7

(224)

224

-: HAND WRITTEN NOTES:-

OF

ELECTRICAL ENGINEERING

(1)

-: SUBJECT:-

SIGNALS & SYSTEMS

7

2

SIGNALS & SYSTEM

3

1. Signal defination & its classification
2. Different operations on signals
   - → Scaling → Integration
   - → Shifting → Differentiation
   - → Reversal → convolution
3. Basic System Properties
   - → Static / Dynamic
   - → Causal / Non - causal
   - → Linear / Non - linear
   - → Time invariant / time variant
   - → Stable / Unstable
4. Fourier Series
5. Fourier Transform
6. Laplace Transform
7. Sampling Theorem
8. Discrete Time System
9. Z - Transform

Continuous Time System (items 1–5)

Discrete Time System (items 7–9)

Important Topics (GATE):

1. Z - transform
2. Discrete Time System
3. Laplace Transform
4. Fourier Transform
5. Basic System properties
6. Fourier Series
7. RMS / Power Calculations

DIFFERENT OPERATIONS ON SIGNALS

→ Shifting
→ Scaling
→ Reversal

(4)

1 Shifting → Time shifting
→ Amplitude shifting.

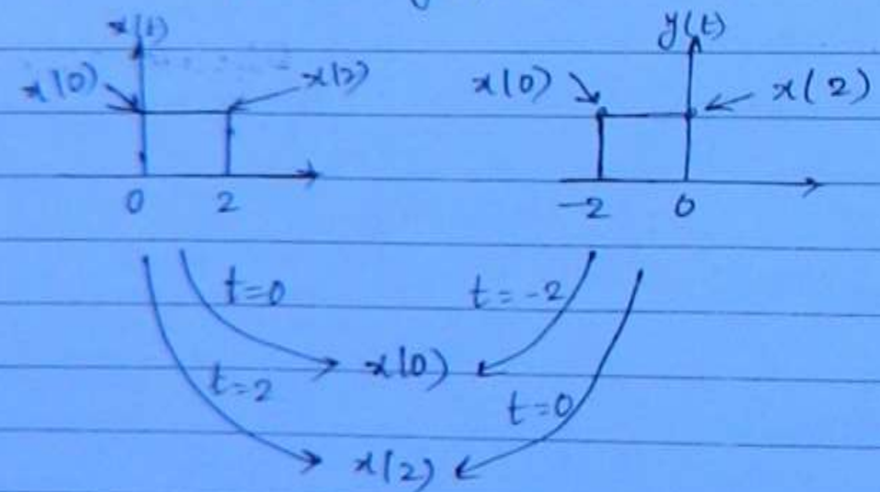
(1). Time Shifting -

$x(t) \longrightarrow x(t+k) = y(t)$

case (a) <u>when $k > 0$</u>

Ex $k = 2$.

$x(t) \longrightarrow y(t) = x(t+2)$



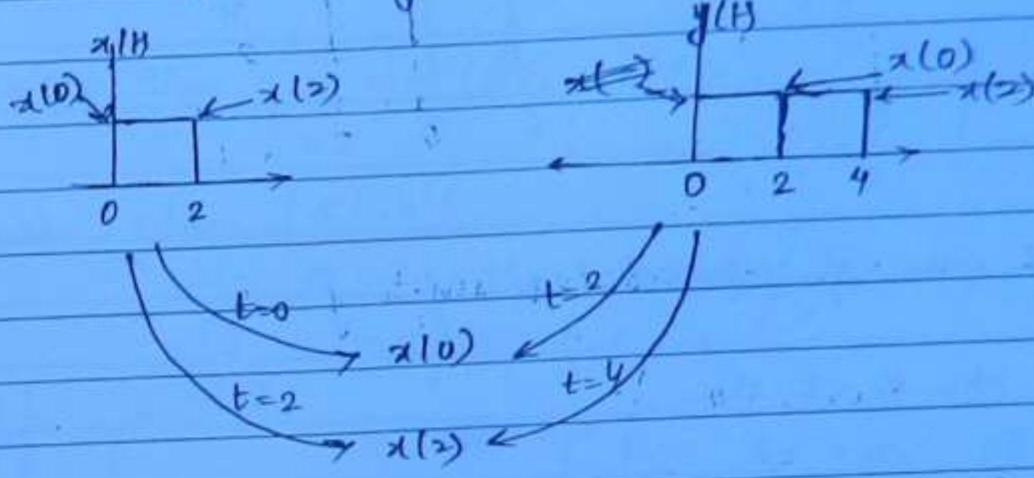
∴ Its a case of <u>left shifting</u> (or)
<u>time advance</u>

case (b) <u>when $k<0$</u>

Ex $k = -2$

(5)

$$x(t) \longrightarrow y(t) = x(t-2)$$



∴ Its a case of <u>right shifting</u> or <u>time delay</u>

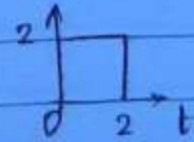
(ii) Amplitude Shifting -

$$x(t) \longrightarrow y(t) = x(t) + k$$

case (a) <u>when $k>0$</u>

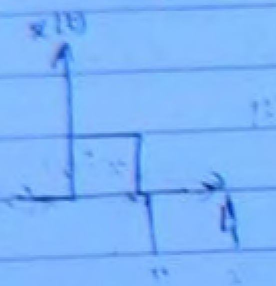
Ex $k = +2$

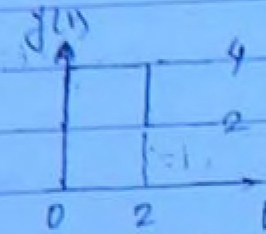
$$x(t) \longrightarrow y(t) = x(t) + 2$$



$$x(t) = \begin{cases} 0 & t<0 \\ 2 & 0 \le t \le 2 \\ 0 & t>2 \end{cases} \qquad y(t) = x(t) + 2 \begin{cases} 0+2 & t<0 \\ 2+2 & 0 \le t \le 2 \\ 0+2 & t>2 \end{cases}$$

$$y(t) = \begin{cases} 2 & t < 0 \\ 4 & 0 \le t \le 2 \\ 2 & t > 2 \end{cases}$$

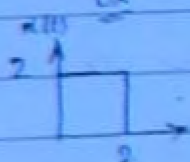




It's a case of <u>upward shifting</u>.

case (b) <u>when $K < 0$</u>

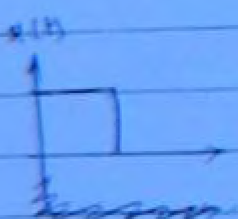
Ex $K = -2$

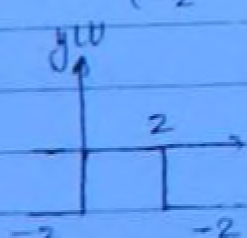


$x(t) \longrightarrow y(t) = x(t) - 2$

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases} \qquad y(t) = x(t) - 2 = \begin{cases} 0 - 2 & t < 0 \\ 2 - 2 & 0 \le t \le 2 \\ 0 - 2 & t > 2 \end{cases}$$

$$y(t) = \begin{cases} -2 & t < 0 \\ 0 & 0 \le t \le 2 \\ -2 & t > 2 \end{cases}$$





It's a case of downward shifting.

2. Scaling → Time Scaling
→ Amplitude Scaling

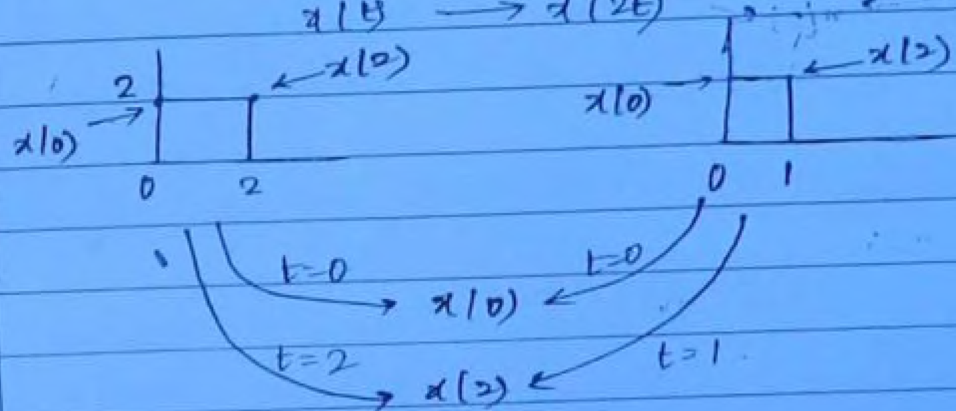
(i) Time Scaling

$x(t) \longrightarrow y(t) = x(\alpha t)$, $\alpha \neq 0$

case (a) <u>when $|\alpha| > 1$</u>

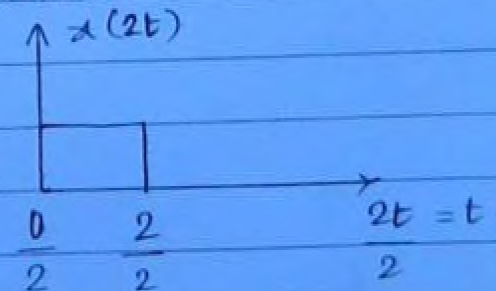
<u>Ex</u> $\alpha = 2$

$x(t) \longrightarrow x(2t)$



Its a case of <u>time compression</u>

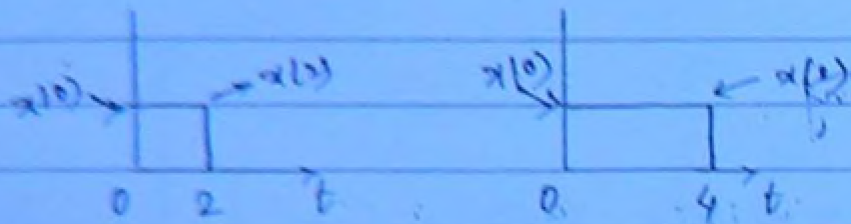
2^{nd} method : Shortcut



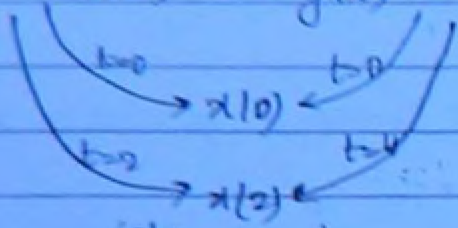
Divide axis of αt & its elements by α

case (b) when $|\alpha| < 1$

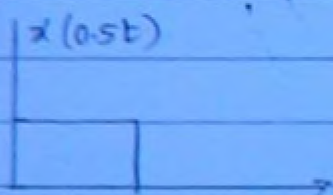
Ex $\alpha = 0.5$



$x(t) \rightarrow y(t) = x(0.5t)$



2nd method : Shortcut



$\frac{0}{0.5} = 0 \qquad \frac{2}{0.5} = 4 \qquad \frac{0.5t}{0.5} = t$

It's a case of time expansion

(ii) Amplitude Scaling

$x(t) \rightarrow y(t) = \alpha\, x(t)$ Ampli

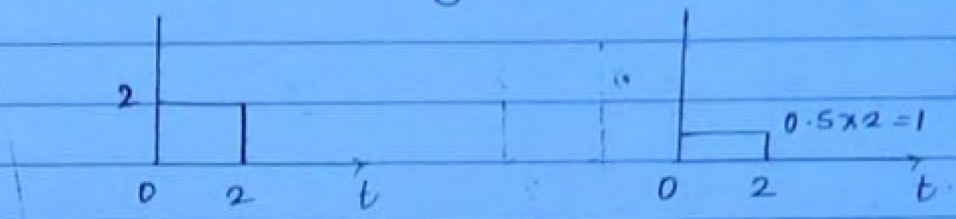
case (a) when $|\alpha| > 1$ Amplification

Ex $\alpha = 2$

$x(t) \rightarrow y(t) = 2x(t)$

Case (b) : when $|\alpha| < 1$: Attenuation
Ex $\alpha = 0.5$

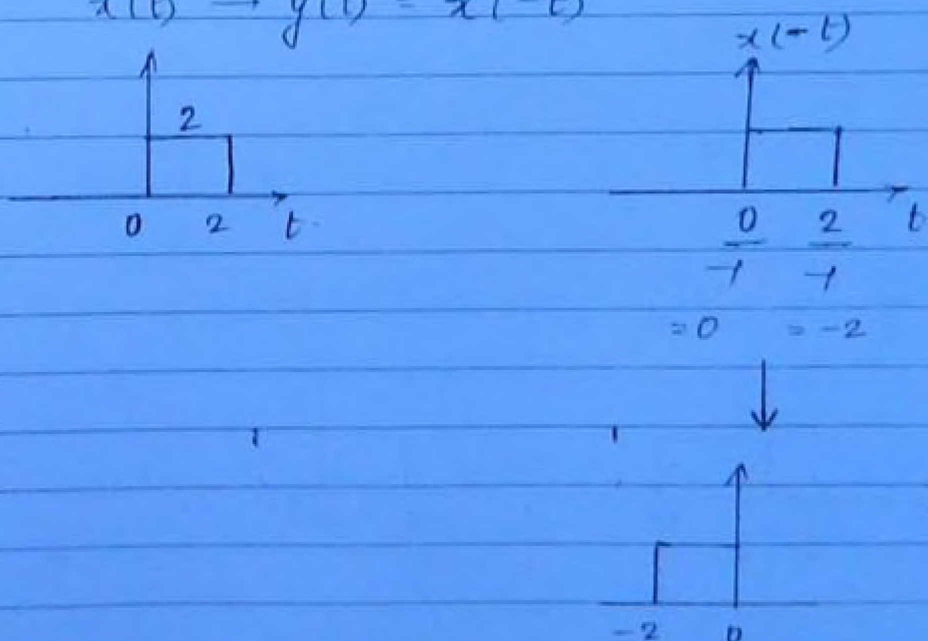
$x(t) \to y(t) = 0.5\,x(t)$



3. Reversal → Time Reversal
→ Amplitude Reversal

Its a special case of Scaling with "$\alpha = -1$"

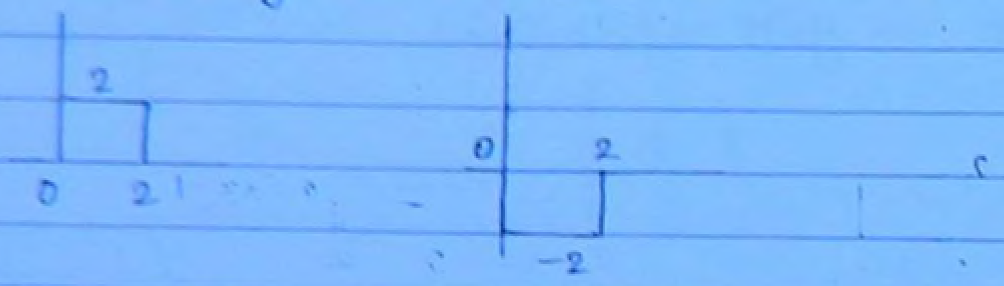
case (a)
(i) Time Reversal
$x(t) \to y(t) = x(-t)$



Signal folding takes place about y-axis

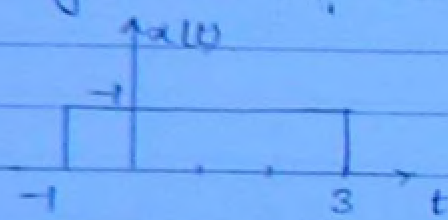
(ii) Amplitude Reversal

$$x(t) \rightarrow y(t) = -x(t)$$



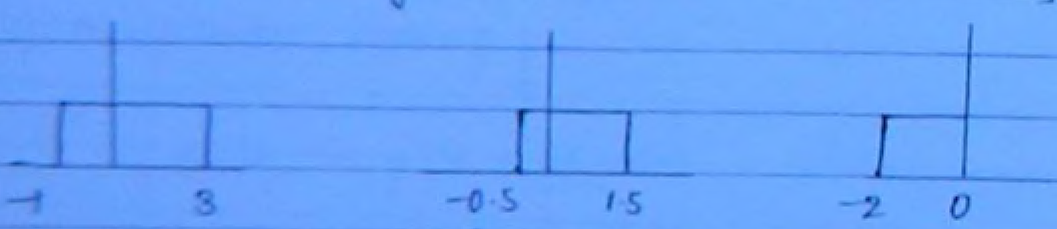
Signal folding takes place about x-axis.

Q Draw signal $x(2t+3)$



sol

I $x(t) \xrightarrow{\text{time scaling}} x(2t) \longrightarrow x[2(t+1.5)]$

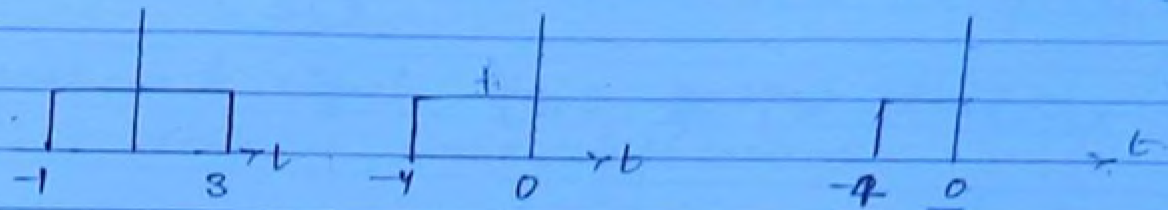


$x(t) \rightarrow x(t+1.5) \rightarrow x(2(t+1.5))$

not correct ans on the Axis

$(t+1.5)$ is not available.

II $x(t) \longrightarrow x(t+3) \longrightarrow x(2t+3)$ (11)

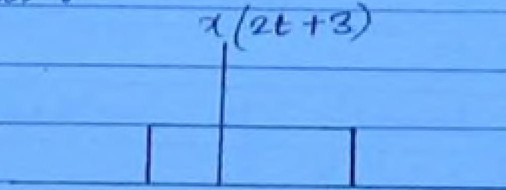


$\frac{-4}{2}$ $\frac{0}{2}$

$= -2$ $= 0$

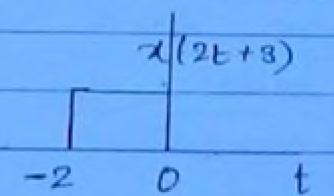
III

Shortcut :

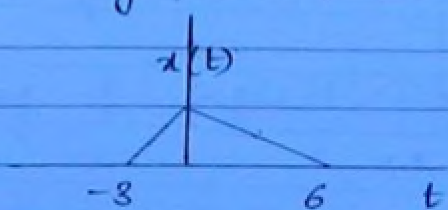


$\frac{-1-3}{2}$ $\frac{3-3}{2}$ $\frac{\cancel{(2t+3)}-\cancel{3}}{\cancel{2}} = t$

$= -2$ $= 0$



Q Draw signal $x(-2t+1)$



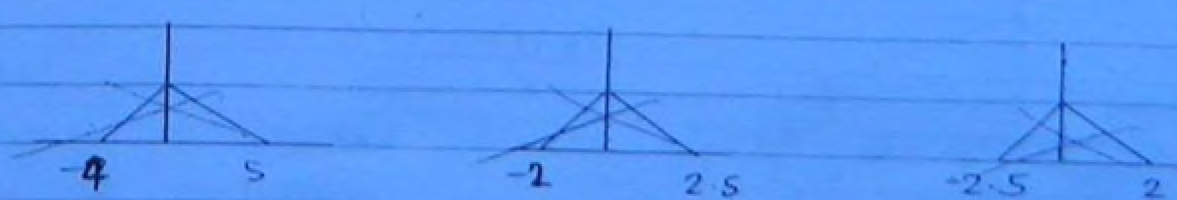
$\frac{-3-1}{-2}$ $\frac{6-1}{-2}$

$\frac{-4}{-2}$ $\frac{5}{-2}$

$= 2$ $= -2.5$

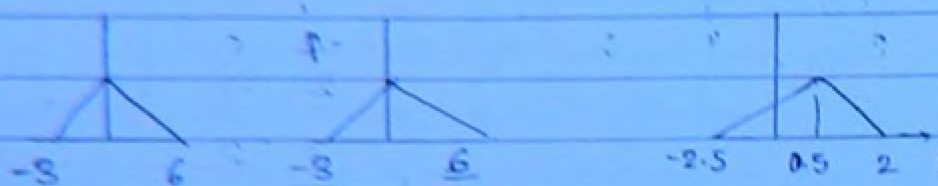
$\frac{-4}{-2}$ $\frac{5}{-2}$

Sol. $= 2$ $= -2.5$



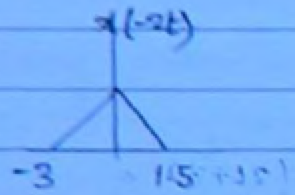
I $x(-2t+1) = x[-2(t-0.5)]$

$x(t) \rightarrow x(-2t) \rightarrow x[-2(t-0.5)]$

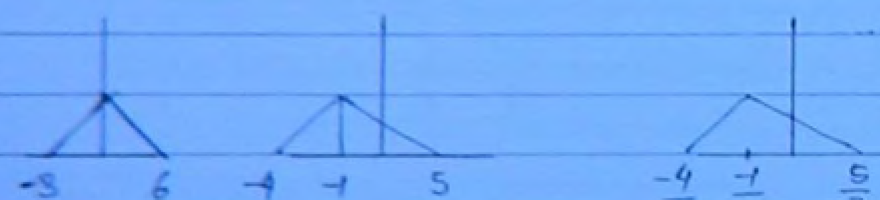


$\frac{-3}{-2}$ $\frac{6}{-2}$

$= 1.5$ $= -3$

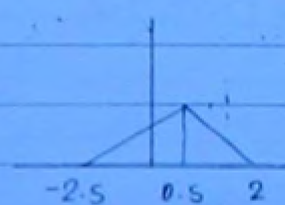


II $x(t) \longrightarrow x(t+1) \longrightarrow x(-2t+1)$



$\frac{-4}{-2}$ $\frac{-1}{-2}$ $\frac{5}{-2}$

$= 2$ $= 0.5$ $= -2.5$



III

$\frac{-3-1}{-2}$ $\frac{0-1}{-2}$ $\frac{6-1}{-2}$

$= 2$ $= 0.5$ $= -2.5$

$\frac{(-2t+1)-1}{-2} = t$

SIGNAL DEFINITION & ITS CLASSIFICATION

Signal → A signal is a ~~system~~ function which contains some information.

System → A system is interconnection of devices or components which convert signal from one form to another

Classification of Signal -

1. Continuous & Discrete -

Continuous time signal -
A signal is called continuous if variable on x axis is continuous in nature

Discrete time signal -
A signal is called discrete if variable on x axis is integral in nature.

2. Analog & Digital Signals.

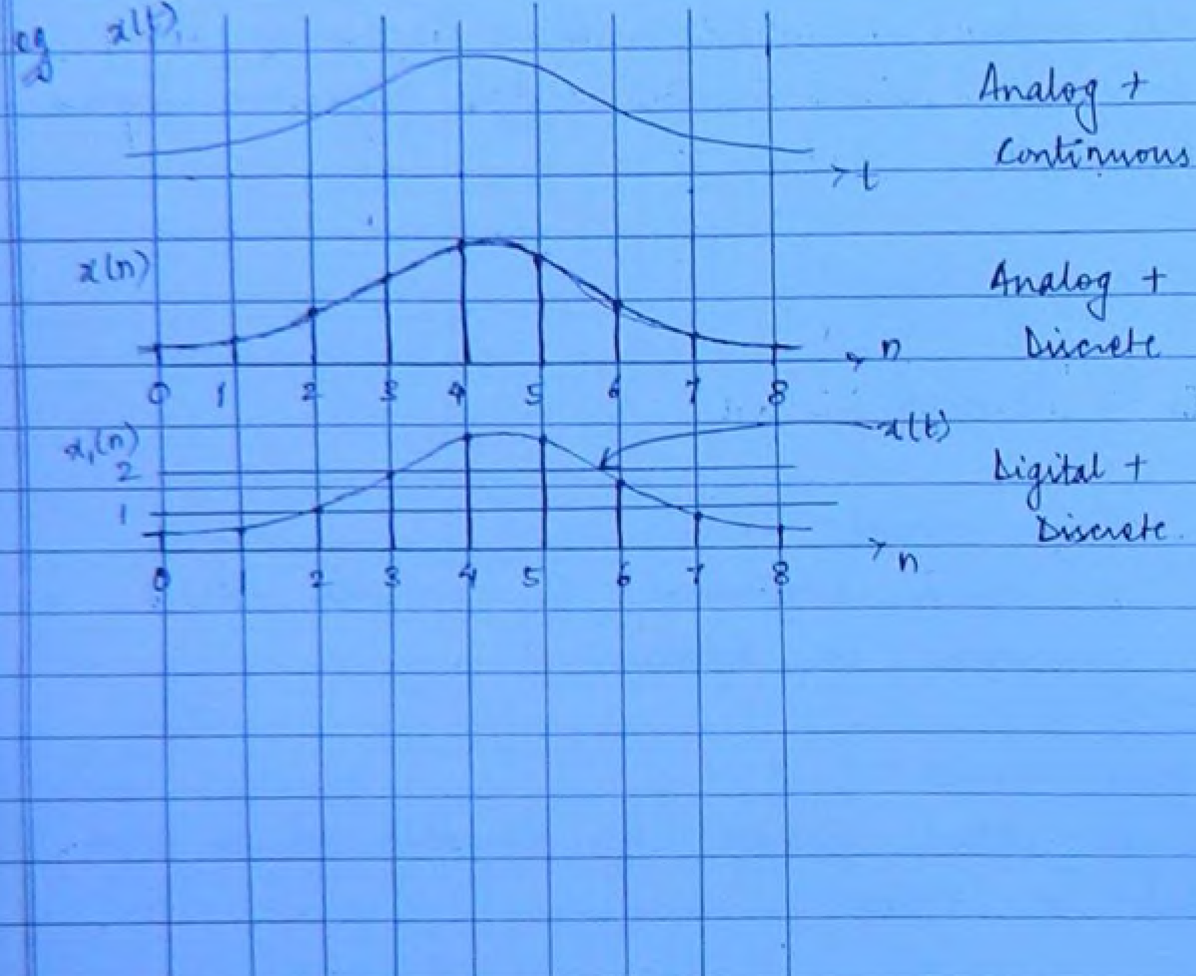
Analog -
A signal is called analog signal if it can take any value on y-axis or magnitude axis

Digital -
A signal is called digital signal if it can take only finite no. of values on y-axis

Analog & Digital terms are related to y-axis or magnitude axis (T4)

Continuous & Discrete terms are related to x-axis

eg



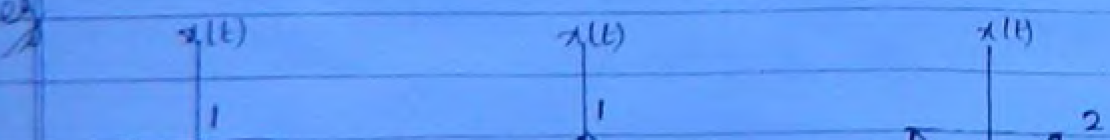
3 Odd & Even Signals

Even Signal

$$\boxed{x(t) = x(-t)}$$

Even signals are symmetrical or mirror images about y-axis

eg



eg $x(t) = \cos \omega_0 t \longleftarrow$ Even signal.

$$\Big\downarrow t = -t$$

$x(-t) = \cos(-\omega_0 t)$

$= \cos \omega_0 t$

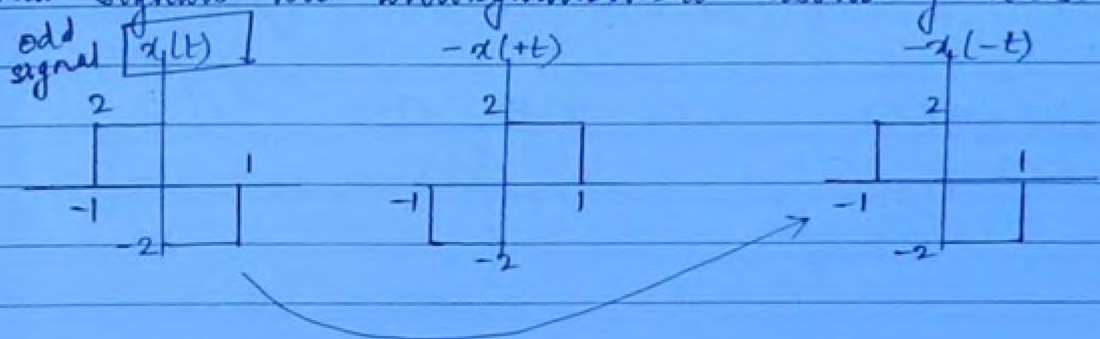
$x(-t) = x(t)$

Odd Signal

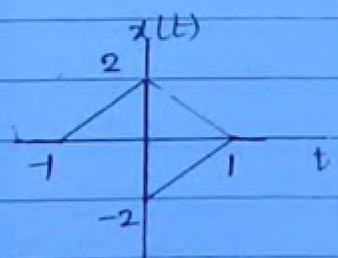
$$\boxed{\begin{array}{c} x(t) = -x(-t) \\ \text{or} \\ x(-t) = -x(t) \end{array}}$$

Odd signals are antisymmetrical about y-axis

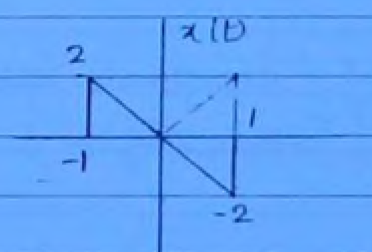
eg



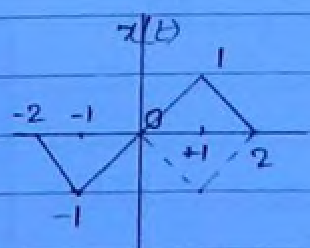
eg



eg



eg



eg $x(t) = \sin \omega_0 t$

$$\Big\downarrow t = -t$$

$x(-t) = \sin(-\omega_0 t)$

$= -\sin \omega_0 t$

$x(-t) = -x(t)$

* The average (or) mean (or) dc value of any odd signal is 0.

* The value of odd signal at origin is 0.

* Odd signal passes through origin.

* Any signal can be represented as a sum of 2 signals, one is even & other is odd.

$$x(t) = \underbrace{x_e(t)}_{\text{even part of } x(t)} + \underbrace{x_o(t)}_{\text{odd part of } x(t)}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

IMP:

* $x(t) = 2$ = DC / constant value signal
[Even signal]

* dc + even = even

eg $2 + t^2 = x(t)$

$\downarrow t = -t$

$2 + t^2 = x(-t)$

* dc + odd = neither even nor odd

eg $x(t) = 2 + t^3$

$\downarrow t = -t$

$x(-t) = 2 - t^3$

$E \times E = E$

eg $t^2 \times t^4 = t^6$

$O \times O = E$

eg $t^3 + t^5 = t^8$

$O \times E = O$

eg $t^3 \times t^2 = t^5$

$\frac{d}{dt}[\text{even}] = \text{odd}$ except dc signal

$\frac{d}{dt}[\text{odd}] = \text{even}$

$\int \text{even}\, dt = \text{odd}$

$\int \text{odd}\, dt = \text{even}$

$\frac{1}{O} = O$ $\frac{1}{E} = E$

Find $x_e(t)$ & $x_o(t)$ parts of

$$x(t) = t^2 \sin t - \frac{t^3}{\sin^2 t} + t^3 \cos t - \frac{\cos^3 t}{t^2} + \frac{t^5}{\sin^5 t}$$

$$x_e(t) = -\frac{\cos^3 t}{t^2} + \frac{t^5}{\sin^5 t} \quad \Big| \quad x_o(t) = t^2 \sin t - \frac{t^3}{\sin^2 t} + t^3 \cos t + \cancel{\frac{t^5}{\sin^5 t}}$$

4 Periodic and Non-periodic

Periodic -

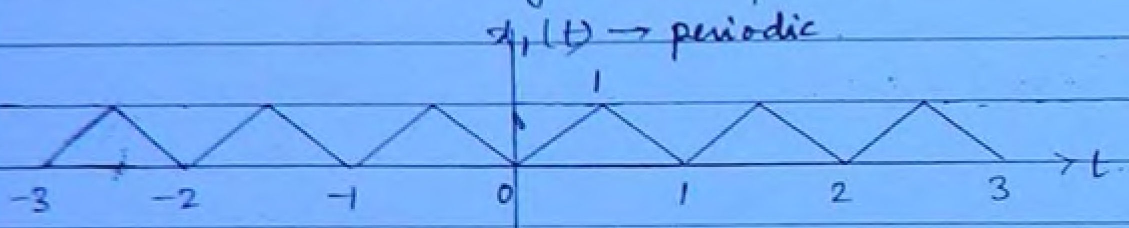
A signal is called periodic if it repeats itself after a certain time period.

ie $x(t) = x(t \pm nT_0)$

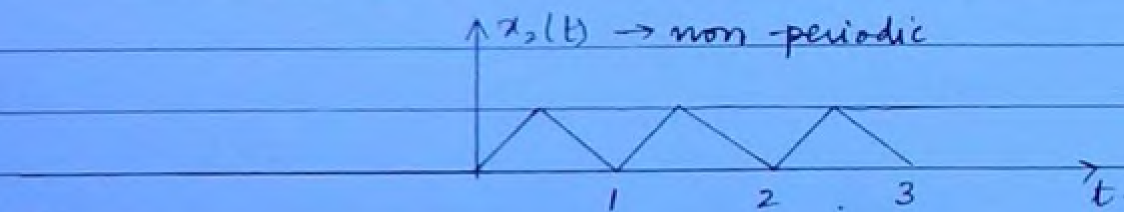
where n = an integer

T_0 = fundamental time period

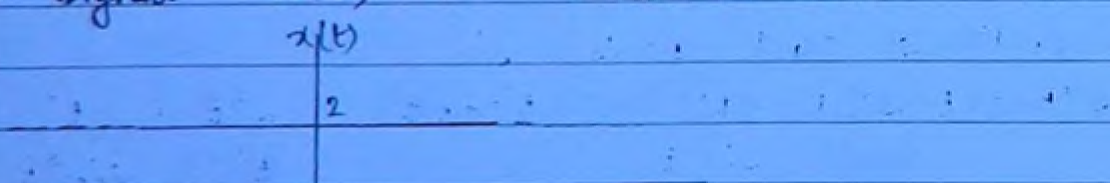
= smaller +ve (fixed) value of time for which signal is periodic



$T_0 = 1$



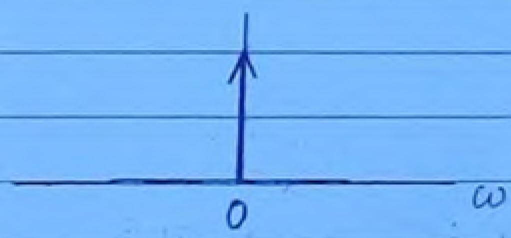
Q DC signal. $x(t) = 2$ Periodic or NP ??



Sol DC signals are periodic signals with undefined fundamental time period

Frequency -

$$x(t) = A_0 \underset{F^{-1}}{\overset{F}{\rightleftharpoons}} 2\pi A_0\, \delta(\omega) = X(\omega)$$



$$x(t) = A_0 = \lim_{\omega_0 \to 0} A_0 \cos \omega_0 t$$

* Frequency of DC signal is 0.

* $\boxed{T_0 = \frac{1}{f_0}}$

[fundamental time period]

$\therefore$ This relation cannot be used for DC
(only for periodic signals whose fundamental time period is defined)

for DC signal T_0 is undefined

* Sum of two or more periodic signals will be periodic if ratios of their fundamental time periods or fundamental frequencies are rational

ie $x(t) = \underbrace{x_1(t)}_{T_1, f_1, \omega_1} + \underbrace{x_2(t)}_{T_2, f_2, \omega_2}$

$\Rightarrow T_0 = LCM[T_1, T_2 \ldots]$

$f_0 = HCF[f_1, f_2 \ldots]$

Q $x(t) = A_0 e^{j\omega_0 t}$

$T_0 = ?$

Sol Let T_0 be the fundamental time period of signal.

ie $x(t) = x(t + T_0)$

$\Rightarrow \cancel{A_0} e^{j\omega_0 t} = \cancel{A_0} e^{j\omega_0(t+T_0)}$

$\Rightarrow \cancel{e^{j\omega_0 t}} = \cancel{e^{j\omega_0 t}} \cdot e^{j\omega_0 T_0}$

$e^{j\omega_0 T_0} = 1 = e^{j2\pi k}$

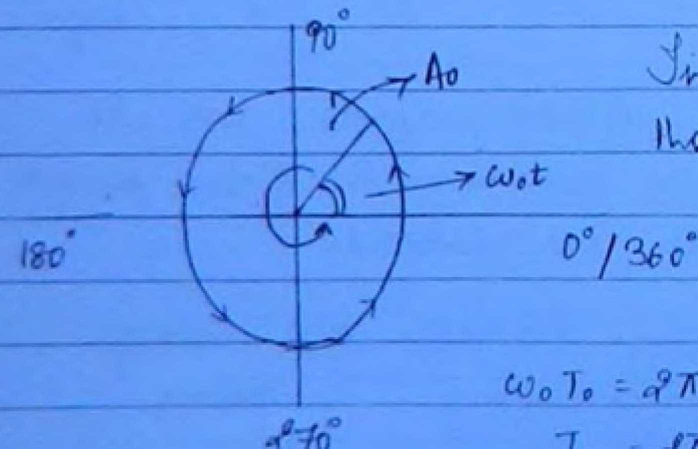
where k = an integer

$\omega_0 T_0 = 2\pi k$

$T_0 = \frac{2\pi}{\omega_0} \times k$

$T_0 = \frac{2\pi}{\omega_0}$

II



Time consumed by the polar plot to complete one rotation

$\omega_0 T_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0}$

6-1-13

Q. Calculate 'T_0'

i) $x(t) = \sin^2(4\pi t)$

ii) $x(t) = \sin 4\pi t + \cos 2t$

iii) $x(t) = \sin 6\pi t + \cos 5\pi t$

(21)

Sol

i) ~~$x(t) = \sin^2(4\pi t)$~~

~~$= \sin 4\pi t \times \sin 4\pi t$~~

~~$\frac{4\pi t}{4\pi} = 1$ ∴ Periodic~~

~~$T_0 = \frac{1}{4}$~~

$x(t) = \sin^2(4\pi t)$

$= \frac{1 - \cos 8\pi t}{2}$

$= \frac{1}{2} - \frac{\cos 8\pi t}{2}$

$\omega_0 = 8\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$

ii) $x(t) = \sin 4\pi t + \cos 2t$

$\omega_1 = 4\pi \qquad \omega_2 = 2$

$\frac{\omega_1}{\omega_2} = \frac{4\pi}{2} = 2\pi = IR$ no

signal $x(t)$ is non-periodic.

iii) $x(t) = \sin 6\pi t + \cos 5\pi t$

$\omega_1 = 6\pi \qquad \omega_2 = 5\pi$

$\omega_0 = HCF[\omega_1, \omega_2]$

$= HCF(6\pi, 5\pi)$

$= \pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$

5. Conjugate Symmetric & Conjugate Antisymmetric

22

Conjugate Symmetric (CS) signal -

$$\boxed{x(t) = x^*(-t)}$$

$x(t) = a(t) + jb(t)$ ----- (1)

$t = -t$

$x(-t) = a(-t) + jb(-t)$

$x^*(-t) = a(-t) - jb(-t)$ ---- (2)

In CS signal $x(t) = x^*(-t)$

From (1) & (2)

$a(t) = a(-t) \rightarrow$ Even

$b(t) = -b(-t) \rightarrow$ Odd

Conjugate Antisymmetric (CAS) signal -

$$\boxed{x(t) = -x^*(-t)}$$

From (2)

$-x^*(-t) = -a(-t) + jb(-t)$ — (3)

From (1) & (3)

$a(t) = -a(-t) \rightarrow$ Odd

$b(t) = b(-t) \rightarrow$ Even

Q. Check CS/CAS.

(23)

i) $x(t) = \underset{t}{2t^2} + j\underset{0}{\sin t}$

$\Rightarrow$ CS.

ii) $x(t) = \underset{0}{\sin t} + j\underset{0}{t^3}$

neither CS nor ACS.

iii) $x(t) = \underset{0}{\sin^3 t} + j\underset{E}{t^2}$

CAS.

— x —

Any signal can be represented as a sum of CS & ACS signal

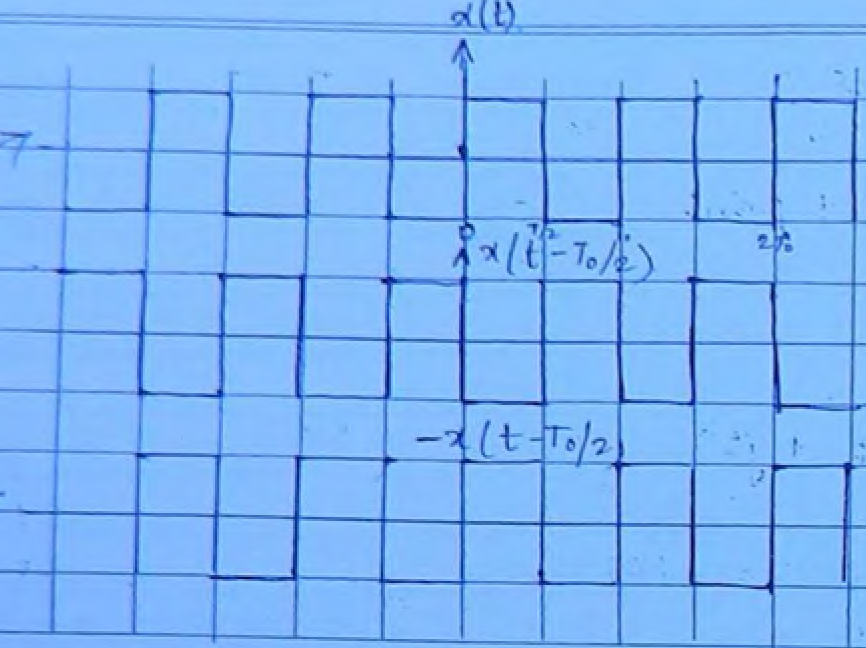
ie $x(t) = x_{cs}(t) + x_{CAS}(t)$

where $x_{cs}(t) = \dfrac{x(t) + x^*(-t)}{2}$ & $x_{CAS}(t) = \dfrac{x(t) - x^*(-t)}{2}$

6. Half Wave Symmetricity - (HWS)

Eq. $\boxed{x(t) = -x\left(t \pm \frac{T}{2}\right)}$

eg



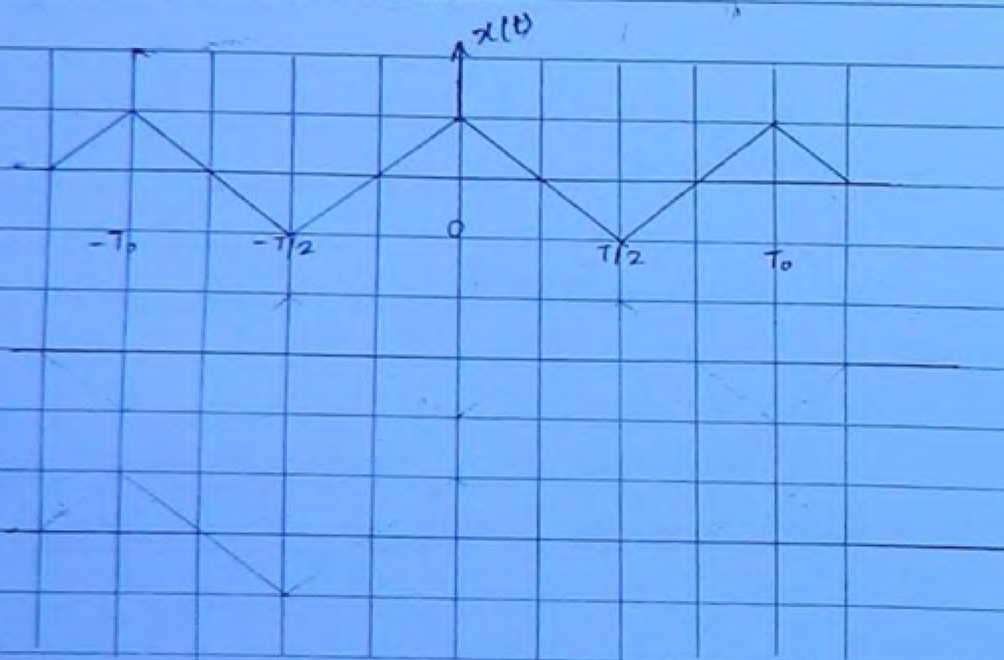
are same

shift either to right or left by T/2

take mirror image about x-axis

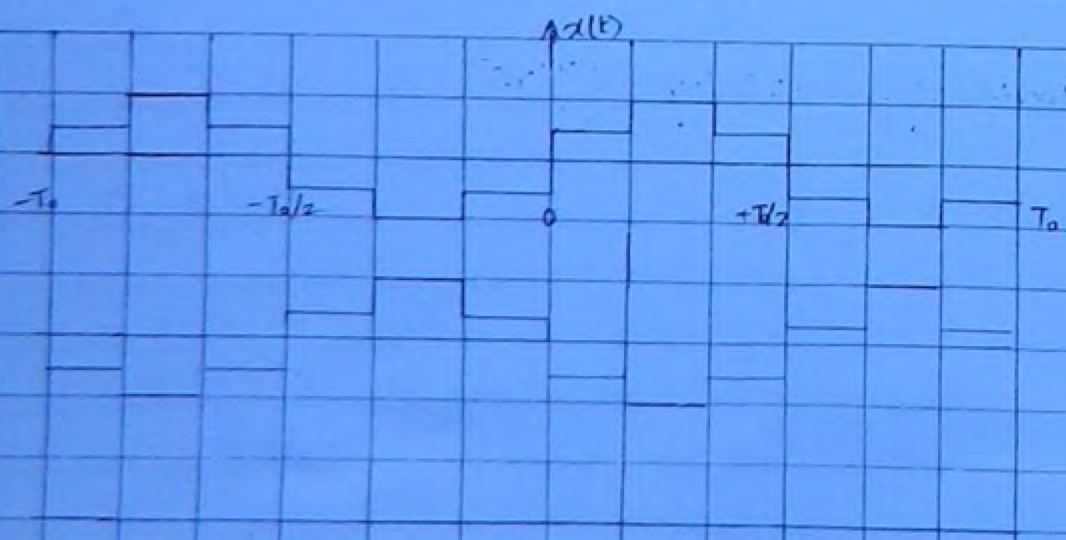
∴ HWS

Q

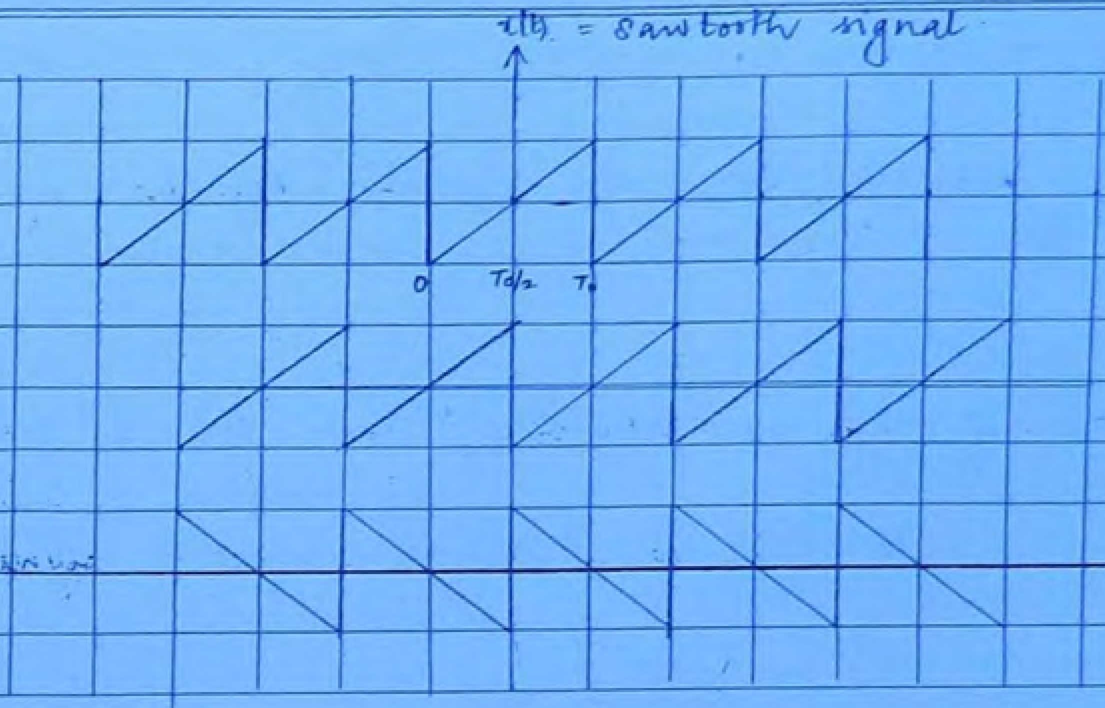


HWS

Q



HWS



NOTE : Half wave symmetric signals have average value = 0 but converse statement is not true.

