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ESE 2018 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

**Test 6 : Renewable Sources of Energy + Industrial and
Maintenance Engineering, Production Engineering**

and Material Science - 1, Strength of Material and Mechanics - 2

Name: ATK

Roll No: M E 1 8 P U M T F 0 0 1

Test Centres

Delhi Bhopal Noida Jaipur Indore
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 Hyderabad

Student's Signature

Instructions for Candidates

- Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
- Answer must be written in English only.
- Use only black/blue pen.
- The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
- Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
- Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

• Always Be Specific in Theory questions.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	41
Q.2	40
Q.3	42
Q.4	
Section-B	
Q.5	28
Q.6	
Q.7	57
Q.8	
Total Marks Obtained	208

Signature of Evaluator

Cross Checked by

Section A : Renewable Source of Energy, Industrial and Maintenance Engineering

- Q.1 (a) Find out solar altitude angle on a horizontal surface at 2 hours, after local noon on June 16 for a city located at 40°N (latitude). Also, find out the solar time (or local apparent time) at sunrise and at sunset and day length. Use following relation:

$$\cos\theta_i = \cos\phi \cos\delta \cos\omega + \sin\delta \sin\phi$$

where, θ_i - angle of incidence; ϕ - latitude angle; δ - declination angle

ω - hour angle; α - solar altitude angle

[12 marks]

Date given - 16 June

$$\therefore n = 31 + 28 + 31 + 30 + 31 + 16$$

$$= 167$$

$$\therefore S = 23.45 \sin\left(\frac{360 \times (284+n)}{365}\right)$$

(11) $\phi = 40^{\circ}$

$$\therefore S = 23.354$$

$$\omega = 2 \times 15 = 30^{\circ}$$

we know

$$\cos\theta_2 = \sin\delta \sin\phi + \cos\delta \cos\phi \cos\omega$$

$$\cos\theta_2 = \sin(23.354^{\circ}) \sin(40^{\circ}) + \cos(23.354^{\circ}) \cos(40^{\circ}) \cos(30^{\circ})$$

$$= 0.863872$$

$$\therefore \theta_2 = 30.2488^{\circ}$$

we know that

$$\alpha + \theta_2 = 90^{\circ} \quad \therefore \alpha = 59.754^{\circ}$$

This is the solar altitude angle.

$$\alpha = 59.754^{\circ} = 59^{\circ} 45'$$

to find the sunrise time & sunset time

$$w_s = \cos^{-1}(\tan \delta \tan \vartheta)$$

$$\therefore w_s = \cos^{-1}(-\tan 40^\circ \tan 28.354^\circ)$$

$$\boxed{\therefore w_s = 111.2422^\circ}$$

Thus for this w_s

$$\begin{aligned}\text{local sunrise time} &= 12:00 - \frac{111.2422}{15} \\ &= 12:00 - 7 \text{ hr } 24 \text{ min } \\ &= \underline{\underline{04:36}}\end{aligned}$$

$$\begin{aligned}\text{local sunset time} &= 12:00 + 7 \text{ hr } 24 \text{ min } \\ &= \underline{\underline{19:24}}\end{aligned}$$

$$\begin{aligned}\text{Day length} &\approx \frac{2w_s}{15} = \frac{2 \times 111.2422}{15} \\ &= \underline{\underline{14 \text{ hours } 49 \text{ min } 56 \text{ sec}}}\end{aligned}$$

- Q.1 (b) A biomass gasifier is used to run a CI engine in a dual fuel model with 80% diesel replacement. The gasifier engine system produces 200 kW of power at 800 rpm. Calculate the biomass feeding rate of the gasifier if the efficiency of the engine is 35% and the calorific value of the producer gas is 17000 kJ/kg, assuming the efficiency of gasifier to be 75%.

[12 marks]

$$\text{power produced} = \underline{\underline{200 \text{ kW}}}$$

Thus heat required to supplied.

$$Q_s = \frac{200}{\eta} = \frac{200}{0.35} = \underline{\underline{571.42857 \text{ kW}}}$$

now total producer gas require

$$m_f = \frac{Q_s}{\text{C.V.}} = \frac{571.42857}{17000}$$

$$\therefore m_f = \underline{\underline{0.03361344 \text{ kg/s}}}$$

(11) out of total 80% diesel is replaced

by biomass gasifier

- The biomass burnt

$$= m_f \times 0.8 = \underline{\underline{0.02689 \text{ kg/s}}}$$

Now Biomass feeding rate

$$= \frac{\text{biomass burnt}}{\eta_{\text{gasifier}}} = \frac{0.02689}{0.75}$$

$$\underline{\underline{m_b = 0.035854 \text{ kg/s}}}$$

$$\text{also } m_b = 2.689 \times 10^{-3} \text{ kg/cycle}$$

Q.1 (c) What is ABC analysis? Explain with the help of Pareto chart.

[12 marks]

In the any inventory control system there are various kind of products or parts which have various prices.

There are some products

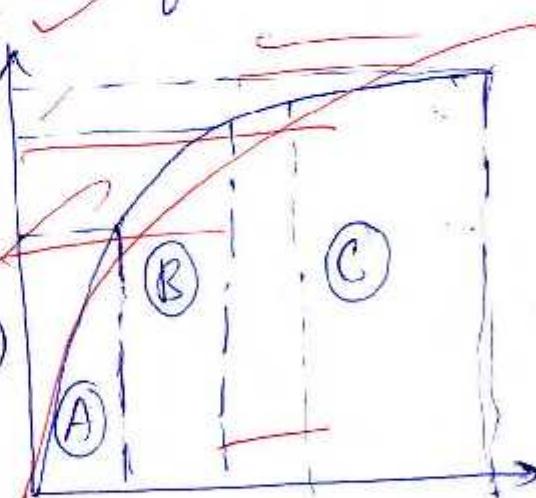
A - Very high price (~~70%~~ of total price)

B - moderate price (~~20%~~ of total price)

C - very low price (~~10%~~ of total price)

Due to very high cost

→ A type of products in stock are very less in stock (~~lot of~~ even then they cover 60% of the cost of total).



which can be easily explained by quantity →

by pareto chart given.

→ Product C are having very low price thus even for large stocking their total cost is ~~10%~~ of total so they are stored in large numbers.

