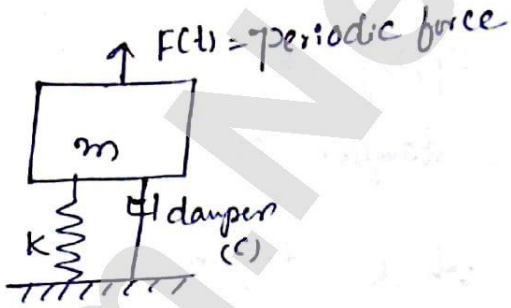
Mechanical Vibrations

Flow chart of vibrations analysis :-

1) Physical model

2) Schematic model



(3) Mathematical model

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

(4) Solution

(5) Analysis

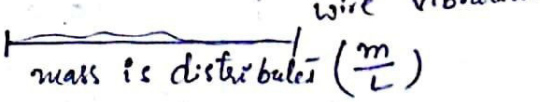
Mechanical vibrations

Discrete / lumped Parameter System

- Single degree
- 2 degree of freedom
- Multi dof

(- Coupled differential Eqns
- Solution comes Exactly

Continuous / discrete tributed Parameters system



Infinite dof

- coupled
- partial differential Eqns
- no guarantee for solution.

Finite Element method is used to convert continuous system into distributed system.

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Single degree of freedom system :-

Vibration system

Input :-

mass (m)

Spring (K)

damper (C)

output :- $x, \dot{x}, \ddot{x}$

vibrations are due to F(t) (Input)

which deforms under the action of force - It's a structural member called Spring.

If friction is involved - dissipation of Energy - Non Conservative force

mass = kg

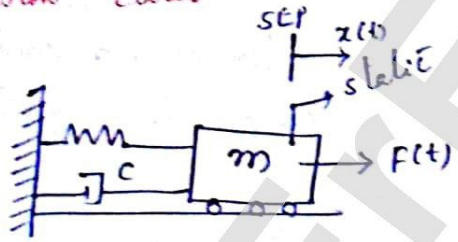
$K = \frac{N}{m}$

$C = \frac{N}{m/s} = \frac{N\text{-sec}}{m}$

$= \frac{N\text{-sec}^2}{m}$

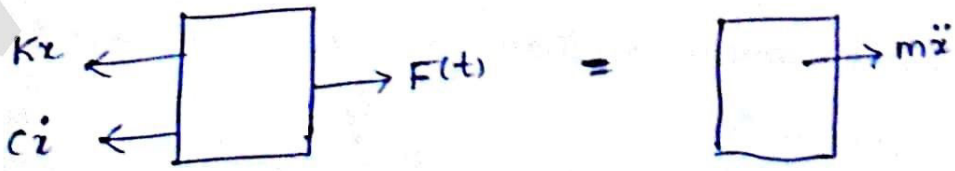
Formulation of Equation of motion :-

[Newton Euler method / Euler method / Free body diagram]



In static Equilibrium; No force $\Rightarrow F(t) = 0$.

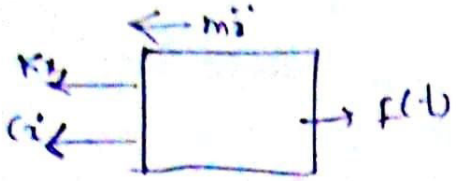
SEP is reference point. The positions the mass passes through is dynamic Equilibrium. for some duration body waits, then to next position.



$$-Kx - C\dot{x} + F(t) = m\ddot{x}$$

$$\boxed{m\ddot{x} + Kx + C\dot{x} = F(t)}$$

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when there is not external forces - free vibrations. obviously

$$m\ddot{x} + c\dot{x} + kx = 0$$

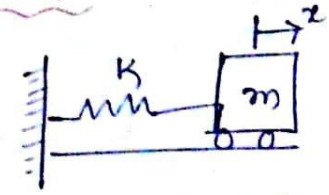
System without damping (undamped)

$$m\ddot{x} + kx = f(t) \quad \text{— forced}$$

if no external force

$$m\ddot{x} + kx = 0 \quad \text{— free}$$

Undamped free vibrations :-



$$m\ddot{x} + kx = 0$$

Boundary conditions :- $x(0) = x_0$ $\quad \dot{x}(0) = V_0$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$(D^2 + \frac{k}{m})x = 0$$

$$D = \pm i\sqrt{\frac{k}{m}} = \pm i\omega_n$$

$$x(t) = A\cos\omega_n t + B\sin\omega_n t \quad \text{— (1)}$$

$A = X\cos\phi$ $\quad B = X\sin\phi$

$$x(t) = X\cos\omega_n t\cos\phi + X\sin\omega_n t\sin\phi$$

$$= X\cos\omega_n t\cdot\cos\phi + X\sin\omega_n t\cdot\sin\phi$$

$$\boxed{x(t) = X\cos(\omega_n t - \phi)}$$

from (1)

$$x(0) = x_0$$

$$x_0 = A$$

$$\dot{x}(t) = -X\sin(\omega_n t - \phi)\cdot\omega_n = -A\sin\omega_n t\cdot\omega_n + B\omega_n\cos\omega_n t$$

$\dot{x}(0) = V_0$

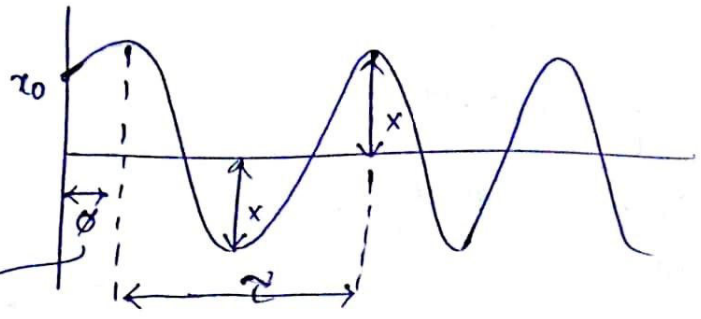
$$V_0 = B\omega_n$$

$$B = \frac{V_0}{\omega_n}$$

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$$X = \sqrt{A^2+B^2} = \sqrt{x_0^2+\left(\frac{V_0}{\omega_n}\right)^2} = \text{Max amplitude}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{V_0}{\omega_n x_0}\right)$$



Initial phase angle

Time period:- Time taken by system for 1 complete oscillation. $T = \frac{2\pi}{\omega_n}$