AVERAGE & AREA OF SIGNAL -

Area of signal → $= \int_{-\infty}^{\infty} x(t)\, dt$

Area of signal x(t) over (t_1, t_2)
$$= \int_{t_1}^{t_2} x(t)\, dt$$

Average value of signal x(t)

$$\begin{cases} = \dfrac{1}{T_0} \displaystyle\int_{T_0} x(t)\, dt & \text{for periodic signal} \\ = \displaystyle\lim_{T_0 \to \infty} \dfrac{1}{T_0} \int_{-T_0/2}^{T_0/2 \text{ [fixed]}} x(t)\, dt & \text{for non-periodic signal} \end{cases}$$

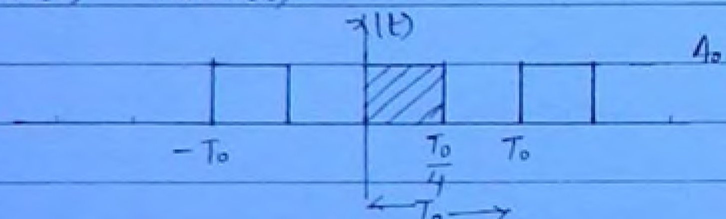
Average of $x(t)$ →

$$\begin{cases} = \dfrac{\text{area of } x(t) \text{ over } T_0}{T_0} & \text{for periodic signal} \\[2ex] = \dfrac{\lim\limits_{T_0 \to \infty} \text{area of } x(t) \text{ over } \left(-\frac{T_0}{2}, \frac{T_0}{2}\right)}{T_0} & \text{for non periodic signal.} \end{cases}$$

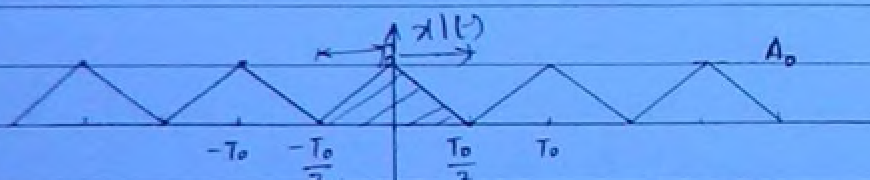
Q Calculate avg value of $x(t)$

i) $x(t) = A_0 u(t)$

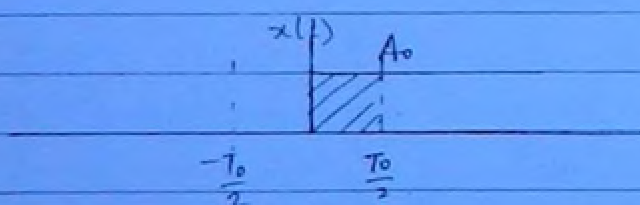
ii)



iii)



i)



$$\text{Avg of } x(t) = \frac{\lim\limits_{T_0 \to \infty} \text{area over } \left(-\frac{T_0}{2}, \frac{T_0}{2}\right)}{T_0}$$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \times \left(\frac{A_0 T_0}{2}\right) = \frac{A_0}{2}$$

ii) Avg of $x(t) = \frac{\text{Area over } 'T_0'}{T_0}$

$$= \frac{A_0 \times \frac{T_0}{4}}{T_0} = \frac{A_0}{4}$$

iii) Avg of $x(t) = \frac{\text{Area over } T_0}{T_0}$

$$= \frac{\frac{1}{2} \times A_0 \times T_0}{T_0} = \frac{A_0}{2}$$

Energy & Power Signal

Energy of signal $x(t)$ –

$$E = \int_{-\infty}^{\infty} (x(t))^2 \, dt$$

Power of signal →

$$P = \begin{cases} \frac{1}{T_0} \int_{T_0} (x(t))^2 \, dt & \text{for periodic signal} \\ \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (x(t))^2 \, dt & \text{for non-periodic signal} \end{cases}$$

* The amount of energy dissipated by a load resistor of R ohms when a v/g signal $V(t) = x(t)$ is applied across the resistor is

$$E = \int^{\infty} \frac{(V(t))^2}{} \, dt$$

$$E = \int_{-\infty}^{\infty} \frac{(x(t))^2}{R} \, dt$$

If $R = 1\,\Omega$

$$E = \int_{-\infty}^{\infty} (x(t))^2 \, dt$$

The energy & power expression written above represent normalised energy & normalised power as they are calculated for $1\,\Omega$ load resistance & $v(t) = x(t)$

Energy signal

$E =$ finite   Power of signal $= 0$

$$\boxed{P = \lim_{T_0 \to \infty} \frac{E}{T_0}}$$

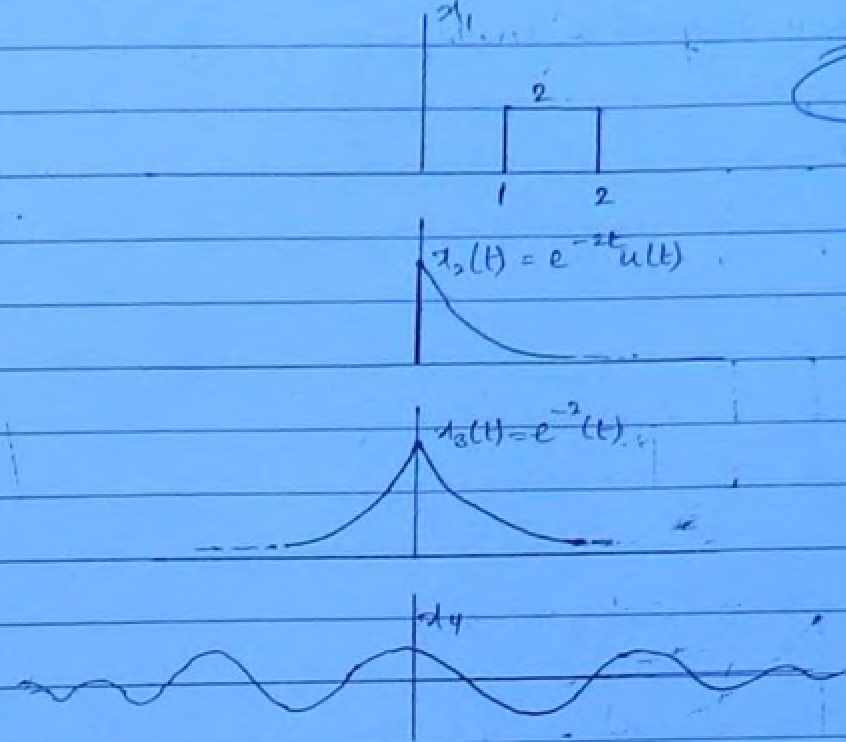
→ this expression is used for calculation of power & energy signal

Energy signals are absolutely integrable

ie $\boxed{\int_{-\infty}^{\infty} |x(t)| \, dt < \infty}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \text{Area of } |x(t)|^2 \text{ graph}$$

eg



eg

$$y(t) = \begin{cases} \frac{1}{t} & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Ans The signal is finite
but at $t = 0$ $y(t) = \infty$
$\therefore$ its not an energy signal

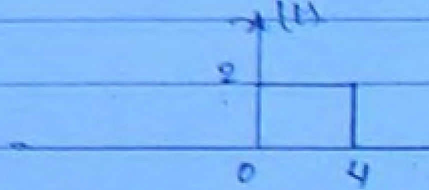
* Finite duration signal having finite values at each & every instant of time are energy signals

* If signal is having infinite extension & signal amplitude is decreasing in nature then signal will be energy signal

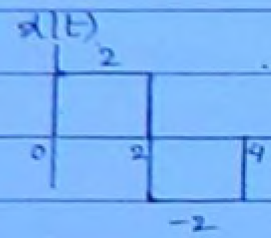
* If any signal is having infinite value at any instant of time then signal is neither energy

Q Calculate energy of signal

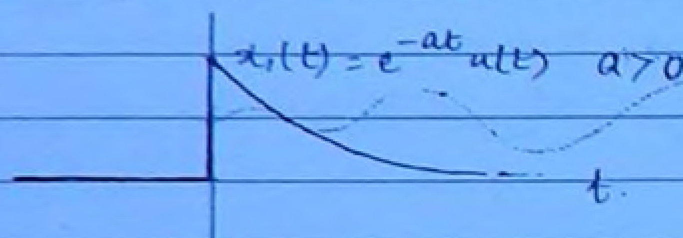
i)



ii)

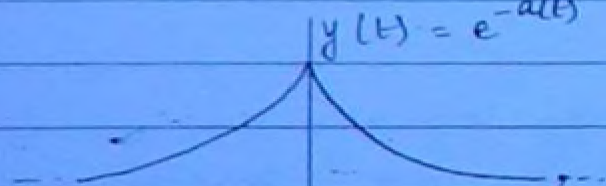


iii) $x_1(t) = e^{-at}u(t) \quad a>0$



iv) $x_2(t) = e^{-at}u(-t) \quad a>0$

v) $y(t) = e^{-a|t|} \quad a>0$



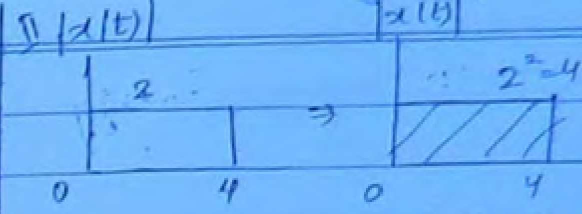
Sol. (i) $\cancel{\int_{-\infty}^{\infty} |2|^2 dt} = \int_0^4 2^2\, dt$

$= 4(t)_0^4$

$= 16$

ii) $\cancel{\int_0^4 [2^2 + (-2)^2]\, dt = (8t)_0^4 =}$

ii) $\int_0^2 2^2\,dt + \int_2^4 2^2\,dt = 16$



$A = 4 \times 4 = 16$

iii) $x(t) = e^{-at}u(t) \quad a > 0$

$$E = \int_0^{\infty} (e^{-at})^2\,dt = \left| \frac{e^{-2at}}{-2a} \right|_0^{\infty}$$

$$= \frac{1}{2a}\left(e^0 - e^{\infty}\right)$$

$$= \frac{1}{2a}$$

Area of $x(t)$

$$= \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_0^{\infty} e^{-at}\,dt = \frac{1}{a}$$

iv) $x(t) = e^{-at}u(-t) \quad a > 0$

$= x_1(-t)$

$$E = \int_{-\infty}^{0} (e^{-at})^2\,dt = \left| \frac{e^{-2at}}{-2a} \right|_{-\infty}^{0}$$

$$= \frac{1}{2a}\left(e^{-\infty} - e^0\right)$$

$$= \frac{1}{2a}$$

Area $= \frac{1}{a}$

By time reversal neither area nor energy changes.

so $E = \frac{1}{2a}$ $\quad A = \frac{1}{a}$

v) $y(t) = e^{-a|t|} \quad a > 0$

$$= \begin{cases} e^{at} & t < 0 \\ e^{-at} & t > 0 \end{cases}$$

$$y(t) = e^{at}u(-t) + e^{-at}u(t)$$

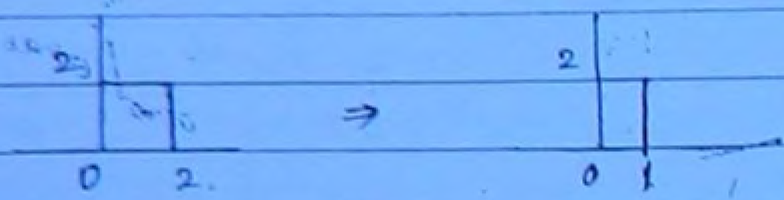
Q. $x(t) \longrightarrow$ Energy E.

(32)

$x(2t) \longrightarrow$?

a) $\frac{E}{4}$ b) $\frac{E}{2}$ c) $2E$ d) $4E$.

Sol.



$E = 8$ $E' = 4$.

By time reversal neither area nor energy changes

$\therefore$ Ans (b)

$x(at)$ $a \neq 0$ $\longrightarrow$ $\frac{E}{|a|}$

Power Signal -

$P =$ finite $E = \infty$

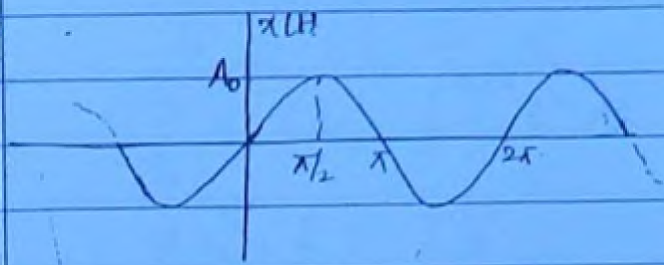
Condition for a periodic signal to be a power signal

$$\int_{T_0} |x(t)|\, dt < \infty$$

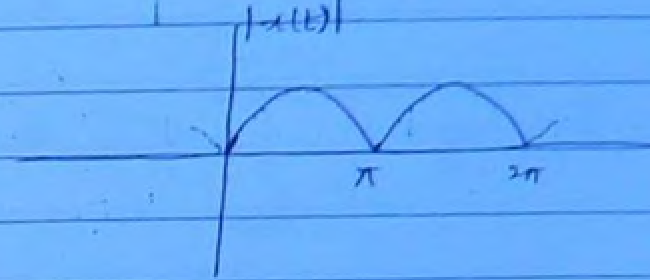
Q. Calculate power of signal

$$x(t) = A_0 \sin \omega_0 t$$

Sol



⇒



$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2\, dt = \frac{1}{T_0}\int_0^{T_0} A_0^2 \sin^2 \omega_0 t \, dt$$

$$= \frac{A_0^2}{2T_0}\int_0^{T_0} (1 - \cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{2T_0}\left[t - \frac{\sin 2\omega_0 t}{2\omega_0}\right]_0^{T_0}$$

$$= \frac{A_0^2}{2T_0}\left[(T_0 - 0) - \frac{1}{2\omega_0}(\sin 2\omega_0 T_0 - 0)\right]$$

$$= \frac{A_0^2}{2T_0}\left[T_0 - \frac{\sin 2\omega_0 T_0}{2\omega_0}\right] \qquad (\omega_0 T_0 = 2\pi)$$

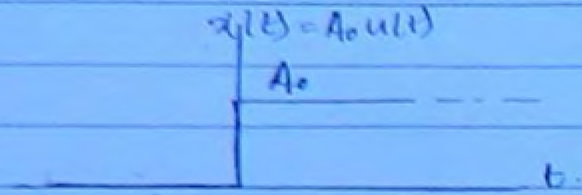
$$= \frac{A_0^2}{2T_0}\left[T_0 - \frac{\sin 4\pi}{2\omega_0}\right]^{0}$$

$$= \frac{A_0^2}{2T_0}[T_0]$$

$$\boxed{P = \frac{A_0^2}{2}}$$

* POWER is also known as Mean Square Value
  & Power = $(RMS)^2$

8



$P = ?$

Sol

$$P = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (x(t))^2 \, dt$$

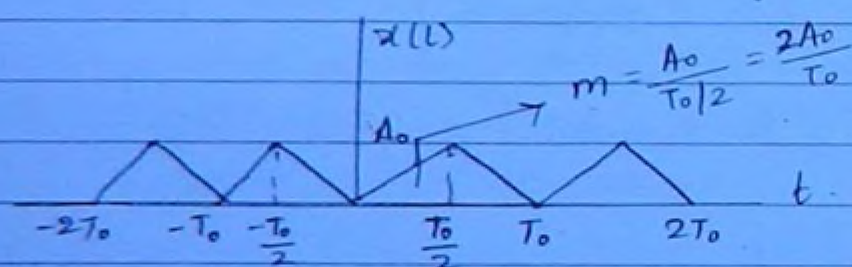
$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A_0^2 \, dt = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{0}^{T_0} A_0^2 \, dt$$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} A_0^2 \left(t \right)_{0}^{T_0/2}$$

$$= \lim_{T_0 \to \infty} \frac{A_0^2}{T_0} \left[\frac{T_0}{2} \right]$$

$$= \frac{A_0^2}{2}$$

8



Sol

$$P = \frac{1}{T_0} \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \frac{1}{T_0} \int_{-\infty}^{\infty} \quad \frac{1}{T_0} \left[\int_{0}^{T_0/2} + \int_{T_0/2}^{T_0} \right.$$

$$= \frac{1}{T_0} \int_{-T}^{T_0/2} (x(t))^2 \, dt$$

$$P = \frac{1}{T_0} \times 2 \times \int_0^{T_0/2} (x(t))^2 \, dt$$

$$= \frac{2}{T_0} \int_0^{T_0/2} \left(\frac{2A_0 \cdot t}{T_0} \right)^2 \cdot dt$$

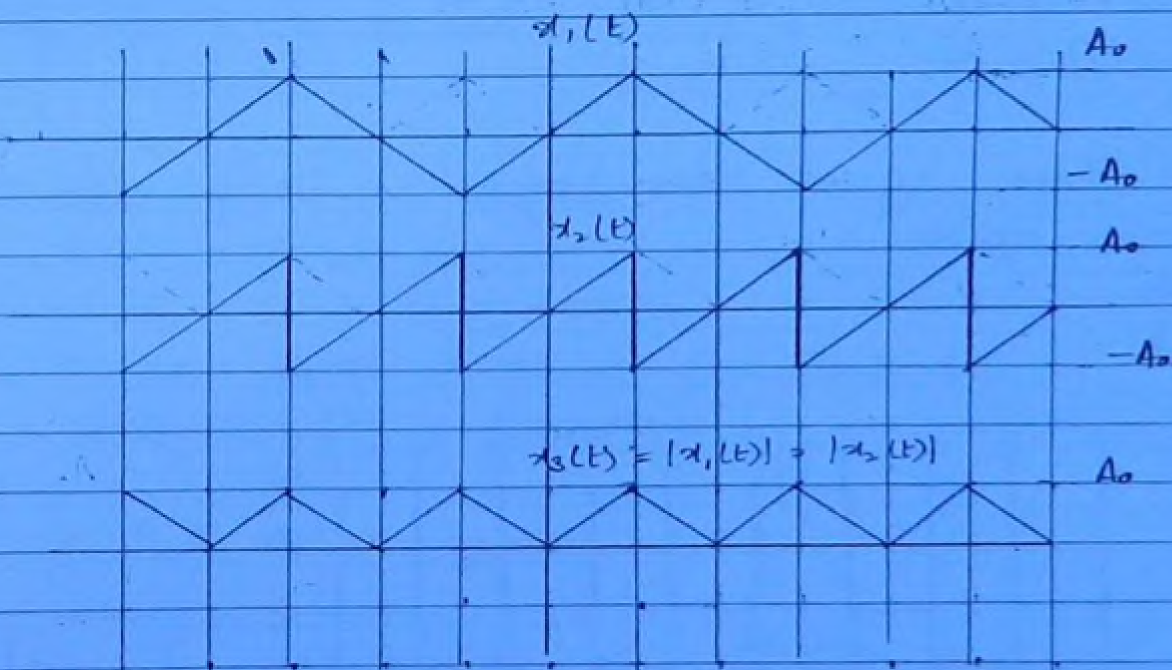
$y = mx + c$
$= mx$
when slope passes through origin.

$$= \frac{2}{T_0} \times \frac{4 A_0^2}{T_0^2} \int_0^{T_0/2} t^2 \, dt$$

$$= \frac{8 A_0^2}{T_0^3} \left[\frac{t^3}{3} \right]_0^{T_0/2}$$

$$= \frac{\cancel{8} A_0^2}{\cancel{T_0^3}} \times \frac{1}{3} \times \frac{\cancel{T_0^3}}{\cancel{8}}$$

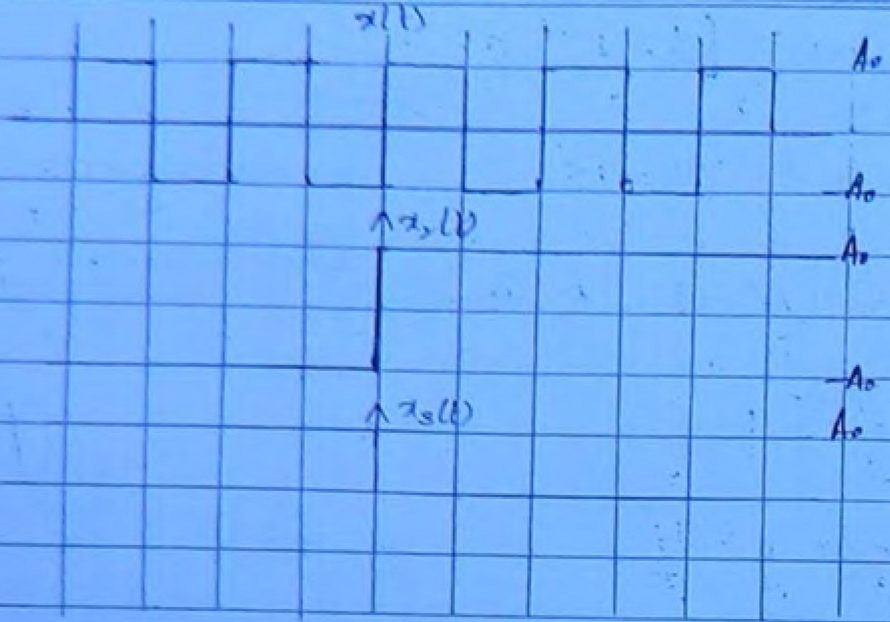
$$= \cancel{A_0^3} \qquad \boxed{P = \frac{A_0^2}{3}}$$



Power of signals with same modulus value are equal.

$$P = \frac{A_0^2}{3} \qquad RMS = \frac{A_0}{\sqrt{3}}$$

Q



(36)

$$x_4(t) = A_0 e^{j\omega_0 t}$$

complex ~~sig~~ exponential signal

$\therefore \ |x_4(t)| = A_0$

$\therefore$ All these signals have same modulus value

$\therefore \ P = A_0^2$

$\therefore$ RMS $= A_0$

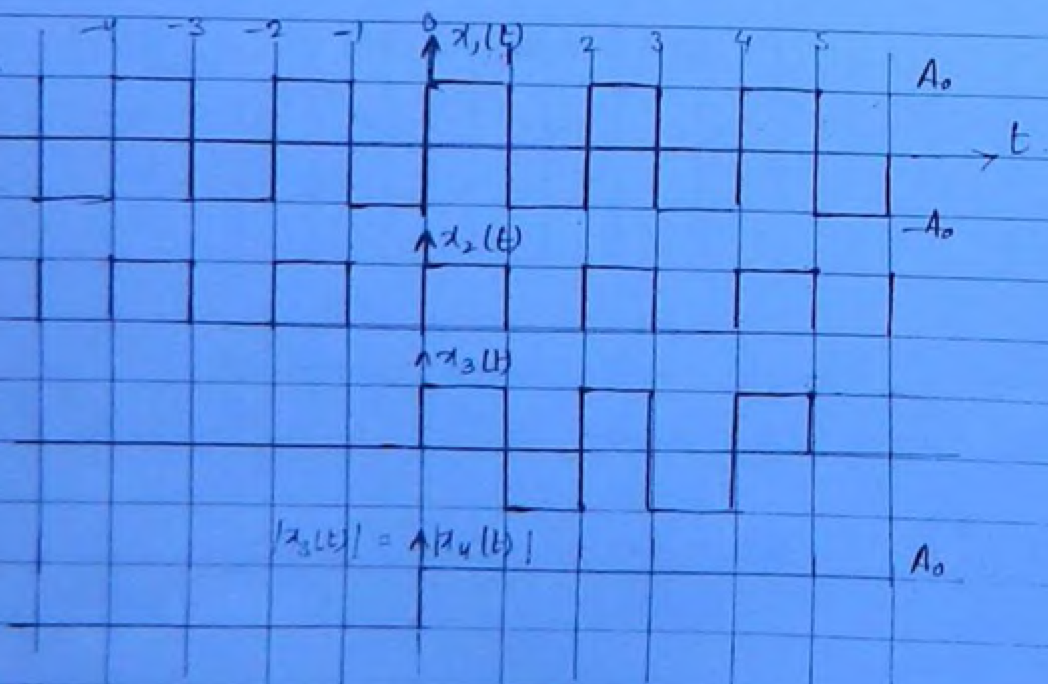
Q

$P_{x_1} = A_0^2$

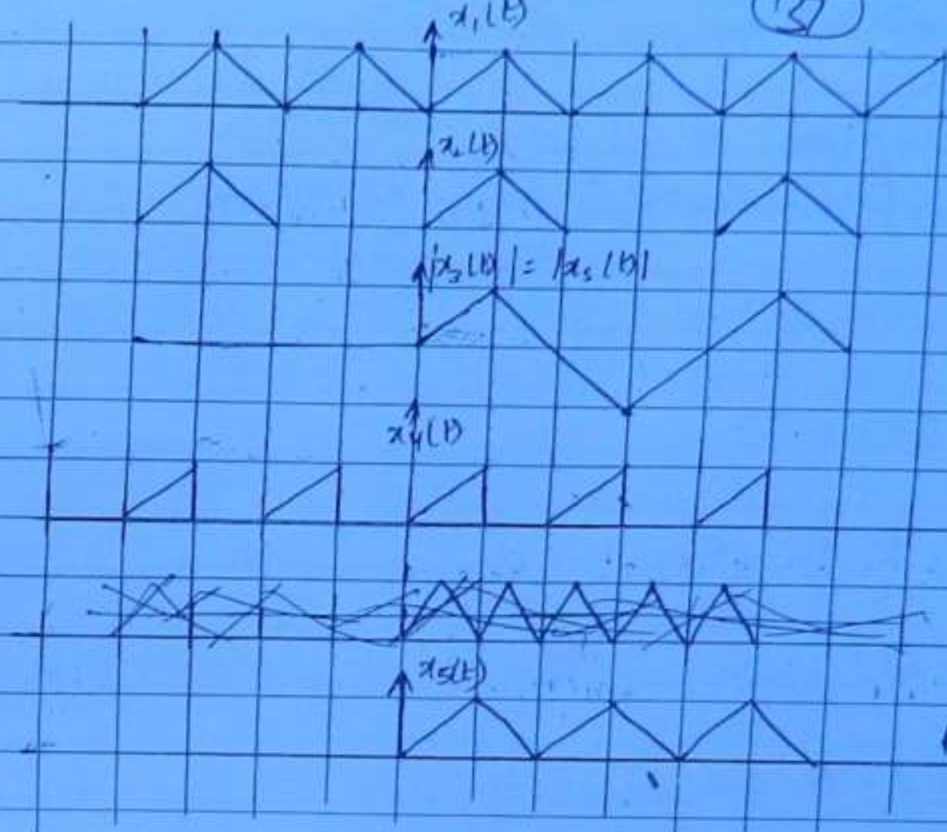
$P_{x_2} = \dfrac{P_{x_1}}{2} = \dfrac{A_0^2}{2}$

$P_{x_3} = \dfrac{P_{x_1}}{2} = \dfrac{A_0^2}{2}$

$P_{x_4} = P_{x_3} = \dfrac{A_0^2}{2}$



Q



$P_{x_1} = \frac{A_0^2}{3}$

$P_{x_2} = \frac{A_0^2}{6} = \frac{P_{x_1}}{2}$

$P_{x_3} = P_{x_5} = \frac{A_0^2}{6}$

$P_{x_4} = P_{x_5} = \frac{A_0^2}{6}$

$P_{x_5} = \frac{P_{x_1}}{2} = \frac{A_0^2}{6}$

Q Calculate power of signal

i) $x_1(t) = A_0 \sin \omega_0 t$

ii) $x_2(t) = A_0 \sin(\omega_0 t + \phi)$

iii) $x_3(t) = \cancel{x_1(t)}\ x_1(t - t_1) = A_0 \sin(\omega_0(t - t_1))$

iv) $x_4(t) = x_1(2t) = A_0 \sin 2\omega_0 t$

Sol

i) $P_{x_1} = \frac{A_0^2}{2}$

ii) Power is unaffected by time scaling & shifting

$P_{x_2} = \frac{A_0^2}{2}$

iii) $P_{x_3} = \frac{A_0^2}{2}$

iv) $P_{x_4} = \frac{A_0^2}{2}$

Power is unaffected by
→ Time shifting
→ Change in phase of signal
→ Change in fundamental time period / freq of the signal.

IMPORTANT RESULTS –

$$\int_{T_0} \cos(n\omega_0 t + \phi)\, dt = 0$$

where n = an integer

$$\int_{T_0} \sin(n\omega_0 t + \phi)\, dt = 0$$

$$\int_{T_0} \sin^2(n\omega_0 t + \phi)\, dt = \frac{T_0}{2}$$

$$\int_{T_0} \cos^2(n\omega_0 t + \phi)\, dt = \frac{T_0}{2}$$

Orthogonal Signals

Two signals $x_1(t)$ & $x_2(t)$ are said to be orthogonal if

$$\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt = 0 \quad \text{for non periodic signals}$$

$$\int_{T_0} x_1(t)\, x_2(t)\, dt = 0 \quad \text{for periodic signals}$$

Effect of orthogonality on energy & power calculations -

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Any signal will be either energy or power

If $x_1(t)$ & $x_2(t)$ are orthogonal

& $y(t) = x_1(t) + x_2(t)$

then $\boxed{P_y = P_{x_1} + P_{x_2}}$] If $x_1(t)$ & $x_2(t)$ are power signals ~~also~~.

$\boxed{E_y = E_{x_1} + E_{x_2}}$] If $x_1(t)$ & $x_2(t)$ are energy signals

Q. $x(t) = 2\sin(8\omega_0 t + 45°) + 4\sin(4\omega_0 t + 35°)$

$$\int_{T_0} \sin(n\omega_0 t + \phi_1)\sin(m\omega_0 t + \phi_2)\,dt = 0 \quad m \neq n$$

m & n are integers

Sol. $P = \underbrace{P_1 + P_2}_{\text{b/z they are orthogonal}} = \frac{2^2}{2} + \frac{4^2}{2} = 10$ ($A^2/2$)

Q. $x(t) = 2\cos(3\omega_0 + 75°) + 4\cos(5\omega_0 + 85°)$

$$\int_{T_0} \cos(n\omega_0 t + \phi_1)\cos(m\omega_0 t + \phi_2)\,dt = 0 \quad m \neq n$$

Sol. $P = P_1 + P_2 = \frac{2^2}{2} + \frac{4^2}{2} = 10$

Q. ~~$x(t) = 2\cos(5\omega_0 t + 35°)$~~ $+ 3\sin(9\omega_0 t + 65°)$

Q $x(t) = 2\cos(5\omega_0 t + 35^\circ) + 3\sin(9\omega_0 t + 65^\circ)$

(40)

$$\int_{T_0} \cos(n\omega_0 t + \phi_1)\sin(m\omega_0 t + \phi_2)\, dt$$

$= 0 \rightarrow m \neq n$

$\rightarrow m = n \quad \phi_1 = \phi_2$

Sol $P = P_1 + P_2 = \frac{2^2}{2} + \frac{3^2}{2}$

Q $x(t) = 2\cos(2\omega_0 t + 45^\circ) + 3\sin(2\omega_0 t + 45^\circ)$

Sol $P = P_1 + P_2 = \frac{2^2}{2} + \frac{3^2}{2}$

Q $x(t) = 2 + 4\sin(3\omega_0 t + 35^\circ)$

$$\int_{T_0} A_0 \sin(n\omega_0 t + \phi)\, dt = 0 \times A_0 = 0$$

Sol $P = P_1 + P_2 = 2^2 + \frac{4^2}{2}$

* 2 harmonics of different frequencies are always orthogonal.

* sin function & cosine function of same phase & same frequencies are orthogonal

* dc & sin signal are orthogonal

Q. $x(t) = A_1 \sin(\omega_0 t + \phi_1) + A_2 \sin(\omega_0 t + \phi_2) \quad \phi_1 \neq \phi_2.$

Sol $\cancel{P \neq P_1 + P_2 \neq A_0}$

$x(t) = A_0 \sin(\omega_0 t + \theta)$

$$P = \frac{A_0^2}{2} \quad \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2)} = A_0$$

$$P = \frac{A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_1 - \phi_2)}{2}$$

Q. $x(t) = 2 \sin 3t + 3 \cos\left(3t + \frac{\pi}{3}\right)$

Calculate RMS value of signal.

Sol $A_0^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2)$

$$x(t) = 2 \sin 3t + 3 \sin\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$= 2 \sin 3t + 3 \sin\left(3t + \frac{5\pi}{6}\right)$$

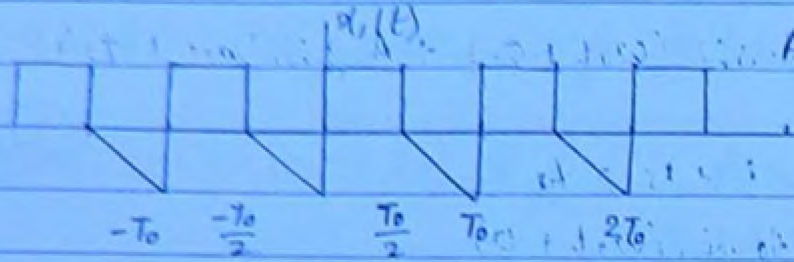
$$A_0^2 = 2^2 + 3^2 + 2(2)(3) \cos\left(\frac{5\pi}{6}\right)$$

$$= 13 + 12 \cos \frac{5\pi}{6}$$

$$P = \frac{A_0^2}{2} = \left(13 + 12 \cos \frac{5\pi}{6}\right) \frac{1}{2} = 1.3038$$

$$RMS = \sqrt{P} = 1.141$$

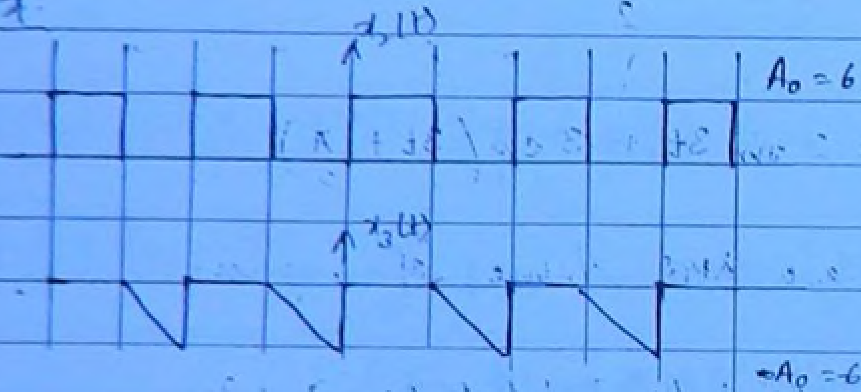
8



Calculate RMS value of signal

a) $6\sqrt{2}$ b) $2\sqrt{6}$ c) $\sqrt{27}$ d) $4\sqrt{3}$

sol $B = \lambda$



$$\int_{T_0} x_2(t)\, x_3(t)\, dt = 0$$

orthogonal..

$$P_{x_1} = P_{x_2} + P_{x_3}$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{6}$$

$$= \frac{6^2}{2} + \frac{6^2}{6} = 24$$

$$RMS = \sqrt{P_{x_1}} = \sqrt{24} = 2\sqrt{6} \quad (b)$$

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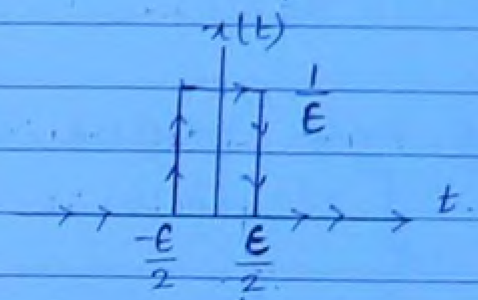
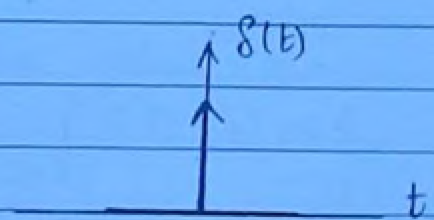
Basic Signals -

1. Unit Impulse : $\delta(t)$

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



Area under impulse = 1.



$$\delta(t) = \lim_{\epsilon \to 0} x(t)$$

$$= \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Properties -

1. $\int_{-\infty}^{\infty} \delta(t)\, dt$ = Area under impulse

$$= \int_{-\infty}^{\infty} \lim_{\epsilon \to 0} x(t)\, dt$$

$$= \lim_{\epsilon \to 0} \left[\int_{-\infty}^{\infty} x(t)\, dt \right]$$

$$= 1$$

2. $\delta(t)$ is an even signal

3. 1^{st} derivative of $\delta(t)$ is known as "doublet function" which is an odd signal.

$$\frac{d\delta(t)}{dt}$$

4. $\delta(at),\ a \neq 0 = \frac{1}{|a|} \delta(t)$.

5. Weight of Impulse / Strength of Impulse -

$$x(t) = A_0\, \delta(t)$$

Area under weighted impulse

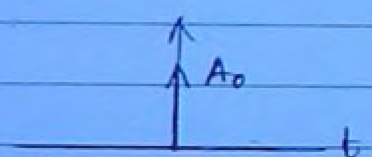
$$= \int_{-\infty}^{\infty} x(t)\, dt$$

$$= \int_{-\infty}^{\infty} A_0 \cdot \delta(t)\, dt$$

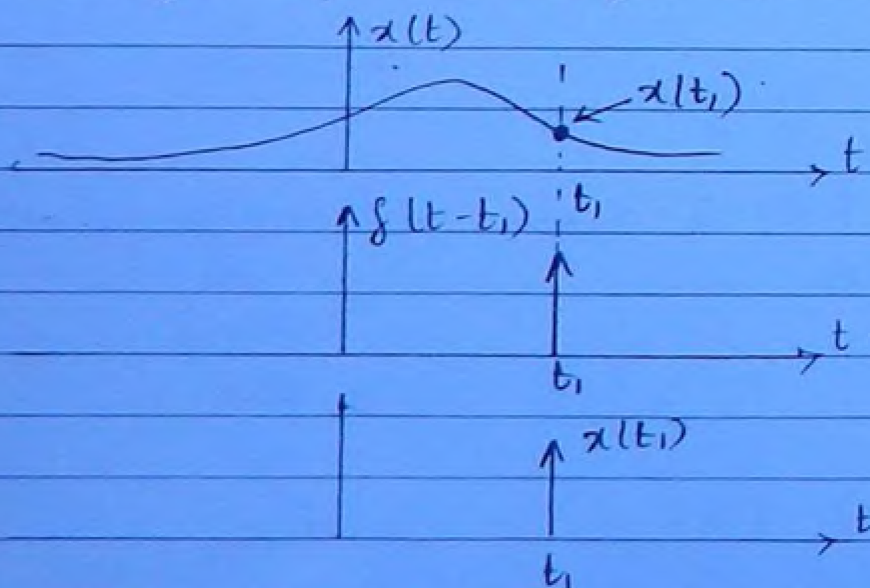
$$= A_0 \left[\overset{1}{\cancel{\int_{-\infty}^{\infty} \delta(t)\, dt}} \right]$$

$$\boxed{\text{Area under weighted impulse} = \text{weight of impulse}}$$

$$x(t) = A_0\, \delta(t)$$



6. $x(t)\, \delta(t - t_1) = x(t_1)\, \delta(t - t_1)$



7. $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_1)\,dt$

$= \int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt$

$= x(t_1)\left[\cancelto{1}{\int_{-\infty}^{\infty}\delta(t-t_1)\,dt}\right]$

$= x(t_1)$

8. Find the value of -

i) $I = \int_{-5}^{4}\delta(t-5)\,dt$

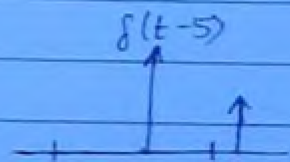
ii) $I = \int_{-5}^{4}\delta(t-2)\,dt$

iii) $I = 2t\,\delta\left(t-\frac{1}{2}\right) \rightarrow t-\frac{1}{2} = 0 \Rightarrow t = \frac{1}{2}$

iv) $I = \sin t\,\delta(2t-\pi)$

v) $I = \int_{-\infty}^{\infty} e^{-2t}\,\delta(-2t+1)\,dt$

Sol i) 0



ii) 1

iii) $2\left(\frac{1}{2}\right)\delta\left(t-\frac{1}{2}\right) = \delta\left(t-\frac{1}{2}\right)$

~~iv) $\sin\left(\frac{\pi}{2}\right) = 1$~~

~~v) $e^{-2(1/2)} = e^{-1} = \frac{1}{e}$~~

iv) $\delta(2t-\pi) = \delta[2(t-\pi/2)]$

$= \frac{1}{2}\delta\left(t-\frac{\pi}{2}\right)$

$I = \frac{1}{2}\sin\frac{\pi}{2}\,\delta\left(t-\frac{\pi}{2}\right)$

$= \frac{1}{2}\delta\left(t-\frac{\pi}{2}\right)$

v) $\delta(-2t+1) = \delta\left[-2\left(t-\frac{1}{2}\right)\right]$

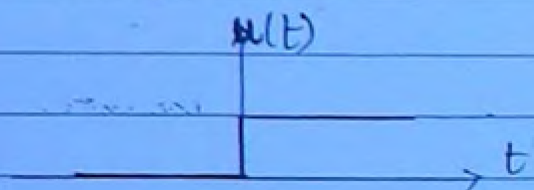
$= \frac{1}{2}\delta\left(t-\frac{1}{2}\right)$

$$I = \frac{1}{2} e^{-2(1/2)} \left[\int_{-\infty}^{\infty} \delta\left(t - \frac{1}{2}\right) dt \right]$$

$$I = \frac{1}{2e}$$

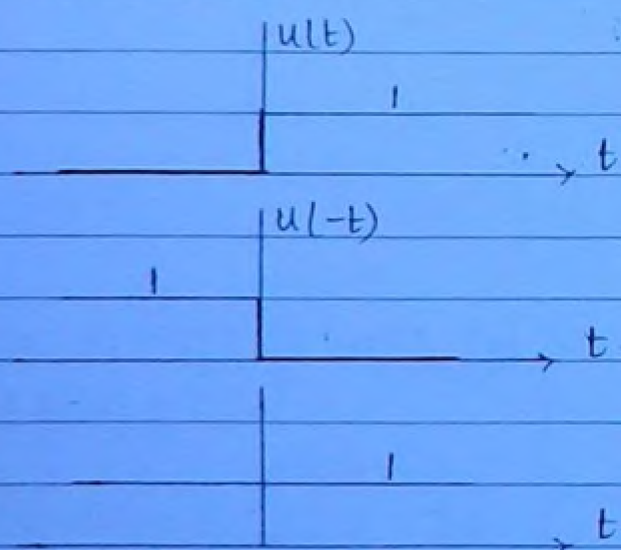
2. Unit Step Signal $u(t)$

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



Properties -

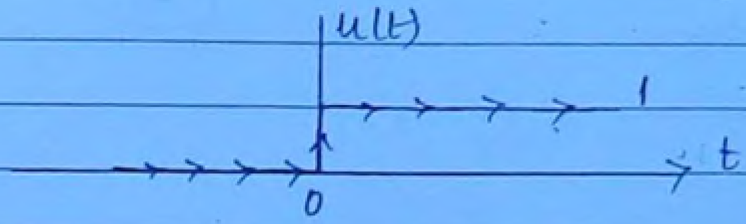
1. $u(t) + u(-t) = 1$



GIBBS PHENOMENON

At the point of discontinuity, signal value is given by the average of signal value taken just before & after the point of discontinuity

2. $\frac{du(t)}{dt}$



$$\boxed{\frac{d\,x(t)}{dt} = \text{slope of } x(t) \text{ w.r.t time 't'}}$$

$\frac{du(t)}{dt} = \delta(t)$

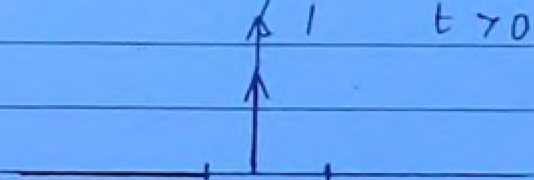
At $t=0$ $u(t)$ is pointing upwards forming $90°$ with x-axis
differentiation $\frac{du(t)}{dt}$ = slope
ie tan value
of $\tan 90° = \infty$
so $\frac{du(t)}{dt}$ is ~~...~~ at $t=0$
pointing up till ∞
which is an impulse signal.