→ In case of product B they are having moderate stock and moderate cost (~~20%~~ of total.)

Q.1 (d) Name the different techniques used for condition monitoring and list its main objective.

[12 marks]

Condition monitoring

- Monitoring the condition of the machine using the information of the parameters which influence the condition of it is called as condition based monitoring.
- It is based on the predictive maintenance a type of preventive maintenance.

Techniques used

- 1) wear Debris analysis
- 2) noise
- 3) ?
- 4) ?

⑥

Main objectives

- # 1) To reduce or eliminate the breakdown of machine and cost related to it
- 2) To reduce unwanted cost of maintenance due to a regular check up.
- 3) To reduce the cost of the maintenance using advance and easy to operate techniques
- 4) Eliminate break down of machine

- Q.1 (e) Differentiate between zero wear and measurable wear. Name all the types of possible common wear mechanisms.

[12 marks]

- When the wear of the machine part is measurable with instruments ie. can be detected by the equipment
~~then using some techniques and sensors~~
then it is called measurable wear
- zero wears can not be detected by the instruments so called zero wear.

Types of wear mechanisms

- ✓ 1) Fitting wear o, Adhesion wear.
- ✓ 2) Corrosive wear (b)
- ✓ 3) erosive wear
- ✓ 4) scuffing wear
- ✓ 5) Abrasive wear
- ✓ 6) Electrolytic wear
- ✓ 7) Diffusive wear
- ✓ 8) Sliding wear.

Q.2 (a) Derive the relationship between the maximum power developed by the turbine in comparison to the total available power in wind?

If the inlet wind velocity (upstream) is 18 m/s and drop to 60% of wind velocity over blade and drop to 33% at downstream wind velocity, then determine the ratio maximum thrust on turbine rotor at maximum power condition to at maximum torque condition. Atmospheric condition are $T_1 = 27^\circ\text{C}$ and $P_1 = 1.012 \text{ bar}$.

[20 marks]

Assumption

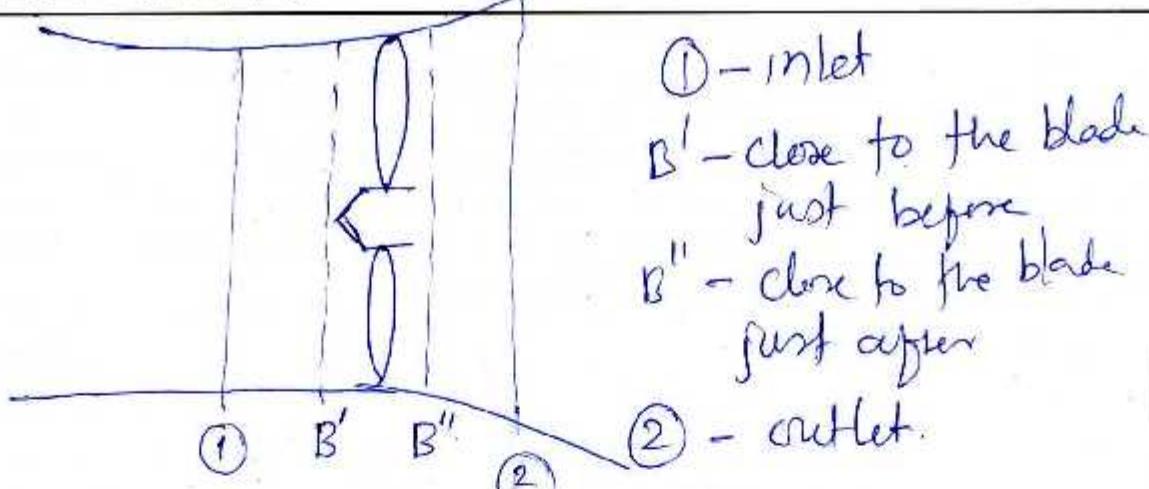
- 1) wind velocity is constant v_∞
- 2) There is ~~no friction loss over the~~ ~~turbine blade or other losses due~~ ~~to friction neglected.~~
- 3) flow of velocity is normal to the plane of rotation. & ~~s is constant~~

when air flows through the area A with ~~v₀ velocity~~ the energy passes by it

$$P = \frac{1}{2} m v_0^2 W$$

$$P = \frac{1}{2} \rho A v_0^3 \quad \text{--- (1)}$$

now considering the wind turbine of some radius such that flow area B A as shown in figure



① - inlet

B' - close to the blade
just before

B'' - close to the blade
just after

② - outlet.

now applying the bernoulli's law

① & B' & B'' & ②

$$P_1 + \frac{1}{2} \rho V_1^2 = P_{B'} + \frac{1}{2} \rho V_{B'}^2 \quad \text{--- } ②$$

$$P_{B''} + \frac{1}{2} \rho V_{B''}^2 = P_2 + \frac{1}{2} \rho V_2^2 \quad \text{--- } ③$$

by ~~subtracting~~
adding ③ & ②

$$P_{B''} + P_1 + \frac{1}{2} \rho V_1^2 + \frac{1}{2} \rho V_{B''}^2 = P_2 + P_{B'} + \frac{1}{2} \rho V_{B'}^2 + \frac{1}{2} \rho V_2^2$$

as A remains same for B' & B''

$$V_{B'} = V_{B''} \text{ also } P_1 = P_2 = P_{atm.}$$

$$\therefore P_{B''} + \frac{1}{2} \rho V_1^2 = P_{B'} + \frac{1}{2} \rho V_2^2$$

$$\therefore P_{B'} - P_{B''} = \frac{1}{2} \rho (V_1^2 - V_2^2)$$

from mass flow rate $m = \rho A \times V_B$

$$F_{drag} = A(P_{B'} - P_{B''}) = \rho A \left(\frac{V_1^2 - V_2^2}{2} \right) \quad ④$$

$$\text{also } F_{drag} = \rho A \left(\frac{V_1 - V_2}{2} \right)^2 \quad ⑤ \text{ (by change of momentum eqn)}$$

$$\text{Thus } V_B = \frac{V_1 + V_2}{2} \quad \text{--- (using ④ & ⑤)}$$

Finally

$$\text{Pactual} = \frac{1}{2} \rho A v_B (v_1^2 - v_2^2) = \frac{1}{2} \rho A \frac{(v_1 + v_2)}{2} \times \frac{(v_1^2 - v_2^2)}{2}$$

$$\frac{dP}{dv_2} = \frac{1}{4} \rho A \left[(v_1^2 - v_2^2) \times \frac{1}{2} - 2v_2 \times (v_1 + v_2) \right]$$

for P_{\max}

$$\frac{dP}{dv_2} = 0$$

$$v_1^2 - v_2^2 = 2v_2(v_1 + v_2) \Rightarrow v_2 = \frac{v_1}{3}$$

(10)

$$P_{\max} = \frac{1}{4} \rho A \left(1 + \frac{1}{3}\right) \times \left(1 - \frac{1}{3}\right) v_1^3 = \frac{8}{27} \rho A v_1^3$$

