, 3 and 4 (c) 2, 3 and 5 (d) 3, 4 and 5.

IES-3. Ans. (d) Design of pin of a rocker arm of an I.C. engine is based on bearing, shearing, and bending failures.

IES-4. Consider the following statements regarding the differential of an automobile: [IES-1994]

1. The speed of the crown wheel will always be the mean of the speeds of the two road wheels.
2. The road wheel speeds are independent of the number of teeth on the planets.
3. The difference between the speeds of the road wheels depends on the number of teeth on the planets.
4. The ratio of speeds of the road wheels depends upon the number of teeth on the gear wheels attached to them and on the crown wheel.

Of these statements

- | | |
|-------------------------|--------------------------|
| (a) 1 and 2 are correct | (b) 3 and 4 are correct |
| (c) 1 and 3 are correct | (d) 2 and 4 are correct. |

IES-4. Ans. (d)

IES-5. Interchangeability can be achieved by [IES-1993]

- | | |
|---------------------|-----------------------------|
| (a) Standardisation | (b) better process planning |
| (c) Simplification | (d) better product planning |

IES-5. Ans. (a) Interchangeability can be achieved by standardisation.

IES-6. In an automobile service station, an automobile is in a lifted up position by means of a hydraulic jack. A person working in the service station gave a tap to one rear wheel and made it rotate by one revolution. The rotation of another rear wheel is [IES-1993]

- (a) Zero
- (b) Also one revolution in the same direction
- (c) Also one revolution but in the opposite direction
- (d) unpredictable

IES-6. Ans. (a) When one rear wheel is rotated, other is free.

IES-7. Which of the following stresses are associated with the tightening of a nut on a stud?

1. Tensile stresses due to stretching of stud.
2. Bending stresses of stud. [IES-1993]
3. Transverse shear stresses across threads.
4. Torsional shear stresses in threads due to frictional resistance.

Select the correct answer using the codes given below:

- | | | | |
|----------------|----------------|----------------|----------------|
| (a) 1, 2 and 3 | (b) 1, 2 and 4 | (c) 2, 3 and 4 | (d) 1, 3 and 4 |
|----------------|----------------|----------------|----------------|

IES-7. Ans. (a)

IES-8. Match the following [IES-1992]

List -I (Dynamometer)

A. Torsion Dynamometer

B. Tesla fluid friction dynamometer

C. Prony brake

D. Swinging field dynamometer

List - II (Characteristics)

1. High speeds and low power

2. Power absorbed independent of size of flywheel.

3. Power absorbed available for useful applications

4. Large powers

IES-8. Ans. (d)

Previous 20-Yrs IAS Questions

IAS-1. Rope brake dynamometer uses [IAS-2001]

- (a) Water as lubricant (b) oil as lubricant
(c) Grease as lubricant (d) no lubricant

IAS-1. Ans. (d)

IAS-2. Consider the following statements regarding power:

1. It is the capacity of a machine. [IAS-1997]
2. The efficiency is always less than unity as every device operates with some loss of energy.
3. A dynamometer can measure the power by absorbing it.
4. Watt-hour is the unit of power.

Of these statements:

- (a) 1, 2 and 3 are correct (b) 2, 3 and 4 correct
(c) 1, 3 and 4 are correct (d) 1, 2 and 4 are correct

IAS-2. Ans. (a)

Answer with Explanation