3. $u(t)$ is a power signal

$P = \frac{1}{2}$

$RMS = \frac{1}{\sqrt{2}}$ $\quad$ $Avg = \frac{1}{2}$

4. $$\int_{-\infty}^{t} \delta(t)\,dt = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} = u(t)$$



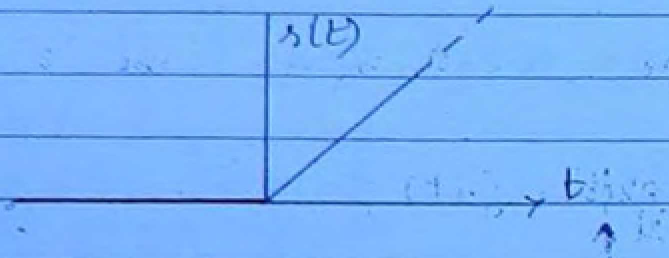
$-\infty \longrightarrow t \quad t<0$

$-\infty \longrightarrow t \quad t>0$

3. Unit Ramp Signal -

$$r(t) = \int_{-\infty}^{t} u(t)\, dt$$

$$= t\, u(t)$$

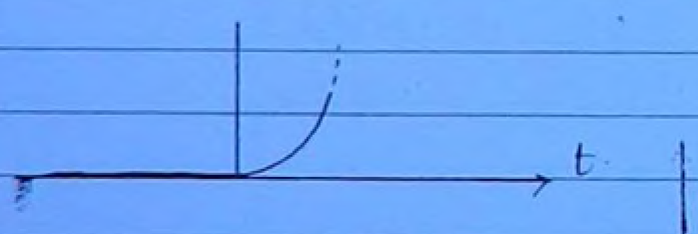


⇒ neither energy ~~not~~ nor power.

4. Unit Parabolic Signal -

$$P(t) = \int_{-\infty}^{t} r(t)\, dt$$

$$= \frac{t^2}{2}\, u(t)$$



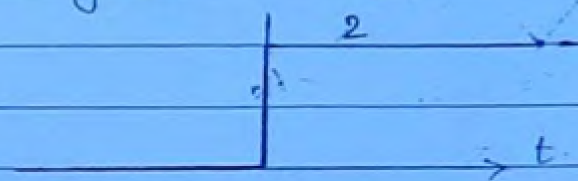
⇒ neither energy nor power.

5. Singnum Function -

$$Sgn(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

Properties.

1. $1 + sgn(t) = 2u(t)$

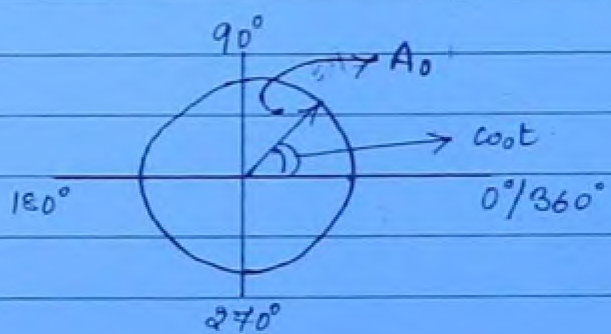


$$\boxed{u(t) = \frac{1 + sgn(t)}{2}}$$

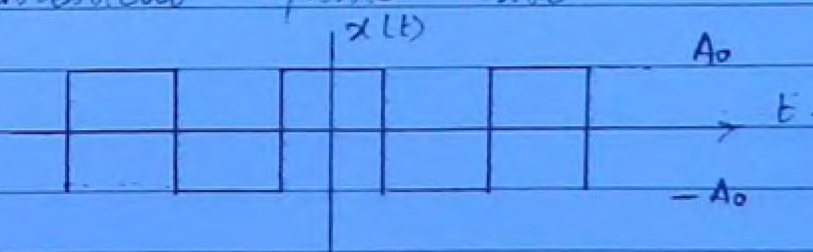
6. Complex Exponential

$x(t) = A_0 e^{j\omega_0 t}$

$\Rightarrow P = A_0^2$



7. Symmetrical Square Wave



$\rightarrow P = A_0^2 \quad \rightarrow RMS = A_0$

$\rightarrow Avg = 0 \quad \rightarrow HWS$

8. Symmetrical Triangular Wave -

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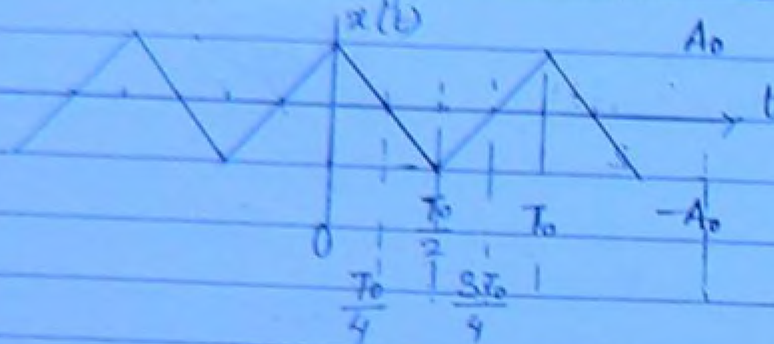
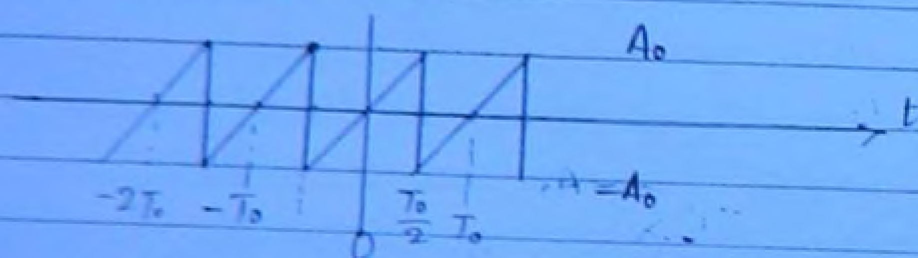


→ $P = \frac{A_0^2}{3}$ → $RMS = \frac{A_0}{\sqrt{3}}$

→ Avg = 0 → HWS.

9. Sawtooth Wave -



→ $P = \frac{A_0^2}{3}$ → $RMS = \frac{A_0}{\sqrt{3}}$

→ Avg = 0 → not HWS.

10. Sampling Function -

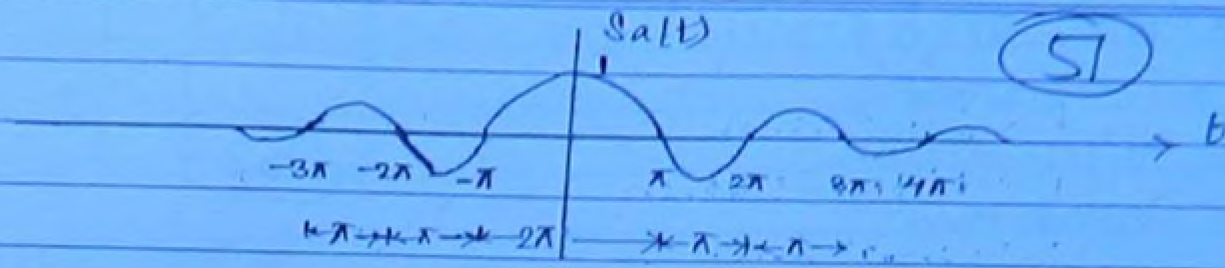
$$x(t) = Sa(t) = \frac{\sin t}{t}$$

→ $Sa(0) = 1$

→ $Sa(\infty) = 0$

If $Sa(t) = 0$

then $\frac{\sin t}{t} = 0 \Rightarrow \sin t = 0 \Rightarrow \boxed{t = n\pi, \ n \neq 0}$



→ Energy of $Sa(t) = \pi$ [F.T]

11. Sinc function -

$$x(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$
$$= Sa(\pi t)$$

→ $\text{sinc}(0) = 1$
→ $\text{sinc}(\infty) = 0$

$x(t) = Sa(t)$ → Energy. $'E' = \pi$

$a = \pi$ $\quad x(at) \rightarrow \frac{E}{a} = \frac{\pi}{\pi} = 1$

$= Sa(\pi t)$
$= \text{sinc}(t)$

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- DIFFERENT OPERATIONS ON SIGNALS -

- → Integration
- → Differentiation
- → Convolution

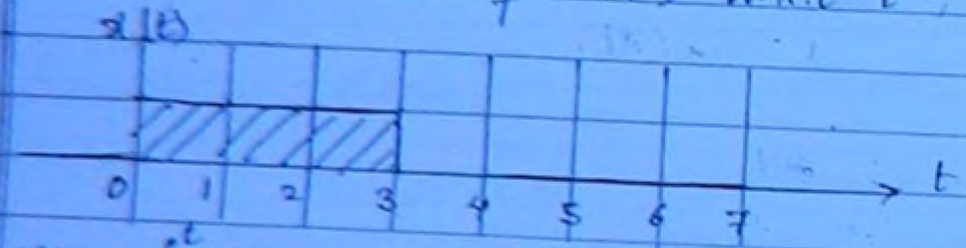
1. Integration - (Graphical)

↳ only for rectangular pulses.

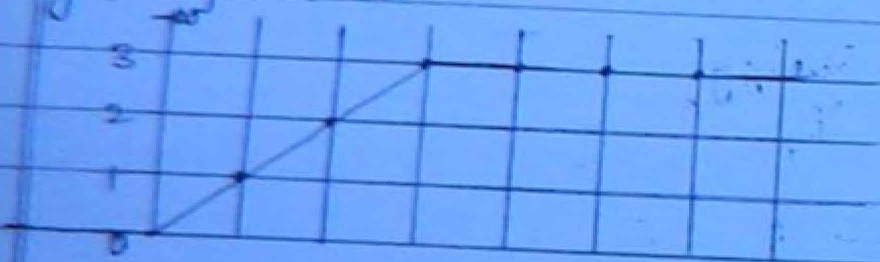
$$y(t) = \int_{-\infty}^{t} x(t)\,dt$$

= Area of $x(t)$ w.r.t 't'

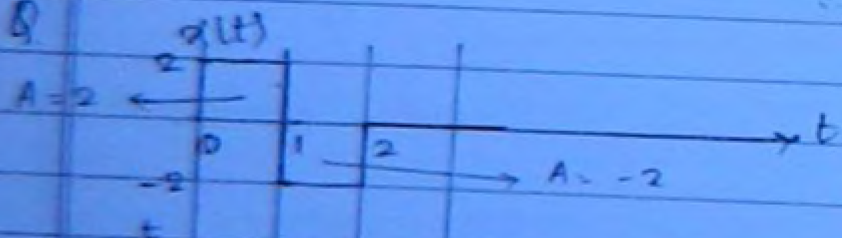
eg.



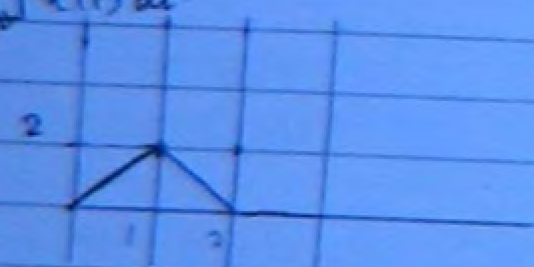
$$y(t) = \int_{-\infty}^{t} x(t)\,dt$$



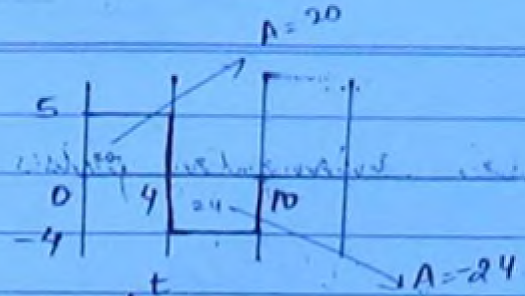
Q.



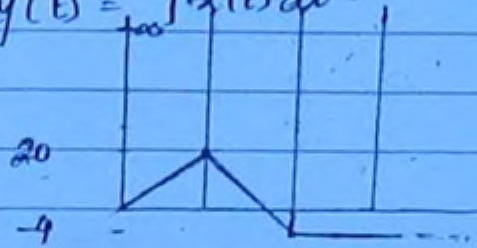
$$\int_{-\infty}^{t} x(t)\,dt$$



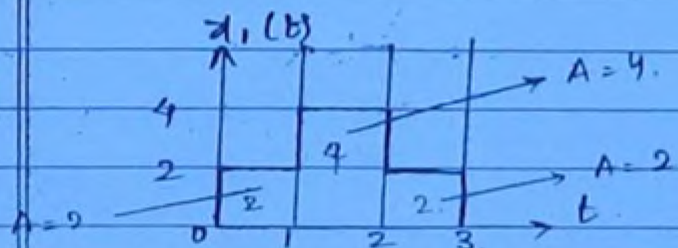
Q

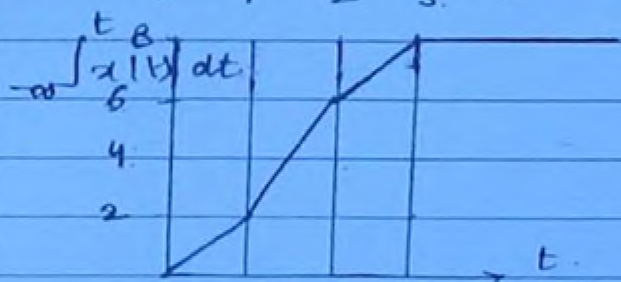


$y(t) = \int_{-\infty}^{t} x(t)\,dt$

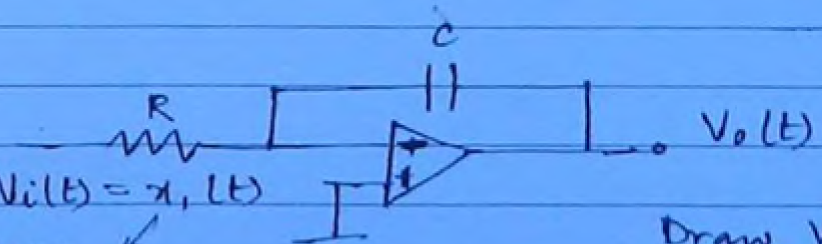


Q





Q

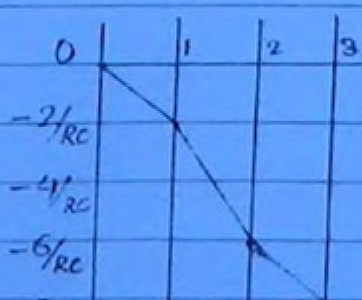


$V_i(t) = x_1(t)$

prev. ques pulse.

Draw Vot

Sol



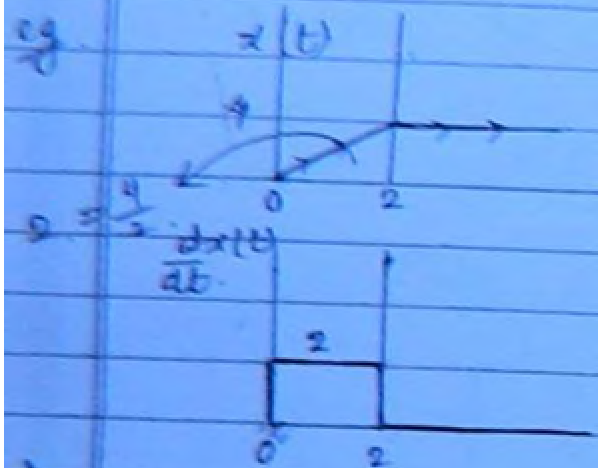
$$V_0(t) = \frac{-1}{RC} \int_{-\infty}^{t} V_i(t)\,dt$$

$$= \frac{-1}{RC} \left[\int_{-\infty}^{t} x_1(t)\,dt \right]$$

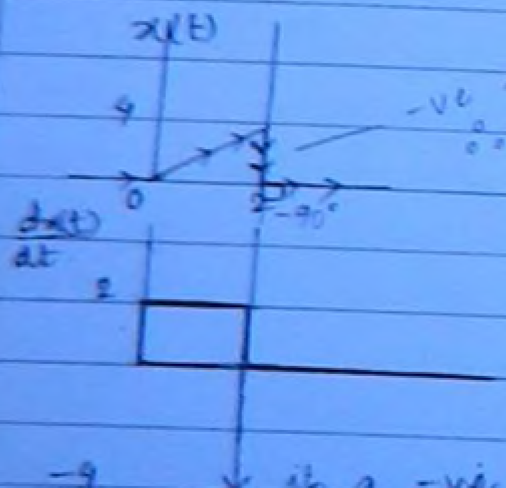
2 Differentiation – (Graphical) –
↳ only for rectangular triangular pulses.

$\frac{dx(t)}{dt}$ = slope of $x(t)$ w.r.t 't'

eg.

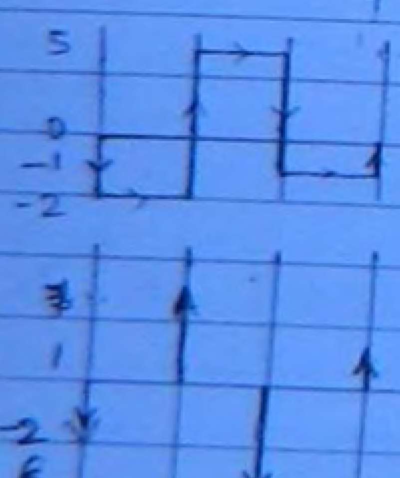


eg.



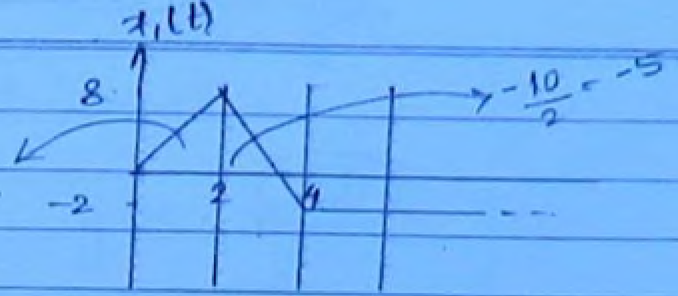
-ve slope ∴ tan(-90°) = -∞ represented by falling impulse

is a -ve impulse

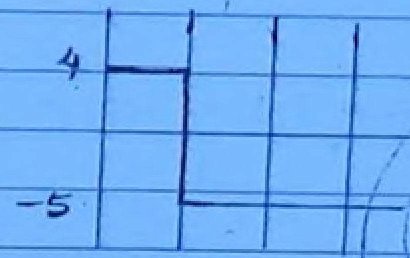


Q

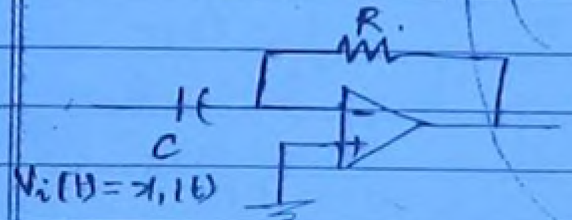
$y = \frac{8}{2}$



55



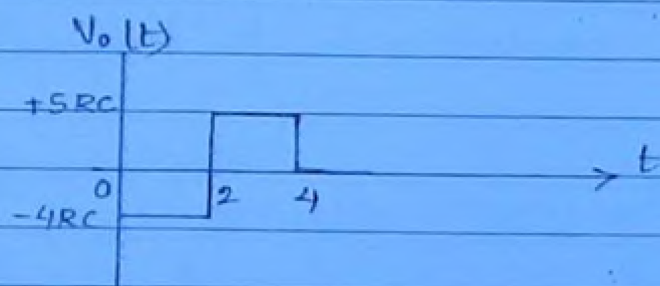
Q



Draw $V_o(t)$

$$V_o(t) = -RC \frac{d\,V_i(t)}{dt}$$

$$= -RC \frac{d\,x_1(t)}{dt}$$



-1-13

3. Convolution –

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, \underbrace{x_2(t-\tau)}_{x_2[-(\tau-t)]}\, d\tau$$

Steps (obj) –

$x_1(t)$ $\downarrow t=\tau$ $x_1(\tau)$

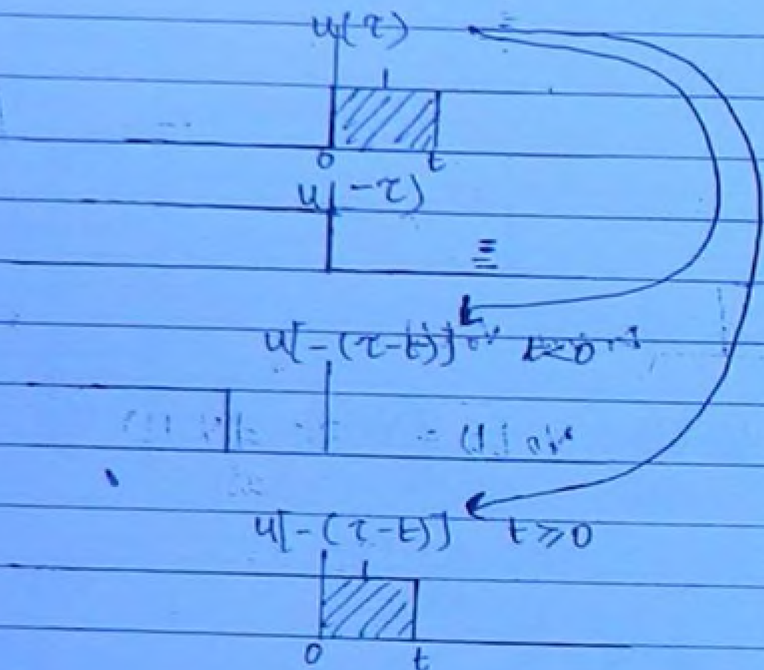
$x_2(t)$ $\downarrow t=\tau$ $x_2(\tau)$ $\downarrow$ $x_2(-\tau) \rightarrow$ folding $\downarrow$

Steps (stof)

Q. $y(t) = u(t) * u(t)$

$= \int_{-\infty}^{\infty} u(\tau)\, \underline{u(t-\tau)}\, d\tau$

$\quad \hookrightarrow u(-(\tau - t))$



$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t dt & t \geq 0 \end{cases}$$

$$= \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

$= t\, u(t)$

$= r(t)$

2^{nd} method -

S7

$$y(t) = \underset{\downarrow \atop u(t)}{x_1(t)} * \underset{\downarrow \atop u(t)}{x_2(t)}$$

$$Y(s) = X_1(s) . X_2(s)$$

$$= \frac{1}{s} . \frac{1}{s} = \frac{1}{s^2}$$

$$Y(s) = \frac{1}{s^2}$$

$$y(t) = r(t)$$

Properties of Convolution -

1. Commutative -

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

$$\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

2. Associative -

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

3. Distributive -

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

4. Impulse Response - (VV IMP)

$$x_1(t) * \delta(t - t_1) = x(t - t_1)$$

$$\downarrow t_1 = 0$$

5. Derivative Property -

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If $y(t) = x_1(t) * x_2(t)$

then $\frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} * x_2(t)$

$= x_1(t) * \frac{dx_2(t)}{dt}$

6) Step Response (VVIMP)

$y(t) = x(t) * u(t)$

$\frac{dy(t)}{dt} = x(t) * \frac{du(t)}{dt} = x(t) * \delta(t)$

$\frac{dy(t)}{dt} = x(t)$

$\Rightarrow y(t) = \int_{-\infty}^{t} \left[\frac{dy(t)}{dt}\right] dt = \int_{-\infty}^{t} x(t)\, dt$

$\Rightarrow \boxed{y(t) = \int_{-\infty}^{t} x(t)\, dt}$

7) Duration Property (VVIMP)

$y(t) = x_1(t) * x_2(t)$

s/g	extension
$x_1(t)$	$t_1 \leq t \leq t_2$
$x_2(t)$	$t_3 \leq t \leq t_4$
$y(t)$	$(t_1 + t_3) \leq t \leq (t_2 + t_4)$

8. Scaling Property -

If $x_1(t) * x_2(t) = y(t)$

then $x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$

$a \neq 0$

9. Delay Property -

If $x_1(t) * x_2(t) = y(t)$

then $x_1(t-t_1) * x_2(t-t_2) = y[t-(t_1+t_2)]$

Q. $r(t) * u(t)$

$$= \int_{-\infty}^{t} r(t)\, dt = P(t) = \frac{t^2}{2} u(t)$$

$P(t)$ = parabolic signal

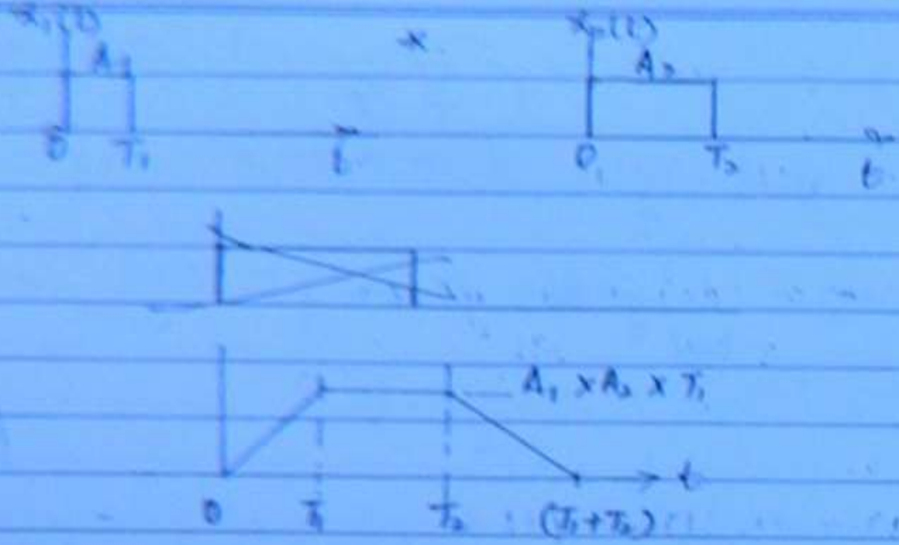
Q. $r(t-3) * u(t-2) = P(t-5)$

$$= \frac{(t-5)^2}{2} u(t-5)$$

* If two rectangular pulses of unequal duration are convolved then resultant signal is a trapezoid

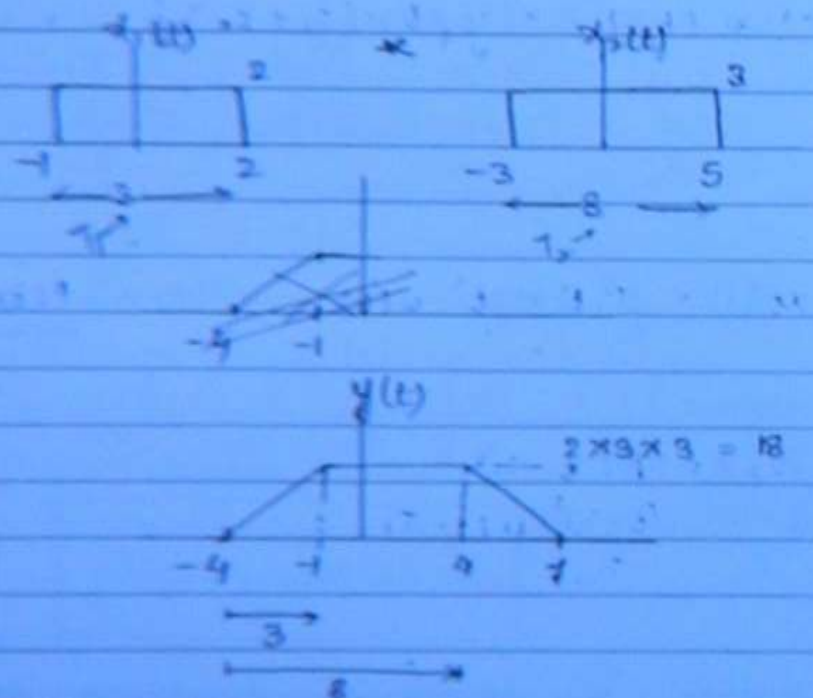
* Convolution of two rectangular pulses of equal duration will be a triangle

Eg 1

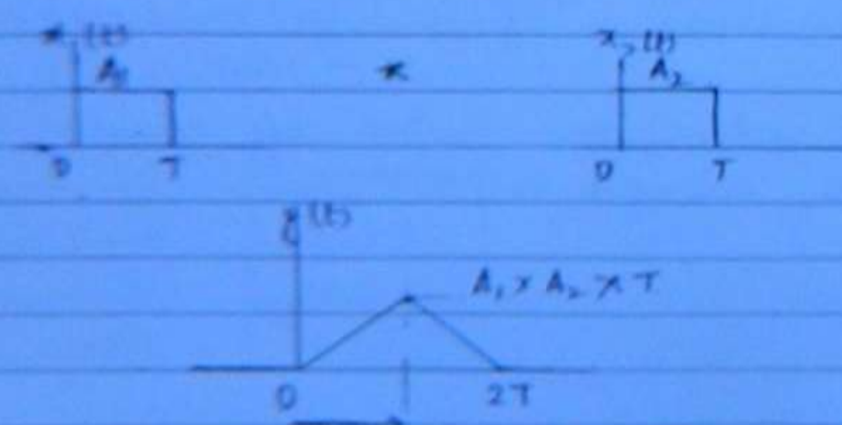


$T_1 \to$ smaller duration.
$T_2 \to$ larger duration.

Eg 2



Eg 3



BASIC SYSTEM PROPERTIES

Basic System Properties -

61

$x(t) \longrightarrow$ system $\longrightarrow y(t)$

$t = 1$ sec.

$$y(1) = y(t) \begin{cases} \text{present} & x(t) = x(1) \\ \text{past} & x(t-1) = x(0) \\ \text{future} & x(t+1) = x(2) \end{cases}$$

1. Static / Dynamic -

Static system → If output of system depends on only present values of input, system is static
→ These systems are also known as memoryless system.

Dynamic system → If output of system depends on past or future values of input at any instant of time, system is dynamic
→ These systems are also known as systems with memory.

For static system there should not be any scaling / shifting in time either in $y(t)$ or $x(t)$.

Q. Check static / dynamic system -

Derivative / Integrating systems are dynamic system.

1. $y(t) = x(t) + x(t-1)$ D
2. $y(t) = (t+1)\,x(t)$ S (irrespective of co-eff)
3. $y(t) = x(\cos t)$ D
4. $y(t) = \cos[x(t)]$ S
5. $y(t) = \cos t \; x^2 t$ S
6. $y(t) = \int_{-\infty}^{t} x(k)\,dk \rightarrow x(-\infty) + \ldots + x(t-1) + \ldots x(t)$ D

8. $y(t) = \text{Real}[x(t)] \xrightarrow{\text{conjugate}}$

$= \dfrac{x(t) + x^*(t)}{2} \Rightarrow S.$ (62)

2. Causal / Non-causal / Anti-causal system -

Causal → If output of the system does not depend on future values of input then system is called causal.

→ They are practical systems or physically realisable systems.

eg $y(t) = x(t)$

$y(t) = x(t) + x(t-1)$

$y(t) = x(t-1)$

Anticausal → If output of the system depends only on future values of input, its anticausal.

eg $y(t) = x(t+1)$

Non-causal → If output of the system depends on future values of input at any instant of time, system is called non-causal.

eg $y(t) = x(t+1)$

$y(t) = x(t+1) + x(t-1)$

$y(t) = x(t+1) + x(t)$

$y(t) = x(t) + x(t-1) + x(t+1)$

* All anticausal systems are non-causal but

Q. Check causal / non-causal systems -

63

1. $y(t) = x(-t)$ $\xrightarrow{t=-1}$ $y(-1) = x(1)$ NC

2. $y(t) = \cos[x(t)]$ C

3. $y(t) = x(\sin t)$ $\xrightarrow{t=-\pi}$ $y(-\pi) = x(0)$ NC

4. $y(t) = \begin{cases} x(t), & t < 0 \\ x(2t) & t \geq 0 \end{cases}$ NC

$x(2t) \rightarrow$ future

5. $y(t) = \begin{cases} x(t) & t < 0 \\ x(2t) & t = 0 \\ x(t-1) & t > 0 \end{cases}$ $\Rightarrow y(0) = x(0) \rightarrow$ present. C

$x(t-1) \rightarrow$ past

6. $y(t) = \int_{-\infty}^{t} x(k)\,dk.$ C. $x(-\infty) + \ldots\ldots x(t)$

7. $y(t) = \int_{-\infty}^{-t} x(k)\,dk.$ NC $x(-\infty) + \ldots\ldots x(-t)$

At $t = -1$

$y(-1) = \ldots\ldots x(1)$

8. $y(t) = \int_{-\infty}^{t} x(-k)\,dk$ NC

$x(-t)$

At $t = -1$

$y(-1) = x(1)$

9. $y(t) = CS[x(t)]$ CS = conjugate symmetric

$= \dfrac{x(t) + x^*(-t)}{2}$

At $t = -1$

$y(-1) = \dfrac{x(-1) + x^*(1)}{2}$ $\therefore$ NC.

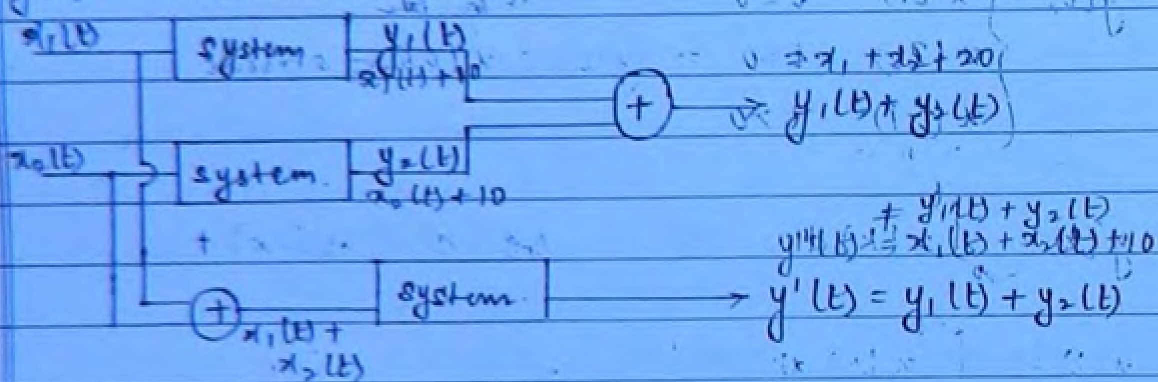
3. Linear / Non-linear system :- (64)

Linear system → follows law of superposition

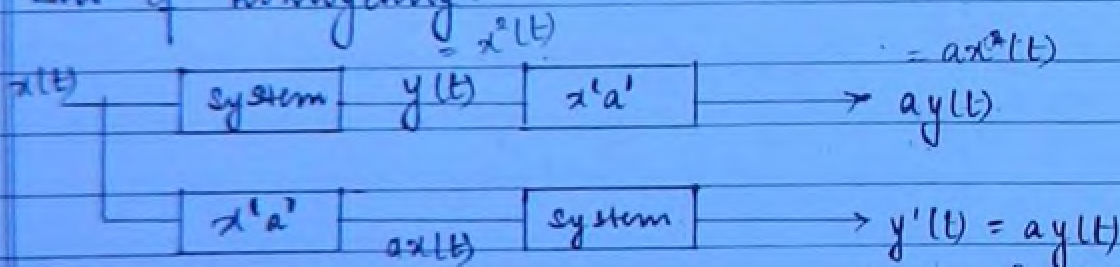
→ Law of superposition = Law of additivity & Law of homogenity

Law of additivity

eg. $y(t) = x(t) + 10$ NL



Law of homogenity



eg $y(t) = x^2(t)$ NL

Q. $y(t) = \cos[x(t)]$

additivity $\cos[x_1(t)]$, $\cos[x_2(t)]$ → + → $\cos[x_1(t)] + \cos[x_2(t)]$

$x_1(t) + x_2(t) \rightarrow y'(t) = \cos[x_1(t) + x_2(t)]$

Q $y(t) = x[\cos(t)]$ (65)

additivity $x_1 \cos t + x_2 \cos t = y_1(t) + y_2(t)$

$$y'(t) = \cancel{x_1 \cos t + x_2 \cos t} \cdot (x_1 + x_2)\cos t$$
$$= x_1 \cos t + x_2 \cos t$$

homogenity. $a x \cos(t) = a y(t)$

$$y'(t) = a x \cos t$$

Linear

* Alternate method –

For linear system

1. o/p should be 0 for 0 i/p.
2. No non-linear operator.
   [eg trignometric fn^n & inverse trig fn^n, log, exp, modulus, square, cube, root or power, sampling, sinc ( ), u ( ), sgn ( ) etc]
   should operate either on 'x' or 'y'

Integral & Differential operation are linear

Q Check linear / non-linear.

1. $y(t) = \log t \cdot x(t)$ L
2. $y(t) = x \log t$ L
3. $y(t) = \log[x(t)]$ NL
4. $y(t) = \begin{cases} x(t) & t < 0 \\ x(t+1) & t \geq 0 \end{cases}$ L for no input applied o/p is 0

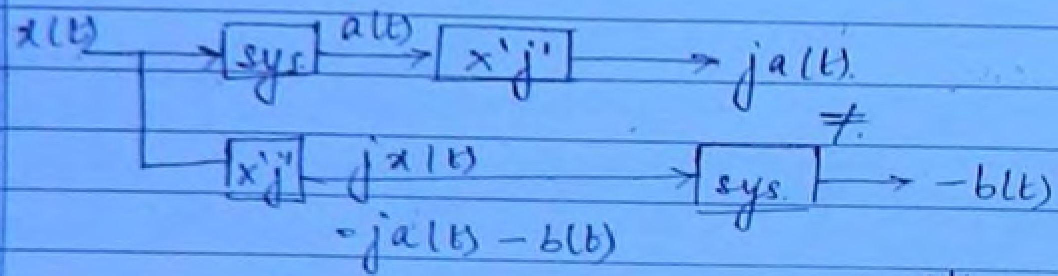
6. $y(t) = \text{odd}[x(t)] = \dfrac{x(t) + x(-t)}{2}$ L

7. $y(t) = \text{Real}[x(t)] = \dfrac{x(t) + x^{*}(t)}{2}$ NL

NL operator

$x(t) = a(t) + j\,b(t)$

Law of homogenity



NL

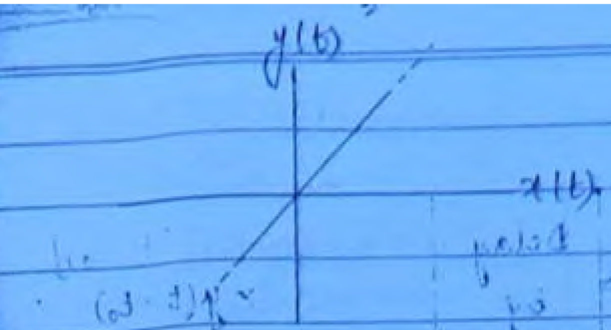
* Real, imaginary & conjugate operators are non-linear

* Even & Odd operators are linear operators

* Integral & derivative operators are linear

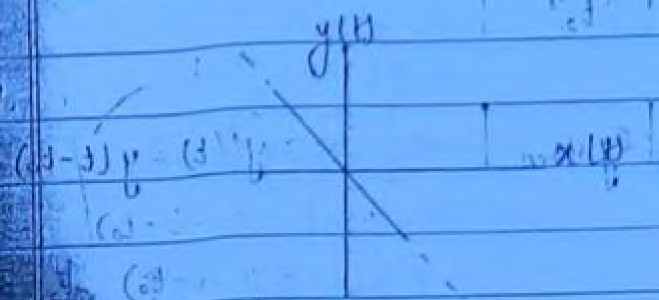
* System linearity is independent of time-scaling, time shifting & co-efficients of the system

8.



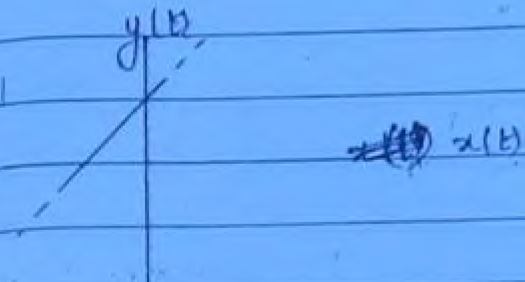
L $y(t) = mx(t)$

9.



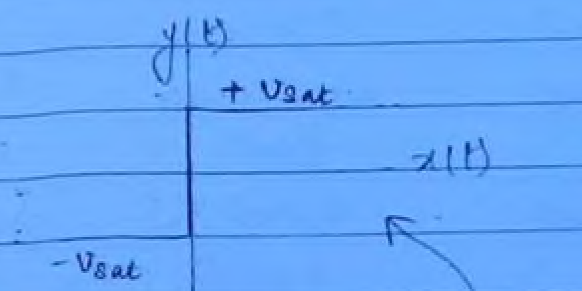
L $y(t) = -mx(t)$

10.



NL. $y(t) = mx(t) + c$

11.



NL. $V_{sat} = sgn[x(t)]$

$t=0$ $y(0) = +V_{sat}$ n $-V_{sat}$

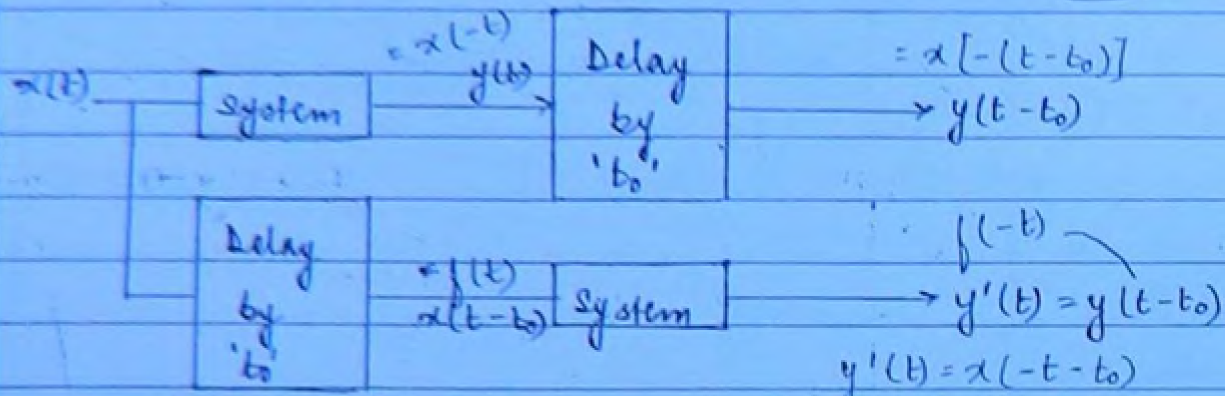
graph.

12. Comparator.

NL

* Virtual ground concept is applicable only for linear op-amps. This concept is not applicable to comparator & schmitt trigger coz they are NL.

4. Time invariant / Time variant system -



$y'(t) = x(-t-t_0)$

$\neq y(t-t_0)$

eg $y(t) = x(-t)$ TV.

Q $y(t) = x(\cos t)$

$y(t) = x \cos t$ $= x \cos(t-t_0)$

~~$f(t) = x\cos(t-t_0)$~~ ~~$= x\cos(t-t_0)$~~

$x(t-t_0)$ $f(\cos t)$

~~Ex~~ $= f(t)$ $= x[\cos t - t_0]$

$\neq y'(t)$

TV

Q $y(t) = x[t^2]$

$y(t) = x[t^2]$ $= x(t^2 - t_0)$

$f(t) = x(t-t_0)$ $= x(t - t_0)^2$

TV

$y(t) = t\,x(t)$

$y(t) = t\,x(t) \longrightarrow (t-t_0)\,x(t-t_0) = y(t-t_0)$

$x(t-t_0) = f(t)$ $\qquad$ $\cancel{(t-t_0)}\, x(t-t_0) = t\,f(t)$

~~TIV~~ TV.

For TIV system

1. There should not be any scaling of time either in $x(t)$ or $y(t)$.
2. All the co-efficients in system relationship should be independent on time.

$y(t) = \cos[x(t)]$ $\qquad$ TIV.

$y(t) = [x\underbrace{|t|}_{\text{scaling}}]$ $\qquad$ ~~TIV~~ TV

$y(t) = |t|\,x(t)$ $\qquad$ TV

$y(t) = \begin{cases} x(t) & t<0 \\ x(t-1) & t\geq 0 \end{cases}$ $\qquad$ TV $\qquad$ Conditions on time are imposed → TV.

