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ESE 2018 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering
Test 1 : Heat Transfer + Theory of Machines

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

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Question No.	Marks Obtained
Section-A	
Q.1	32
Q.2	48
Q.3	
Q.4	44
Section-B	
Q.5	39
Q.6	
Q.7	
Q.8	32
Total Marks Obtained	~~185~~ (195)

Signature of Evaluator: Ankit

Cross Checked by

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Section A : Heat Transfer

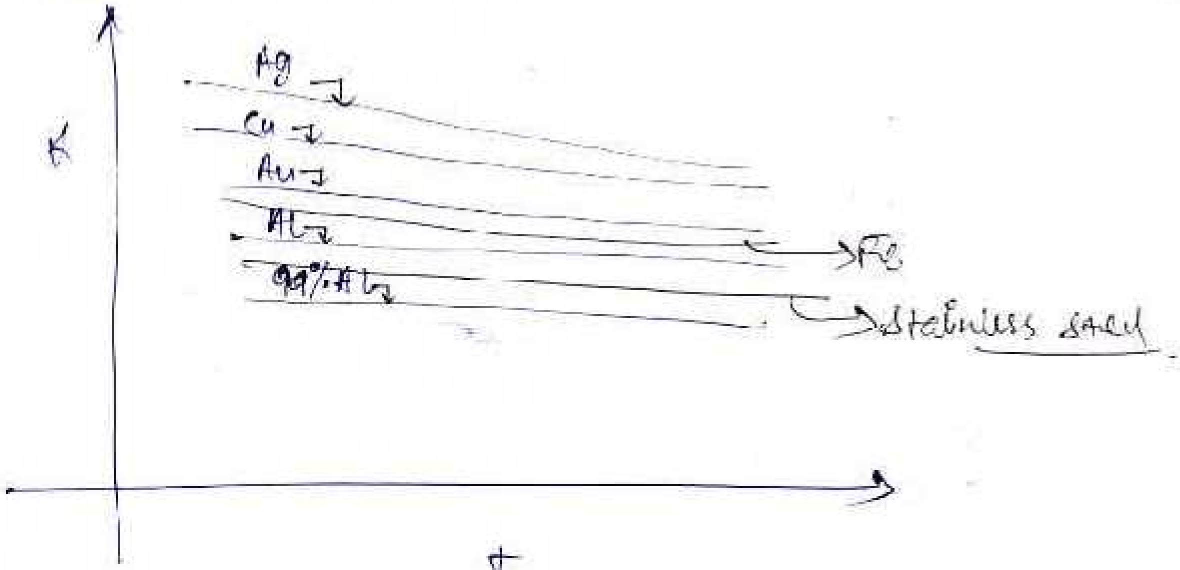
Q.1 (a) (i) Show the variation of thermal conductivity of following metallic solids with temperature: Pure silver, Copper, Gold, Aluminium, 99% pure aluminium, Brass, Platinum, Uranium, Iron, Stainless steel.

(ii) Show that no surface at any temperature can have emissivity greater than unity.

[6 + 6 marks]

Sol. i) Metals have thermal conductivity greater than its alloys.

Also thermal conductivity of metals and alloys so a decrease with increase in temperature.



2

Q.1 (b) A thin-walled copper tube of outside metal radius $r = 0.01$ m carries steam at 400 K. It is placed inside a room where the surrounding air temperature is 300 K. The tube is insulated with magnesia insulation of an approximate thermal conductivity of 0.07 W/mK.

(i) What is the critical thickness of insulation for an external convective coefficient $h = 4$ W/m^{2}K?

(ii) Determine the rate of heat transfer per meter of tube length for conditions:

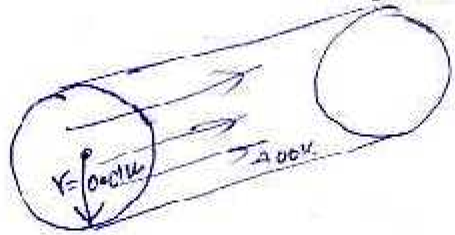
1. a 0.002 m thick layer of insulation.
2. the critical thickness of insulation.
3. a 0.05 m thick layer of insulation.

Comment on the result.

[12 marks]

Sol. $T_0 = 300K$, $K_{ins} = 0.07$ W/mK.

$r = 0.01m = 10$ mm



i) $R_{crit} = \frac{K}{h}$

$= \frac{0.07}{4} = 0.0175m$

12.

$\therefore R_{crit} = 17.5mm$.

$\therefore$ Critical thickness $= 17.5mm - 10mm$
$= 7.5mm$.

ii) ① 0.002m or 2mm thick insulation.

$\therefore q = \dfrac{\Delta T}{\dfrac{\ln(r_2/r_1)}{2\pi KL} + \dfrac{1}{h \times 2\pi r_2 L}}$

$\therefore r_1 = 0.01m = 10$ mm;
$r_2 = 10 + 2 = 12mm$

$\therefore q/L = \dfrac{400-300}{\dfrac{\ln(12/10)}{2\pi \times 0.07} + \dfrac{1}{4 \times 2\pi \times 0.012}} = 26.807$ W/m

② $r_1 = 10mm$, $r_2 = 17.5mm$

$\therefore q/L = \dfrac{400-300}{\dfrac{\ln(17.5/10)}{2\pi \times 0.07} + \dfrac{1}{4 \times 2\pi \times 0.0175}} = 28.2007$ W/m

(3) 0.05m thick layer of insulation.

⇒ $r_1 = 10mm$, $r_2 = 10 + 50 = 60mm$

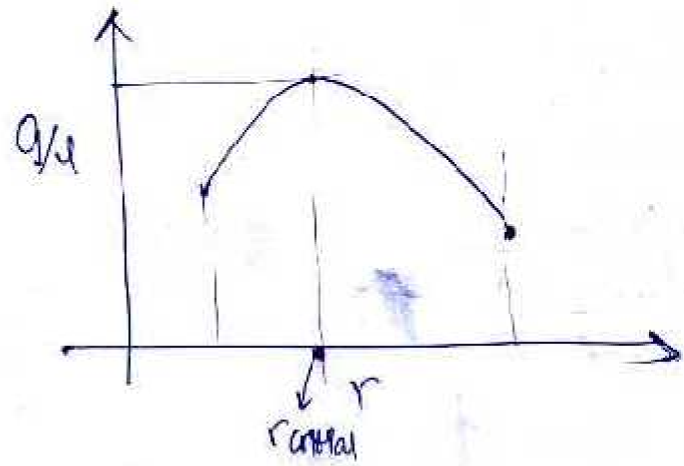
$$\therefore q/l = \frac{400 - 300}{\frac{\ln(60/10)}{2\pi \times 0.07} + \frac{1}{4 \times 2 \times \pi \times 0.06}}$$

$= \underline{21.1105}$ w/m.