But $v_1 = v_\infty$

$$\therefore P_{\max} = \frac{8}{27} \rho A v_\infty^3 = \frac{16}{27} \text{ Parabrate}$$

We have

$$v_\infty = 18 \text{ m/s}$$

Numerical ?

Q.2(b) What is a fuel cell and what are its main advantages? Classify fuel cells based on

1. Type of electrolyte
2. Types of the fuel and oxidant
3. Operating temperature
4. Application
5. Chemical nature of electrolyte

[20 marks]

Fuel cell - It is the device which uses fuels to combine in presence of electrolyte to convert its chemical energy into heat energy and electrical energy.

Advantages

- 1) No moving part so no vibration, low maintenance
- 2) very high efficiency
- 3) By product water (generally) is useful in space
- 4) Can produce heat as well as electricity which can be used in powerplant applications.

Classification

I) Type of electrolyte

(i) Alkaline fuel cell (KOH)

(ii) Phosphoric acid fuel cell (H_3PO_4)

(iii) Molten carbonate fuel cell (H_2CO_3)

(iv) Polymer electrolytic membrane fuel cell

(v) proton exchange fuel cell. (H^+)

(vi) Solid oxide fuel cell (O^{2-})

2) Type of fuel cell -

There are various types of fuel cell which use different fuels

- (i) H₂ and O₂ fuel cell.
- (ii) Carbon and oxygen fuel cell
- (iii) H₂ and air fuel cell
- (iv) methane and O₂ fuel cell etc. *same*
- (v) methanol & O₂ fuel cell

3) Operating temperature

- (i) low temperature fuel cell ($80^{\circ}-100^{\circ}\text{C}$)
- (ex) polymer membrane fuel cell.
- (ii) medium temperature fuel cell ($100^{\circ}-300^{\circ}\text{C}$)
- (ex) Alkaline fuel cell, phosphoric acid fuel cell
- (iii) High temperature fuel cell ($500^{\circ}-1000^{\circ}\text{C}$)
- (ex) molten carbonate fuel cell, solid oxide fuel cell.

4) Application

- (i) Domestic application fuel cell
 - to produce small amount of electricity
 - these are basic fuel cells
- (ii) Commercial fuel cells.
 - They generate moderate amount of electric power generally used in buses & vehicles.

(iii) Industrial fuel cells

They produce large amount of power hence used in power generating industries to run electrical applications.

5) Chemical nature of electrolyte

- 1) +ve ion electrolyte en- H^+
- 2) -ve ion electrolyte en- OH^- , O_2^{2-} , CO_3^{2-}

(14) more?
Z

Q.2 (c) Discuss the relative merits and limitations of tidal power. What are the difficulties in tidal power developments? For a typical tidal power plant shown below, the basin area is $25 \times 10^6 \text{ m}^2$. The tide has a range of 10 m. However, turbine stops working when the head on it falls below 2 m. Assume that density of seawater is 1025 kg/m^3 , acceleration due to gravity is 9.81 m/s^2 , combined efficiency of turbine and generator is 75% and period of energy generation is 6h and 12.5 min.

Determine:

1. Work done in filling or emptying the basin
2. Average power
3. The energy generated in one filling process (in kWh)

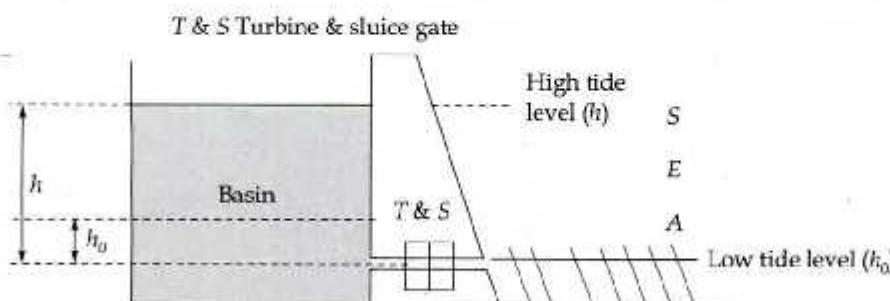


Figure: Single Basin tidal plant

[20 marks]

Merits

- 1) It is renewable form of energy thus there is no harm to the environment.
- 2) At coastal area lands near sea are totally can be utilized.
- 3) With very low investment huge amount of energy can be capture with no pollution.

Demerits

- 1) Direction of tides and the head available is always varying with time so very complex power generation process.
- 2) Due to reduction in tidal height due to disturbance it affect the ecosystem at coastal region.
- 3) It is not continuous and reliable source of energy.

Difficulties in developing tidal power plant

- 1) Tidal height and the velocity is always time dependant and vary continuously so difficulty in designing the turbine generator.
- 2) Seawater contain huge amount of salt thus very high danger of corrosion.
- 3) Due to the high impact of wave erosion of the turbine takes place.