$y(t) = \int_{-\infty}^{t} x(k)\,dk.$ $\qquad$ ~~TV~~ TIV $\qquad$ Integrator is LTI system

$y(t) = \int^{2t} x(k)\,dk$ $\qquad$ TV

Q. $y(t) = \int_{-\infty}^{t} x(-k)\,dk$. TV.

$\downarrow$

$x(\pm t)$
scaling.

70

Q $y(t) = \int_{-\infty}^{t} \underline{\cos k\, x(k)}\,dk$. TV.

$\downarrow$

on putting limits.
$\underline{\cos t\; x(t)}$
$\hookrightarrow$ TV.

Q $y(t) = \text{Real}\,[x(t)] = \dfrac{x(t) + x^*(t)}{2}$ TIV.

Q $y(t) = CS\,[x(t)] = \dfrac{x(t) + x^*(-t)}{2}$ TV.

Differential Equations for LTI systems:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t)$$
$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1}\frac{d^{m-1} x(t)}{dt^{m-1}} + \ldots + b_0 x(t)$$

For time invariance –
$a_n, a_{n-1} \ldots a_0,\ b_m, b_{m-1} \ldots b_0$ should be independent of time

For linearity –
Initial conditions/states should be zero

Q1. $2\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = x(t)$. L, TIV. 71

2. $\frac{d^2y(t)}{dt^2} + 2t\frac{dy(t)}{dt} + y(t) = x(t)$. TV.

3. $\left[\frac{dy(t)}{dt}\right]^{2} + 2\frac{dy(t)}{dt} + y(t) = x(t)$. TIV, NL.

$\rightarrow$ NL.

4. $y(t) = x(t).x(t)$. NL, TIV.

$y(t) = x^2(t)$

$x(t) \longrightarrow y(t) = x^2(t) \longrightarrow x^2(t-t_0)$

$x(t) \longrightarrow x(t-t_0) \longrightarrow x^2(t-t_0)$

TIV.

* Any delay provided in input must be reflected in output for a TIV system.

5. Stable / Unstable System -

BIBO (Bounded Input Bounded Output) Criteria -

bounded / finite in amplitude.

eg $u(t)$, $\sin t$, $\cos t$, dc, $sgn(t)$

Output should be bounded and finite for finite & bounded input / for all instants of time.

Q1. $y(t) = x(t) + 2$ → Stable

$x(t)$	$y(t)$
2	4

2. $y(t) = t\, x(t)$ → Unstable

$x(t)$	$y(t)$
2	2t

3. $y(t) = x(t) + t$ → Unstable

$x(t)$	$y(t)$
2	2+t

4. $y(t) = \cos t\; x(t)$ → Stable

$x(t)$	$y(t)$
2	2cost

5. $y(t) = \frac{x(t)}{t-1}$ → Unstable

$x(t)$	$y(t)$
2	$\frac{2}{t-1}$

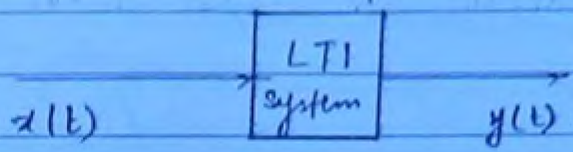
6. $y(t) = \frac{x(t)}{\sin t}$ → Unstable

$x(t)$	$y(t)$
2	$\frac{2}{\sin t}$

7. $y(t) = \text{Real}\,[x(t)]$ → Stable

$= \frac{x(t) + x^*(t)}{2}$

Linear Time Invariant (LTI) System -



$y(t) = x(t) * h(t)$

$h(t)$ = Impulse response of system.

$H(\omega)$ or $H(s)$ = Transfer function.

Impulse Response -

If input to the LTI system is unit impulse then response of the system is known as impulse response.

Convolution -

* Convolution is used to calculate response of LTI system.
* Convolution is a LTI operator.

$$y(t) = x(t) * h(t)$$

Transfer function

Its defined as the ratio of the laplace transform of output to the laplace transform of input when initial conditions are assumed to be zero.

$y(t)$ = Zero input response + Zero state response

ie) $y(t)$ = ZIR + ZSR

ZIR: due to initial conditions or states

ZSR: due to applied input

For linear system initial condition should be zero because non-zero initial conditions make a system non-linear.

1. Condition for LTI system to be static.-

$$y(t) = x(t) * h(t)$$

$$\Rightarrow \quad y(t) = \int_{-\infty}^{\infty} h(\tau) . \underbrace{x(t-\tau)}_{\text{past / future values}} d\tau$$

$$h(\tau) = 0 \quad \tau \neq 0 \xrightarrow{\tau = t}$$

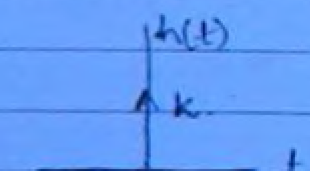
$$h(t) = 0 \quad t \neq 0$$

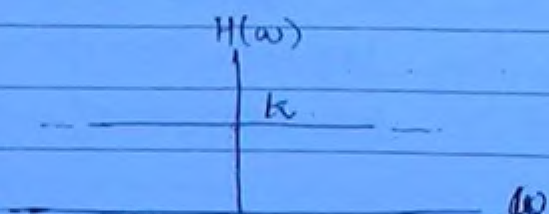
$$\boxed{h(t) = K\delta(t)}$$

Condition for static LTI system

$$h(t) = K\delta(t)$$

$$H(\omega) \text{ or } H(s) = K$$



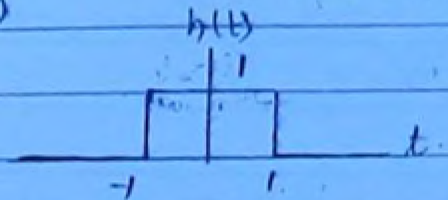


For static LTI system transfer function should be independant of frequency

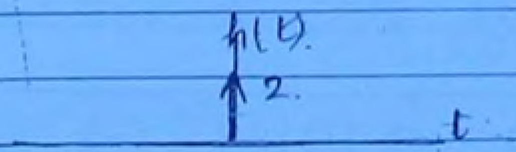
Q Check static / dynamic LTI system

1. $h(t) = \sin t$ D.   75

2. $h(t)$ D.



3. S.



4. $h(t) = \delta(t-2)$ D.

5. $H(s) = \frac{s}{s+1}$ HPF   D.

6. $H(s) = \frac{1}{s}$ LPF   D

7. $H(s) = \frac{s-1}{s+1}$ APF   D.

* Filters are dynamic system.

Condition for LTI system to be causal

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, \overbrace{x(t-\tau)}^{\text{future input}}\, d\tau \qquad \tau < 0$$

$$h(\tau) = 0 \qquad \tau < 0$$

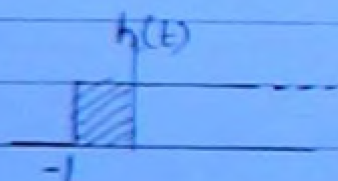
$\downarrow$ $\tau = t$

$$\boxed{h(t) = 0 \qquad t < 0}$$

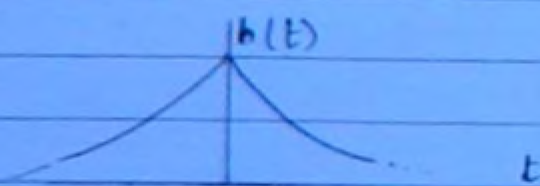
Q Check causal / non-causal system.

1. $h(t) = u(t)$ C

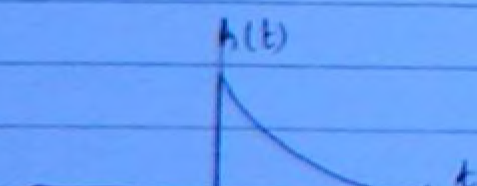
2. $h(t) = (t+1)\,u(t)$ C

3. 

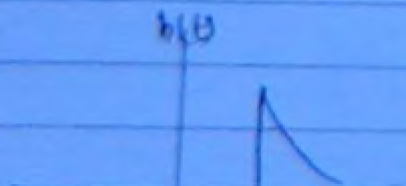
NC

4. 

NC

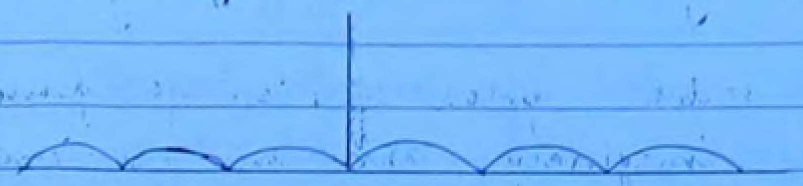
5. 

C

6. 

C

7. $h(t) = |\sin(t)|$ NC



Condition for LTI system to be stable.

$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

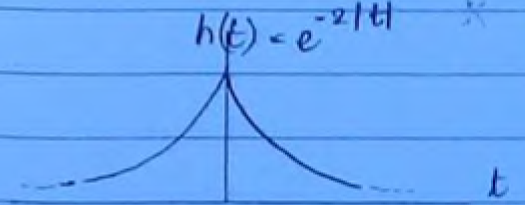
ie impulse response should be absolutely integrable.

* If impulse response of system is represented by an energy signal then system will be stable.

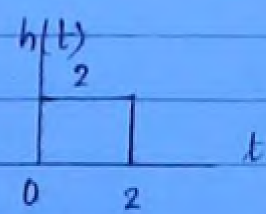
** Impulse response & transfer function terms are used only for LTI systems.

eg.

1. $h(t) = e^{-2|t|}$



2.



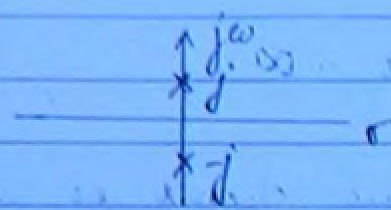
3. $h(t) = Sa(t)$

Condition for LTI system to be marginally stable :-

For marginally stable, poles of transfer function are located on imaginary axis in pole-zero plot.

eg 1. $H(s) = \frac{1}{s^2+1}$

poles : $s = \pm j$

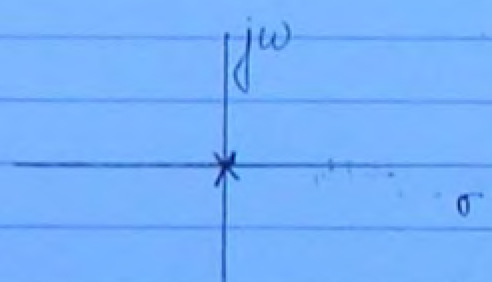


System is marginally stable.

$h(t) = \sin t \, u(t)$ ← power signal

2. $H(s) = \frac{1}{s}$

pole $s = 0$



System is marginally stable.

$h(t) = u(t)$ ← power signal

If impulse response of LTI system is represented by a power signal, then system will be marginally stable.

Q Check stability of system:-

1. $h(t) = \cos\omega_0 t \cdot u(t)$ → power signal
System is marginally stable

2. $h(t) = r(t)$ → unstable

3. $h(t) = e^{-2t}u(t)$ → energy signal

4. $h(t) = e^{2t}u(t)$ → unstable

Integration :-

$$y(t) = \int_{-\infty}^{t} x(k)\, dk.$$

$$\Rightarrow Y(s) = \frac{X(s)}{s}$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s}$$

$$\boxed{H(s) = \frac{1}{s}}$$ → marginally stable

BIBO:

$\Rightarrow x(t) = u(t)$ = bounded input.

$$\Rightarrow y(t) = \int_{-\infty}^{t} x(k)\, dk$$

$$= \int_{-\infty}^{t} u(k)\, dk.$$

$= r(t)$ → unbounded

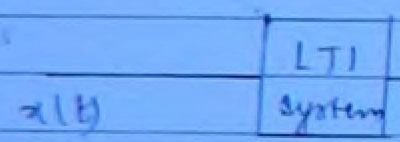
All marginally stable LTI systems are unstable according to BIBO criteria. This distortion is

Distortion in LTI site -

- magnitude distortion.
- phase / delay distortion.

Magnitude distortion.-

A system provides unequal amount of amplification or attenuation to different frequency ~~...~~ components present in input signal. Then system is having magnetotude distortion.



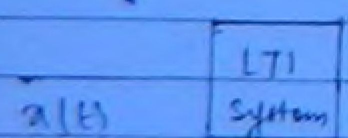
$y(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$

$= \sin \omega_1 t + \sin \omega_2 t$ $\quad A_1 \neq A_2$.

$\omega_1 \neq \omega_2$

Phase / Delay Distortion -

A system provided unequal amount of time delay to different frequency components present in input signals then system is having phase/delay distortion.



$y(t) = \sin[\omega(t - t_1)]$
$+ \sin[\omega_2 (t - t_2)]$

$= \sin \omega_1 t + \sin \omega_2 t$ $\quad t_1 \neq t_2$

Condition for distortionless LTI system -

$x(t) \rightarrow \boxed{\text{LTI system}} \rightarrow y(t) = kx(t-t_0)$

$= \sin\omega_1 t \qquad = k\sin[\omega_1(t-t_0)]$

$+ \sin\omega_2 t. \qquad = k\sin(\omega_2(t-t_0))]$

$$y(t) = k\,x(t-t_0)$$

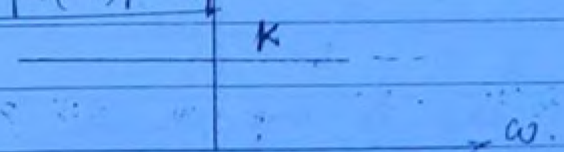
$$Y(s) = k \cdot X(s)\, e^{-st_0}$$

$$H(s) = \frac{Y(s)}{X(s)} = k e^{-st_0}$$

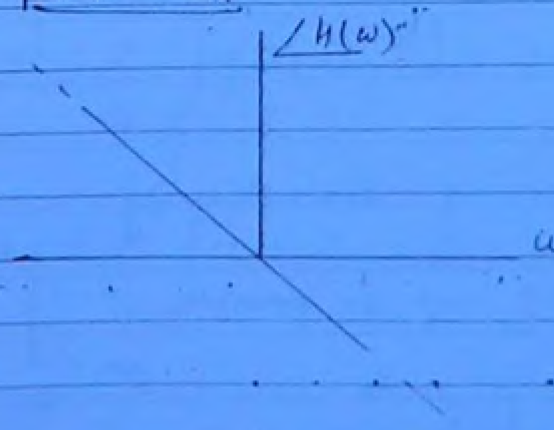
$$\downarrow s = j\omega$$

$$\boxed{H(\omega) = k e^{-j\omega t_0}}$$

$$\boxed{|H(\omega)| = k}$$



$$\boxed{\angle H(\omega) = -\omega t_0}$$



For distortionless LTI system, mag^n of transfer function should be independent of freq & phase of transfer function should be linear

FOURIER SERIES

* Fourier Series expansion is used for periodic power signals.

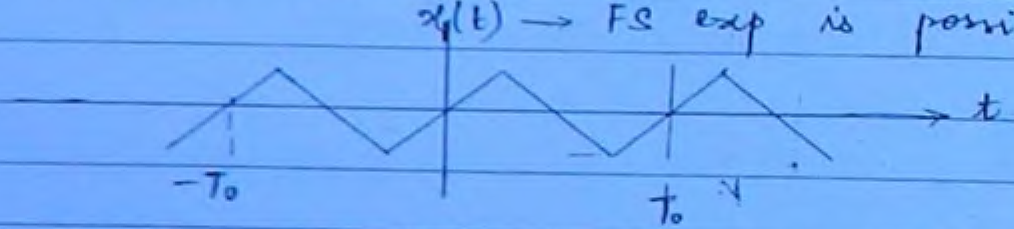
* In fourier series signal is expanded in terms of its harmonics which are sinusoidal & orthogonal to one another.

Condition for existance of FS expansion
- (Dirichlet Conditions).

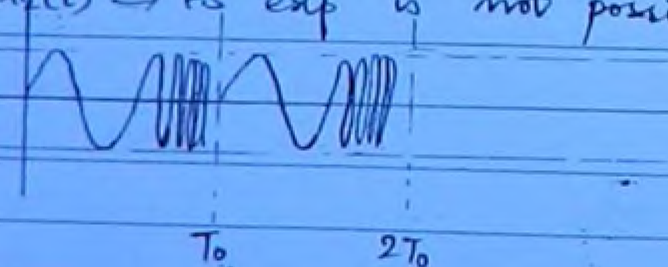
1. Signal should be deterministic over its time period.

   a) s/g should have finite no. of maxima & minima over 'T_0'.

$x_1(t) \rightarrow$ FS exp is possible



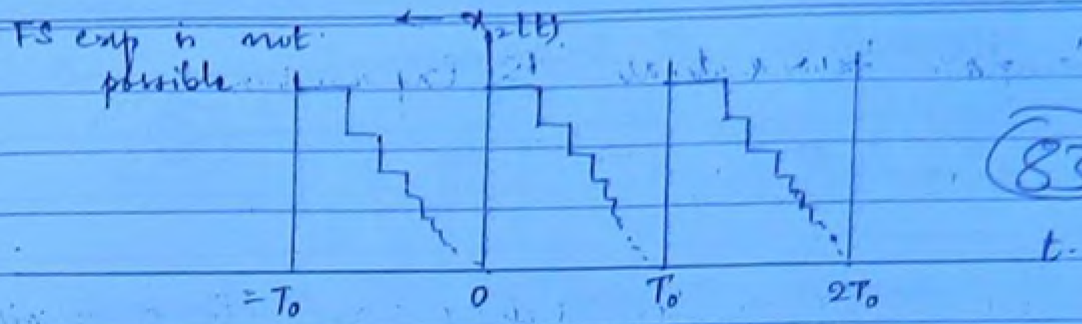
$x_2(t) \rightarrow$ FS exp is not possible.



   b) s/g should have finite no. of discontinuity over 'T_0'

$x_1(t) \rightarrow$ FS exp is possible.

FS exp is not possible



2. Signal should be absolutely integrable over its time period.

ie $$\boxed{\int_{T_0} |x(t)|\, dt < \infty}$$

Types of FS expansion -

1. Trignometric FS expansion
2. Complex Exponential FS expansion

1. Trignometric FS expansion -

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where a_0 = avg / dc value of $x(t)$

$$= \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt$$

$a_n \to n\omega_0$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\, dt$$

$b_n \to n\omega_0$

2. Complex Exponential FS Expansion

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$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where C_n = complex exponential FS co-efficients

$$= \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t} dt \quad —(1)$$

$$C_{-n} = \frac{1}{T_0}\int_{T_0} x(t) e^{jn\omega_0 t} dt.$$

$$C_{-n}^{*} = \frac{1}{T_0}\int_{T_0} x^{*}(t) e^{-jn\omega_0 t} dt \quad —(2)$$

For CS C_n :

$$C_n = C_{-n}^{*}$$

From (1) & (2)

$$x(t) = x^{*}(t)$$

$$C_n = |C_n| e^{j\angle C_n} \quad —(3)$$

$$C_{-n} = |C_{-n}| e^{j\angle C_{-n}}$$

$$C_{-n}^{*} = |C_{-n}| e^{-j\angle C_{-n}} \quad —(4)$$

For CS C_n

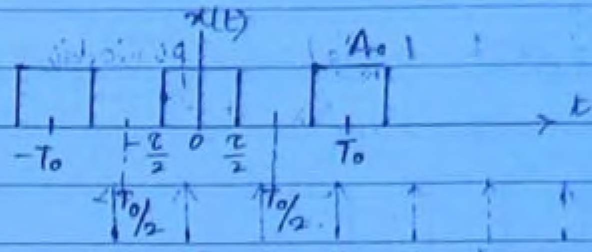
$$C_n = C_{-n}^{*}$$

From (3) & (4)

$$|C_n| = |C_{-n}| \quad \rightarrow \text{Even symmetry}$$

$$\angle C_n = -\angle C_{-n}$$

Q. Find C_n



Sol.

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,e^{-jn\omega_0 t}\,dt$$

$$C_n = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A_0\,e^{-jn\omega_0 t}\,dt$$

$$= \frac{A_0}{T_0}\left[\frac{-e^{-jn\omega_0 t}}{jn\omega_0}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{A_0}{T_0} \times \frac{1}{jn\omega_0}\left[e^{jn\omega_0\tau/2} - e^{-jn\omega_0\tau/2}\right]$$

$$C_n = \frac{A_0}{T_0} \times \frac{1}{jn\omega_0} \times \frac{2j\sin\left(\frac{n\omega_0\tau}{2}\right)}{\frac{n\omega_0\tau}{2}} \times \frac{n\omega_0\tau}{2}$$

$$C_n = \frac{A_0\tau}{T_0}\,Sa\left(\frac{n\omega_0\tau}{2}\right) \longrightarrow CS. \qquad Sa(c) = \frac{\sin c}{c}$$

$$|C_n| = \frac{A_0\tau}{T_0}\left|Sa\left(\frac{n\omega_0\tau}{2}\right)\right| \longrightarrow \text{Even symmetry}$$

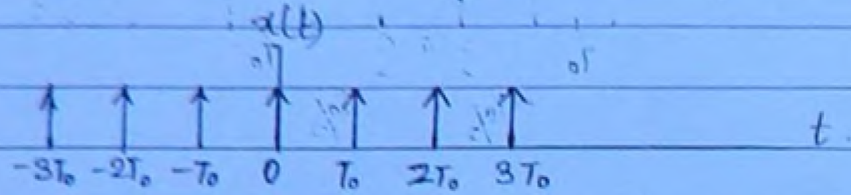
= magnitude of n^{th} harmonic $(n\omega_0)$

$$\angle C_n = \frac{n\omega_0\tau}{2} \longrightarrow \text{Odd symmetry}$$

= phase of n^{th} harmonic $(n\omega_0)$

Q Find C_n.

$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ ⇒ periodic impulse train



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Sol

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{\delta(t)\, e^{-jn\omega_0 t}}_{\substack{f(t)\cdot\delta(t) \\ = f(0)\,\delta(t)}}\, dt \qquad \because f(t)\,\delta(t) = f(0)\,\delta(t)$$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \overbrace{\delta(t)\, dt}^{1}$$

$$C_n = \frac{1}{T_0}$$

Q

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos\left[2\omega_0 t + \frac{\pi}{4}\right]$$

Find C_n

Sol

$$x(t) = 1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{1}{2}\left[e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right]$$

$$x(t) = 1 + e^{j\omega_0 t}\left[1 + \frac{1}{2j}\right] + e^{-j\omega_0 t}\left[1 - \frac{1}{2j}\right] +$$
$$\left[\frac{1}{2} e^{j\pi/4}\right] e^{j2\omega_0 t} + \left[\frac{1}{2} e^{-j\pi/4}\right] e^{-j\omega_0 t}$$

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C_n are $\boxed{C_n e^{jn\omega_0 t}, \; C_{-2} e^{-j2\omega_0 t} \\ C_2 e^{j2\omega_0 t}}$

$$\Rightarrow x(t) = C_0 + C_1 e^{j\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_{-2} e^{-j2\omega_0 t}$$

$C_0 = 1$

$C_1 = 1 + \frac{1}{2j}$ | $C_{-1} = 1 - \frac{1}{2j}$

$C_2 = \frac{1}{2} e^{j\pi/4} = \frac{1}{2}\left(\frac{1+j}{\sqrt{2}}\right)$ | $C_{-2} = \frac{1}{2} e^{-j\pi/4} = \frac{1}{2}\left(\frac{1-j}{\sqrt{2}}\right)$

Q. Consider a periodic sig $x(t)$ with $T_0 = 8$ of FS co-eff.

$C_1 = C_{-1} = 2$

$C_3 = 4j \qquad C_{-3} = -4j$

Find $x(t)$

Sol $\cancel{x(t) = 2}$ $\qquad T_0 = 8 \qquad \frac{2\pi}{\omega_0} = 8 \qquad \omega_0 = \frac{\pi}{4}$

$\cancel{\frac{2\omega_0}{2\pi} = 8}$

$\cancel{\omega_0 = 16\pi}$

$$x(t) = C_1 e^{j\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_3 e^{3j\omega_0 t} + C_{-3} e^{-3j\omega_0 t}$$

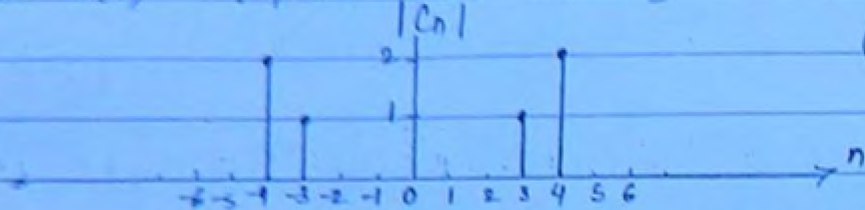
$$x(t) = 2e^{j\omega_0 t} + 2e^{-j\omega_0 t} + 4je^{3j\omega_0 t} - 4j e^{-3j\omega_0 t}$$

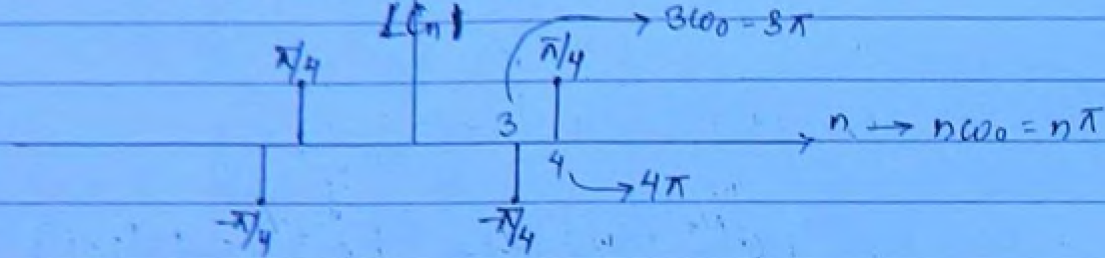
$$\Rightarrow x(t) = 2(2\cos\omega_0 t) + 4j(2j\sin 3\omega_0 t)$$

$$= 4\cos\omega_0 t - 8\sin 3\omega_0 t$$

$$= 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t$$

Q. Find $x(t)$ [Given that $\omega_0 = \pi$]





Ans

a) $6\cos\left(2\pi t + \frac{\pi}{4}\right) - 3\cos\left(3\pi t - \frac{\pi}{4}\right)$ ✓ ✗

b) $4\cos\left(4\pi t - \frac{\pi}{4}\right) - 2\cos\left(3\pi t + \frac{\pi}{4}\right)$ ✗ ✗

c) $2\cos\left(2\pi t + \frac{\pi}{4}\right) - 2\cos\left(3\pi t - \frac{\pi}{4}\right)$ ✓ ✗

d) $4\cos\left(4\pi t + \frac{\pi}{4}\right) + 2\cos\left(3\pi t - \frac{\pi}{4}\right)$ ✓ ✓

Sol

$$C_n = |C_n| e^{j\angle C_n}$$

Solve by elimination

Relation between a_n, b_n & C_n

→ $C_n = \frac{1}{2}(a_n - jb_n)$

→ $a_n = 2\,\text{Real}[C_n]$

→ $b_n = -2\,\text{Img}[C_n]$

→ $a_0 = C_0$ = avg / dc value of $x(t)$

Q If FS expansion of a s/g f(t) is (89)

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(3n\pi)^2} \cdot e^{j\pi n t}$$

Determine :

a) Time period of f(t)

b) A term in that ~~expression~~ expansion is $A_0 \cos 6\pi t$, then calculate the value of A_0

c) Repeat part (b) for $A_0 \sin 6\pi t$

Sol a) $\omega_0 = \pi$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\frac{2\pi}{T_0} = \pi \qquad T_0 = 2$$

b) $$C_n = \frac{3}{4+(3n\pi)^2}$$

$$C_6 = \frac{3}{4+(18\pi)^2}$$

$A_0 \cos 6\pi t = a_n \cos n\omega_0 t$

$= a_n \cos n\pi t$

$n=6$

$a_n = 2 \text{ Real } [C_n]$

$$A_0 = a_6 = 2C_6 = \frac{2\times 3}{4+(18\pi)^2}$$

$a_n = 2 \text{ Real}[C_n] = 2C_n$

c) $A_0 \sin 6\pi t = b_n \sin n\omega_0 t$

$= b_n \sin n\pi t$

$n=6$

$A_0 = b_6 = 0$

$x(t) \div C_n$ pairs

	$x(t)$		C_n
1.	Real	$\longrightarrow$	CS
2.	CS	$\longrightarrow$	Real
3.	Img	$\longrightarrow$	CAS
4.	CAS	$\longrightarrow$	Img
5.	Real + even	$\longrightarrow$	Real + even
6.	Img + even	$\longrightarrow$	Img + even
7.	Real + odd	$\longrightarrow$	Real + odd
8.	Img + odd	$\longrightarrow$	Img + odd.

Q. $f(t) = \sum_{n=-100}^{100} \cos n\pi \, e^{jn\pi t}$

$\hookrightarrow C_n = R + E$.

then $f(t)$ will be

a) R + O
b) I + O
c) R + E
d) I + E.

Q. $f(t) = \sum_{n=-100}^{100} j\sin n\pi \, e^{jn\pi t}$

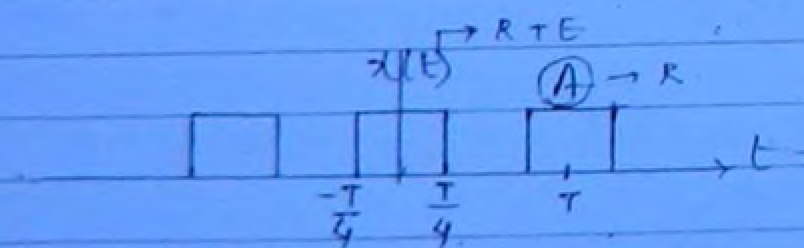
$\hookrightarrow C_n = I + O$

then $f(t)$ will be

a) R + O
b) I + O
c) R + E
d) I + E

CWB chapter 3 Q 14

Q.



$C_k = ? = R + E$

a) $\frac{A}{j\pi k} \sin\left(\frac{\pi}{2} k\right)$

b) $\frac{A}{2\pi k} \cos\left(\frac{\pi}{2} k\right)$

Sol
$$f(k) = k$$
$$f(-k) = -k = -f(k)$$
$\Big] \rightarrow$ odd signal.

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Symmetricities in FS -

1. • Even symmetry -

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \underset{E \quad O}{x(t)} \, dt = \frac{2}{T_0} \int_{0}^{T_0/2} x(t) \, dt$$

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{E \quad O} \underbrace{\cos n\omega_0 t}_{E \quad E = E \quad O} \, dt$$

$$= \frac{4}{T_0} \int_{0}^{T_0/2} x(t) \cos n\omega_0 t \, dt$$

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{E \quad O} \underbrace{\sin n\omega_0 t}_{O \quad O = O \quad E} \, dt$$

$$= 0$$

$$\Rightarrow \quad x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \underbrace{\sum_{n=1}^{\infty} b_n \sin n\omega_0 t}_{\to 0}$$

Fourier series expansion of any even signal does not contain any sine term.

2. Odd symmetry -

$a_0 = 0$

$a_n = 0$

$b_n \neq 0$

Fourier series expansion of any odd signal will contain sine term.

3. Half wave symmetry - (HWS)
Fourier series expansion of any half wave symmetry contains only odd harmonics.

4) Even + HWS -

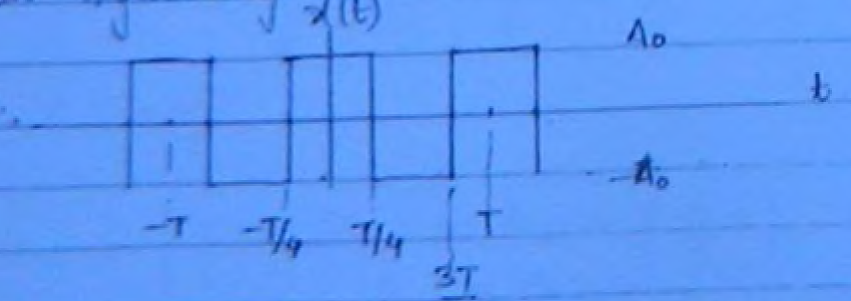
HWS → adv value is 0
odd harmonics present

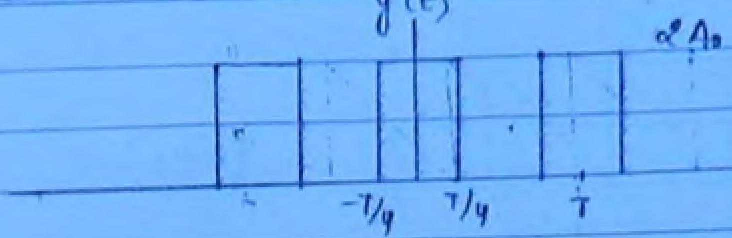
Even → cosine terms present

FS expansion of Even HWS signal contains cosine term with odd harmonics.

5) Odd + HWS -
FS expansion of odd HWS signal contains sine terms with odd harmonics.

6. Hidden symmetry -

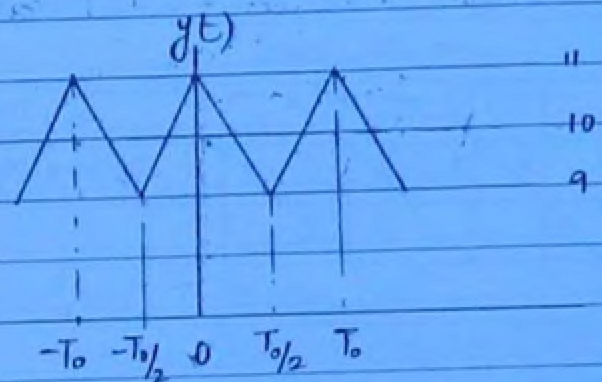




$$y(t) = x(t) + \underset{\substack{\downarrow \\ dc}}{A_0}$$

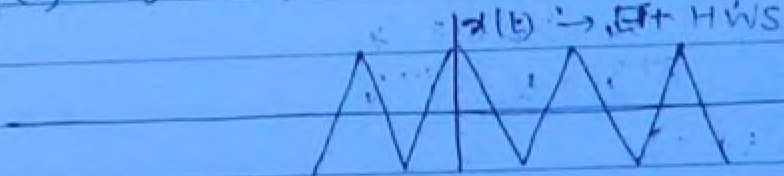
$$y(t) = dc + a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t + a_5 \cos 5\omega_0 t + \ldots$$

Q.

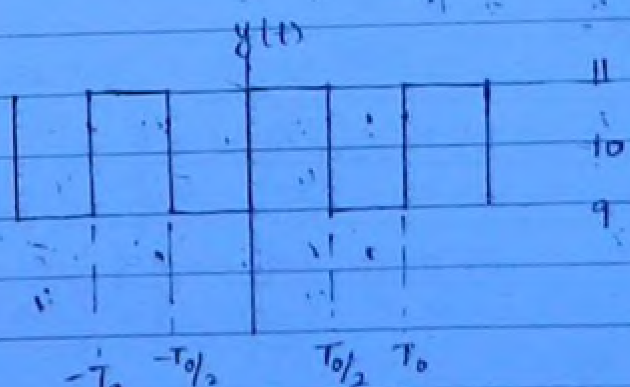


Sol $y(t) = 10 + x(t) = 10 + a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t + a_5 \cos 5\omega_0 t + \ldots$

$x(t) \rightarrow$ Eff + HWS $\rightarrow$ odd harmonics of cos

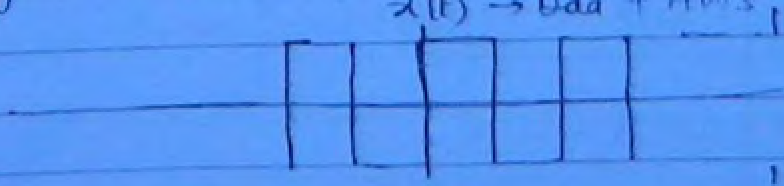


Q.

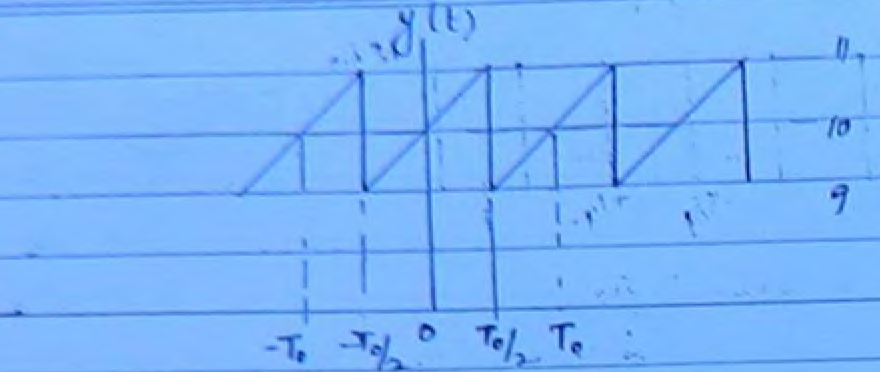


Sol $y(t) = 10 + x(t) = 10 + b_1 \sin \omega_0 t + b_3 \sin 3\omega_0 t + \ldots$

$x(t) \rightarrow$ Odd + HWS $\rightarrow$ odd harmonic of sine



Q



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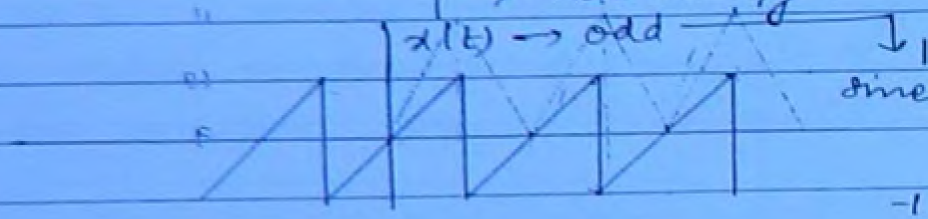
Sol.

$y(t) = 10 + x(t)$

$= 10 + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + b_3 \sin 3\omega_0 t + \ldots$

→ Sawtooth s/g

$x(t) \rightarrow$ odd → sine terms



Q A s/g $x(t)$ is given by

$$x(t) = \begin{cases} +1 & -T/4 < t \leq 3T/4 \\ -1 & 3T/4 < t \leq 7T/4 \\ -x(t+T) & \end{cases}$$

Which among the following gives the fundamental fourier terms of $x(t)$?

t = 0

+ve a) $\frac{4}{\pi} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

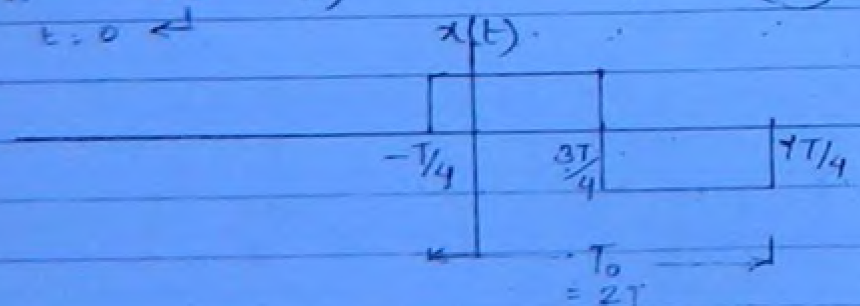
b) $\frac{\pi}{4} \cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

c) $\frac{4}{\pi} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

d) $\frac{\pi}{4} \sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

-ve t = 0

Sol



$$T_0 = \frac{7T}{4} + \frac{T}{4} = \frac{8T}{4} = 2T$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

$x(t) \rightarrow$ sq. wave shifted by $\frac{\pi}{4}$ $\frac{\pi}{4}$ $\frac{\pi}{4}$.

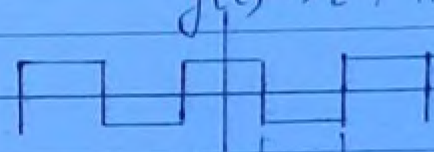
$x(t) \rightarrow$ HWS ~~+ even~~

$x(t) \rightarrow$ contains only cos terms.

$$x(t) = \text{HWS}$$
$$= -x\left(t + \frac{T_0}{2}\right) = -x(t+T)$$

$$x(t) = a_1 \cos\omega_0 t + b_1 \sin\omega_0 t + a_3 \cos 3\omega_0 t + b_3 \sin 3\omega_0 t + \ldots$$

$y(t) \rightarrow E + \text{HWS}$.



$$y(t) = a_1 \cos\omega_0 t + a_3 \cos 3\omega_0 t + \ldots$$

$$x(t) = y\left(t - \frac{T}{4}\right)$$

$$= a_1 \cos\left[\omega_0\left(t - \frac{\pi}{4}\right)\right] + a_3 \cos\left[3\omega_0\left(t - \frac{T}{4}\right)\right] + \ldots$$

$$a_1 \cos\left[\omega_0\left(t - \frac{\pi}{4}\right)\right] = a_1 \cos\left[\frac{\pi}{T}\left(t - \frac{\pi}{4}\right)\right]$$

$$= a_1 \cos\left[\frac{\pi t}{T} - \frac{\pi}{4}\right]$$

* The polarity of periodic signal at any time instant is decided by the polarity of fundamental fourier term or term with fundamental frequency because this term is dominant in the expansion of periodic signal. (signal should have infinite number of harmonics) (96)

By this method.
At $t = 0$
polarity of $x(t) =$ +ve.

Substitute in options (a) & (c).
Ans (a) as its +ve

Properties of Fourier Series -

1. Linearity -

$$a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 C_{1n} + a_2 C_{2n}$$

where $x_1(t) \rightleftharpoons C_{1n}$
$x_2(t) \rightleftharpoons C_{2n}$

2. Time Reversal -
$$x(-t) \rightleftharpoons C_{-n}$$

3. Time shifting
$$x(t - t_0) \rightleftharpoons C_n e^{-jn\omega_0 t_0}$$

4. Conjugation
$$x^*(t) \rightleftharpoons C_{-n}^*$$

5. Frequency shifting -

$$e^{jm\omega_0 t}x(t) \rightleftharpoons C_{n-m}$$

6. Convolution in time -

$$\underbrace{x_1(t)}_{\downarrow \; T_1} * \underbrace{x_2(t)}_{\downarrow \; T_2} \rightleftharpoons T\left[C_{1n} \cdot C_{2n}\right]$$

$$T = LCM\left[T_1, T_2\right]$$

7. Multiplication in Time -

$$x_1(t)\, x_2(t) \rightleftharpoons C_{1n} * C_{2n}$$

8. Differentiation in Time -

$$\frac{d^m x(t)}{dt^m} \rightleftharpoons (jn\omega_0)^m C_n$$

9. Integration in time -

$$\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \frac{C_n}{jn\omega_0}$$

10. Parsevals power theorem -

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

11. Time Scaling -

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(\alpha t) = \sum_{n=-\infty}^{\infty} C_n e^{jn(\alpha\omega_0)t}$$

$x(t) \rightarrow$

$x(\alpha t) \rightarrow$

Q Find C_n in terms of where

$$x(t) \rightleftharpoons C_n$$

$$y(t) \rightleftharpoons C_n'$$

(98)

i) $y(t) = x(t-t_0) + x(t+t_0)$

ii) $y(t) = e^{j2\omega_0 t} x(t)$

iii) $y(t) = \text{Real}\,[x(t)]$

iv) $y(t) = \text{Odd}\,[x(t)]$

Sol

i) $C_n' = C_n e^{-jn\omega_0 t_0} + C_n e^{jn\omega_0 t_0}$

$= C_n \left[e^{jn\omega_0 t_0} + e^{-jn\omega_0 t_0}\right]$

$= 2C_n \cos n\omega_0 t_0$

ii) $C_n' = C_{n-2}$

iii) $\text{Real}\,[x(t)] = y(t)$

$\dfrac{x(t) + x^*(t)}{2} = y(t)$

$\cancel{C_n' = \dfrac{C_n + C_n^*}{2}}$ $\qquad C_n' = \dfrac{C_n + C_{-n}^*}{2}$

iv) $y(t) = \text{Odd}\,[x(t)]$

$y(t) = \dfrac{x(t) - x(-t)}{2}$

$C_n' = \dfrac{C_n - C_{-n}}{2}$

Q. Let $x(t)$ be a periodic signal with fundamental time period 'T' & ~~$y(t) = x(t) +$~~

if $y(t) = x(t-t_0) + x(t+t_0)$

The FS coefficients (exponential) of $y(t)$ are denoted by 'b_k'. If $b_k = 0$ for odd k

then t_0 can be equal to.

a) $\frac{T}{8}$ b) $\frac{T}{4}$ c) $\frac{T}{2}$ d) $2T$

sol.

$$y(t) = x(t-t_0) + x(t+t_0)$$

$$b_k = C_k e^{-jk\omega_0 t_0} + C_k e^{jk\omega_0 t_0}$$

$$= C_k \left[e^{-jk\omega_0 t_0} + e^{jk\omega_0 t_0} \right]$$

$$b_k = 2C_k \cos k\omega_0 t_0$$

for odd k $b_k = 0$

$$0 = 2C_k \cos k\omega_0 t_0$$

$$\cos k\omega_0 t_0 = 0$$

$$k\omega_0 t_0 = k\frac{\pi}{2}$$ → odd integer (given)

$$\omega_0 t_0 = \frac{\pi}{2}$$

$$\frac{2\pi}{T} t_0 = \frac{\pi}{2}$$

$$\cancel{t_0} = \frac{\pi}{4} \quad \boxed{t_0 = \frac{T}{4}} \rightarrow (b)$$

CNB. chapter 8.

Q17. The s/g $x(t)$ has $T_0 = 2$ & the foll FS co-effs

$$C_k \begin{cases} = \left(\frac{1}{2}\right)^k & k \geq 0 \\ = 0 & k < 0 \end{cases}$$

The value of $x(0)$ will be

a) 1 b) 2 c) 3 d) 4.