∴ we from the above 3 results, we observe that;

(1) upto $r_2 = r_{critical}$, heat transfer rate increases as rate of decrease of convection resistance is more than rate of increase of conduction resistance, thus total resistance decreases and heat transfer increases

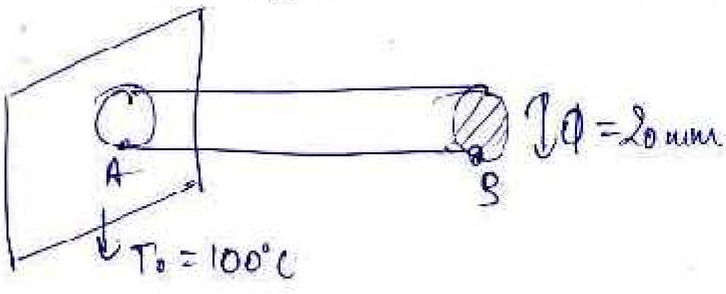
(2) At $r = r_{critical}$, heat transfer reaches its maximum and after that it starts to decrease.



Q.1 (c) A very long copper rod 20 mm in diameter extends horizontally from a plane heated wall maintained at 100°C. The surface of the rod is exposed to an air environment at 20°C with convective heat transfer coefficient of 9 W/m²K. Determine the heat loss if the thermal conductivity of Copper is 410 W/mK. Further estimate how long the rod should be in order to be considered infinite.

[12 marks]

→



$T_\infty = 20°C$
$h = 9$ W/m²K
$K_{cu} = 410$ W/mK.

Assuming the ~~fluid~~ Rod to be ~~insulated at its tip (B)~~ infinitely long:

~~$q_{loss} = \sqrt{hPKA}\;\theta_0 \tanh mL$~~

$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{9 \times \pi \times 0.02 \times 4}{410 \times \pi \times (0.02)^2}} = \underline{2.0952}$ /m

$\Rightarrow q_{loss} = (\sqrt{hPKA} \cdot \theta_0)$

$\therefore q_{loss} = \sqrt{\frac{9 \times \pi \times 0.02 \times 410 \times \pi \times (0.02)^2}{4}}\,(100 - 20)$ ~~tanh(...)~~

$= 21.5907$ W. 21.67 W

9

$\therefore$ for Rod to be considered infinitely long.

$mL \geq 2.65$

$\Rightarrow (2.0952) L \geq 2.65$

✓ $\therefore L \geq 1.2647$ m

Q.1 (d) (i) What is the significance of Biot number in lumped system analysis?
(ii) Explain the phenomenon of heat transfer by free convection. What forces control the fluid motion? Can free convection occur in space vehicles with zero gravity trajectory?

[4 + 8 marks]

i) Biot Number $= \frac{hL_c}{k} = \frac{\text{Internal conduction resistance}}{\text{External convection resistance}}$

For $Bi < 0.1$, the internal ~~heat~~ conduction resistance is assumed as negligible compared to external convection resistance and thus the whole body is assumed to be at a uniform temperature T.

3

$$\Rightarrow -mc_p dT = hA(T - T_\infty)$$

ii) Heat transfer by free convection

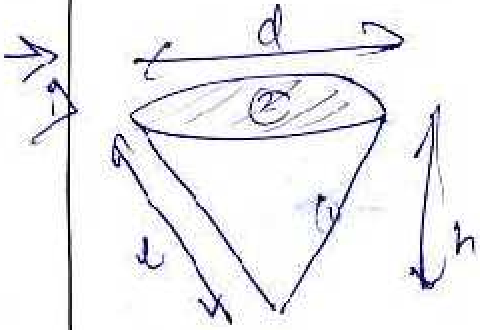
Q.1 (e) Establish a relation for the shape factor of following cavities with respect to itself:

(i) Conical cavity of diameter d and depth h.

(ii) Hemispherical bowl of diameter d

Cavities are closed on its outer surface with a flat surface.

[6 + 6 marks]



$l = \sqrt{h^2 + d^2/4}$

* $F_{11} + F_{12} = 1$ ——— ①

* $F_{22} + F_{21} = 1$ ——— ②

$F_{22} = 0 \Rightarrow F_{21} = 1.$

* $A_1 F_{12} = A_2 F_{21}$ ——— ③

$\Rightarrow \pi \frac{D}{2} l \, F_{12} = \frac{\pi d^2}{4} F_{21}$

$\Rightarrow \sqrt{h^2 + d^2/4} \, F_{12} = \frac{d}{4} F_{21}.$

$\therefore F_{12} = \frac{d/4}{\sqrt{h^2 + d^2/4}}$

$\Rightarrow F_{11}$ from eqn ①;

$F_{11} = 1 - \frac{d/4}{\sqrt{h^2 + d^2/4}}.$ d/2

$= \frac{2\sqrt{4h^2 + d^2} - d}{2\sqrt{4h^2 + d^2/4}}$

$\therefore F_{11} = \frac{2\sqrt{4h^2 + d^2} - d}{2\sqrt{4h^2 + d^2}}$

4

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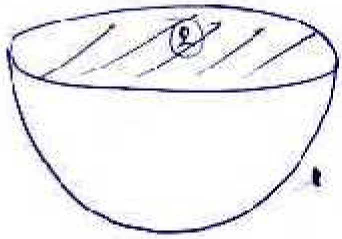
ii)



$+ F_{22} = 0$

$\Rightarrow F_{21} = 1.$

$* \; F_{11} + F_{12} = 1$

$* \; A_1 F_{12} = A_2 F_{21}$

$\frac{\pi D^2}{2} F_{12} = \frac{\pi D^2}{4} f_{21}$

6

$\therefore F_{12} = 0.5$

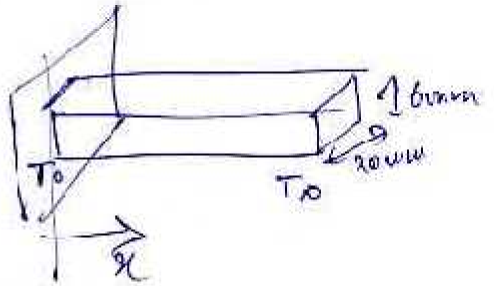
$\therefore F_{11} + F_{12} = 1$

$\Rightarrow F_{11} = 0.5$

Q.2 (a) A handle is used in the frying, it is required to heat oil to about 300°C for frying purpose. The section of the handle is 6 mm × 20 mm. The surroundings is at 30°C. The conductivity of the material is 205 W/mK. If the temperature at a distance of 380 mm from the oil should not reach above 40°C, determine the convective heat transfer coefficient. Assume the handle as long fin.