ω_n = natural frequency $= \sqrt{\frac{K}{m}}$ — K: N/m, m: kg

Energy method :-

$$E = K.E + P.E = \text{constant}$$

$$\frac{1}{2}ml^2\dot{\theta}^2 + mgl(1-\cos\theta) = \text{Constant}$$

$$\frac{dE}{dt} = 0$$

$$\frac{dE}{d\dot{\theta}}\cdot\frac{d\dot{\theta}}{dt} + \frac{dE}{d\theta}\cdot\frac{d\theta}{dt} = 0$$

$$\frac{1}{2}ml^2 2\dot{\theta}\ddot{\theta} + mgl(\sin\theta)\dot{\theta} = 0$$

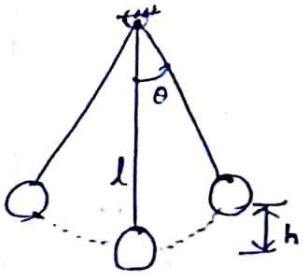
$$ml^2\ddot{\theta} + mgl\sin\theta = 0 \qquad (\dot{\theta}\neq 0)$$

$$ml^2\ddot{\theta} = -mgl\sin\theta$$

$$ml^2\ddot{\theta} + mgl\theta = 0 \qquad (\theta \text{ is small}, \sin\theta = \theta)$$

ml^2 → Inertia, mgl → Stiffness

$$\ddot{\theta} + \frac{mgl}{ml^2}\theta = 0 \qquad \left[\omega_n^2 = \frac{g}{l}\right]$$



$h = l - l\cos\theta$

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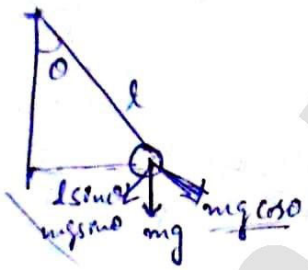
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$$T = 2\pi\sqrt{\frac{l}{g}}$$

Equilibrium method for simple pendulum :-

Take moment

$$ml^2\ddot{\theta} + mgl\sin\theta = 0$$



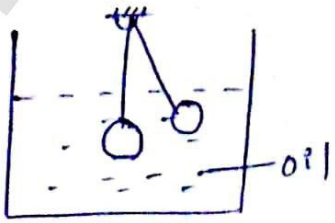
$I = ml^2$

$\ddot{\theta}$

Damped free vibrations :-

$T_d > T_{undamped}$

$\omega_d < \omega_n$



as viscosity ↑, damping ↑

Time period ↑, f ↓

Critical damping :-

$$c_c = 2\sqrt{km}$$

where the pendulum losses complete Energy and comes to Equilibrium - critical damping

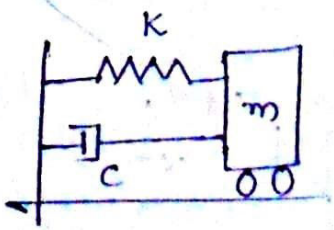
$\xi < 1$ $C < C_c$ - under damped — oscillations present here only

$\xi = 1$ $C = C_c$ - critical damped } No oscillations

$\xi > 1$ $C > C_c$ - over damped.

Always damping Expressed as percentage of critical damping.

$$\xi = \frac{C}{C_c} = \text{damping ratio}$$



$$m\ddot{x} + c\dot{x} + kx = 0$$

$x(0) = x_0$; $\dot{x}(0) = v_0$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

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$\omega_n^2 = \frac{k}{m}$; $\frac{c}{m} = \frac{c}{\sqrt{m}} \cdot \frac{c}{\sqrt{m}} \frac{\sqrt{k}}{\sqrt{k}} \cdot \frac{2}{2}$

$= 2 \cdot \frac{c}{c_c} \cdot \omega_n$

$\frac{c}{m} = 2\xi\omega_n$

$\boxed{c = 2m\xi\omega_n}$

$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$

$D^2 + 2\xi\omega_n + \omega_n^2 = 0$

$D = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$

$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$\xi = 1$, $\xi > 1$ no oscillations.

when $\xi < 1$ – under damped – oscillation

then $D = -\xi\omega_n \pm i\omega_n\sqrt{1-\xi^2}$

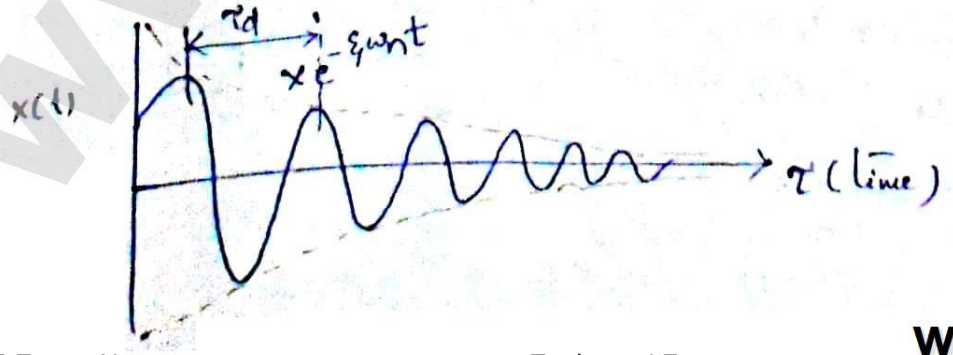
$= -\xi\omega_n \pm i\omega_d$ $\boxed{\omega_d = \sqrt{1-\xi^2}\,\omega_n}$

$\boxed{\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\tau_n}{\sqrt{1-\xi^2}}}$

$x(t) = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$

$x(t) = X e^{-\xi\omega_n t}\cos(\omega_d t - \phi)$ $X = \sqrt{A^2 + B^2}$

$\phi = \tan^{-1}\left[\frac{B}{A}\right]$



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Amplitude of vibrations decays Exponentially in damped free vibrations.