Given

$$A_b = 2.5 \times 10^6 \text{ m}^2$$

$$R_1 = 10 \text{ m}, R_2 = 2 \text{ m}$$

$$\rho = 1025 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

$$\eta_o = 0.75$$

$$\text{Period} = 6 \text{ h } 12.5 \text{ min} = \underline{\underline{22350 \text{ sec}}}$$

(i) work done on ~~filling~~ ^{emptying} the basin

$$W = - \int_{R_1}^{R_2} \rho A_b g h dh = \int_{R_2}^{R_1} \rho A_b g h dh$$

$$W = \rho A_b g \left[\frac{R_1^2 - R_2^2}{2} \right]$$

$$W = 1025 \times 2.5 \times 10^6 \times 9.81 \left[\frac{10^2 - 2^2}{2} \right]$$

$$\boxed{W = 12066.3 \times 10^9 \text{ Joule}}$$

Thus work done on filling or emptying the basin

$$\boxed{W = 120.663 \times 10^{12} \text{ Joule}}$$

(ii) Average power

$$P = \frac{W}{T} = \frac{120.663 \times 10^{12}}{22350}$$

$\therefore P = 0.53988 \times 10^9 \text{ W}$

$$\boxed{\therefore P = 539.88 \text{ MW}} \quad \cancel{\times 0.75} \quad \cancel{539.88 \text{ MW}}$$

Thus average power of plant is 539.88 MW

(iii) After the feeding to turbine

~~per~~ energy generated

$$E = \eta_0 \times P W$$

$$E = \frac{0.75 \times 12066.3 \times 10^9}{3600 \times 10^3}$$

$$\boxed{E = 2513812.5 \text{ kwh}}$$

✓ Energy generated in one filling

process

$$\boxed{1) 251.3812 \times 10^3 \text{ kwh}}$$

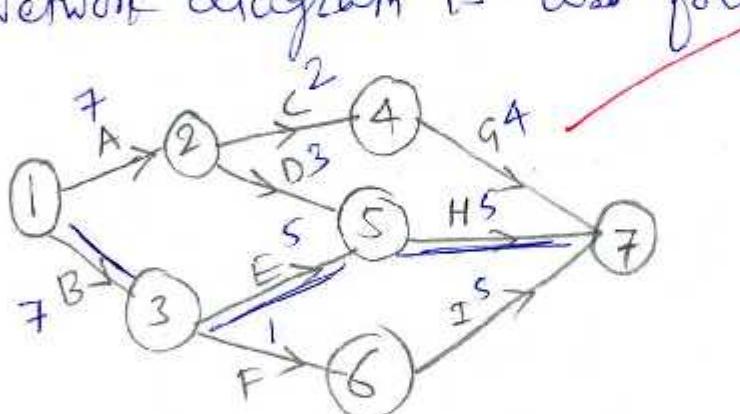
Q.3 (a) The data shown below is for a PERT network.

Predecessor Event	Successor Event	Activity Time		
		t_0	t_m	t_p
1	2	5	6	13
1	3	2	7	12
2	4	1.5	2	2.5
2	5	1	3	5
3	5	4	5	6
3	6	1	1	1
4	7	2	3	10
5	7	4	5	6
6	7	3	5	7

1. Draw the network
2. Each day the project can be shortened is worth ₹ 5000. Should the firm pay ₹12500 to reduce duration of activity 3 - 5 to 2 days?

[20 marks]

(i) Network diagram is as follow:



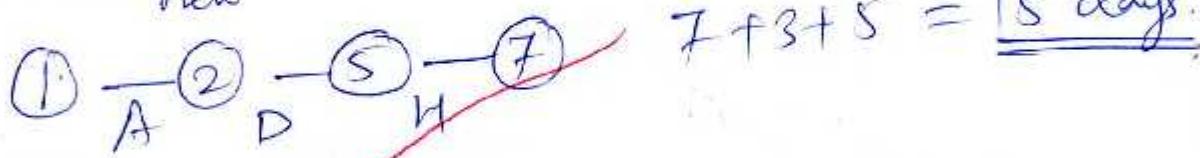
Activity	$\frac{(t_0+t_p+4t_m)}{6}$	$(t_p-t_0)_e$
A	7	8/6
B	7	10/6
C	2	1/6
D	3	4/6
E	5	2/6
F	1	0
G	4	8/6
H	5	2/6
I	5	4/6

Critical path is



$$\text{Duration of critical path} = 7 + 5 + 5 = \underline{\underline{17 \text{ days}}}$$

When firm reduce the duration of the activity 3-5 to 2 days critical path will be ~~not~~ changed and new will become



Thus firm should not reduce the duration ~~to~~ 2 days. ~~McIntosh~~
~~Cost~~
~~Also~~

- Q.3 (b)** For a product, the price discount is as follows:

Order quantity	Unit price (₹)
$0 \leq Q < 550$	12
$550 \leq Q < 700$	11.20
$700 \leq Q$	10.70

Determine the EOQ considering discount and total inventory cost. The monthly demand for the product is 225 units, carrying cost is 2% of unit cost per month and cost of ordering is ₹ 100.

$$D = 225 \times 12 = 2700 \text{ units} \quad [20 \text{ marks}]$$

$$C_o = 100 \text{ ₹}$$

$$C_h = 12 \times 0.02 C_p \text{ ₹/unit/year}$$

~~$$\text{Let } C_p = 12 \text{ ₹} \therefore C_h = 0.88 \text{ ₹/unit/year}$$~~

~~$$\therefore Q^* = \sqrt{\frac{2 \cdot D \cdot C_o}{C_h}} = \sqrt{\frac{2 \cdot 2700 \times 100}{2.88}}$$~~

$$Q^* = 433.01 \approx 433.$$

$$TIC = \sqrt{2 D C_o C_h} = 1247.0765 \text{ ₹}$$

$$T.C. = TIC + 12 \times 2700 = \underline{\underline{33647.0765}} \text{ ₹}$$

Now Assuming

$$C_p = 11.2 \text{ ₹} \quad C_h = 2.688 \text{ ₹/unit/year}$$

$$\therefore Q^* = \sqrt{\frac{2 \times 2700 \times 100}{2.688}} = 448.2$$

but for $C_p = 11.2$, $550 \leq Q \leq 700$

Thus taking $EoQ = 550 \text{ units/order}$

$$TC = 11.2 \times 2700 + \frac{2700}{550} \times 100 + \frac{550}{2} \times 2.688$$

$$\boxed{TC = 31479.209 \text{ ₹}}$$

Now for

$$C_p = 10.7 \text{ ₹}, C_h = 2.568 \text{ ₹/unit/year}$$

$$\therefore Q^* = \sqrt{\frac{2 \times 2700 \times 100}{2.568}} = 458.5633 \text{ units/order}$$

$\cancel{Q^*}$ does not belong to ≥ 700

Thus taking $Q = 700$
no. of orders $\rightarrow D/Q = 3857 \approx 4$

$$TC = 10.7 \times 2700 + 4 \times 100 + \frac{700}{2} \times 2.568$$

$$\boxed{TC = 30188.8 \text{ ₹}}$$

Thus EoQ considering discount is 700 unit/order

and total cost for inventory \rightarrow

$$\boxed{\cancel{TC = 30188.8 \text{ ₹}}}$$

- Q.3 (c) (i) Define production planning and control and identify its objectives.