[20 marks]

Sol. Assuming the handle as long fin;



$T_0 = 300°C$

$T_\infty = 30°C$

$T @ x = 380 mm = 40°C$

Now,

$T @ x = 380 mm \leq 40°C;$

18

So for the limiting case, $T_{@x=380mm} = 40°C$

$\therefore \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$

$\Rightarrow \frac{40 - 30}{300 - 30} = e^{-m \times 0.38}$

$\Rightarrow m = 8.6732.$

& $m = \sqrt{\frac{hP}{KA}}$

$\Rightarrow 8.6732 = \sqrt{\frac{h \times 2(20+6) \times 10^{-3}}{205 \times 20 \times 6 \times 10^{-6}}}$

$\therefore h = 35.5873 \ W/m^2K.$

Q.2 (b) Air at 20°C and maintained at 1 bar is flowing over a flat plate at a velocity of 4 m/s. If the plate is 300 mm wide and at 56°C, calculate the following quantities at $x = 300$ mm, given that properties of air at the bulk mean temperature, 38°C are:

$\rho = 1.1374$ kg/m³; $k = 0.02732$ W/mK, $c_p = 1.005$ kJ/kgK; $\nu = 16.768 \times 10^{-6}$ m²/s, Pr = 0.7

(i) Boundary layer thickness

(ii) Local friction coefficient

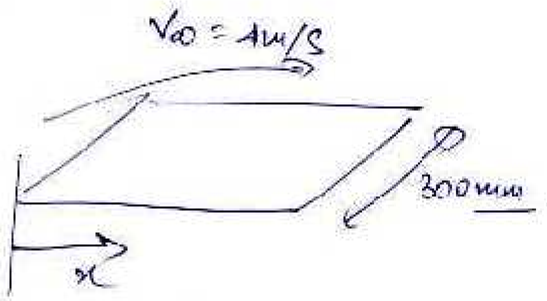
(iii) Average convective heat transfer coefficient

(iv) Rate of heat transfer by convection

(v) Total mass flow rate through the boundary

[20 marks]

→ $T_\infty = 20°C$, $V_\infty = 4$ m/s.



i) ~~$\delta = \frac{5x}{\sqrt{Re_x}}$~~

18.

$Re_{x=300mm} = \frac{Vx}{\nu}$

$= \frac{4 \times 0.3}{16.768 \times 10^{-6}} = \underline{71544.8855}$

∴ flow is laminar.

→ $\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.3}{\sqrt{71544.8855}}$

∴ $\delta = \underline{5.6071 mm}$.

ii). $C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{71544.8855}}$ = ~~2.4~~

∴ $C_{fx} = \underline{2.4824 \times 10^{-3}}$.

iii). $\overline{Nu} = 0.664 (Re)^{1/2} (Pr)^{1/3} = \frac{\bar{h} L}{k}$

$\therefore \frac{\bar{h} \times 0.3}{0.02732} = 0.664 \, (71544.2855)^{1/2} \, (0.7)^{1/3}$

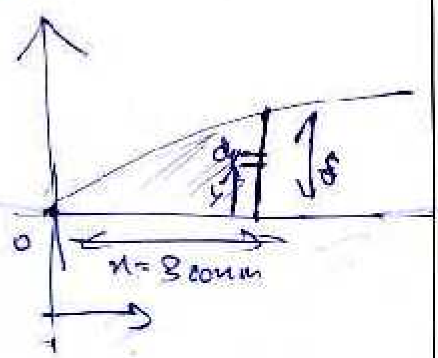
$\therefore \bar{h} = 14.3609 \, W/m^2K$

iv) $Q_{convection} = h A \Delta T$

$\Rightarrow 14.3609 \times (0.3 \times 0.3) \times (56 - 20)$

$= 46.5294$ Watts

v) $\dot{m} = \rho A V = \int \rho \, (b\,dy) \, V$



Assuming cubic velocity profile,

$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$\therefore \dot{m} = \int_0^\delta \rho b U_\infty \left(\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right) dy$

$= \rho b U_\infty \int_0^\delta \frac{3}{2}\frac{y}{\delta} dy - \frac{1}{2}\frac{y^3}{\delta^3} dy$

$\Rightarrow \rho b U_\infty \left[\frac{3}{2} \times \frac{y^2}{2} \times \frac{1}{\delta} - \frac{1}{2} \times \frac{y^4}{4} \times \frac{1}{\delta^3}\right]_0^\delta$

$\rho b U_\infty \left[\frac{3 \times \delta}{4} - \frac{1 \times \delta}{8}\right] = \frac{5}{8} \rho b U_\infty \delta$

$\therefore \dot{m} = \frac{5}{9} \times 1.1371 \times 0.3 \times 4 \times 5.6071 \times 10^{-3}$

$= \underline{4.7831 \times 10^{-3} \text{ kg/s}}$

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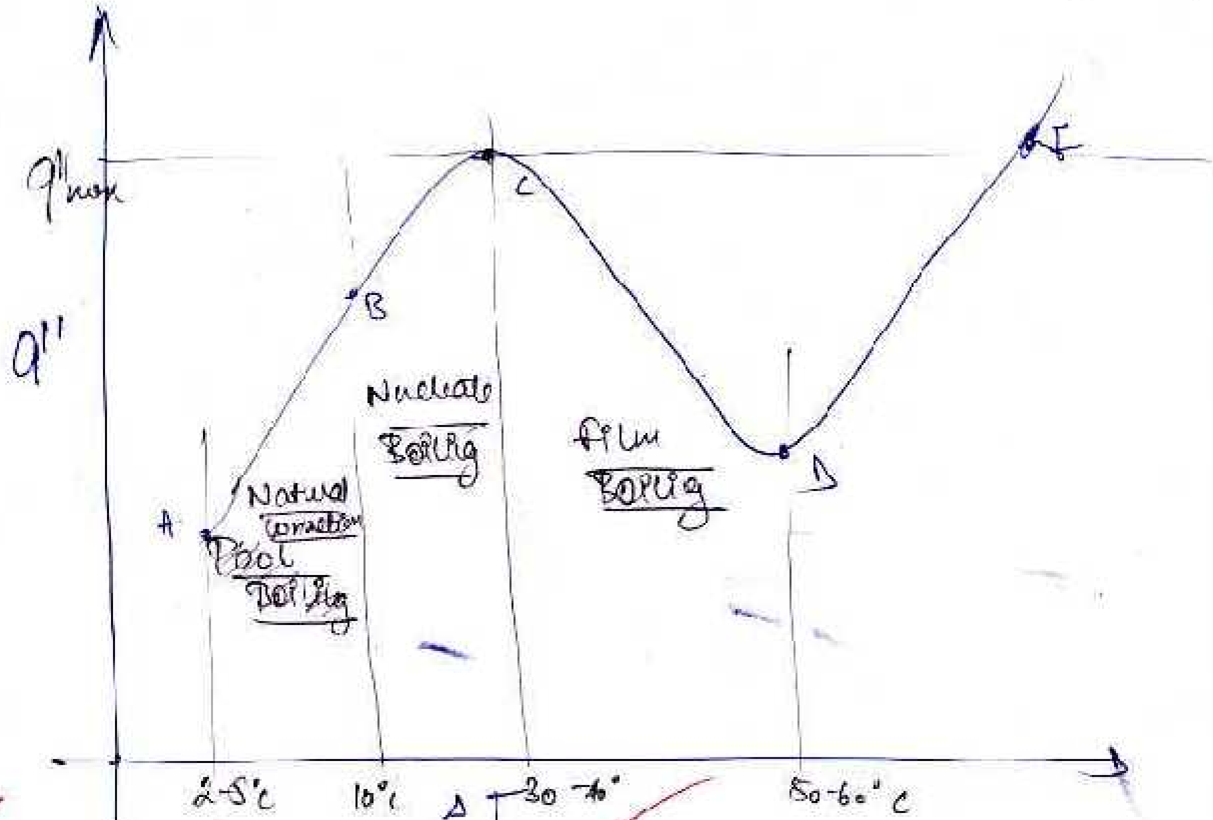
Q.2 (c) Draw the typical boiling curve for pool boiling of water at saturation temperature and atmospheric pressure with schematic representation of each boiling curve.