If 2 successive amplitudes are taken

$$\frac{x_i}{x_{i+1}} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\ln \frac{x_i}{x_{i+1}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \delta : \text{logarithmic decrement}$$

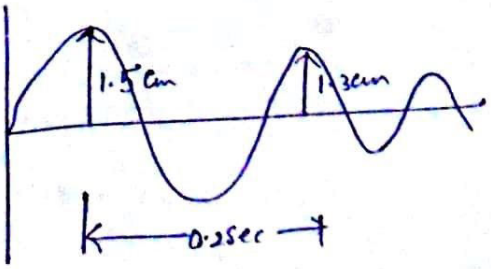
$$\delta = \ln\left(\frac{x_i}{x_{i+1}}\right)$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad ; \quad \xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \cdots$$

$$\delta = \frac{1}{n}\ln\left(\frac{x_i}{x_{i+n}}\right) \qquad \frac{x_i}{x_{i+n}} = e^{n\delta}$$

(1) The free vibration decay curve for spring mass damper system. If mass of system 1kg, determine (a) K (b) damping constant ?



$$\tau_d = 0.2\,\text{sec}$$

$$\frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = 0.2$$

$$\omega_n = \frac{2\pi}{0.2\sqrt{1-0.24^2}}$$

$$= 32.42\ \text{rad/sec}$$

$$\ln\left[\frac{1.5}{1.3}\right] = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

$$0.02278 = \frac{\xi}{\sqrt{1-\xi^2}}$$

$$\xi = 0.248 = \frac{\xi^2}{\frac{1-\xi^2}{\xi^2}}$$

$$\omega_n^2 = \frac{K}{m}$$

$$K = 1051\ N/m$$

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$$\frac{c}{c_c} = \zeta$$

$$c = \zeta\, 2\sqrt{km}$$

$$= 0.248 \times 2 \times \sqrt{1051}$$

$$= 16.08 \ \frac{N\text{-}sec}{m}$$

(2) In a spring mass damper system, the mass completes 5 oscillations in 1 sec. During these 5 oscillations the amplitude decays to 5% of initial value. If the stiffness of spring is 1N/mm. Determine a) mass of system b) c :

A)

$$T = \frac{1}{5} = 0.2 \text{ sec}$$

$$\omega = \frac{2\pi N}{T_d}$$

$$\omega_d = \frac{2\pi}{0.2} = 10\pi$$

$$\delta = \frac{1}{n} \cdot \ln\left[\frac{x_i}{x_{i+n}}\right]$$

$$= \frac{1}{5} \ln\left[\frac{x_i}{x_{i+5}}\right]$$

$$= \frac{1}{5} \ln\left[\frac{1}{0.05}\right]$$

$$= 0.598$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\delta^2 = \frac{4\pi^2\zeta^2}{1-\zeta^2}$$

$$\frac{1-\zeta^2}{\zeta^2} = \frac{4\pi^2}{\delta^2}$$

$$\frac{1}{\zeta^2} \cdot 01 = \frac{4\pi^2}{\delta^2} + 1$$

$$\zeta = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}}$$

$$\zeta = 0.052$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{10\pi}{\sqrt{1-0.052^2}}$$

$$= 31.47 \text{ radlse}$$

$$\omega_n^2 = \frac{1\times10^3}{m}$$

$$m = 1.007 \text{ kg}$$

$$C = 2\sqrt{km}\ \zeta$$

$$= 2\sqrt{1000 \times 1.007} \times 0.052$$

$$= 6.60 \ \frac{N\text{-}sec}{m}$$

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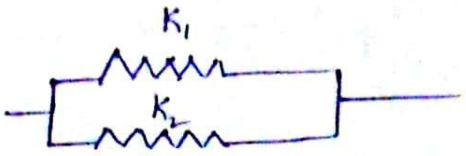
System of Springs :-

Series



$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$ 'δ' are sum up

Parallel



$K_{eq} = K_1 + K_2$

deflection is same throughout

$$\text{Stiffness of Spring} \propto \frac{1}{l}$$

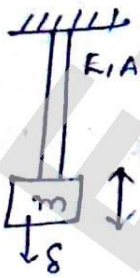
when Spring cut into 2 pieces - deflection will become 2 times that of initial K for each spring.

$\frac{\delta}{2}$, $\frac{\delta}{2}$. $\Rightarrow$ 2 times that of initial K for each spring.

$$K = \frac{F}{\delta/2} = \frac{2F}{\delta}$$

If Spring cut into n pieces, each spring has 'nK' value.

Structural members as Springs :-



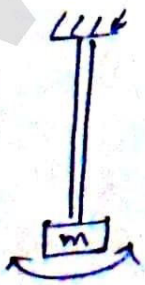
(Axial vibrations)

$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\sigma}{E} = \frac{F}{EA}$$

$$\boxed{\delta = \epsilon \cdot L = \frac{FL}{EA}}$$

$$K = \frac{F}{\delta} = \frac{EA}{L}$$



Acts as cantilever

$$\delta = 3\frac{Fl^3}{3EI}$$

Bending (δ)

(Transverse vibrations) perpendicular to the axis of Sping.



disc is having some twist 'θ'.

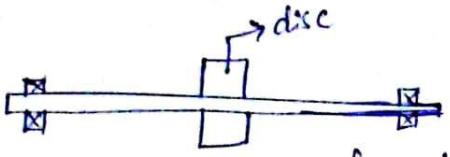
$$\frac{T}{J} = \frac{G\theta}{L}$$

$$K_t = \frac{T}{\theta} = \frac{JG}{L}$$

$$I_p\ddot{\theta} + K_t\theta = 0$$

$$\omega_n = \sqrt{\frac{K_t}{I_p}}$$

Critical Speeds of shafts :-



Bearings are referred as long bearings. Bearings are fixed end conditions.



Long bearings - fixed end conditions.

$$\delta = \frac{Fl^3}{192EI}$$ Long bearing

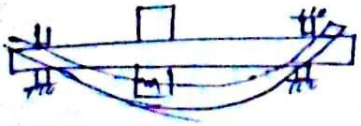
(Both slope zero at fixed ends)

$$K = \frac{192EI}{l^3}$$

$$\omega_n = \sqrt{\frac{192EI}{l^3 m}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

Consider, short bearings



Simply supported beam.

$$\delta = \frac{Fl^3}{48EI}$$ short bearing

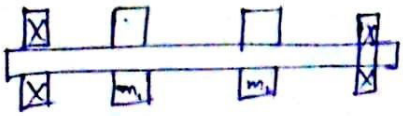
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If $\frac{l}{d} \geq 0.7$ long bearing
$\frac{l}{d} \leq 0.4$ short bearing
} whether short or long bearing determination.

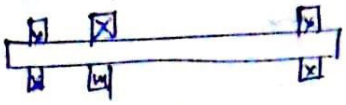
In bln 0.4 to 0.7 we should go for experimental values.

→ If shaft carries 2 discs, it will have 2 ω_n's.

for ω_n estimation Dunkerlay's method.