[10 marks]

~~Production planning and control~~

~~It is the~~

- Q.3 (c) (ii) Distinguish between CPM and PERT in tabular form. Explain with the help of practical examples.

[10 marks]

PERT

- 1) It is project evaluation Review technique
- 2) uses three time estimation system
- 3) It is event oriented
- 4) It is used for performing new projects
- 5) crashing analysis is not performed
- 6) Based on the probability approach and activities have beta distribution

ex

New projects like R&D and huge project having no previous data. Thus there is no absolute idea about the completion of project time

CPM

- 1) It is critical path method.
- 2) As activity period is known so used only single time estimation
- 3) It is activity oriented.
- 4) It is used for well known project
- 5) crashing is performed to reduce the cost and time of project
- 6) Based on Deterministic model and ~~probabilistic~~ project time is well known

Ex

Regular Construction projects which are well known and huge data is present to evaluate. Thus there is well known structure of the cost and period of each activity.

Section B : Production Engineering & Material Science - 1, SOM & Mechanics - 2

Q.5 (a) Compare sand, die, investment, lost foam and continuous casting techniques.

[12 marks]

Sand Casting

- It is the casting technique used for the casting of the huge products like machine beds. (6)
- Due to lack of the availability of the complex patterns, simple casting can be performed.
- ~~low surface finish & Dimensional Accuracy~~

Die Casting

- It is the casting process used to cast the small and low melting point products.
- They have well established dies so complex shaped casting can be performed with very high dimensional accuracy and surface finish.
- They have directional solidification due to die material and cooling provided.
- ex - Casting Al, Zn, Cu, IC engine Cam and Al piston.

Investment Casting (Wax used as pattern)

- Res Casting of the high temperature melting point metals like tungsten
- They have very high surface finish due to ceramic coated surface.
- There is dimensional accuracy and can be suitable for complex parts also.
- ex: gold ornaments, turbine blades, body implants etc.
- There is limit of size (20 kg) - !!

Lost foam casting

- Disposable material like foam, is used as pattern which gets evaporated when mould is poured.
- Complex shape and internal parts with small inserts can be easily produced.
- Once used pattern is not used again.

Continuous Casting

- This type is used to produce the continuous casting of long length having same cross section area.
- Mass production of Casting blooms and ingot is performed.

Q.5(b) What are 'hybrid composites'? State the principal applications of hybrid composites.

[12 marks]

Hybrid Composite

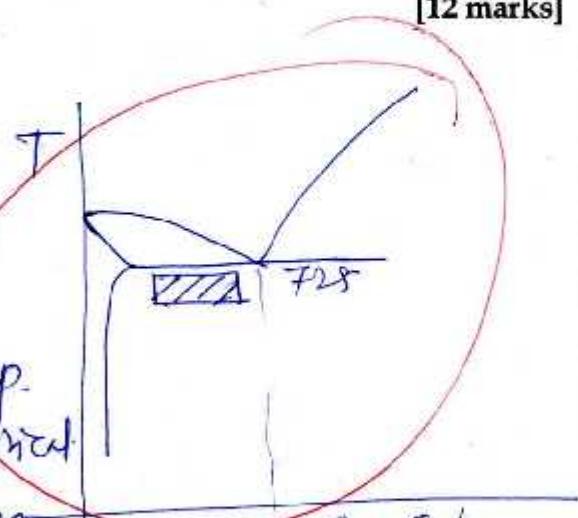
Q.5 (c) Explain the following:

1. Process annealing
2. Normalising
3. Spheroidizing

[12 marks]

1) Process annealing

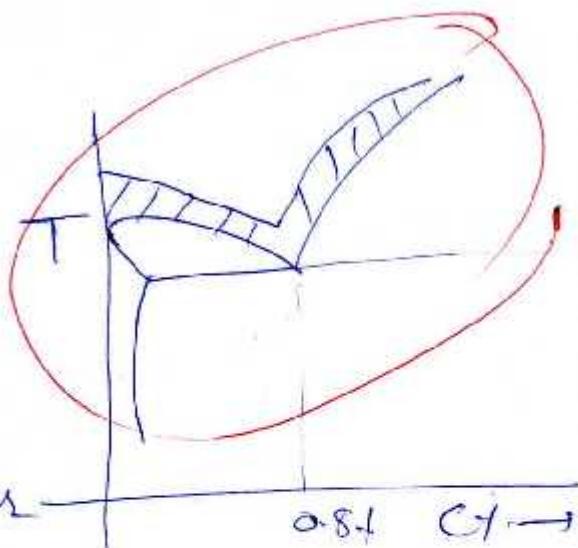
When the iron with low carbon percentage is heated to the temp. lower than the lower critical temp and higher than Rx temperature ^{and cooled} then in furnace then



Process annealing is performed to improve the grain structure and improve the ductility of low carbon steel.

2) Normalising

When low or medium carbon containing iron is heated above the upper critical temperature



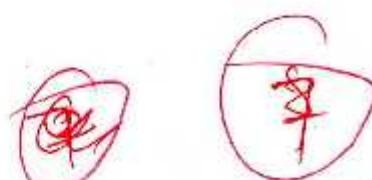
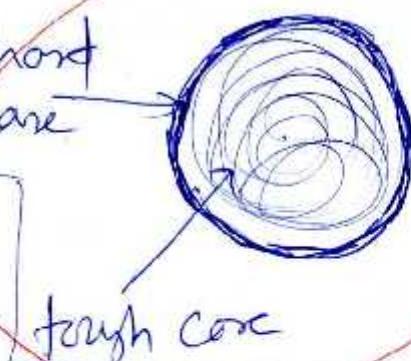
and cooled in the atmospheric air then outer body is cooled at very high rate & more than critical cooling rate) and case (on outer party becomes hard and inner part remains tough.)