[20 marks]

(*).



12

$\Delta T = T - T_s$.

* When T becomes more than T_s; then water begins to boil and initially heat transfer occurs by natural convection. Bubbles form but can't reach the free surface.

* As ΔT increases, nucleate boiling starts and bubbles start to reach the free surface.

* Then at point C, we get maximum heat flux as the critical heat flux point C.

* From C to D, first unstable film boiling occurs followed by stable film boiling.

* Point D is called as Leidenfrost point and after that radiation effects become dominant.

* If q'' gets more than critical heat flux, the wire responsible for heating melts itself.

Q.4 (a) A glass of diameter 50 mm contains some hot milk. The height of the milk in the glass is 100 mm. To cool the milk, the glass is placed into a large pan filled with cold water at 25°C. The initial temperature of the milk is 80°C. The milk is stirred slowly and continuously so that its temperature remains uniform at all times. The heat transfer coefficient between the water and the glass is 100 W/m^{2}K. Can the milk in this case be treated as a lumped-parameter system? If so, determine (i) the time taken for the milk to cool from 80°C to 30°C and (ii) the total amount of energy transferred from milk to water during the cooling process.

Thermal conductivity of milk = 0.6 W/mK

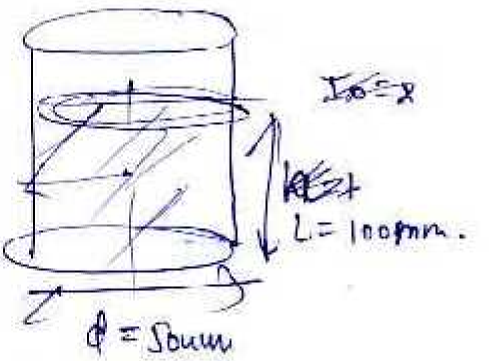
Density of milk = 900 kg/m^3

Specific heat capacity of milk = 4.2 kJ/kgK

Neglect the effect of stirring work and heat loss from the ends.

[20 marks]

Sol.



$T_\infty = 25°C$

$T_i = 80°C$.

$h = 100 W/m^2K$.

for it to be assumed lumped system,

$Bi < 0.1$

$\therefore Bi = \frac{hL_c}{K}$ & $L_c = V/A$

~~$\therefore L_c = \frac{\frac{\pi}{4}D^2H}{\pi DH + 2\times\pi D^2/4}$~~

~~$\therefore$ Since heat loss from ends is to be neglected, Area (A)~~

~~becomes $\pi D H$.~~

~~$\therefore L_c = \frac{\pi D^2/4 H}{\pi D H} = D/4$.~~

$\therefore Bi = \frac{h L_c}{k} = \frac{100 \times (0.05/4)}{0.6}$

$\therefore L_c = \frac{V}{A} = \frac{\pi R^2 H}{2\pi R^2 + 2\pi R H} = \frac{HR}{2(H+R)}$

$L_c = \frac{0.1 \times 0.025}{2(0.125)} = \underline{0.01\ m}.$

$\Rightarrow \underline{Bi < 0.1}$.

$\therefore$ i) $\frac{T - T_0}{T_i - T_0} = e^{-\frac{hAt}{\rho V C_p}} = e^{-\frac{ht}{\rho L_c C_p}}$

$\frac{30 - 25}{80 - 25} = e^{-\frac{100 \times t}{900 \times 0.01 \times 4200}}$

$\frac{5}{55} = e^{-\frac{t}{378}}$

$\therefore t = 906.404$ secs.

13.

ii) $q = m C_p \Delta T$ (heat lost by milk)

$= (\rho V) C_p \Delta T = \rho \times \frac{\pi D^2}{4} \times H \times C_p \times \Delta T$

$= 900 \times \left(\pi \times \frac{0.05^2}{4} \times 0.1\right) \times 4200 \times (80 - 30)$

$= \underline{37.110\ kJ}$

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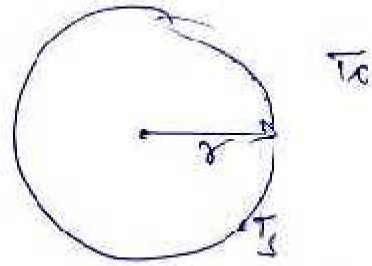
Q.4 (b) A metal sphere of surface area 0.03 m^2 is in an evacuated enclosure whose walls are held at very low temperature. Electric current is passed through resistors embedded in the sphere causing electrical energy to be dissipated at the rate of 85 W. If the sphere surface temperature is measured to be 640 K, while in steady state, calculate emissivity of the sphere surface and state the assumptions made in the estimation.

[20 marks]

Sol. Area = 0.03 m^2

$4\pi R^2 = 0.03$

$\therefore R = 48.86$ mm.



$q_{generation} = 85$ W.

Assumptions ??

$\therefore \quad \dot{E}_{in} - \dot{E}_{out} + \dot{q}_{gen} = \dot{q}_{storage}$

At steady state,

$\dot{q}_{storage} = 0$

$\Rightarrow \dot{q}_{gen} = \dot{E}_{out} - \dot{E}_{in}$

$\dot{q}_{gen} = hA(T_s - T_\infty) + \varepsilon \sigma A (T_s^4 - T_\infty^4)$

Now, $T_s = 640$ K and T_∞ is very low;

So due to a large temperature difference; major heat transfer occurs by radiation.

So, neglecting the convection heat transfer.