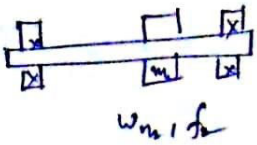


(i)



ω_{n_1}, f_{n_1}

(ii)

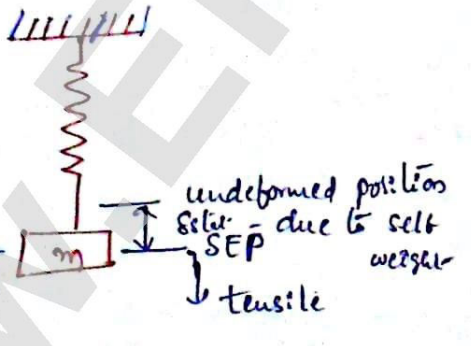


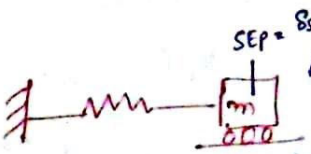
ω_{n_2}, f_{n_2}

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2}$$

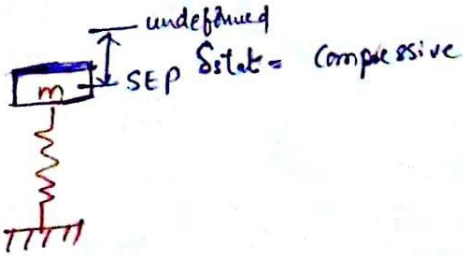
$$\frac{1}{f_n^2} = \frac{1}{f_{n_1}^2} + \frac{1}{f_{n_2}^2}$$

Eg:





No δ_{stat}.



Choose static Equilibrium position.

$$m\ddot{x} + k(x + \delta_{stat}) - mg = 0$$

$$\delta_{st} = \frac{mg}{k}$$

$k\delta_{st} = mg$

So, these are not considered in problem.

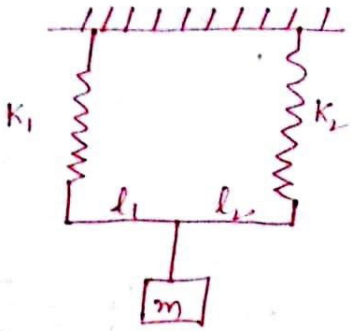
They are cancelling.

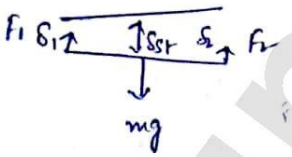
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$$\omega_n = \sqrt{\frac{g}{\delta_{stat}}}$$

$$K_{eq} = \frac{mg}{\delta_{stat}}$$

*





$$\delta_1 = \frac{F_1}{K_1}$$

$$\delta_2 = \frac{F_2}{K_2}$$

$$F_1 = \frac{mg\,l_2}{l_1 + l_2}$$

$$F_2 = \frac{mg\,l_1}{l_1 + l_2}$$

for springs to be parallel

$$\delta_1 = \delta_2$$

$$\frac{mg\,l_2}{(l_1+l_2)K_1} = \frac{mg\,l_1}{(l_1+l_2)K_2}$$

$$l_2 K_2 = K_1 l_1$$

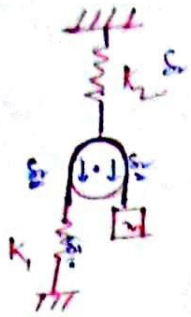
$$\delta_{st} = \delta_1 + \left(\frac{\delta_2 - \delta_1}{l_1 + l_2}\right) l_1$$

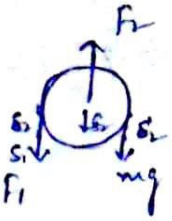
$$K_{eq} = \frac{mg}{\delta_{st}}$$

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}}$$

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mass of pulley negligible





$F_1 = mg$

$F_2 = 2mg$

$\delta_1 = \frac{F_1}{K_1}$; $\delta_2 = \frac{F_2}{K_2}$

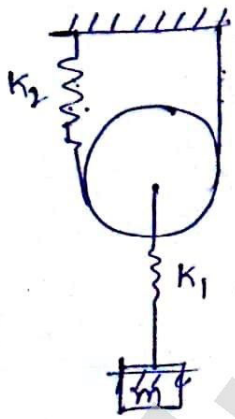
as centre of pulley goes down by δ_2.

deflection δ_2 in K_1, mg.

δ_1 in K_1.

$\delta_{st} = \delta_1 + 2\delta_2$

*



$F_2 = T = \frac{mg}{2}$

$\delta_{st} = \delta_1 + \frac{\delta_2}{2}$

If K_2 goes δ_2, centre of pulley deflects by $\frac{\delta_2}{2}$

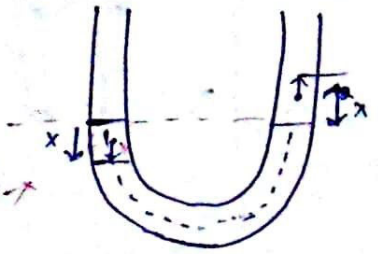


→ U-tube manometer :-

A = A. CSA of column

L = length of column

ρ = density



$PE = (\rho Ax)gx = \frac{1}{2}(2\rho Ag)x^2$ ↓ K_{eq}

$K.E = \frac{1}{2}(\rho AL)\dot{x}^2$

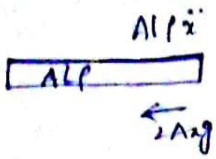
centre of column down by $\frac{x}{2}$, up by $\frac{x}{2}$. So, total = x.