These parts are suitable for the direct engineering applications as core is

hard so no wear and

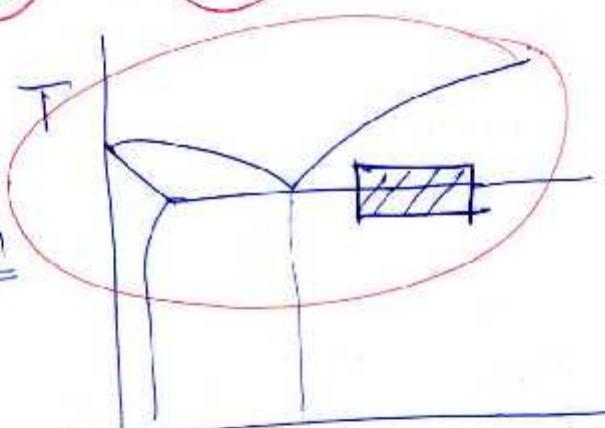
Due to tough core they can absorb shocks.

Thus normalizing is also called as final heat treatment.



(3) Spheroidizing

When iron having high content of carbon is heated to the temp very close to



$0.8 + C_f$

lower critical temperature and cooled very slowly non carbons combine together to form spheroids and thus medium and high carbon steels becomes ductile.

This heating process is used to improve the ductility of the high & medium carbon steel.

Q.5(d) What are the laws of Coulomb's friction?

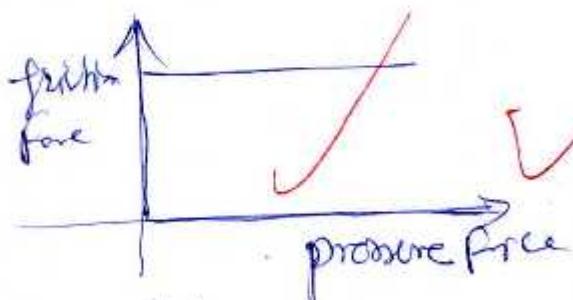
(Be Specific)

[12 marks]

When there is friction between the solid parts at high temperature then there will be sticking of metals occurs due to welding action. or there will be sliding friction due to presence of normal pressure force.

In sticking zone

Friction is due to welding of metal parts thus to over come this kind of friction Constant shear stress (τ) is required.



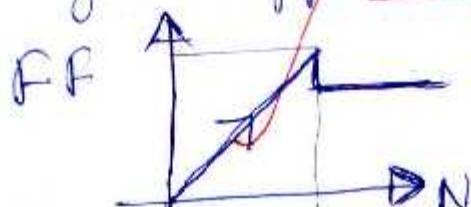
$$\boxed{\text{Friction force} = k \times A_s}$$

In sliding zone \rightarrow

$$\boxed{F_f = \mu_s \times N}$$

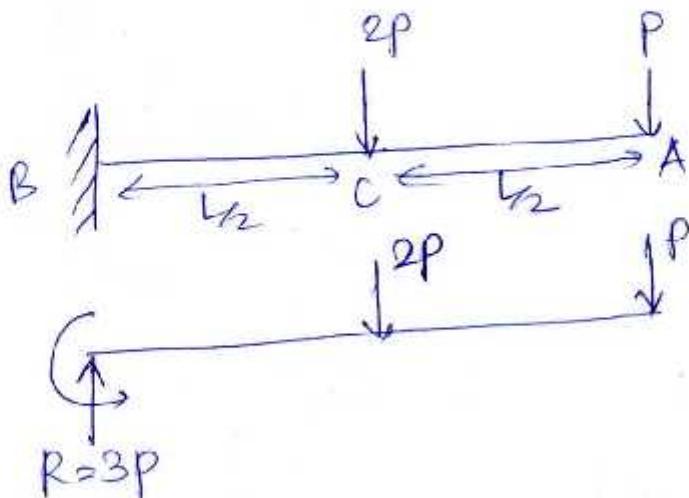
The friction is due to rough surfaces of mating solids. Then friction is due to

• the coef of friction proportional to normal force applied. Once there is relative motion then μ_s becomes μ_k and that friction force becomes constant. (generally $\mu_s > \mu_k$)



- Q.5 (e)** A cantilever beam of length L , is subjected to two concentrated loads of $2P$ and P at its mid length and free end respectively. If the deflection at its free end is limited to $\frac{L}{500}$. What should be the value of P ? Take flexural rigidity of the beam as EI .

[12 marks]



~~we have for cantilever beam with concentrated load.~~

$$\text{Slope} = \frac{WL^3}{3EI} \quad \text{and slope} = \frac{WL^2}{2EI}$$

(1)

~~now considering load P~~

$$S_{\text{at } A} = \frac{PL^3}{3EI} \quad \text{(1)}$$

~~now considering load $2P$~~

$$S_{\text{at } A} = S_{\text{at } C} + \text{slope at } C \times \frac{L}{2}$$

$$= \frac{(2P)(\frac{L}{2})^3}{3EI} + \frac{(2P)(\frac{L}{2})^2 \times \frac{L}{2}}{2EI}$$

$$= \frac{PL^3}{EI} \left(\frac{1}{12} + \frac{1}{8} \right) = \frac{5}{24} \frac{PL^3}{EI}$$

$$\text{Total deflection } S_A = \frac{PL^3}{3EI} + \frac{5}{24} \frac{PL^3}{EI} = \frac{L}{500}$$

$$\therefore P = \frac{6}{1625} \frac{EI}{L^2}$$

Q.7 (a) A solid shaft is to transmit 310 kW at 117 rpm. If the shear stress is not to exceed 100 MPa, find the diameter of the shaft. What percent saving in material would be obtained if this shaft is replaced by a hollow one whose internal diameter equals 0.65 of its external diameter? Assume the length, material and maximum allowable shear stress of both the shaft being the same.