$\therefore \dot{q}_{gen} = \varepsilon \sigma A (T_s^4 - T_\infty^4)$

$\because T_\infty << T_s \Rightarrow T_s^4 - T_\infty^4 \approx T_s^4$

$\Rightarrow \dot{q}_{gen} = \varepsilon \sigma A T_s^4$

$\Rightarrow 85 = \varepsilon\ (5.67\times10^{-8})\times 0.03 \times (640)^4$

13

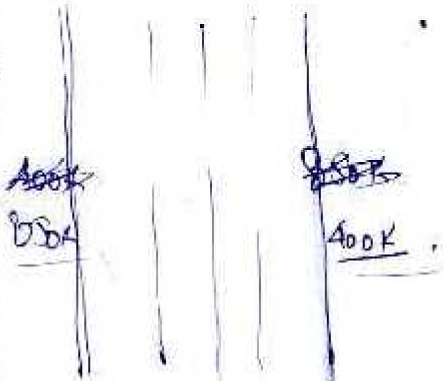
$\therefore\ \varepsilon = 0.2978$

Q.4 (c) An industrial furnace employs a hollow brick lining. The inside and outside surfaces of hollow brick lining are maintained at 850 K and 400 K by placing the radiation shields in between the hollow space. The heat loss to the furnace surroundings at 300 K is both by radiation and natural convection. Calculate the number of shields needed. The emissivity of walls and shields can be taken as 0.8. The convective heat transfer coefficient is governed by the expression $h = 1.5(\Delta T)^{0.33}$ W/m²K.

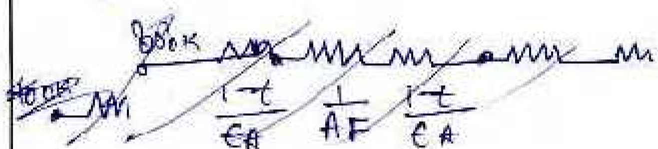
[20 marks]

Sol.

$T_\infty = 300K$.



~~Drawing the resistance circuit;~~



~~For n shields;~~

→ For 1 shield, $R_{thermal} = \frac{1-\epsilon}{\epsilon A} + \frac{1}{AF} + \frac{1-\epsilon}{\epsilon A}$

$\because F = 1 \Rightarrow R = \frac{1}{A}\left(\frac{1}{\epsilon} - 1 + 1 + \frac{1}{\epsilon} - 1\right)$

$R = \frac{1}{A}\left(\frac{2}{\epsilon} - 1\right)$

For n shields, we have (n+1) resistances; so

$R = \frac{n+1}{A}\left(2/\epsilon - 1\right)$.

$\therefore q_{loss} = \frac{\sigma (T_0^4 - T_i^4)}{R} = \frac{(850^4 - 400^4) \times 5.67 \times 10^{-8}}{\frac{n+1}{A}(2/\epsilon - 1)}$

$\therefore \frac{q_{loss}}{A} = \frac{450}{(n+1)\left(\frac{2}{0.8} - 1\right)} = \frac{300}{n+1}$ ~~3.3099 × 10^11 / n+1~~

$= \frac{18764.15895}{n+1}$

$\therefore q_{loss} = \frac{800}{n+1} = q_{rad} + q_{convection}$.

<u>$q_{radiation}$</u> $= \varepsilon \sigma A (T^4 - T_0^4)$

$= 0.8 \times 5.67 \times 10^{-8} \times (400^4 - 300^4)$

$= ~~70.9383~~ \; \underline{793.8}\ W/m^2$.

$q_{convection} = h \Delta T$

$\Rightarrow 1.5 (\Delta T)^{0.33} \times \Delta T$

$= 1.5 (\Delta T)^{1.33}$

$= 1.5 (400 - 300)^{1.33} = \underline{685.632\ W/m^2}$

18

~~$\therefore \frac{800}{n+1} = 793.8 + 685.632$~~

~~$\frac{3.3893 \times 10^{11}}{n+1} = 793.8 + 685.632$~~

$\frac{18764.15625}{n+1} = 793.8 + 685.632$

$\therefore n = 11.683$

$\therefore n = 12$

$\therefore$ 12 shields are needed

Section B : Theory of Machines

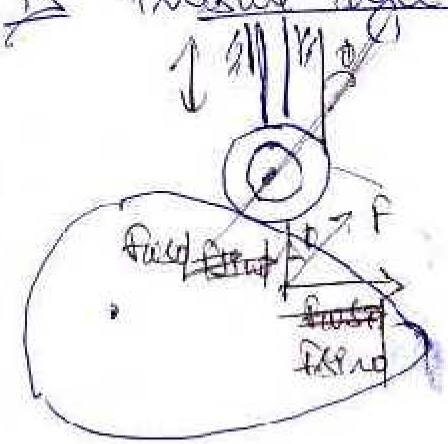
Q.5 (a) **(i)** What is the significance of pressure angle in Cam follower mechanism?

(ii) Define the following terms frequently being used to describe the geometry of a radial cam:

1. Trace point
2. Pitch point
3. Prime circle

[6 + 6 marks]

(a) 1) Pressure angle.



* Pressure angle is angle b/w the line of motion of follower and common normal to the cam & follower surfaces.

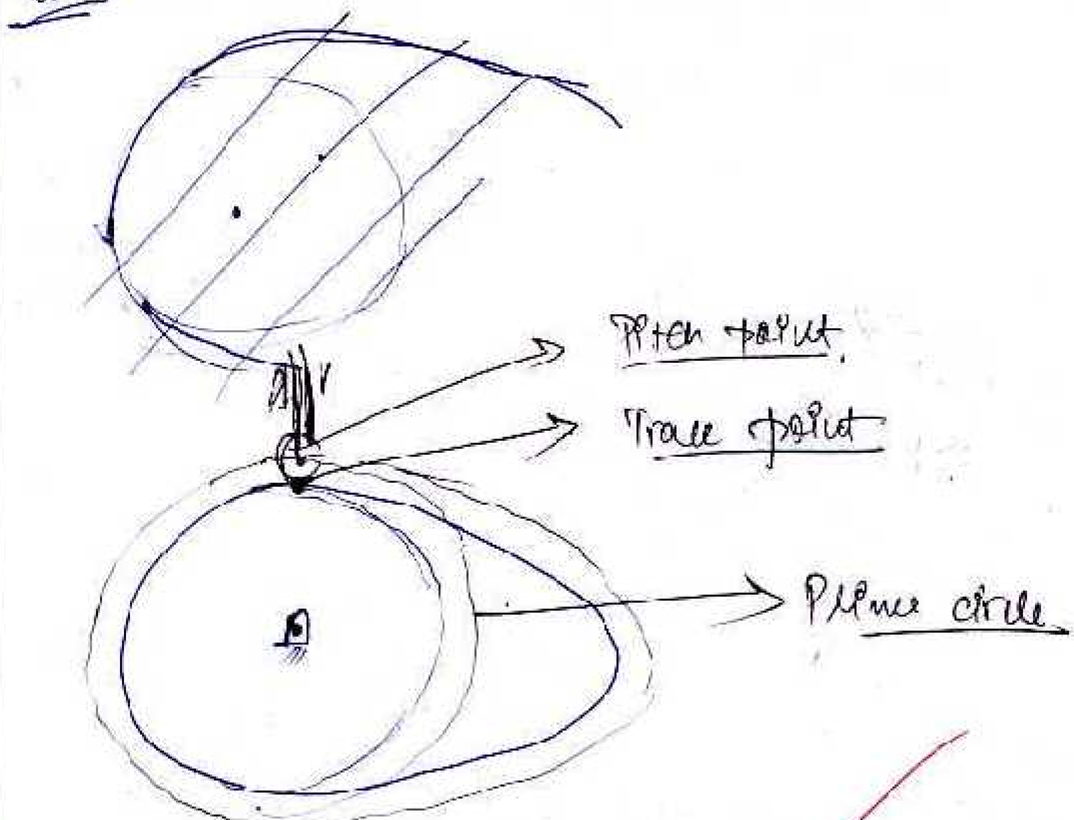
4

* As ϕ increases, the component responsible for follower motion Fcosϕ decreases and Fsinϕ increases.