Sol. ~~$a_k = C_0 + C_1 e^{-jk\omega_0 t_0} +$~~

$$x(t) = \sum_{k=-\infty}^{\infty} C_n e^{jk\omega_0 t_0}$$

$$\Big\downarrow t=0$$

$$\boxed{x(0) = \sum_{k=-\infty}^{\infty} C_k}$$ VVIMP

$$x(0) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \ldots$$

$$= \frac{a}{1-r}$$

$$= \frac{1}{1-1/2}$$

$$= 2$$

Q 18 The s/g $x(t)$ has $T_0 = 1$ & the following FS co-eff.

$$C_k = \begin{cases} \left(\frac{-1}{3}\right)^k & k \geq 0 \\ 0 & k < 0 \end{cases}$$

Find $x(t)$

a) $\dfrac{1}{1 - \frac{1}{3} e^{j2\pi t}}$ b) $\dfrac{1}{1 + \frac{1}{3} e^{j2\pi t}}$

c) $\dfrac{1}{1 + \frac{1}{3} e^{-j2\pi t}}$ d) $\dfrac{1}{1 - \frac{1}{3} e^{-j2\pi t}}$

Sol

$$x(t) = \sum_{k=-\infty}^{\infty} C_n e^{-jk\omega_0 t}$$

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k e^{jk\omega_0 t}$$

$$= \cancel{1 - \frac{1}{3} + \left(\frac{+1}{3}\right)^2} = \sum_{k=0}^{\infty} \left(\frac{-1}{3} e^{j\omega_0 t}\right)^k$$

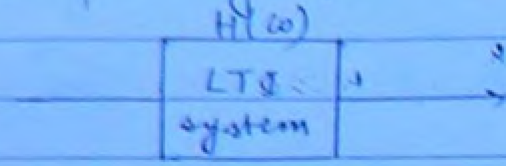
$$= 1 + \left(\frac{-1}{3} e^{j\omega_0 t}\right) + \left(\frac{-1}{3} e^{j\omega_0 t}\right)^2 + \ldots$$

$$= \frac{1}{1 - \left[\frac{-1}{3} e^{j\omega_0 t}\right]} = \frac{1}{1 + \frac{1}{3} e^{j\omega_0 t}}$$

$$\omega_0 = \frac{2\pi}{T_0} = 2\pi$$

Ans (b)

FS for LTI system -



$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \qquad y(t) = \sum_{n=-\infty}^{\infty} C_n' e^{jn\omega_0 t}$$

$$\boxed{C_n' = C_n H(n\omega_0)}$$

Q Consider the differential given below.

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

where $x(t) = i/p = \cos 2\pi t$

$y(t) = o/p$

Find o/p co-eff. C_n' for $y(t)$

Sol

$$\underset{=\cos 2\pi t}{x(t)} = \frac{dy(t)}{dt} + 4y(t) = sY(s) + 4Y(s) = \frac{s}{s^2 + 4\pi^2}$$

$sY(s) + 4Y(s) = X(s)$ ~~H(s)~~

$$\boxed{H(s) = \frac{1}{s+4}}$$

$$H(j\omega_0) = \frac{1}{j\omega_0 + 4}$$

$$x(t) = \cos \omega_0 t$$

$$= \frac{1}{2} e^{+j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$= C_1 e^{j\omega_0 t} + C_{-1} e^{-j\omega_0 t}$$

$$C_1 = \frac{1}{2} \qquad C_{-1} = \frac{1}{2}$$

~~x(t) =~~

$$C_n' = C_n H(n\omega_0)$$

$n=1$

$$C_1' = C_1 H(\omega_0) = \frac{1}{\ } \left(\frac{1}{\ } \right) = \frac{1}{\ } \left[\frac{1}{\ } \right]$$

FOURIER TRANSFORM

Fourier Transform is used for frequency domain analysis of energy & power signals, whereas Laplace transform can be used for analysis of neither energy nor power signals also. (upto certain extent)

Laplace transform is used for circuit analysis & fourier transform is used for signal analysis.

$$x(t) \rightleftharpoons \underset{\text{rad/sec}}{X(\omega)} \text{ or } \underset{\text{Hz}}{X(f)}$$

$$\boxed{H(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega}$$

Condition for existence of FT
(Dirichlet conditions)

1. Signal should be deterministic over any finite interval.
   i) Signal should have finite no. of minima & maxima over finite interval.
   ii) Signal should have finite no. of discontinuity over finite interval.

2. Signal should be absolutely integrable

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

Q $x(t) = e^{-at} u(t) \quad a > 0$

$X(\omega) = ?$

(104)

Sol: Replacement $s = j\omega$ is valid only for energy s/g

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \, dt$$

$$X(\omega) = \frac{\left[e^{-(a+j\omega)t} \right]_0^{\infty}}{-(a+j\omega)}$$

$$= \frac{e^{-(a+j\omega)\infty} - \cancel{e^{0}}^{\,1}}{-(a+j\omega)}$$

$$\cancel{e^{-(a+j\omega)\infty}}^{\,0} = \cancel{e^{-a\infty}}^{\,0} \cdot \cancel{e^{-j\infty}}^{\,\text{undefined}}$$

$$X(\omega) = \frac{0 - 1}{-(a+j\omega)}$$

$$= \frac{1}{a+j\omega}$$

Properties of fourier transform -

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1. Linearity -

$$a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 X_1(\omega) + a_2 X_2(\omega)$$

where $x_1(t) \rightleftharpoons X_1(\omega)$

$x_2(t) \rightleftharpoons X_2(\omega)$

2. Time reversal -

$$x(-t) \rightleftharpoons X(-\omega)$$

3. Conjugation -

$$x^*(t) \rightleftharpoons X^*(-\omega)$$

4. Time shifting -

$$x(t-t_0) \rightleftharpoons X(\omega) \cdot e^{-j\omega t_0}$$

5. Time scaling -

$$x(at) \quad a \neq 0 \quad \rightleftharpoons \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

6. Frequency shifting -

$$e^{j\omega_0 t} x(t) \rightleftharpoons X(\omega - \omega_0)$$

7. Convolution in time -

$$x_1(t) * x_2(t) \rightleftharpoons X_1(\omega) \cdot X_2(\omega)$$

8. Multiplication in time -

$$x_1(t) \cdot x_2(t) \rightleftharpoons \frac{1}{2\pi} [X_1(\omega) * X_2(\omega)]$$

or

(106)

9. Differentiation in time -

$$\frac{d^n x(t)}{dt} \rightleftharpoons (j\omega)^n X(\omega)$$

10. Integration in time -

$$\int_{-\infty}^{t} x(t)\, dt \rightleftharpoons \frac{X(\omega)}{j\omega} + \cancel{\pi X(\omega)}\ \pi X(0)\delta(\omega)$$

where $X(0) = X[\omega]\Big|_{\omega=0}$

11. Differentiation in frequency -

$$t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(\omega)}{d\omega^n}$$

12. Modulation property -

$$x(t)\cos\omega_0 t \rightleftharpoons \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$$

$$x(t)\sin\omega_0 t \rightleftharpoons \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$$

13. Area under time domain -

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\downarrow \omega = 0$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt$$

eg. $x(t) = e^{-at}u(t) \quad a>0$

$X(\omega) = \dfrac{1}{a+j\omega}$

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Area of $x(t) = X(0) = \dfrac{1}{a}$

14. Area under frequency domain -

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

$\downarrow t=0$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)\,d\omega = 2\pi x(0)$$

$$\boxed{\text{Area under s/g } X(\omega) = 2\pi x(0) = 2\pi x(t)\Big|_{t=0}}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}\,df$$

$\downarrow t=0$

$$x(0) = \int_{-\infty}^{\infty} X(f)\,df$$

$$\boxed{\text{Area under } X(f) = x(0) = x(t)\Big|_{t=0}}$$

15. Parseval's energy theorem :-

(108)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega.$$

$$= \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

Q. Find $Y(\omega)$ in terms of $X(\omega)$ where $y(t) \rightleftharpoons Y(\omega)$

$x(t) \rightleftharpoons X(\omega)$

i) $y(t) = x(t - t_0) + x(t + t_0)$

ii) $y(t) = e^{-j2t} x(t)$

iii) $y(t) = x(-3t)$

iv) $y(t) = x(2t - 1)$

v) $y(t) = x(-3t + 2)$

Sol

i) $Y(\omega) = X(\omega) e^{-j\omega t_0} + X(\omega) e^{j\omega t_0}$

$= X(\omega) \left[e^{-j\omega t_0} + e^{j\omega t_0} \right]$

$= X(\omega) \left[2 \cos j\omega t_0 \right]$

ii) $Y(\omega) = X(\omega + 2)$

~~$e^{j\omega_0 t} x(t) \rightleftharpoons$~~

iii) $Y(\omega) = \frac{1}{3} X\left(-\frac{\omega}{3}\right)$

~~iv) $Y(\omega) = \frac{1}{2} X\left(\frac{-\omega}{2}\right)$~~

iv) 1st method -

$$x(t) \longrightarrow \underbrace{x(t-1)}_{=f(t)} \longrightarrow \underbrace{x(2t-1)}_{=f(2t)} = y(t)$$

$X(\omega) =$

$F(\omega) = X(\omega)e^{-j\omega}$ $\quad$ $Y(\omega) = \frac{1}{2}F\left(\frac{\omega}{2}\right)$

$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j\omega/2}$

2nd method -

$$y(t) = x(2t-1) = x\left[2\left(t-\frac{1}{2}\right)\right]$$

$$x(t) \longrightarrow \underbrace{x(2t)}_{=f(t)} \longrightarrow \underbrace{x\left[2\left(t-\frac{1}{2}\right)\right]}_{=f\left(t-\frac{1}{2}\right)}$$

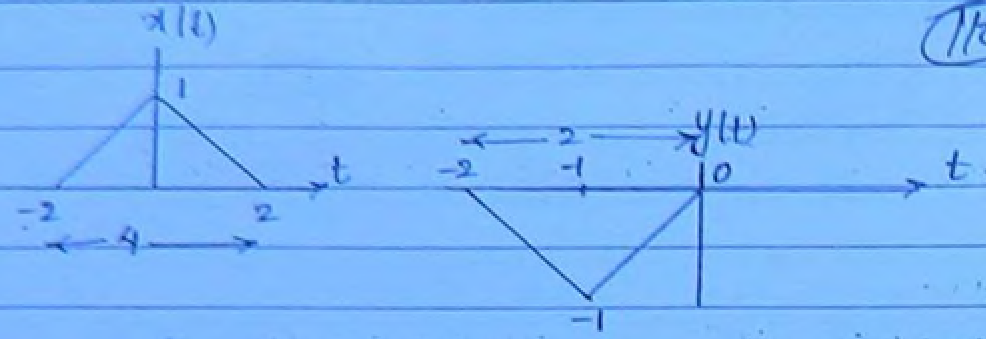
$F(\omega) = \frac{1}{2}X\left(\frac{\omega}{2}\right)$ $\quad$ $F(\omega)e^{-j\omega/2}$

$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j\omega/2}$

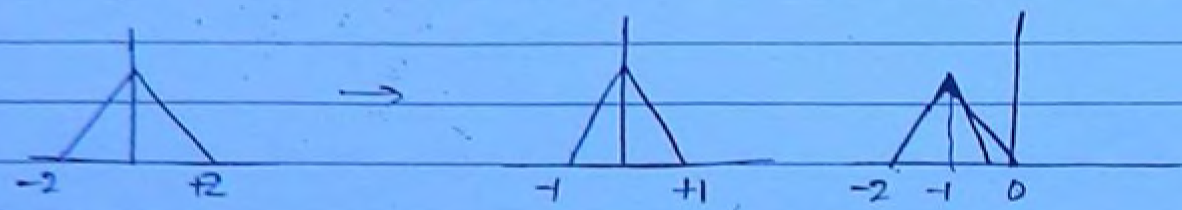
v) $y(t) = x(-3t+2) = x\left[-3\left(t-\frac{2}{3}\right)\right]$

$\cancel{x(t)} \rightarrow x(-3t)$ $\quad$ $Y(\omega) = \frac{1}{3}X\left(-\frac{\omega}{3}\right)e^{-j\frac{2}{3}\omega}$

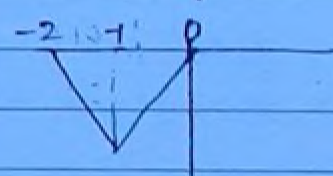
vi)



Sol $x(t) \longrightarrow x(2t) \longrightarrow x(2t+1)$



$\Big\downarrow -x[+(2t+1)] = y(t)$



$$y(t) = -x[+(2t+1)]$$

$$Y(\omega) = \frac{-1}{2} X\left(\frac{\omega}{2}\right) e^{j\omega}$$

Q. $x(t) = e^{at}\, u(-t)$ ~~at→0~~ $a > 0$

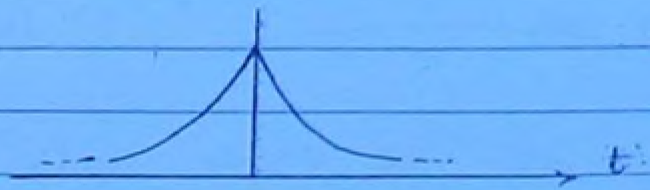
Sol $e^{-at}\, u(t) \rightleftharpoons \dfrac{1}{a + j\omega}$

$\big\downarrow t = -t \qquad \big\downarrow \omega = -\omega$

$$e^{at}\, u(-t) \rightleftharpoons \frac{1}{a - j\omega}$$

Q $x(t) = e^{-a|t|}$ $\quad a > 0$

Sol.



$$x(t) = e^{at}u(-t) + e^{-at}u(t)$$

$$X(\omega) = \frac{1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{2a}{a^2 + \omega^2}$$

$$\boxed{\underset{a>0}{e^{-a|t|}} \rightleftharpoons \frac{2a}{a^2+\omega^2}}$$

→ Area under s/g $x(t)$

$$= X(\omega)\Big|_{\omega=0}$$

$$= \frac{2a}{a^2} = \frac{2}{a}$$

Q $$\int_{-\infty}^{\infty} \Big(\underbrace{\frac{2a}{a^2+\omega^2}}_{X(\omega)}\Big) d\omega = ?$$

Sol. Area under $X(\omega) = 2\pi\, x(t)\Big|_{t=0}$

$$= 2\pi\, x(t)\Big|_{t=0}$$

$$= 2\pi \times 1$$

$$= 2\pi$$

Property of duality :-

$\rightarrow \quad x(t) \rightleftharpoons X(\omega) \qquad t = -\omega$

$\omega = t$

$X(t) \rightleftharpoons 2\pi x(-\omega)$

$\rightarrow \quad x(t) \rightleftharpoons X(f) \qquad t = -f$

$f = t$

$X(t) \rightleftharpoons x(-f)$

Q. $x(t) = \dfrac{2a}{a^2 + t^2}$

Sol. $t \to \omega$

$= \dfrac{2a}{a^2 + \omega^2}$

$e^{-a|t|} \quad a > 0 \rightleftharpoons \dfrac{2a}{a^2 + \omega^2} \qquad t = -\omega$

$\omega = t$

$\dfrac{2a}{a^2 + t^2} \rightleftharpoons 2\pi e^{-a|-\omega|}, \; a > 0$

$= 2\pi e^{-a|\omega|}, \; a > 0$

Q. $x(t) = \dfrac{1}{a+jt}$

$X(\omega) = ?$

Sol. $e^{-at}u(t) \quad a>0 \rightleftharpoons \cancel{\dfrac{2a}{a^2+\omega^2}} \quad \dfrac{1}{a+j\omega} \quad t=-\omega$

$\omega = t$

$\dfrac{1}{a+jt} \rightleftharpoons 2\pi e^{a\omega}u(-\omega) \quad a>0$

Q. $x(t) = A_0 = $ dc signal

$X(\omega) = ?$

Sol. $\delta(t) \rightleftharpoons A_0$

$\omega = t \qquad t=-\omega$

$A_0 \rightleftharpoons 2\pi A_0 \delta(-\omega)$

$\boxed{A_0 \rightleftharpoons 2\pi A_0 \delta(\omega)}$ $\qquad$ δ.. is an even fn

Q. $x(t) = \cos\omega_0 t$

$X(\omega) = ?$

Sol $x(t) = \cos\omega_0 t = \dfrac{1}{2}e^{j\omega_0 t} + \dfrac{1}{2}e^{-j\omega_0 t}$

$A_0 \rightleftharpoons 2\pi A_0 \delta(\omega)$

$\downarrow A_0 = \frac{1}{2}$

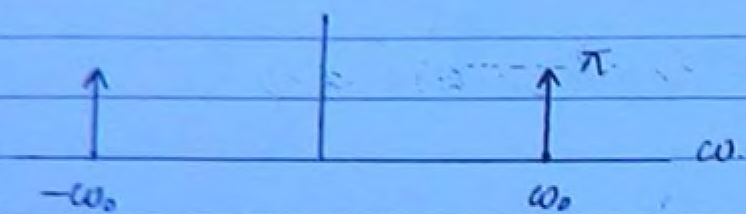
$\frac{1}{2} e^{j\omega_0 t} \rightleftharpoons \pi \delta(\omega - \omega_0)$

$\frac{1}{2} e^{j\omega_0 t} \rightleftharpoons \pi \delta(\omega + \omega_0)$

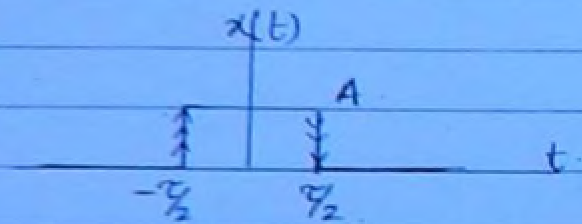
using linearity property.

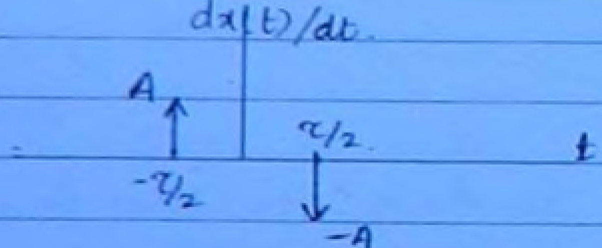
$X(\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$

$X(\omega) = \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$



* Q





$\frac{dx(t)}{dt} = A\,\delta\left(t + \frac{\tau}{2}\right) - A\,\delta\left(t - \frac{\tau}{2}\right)$

$\downarrow\uparrow$ FT

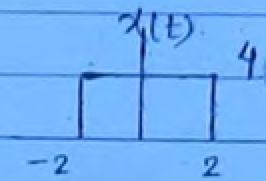
$j\omega X(\omega) = A e^{j\omega\tau/2} - A e^{-j\omega\tau/2}$

$X(\omega) = \frac{A}{\cancel{j\omega}} \cancel{2j} \left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\quad}\right] \times \omega\tau/2$

$x(t)$ = rectangular pulse
(even or symmetrical about y-axis) $= A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$

$X(\omega) = \text{Area}.\ Sa\left(\frac{\omega.\ \text{Duration}}{2}\right)$

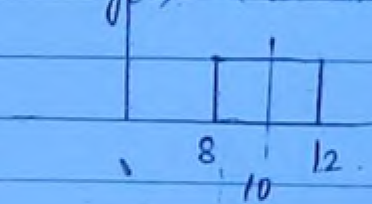
Q.



sol $16\ Sa\left(\omega \cdot \frac{4}{2}\right)$

$= 16\ Sa\,(2\omega)$

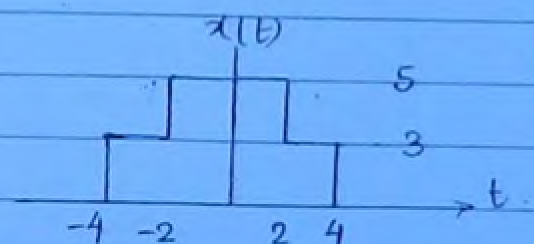
$y(t) = x(t-10)$



$Y(\omega) = X(\omega)\,e^{-j10\omega}$

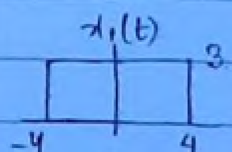
$= 16\,Sa\,(2\omega)\,e^{-j10\omega}$

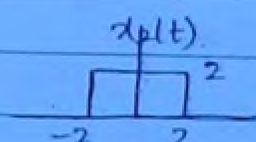
Q.

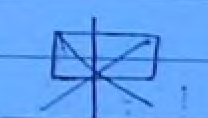


$X(\omega) = ?$

sol





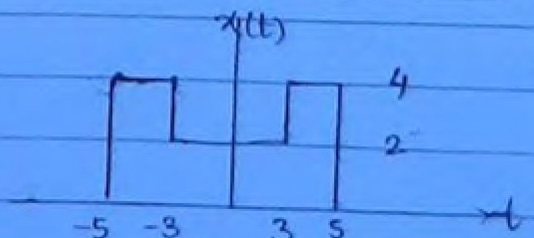


$24\ Sa\left(\omega \times \frac{8}{2}\right)$ $\qquad$ $8\ Sa\left(\omega \times \frac{4}{2}\right)$

$= \boxed{24\,Sa(4\omega)} + \boxed{8\,Sa(\omega 2)}$

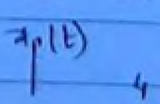
$X(\omega) = 24\ Sa(4\omega) + 8\,Sa(2\omega)$

Q.



$X(\omega) = ?$

sol



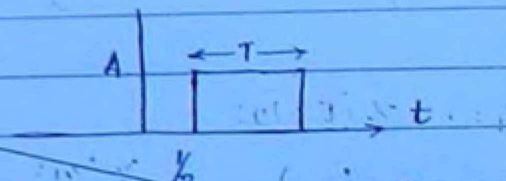
Q $x(t) = \text{rect}\left(t - \frac{1}{2}\right)$

$y(t) = x(t) + x(-t) \Rightarrow Y(\omega) = ?$

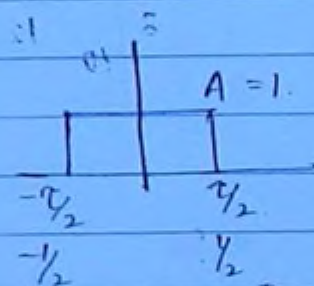
a) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$ b) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)$

c) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)$ d) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\,\sin\left(\frac{\omega}{2}\right)$

Sol $\cancel{y_1(t) = x(t) =}$



$= AT$



$f(t) = \text{rect}\ t$

A = 1, $\tau = 1$ $= A\,\text{rect}\left(\frac{t}{\tau}\right)$

$F(\omega) = Sa\left(\frac{\omega}{2}\right)$

$x(t) = f\left(t - \frac{1}{2}\right) \Rightarrow X(\omega) = F(\omega)e^{-j\omega/2}$

$= Sa\left(\frac{\omega}{2}\right)e^{-j\omega/2}$

$y(t) = x(t) + x(-t)$

$Y(\omega) = X(\omega) + X(-\omega)$

$= Sa\left(\frac{\omega}{2}\right)e^{-j\omega/2} + Sa\left(-\frac{\omega}{2}\right)e^{j\omega/2}$ [(Sa) is an even fn]

$= Sa\left(\frac{\omega}{2}\right)\left[e^{-j\omega/2} + e^{j\omega/2}\right]$

$= Sa\left(\frac{\omega}{2}\right)(2\cos\omega/2) = 2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\frac{\omega}{2}$ — (C)

$\boxed{Sa\left(\frac{\omega}{2}\right) = \text{sinc}\left(\frac{\omega}{2\pi}\right)}$

CWB chapter 4.

Q29 $h(t) \rightleftharpoons H(\omega) = \dfrac{2\cos\omega \sin 2\omega}{\omega}$

Find value of $h(t)$ at origin. ie $h(0)$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 2.

sol. $h(t) \rightleftharpoons H(\omega) = \dfrac{2\cos\omega \sin 2\omega}{\omega}$

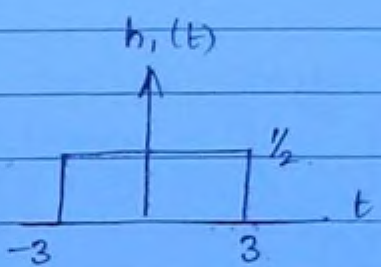
$= \dfrac{\sin 3\omega}{\omega} + \dfrac{\sin(+\omega)}{\omega}$

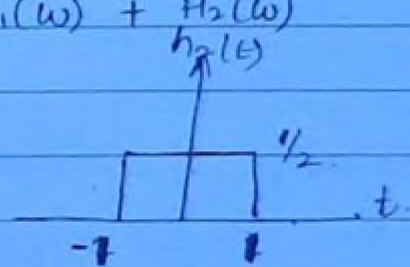
$= \dfrac{\sin 3\omega}{\omega} + \dfrac{\sin\omega}{\omega}$

$= 3\left(\dfrac{\sin 3\omega}{3\omega}\right) + \dfrac{\sin\omega}{\omega}$

$= 3\,Sa(3\omega) + Sa(\omega)$

$= H_1(\omega) + H_2(\omega)$





$h(t) = h_1(t) + h_2(t)$

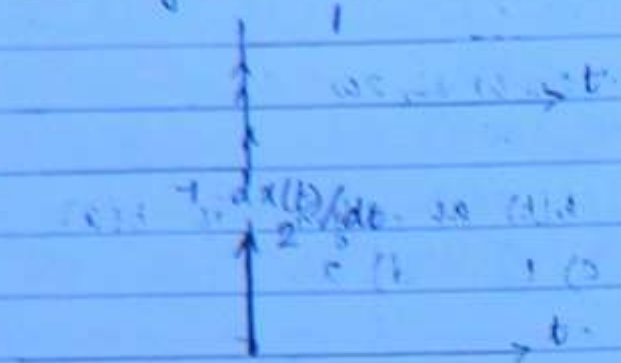
$h(0) = h_1(0) + h_2(0)$

$= \frac{1}{2} + \frac{1}{2}$

$h(0) = 1 \longrightarrow$ (c)

Q $x(t) = sgn(t)$

Sol.



$$\frac{dx(t)}{dt} = 2\,\delta(t)$$

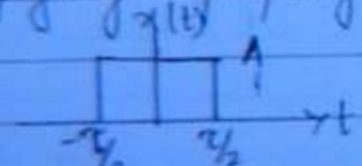
$$\Big\Updownarrow FT$$

$$j\omega X(\omega) = 2$$

$$X(\omega) = \frac{2}{j\omega}$$

$$\boxed{sgn(t) \rightleftharpoons \frac{2}{j\omega}}$$

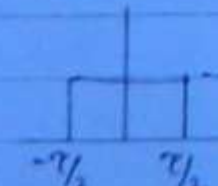
* Applying property of duality



$$\rightleftharpoons X(\omega) = A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$A\tau\ Sa\left(\frac{t\tau}{2}\right) \rightleftharpoons 2\pi x(-\omega) = 2\pi x(\omega)$$

$= A_0\, Sa(Kt)$



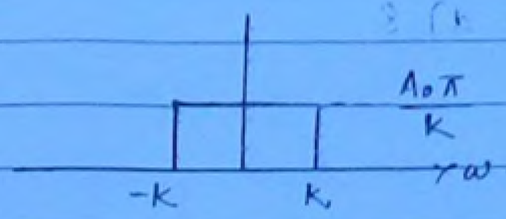
$$2\pi A = 2\pi \times \frac{A_0}{\pi K} = \frac{A_0 \pi}{K}$$

$$\frac{\tau}{2} = k$$

$$\tau = 2k$$

$$A\tau = A_0$$

$$A = \frac{A_0}{\tau} = \frac{A_0}{2k}$$



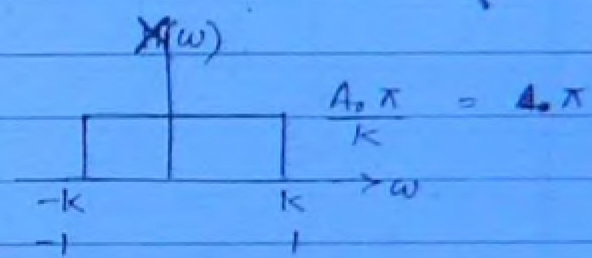
$\rightleftharpoons A_0 \, Sa(kt)$

Calculate energy & area of

$$x(t) = Sa(t)$$

$$x(t) = Sa(t)$$

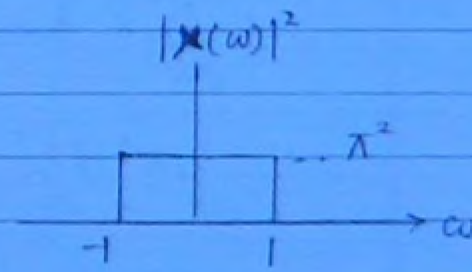
$$\xrightarrow{A_0=1,\ k=1} \; = A_0 \, Sa(kt)$$



Parseval's Energy Theorem -

Energy of $Sa(t)$ $= \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega\right]$

$$= \frac{1}{2\pi} \times \text{area of } |X(\omega)|^2$$



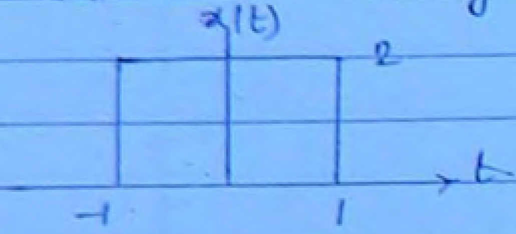
$$= \frac{1}{2\pi} \times (2\pi^2)$$

$$= \pi$$

Area of $Sa(t)$ Area of $x(t) = X(\omega)\big|_{\omega=0} = \pi$

Q Calculate $\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$ (120)

where $x(t)$ is a rectangular pulse



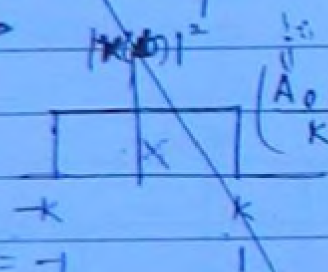
a) 16π b) 8π c) 4 d) 8

Sol

$$\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$= \text{Area of } |X(\omega)|^2$

$\Rightarrow$



$$\left(\frac{A_0 \pi}{k}\right)^2 = (2\pi)^2 = 4\pi^2$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi} \times \text{Area of } |X(\omega)|^2$$

$$= \frac{1}{2\pi} \times 8\pi^2$$

$$= 4\pi$$

Energy of $x(t)$ $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega$

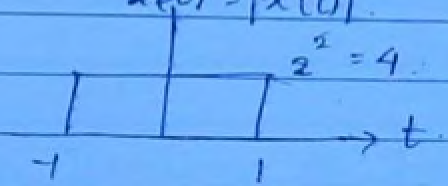
$$\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = 2\pi E$$

$$= 2\pi \int_{-\infty}^{\infty}|x(t)|^2 dt$$

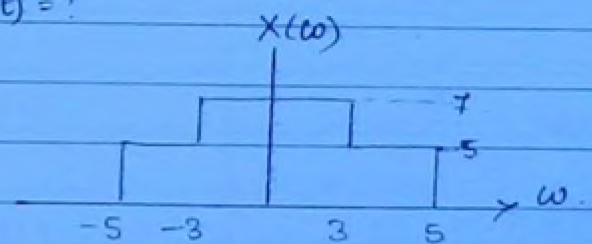
$$= 2\pi \times \text{area of } |x(t)|^2$$

$$= 2\pi \times 8 = 16\pi \rightarrow (a)$$

$\cancel{z(t)} = |x(t)|^2$



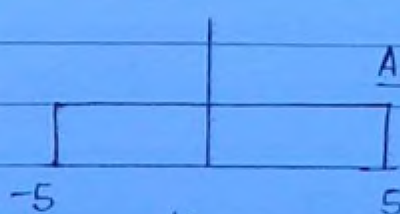
Q $x(t) = ?$

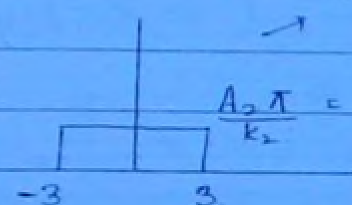


Sol

$A_1 = \frac{5 \times k_1}{\pi}$ $\qquad$ $A_2 = \frac{2 \times k_2}{\pi}$

$\frac{A_1 \pi}{k_1} = 5$ $\qquad$ $\frac{A_2 \pi}{k_2} = 2$





$\Updownarrow$ $\qquad$ $\Updownarrow$

$x_1(t) = A_1 Sa(k_1 t)$

$= \frac{25}{\pi} Sa(5t)$

$x_2(t) = A_2 Sa(k_2 t)$

$= \frac{6}{\pi} Sa(3t)$

$$x(t) = x_1(t) + x_2(t)$$

$$= \frac{25}{\pi} Sa(5t) + \frac{6}{\pi} Sa(3t)$$

$\frac{1}{2} + \frac{1}{2} sgn(t)$

Q $\quad u(t) = \frac{1 + sgn(t)}{2} = x(t)$ 122

$X(\omega) = ?$

$A_0 \rightleftharpoons 2\pi A_0 \delta(\omega)$

$A_0 = \frac{1}{2}$

$\frac{1}{2} \rightarrow \pi\delta(\omega)$

$$X(\omega) = \pi\delta(\omega) + \frac{1}{2}\left(\frac{2}{j\omega}\right)$$

$$u(t) \rightleftharpoons \frac{1}{j\omega} + \pi\delta(\omega)$$

FT for periodic signal

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$1 \rightleftharpoons 2\pi\delta(\omega)$$

$$\sum_{n=-\infty}^{\infty} C_n \rightleftharpoons \sum_{n=-\infty}^{\infty} C_n 2\pi\delta(\omega)$$

$$\boxed{\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \rightleftharpoons 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)}$$

$$\Rightarrow c_n = \frac{1}{T_0}$$

Q. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ $\quad X(\omega) = ?$ (123)

Sol. $X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \, \delta(\omega - n\omega_0)$

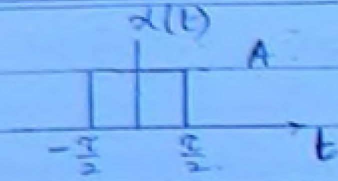
$$= 2\pi \sum \frac{1}{T_0} \delta(\omega - n\omega_0)$$

$$= \frac{2\pi}{T_0} \sum \delta(\omega - n\omega_0)$$

$$X(\omega) = \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

9-1-13 Fourier Transform for Important Signals -

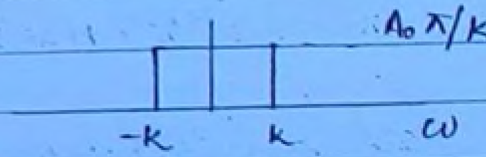
	$f(t)$	$F(\omega)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
3.	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
4.	A_0	$2\pi A_0 \delta(\omega)$
5.	$e^{-at}u(t)$ $\quad a>0$	$\frac{1}{a+j\omega}$
6.	$e^{-a\|t\|}$ $\quad a>0$	$\frac{2a}{a^2+\omega^2}$

9. 

$A\tau \, Sa\left(\frac{\omega\tau}{2}\right)$

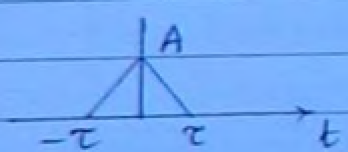
$x(t) = A \, rect\left(\frac{t}{\tau}\right)$

10. $A_0 \, Sa(kt)$



11. $\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ $\qquad$ $\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

12. Periodic signal $\qquad$ $2\pi \sum_{n=-\infty}^{\infty} (C_n \delta(\omega - n\omega_0))$

13. 

$A\tau \, Sa^2\left(\frac{\omega\tau}{2}\right)$

$f(t) - F(\omega)$ pairs -

	$f(t)$		$F(\omega)$
1.	Real	$\longrightarrow$	CS
	CS	$\longrightarrow$	Real
2.	Img	$\longrightarrow$	CAS
	CAS	$\longrightarrow$	Img
3.	R + E	$\longrightarrow$	R + E
4	I + E	$\longrightarrow$	I + E

5. $R+0 \longrightarrow I+0$

6. $I+0 \longrightarrow R+0$

7. $D \longrightarrow P$

$P \longrightarrow D$

8. $C \longrightarrow NP$

$NP \longrightarrow C$

9. $C+P \longrightarrow D+NP$

$C+NP \longrightarrow C+NP$

10. $D+P \longrightarrow D+P$

$D+NP \longrightarrow C+P$

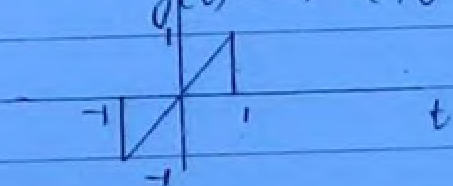
CWB chap. 5

26. $Y(\omega) = ?$

$\downarrow$

$I+0$

$y(t) \longrightarrow R+0$



a) $4\pi j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega}\right]$ $\quad \frac{0}{0} = E$

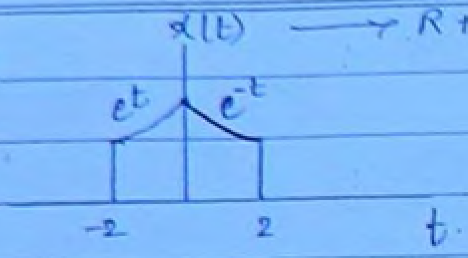
b) $2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$ $\quad \frac{E}{0} = 0 \quad \frac{0}{E} = 0$

c) $4\pi j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right] \to E$

d) $2j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right] \to E$

Q $X(\omega) = ?$ $\downarrow$ R+E

$x(t) \longrightarrow$ R + E



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a) $2 - 2e^{-2}\sin 2\omega + 2\omega e^{-2}\sin 2\omega$ $\rightarrow 0$

b) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\cos 2\omega$.

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\downarrow \quad\quad \downarrow$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 0 \times E = 0$

c) $\dfrac{\overset{E}{2} - \overset{E}{2e^{-2}}\cos 2\omega + \overset{0\times 0}{2\omega e^{-2}}\sin 2\omega}{1+\omega^2} = E$

At $\omega = 0$
$X(0) = 2 - 2e^{-2}$ $\rightarrow$ Ans

d) $\dfrac{2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\sin 2\omega}{1+\omega^2}$

At $\omega = 0$
$X(0) = 2 + 2e^{-2}$.

Sol. Area under time domain : $\displaystyle\int_{-\infty}^{\infty} x(t)\,dt = X(\omega)\Big|_{\omega=0}$

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_{-2}^{2} x(t)\,dt = 2\int_{0}^{2} x(t)\,dt$$

$$= 2\int_{0}^{2} e^{-t}\,dt = 2\left[1 - e^{-2}\right]$$

$$X(0) = 2 - 2e^{-2}$$

put $\omega = 0$ in options.

Ans (c)

Q27. $y(t) = x(t)\cos t \rightleftharpoons Y(\omega)$

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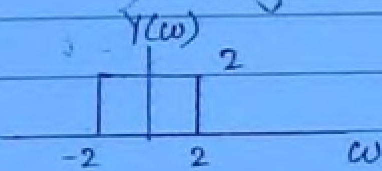
$Y(\omega) = \begin{cases} 2 & |\omega| \leq 2 \\ 0 & \text{otherwise} \end{cases}$

then $x(t)$ will be

$x(0)$ $\boxed{\frac{4}{\pi}}$ a) $\frac{4}{\pi}\frac{\sin t}{t}$ b) $2\frac{\sin t}{t}$ $x(0)$ is 2.

4. c) $4\frac{\sin t}{t}$ d) $2\pi\frac{\sin t}{t}$ 2π.

Sol



$y(t) = x(t)\cos t$

$\downarrow t=0$

$y(0) = x(0)$ — (1)

Area under frequency domain : $\int_{-\infty}^{\infty} Y(\omega)\, d\omega = 2\pi y(0)$

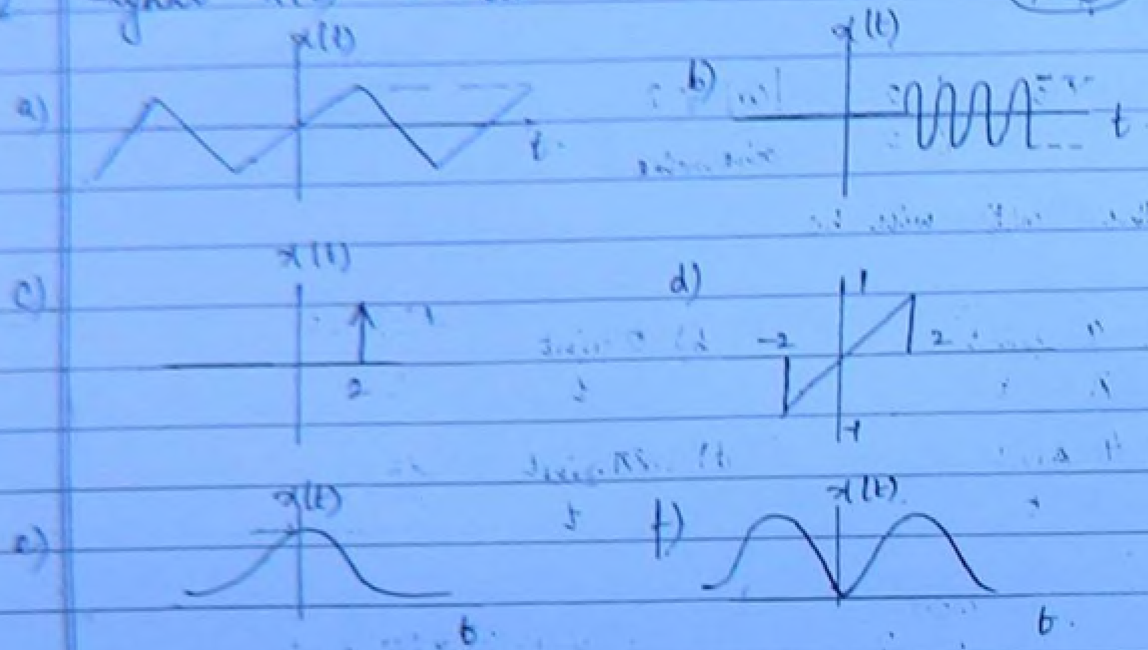
$\Rightarrow \overset{\text{area}}{8} = 2\pi y(0)$

$\Rightarrow y(0) = \frac{4}{\pi} = x(0)$ from (1)

$x(t)_{t=0} = \frac{4}{\pi}$

Ans (a)

§ Signal $x(t)$ is real



1. $\text{Real}[X(\omega)] = 0$

A. a, d B. e, f.

C b, c D. b, d.

Sol $x(t) = R + O \longrightarrow X(\omega) = CS \rightarrow I + O$

$= \underbrace{\cancel{\text{Real}}[X(\omega)]}_{E}^{=0} + j\,\underbrace{\text{Img}[X(\omega)]}_{O}$

O odd

check for odd s/g.

a, d

Ans A.

2 $\text{Img}[X(\omega)] = 0$

A a, d B e, f

C b, c D b, d

Sol $x(t) = R + E \longrightarrow X(\omega) = C.S. \rightarrow R + E$ $=0$

3. $\int_{-\infty}^{\infty} X(\omega)\, d\omega = 0$

A. e B. a, b, c, d, f.
C. b, c D. a, d, e, f.

sol. Area under freq domain

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi x(0)$$

$$= x(0) = 0$$

$$= x(t)\Big|_{t=0} = 0$$

$$\downarrow$$

a, b, c, d, f. Ans B.

4. $\int_{-\infty}^{\infty} \omega X(\omega)\, d\omega = 0$

A. a, b, c, d, f B. e
C. b, c, e, f D. b, c

sol $\int_{-\infty}^{\infty} \omega X(\omega) d\omega$ means slope at origin is 0

tangent at orig at $x=0$ is either x-axis or parallel to it

$$x(t) \rightleftharpoons X(\omega)$$

$$f(t) = \frac{1}{j} \frac{dx(t)}{dt} \rightleftharpoons \frac{j\omega X(\omega)}{j} = F(\omega)$$

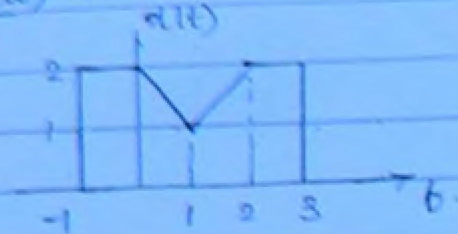
Area under frequency domain

$$\int_{-\infty}^{\infty} F(\omega)\, d\omega = 2\pi f(0)$$

$$\Rightarrow \int_{-\infty}^{\infty} \omega X(\omega)\, d\omega = 2\pi f(0)$$

$$f(0) = 0 \quad f(t)\Big|_{t=0} \Rightarrow \frac{1}{j} \frac{dx(t)}{dt}\Big|_{t=0} = 0$$

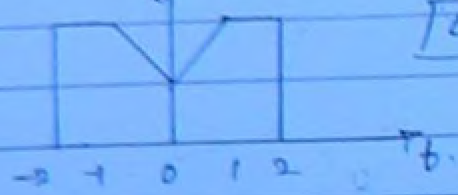
Q $\angle X(\omega) = ?$



Sol

$y(t) = R+E \implies Y(\omega) = R+E$

$\boxed{\angle Y(\omega) = 0}$



$x(t) = y(t-1)$

$X(\omega) = Y(\omega) e^{-j\omega}$

$\angle X(\omega) = \angle Y(\omega) + (-\omega)$, where $\angle Y(\omega) \to 0$

$= -\omega$

1st general real & even s/g

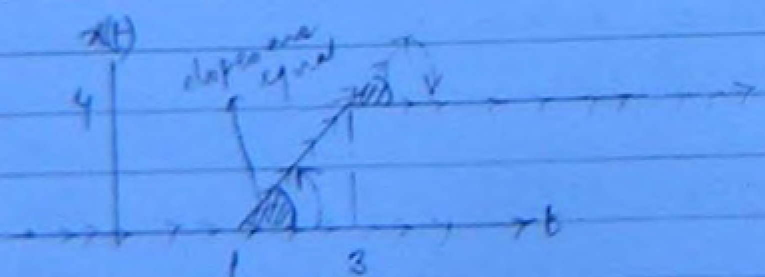
then provide time shifting to generate s/g (given)

& then calculate phase according to

the real s/g drawn

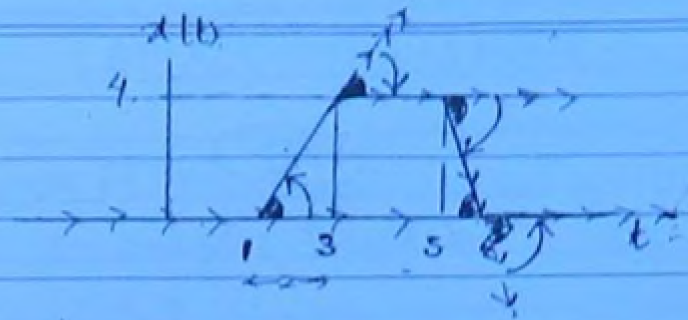
Mathematical Representation of waveform :-

|slope| ↑ = +ve sign ↓ = -ve sign



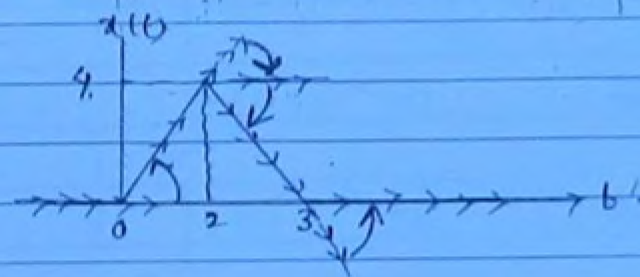
$x(t) = 0 + \frac{4}{2} r(t-1) - \frac{4}{2} r(t-3)$

2.