$$P = 310 \text{ kW} \quad N = 117 \text{ rpm} \quad [20 \text{ marks}]$$

$$T = \frac{P}{W} = \frac{P \times 60}{2\pi \times N} = 25.3015 \text{ kNm}$$

$$\boxed{T = 25.3015 \times 10^6 \text{ Nmm}}$$

we have

$$T_{max} = \frac{16T}{\pi D^3} \quad \therefore D = \sqrt[3]{\frac{16 \times T}{\pi T_{max}}}$$

$$\boxed{\therefore D = 108.819 \text{ mm}} \quad \text{--- } ①$$

Now this shaft is to be replaced by shaft (hollow) having $D_i = 0.65 D_o$

For same strength

$$T_{max} = \frac{16 T}{\pi D_o^3 (1 - K^4)} \quad (K = \frac{D_i}{D_o})$$

$$\therefore D_o^3 = \frac{16 \times 28.3015 \times 10^6}{\pi \times (1 - 0.65^4) \times 100}$$

$$\therefore D_o = 116.1905 \text{ mm}$$

$$\therefore D_i = 75.523839 \text{ mm.}$$

Now % Saving in material = $\frac{A_{solid} - A_{hollow}}{A_{solid}} \times 100$

$$= \frac{D_s^2 - (D_o^2 - D_i^2)}{D_s^2}$$

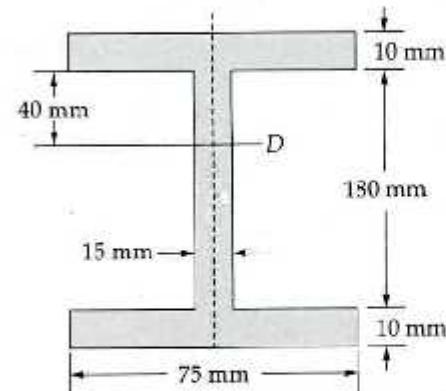
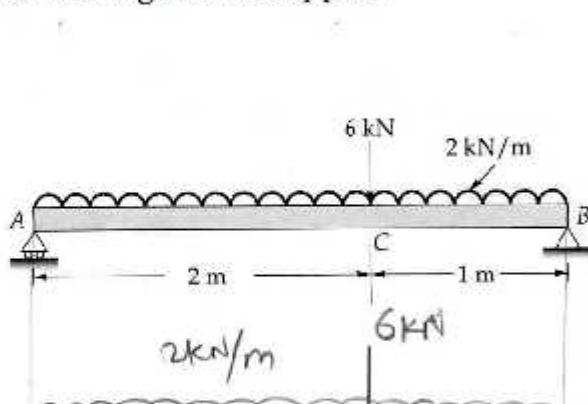
$$= \frac{108.819^2 - (116.1905^2 - 75.524^2)}{108.819^2}$$

$$= \frac{4045.1928}{11841.57476} \times 100$$

$$= \underline{\underline{34.1609}}$$

Thus ~~34.1609~~ % of material is saved.

- Q.7(b) Simply supported beam of 3 m span is subjected to loads as shown below. Determine the principal stresses at point D in the web. The section is located at a distance of 1 m from the right hand support.



[20 marks]

applying moment sign
 $\Sigma M_A = 0$

$$\therefore R_B \times 3 = 6 \times 2 + \frac{2 \times 3^2}{2}$$

$$\therefore R_B = 7 \text{ kN.}$$

Let us find the bending moment at 1 meter from right hand support

Thus bending moment at C

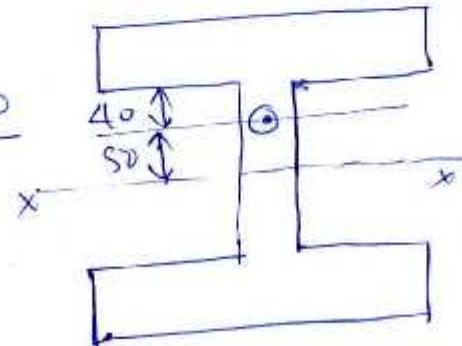
$$BM_C = R_B \times 1 - \frac{2 \times 1}{2} = 7 - 1 = \underline{\underline{6 \text{ kNm}}} \text{ (sagging)}$$

$$\& F_C = R_B - 2 \times 1 = 7 - 2 = \underline{\underline{5 \text{ kN.}}}$$

First we will find the moment arm about natural axis.

$$I_{xx} = \frac{200^3 \times 75}{12} - \frac{180^3 \times 60}{12}$$

$$\therefore I_{xx} = 20840000 \text{ mm}^4$$



Thus

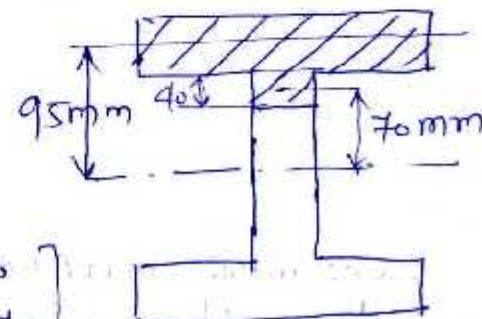
$$\sigma_D = \frac{BM \times y}{I_{xx}} = \frac{6 \times 10^6 \times 50}{20.84 \times 10^6} = 14.3954 \text{ MPa}$$

σ_D is compressive in nature

$$\boxed{\therefore \sigma_D = -14.3954 \text{ MPa}}$$

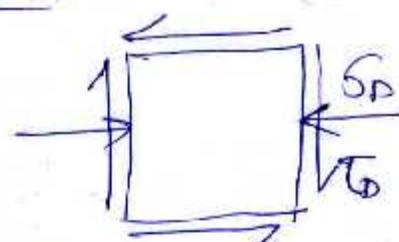
Now for shear stress at D

$$\tau_D = \frac{F_c \times A_y}{b \times I_{xx}}$$



$$(Q) \quad \tau_D = \frac{5000 \times [95 \times 75 \times 10 + 15 \times 40 \times 70]}{15 \times 20.84 \times 10^6}$$

$$\boxed{\tau_D = 1.81142 \text{ MPa}}$$



Now To determine principle stresses

$$\sigma_1 / \sigma_2 = \frac{-14.3954}{2} \pm \sqrt{\left(\frac{14.3954}{2}\right)^2 + (1.81142)^2}$$