* As Fsinϕ increases, follower motion gets difficult.

Thus for translating followers, a pressure angle of more than 30° is not recommended.

iv)



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Trace point.

Point of contact of cam with follower is known as trace point.

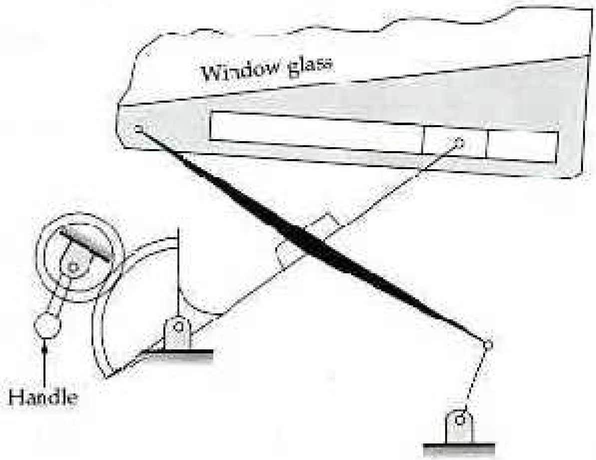
Pitch point.

Point of contact of pitch curve with follower is pitch point. for knife edge follower trace and pitch point are same.

Prime circle.

Smallest circle on the pitch curve which is concentric with base circle. ~~it~~ ~~[illegible]~~

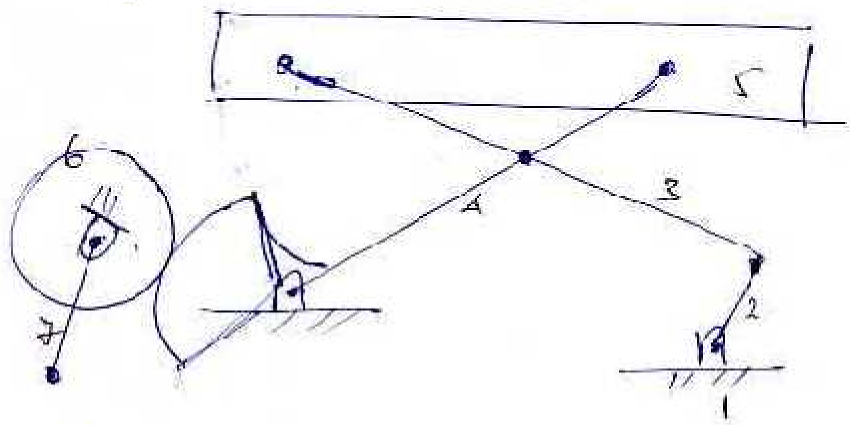
Q.5 (b) Show that the automobile window glass guiding mechanism given below has a single degree of freedom.



Automobile window guidance linkage

[12 marks]

b. Drawing a simpler version of linkage.



DoF = 3(n−1) − 2j − h.

n = 7, j = 6 Binary joints + 1 Ternary joint b/w link 6, 7 & 1

= 6 + 2(1) = 8

h = 1 (b/w link 4 & 6)

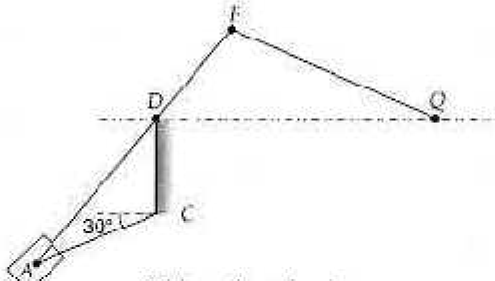
∴ DoF = 3(7−1) − 2×8 − 1

= 18 − 16 − 1

= 1. Ans

10

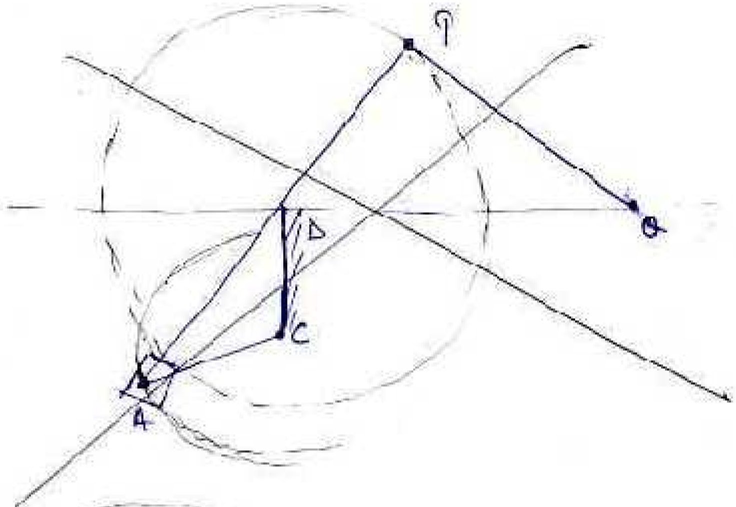
Q.5 (d) In a Whitworth quick return motion mechanism as shown below, the distance between the fixed centres is 100 mm and the length of the driving crank is 120 mm. The length of the slotted lever is 200 mm and the length of the connecting rod is 170 mm. Calculate the ratio of the time of cutting to return stroke.