$AL\rho\ddot{x} + 2\rho Agx = 0$

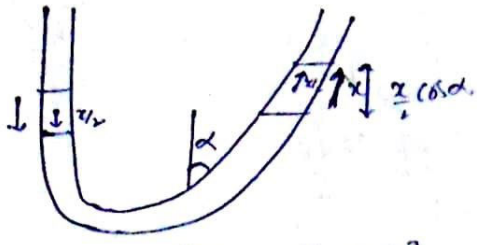
$\omega_n = \sqrt{\frac{2\rho Ag}{AL\rho}} = \sqrt{\frac{2g}{L}}$

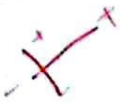
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In Equilibrium method



$$AL\rho\ddot{x} + 2A\rho gx = 0$$





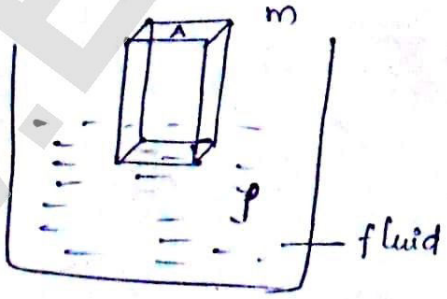
$$PE = (A\rho x)g\left[\frac{x}{2} + \frac{x}{2}\cos\alpha\right]$$

$$= \frac{1}{2}A\rho g(1+\cos\alpha)x^2$$

K.E : $\frac{1}{2}(A\rho L)\dot{x}^2$ (m)

$$A\rho L\ddot{x} + A\rho g(1+\cos\alpha)x = 0$$

$$\omega_n = \sqrt{\frac{g(1+\cos\alpha)}{L}}$$



Self weight, buyonacy forces cancelled.



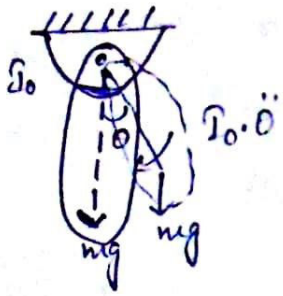
$$m\ddot{x} + A\rho xg = 0$$

$$\omega_n = \sqrt{\frac{A\rho g}{m}}$$

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Compound pendulum :-

Any object / shape pivoted at any point other than center of gravity is called compound object



Taking moment

$$I_0 \ddot{\theta} + mgl\sin\theta = 0$$

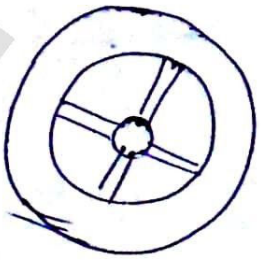
$$I_0 \ddot{\theta} + mgl\theta = 0 \qquad [\sin\theta \simeq \theta \text{ small}]$$

$$\omega_n = \sqrt{\frac{mgl}{I_0}}$$

$$I_0 = I_{cg} + ml^2$$

At the center of percussion, max momentum transfer.

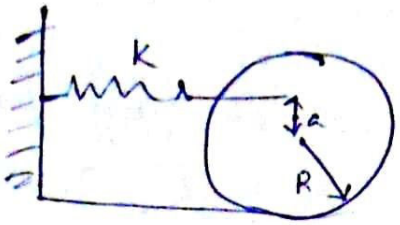
Fly wheel :-



(Refer text-book)

$$I_0 \ddot{\theta} + \dots$$

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Let centre of disc moved by x, angular displacement $= \theta$.

$$\delta = x + a\theta$$
$$= R\theta + a\theta$$
$$= (R+a)\theta$$

$$K.E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}mR^2\dot{\theta}^2$$

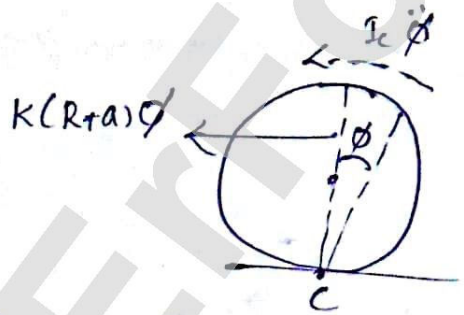
$x = R\theta$

$$= \frac{1}{2}m(R\dot{\theta})^2 + \frac{1}{2}mR^2\dot{\theta}^2 = \frac{1}{2}\left[\frac{3}{2}mR^2\right]\dot{\theta}^2$$

$$P.E = \frac{1}{2}k(R+a)\theta^2$$

$$\omega_n = \sqrt{\frac{k(R+a)^2}{3/2\, mR^2}}$$

[Take moment Equilibrium about contact point when both rotation & Translations are there]



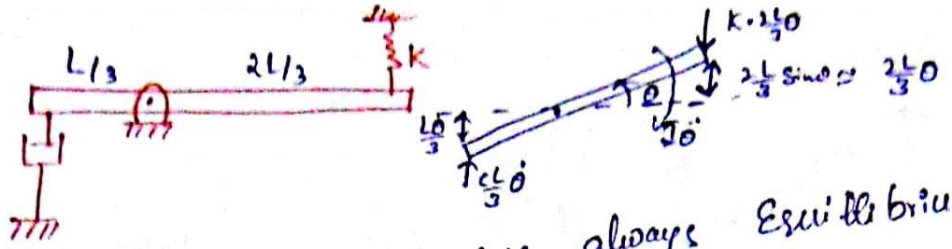
$$I_c = I_{CG} + mR^2$$

$$= \frac{mR^2}{2} + mR^2 = \frac{3}{2}mR^2$$

$$I_c\ddot{\phi} + k(R+a)\phi(R+a) = 0$$

$$\frac{3}{2}mR^2\ddot{\phi} + k(R+a)^2\phi = 0$$

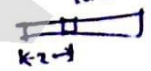
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when damper is There., use always Equilibrium method

Consider moment of es about a fixed end point

$$I\ddot{\theta} + K\frac{2l}{3}\theta\cdot\frac{2l}{3}\cdot\theta + \left(\frac{cL}{3}\right)\cdot\frac{L}{3}\cdot\dot{\theta} = 0$$



$\rho = \frac{m}{l}$

$$\left(\frac{mL^2}{9}\right)\ddot{\theta} + \left(\frac{cL^2}{9}\right)\dot{\theta} + \left(\frac{4KL^2}{9}\right)\theta = 0$$

$$\left.\frac{\rho x^3}{3}\right]_{-\frac{l}{3}}^{2l/3} = \frac{ml^2}{9}$$

$$\omega_n^2 = \frac{\frac{4KL^2}{9}}{\frac{mL^2}{9}} = \frac{4K}{m}$$

$$\int \rho x^2 dx$$

$$\left.\frac{\rho x^3}{3}\right|_0^l = \frac{ml^2}{3}$$

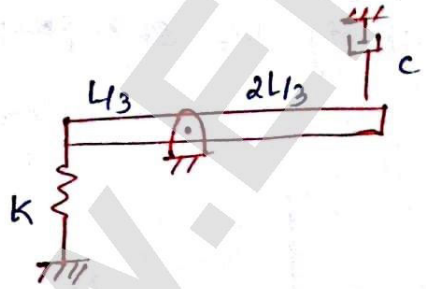
$$\omega_n = 2\sqrt{\frac{K}{m}}$$

$$\left.\frac{\rho x^3}{3}\right|_{-\frac{l}{2}}^{l/2} = \frac{\rho l^3}{12} = \frac{ml^2}{12}$$