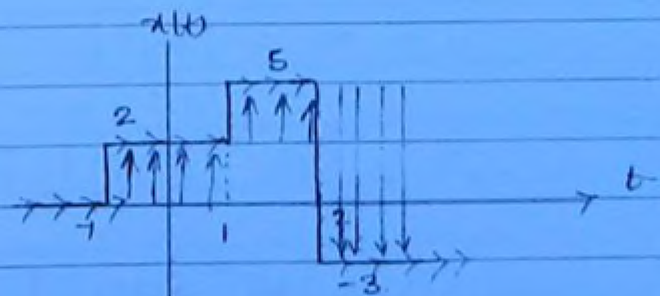
$$x(t) = 0 + \frac{4}{2} r(t-1) - \frac{4}{2} r(t-3) - \frac{4}{1} r(t-5) + \frac{4}{1} r(t-6)$$

3.



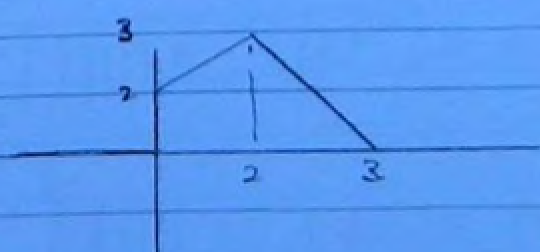
$$x(t) = 0 + \frac{4}{2} r(t-0) - \frac{4}{2} r(t-2) - \frac{4}{1} r(t-2) + \frac{4}{1} r(t-3)$$

4.



$$x(t) = 0 + 2u(t-(-1)) + 3u(t-(1)) - 8u(t-(2)) \cancel{- 3u(t-(2))}$$

5



$$2u(t) + \frac{1}{2}[r(t-0) - r(t-2)] - 3[r(t-2) - r(t-3)]$$

$$\Rightarrow 2u(t) + \frac{1}{2}[r(t) - r(t-2)] - 3[r(t-2) - r(t-3)]$$

8

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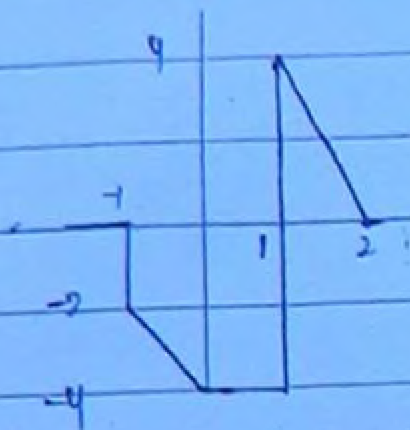


$$x(t) = -2u(t+1) - 2[r(t+1) - r(t)] + 8u(t-1) - 4[r(t-1) - r(t-2)]$$

LAPLACE TRANSFORM -

$f(t) \rightleftharpoons F(s)$

where s = complex variable

$= \sigma + j\omega$

$\sigma \rightarrow$ represents damping factor

$\omega \rightarrow$ represents oscillation frequency.

$$F(s) = \begin{cases} \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt & \text{Bilateral Laplace Transform} \\ \int_{0}^{\infty} f(t)\, e^{-st}\, dt & \text{Unilateral Laplace Transform} \end{cases}$$

Condition for Existence of Laplace Transform -

$$|F(s)| < \infty$$

$$\Rightarrow \left| \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt \right| < \infty$$

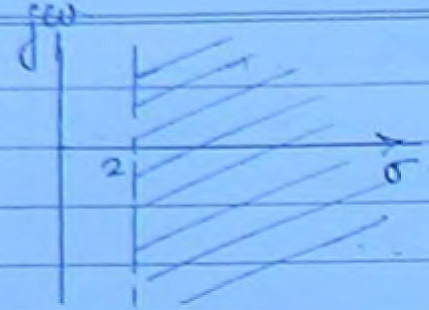
$$\Rightarrow \int_{-\infty}^{\infty} |f(t)\, e^{-st}\, dt| < \infty$$

$$\Rightarrow \int_{-\infty}^{\infty} |f(t)\, e^{-(\sigma+j\omega)t}|\, dt < \infty$$

$$\Rightarrow \int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, \cancelto{1}{|e^{-j\omega t}|}\, dt < \infty$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty}$$

eg



$f(t) = e^{2t} u(t)$

13B

LHS $= \int_{-\infty}^{\infty} |f(t) e^{-\sigma t}| \, dt$

$= \int_{0}^{\infty} |e^{2t} e^{-\sigma t}| \, dt$

$= \int_{0}^{\infty} e^{(2-\sigma)t} \, dt < \infty$

only when $2-\sigma < 0$

$\boxed{\sigma > 2}$ ROC

eg $f(t) = e^{t^2} u(t)$

↳ FT & LT will not exist.

Q. $f(t) = e^{-at} u(t)$

$F(s) = ?$ ROC $= ?$

sol. $= \cancel{\int_{-\infty}^{\infty} |f(t) e^{-\sigma t}| \, dt}$ $\qquad = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt$

$= \cancel{\int_{0}^{\infty} |e^{-(a+\sigma)t}| \, dt}$ $\qquad = \int_{-\infty}^{\infty} e^{-at} e^{-st} \, dt$

$=$ $\qquad = \int_{0}^{\infty} e^{-(s+a)t} \, dt$

$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty}$

$= \frac{e^{-(s+a)t} - e^{0} \to 1}{-(s+a)}$

$e^{-(s+a)\infty} = e^{-(\sigma + j\omega + a)\infty}$ undefined

$= e^{-(\sigma+a)\infty} \cancel{e^{-j\omega\infty}}$

$-(\sigma+a)\infty = 0$ $\qquad (\sigma + a) > 0$

$$\therefore \quad F(s) = \frac{e^{-(s+a)\infty} - \cancel{e^{0}}^{\,1}}{-(s+a)} \qquad \text{at } (\sigma + a) > 0$$

$$\sigma > -a \quad \underline{\underline{ROC}}$$

$$\cancel{e^{-(s+a)\infty}} \to 0$$

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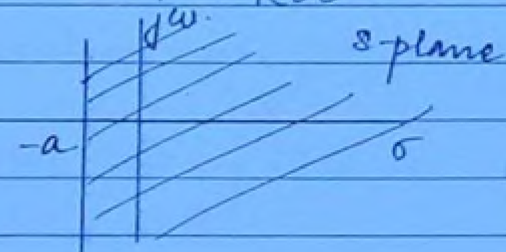
ROC



$$\boxed{\begin{array}{l} e^{-at}u(t) \rightleftharpoons \dfrac{1}{s+a} \\ ROC \quad \sigma > -a \end{array}}$$

Properties of Laplace Transform -

1. Linearity -

$$a_1 f_1(t) + a_2 f_2(t) \rightleftharpoons a_1 F_1(s) + a_2 F_2(s)$$

2. Time - reversal -

$$f(-t) \rightleftharpoons F(-s)$$

3. Conjugation -

$$f^*(t) \rightleftharpoons F^*(s^*)$$

4. Time shifting -

$$f(t - t_0) \rightleftharpoons F(s)\, e^{-st_0}$$

5. Time scaling -

$$f(at) \quad a \neq 0 \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right)$$

6. Convolution - theorem

$$f_1(t) * f_2(t) \rightleftharpoons F_1(s) \cdot F_2(s)$$

7. Convolute Multiplication in time.

$$f_1(t) \cdot f_2(t) \rightleftharpoons \frac{1}{2\pi j}\left[F_1(s) * F_2(s)\right]$$

8. Frequency shifting -

$$e^{-at} f(t) \rightleftharpoons F(s+a)$$

9. Differentiation in time -

$$\frac{d^n f(t)}{dt^n} \rightleftharpoons s^n F(s) \qquad \text{Bilateral LT}$$

$$\frac{d^n f(t)}{dt^n} \rightleftharpoons s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - s^{n-3} f''(0^-) \ldots$$

where $f(0^-) = f(t)\big|_{t=0^-}$

$$f'(0^-) = \frac{df(t)}{dt}\bigg|_{t=0^-}$$

$$f''(0^-) = \frac{d^2 f(t)}{dt^2}\bigg|_{t=0^-}$$

10. Integration in time

$$\int_{-\infty}^{t} f(t)\,dt \rightleftharpoons \frac{F(s)}{s}, \qquad \text{Bilateral LT}$$

$$\int_{-\infty}^{t} f(t)\,dt \rightleftharpoons \frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(t)\,dt}{s} \qquad \text{Unilateral}$$

11. Differentiation in frequency -

$$t^n f(t) \rightleftharpoons (-1)^n \frac{d^n F(s)}{ds^n}$$

12. Integration in frequency -

$$\frac{f(t)}{t} \rightleftharpoons \int_s^{\infty} F(s)\, ds.$$

13. Initial value theorem -

$$f(0) = \lim_{s \to \infty} s \cdot F(s)$$

→ Applicable only for causal type signals
ie $f(t) = 0 \quad t < 0$

14. Final value theorem -

$$f(\infty) = \lim_{s \to 0} s\, F(s)$$

→ Applicable only for causal type signals
ie $f(t) = 0 \quad t < 0$

→ The term $sF(s)$ should have LHS poles only (in pole - zero plot)

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13. $f(t) \rightleftharpoons F(s) = \dfrac{\omega_0}{s^2+\omega_0^2}$ $\quad \sigma < 0$

$f(\infty) = ?$

a) 0 $\quad$ b) 1

c) $-1 \le f(\infty) \le 1$ $\quad$ d) ∞

Sol. $f(\infty) = sF(s)$

$= s\,\dfrac{\omega_0}{s^2+\omega_0^2} \Rightarrow$ poles, $s = \pm j\omega_0$

$\neq$ LHS.

$\cancel{f(t) = \cos\omega_0 t}$

FVT is not applicable.

$\therefore$ calculate L^{-1}.

$f(t) = \sin\omega_0 t\ u(t)$

$f(\infty) = (-1, 1)$

Ans (C)

14. $y(t) \rightleftharpoons Y(s) = \dfrac{1}{s(s-1)}$

$y(\infty) = ?$

a) -1 $\quad$ b) 0 $\quad$ c) 1 $\quad$ d) unbounded

Sol $\cancel{f(\infty)}$ $sY(s) = \dfrac{1}{s-1}$ $\quad$ pole $\Rightarrow s = 1$

FVT not applicable

$$Y(s) = \frac{1}{s(s-1)} = \frac{1}{s-1} - \frac{1}{s}$$

$$y(t) = e^{t}u(t) - u(t)$$
$$= u(t)(e^{t}-1)$$
$$y(\infty) = \text{unbounded.} \Rightarrow \infty - 1$$

Ans (d)

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Q Find $Y(s)$ in terms of $F(s)$.
where $y(t) \rightleftharpoons Y(s)$
$f(t) \rightleftharpoons F(s)$

i) $y(t) = f(t-1) + f(t+1)$
$y(t) = e^{2t} f(t)$
$y(t) = f(-2t+1)$
$y(t) = f(-2t)$

Sol
$$y(t) = f(t-1) + f(t+1)$$
$$Y(s) = F(s)e^{-s} + F(s)e^{s}$$

ii) $Y(s) = F(s-2)$

iii) $Y(s) = \frac{1}{2} F\left(\frac{-s}{2}\right)$

iv) $y(t) = f(-2t+1)$
$= f\left(-2\left(t - \frac{1}{2}\right)\right)$

$$Y(s) = \frac{1}{2} F\left(-\frac{s}{2}\right)e^{-s/2}$$

Q $f(t) = -e^{-at} u(-t)$

Sol $e^{-at} u(t) \rightleftharpoons \frac{1}{s+a} \quad \sigma > -a$

$\downarrow t = -t \qquad \downarrow \qquad \downarrow \sigma = -\sigma$

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$e^{at}u(-t) \rightleftharpoons \frac{1}{-s+a}$, $-\sigma > -a$

$\downarrow a = -a$

$e^{-at}u(-t) \rightleftharpoons \frac{1}{-(s+a)}$, $-\sigma > a$

$-e^{-at}u(-t) \rightleftharpoons \frac{-1}{-s-a}$, $\sigma > a$

$\hookrightarrow$ coz ROC remains same as power remains same.

$$\boxed{\begin{array}{ll} \Rightarrow -e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a} & \sigma < -a \\ \Rightarrow e^{-at}u(t) \rightleftharpoons \frac{1}{s+a} & \sigma > -a \end{array}}$$

There are 2 possible answers laplace inverse of $\frac{1}{s+a}$

Select correct option

according to ROC given in question.

Q $f(t) = \cos\omega_0 t \, u(t)$

$F(s) = ?$ ROC = ?

Sol $f(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} u(t)$

$= \frac{1}{2}e^{j\omega_0 t}u(t) + \frac{1}{2}e^{-j\omega_0 t}u(t)$

$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}$ $\sigma > \underbrace{-a}_{\text{Real}}$

$a = -j\omega_0$

$e^{j\omega_0 t}u(t) \rightleftharpoons \frac{1}{s-j\omega_0}$ $\sigma > 0$

here a is Img

Real part = 0

$$f(t) = \frac{1}{2} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-j\omega_0 t} u(t)$$

$$F(s) = \frac{1}{2}\left[\frac{1}{s - j\omega_0} + \frac{1}{s + j\omega_0}\right] \quad \sigma > 0$$

$$F(s) = \frac{s}{s^2 + \omega_0^2} \quad \sigma > 0$$

$$f(t) = \sin \omega_0 t \; u(t)$$

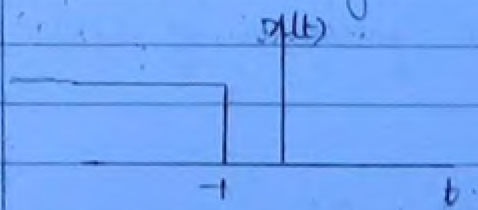
$$F(s) = \frac{\omega_0}{s^2 - \omega_0^2} \quad \sigma > 0$$

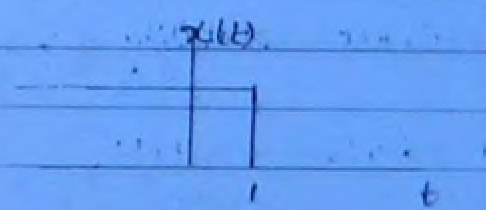
Region of Convergence (ROC)

ROC excludes values where LT is ∞

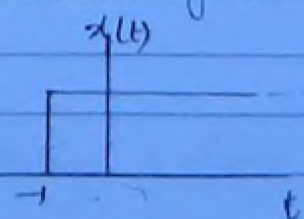
Its defined as the range of complex variable 's' in s-plane for which LT of s/g is convergent or finite.

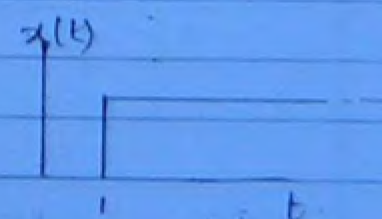
Left Sided Signal



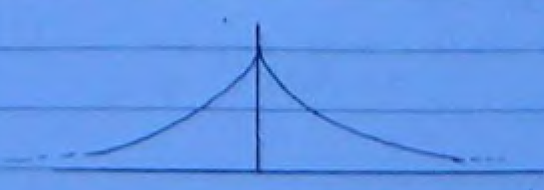


Right Sided Signal





Both Sided



Properties of ROC -

1. ROC does not include any pole.

2. For right sided signal ROC is right side to the right ~~p~~ most pole.

3. For left sided signal ROC is left side to the left most pole.

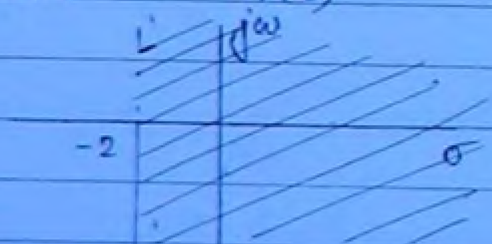
4. For both sided signal ROC is a "strip in s-plane"

5. For stability ROC includes imaginary axis

6. For finite duration signal ROC is entire s-place excluding possibly $s = 0$ &/or $\pm\infty$.

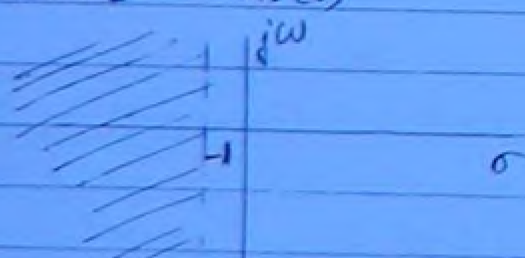
Q. Check stability of system at extension of $h(t)$

1. $ROC_1 \longleftarrow h_1(t)$



$\Rightarrow$ Stable, R.S.

2. $ROC_2 \longleftarrow h_2(t)$

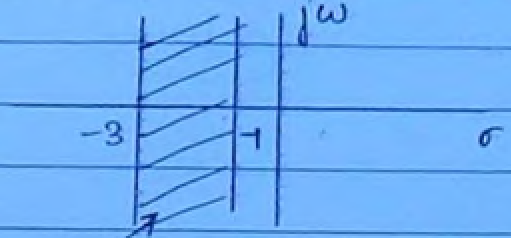


$\Rightarrow$ unstable, L.S.

$\downarrow$

does not include imaginary axis

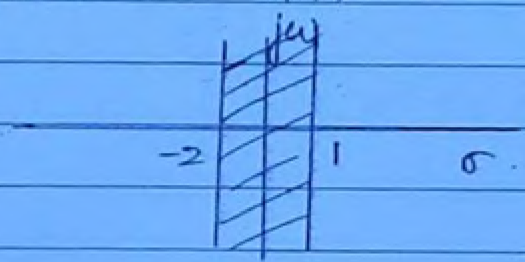
3. ROC 3 ⟵ $h_3(t)$



strip

⇒ unstable, both sided.

4. ROC 4 ⟵ $h_4(t)$

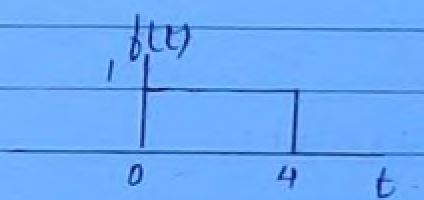


⇒ stable, both sided.

Q. $f(t) = u(t) - u(t-4)$

$F(s) = ?$ ROC = ?

A.



$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{0}^{4} 1 \cdot e^{-st}\, dt$$

$$= \left. \frac{e^{-st}}{-s} \right|_{0}^{4}$$

$$= \frac{1 - e^{-4s}}{s}$$

ROC. Entire s-plane excluding $s = -\infty$

because.

↳ $s = 0$ $F(0) = 4$ $\frac{1-e^{-4s}}{s} = \frac{0}{0} \Rightarrow 4e^{-4s}$.

↳ 4 at $s = 0$

↳ $s = \infty$ $F(\infty) = 0$

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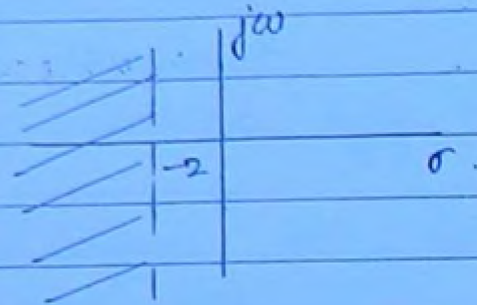
* $e^{-2t} u(-t)$

$\downarrow$ LS

$\sigma < -2$



* $\dfrac{1}{S+2}$

$\xrightarrow{RS} e^{-2t} u(t)$

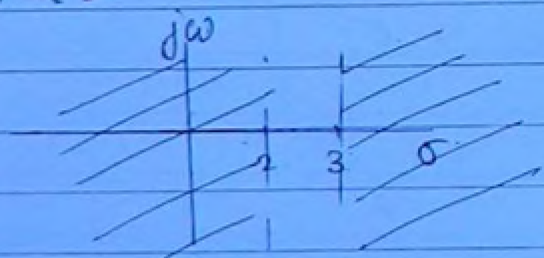
$\xrightarrow{LS} -e^{-2t} u(-t)$

Q. $f(t) = e^{2t} u(-t) + e^{3t} u(t)$

$F(s) = ?$ ROC $= ?$

Sol $f_1(t) = e^{2t} u(-t)$ $\quad f_2(t) = e^{3t} u(t)$

ROC $\sigma < 2$ $\quad \sigma > 3$



Since ROC is not common
LT does not exist

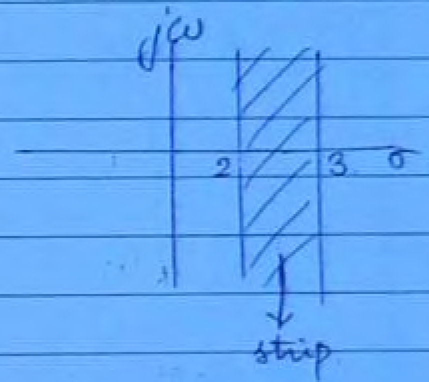
$\hookrightarrow$ both sided.

Q $f(t) = e^{3t}u(-t) + e^{2t}u(t)$

Sol ROC $\quad \sigma < 3 \qquad \sigma > 2$

$\therefore$

ROC $\rightarrow (2 \text{ to } 3) = \sigma \Rightarrow 2 < \sigma < 3$



$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(3-s)} u(-t) + e^{(2-s)} u(t)$$

$$\boxed{e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}}$$

$\downarrow t = -t \qquad \downarrow s = -s$

$$e^{at}u(-t) \rightleftharpoons \frac{1}{-s+a}$$

$$F(s) = \frac{1}{-s+3} + \frac{1}{s-2}$$

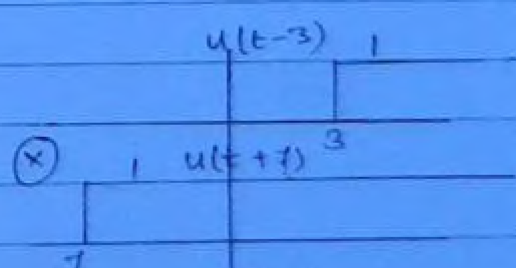
Q $f(t) = u(t-3) \cdot u(t+7)$

$F(s) = ?$

Sol $F_1(s) = \dfrac{e^{-3s}}{s} \qquad F_2(s) = \dfrac{e^{7s}}{s}$

~~$F_{s2}(s)$~~

~~$f(t) = f_1(t) f_2(t)$~~



$f(t) = u(t-3)\,u(t+7)$
$= u(t-3)$

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$F(s) = \cancel{\frac{1}{s+3}}\ \frac{e^{-3s}}{s}$

LT & ROC for important signals -

	$f(t)$	$F(s)$	ROC
1.	$\delta(t)$	1	entire s-plane
2.	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma > -a$
3.	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\sigma < -a$
4.	$\cos\omega_0 t\ u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\sigma > 0$
5.	$\sin\omega_0 t\ u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\sigma > 0$
6.	$u(t)$	$\frac{1}{s}$	$\sigma > 0$
7.	$t^n u(t)$, $n \geq 0$	$\frac{n!}{s^{n+1}}$	$\sigma > 0$

Q $r(t) = t\,u(t) \rightleftharpoons \frac{1}{s^2}$

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$r(t-1) = (t-1)u(t-1) \rightleftharpoons \frac{1}{s^2}e^{-s}$

$f(t) = t\,u(t-1)$

$= (t-1+1)\,u(t-1)$

$= (t-1)u(t-1) + u(t-1)$

$F(s) = \frac{1}{s^2}e^{-s} + \frac{1}{s}e^{-s}$

$f(t) = (3t^2 + 2t + 5)\,u(t-1)$

$f(t) = 3t^2 u(t-1) + 2t\,u(t-1) + 5u(t-1)$

$= \cancel{3(t-1)}$

$f(t) = 3(t-1)^2$

$f(t) = (3t^2 + 2t + 5)\,u(t-1)$

$\big\downarrow t = t+1$

$f(t+1) = [3(t+1)^2 + 2(t+1) + 5]\,u(t)$

$f(t+1) = [3t^2 + 8t + 10]\,u(t)$

$\Downarrow LT$

$F(s)e^{s} = \frac{6}{s^3} + \frac{8}{s^2} + \frac{10}{s}$

$F(s) = \left[\frac{6}{s^3} + \frac{8}{s^2} + \frac{10}{s}\right]e^{-s}$

Q $f(t) = [t^3 + 5t^2 + 3t + 1]\, u(t-1)$

Sol $f(t) = (t^3 + 5t^2 + 3t + 1)\, u(t-1)$

$\downarrow t = t+1$

$$f(t+1) = [(t+1)^3 + 5(t+1)^2 + 3(t+1) + 1]\, u(t)$$

$$= [t^3 + 1 + 3t^2 + 3t + 5(t^2 + 2t + 1) + 3t + 3 + 1]\, u(t)$$

$$= [t^3 + 8t^2 + 16t + 10]\, u(t)$$

$$F(s) = \left[\frac{6^{\to 3!}}{s^4} + \frac{16^{\to 8\times 2!}}{s^3} + \frac{16}{s^2} + \frac{10}{s} \right] e^{-s}$$

Shortcuts for partial fractions -

Quadratic factors -

Q $F(x) = \dfrac{4x^2 + 2x + 18}{(x+1)(x^2 + 4x + 13)}$

$$= \frac{A^{\to 2}}{x+1} + \frac{Bx + C}{x^2 + 4x + 13} \quad \text{---} (1)$$

i) $\lim\limits_{x \to \infty} x F(x)$ $\quad \dfrac{4x^3}{x^3} = \dfrac{Bx^2}{x^2} + \dfrac{2x}{x}$

$$4 = B + 2$$

$$B = 2.$$

ii) put $x = 0$ in (1)

$$\frac{18}{13} = 2 + \frac{C}{13} \Rightarrow C = -8.$$

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Q $F(x) = \dfrac{2x^2+4x+5}{x(x^2+2x+5)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2x+5}$ — (1)

Sol i) $A = 1$

ii) $\dfrac{2x^3}{x^3} = \dfrac{Bx^2}{x^2} + \dfrac{1x}{x}$

$2 = B+1 \quad B=1$

iii) $\dfrac{11}{8} = 1 + \dfrac{1+C}{8} \Rightarrow C = 2$

Repeated factor-

Q $F(x) = \dfrac{4x^3+16x^2+23x+13}{(x+1)^3(x+2)} = \dfrac{\overset{1}{\cancel{A}}}{x+2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} + \dfrac{\overset{2}{\cancel{D}}}{(x+1)^3}$ — (1)

i) $\lim_{x\to\infty} x\,F(x)$ $\quad 4 = 1 + B + 0 + 0$

$B = 3$

ii) Put $x=0 \Rightarrow \dfrac{13}{2} = \dfrac{1}{2} + 3 + C + 2$

$\Rightarrow C = 1$

CWB.

Q 20 $x(t) \rightleftharpoons X(s) = \dfrac{s^2+2s+5}{(s+3)(s+5)^2}$ $\quad \sigma > -3$

RS

a) $[e^{-3t} + 3e^{-5t} + 2te^{-5t}]u(t)$ $\quad x(0) \rightarrow 1+3\cdot 4$

b) $[2e^{-3t} - 4e^{-5t} - 5te^{-5t}]u(t)$ — $2-4=-2$

c) $[2e^{-3t} - e^{-5t} - 10te^{-5t}]u(t)$ — $2-1=1$

d) $[e^{-3t} - 4e^{-5t} - 5te^{-5t}]u(t)$ — $1-4=-3$

Sol. $\dfrac{s^2+2s+5}{(s+3)(s+5)^2} = \dfrac{A^{\to 2}}{s+3} + \dfrac{B^{\to -1}}{s+5} + \dfrac{C^{\to -10}}{(s+5)^2}$

$= \dfrac{2}{s+3} - \dfrac{1}{s+5} - \dfrac{10}{(s+5)^2}$

poles $= -5, -5, \underline{\underline{-3}} \to$ right most pole $= s$

$x(t) = 2e^{-3t}u(t) - e^{-5t}u(t) - 10\,e^{-5t}[t\,u(t)] \longrightarrow (C)$

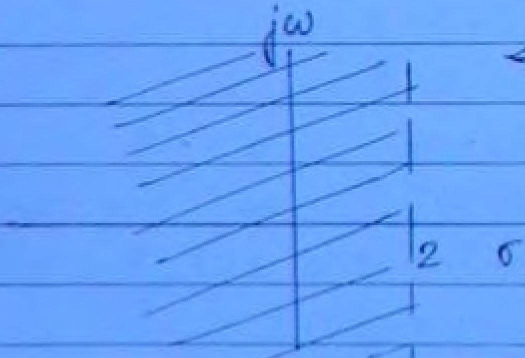
$t\,u(t) \rightleftharpoons \dfrac{1}{s^2}$

$e^{-5t}[t\,u(t)] \rightleftharpoons \dfrac{1}{(s+5)^2}$

CAUSAL & STABLE SYSTEM

- For a causal system ROC is right side to the right most pole
- For the stability of causal system all the poles of system T.F should lie in the LHS of s-plane

eg.



$\longleftarrow h(t)$

$\Rightarrow$ stable (as ROC includes imaginary axis)

$\Rightarrow$ Anticausal system

Anticausal System -

* For anticausal system, ROC is left side to the left most pole.
* For stability of anticausal system, all the poles of T.F should lie in the RHS of s-plane.

CWB. L.T.

18. $x(t) \rightleftharpoons X(s) = \log\left(\frac{s+5}{s+6}\right)$

$x(t) = ?$

a) $\frac{1}{t}[e^{-6t} - e^{-5t}]u(t)$ b) $\frac{1}{t}[e^{-6t} + e^{-5t}]u(t)$

c) $t[e^{-6t} - e^{-5t}]u(t)$ d) $\frac{1}{t}[e^{-5t} - e^{-6t}]u(t)$

Sol $X(s) = \log(s+5) - \log(s+6)$

$$x(t) \rightleftharpoons X(s)$$

$$t^n x(t) \rightleftharpoons (-1)^n \frac{dX(s)}{ds}$$

$$\downarrow n=1$$

$$t\,x(t) \rightleftharpoons -\frac{dX(s)}{ds}$$

$$= -\left[\frac{1}{s+5} - \frac{1}{s+6}\right]$$

$$= \frac{1}{s+6} - \frac{1}{s+5}$$

$$x(t) = \frac{1}{t}[e^{-6t} - e^{-5t}]u(t) \quad -(a)$$

Q19. $x(t) \rightleftharpoons \frac{(1-e^{t})\,u(t)}{t}$ (152)

$X(s)$

= a) $\log\left(\frac{s}{s-1}\right)$ b) $\log\left(\frac{s-1}{s}\right)$

c) $\log\left(\frac{s-1}{s+1}\right)$ d) $\log\left(\frac{s+1}{s-1}\right)$.

Sol

$$-\frac{1}{t}e^{-at}u(t) \rightleftharpoons -\int \frac{1}{s+a} \Rightarrow -\int \frac{1}{s+1}$$

$$+\frac{1}{t}u(t) \rightleftharpoons +\int \frac{1}{s} \Rightarrow \int \frac{1}{s}$$

$$X(s) = \int_s^\infty \frac{1}{s} - \int_s^\infty \frac{1}{s+1}$$

$$= \cancel{\log s - \log(s+1)}$$

$$= \left[\log s - \log(s+1)\right]_s^\infty$$

$$= \log\left[\frac{s}{s-1}\right]_s^\infty$$

$$= 0 - \log\frac{s}{s-1}$$

$$X(s) = \log\left(\frac{s-1}{s}\right) \quad \text{(b)}$$

Q The differential eq^n for an LTI system is given below –

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

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Determine h(t) for each of the foll cases –

i) when the system is stable.

ii) when the system is causal.

iii) when the system is neither causal nor stable.

Sol.

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = x$$

$$(D^2 - D - 2)y = x \qquad s^2Y(s) - sY(s) - 2Y(s) = X(s)$$

$$(s^2 - s - 2)Y(s) = X(s)$$

$$\cancel{m^2 - m - 2 = 0}$$

$$\cancel{(m-2)(m+1) = 0}$$

$$\cancel{m = 2, -1}$$

$$\cancel{CF = C_1e^{2t} + C_2e^{-t}}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(s-2)(s+1)}$$

$$H(s) = \frac{1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1} \qquad A \to \tfrac{1}{3},\ B \to -\tfrac{1}{3}$$

$$= \left[\underbrace{\frac{1/3}{s-2}}_{\downarrow} - \underbrace{\frac{1/3}{s+1}}_{\downarrow\ RS}\right]$$

Poles ⇒ −1, 2

i) When system is stable

ROC $-1 < \sigma < 2$

ROC$_1$ $-1 < \sigma$, $\underbrace{\sigma > -1}_{\downarrow\ RS}$ ROC$_2$ $\underbrace{\sigma < 2}_{\downarrow\ LS}$

$$h(t) = \frac{-1}{3}e^{-t}u(t) + \frac{1}{3}\left[-e^{2t}u(-t)\right]$$

ii) When system is causal.

ROC: $\sigma > 2$

$\text{ROC}_1 \Rightarrow \underbrace{\sigma > -1}_{\downarrow\, \text{RS}}$ $\quad$ $\text{ROC}_2 \Rightarrow \underbrace{\sigma > 2}_{\downarrow\, \text{RS.}}$

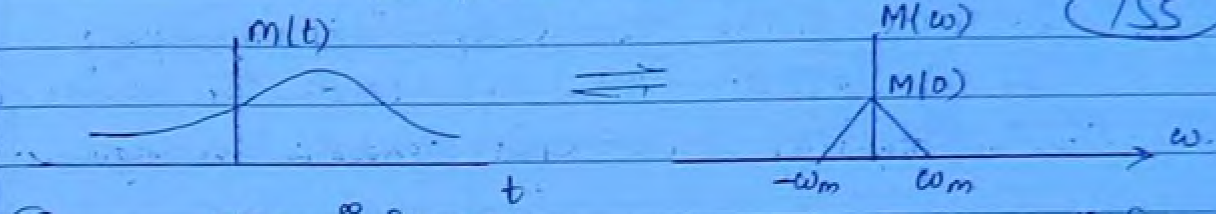
$$h(t) = \frac{-1}{3}\left[e^{-t}u(t)\right] + \frac{1}{3}\left[e^{2t}u(t)\right]$$

iii) ROC: $\sigma < -1$

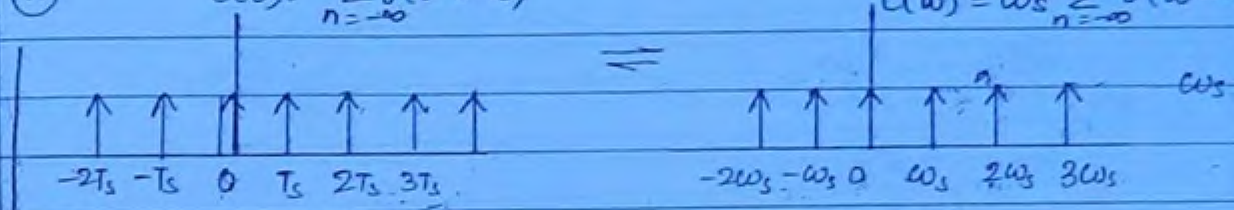
$\text{ROC}_1 \Rightarrow \underbrace{\sigma < -1}_{\downarrow\, \text{LS}}$ $\quad$ ROC_2 $\underbrace{\sigma \cancel{<} 2}_{\downarrow\, \text{LS}}$

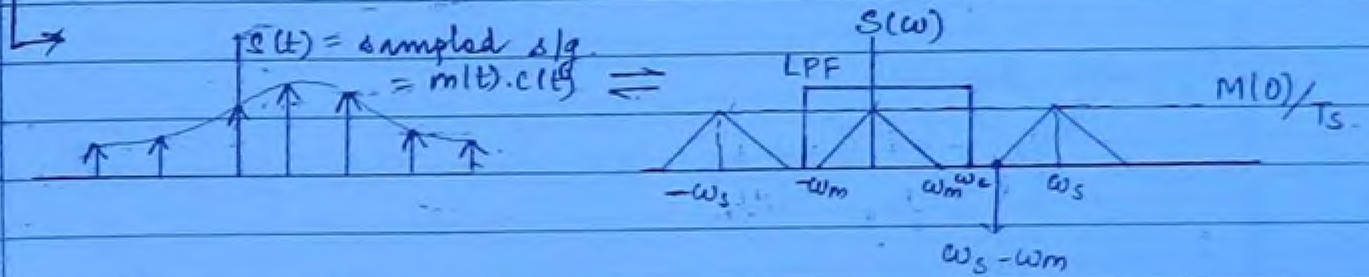
$$h(t) = \frac{-1}{3}\left[-e^{-t}u(-t)\right] + \frac{1}{3}\left[-e^{+2t}u(-t)\right]$$

SAMPLING THEOREM



(×) $c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ ⇌ $C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$





$s(t) = m(t)c(t)$

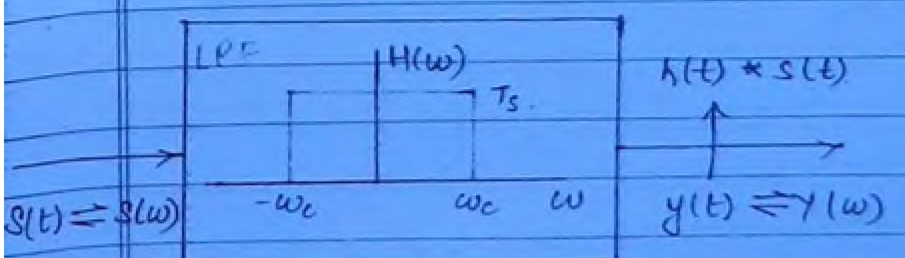
$S(\omega) = \frac{1}{2\pi}\left[m(\omega) * C(\omega)\right]$

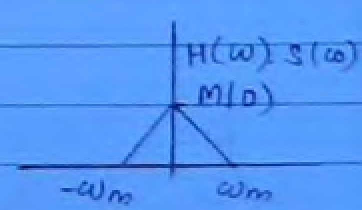
$= \frac{1}{2\pi}\left[m(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$

$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} m(\omega - n\omega_s)\right]$

$= \frac{1}{T_s}\left[\ldots + m(\omega + \omega_s) + m(\omega) + M(\omega - \omega_s) + M(\omega \mp 2\omega_s) + \ldots\right]$

ω_s : sampling freq $= \frac{2\pi}{T_s}$





→ * $\boxed{\omega_m \le \omega_c \le \omega_s - \omega_m}$

→ To avoid spectral overlapping of adjacent spectrum

* A signal can be represented by its samples or recovered back from its samples if sampling frequency is greater than or equal to twice of maximum frequency component present in signal.

* Nyquist Rate -

$$f_{ny} = 2f_m$$

or

$$\omega_{ny} = 2\omega_m .$$

Nyquist Interval -

$$T_{ny} = \frac{1}{f_{ny}} = \frac{1}{2f_m}$$

Oversampling -

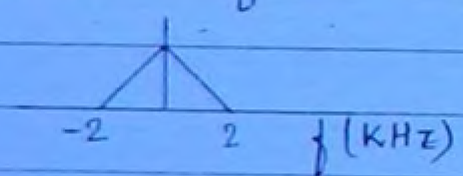
$$\omega_s > 2\omega_m$$

Undersampling - objective

$$\omega_s < 2\omega_m$$

eg

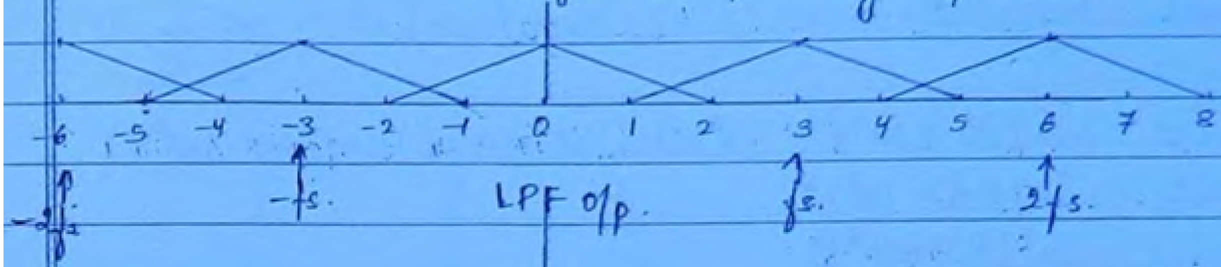
$$m(t) \rightleftharpoons m(f)$$

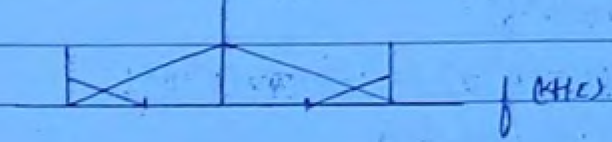


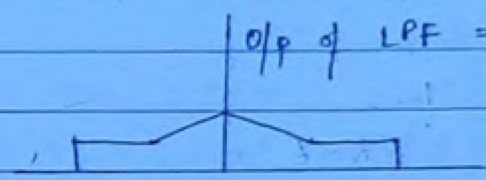
$f_m = 2$ KHz

$f_s = 3$ KHz $< 2 f_m$

S(f) = sampled s/g spectrum.