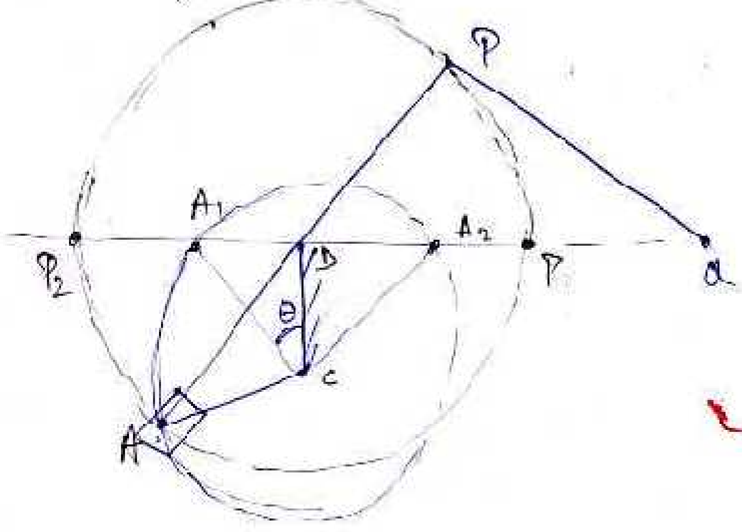


Whitworth mechanism

[12 marks]

(d)





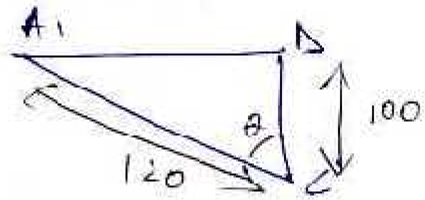
CD = 100 mm

AC = 120 mm

AP = 200 mm

PQ = 170 mm.

∴ from ΔA_1DC →



$\cos\theta = \frac{100}{120}$

$\Rightarrow \theta = 33.557°$

$\Rightarrow \alpha = 2\theta = 67.1146°$

where α is angle turned during return stroke

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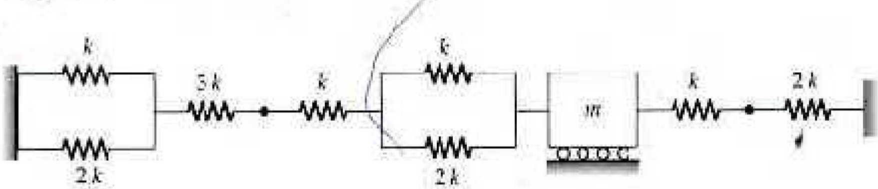
$\leftarrow \beta = 360 - \alpha = 292.8853°$

$\therefore Q_{RR} = \dfrac{\text{time (cutting stroke)}}{\text{time (return stroke)}} = \dfrac{\beta}{\alpha}$

$= \dfrac{292.8853}{67.114} = 4.3639$

10

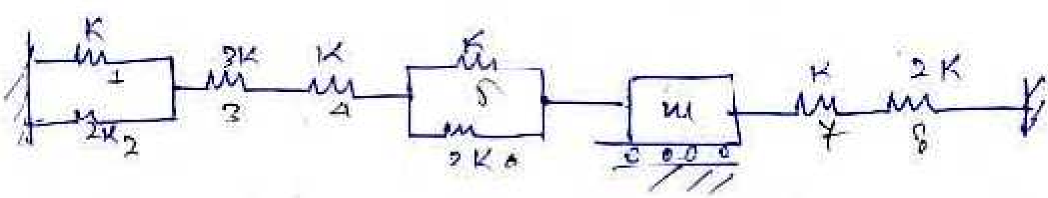
Q.5 (e) The system shown below by a block attached to a single spring. Find the equivalent stiffness of combined system.

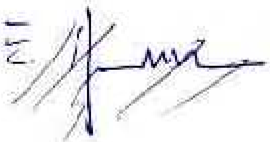


10

[12 marks]

$\Rightarrow$



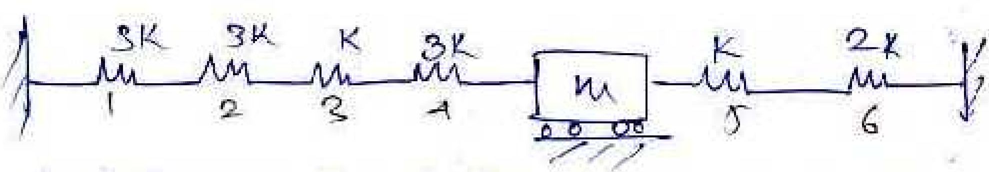


1 & 2 are in parallel, and 5, 6 are also in parallel.

$\Rightarrow K_{eq(1,2)} = K_1 + K_2 = 3K$

$K_{eq(5,6)} = K_1 + K_2 = 3K.$

$\equiv$



* 1, 2, 3, 4 are in series. 5, 6 are in series.

$\Rightarrow K_{eq}(1,2,4) =$ ~~K~~

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{K}{K_2} + \frac{1}{K_4}$$

$$\frac{1}{K_{eq}} = \frac{1}{3K} + \frac{1}{3K} + \frac{1}{3K} = \frac{3}{3K}$$

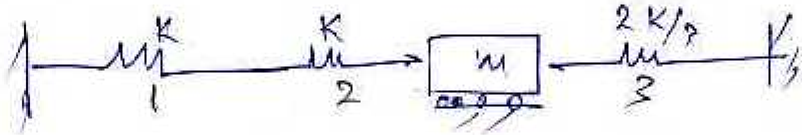
$\Rightarrow K_{eq\,(1,2,4)} = K$.

$\therefore$ ~~K_{eq}~~ $K_{eq}(5,6) =$

$$\frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{2K} = \frac{3}{2K}$$

$\therefore K_{eq} = \frac{2K}{3}$.

$\equiv$



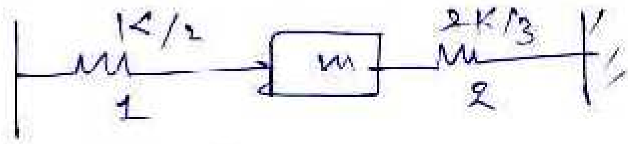
1 & 2 are in series

$$\Rightarrow \frac{1}{K_{eq(1,2)}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$= \frac{1}{K} + \frac{1}{K} = \frac{2}{K}$$

$\therefore K_{eq} = K/2$.

$\equiv$



1 & 2 are in parallel;

$$\Rightarrow K_{eq} = \frac{K}{2} + \frac{2K}{3}$$

$$\Rightarrow \frac{3K}{6} + \frac{4K}{6}$$

$\therefore K_{eq} = \frac{7K}{6}$.

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Q.7 (a) Two gear wheels mesh externally and are to give a velocity ratio of 3 : 1. The teeth are of involute form; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 120 rpm. Determine :

(i) the number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel

(ii) the length of path and arc of contact

(iii) the number of pairs of teeth in contact

(iv) the maximum velocity of sliding

[20 marks]

* → $m = 6$ mm $N_p = 120$ RPM $= 120 \times \frac{2\pi}{60} = \underline{12.566 \text{ rad/s}}$.