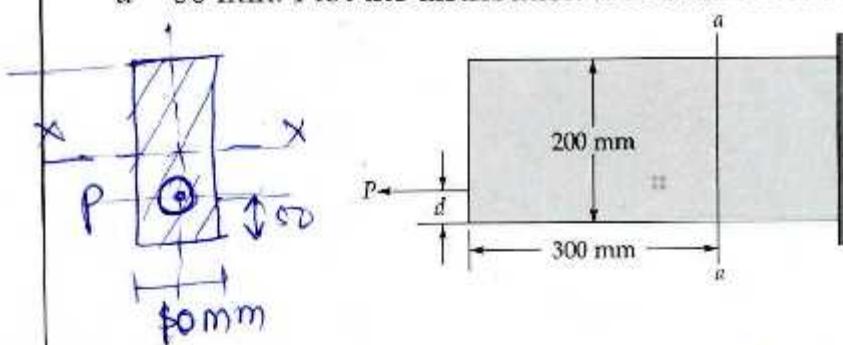
$$\sigma_{1/2} = -7.1977 \pm 7.4224 \text{ MPa}$$

$$\therefore \sigma_1 = -14.61984 \text{ MPa}$$

$$\therefore \sigma_2 = +0.92444 \text{ MPa}$$

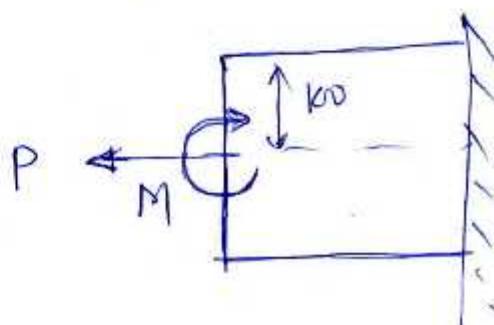
$$\therefore \tau_{\max} = \pm 7.4224 \text{ MPa}$$

- Q.7 (c) (i) The horizontal force of $P = 80 \text{ kN}$ acts at the end of the plate. The plate has a thickness of 10 mm and P acts along the centerline of this thickness such that $d = 50 \text{ mm}$. Plot the distribution of normal stress acting along section a-a.



[10 marks]

Due to eccentric loading there will be moment due to P on natural axis along with pull P
The new equivalent diagram for section a-a



$$M = P \times (100 - d)$$

$$M = 80 \times 10^3 \times 50$$

$$\underline{\underline{M = 4 \times 10^6 \text{ Nmm}}}$$

For section $x-x$

$$I_{xx} = \frac{200^3 \times 10}{12} = \underline{\underline{6.667 \times 10^6}}$$

σ_{max} at (a-a) Due to bending

$$\sigma_{max} = \frac{4 \times 10^6 \times Y_{max}}{I_{xx}} = \frac{4 \times 10^6 \times 100}{6.667 \times 10^6}$$

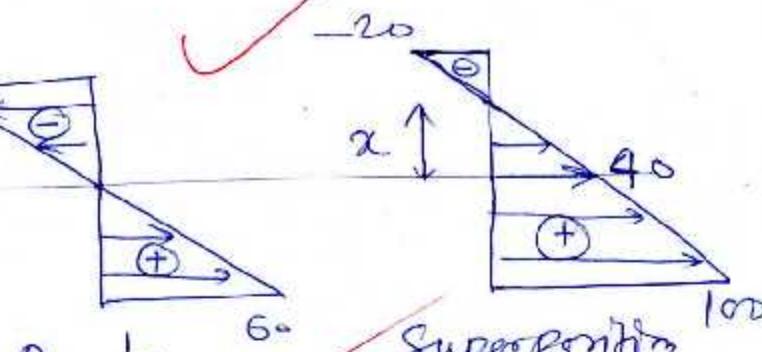
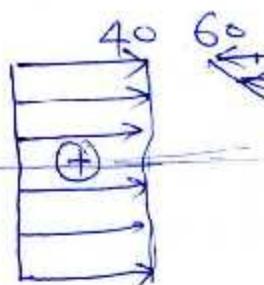
$\sigma_{max} = 60 \text{ MPa}$

[tensile at bottom]
[& compressive at top]

Due to P

$$\sigma_{Ax} = \frac{P}{10 \times 20} = \frac{80000}{20000} = 40 \text{ MPa.}$$

(10)



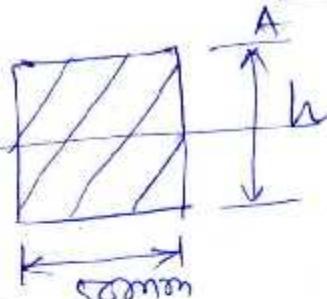
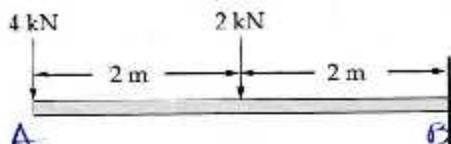
Due to
P

Due to
M

Superposition
of P & M.

$$x = \cancel{\frac{40}{60} \times 100} = \underline{\underline{66.67 \text{ mm}}}$$

- Q.7 (c) (ii) The cantilever beam shown below has a rectangular cross-section of 50 mm width and h mm depth. Find the depth h if the maximum deflection is not to exceed 10 mm. (Use $E = 10 \text{ GPa}$.)



[10 marks]

Assuming ~~after~~ EI as constant over length.

Due to load 4 kN Deflection at free end

$$\delta'_A = \frac{(4000) \times (4000)^3}{3EI} = \frac{8.5333 \times 10^{13}}{EI}$$

Due to load 2 kN Deflection at free end

$$\delta''_A = \frac{(2000)(2000)^3}{3EI} + \frac{2000(2000)^3}{2EI}$$

~~(10)~~

$$= \frac{1.3333 \times 10^{13}}{EI}$$

$$\therefore \delta_A = \delta''_A + \delta' = \frac{9.8666}{EI} \times 10^{13}$$

For $\delta_A = 10 \text{ mm}$

$$EI = 9.8666 \times 10^2 \quad \therefore I = 9.8666 \times 10^8 \text{ mm}^4$$

$$I = \frac{bh^3}{12} = \frac{50}{12} \times h^3 \quad (E = 10^4 \text{ MPa})$$

$$\therefore h = 618.6707 \text{ mm}$$