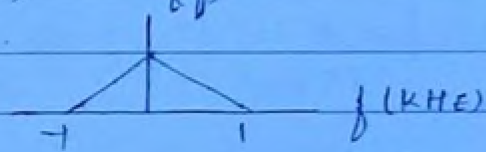






The o/p of LPF will be distorted cuz of undersampling

$m(t) \rightleftharpoons m(f)$



$T_s = 1$ msec

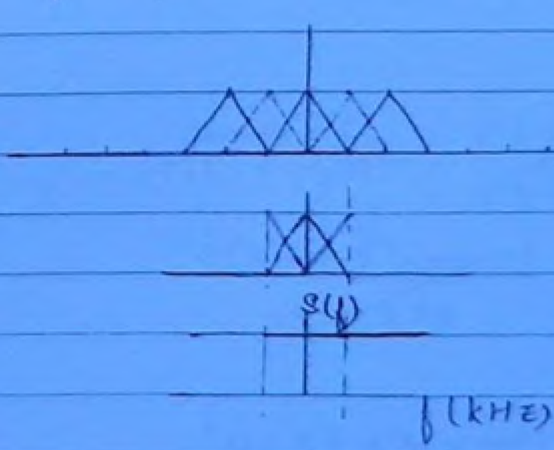
Draw sampled s/g spectrum S(f)

$\frac{\omega}{2\pi} = \frac{1}{T_s}$

$f_m = 1$ KHz $\qquad T_s = 1$ msec

$f_s = 1$ KHz $< 2f_m \qquad \frac{2\pi}{\omega} = \cancel{1 \times 10} \qquad f_s = 10^3$ Hz

$= 1$ KHz



dc signal in s domain = o/p

Q. Calculate nyquist rate (in rad/sec)

i) $m(t) = \sin(2\pi \times 10^6 t) + 2\sin(4\pi \times 10^6 t)\cos(3\pi \times 10^6 t)$

ii) $m(t) = Sa(4\pi \times 10^6 t)$

iii) $m(t) = Sa^2(4\pi \times 10^6 t)$

iv) $m(t) = Sa^3(4\pi \times 10^6 t)\, Sa^4(3\pi \times 10^6 t)$

v) $m(t) = \underbrace{m_1(t)}_{f_{m_1} = 2\,kHz} * \underbrace{m_2(t)}_{f_{m_2} = 8\,kHz}$

sol

i) $m(t) = \sin(2\pi \times 10^6 t) + \sin(\cancel{5\pi}\, 7\pi \times 10^6 t) + \sin(\pi \times 10^6 t)$

$f_1 = \dfrac{1}{2\pi \times 10^6}$ $\quad f_2 = \dfrac{1}{7\pi \times 10^6}$ $\quad f_3 = \dfrac{1}{\pi \times 10^6}$

$\omega_m = 7\pi \times 10^6$

$\omega_{ny} = 2 \times 7\pi \times 10^6$

$= 14\pi \times 10^6$

ii) $m(t) = Sa(4\pi \times 10^6 t) = A_0\, Sa(kt)$ $\quad \omega_m = k$

$= \cancel{\sin\left(\dfrac{4\pi \times 10^6 t}{2\pi}\right)}$ $\quad \dfrac{\sin(4\pi \times 10^6 t)}{4\pi \times 10^6 t}$

$\omega_m = 4\pi \times 10^6$

$\omega_{ny} = 8\pi \times 10^6$

iii) $m(t) = Sa^2(4\pi \times 10^6 t)$ $\quad x(t) \rightarrow \boxed{NL} \rightarrow x^2(t)$ $\quad f_m \qquad n \times f_m$

$= \dfrac{\sin^2(4\pi \times 10^6 t)}{4\pi \times 10^6 t}$ $\quad = \dfrac{1 - \cos(8\pi \times 10^6 t)}{8\pi \times 10^6 t}$

$\cancel{\omega_m = 8\pi \times 10^6}$ $\quad \omega_m = 2 \times 4\pi \times 10^6$

iv) $m(t) = Sa^3(4\pi \times 10^6 t)\, Sa^4(3\pi \times 10^6 t)$

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$$m(t) = m_1(t) \cdot m_2(t)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$f_m \qquad f_{m_1} \qquad f_{m_2}$$

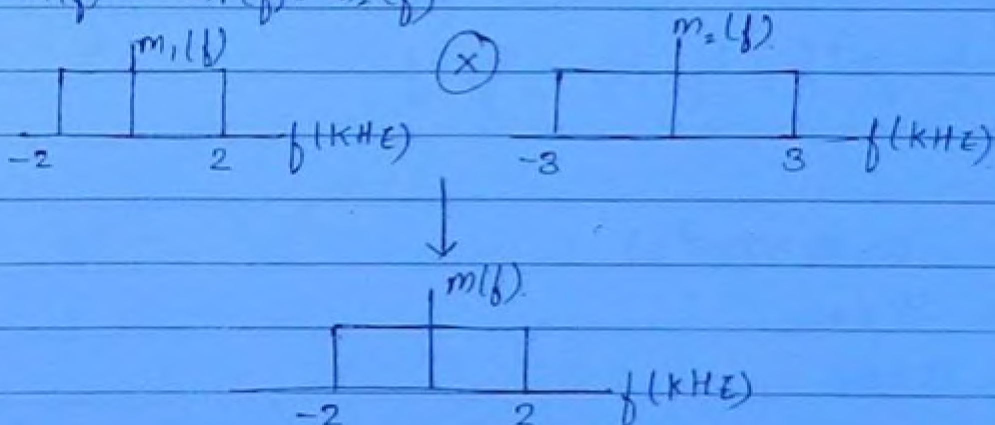
$$f_m = f_{m_1} + f_{m_2}$$

$$\omega_m = \omega_{m_1} + \omega_{m_2}$$
$$= 3 \times 4\pi \times 10^6 + 4 \times 3\pi \times 10^6$$
$$= 24\pi \times 10^6$$

$$\omega_{ny} = 48\pi \times 10^6 \text{ rad/sec}$$

v) $m(t) = m_1(t) * m_2(t)$

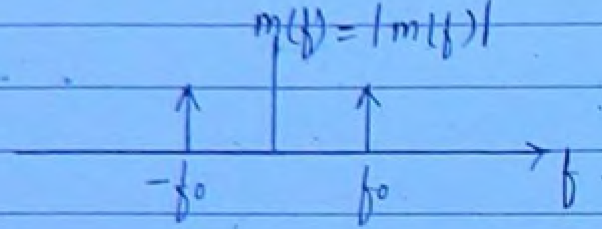
$$m(f) = m_1(f) \cdot m_2(f)$$

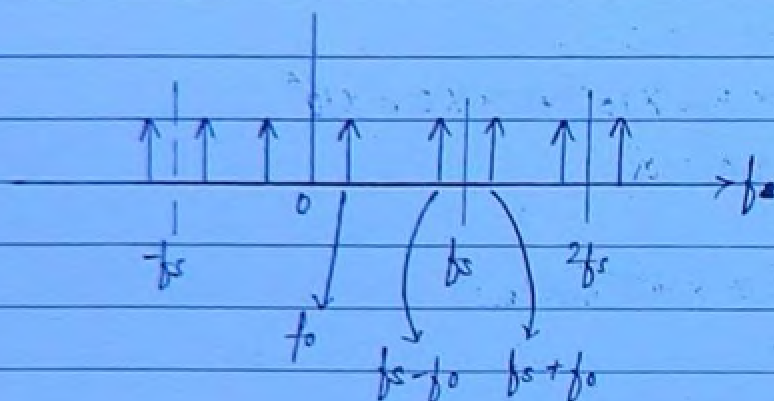


$$f_m = 2 \text{ KHz}$$
$$f_{ny} = 4 \text{ KHz}$$

* $m(t) = \cos 2\pi f_0 t = \cos \omega_0 t$

$$m(\omega) = \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$$





frequency component present in sampled s/g $S(f)$
$f_0,\ f_s \pm f_0;\ 2f_s \pm f_0;\ 3f_s \pm f_0 \ldots$

Q) $m(t) = \cos(24\pi \times 10^3 t)$
$T_s = 50\,\mu sec$
LPF $f_c = 15\,kHz$.

Which of the foll freq component are present at the o/p of the LPF.

a) 8 KHz b) 12
c) 12 & 8 d) 12 & 9

sol $f_s = \frac{10^6}{50} = 0.2 \times 10^5 = 20\ kHz$

$\cancel{f_s} < 2f_m$

$\downarrow$

~~undersampling.~~

$\overset{20\,KHz}{\uparrow}$

$f_s < 2f_0 \rightarrow 24\,KHz$ undersampling.

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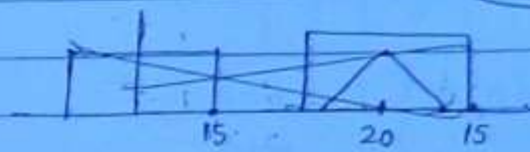
$f_0,\ f_s \pm f_0,\ 2f_s \pm f_0$



12 ; 20 ± 12 ; 40 ± 12 ...

12 ; 8 | 32, 28, 52 ....

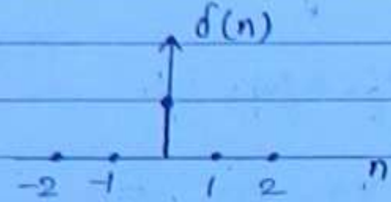
< 15

→ Ans

DISCRETE TIME SYSTEM

1. Unit Impulse : $\delta(n)$

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Properties of $\delta(n)$:

1. $\delta(n)$ is an even signal.

2. $\delta(an) \quad a \neq 0 \quad = \delta(n)$

$$\delta(at) \quad a \neq 0 \quad = \frac{1}{|a|}\delta(t)$$

3. $\delta(n)$ is an energy signal $[E = 1]$
$\delta(t)$ is neither energy nor power

4. $x(n) . \delta(n-n_1) = x(n_1) . \delta(n-n_1)$

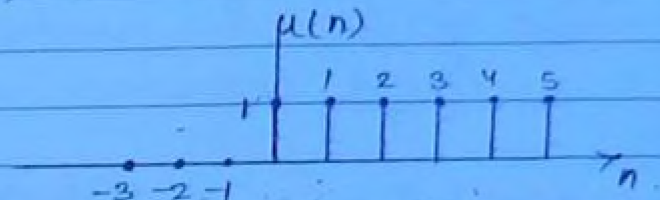
5. $x(n) * \delta(n-n_1) = x(n-n_1)$

6. $\int_{-\infty}^{t} \delta(k)\,dk = u(t)$

$$\sum_{k=-\infty}^{\infty} \delta(k) = u(n)$$

Unit Step Signal : $u(n)$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

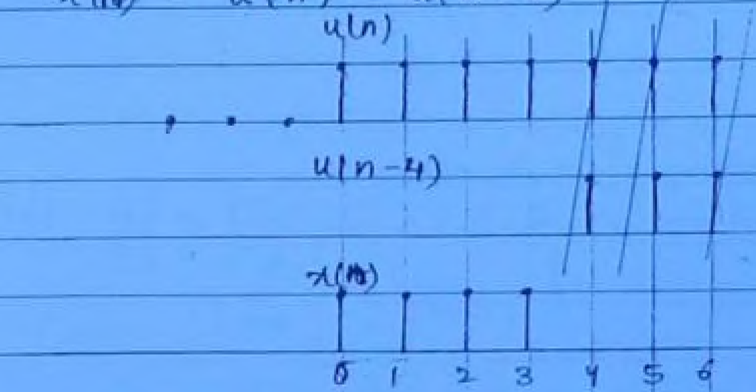


Properties -

1. $u(n)$ is power s/g $\left(P = \frac{1}{2}\right)$

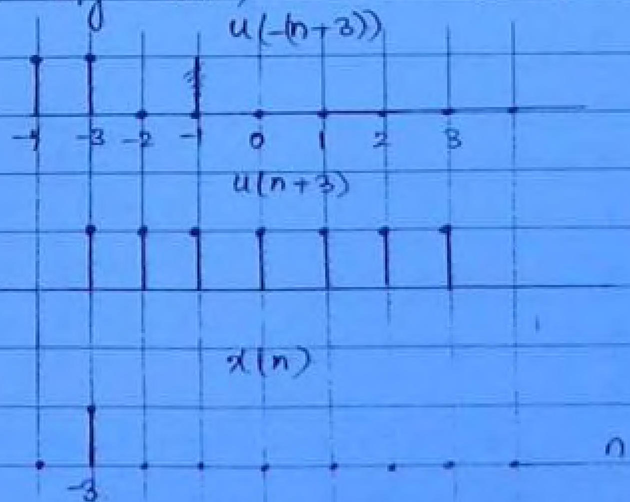
Q Draw s/g $x(n)$

$x(n) = u(n) - u(n-4)$



$\rightarrow u(-(n+3)).\,u(n+3)$

Q Draw s/g $x(n) = u(-n-3)\,.\,u(n+3)$



Operation on Signals -

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1. Time shifting -

$x(n) = \{-1 \quad 2 \quad \underset{\uparrow}{5} \quad 0 \quad 3\}$

n = -2   n = 2

$x(n-1) = \{-1, \underset{\uparrow}{2}, 5, 0, 3\}$

$x(n+2) = \{-1, 2, 5, 0, \underset{\uparrow}{3}\}$

2. Time Scaling -

i) Time compression / Decimation

$x(n) = \{-1, 3, 0 \quad 1 \quad \underset{\uparrow}{-1} \quad 5 \quad 3 \quad 2 \quad 4\}$

n = -4   n = 4

$f(n) = x(2n) = \{-1, 0, \underset{\uparrow}{1}, 3, 4, 0\}$

$f(-3) = x(-6) = 0$
$f(-2) = x(-4) = -1$
$f(-1) = x(-2) = 0$
$f(0) = x(0) = 1$
$f(1) = x(2) = 3$
$f(2) = x(4) = 4$
$f(3) = x(6) = 0$

$x(3n) = \{3, \underset{\uparrow}{-1}, 2\}$

Time compression leads to loss of information in discrete time system whereas no loss of information takes place in case of continuous time.

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ii) Time expansion - Interpolation

$$x(n) = \{4, \underset{\uparrow}{2}, 7\}$$

$n=-1$ $n=1$

$$f(n) = x\left(\frac{n}{2}\right) = \{4, 0, \underset{\uparrow}{2}, 0, 7\}$$

$\hookrightarrow 2-1 = 1$

$\hookrightarrow$ no. of zeros.

$f(-2) = x(-1) = 4$

$f(-1) = x(-1/2) = 0$

$f(0) = x(0) = 2$

$f(1) = x(1/2) = 0$

$f(2) = x(1) = 7$.

$$f(n) = x\left(\frac{n}{4}\right) = \{4, 0, 0, 0, 2, 0, 0, 0, 7\}$$

$\hookrightarrow 4-1 = 3$ zeros.

Q. Find $x(n)$

if $f(n) = \{3, 4, \underset{\uparrow}{5}, 6, 7\}$

i) $x(n) = f(-2n)$

ii) $x(n) = f(2n/3)$

iii) $x(n) = f(2n-1)$

iv) $x(n) = f(-2n-1)$

Sol. i) ~~$x(-4)$~~ $= f(-2(2)) = x(+4) = 0$ $\quad$ ~~$x(n) = \{0, \underset{\uparrow}{5}, 0\}$~~

$f(-2(1)) = x(3) = 0$

i) $f(n) \longrightarrow f(2n) \longrightarrow f(-2n)$
$f(2n) \Rightarrow \{3, \underset{\uparrow}{5}, 7\}$ $\quad f(-2n) = \{7, \underset{\uparrow}{5}, 3\}$

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ii) $x(n) = f\left(\frac{2n}{3}\right)$

$f(n) \longrightarrow f(2n) \longrightarrow f\left(\frac{2n}{3}\right)$

$f(2n) = \{3, \underset{\uparrow}{5}, 7\}$

$f\left(\frac{2n}{3}\right) = \{3, 0, 0, \underset{\uparrow}{5}, 0, 0, 7\}$

$f(n-1) \longrightarrow f(2n-1)$

~~iii) $f(n) \rightarrow f(2n) \rightarrow f(2n-1)$~~
~~$\{3, 4, 5, 6, 7\} \rightarrow \{3, 4, 5, 6, 7\} \rightarrow \{\underset{\uparrow}{4}, 6\}$~~
~~$f(2n) = \{3, \underset{\uparrow}{5}, 7\}$ $\quad f(2n-1) = \{\underset{\uparrow}{3}, 5, 7\}$~~

~~iv) $f(n) \rightarrow f(2n+1) \longrightarrow f(-(2n+1))$~~
~~iv) $f(n) \rightarrow f(n-1) \rightarrow f(2n-1) \rightarrow f(-2n-1)$~~
~~$f(2n) = \{3, \underset{\uparrow}{5}, 7\}$ $\quad f(2n) = \{3, \underset{\uparrow}{4}, 5, 6, 7\} = f(n-1)$~~
~~$f(2n+1) = \{3, 5, \underset{\uparrow}{7}\}$~~

~~$f(2n-1) = \{\underset{\uparrow}{4}, 6\}$~~

~~$f(-(2n+1)) = \{\underset{\uparrow}{7}, 5, 3\}$~~

~~$f(-2n-1) = \{6, \underset{\uparrow}{4}\}$~~

---

iii) $f(n) \rightarrow f(n-1) \rightarrow f(2n-1)$
$\{3, 4, \underset{\uparrow}{5}, 6, 7\} \rightarrow \{3, \underset{\uparrow}{4}, 5, 6, 7\} \rightarrow \{\underset{\uparrow}{4}, 6\}$

iv) $f(n) \rightarrow f(n-1) \rightarrow f(2n-1) \rightarrow f(-2n-1)$
$\rightarrow \{6, \underset{\uparrow}{4}\}$

3. Convolution -

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

$$t = n \qquad \tau = k$$

$$y(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$= x_1(n) * x_2(n)$$

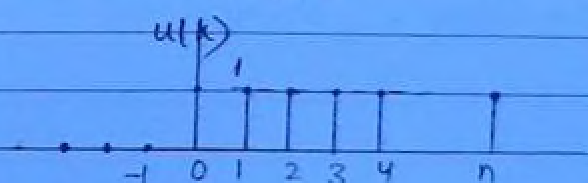
Signal	Extension	Length
$x_1(n)$	$n_1 \le n \le n_2$	L_1
$x_2(n)$	$n_3 \le n \le n_4$	L_2
$y(n)$	$(n_1+n_3) \le n \le (n_2+n_4)$	$\boxed{L = L_1 + L_2 - 1}$ * VVIMP

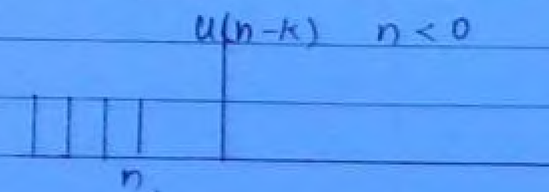
Q. $y(n) = u(n) * u(n)$

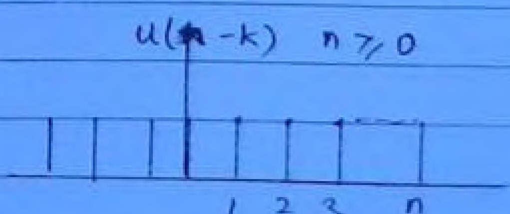
$$= \sum_{k=-\infty}^{\infty} u(k)\, u(n-k)$$

$k = n$

$n - k = 0$







$$y(n) = \begin{cases} 0 & n < 0 \\ \sum_{k=0}^{n} 1 & n \ge 0 \end{cases}$$

$$= \begin{cases} 0 & n < 0 \\ n+1 & n \ge 0 \end{cases}$$

Q. $y(n) = x_1(n) * x_2(n)$

$x_1(n) = \{1, -1, \underset{\uparrow}{1}\}$ $\quad x_2(n) = \{\underset{\uparrow}{2}\ -1\ 1\}$

Sol. Tabular Method

$\rightarrow$ or 3rd element + 1st element − 1

$3 + 1 - 1 = 3$

$x_2(n) \backslash x_1(n)$	1	−1	1
2	2	−2	2
−1	−1	1	−1
1	1	−1	1

$y(n) = \{2, -3, \underset{\uparrow}{4}, -2, 1\}$

Q. $y(n) = x_1(n) * x_2(n)$

$x_1(n) = -1\ \underset{\uparrow}{2}\ 0\ 1$

$x_2(n) = 3\ 1\ \underset{\uparrow}{0}\ -1$

$2 + 3 - 1 = 4$

	−1	2	0	1
3	−3	6	0	3
1	−1	2	0	1
0	0	0	0	0
−1	1	−2	0	−1

$y(n) = -3\ 5\ 2\ \underset{\uparrow}{4}\ -1\ 0\ -1$

Q $x(n) = \{1, -1\}$ → LTI system → $y(n) = \{1, 0, 0, 0, -1\}$
$\uparrow$ $\uparrow$

$h(n) = ?$

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$y(n) = x(n) * h(n)$

solve by the method

		1	+1	1	1
→	1	1	1	1	1
	−1	−1	+1	−1	−1

$1 + x - 1 = 1$
$x = 1$

1 0 0 0 −1
$\uparrow$

a) {1 0 0 1} b) 1 0 1
c) {1 1 1 1} d) 1 1 1

Sol
$L_1 = 2$
$L_2 = ?$
$L = 5 = L_1 + L_2 - 1$
$5 = 2 + L_2 - 1$
$L_2 = 4$ (a) or (c)

CWB Z-T

52 Let $y(n)$ denote convolution of $h(n)$ & $g(n)$ where $h(n) = \left(\frac{1}{2}\right)^n u(n)$

& $g(n)$ is causal sequence

If $y(0) = 1$ $y(1) = \frac{1}{2}$ then $g(1)$ is sol $h(n) = \left(\frac{1}{2}\right)^n u(n)$ $\qquad g(n) =$

$= \cancel{\tfrac{1}{8}} \quad 2 \quad \underset{\uparrow}{1} \quad \frac{1}{2} \quad \frac{1}{4} \ldots$ $\qquad$ 171

$g \backslash h(n)$	1	1/2	1/4	1/8
$g(0) \to 1$	1	1/2	1/4	1/8
$\boxed{g(1) \to 0}$	0			

Energy & Power Signal -

Energy signal -

→ $E = \text{finite} \qquad P = 0$

→ Energy signals are absolutely summable

ie $\sum_{n=-\infty}^{\infty} |x(n)| < \infty$

→ $\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$

Q Calculate E of s/g

i) $x(n) = \delta(n)$

ii) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

iii) $x(n) = \{-1, \underset{\uparrow}{1+2j}, 2j, 1\}$

sol i) $E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |\delta(n)|^2$

ii) $E = \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n u(n) \right]^2$ (172)

$E = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n \right]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

$= 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \ldots = \frac{1}{1 - 1/4} = \frac{4}{3}$

HP. $\frac{a}{a-r}$

iii) $E = \sum_{n=-1}^{2} \{-1, 1+2j, 2j, 1\}^2$

$= |-1|^2 + (|1+2j|)^2 + (|2j|)^2 + |1|^2$

$= \cancel{1 + (1+2j-4) + (-4)} + 1$

$= 1 + 5 + 4 + 1$

$= 11$

Power Signal -

→ $P =$ finite $E = \infty$

→ $P = \begin{cases} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 & \text{for periodic signal} \\ \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 & \text{for non-periodic signal} \end{cases}$

Q Calculate Power of Signal

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i) $x(n) = A_0 u(n)$

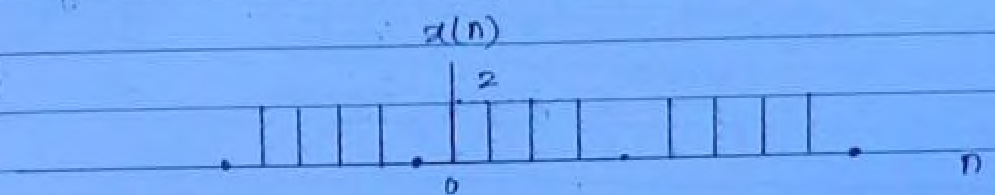
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} A_0^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} A_0^2 (N+1)$$

$$P = \frac{A_0^2}{2}$$

ii)



$$P = \frac{|2|^2 + |2|^2 + |2|^2 + |2|^2 + |0|^2}{5}$$

$$= \frac{16}{5}$$

Q $x(n) = \{-4-5j,\ \underset{\uparrow}{1+2j},\ 4\}$

i) $$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{\{-4-5j \quad \underset{\uparrow}{1+2j} \quad 4\} + \{4 \quad \underset{\uparrow}{1+2j} \quad -4-5j\}}{2}$$

$$= -5j \quad \underset{\uparrow}{2(1+2j)} \quad -5j$$

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ii) $x_o(n) = \frac{x(n) - x(-n)}{2}$

$$= \frac{\{-4-5j \quad \underset{\uparrow}{1+2j} \quad 4\} - \{4 \quad \underset{\uparrow}{1+2j} \quad -4-5j\}}{2}$$

$$= \frac{-8-5j \quad \underset{\uparrow}{0} \quad 8+5j}{2}$$

iii) $x_{cs}(n) = \frac{x(n) + x^*(-n)}{2}$

$$= \frac{\{-4-5j \quad 1+2j \quad 4\} + \{4 \quad 1-2j \quad -4+5j\}}{2}$$

$$= \frac{-5j \quad \underset{\uparrow}{2} \quad 5j}{2}$$

iv) $x_{cAs}(n) = \frac{x(n) - x^*(-n)}{2}$

$$= \frac{\{-4-5j \quad \underset{\uparrow}{1+2j} \quad 4\} - \{4 \quad \underset{\uparrow}{1-2j} \quad -4+5j\}}{2}$$

$$= \frac{-8-5j \quad \underset{\uparrow}{4j} \quad 8-5j}{2}$$

PERIODIC / NON-PERIODIC SIGNAL -

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Periodic Signal -

$$x(n) = x(n \pm kN)$$

where K = an integer

N = fundamental time period

⇒ an integer

$$\boxed{x(n) = \underbrace{x_1(n)}_{\downarrow\ N_1} + \underbrace{x_2(n)}_{\downarrow\ N_2}}$$

$\Rightarrow \dfrac{N_1}{N_2} = \dfrac{\text{integer}}{\text{integer}}$ = Rational number (always)

∴ there is no need to check for N_1 & N_2 to be rational ratio.

Sum of 2 or more periodic signals in discrete time ~~sig~~ system will be always periodic

$$\boxed{N = LCM[N_1, N_2]}$$

Complex Exponential & sinusoidal signals are always period in continuous time system.

In discrete time system -

$$x(n) = A_0 e^{j\omega_0 n}$$

Let N be the fundamental time period of $x(n)$

ie $x(n) = x(n+N)$

$$\Rightarrow \cancel{A_0} e^{j\omega_0 n} = \cancel{A_0} e^{j\omega_0(n+N)}$$

$$\Rightarrow \cancel{A_0}\cancel{e^{j\omega_0 n}} = \cancel{e^{j\omega_0 n}} e^{j\omega_0 N}$$

$e^{j\omega_0 N} = 1 = e^{j2\pi k}$ $\quad k =$ an integer.

$\omega_0 N = 2\pi K$.

$\Rightarrow \boxed{\frac{2\pi}{\omega_0} = \frac{N}{K}} = \frac{int}{int} =$ Rational no.

Complex exponential & sinusoidal signals in discrete time system will be periodic only when $\frac{2\pi}{\omega_0}$ is rational.

$$\boxed{N = \frac{2\pi}{\omega_0} K.}$$

where $K =$ a least integer for which 'N' is an integer.

eg $\quad x(n) = e^{j\frac{3\pi}{5}n}$

$\omega_0 = \frac{3}{5}\pi$

$\Rightarrow \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi} \times 5 = \frac{10}{3} \rightarrow$ R.no

$\therefore x(n) \rightarrow$ periodic.

$N = \frac{2\pi}{\omega_0} \times K = \frac{2\pi}{3\pi} \times 5 \times K$

$N = \frac{10}{3} K$

for $k = 3$ $\quad \boxed{N = 10}$

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Q1. i) $x(n) = \sin 2n$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi \rightarrow \text{I.R no}$$

$\therefore$ $x(n) \rightarrow$ Non-periodic.

ii) $x(n) = \sin \frac{3\pi}{5} n + \cos \frac{7\pi}{3} n$

$$N_1 = \frac{2\pi}{\omega_1} \times k_1 = \frac{2\pi}{3\pi} \times 5 \times k_1$$

$$N_1 = 10$$

$$N_2 = \frac{2\pi}{\omega_2} \times K_2 = \frac{2\pi}{7\pi} \times 3 \times K_2$$

$$N_2 = 6 \qquad K_2 = 7.$$

$N = \text{LCM}(6, 10)$

$N = 30$

Basic System Properties -

1. Static / Dynamic System -

1. $y(n) = x(-n)$ D

2. $y(n) = x(n+1)$ D

accumulator

3. $y(n) = \sum_{k=-\infty}^{\infty} x(k) = x(-\infty) + \underbrace{\cdots\cdots}_{\text{past}} x(n-1) + \underset{\text{present}}{x(n)}$ & D

4. $y(n) = odd[x(n)] = \dfrac{x(n) - x(-n)}{2}$ D

5. $y(n) = Real[x(n)] = \dfrac{x(n) + x^*(+n)}{2}$ S

2. Causal / Non-causal -

1. $y(n) = x(n) + x(n-1)$ C

2. $y(n) = x(n+1) + x(n)$ NC

3. $y(n) = CS[x(n)] = \dfrac{x(n) + x^*(-n)}{2}$ NC

4. $y(n) = x(-n)$ NC

5. $y(n) = \sum_{k=-\infty}^{\infty} x(n)$ C $= x(-\infty) + \cdots\cdots + x(n-1) + x(n)$

6. $y(n) = \sum_{k=-\infty}^{-n} x(n)$ NC $x(-n)$

$y(-1)$ $\xrightarrow{n=-1}$ $= x(1)$

7. $y(n) = \sum_{k=-\infty}^{\infty} x(-k)$ NC $\rightarrow x(-n)$

3. Linear / Non-linear -

1. $y(n) = x(n) + 10$ NL

2. $y(n) = Sa(n) . x(n)$ L

3. $y(n) = Sa[x(n)]$ NL

4. $y(n) = x[Sa(n)]$ L

5. $y(n) = \begin{cases} x(n) & n < 0 \\ x(n-1) & n \geq 0 \end{cases}$ L.

6. $y(n) = Real[x(n)] = \dfrac{x(n) + x^*(n)}{2}$ NL

7. $y(n) = Even[x(n)]$ L

8. $y(n) = \sum_{k=-\infty}^{\infty} x(k)$ L

4. Time-invariant / Time variant -

1. $y(n) = x(2n)$ TV

2. $y(n) = \cos n . x(n)$ TV

3. $y(n) = \cos[x(n)]$ TIV

4. $y(n) = odd[x(n)]$ TV

5. $y(n) = \sum_{k=-\infty}^{n} x(k) \rightarrow x(n)$ TIV

7. $y(n) = \sum_{k=-\infty}^{n} x(2k) \rightarrow x(2n)$ TV

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8. $y(n) = \sum_{k=-\infty}^{n} \cos k . x(k) \rightarrow \cos n . x(n)$ TV.

5. Stable / Unstable -

BIBO system

1. $y(n) = x^2(n)$ S

2. $y(n) = n x(n)$ US $\quad \begin{matrix} x(n) & y(n) \\ 2 & 2n \end{matrix}$

3. $y(n) = \cos n . x(n)$ S $\quad x = 10 \quad y(n) = 10 \cos n$.

4. $y(n) = \dfrac{x(n)}{\sin n}$ US

5. $y(n) = \text{Even}[x(n)] = \dfrac{x(n) + x(-n)}{2}$ S

6. $h(n)$ = Impulse response
$= \left(\frac{1}{2}\right)^n u(n) \rightarrow$ energy s/g S

7. $h(n) = u(n)$ ~~US~~ S marginally stable

8. $h(n) = \cos \omega_0 n . u(n)$ marginally stable

$\rightarrow$ BIBO US

9. $h(n) = 2^n u(n)$ US
↳ neither energy nor power

Z - TRANSFORM

Discrete time fourier transform is used for frequency domain representation of energy & power signals

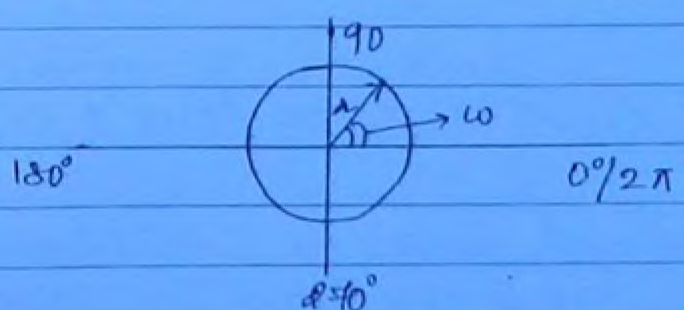
Z-transform is used for frequency domain representation of energy, power & neither energy nor power s/g also (upto some extent)

eg $a^n u(n)$ → neither energy nor power
↳ ZT & DTFT will not exist.

$x(n) \rightleftharpoons X(z)$ $\qquad X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

where z = complex variable

$= \underbrace{r}_{\text{damping factor}} \underbrace{e^{j\omega}}_{\text{oscillation frequency}}$



$x(n) = a^n u(n)$

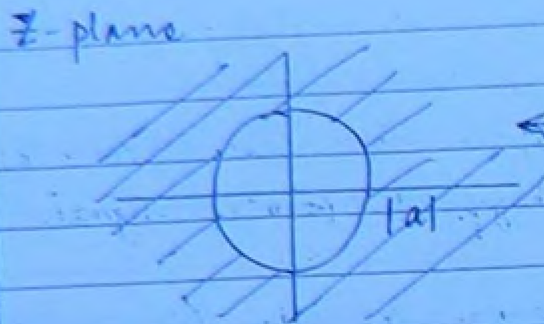
$X(z) = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n = 1 + az^{-1} + (az^{-1})^2 + \ldots$$

$|z| > |a|$

(182)



ROC :

Q $x(n) = -a^n u(-n-1)$

$X(z) = ?$ ROC = ?

sol

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} (a z^{-1})^n$$

$$= -[\cancel{1 + az^{-1} + (az^{-1})^2} + \ldots]$$

$n = -n$

$$= -\sum_{n=1}^{\infty} (a z^{-1})^{-n}$$

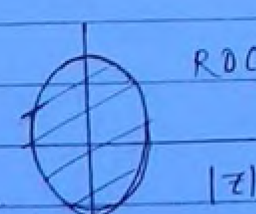
$$= -\sum_{n=1}^{\infty} (a^{-1} z)^n$$

$$= -[a^{-1}z + (a^{-1}z)^2 + (a^{-1}z)^3 + \ldots]$$

$$= -\left[\frac{a^{-1}z}{1 - a^{-1}z}\right] \quad |a^{-1}z| < 1$$

$$= -\left[\frac{z}{a - z}\right] \quad |z| < |a|$$

$$= \frac{1}{1 - az^{-1}} \quad |z| < |a|$$



ROC :

$|z| < |a|$

Properties of Z - transform -

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1. Linearity -

$$a_1 x_1(n) + a_2 x_2(n) \rightleftharpoons a_1 X_1(z) + a_2 X_2(z)$$

2. Time reversal -

$$x(-n) \rightleftharpoons X(z^{-1})$$

3. Time shifting -

$$x(n - n_0) \rightleftharpoons X(z) z^{-n_0}$$

4. Conjugation -

$$x^*(n) \rightleftharpoons x^*(z^*)$$

5. Scaling (of Z)

$$a^n x(n) \rightleftharpoons X(a^{-1} z)$$

6. Convolution ~~of~~ in time

$$x_1(n) * x_2(n) \rightleftharpoons X_1(z) \, X_2(z)$$

7. Differentiation in time / successive difference -

$$\frac{dx(n)}{dn} = \frac{x(n) - x(n-1)}{n-(n-1)}$$

$$= x(n) - x(n-1) \rightleftharpoons X(z) - X(z) z^{-1}$$

$$= (1 - z^{-1}) X(z)$$

8. Integration in time / Accumulation

$$\sum_{k=-\infty}^{\infty} x(k) \rightleftharpoons \frac{X(z)}{1 - z^{-1}}$$

9. Differentiation in frequency -

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$$n\,x(n) \rightleftharpoons -z\,\frac{dX(z)}{dz}$$

10. Initial Value Theorem -

$$x(0) = \lim_{z \to \infty} X(z)$$

$$x(0) = \lim_{s \to \infty} s\,X(s)$$

→ Applicable only for causal type signals
ie $x(n) = 0 \quad n < 0$

11. Final Value Theorem -

$$x(\infty) = \lim_{z \to 1} (1 - z^{-1})\,X(z)$$

→ Applicable only for causal type signals
ie $x(n) = 0 \quad n < 0$

→ The term $(1 - z^{-1})X(z)$ should have poles inside unit circle in z-plane

ROC

It is defined as the range of complex variable z in z-plane for which z-transform of signal is convergent or finite

Properties of ROC -

1. It does not include any pole.

2. For right sided signal, ROC is exterior to a circle.

3. For left sided signal ROC is interior to a circle.

4. For both sided signal ROC is a RING in z-plane.

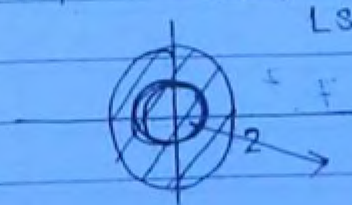
5. For stability ROC includes unit circle.

6. For finite duration signal ROC is entire z-plane excluding possibly $z = 0$ and/or $\pm\infty$

Q Check stability of system & extension of $h(n)$

1. $ROC_1 \rightarrow h_1(n)$

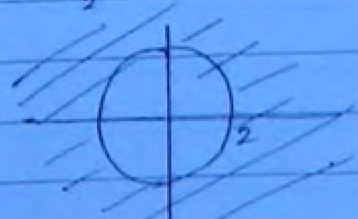
LS.



$|z| < 1$ ∴ stable

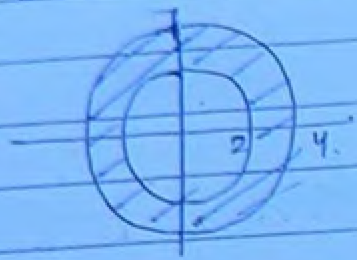
2. $ROC_2 \rightarrow h_2(n)$ RS.

unstable



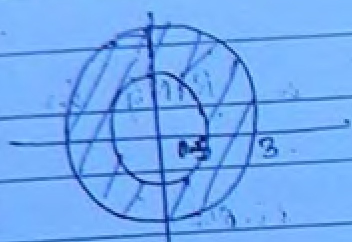
3. $ROC_3 \rightarrow h_3(n)$ Both sided

unstable



4. $ROC_4 \rightarrow h_4(n)$ Both sided

stable



Q $x(n) = \{3 \quad -2 \quad \underset{\uparrow}{1} \quad 4 \quad 7\}$

$X(z) = ?$ ROC = ?

method -1

sol $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-2}^{2} x(n) z^{-n} = x(-2)z^{2} + x(-1)z + x(0) + x(1)z^{-1} + x(2)z^{-2}$

$X(z) = 3z^{+2} - 2z^{1} + 1 + 4z^{-1} + 7z^{-2}$

$= 3z^{2} - 2z + 1 + 4z^{-1} + 7z^{-2}$

ROC entire z-plane excluding

$z = 0, \pm\infty$

Q. 2^{nd} method

$x(n) = \{3, \quad -2 \quad \underset{\uparrow}{1} \quad 4 \quad 7\}$

$3\delta(n+2) - 2\delta(n+1) + \delta(n) + 4\delta(n-1) + 7\delta(n-2)$

CWB chapter ZT

31. $x(n) \rightleftharpoons X(z) \rightarrow$ [LTI system] $\rightarrow y(n) = x(n) * h(n)$

$h(n) = 2\delta(n-3)$

$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

Sol $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

$x(n) = 1 \quad 0 \quad 1 \quad -2 \quad \underset{\uparrow}{2} \quad 0 \quad 0 \quad 0 \quad -3$

$h(n) = 2\delta(n-3)$

$y(n) = x(n) * h(n)$

$Y(z) = X(z)\,H(z)$

$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$

$= 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$

$y(2) = -4$

Ans (d)

Q. i/p $x(n) \rightleftharpoons X(z) = 1 - 3z^{-1}$

o/p $y(n) \rightleftharpoons Y(z) = 1 + 2z^{-2}$

An LTI system has impulse response $h(n)$ defined as

$h(n) = x(n-1) * y(n)$

The o/p of the systems for i/p $\delta(n-1)$

a) has ZT $\Rightarrow z^{-1} X(z) Y(z)$

b) equal $\delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$

Method - 1

Sol.

$$p(n) \rightleftharpoons P(z) \longrightarrow \boxed{\text{LTI system}} \longrightarrow q(n) \rightleftharpoons Q(z)$$

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$$h(n) \rightleftharpoons H(z)$$

$\rightarrow$ $p(n) = \delta(n-1)$

$P(z) = z^{-1}$

$\rightarrow$ $h(n) = x(n-1) * y(n)$

$H(z) = z^{-1} X(z) Y(z)$

$\rightarrow$ $q(n) = p(n) * h(n)$

$$Q(z) = P(z) H(z)$$

$$= z^{-1} z^{-1} X(z) Y(z)$$

$$= z^{-2} X(z) Y(z)$$

$$= z^{-2} [1 - 3z^{-1}][1 + 2z^{-2}]$$

$$= z^{-2} [1 + 2z^{-2} - 3z^{-1} - 6z^{-3}]$$

$$= z^{-2} + 2z^{-4} - 3z^{-3} - 6z^{-5}$$

$$q(n) = \delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$$

Method 2

$X(z) = 1 - 3z^{-1} \rightleftharpoons x(n) = \{\underset{\uparrow}{1}, -3\}$ $\quad x(n-1) = \{\underset{\uparrow}{0}, 1, -3\}$

$Y(z) = 1 + 2z^{-2} \rightleftharpoons y(n) = \{\underset{\uparrow}{1}, 0, 2\}$

$h(n) = x(n-1) * y(n)$

	↓ 0	1	-3
→ 1	0	1	-3
0	0	0	0
2	0	2	-6

~~$h(n) = \delta(n-1) - 3(n-2) + 2(n-3)$~~

$q(n) = h(n) * p(n) = h(n) * \delta(n-1) = h(n-1)$

$h(n-1) = \{ \underset{\uparrow}{0} \quad 0 \quad 1 \quad -3 \quad 2 \quad -6 \}$

$q(n) = \delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$

* $x(n) = (-2)^n \underline{u(-n)}$
LS.

$|z| < |-2|$

$z < |2|$

Q $x(n) = \left(\frac{-1}{2}\right)^{-n} u(-n) + 3^n u(n) \quad = (-2)^n u(-n) + 3^n u(n)$

$X(z) = ? \qquad ROC = ?$

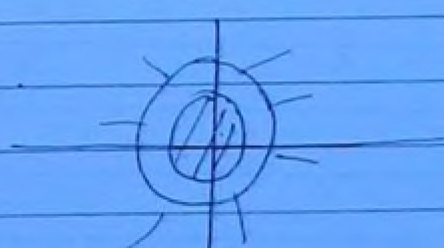
Sol. $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$ROC_1 \Rightarrow |z| < \left|\frac{-2}{1}\right|$

$z < 2$

$ROC_2 \Rightarrow |z| > |3|$

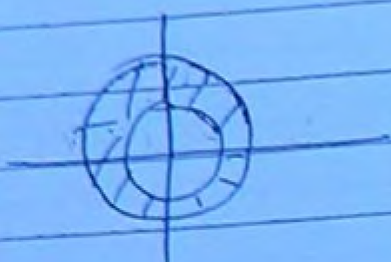
$z > 3$



Z - transform will not exist as theres no common ROC

Q $x(n) = 2^n u(n) + 3^n u(-n)$ (190)

sol ROC$_1$ $\rightarrow$ $|z| > |2|$ ROC$_2$ $\Rightarrow$ $|z| < |3|$



ROC $2 < |z| < 3$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n} + \sum_{n=-\infty}^{\infty} 3^n u(-n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n + \sum_{n=-\infty}^{0} 3^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n + \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^n$$

$$= \frac{1}{1-2z^{-1}} + \frac{1}{1-\frac{1}{3}z}$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}}$$

$\downarrow n = -n$ $\downarrow z = z^{-1}$

$$a^{-n} u(-n) \rightleftharpoons \frac{1}{1-az}$$

CWB.

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Q. $x(n) = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u(n)$

ROC = ?

Sol ROC 1.

$\left(\frac{1}{3}\right)^{|n|} = \begin{cases} \left(\frac{1}{3}\right)^{-n} & n<0 \\ \left(\frac{1}{3}\right)^{n} & n \geq 0 \end{cases}$

ROC 2 = $|z| > \left|\frac{1}{2}\right|$

put in option (C)

$= \begin{cases} 3^{n} & n<0 \\ \left(\frac{1}{3}\right)^{n} & n \geq 0 \end{cases}$

$= 3^{n} u(-n-1) + \left(\frac{1}{3}\right)^{n} u(n)$

$x(n) = \underbrace{3^{n} u(-n-1)}_{|z| < |3|} + \underbrace{\left(\frac{1}{3}\right)^{n} u(n)}_{|z| > \left|\frac{1}{3}\right|} - \underbrace{\left(\frac{1}{2}\right)^{n} u(n)}_{|z| > \left|\frac{1}{2}\right|}$

ROC $\frac{1}{2} < z < 3$

ZT & ROC for important signals. (192)

x(n)	X(z)	ROC
$\delta(n)$	1	entire z-plane
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
* $na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\cos\omega_0 n\ u(n)$	$\frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$	$\lvert z\rvert > 1$
$\sin\omega_0 n\ u(n)$	$\frac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$	$\lvert z\rvert > 1$

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$\therefore x(n) = \cos\omega_0 n \cdot u(n)$

$= \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n} u(n)$

$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \quad |z| > |a|$

$a = e^{j\omega_0}$

$\frac{1}{2}(e^{j\omega_0})^n u(n) \rightleftharpoons \left[\frac{1}{1-e^{j\omega_0} z^{-1}}\right]\frac{1}{2} \quad |z| > 1$

$a = e^{-j\omega_0}$

$\frac{1}{2}(e^{-j\omega_0})^n u(n) \rightleftharpoons \left[\frac{1}{1-e^{-j\omega_0} z^{-1}}\right]\frac{1}{2} \quad |z| > 1$

$X(z) = \frac{1}{2}\left[\frac{1}{1-e^{-j\omega_0} z^{-1}} + \frac{1}{1-e^{j\omega_0} z^{-1}}\right]$

$= \frac{z^2 - 2\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$

Q Find inverse ZT for of

$X(z) = \frac{z}{(z-1)(z-2)^2} \quad |z| > 2$

sol $\cancel{X(z) = \frac{A}{z-1} + \frac{B}{z-2}}$ $\quad \frac{X(z)}{z} = \frac{1}{(z-1)(z-2)^2}$

$\frac{X(z)}{z} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$

$X(z) = \frac{Az}{z-1} + \frac{Bz}{z-2} + \frac{Cz}{(z-2)^2} \times \frac{z^{-2}}{z^{-2}}$

$= \frac{A}{1-z^{-1}} + \frac{B}{z-2z^{-1}} + \frac{C}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$

Since ROC $|z| > 2 \rightarrow$ RS.