$$\xi = \frac{C_{eq}}{C_c} = \frac{C_{eq}}{2\sqrt{k_{eq}\cdot m_{eq}}} = \frac{\frac{cL^2}{9}}{2\sqrt{\frac{mL^2}{9}\cdot\frac{4KL^2}{9}}}$$

$$= \frac{c}{4\sqrt{Km}}$$

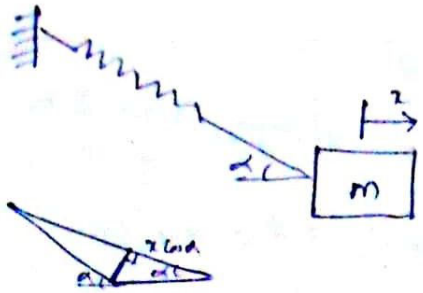
(HW)

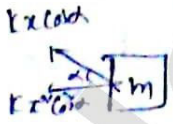


$$\omega_n = \sqrt{\frac{K}{m}} \qquad \xi = \frac{2c}{\sqrt{Km}}$$

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Inclined Springs:-

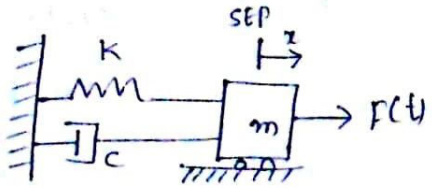




$$m\ddot{x} + kx\cos\alpha = 0$$

$$\omega_n = \sqrt{\frac{k\cos\alpha}{m}}$$

forced vibrations :-



$$m\ddot{x} + c\dot{x} + kx = F(t)$$

sol = C.F + P.I

↳ because of R.H.S term

(C.F) - free vibrations — Complementary function

forced vibrations - P.I — steady state (SS) response

Transient response

Rolling vector diagram:

Generally

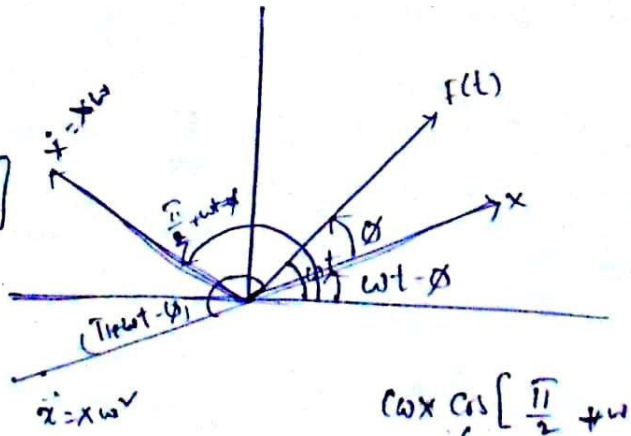
$$x(t) = X\cos(\omega t - \phi)$$

$$\dot{x}(t) = -X\omega\sin(\omega t - \phi)$$

$$= X\omega\cos\left[\frac{\pi}{2} + (\omega t - \phi)\right]$$

$$\ddot{x} = -X\omega^2\cos(\omega t - \phi)$$

$$= X\omega^2\cos(\pi + \omega t - \phi)$$

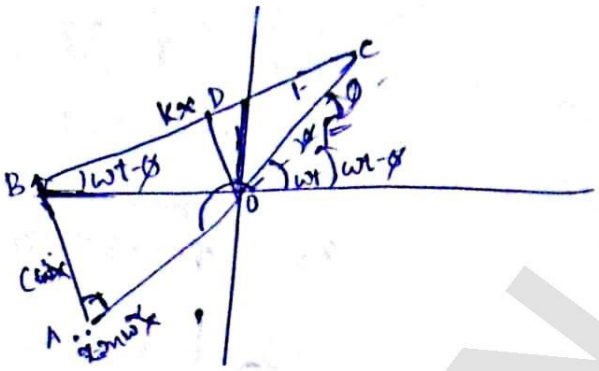


$$m\ddot{x} + kx$$

$$m\omega^2 X\cos(\pi + \omega t - \phi) + kX\cos\left[\frac{\pi}{2} + \omega t - \phi\right]$$

$$+ kX\cos(\omega t - \phi) = F\cos\omega t$$

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At first if is drawn

Δ ODC

$DC^2 + OD^2 = OC^2$

$DC = BC - Bd = kx - m\omega^2 x$

$= (k - m\omega^2)x$

$OD = AB = C\omega x$

$OC = F$

$((k - m\omega^2)x)^2 + (C\omega x)^2 = F^2$

$$x = \frac{F}{\sqrt{(k - m\omega^2)^2 + (C\omega x)^2}}$$

$$\frac{k - m\omega^2}{k} = 1 - \left(\frac{\omega}{\omega_n}\right)^2$$

$$\frac{C\omega}{k} = \frac{C}{m\omega_n^2}\cdot\frac{\omega}{\omega_n} = 2\xi\left(\frac{\omega}{\omega_n}\right)$$

$$\frac{F}{k} = X_{st}$$

$$x = \frac{X_{st}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{C\omega}{k - m\omega^2}\right]$$

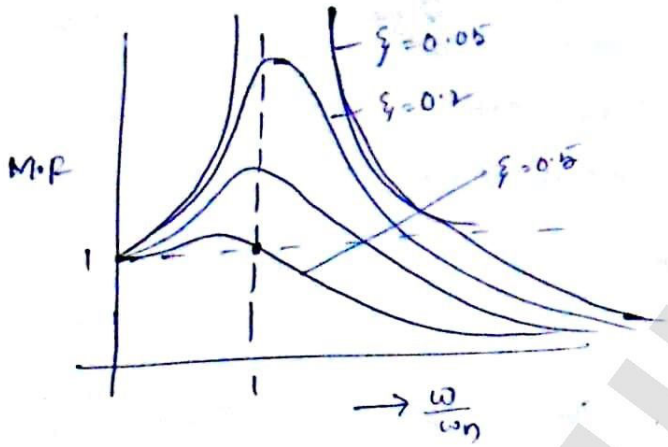
$$\boxed{\frac{x}{x_{st}} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}} = \text{magnification factor}$$