$G = 3:1$; $A_R = A_p = 1$

$\phi = 20°$;

i) $T_{min} = \dfrac{2 A_R}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$

$\Rightarrow \dfrac{2 \times 1}{\sqrt{1 + \frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20} - 1}$

$T_{min} = 44.94 = 45$

∴ $t_{min} = \frac{45}{3} = 15$

∴ Teeth on pinion = 15

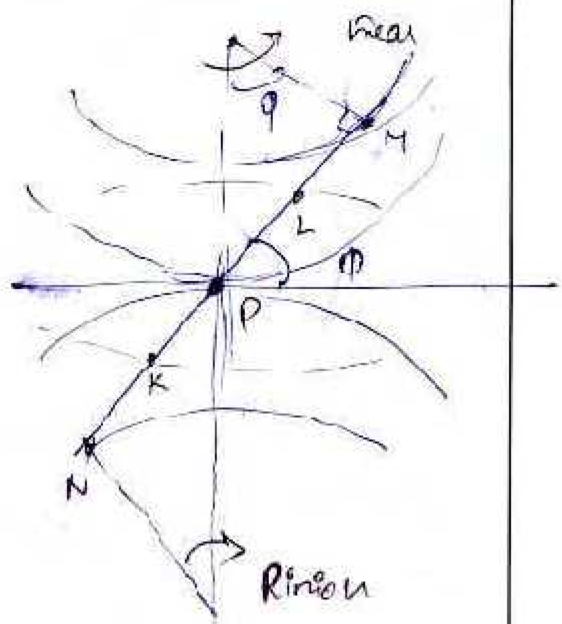
Teeth on gear = 45.

ii)

P.T.O.

(ii)

$KP = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$



$T = 45, \quad t = 15$

$R = \frac{mT}{2}, \quad r = \frac{mt}{2}$

$\Rightarrow R = \frac{6\times45}{2}, \quad r = \frac{6\times15}{2}$

$\therefore R = 135\,mm, \quad r = 45\,mm.$

$\therefore R_a = 135+6 = 141\,mm, \quad r_a = 45+6 = 51\,mm$

$\therefore KP = \sqrt{(141)^2 - (135\cos20)^2} - 135\sin(20)$
$= 15.373\,mm$

$PL = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$

$\Rightarrow \sqrt{(51)^2 - (45\cos20)^2} - 45\sin20$

$= 13.1201\,mm.$

$\therefore$ Path of contact $= KP + PL$
$= 15.373 + 13.1201 = \underline{28.4931\,mm}$

Arc of contact $= \frac{\text{Path of contact}}{\cos\phi}$

$= \frac{28.4931}{\cos20} = 30.3217\,mm.$

iii) Contact ratio (m_c) = $\frac{\text{Arc of contact}}{\pi m}$

$= \frac{30.3217}{\pi \times 6} = 1.6086$

⇒ For 60% of the time, two pair of teeth are in contact and for rest of time, one pair of teeth remain in contact.

iv) $(V_{max})_{slide} = (\omega_1 + \omega_2)(\theta_P)_{max}$.

$KP > PL$

$\Rightarrow (V_{max})_{slide} = (\omega_P + \omega_G) \times KP$

$\omega_P = 12.5663 \text{ rad/s}$

$\Rightarrow \omega_G = \frac{\omega_P}{3} = 4.1887 \text{ rad/s}$

$\therefore (V_{max})_{slide} = (\omega_P + \omega_G) KP$

$= (12.5663 + 4.1887)(15.37?)$

$= \underline{0.257 \text{ m/s}}$.

Q.7 (b) The turbine rotor of a ship has a mass of 2.2 tonnes and rotates at 1800 rpm clockwise when viewed from the aft. The radius of gyration of the rotor is 320 mm. Determine the gyroscopic couple and its effect when

(i) the ship turns right at a radius of 250 m with a speed of 25 km/h

(ii) the ship pitches with the bow rising at an angular velocity of 0.8 rad/s

(iii) the ship rolls at an angular velocity of 0.1 rad/s

[20 marks]

→ $m = 2200 kg$, $N = 1800 rpm$ CW.

$K_{rotor} = 0.32 m$.

∴ $\tau = I \omega \omega_p$.

$I = mk^2 = 2200 \times (0.32)^2 = 225.28 \; kg\text{-}m^2$.

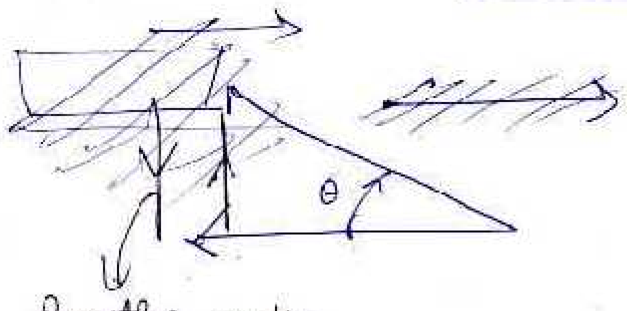
$\omega = \frac{1800 \times 2\pi}{60} = 188.495$ rad/s

i) $\omega_p = \frac{V}{R}$.

$V = \frac{25 \times 5}{18}$ → $\omega_p = \frac{25 \times 5}{18 \times 250} = \frac{1}{36}$ rad/s

∴ $\tau = I \omega \omega_p$

$= 1179.5633 N\text{-}m$.



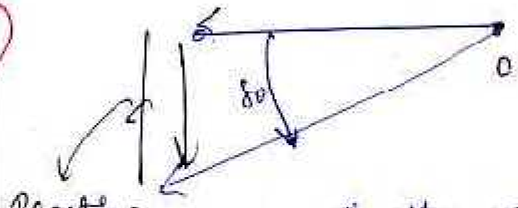
Reactive couple.

∴ The reactive couple will tend to lift the bow and lower the stern.

ii) $\omega_p = 0.8 \text{ rad/s}$

$$\tau = I \omega \omega_p$$
$$= 222.58 \times 188.495 \times 0.8$$
$$= 33644.302 \text{ N-m}$$

8



∴ the reactive couple will make the ship turn left.

iii) $\omega_p = 0.1 \text{ rad/s}$

But since ω & ω_p are parallel to each other.

⇒ $\tau = 0$

⇒ No gyroscopic couple.

Q.7 (c) (i) Why do spur gear make noise?
(ii) What is the key difference between spur gear and helical gear?

[20 marks]

Sol.

i) ~~Spur gears make more noise as two parallel spur gears make a line contact which.~~

i) Spur gears make noise as two parallel spur gears in contact have ~~sudden engagement~~ a higher contact ratio than other gears.

Spur gears can have contact ratio upto 2 to 2.5, which leads to more teeths in contact and hence ~~more~~ ~~also~~ more noise.

Also, spur gears have sudden engagement and disengagement which leads to impacts and more wear and tear of the teeth.

4

ii) * Spur gears have no helix angle provided on the tooth and hence make a sudden engagement.

* In a helical gear, teeths gradually come in contact which leads to lesser impacts and less wear & tear of tooth.

2

* Spur gears are subject to radial and tangential forces whereas helical gears also experience axial forces.