Poles → 1, 2, 2

$x(n) = A\,u(n) + B\,2^n u(n) + \frac{C}{2}\left[n(2)^n u(n)\right]$ → Ans

194

$A = 1 \qquad B = 1 \qquad C = 1$

$x(n) = u(n) + 2^n u(n) + n\,2^{n-1} u(n)$

39. $x(n) \rightleftharpoons X(z) = \dfrac{z^2 + z}{(z-1)^2}$

$\dfrac{X(z)}{z} = \dfrac{z+1}{(z-1)^2}$

$\dfrac{X(z)}{z} = \dfrac{A}{z-1} + \dfrac{B}{(z-1)^2}$

$X(z) = \dfrac{Az}{z-1} + \dfrac{Bz}{(z-1)^2} \times \dfrac{z^{-2}}{z^{-2}}$

$A = 1 \qquad B = 2$

$X(z) = \dfrac{1}{1-z^{-1}} + 2\left[\dfrac{z^{-1}}{(1-z^{-1})^2}\right]$ $\qquad n a^n u(n) = \dfrac{a z^{-1}}{(1-az^{-1})^2}$

$= \cancel{A\,u(n) + B\,(2)^n u(n)}$

$= u(n) + 2n\,u(n)$

$\cancel{X(z) = u(n) + 2^n u(n)}$ $\qquad x(t) = (2n+1)u(n)$ (c)

$\cancel{= (2^n + 1)u(n)}$

40. $x(n) = 4^n u(n) \rightleftharpoons X(z)$

$y(n) \rightleftharpoons Y(z) = X^2(z)$

find $y(n)$

Sol. $x(n) = 4^n u(n) \rightleftharpoons \dfrac{1}{1-4z^{-1}}$

$Y(z) = X^2(z) = \dfrac{1}{\quad} \times z^2 \cdot \dfrac{z^2}{\quad}$

$$\Rightarrow \frac{z^2}{(z-4)^2}$$

$$\Rightarrow \frac{z(z-4)+4z}{(z-4)^2}$$

$$\Rightarrow \frac{z}{z-4} + \frac{4z}{(z-4)^2} \times \frac{z^{-2}}{z^{-2}}$$

$$\Rightarrow \frac{1}{1-4z^{-1}} + \frac{4z^{-1}}{(1-z^{-1})^2}$$

$$x(n) = 4^n u(n) + n4^n u(n)$$
$$= (n+1)4^n u(n)$$
$$= (n+1)4^n u(n+1)$$

$\Rightarrow x(n)=0$ for $n<0$

Ans (b) & (c)

Q Determine z inverse of $X(z)$

$$X(z) = \log(1+az^{-1}) \qquad |z|>|a|$$

Sol

$$X(z) = \log(1+az^{-1})$$

$$\cancel{\frac{dX(z)}{dz} = \frac{-1}{1+az^{-1}} \cdot az^{-2}}$$

$$-z\frac{dX(z)}{dz} \rightleftharpoons nx(n)$$

$$nx(n) \rightleftharpoons -z\frac{dX(z)}{dz}$$

$$= -z\left[\frac{1}{1+az^{-1}}(-az^{-2})\right]$$

$$nx(n) \rightleftharpoons \frac{az^{-1}}{(1+az^{-1})}$$

$$= \frac{-1}{1+az^{-1}} + 1.$$

$$nx(n) = \delta(n) - (-a)^n u(n)$$

$$x(n) = \frac{1}{n}\left[\delta(n) - (-a)^n u(n)\right]$$

Q $x(n) \rightleftharpoons X(z) = \dfrac{1+z^{-1}}{1+\frac{1}{3}z^{-1}}$ (176)

a) Assuming ROC to be $|z| < \frac{1}{3}$ determine $x(0)$, $x(-1)$ & $x(-2)$

b) Assuming ROC to be $|z| > \frac{1}{3}$ determine $x(0)$ $x(1)$ & $x(2)$

Sol.

$$X(z) = \frac{1+z^{-1}}{1+\frac{1}{3}z^{-1}} = \frac{z^0+z^{-1}}{z^0+\frac{1}{3}z^{-1}}$$

a) ROC $|z| < \frac{1}{3}$

→ Slg is left sided

→ Arrange numerator & denominator poly slg in increasing powers of z

$$X(z) = \frac{z^{-1}+1}{\frac{1}{3}z^{-1}+1}$$

$$\frac{1}{3}z^{-1}+1 \;\big)\; \cancel{z^{-1}}+1 \;\big(\; 3-6z+18z^2+\dots$$

$$\cancel{z^{-1}}+3$$

$$-\cancel{2}$$

$$\cancel{2}\;-6z$$

$$6\cancel{z}$$

$$-\cancel{6z}\;\pm 18z^2$$

$$-18z^2$$

$$X(z) = 3 - 6z + 18z^2 + \dots$$

$\to x(0) = 3$

b) ROC $|z| > \frac{1}{3}$.   197

→ s/g is right sided

→ arrange the numerator & denominator polynomial in decreasing powers of z.

$$X(z) = \frac{1 + z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

$$1+\frac{1}{3}z^{-1} \,\big)\, \cancel{1} + z^{-1} \,\big(\, 1 + \frac{2}{3}z^{-1} - \frac{2}{9}z^{-2} + \dots$$

$$\cancel{1} + \frac{1}{3}z^{-1}$$

$$-$$

$$\cancel{\frac{2}{3}z^{-1}}$$

$$\cancel{\frac{2}{3}z^{-1}} + \frac{2}{9}z^{-2}$$

$$-$$

$$-\frac{2}{9}z^{-2}$$

$$X(z) = 1 + \frac{2}{3}z^{-1} - \frac{2}{9}z^{-2} + \dots$$

→ $x(0) = 1$

→ $x(1) = 2/3$

→ $x(2) = -2/9$

Qn 29 $x(n) \rightleftharpoons X(z) = \frac{z}{2z^2 - 3z + 1}$ $|z| < \frac{1}{2}$

find $x(-2)$ (198)

Sol $X(z) = \frac{z}{1 - 3z + 2z^2}$

$$1-3z+2z^2 \,)\, z \,(\, z + 3z^2 + \ldots$$
$$\underline{z - 3z^2 + 2z^3}$$
$$3z^2 - 2z^3$$

$X(z) = z + 3z^2$

$\downarrow$

$x(-2) = 3$

Ans 3 (d)

Causal system -

1. $h(n) = 0 \quad n < 0$

2. $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$

$= \sum_{n=0}^{\infty} h(n) z^{-n}$

$H(z) = h(0) + h(1) z^{-1} + h(2) z^{-2} + \ldots$

* For causal system, expansion of T.F contains only negative powers of z

3. $\lim_{z \to \infty} H(z) =$ 'finite' or '0' (199)

$$= \lim_{z \to \infty} \frac{N(z)}{D(z)}$$

order of $N(z)$ should be $\leq$ order of $D(z)$ for causal system.

4. For causal system ROC is exterior to a circle.

5. For stability of causal system all the poles of transfer function should lie inside unit circle in Z-plane

Anti - causal system

1. $h(n) = 0 \quad n \geq 0$

2. For anti causal system ROC is interior to a circle.

3. For stability of anti causal system, all the poles of transfer function should lie outside unit circle

Q 44. Causal LTI system

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

$$\Rightarrow 2y(z) = \alpha y(z) z^{-2} - 2x(z) + \beta x(z) z^{-1}$$

$$\frac{Y(z)}{\ldots} = \frac{-2 + \beta z^{-1}}{\ldots}$$ Since poles determine stability

For stability $\Rightarrow$ $|pole| < 1$

(200)

$$\Rightarrow \left|\sqrt{\frac{\alpha}{2}}\right| < 1$$

$$\Rightarrow |\alpha| < 2.$$

20) Stable causal system

$$y(n) + \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) = -2x(n) + \frac{5}{4} x(n-1)$$

Sol. $$Y(z) + \frac{1}{4} Y(z) z^{-1} - \frac{1}{8} y(n) z^{-2} = -2X(z) + \frac{5}{4} X(z) z^{-1}$$

$$\frac{Y(z)}{X(z)} = \frac{-2 + 5/4\, z^{-1}}{1 + 1/4\, z^{-1} - \frac{1}{8} z^{-2}}$$

$$= \cancel{-8 + 5z^{-1}} \quad \frac{2(-8 + 5z^{-1})}{8 + 2z^{-1} - z^{-2}}$$

$$=$$

System is causal

$\therefore$ Initial Value Theorem is applicable.

$$h(0) = \lim_{z \to \infty} H(z) = -2$$

In all options check ROC to be inside unit circle.

a) $\left(\frac{1}{4}\right)^n u(n) - 3\left(\frac{-1}{2}\right)^n u(n) \longrightarrow h(0) = -2$ Ans.

d) $\left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n u(n) \longrightarrow h(0) = 1 + 3 = 4$

201

47. $x(z) = \dfrac{0.5}{1-2z^{-1}}$ $\quad x[0] = ?$

ROC includes unit circle.

Sol. ~~$x(z) = \dfrac{0.5}{z}$~~

pole $= 1-2z^{-1} = 0$

$z = 2$

ROC $\quad |z| > 2 \quad$ not possible.

$\therefore \quad \underbrace{|z| < 2}$

$\downarrow$

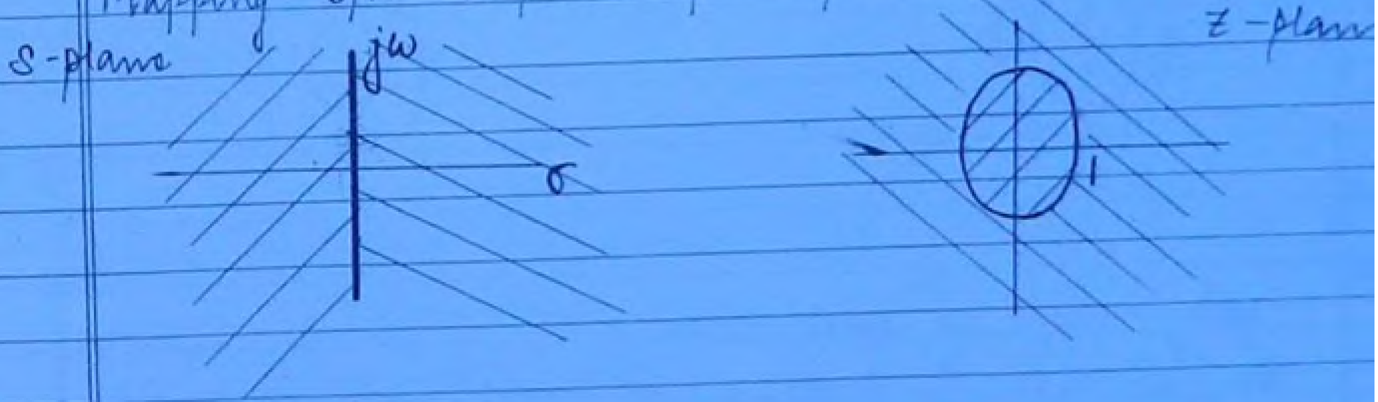
$\therefore$ LS. $x(n)$

$$x(n) = \frac{0.5}{1-2z^{-1}}$$

$$= 0.5\left[-2^n u(-n-1)\right]$$

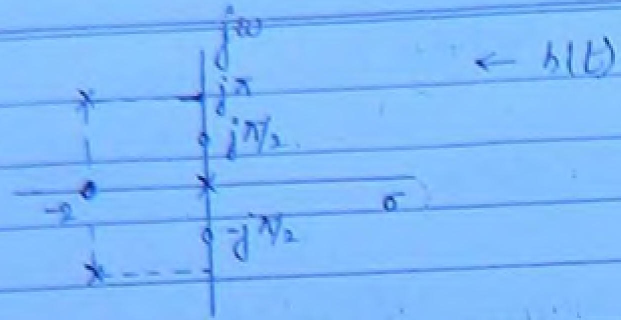
at $n=0 \quad x(n) = 0 \quad$ Ans (b)

Mapping b/w z-plane & s-plane



$z = e^{sT} \rightarrow T =$ sampling time period.

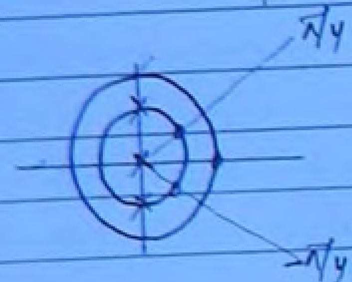
Q.

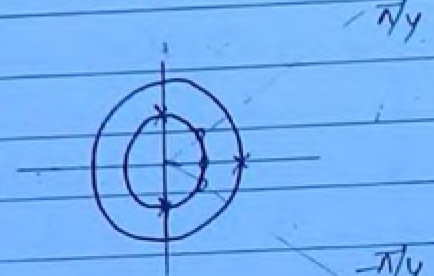


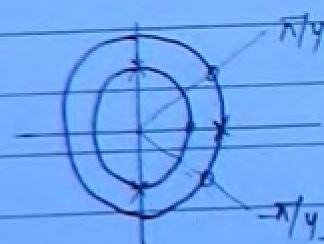
(202)

The corresponding $h(t)$ is sampled at '2Hz' to get $h(n)$. Which one of the following represents the equivalent pole-zero plot of $H(z)$ in z-plane.

The concentric circles are $|z| = 1$ $|z| = \frac{1}{e}$

a) 

b) 

c) 

d) 

Sol. outer circle is unit circle
~~for~~ $j\omega$ axis of s plane is unit circle of z plane
~~2 pole~~ 1 pole & 2 zeros.
$\therefore$ ans (c).

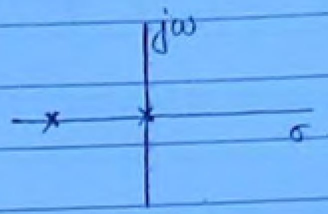
Poles $s = 0, -2 \pm j\pi$

$z = e^{sT} = e^{s/2}$
$= e^{0}, e^{-1 \pm j\pi/2}$
$= 1, \frac{1}{e} e^{j\pi/2}, \frac{1}{e} e^{-j\pi/2}$

54 $G(s) = \dfrac{10}{s(s+5)}$

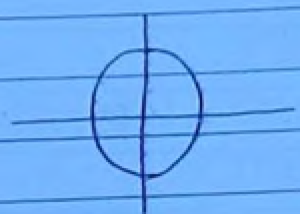
203

sol



Poles $\Rightarrow s = 0$

$s = -5$.



$z = e^{sT} = e^{0}, e^{-5T}$

$G(z)$ poles $\Rightarrow e^{0}, e^{-5T} = 1, e^{-5T}$.

put in options (b)

continuous		discrete
no. of poles	=	no. of poles

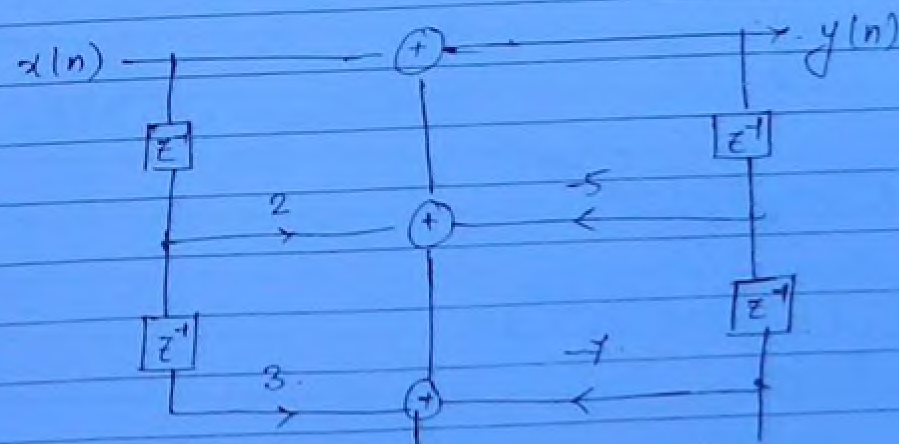
no of zeros may or may not be equal to no of zeros

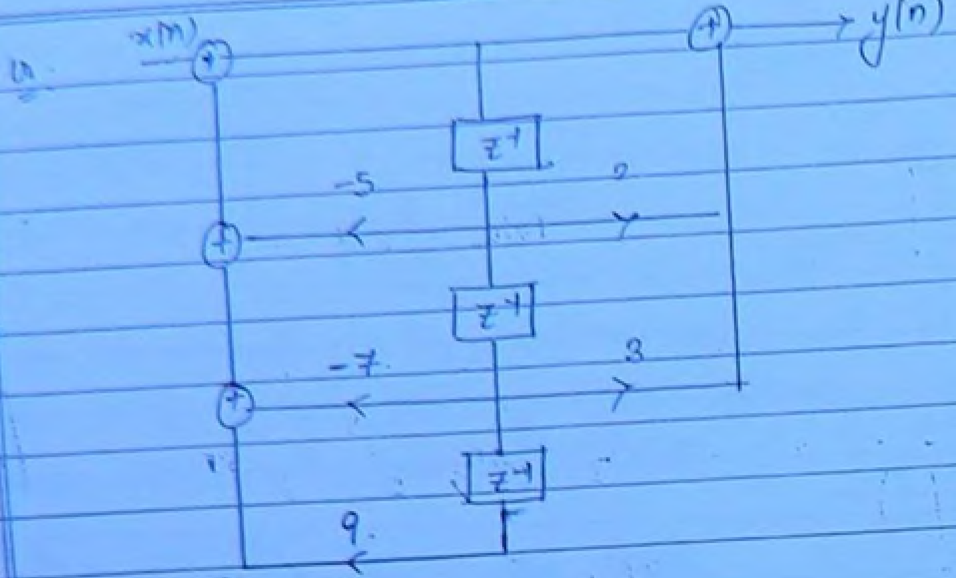
$\therefore$ we map only poles.

8 (b)

BLOCK DIAGRAM REPRESENTATION OF T.F.

$$H(z) = \frac{1+2z^{-1}+3z^{-2}}{1+5z^{-1}+7z^{-2}-9z^{-3}} = \frac{1+2z^{-1}+3z^{-2}}{1-[-5z^{-1}-7z^{-2}+9z^{-3}]}$$





Q04

30 (c)

DTFT

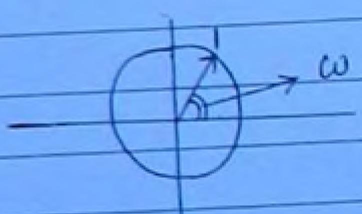
* $\boxed{z = e^{j\omega}}$

$x(n) \rightleftharpoons X(e^{j\omega})$

D Periodic

$e^{j\omega}$

$\boxed{2\pi}$



→ periodic with fundamental period.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$\Big\downarrow z = e^{j\omega}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$\boxed{\begin{aligned} &\int_{2\pi} X(e^{j\omega})\, d\omega = 2\pi\, x(0) \\ &\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi\, x(0) \end{aligned}}$$

$\longrightarrow$ DTFT

$\longrightarrow$ CTFT

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45. $x(n) = \sin(\omega_0 n + \phi) \longrightarrow \boxed{\text{LTI sys}} \longrightarrow y(n) = x(n - n_0)$

$H(e^{j\omega})$

$\angle H(e^{j\omega_0}) = ?$

Sol

$$y(n) = x(n - n_0)$$

$$Y(z) = X(z)\, z^{-n_0}$$

$$H(z) = \frac{Y(z)}{X(z)} = z^{-n_0}$$

$$\Big\downarrow z = e^{j\omega}$$

$$H(e^{j\omega}) = e^{(-j\omega n_0)}$$

$$H(e^{j\omega_0}) = e^{-j\omega_0 n_0}\, \cancel{e^{j2\pi k}}^{\,1} \qquad k = \text{an integer}$$

$$H(e^{j\omega_0}) = e^{j[-n_0\omega_0 + 2\pi k]}$$

$$\angle H(e^{j\omega_0}) = -n_0\omega_0 + 2\pi k$$

16 $x(n) = \left(\frac{1}{2}\right)^n u(n)$ $y(n) = x^n(n) \rightleftharpoons Y(e^{j\omega})$

$Y(e^{j0}) = ?$

sol. $y(n) = x^2 n = \left(\frac{1}{4}\right)^n u(n)$

$$Y(z) = \frac{1}{1 - \frac{1}{4} z^{-1}}$$

$\downarrow z = e^{j\omega}$

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$$

$\downarrow \omega = 0$

$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3} \quad (d)$$

31. $h(n) = \frac{1}{2}\left[\delta[n] + \delta[n-2]\right]$ $|H(e^{j\omega})| = ?$ $\omega = \Omega$

sol $H(z) = \frac{1}{2} z^0 + \frac{1}{2} z^{-2}$

$\downarrow z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{1}{2} + \frac{1}{2} e^{-2j\omega} = \frac{1}{2}\left[1 + e^{-2j\omega}\right]$$

$$= \frac{1}{2}\left[e^{-j\omega} \cdot e^{j\omega} + e^{-2j\omega}\right]$$

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$$|H(e^{j\omega})| = |e^{-j\omega}|\,|\cos\omega|$$

$$|H(e^{j\omega})| = |\cos\omega| \qquad (a)$$

* $$x(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI}\\ \text{system}\end{array}} \longrightarrow y(t) = ?$$

$= A_0 \cos(\omega_0 t + \phi)$ $\quad H(\omega)$

$$y(t) = A_0\, |H(\omega_0)| \cos\left[(\omega_0 t + \phi) + \angle H(\omega_0)\right]$$

CWB chapter 4

21. $$h(t) = e^{-2t} u(t)$$

$$H(\omega) = \frac{1}{2 + j\omega} \qquad (c)$$

22 $$x(t) = 2\cos 2t \qquad \omega_0 = 2$$

$$H(\omega_0) = \frac{1}{2 + j\omega_0} = \frac{1}{2 + 2j}$$

$$|H(\omega_0)| = \frac{1}{2\sqrt{2}}$$

$$\angle H(\omega_0) = -\pi/4$$

$$y(t) = \cancel{A_0}\, 2 \cdot \frac{1}{2\sqrt{2}} \cos\left[2t + \left(-\frac{\pi}{4}\right)\right]$$

$$= 2^{-0.5} \cos(2t - 0.25\pi) \qquad (d)$$

* $x(t) \rightarrow \boxed{\text{LTI system}} \rightarrow y(t)$

$x(t)$	$y(t)$
$\delta(t) \longrightarrow$	$h(t)$ = impulse response $= \frac{dz(t)}{dt}$
$\uparrow \frac{d}{dt}$	$\uparrow \frac{d}{dt}$

$u(t) \longrightarrow z(t)$ = step response

chapter 2.

Q15. $z(t) = 0.5(1 - e^{-2t})\, u(t)$

$h(t) = ?$

sol. I $x(t) = u(t) \rightleftharpoons X(s) = \frac{1}{s}$

$y(t) = z(t) = 0.5(1-e^{-2t})\, u(t)$

$Y(s) = 0.5 \left[\frac{1}{s} - \frac{1}{s+2} \right]$

$Y(s) = \frac{1}{s(s+2)}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$

$h(t) = e^{-2t}\, u(t)$ (a)

$\underline{\Pi}$ $\quad z(t) = 0.5[1-e^{-2t}]u(t)$ (209)

$$h(t) = \frac{dz(t)}{dt}$$

$$= 0.5\frac{d}{dt}[(1-e^{-2t})u(t)]$$

$$= 0.5[2e^{-2t}u(t) + \cancel{(1-e^{-2t})\delta(t)}]$$

$\cancel{(1-e^{-2t})\delta(t)} \to 0$

$f(t).\delta(t)$

$= f(0)\delta(t)$

$= (1-e^{0})\delta(t) = 0$

$$= \frac{1}{2}[2e^{-2t}u(t)]$$

$$h(t) = e^{-2t}u(t)$$

* $x(n)$ / $x(t)$ $\rightarrow$ LTI system $\rightarrow y(n)$

$x(n)$	$y(n)$
$\delta(n)$	$h(n)$ = impulse response $= \frac{dz(t)}{dt} = z(n) - z(n-1)$
$u(n)$	$z(n)$ = step response

CWB ZT

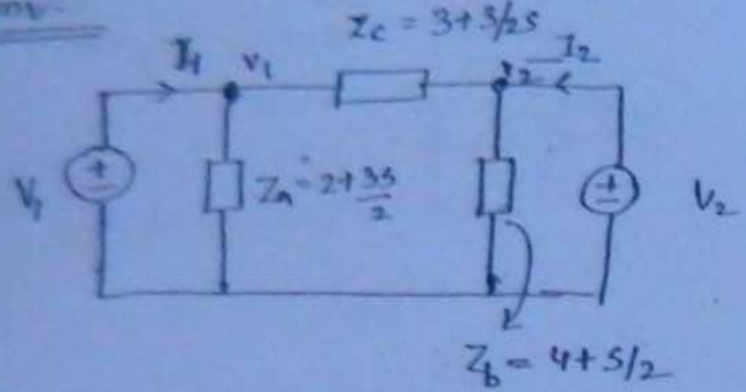
35. $z(n) = \left(\frac{1}{2}\right)^n u(n)$

$h(n) = ?$

Sol $\frac{d}{dn}z(n) = h(n) = n\left(\frac{1}{2}\right)^{n-1}u(n)$

Conv.

210



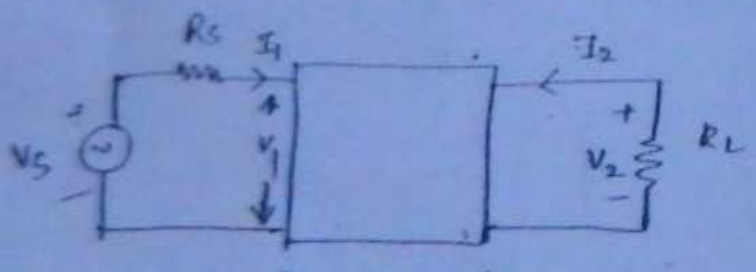
$$I_1 = \frac{V_1}{Z_a} + \frac{V_1 - V_2}{Z_c}$$

$$I_1 = V_1\left[\frac{1}{Z_a} + \frac{1}{Z_c}\right] - \frac{V_2}{Z_c}$$

$$I_1 = V_1 Y_{11} + Y_{12} V_2.$$

$$Y_{11} = \frac{1}{Z_a} + \frac{1}{Z_c}, \quad Y_{12} = \frac{\overline{1}}{Z_c}.$$

Ily apply the same proc. at node 2 V_2. & find Y_{22}, Y_{21}



$$V_s = V_1 + I_1 R_s$$

$~~V_0 = \partial V_s \partial I_1 R_s~~$.

$$V_2 = -I_2 R_L$$

$$V_1 = V_s - I_1 R_s$$

$$V_2(s) = -I_2(s) \cdot 1$$

$$V_1(s) = \frac{1}{s} - 2 I_1(s)$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} & 2 \\ & \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\begin{bmatrix} 1/s - 2 I_1(s) \\ \ldots \end{bmatrix} = \begin{bmatrix} z \end{bmatrix} \begin{bmatrix} I_1(s) \\ \ldots \end{bmatrix}$$

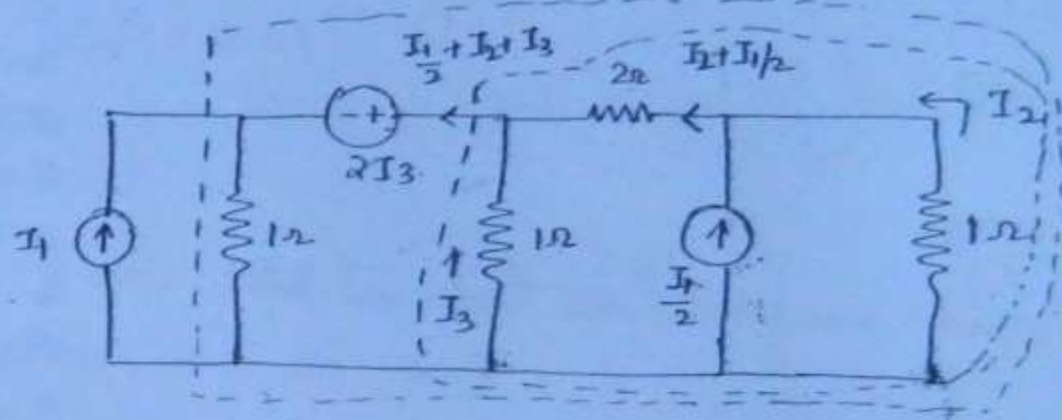
Solve the above matrix & find $I_2(s)$.

(211)

$V_2(s) = -I_2(s)$

$V_2(t) = L^{-1}[V_2(s)] = [0.037 + 0.0456\,e^{-1.9t}$

$- 0.083\,e^{-7.08t}]$.



$$1 \times I_2 + 2\left(I_2 + \frac{I_1}{2}\right) + 2I_3 + 1\left(\frac{3I_1}{2} + I_2 + I_3\right) = 0 \rightarrow (1)$$

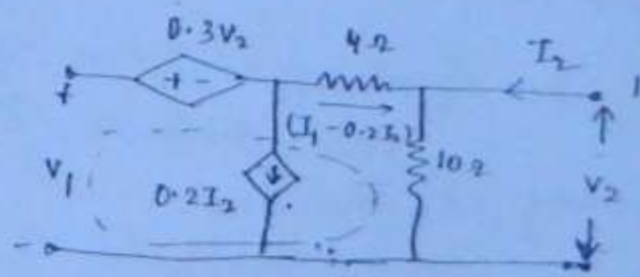
$$(1 \times I_2) + 2\left(I_2 + \frac{I_1}{2}\right) - (I_3 \times 1) = 0 \rightarrow (2)$$

from eq (2)

$$I_3 = I_2 \times 1 + 2\left(I_2 + \frac{I_1}{2}\right) \rightarrow (3)$$

Sub eq (3) in (1) $\qquad$ Ans $= -11/26$

4.



$$V_1 = 0.3V_2 + 4(I_1 - 0.2I_2) + V_2 \rightarrow (1)$$

$$V_2 = 10(I_1 + 0.8I_2) \rightarrow (2)$$

$V_1 = z_{11}I_1 + z_{12}I_2$ $\qquad$ sub (2) in (1)

$V_1 = 17 I_1 + 9.6\,I_2$

$$H_2(z) = \frac{z^{-2}}{H_1(z)} = \frac{z^{-2}(z-0.8)}{z-0.5}$$

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$$= \frac{z^{-1} - 0.8 z^{-2}}{z - 0.5} \times \frac{z^{-1}}{z^{-1}}$$

$$= \frac{z^{-2} - 0.8 z^{-3}}{1 - 0.5 z^{-1}}$$

10. $$X(z) = \frac{z + z^{-3}}{z + z^{-1}}$$

$$z + z^{-1} \,\big)\, z + z^{-3} \,\big(\, 1 - z^{-2} + 2z^{-4} - 2z^{-6} + 2z^{-8} - 2z^{-10} + \ldots$$

$$\underline{-z \pm z^{-1}}$$

$$-z^{-1} + z^{-3}$$

$$-z^{-1} \mp z^{-3}$$

$$2z^{-3}$$

$$2z^{-4} \pm 2z^{-7}$$

$$-2z^{-7}$$

$$X(z) = 1 - z^{-2} + 2z^{-4} - 2z^{-6} + 2z^{-8} - 2z^{-10} + \ldots$$

~~$x(n) =$~~ $\delta(n) - x(n+2) + 2\,x(4)$

$$x(n) = \{ \underset{\uparrow}{1}\ \ 0\ \ -1\ \ 0\ \ 2\ \ 0\ \ -2\ \ 0\ \ 2\ \ 0\ \ -2\ \ldots \}$$

alternate zeros in $x(n)$

Ans (a)

11. $$\lim_{z \to \infty} X(z) = \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2} = 0 \quad \therefore \text{ It's causal}$$

$$= \frac{\cancel{(1-z^{-1})}\, z^{-1} (1-z^{-4})}{4(1-z^{-1})^2}$$

$$= \frac{z^{-1} (1-z^{-2})}{4(1-z^{-1})} \times \frac{(1+z^{-2})}{\cancel{1}}$$

$$= \frac{z^{-1} (1+z^{-2})(1+z^{-1})\cancel{(1-z^{-1})}}{4\cancel{(1-z^{-1})}}$$

pole ~~at~~ is inside unit circle.

$\therefore$ FVT is applicable.

$$c(\infty) = \lim_{z \to 1} (1-z^{-1})\, C(z)$$

$$= \lim_{z \to 1} \frac{2 \times 2}{4} = 1. \quad (C)$$

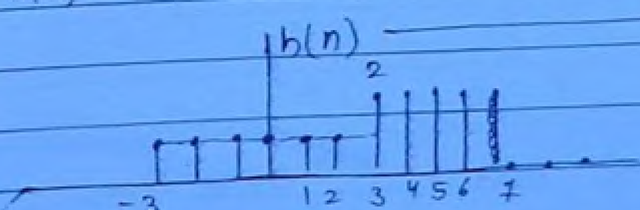
13 $h(n) = -5^n u(-n-1)$

$$H(z) = \frac{1}{1-5z^{-1}} = \frac{z}{z-5}$$

ROC $|z| < 5$ stable

(b)

15 $h(n) = u(n+3) + u(n-3) - 2u(n-7)$



$\rightarrow$ Energy s/g

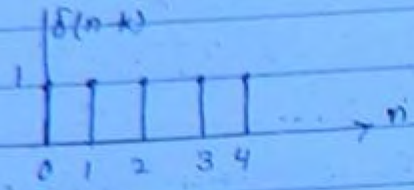
= 8q and add

= 22

not causal (d)

17 $x(n) = \sum_{k=0}^{\infty} \delta(n-k)$ (2.14)

$= \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \dots$



step signal $\frac{z}{z-1}$ (c)

18 $y(n) - 2y(n-1) + y(n-2) = x(n) - x(n-1)$

$\cancel{Y(z) - 2Y(z)z^{-1}}$ $y(n) = u(n)$

(b) $= 1 = y(z)$

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$x_1(n)$ →	1	-2	1
$x_2(n)$			
→ 1	1	-2	1
1	1	-2	1
1	1	-2	1
1	1	-2	1
1	1	-2	1

$x(n) = \{1 \ -1 \ 0 \ 0 \ 0 \ 0 \ -1 \ 1\}$

origin will be any of the 1st three terms.

(a)

25

	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3

~~0~~ 0 1

27. $x(t) * u(t) = \int_{-\infty}^{t} x(k)\,dk$ 215

$x(t) * u(t-t_0) = \int_{-\infty}^{t-t_0} x(k)\,dk$

$x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$ (C)

34. $h_1(n) = \left(\frac{1}{2}\right)^n u(n)$ $H_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$

$h_2(n) = \left(\frac{1}{3}\right)^n u(n)$ $H_2(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$= \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$= \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$\frac{H(z)}{z} = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$\frac{H(z)}{z} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

$$= \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}}$$

$$h(n) = \overset{3}{\cancel{A}}\left(\frac{1}{2}\right)^n u(n) + \overset{-2}{\cancel{B}}\left(\frac{1}{3}\right)^n u(n)$$

42 $x(t) = \sum_{k=100}^{\infty} \cos(k\pi) e^{jk\frac{2\pi}{50}t}$ (216)

$C_k = R + E$

$R + E$

48 $h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n-1]$ → energy signals ∴ stable

$|z| > \left|\frac{1}{2}\right|$ $|z| > \left|\frac{1}{2}\right|$

⇓

$z > \frac{1}{2}$. stable.

49 $H(z) = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$

55 $H(z) = \frac{10}{(z-1)(z-2)}$

$x(n) = u(n) \rightleftharpoons X(z) = \frac{z}{z-1}$

$Y(z) = X(z) \cdot H(z)$

$= \frac{10z}{(z-1)^2(z+2)}$

$\frac{Y(z)}{z} = \frac{10}{(z-1)^2(z+2)}$

$= \frac{A}{z+2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$

$Y(z) = \frac{A}{1+2z^{-1}} + \frac{B}{1-z^{-1}} + \frac{Cz^{-1}}{(1-z^{-1})^2}$

$y(n) = A(-2)^n u(n) + Bu(n) + C\,n\,u(n)$

CWB chapter 5 LT

9. $x(t) = u(t) \qquad y(t) = t^2 e^{-2t} u(t) \qquad H(s) = ?$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{(s+2)^2} \Big/ \frac{1}{s} = \frac{2s}{(s+2)^3}$$ 2/7

6. $x_1(t) = e^{k_1 t} u(t) \qquad x_2(t) = e^{-k_2 t} u(t)$

$y(t) = x_1(t) * x_2(t)$

$$Y(s) = X_1(s)\, X_2(s) = \frac{1}{s-k_1} \cdot \frac{1}{s+k_2} = \frac{\frac{1}{k_1+k_2}}{s-k_1} - \frac{\left(\frac{1}{k_1+k_2}\right)}{s+k_2}$$

$$y(t) = \frac{1}{k_1+k_2}\left[e^{k_1 t} u(t) - e^{-k_2 t} u(t)\right]$$

10. $f(t) \rightleftharpoons F(s) = \frac{s+2}{s+1}$ causal $\qquad g(t) \rightleftharpoons G(s) = \frac{s^2+1}{(s+3)(s+2)}$ causal.

$$h(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau. \quad L[h(t)] \rightleftharpoons H(s) = ?$$

$$= \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau. = f(t) * g(t)$$

$$H(s) = F(s) \cdot G(s) = \frac{\cancel{s+2}}{\cancel{s+1}} \cdot \frac{\cancel{s^2+1}}{(s+3)\cancel{(s+2)}} = \frac{1}{s+3}$$

17. $x(t) = e^{-2t} u(t) + e^{-t} \cos 3t\, u(t)$

$$= \frac{1}{s+2} + \frac{s+1}{(s+1)^2 + 9}.$$

$$= \frac{\quad}{(s+2)(s^2+2s+10)}$$

21. $y(t) = x(t) * u(t) \qquad Y(s) = $

$$x(t) = e^{-2t} u(t) + \delta(t-6) \Rightarrow \left[\frac{1}{s+2} + \frac{e^{-6s}}{s}\right]\frac{1}{s} = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}$$

$$Y(s) = 0.5\left[\frac{1}{\;} - \frac{1}{\;}\right] + \frac{e^{-6s}}{\;}$$

22. $t^n f(t) \rightleftharpoons (-1)^n \frac{d^n F(s)}{ds^n}$

(218)

$\downarrow n=1$

$t f(t) \rightleftharpoons \frac{-dF(s)}{ds}$

23. $H(s) = G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$

$x(t) = \sin \omega t$

$y(t) = 0 \quad \omega = ?$

$Y(s) = 0 \quad \omega = ?$

$H(s) \cdot X(s) = 0 \quad H(s) = 0 \Rightarrow H(j\omega) = 0 \Rightarrow \omega = 3$

24. $\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$

$\rightarrow y(0^-) = -2 \quad y'(0^-) = 0$

$\rightarrow \left.\frac{dy(t)}{dt}\right|_{t=0^+} = ?$

$[s^2 Y(s) - s y(0^-) - y'(0^-)] + 2[sY(s) - y(0^-)] + Y(s) = 1$

$Y(s) = \frac{-3-2s}{s^2+2s+1} \Rightarrow \frac{-3-2s}{(s+1)^2}$

$\Rightarrow \frac{-2(s+1)-1}{(s+1)^2} \Rightarrow \frac{-2}{s+1} - \frac{1}{(s+1)^2}$

$y(t) \rightleftharpoons -2e^{-t}u(t) - te^{-t}u(t)$

$= -2e^{-t} - te^{-t}$

$\frac{dy(t)}{dt} = \underbrace{2e^{-t}}_{1} - [\underbrace{e^{-t}}_{1} - \underbrace{te^{-t}}_{0}] = 2 - [1-0] = 1$

26. $G(s) = \frac{F_2(s)\,F_1^*(s)}{|F_1(s)|^2} = \frac{F_2(s)\,F_1^*(s)}{F_1(s)\,F_1^*(s)} = \frac{F_2(s)}{F_1(s)} = e^{s\tau}$

$g(t) = \delta(t-\tau)$

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29. $y(t) = (1 - 3e^{-t} + 3e^{-2t})\,u(t)$

$Y(s) = 0 \qquad \omega = ?$

$\Rightarrow \frac{1}{s} - \frac{3}{s+1} + \frac{3}{s+2} = 0$

$\Rightarrow s^2 + 2 = 0 \Rightarrow (j\omega)^2 + 2 = 0 \Rightarrow -\omega^2 + 2 = 0 \Rightarrow \omega = \sqrt{2}$

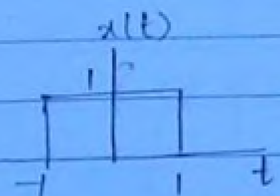
chapter 4. F.T

8. $y(n) - \frac{1}{2}y(n-1) = x(n) \qquad x(n) = k\delta(n)$

$= k\delta(n)$

$Y(z) = \frac{k}{1 - \frac{1}{2}z^{-1}} \Rightarrow y(n) = k\left(\frac{1}{2}\right)^n u(n)$

20.



$\rightleftharpoons X(\omega) = 2Sa(\omega)$

$\Rightarrow X(\omega) = 0 \qquad \omega = ?$

$\Rightarrow 2Sa(\omega) = 0$

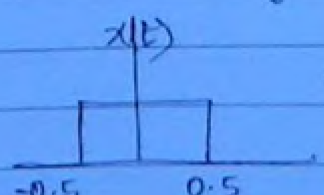
$\Rightarrow Sa(\omega) = 0 \qquad \omega = n\pi \qquad n \neq 0$

25. $x(t) = u(t+0.5) - u(t-0.5)$

$h(t) = e^{j\omega_0 t}$

$y(t) = x(t) * h(t)$

$\omega_0 = ? \qquad y(t) = 0$



$\rightleftharpoons X(\omega) = Sa\left(\frac{\omega}{2}\right)$

$A_0 \rightleftharpoons 2\pi A_0\,\delta(\omega)$

$\downarrow A_0 = 1$

$1 \rightleftharpoons 2\pi\,\delta(\omega)$

$e^{j\omega_0 t} \rightleftharpoons 2\pi\,\delta(\omega - \omega_0)$

$y(t) = 0 \quad \omega = ?$

$Y(\omega) = 0$

$H(\omega).\ X(\omega) = 0$

$\Rightarrow 2\pi\delta(\omega - \omega_0)\, Sa\left(\frac{\omega}{2}\right) = 0$

$\Rightarrow Sa\left(\frac{\omega_0}{2}\right) \delta(\omega - \omega_0) = 0$

$\Rightarrow Sa\left(\frac{\omega_0}{2}\right) = 0 \qquad \Rightarrow \frac{\omega_0}{2} = n\pi \quad n \neq 0$

$\omega_0 = 2n\pi \quad n \neq 0$

28 $X(s) = ?$

$\downarrow s = j\omega$

$X(\omega) = ?$

chapter 3.

9. $x_2(t) = e^{(-2+j)t} = \underbrace{e^{-2t}}_{NP} . \underbrace{e^{jt}}_{P} \qquad x_1(t) = P.$

$\downarrow$

NP

chapter 2

1. $H(z) = \frac{1}{z^2 - 5z + 6} = \frac{1}{(z-3)(z-2)}$

poles $= 2, 3$

4. $y(t) + \int_{0^-}^{\infty} y(\tau)\, x(t-\tau)\, d\tau = \delta(t) + x(t)$

LHS $y(t) + y(t) * x(t) = \delta(t) + x(t)$

$\delta(t) + \delta(t) * x(t) = \delta(t) + x(t)$

17. i) $y(t) = t.\,x(t)$ linear
ii) $y(t) = t\,x^2(t)$ NL
iii) $y(t) = x(2t)$ L.

linearity is unaffected by co-efficients & time scaling.

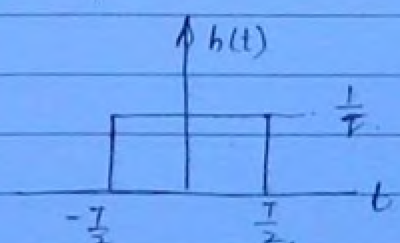
18. LTI $y(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau)\,d\tau$

$\downarrow y(t) = h(t)$ $\quad \downarrow x(t) = \delta(t)$

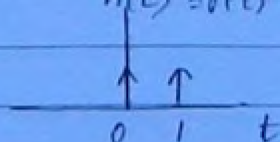
$$h(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(\tau)\,d\tau$$

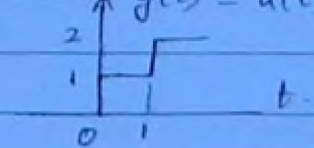
$$= \frac{1}{T}\left[u(\tau)\right]_{t-\frac{T}{2}}^{t+\frac{T}{2}}$$

$$h(t) = \frac{1}{T}\left[u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)\right]$$



19. $h(t) = \delta(t) + \delta(t-1)$ $\qquad y(t) = u(t) + u(t-1)$

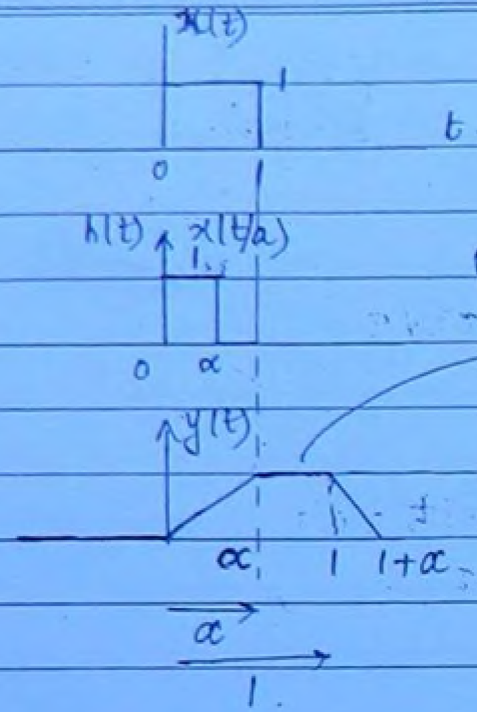




$x(t) = ?$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{\frac{1}{s}\left[1+e^{-s}\right]}{1+e^{-s}} = \frac{1}{s} \Rightarrow x(t) = u(t)$$

20.



$0 < \alpha \le 1$

$\alpha \times |x| = \alpha$.

chapter 1.

14.

$a^n u(n) \qquad b^n u(n)$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$$

16.

$$x(n) = \begin{cases} 0 & n < -2 \quad n > 4 \\ 1 & \text{otherwise.} \end{cases}$$

$n = -n-2$

$$x(-n-2) = \begin{cases} 0 & -n-\cancel{2} < -\cancel{2} \qquad -n-2 > 4 \end{cases}$$

$n > 0 \qquad n < -6$

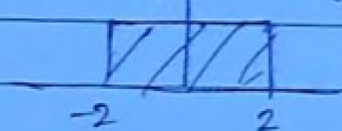
27. $x(t) = \delta(t+2) - \delta(t-2)$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$= \int_{-\infty}^{t} [\delta(\tau+2) - \delta(\tau-2)]\, d\tau$$

$$= u(t+2) - u(t-2)$$

$|y(t)| = |y(t)|^2$



$$E_{y(t)} = \int_{-\infty}^{\infty} |y(t)|^2\, dt$$

$= \text{Area of } |y(t)|^2$

$= 4.$