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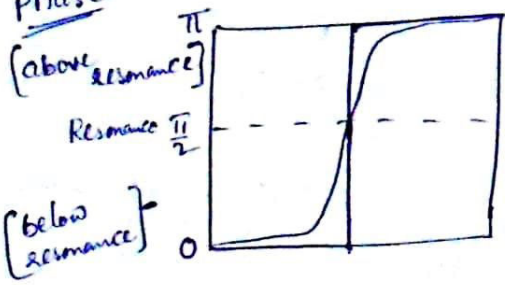
Max. M.F = @ $\frac{\omega}{\omega_n} = \sqrt{1-2\xi^2}$

@ $\frac{\omega}{\omega_n} = 1$ M.F = $\frac{1}{2\xi}$

$\omega_R = \sqrt{1-2\xi^2}\ \omega_n$



Phase

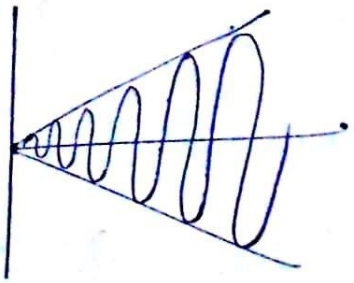


Tachoma narrow bridge.

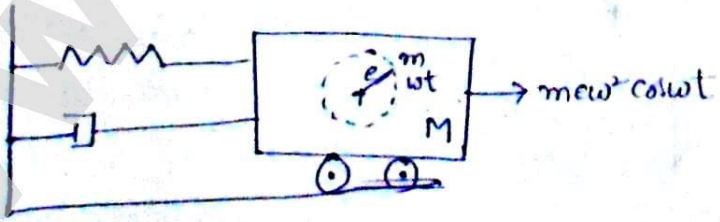
Undamped forced Vibration at resonance :-

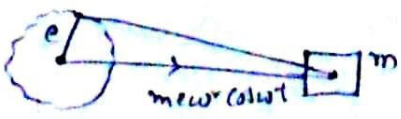
$m\ddot{x} + kx = F\cos\omega_n t$

$x(t) = \frac{F}{2k}\omega_n t \sin\omega_n t$



Rotating / Reciprocating Unbalances :-





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$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \cos\omega t$$

$$x(t) = X \cos(\omega t - \phi)$$

$$X = \frac{1}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}$$

$$\omega_n = \sqrt{\frac{k}{M}}$$

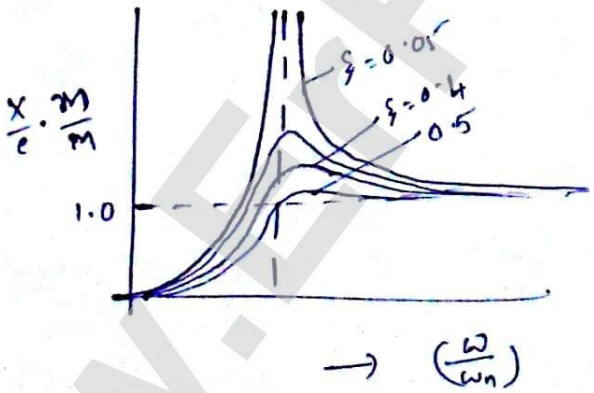
$$\frac{me\omega^2}{k} = \frac{me\omega^2}{M\omega_n^2}$$

$$= \frac{m}{M} \cdot e \cdot \left(\frac{\omega}{\omega_n}\right)^2$$

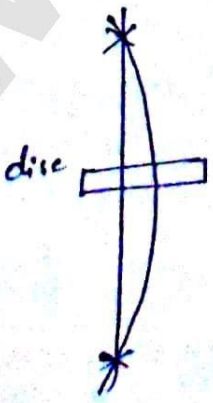
$$X = \frac{\frac{m}{M} \cdot e \cdot \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega^2}{\omega_n^2}\right)\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

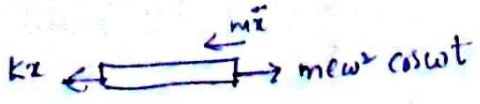
$$\frac{X}{e} \cdot \frac{m}{M} = \left(\frac{\omega}{\omega_n}\right)^2 \cdot (MF)$$

@ $\frac{\omega}{\omega_n} = 1$;



whirling of shafts / critical speeds :-





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$m\ddot{x} + kx = me\omega^2 \cos\omega t$

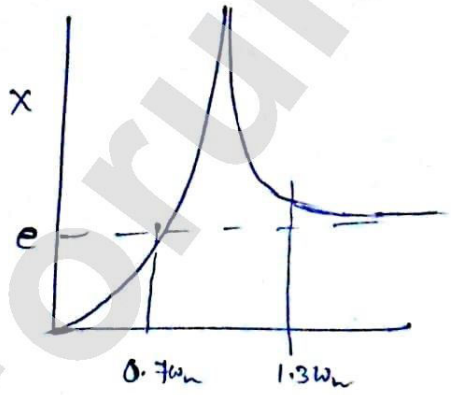
$x(t) = X\cos(\omega t - \phi)$

↳ whirling radius.

$$X = \frac{me\omega^2}{k - m\omega^2}$$

$$= \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

If shaft speed matches with ω_n; $X \to \infty$, called critical speed.



we can run the system

$< 0.7\omega_n$ or $> 1.3\omega_n$.

Rigid rotors $\omega \leq 0.7\omega_n$

flexible rotors $\omega \geq 1.3\omega_n$

If $\frac{\omega}{\omega_n} = 0.5$

$$X = \frac{e[0.5]^2}{1 - (0.5)^2}$$

$$= \frac{e}{3}$$

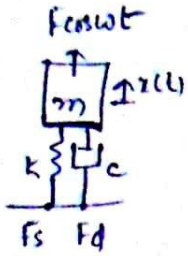
$\frac{\omega}{\omega_n} = 1$ $X = \infty$

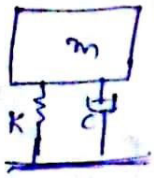
$\frac{\omega}{\omega_n} = 10$. $X = \frac{e(10)^2}{1 - (10)^2} \simeq -e$. [Centre of axis of shaft - C.G - shifts so, -ve]

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Transmissibility :- Transmission of force / Motion

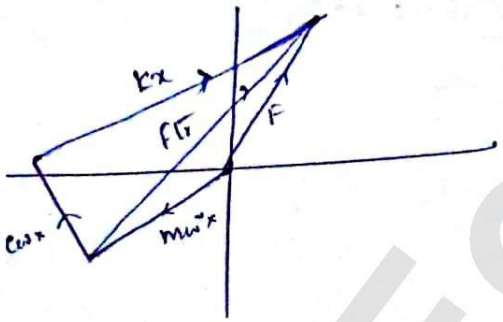
- force
- motion





base is moving $y(t) = y\cos(\omega t - \phi)$

$\bar{F}s + \bar{F}d$ = Total transmission force



$$F_k = \sqrt{(c\omega x)^2 + (kx)^2}$$

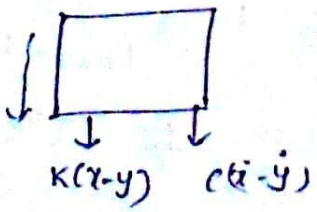
$$\frac{F_k}{x} = \sqrt{k^2 + (c\omega)^2}$$

$$F_t = \frac{\sqrt{k^2 + (c\omega)^2} \cdot F}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$\frac{F_t}{F} = \sqrt{\frac{k^2 + (c\omega)^2}{(k - m\omega^2)^2 + (c\omega)^2}}$$

T.R = Transmissibility ratio $= \dfrac{\sqrt{1 + (2\zeta\frac{\omega}{\omega_n})^2}}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2}}$

when base is in motion



$$m\ddot{x} + k(x-y) + c(\dot{x}-\dot{y}) = 0$$

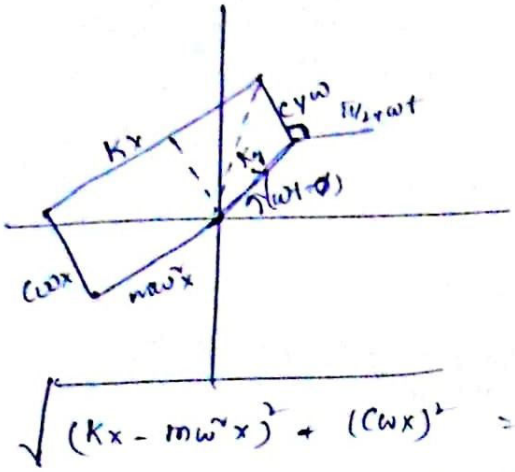
$$m\ddot{x} + c\dot{x} + kx = by + cy$$

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$y(t) = Y\cos(\omega t - \phi)$

$\dot{y}(t) = -Y\sin(\omega t - \phi)\,\omega$

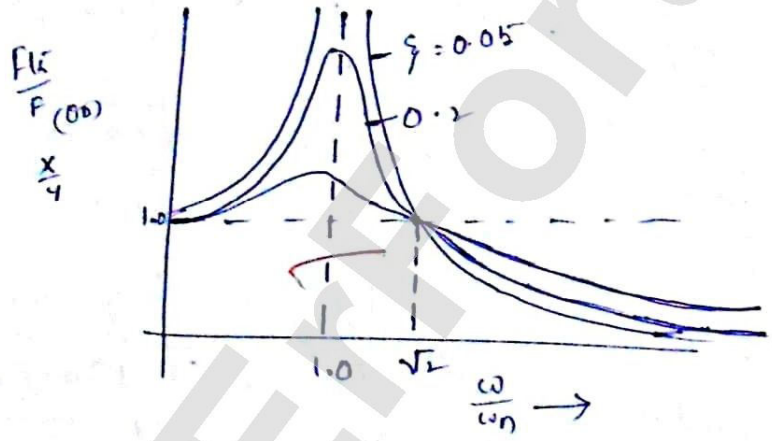
$Y\omega \cos\left[\frac{\pi}{2} + \omega t - \phi\right]$



$$\sqrt{(Kx - m\omega^2 x)^2 + (C\omega x)^2} = \sqrt{(Ky)^2 + (Cy\omega)^2}$$

Amplitude ratio $\frac{X}{Y}$ = amplitude of base

$$= \sqrt{\frac{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$



a) $\frac{\omega}{\omega_n} = 0$ $TR = 1$ for all ζ.

b) $0 < \frac{\omega}{\omega_n} < 1$ then $TR > 1$, $TR \uparrow$ as $\uparrow$ in $\frac{\omega}{\omega_n}$

$TR \downarrow$ with increase in ζ

c) $\omega = \omega_n$: TR is max

d) $1 < \frac{\omega}{\omega_n} \leq \sqrt{2}$, TR is decreases an increase in frequency ratio.

$\omega \downarrow$, $\zeta \downarrow$; $TR \uparrow$, $\zeta \uparrow$ $TR \downarrow$ as $\frac{\omega}{\omega_n} \uparrow$

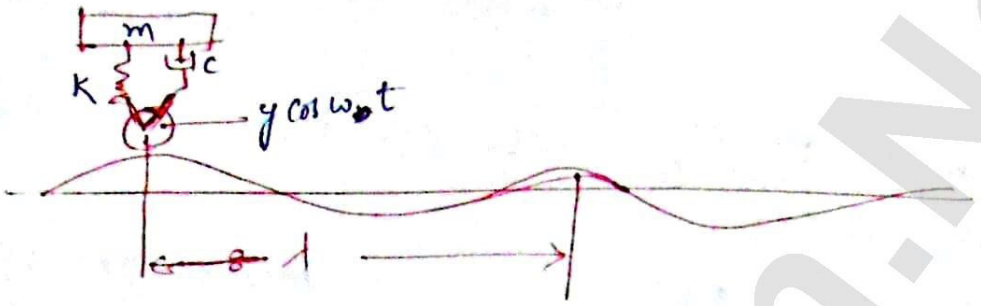
$TR \downarrow$ as $\zeta \uparrow$

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for $\frac{\omega}{\omega_n} = \sqrt{2}$ TR = 1 for all ζ

$\frac{\omega}{\omega_n} > \sqrt{2}$ TR < 1 TR ↓, = $\frac{\omega}{\omega_n}$ ↑, TR ↑, ζ ↑

Eg:



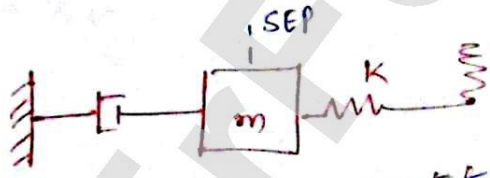
Time taken for 1 wave length to cover = $T = \frac{\lambda}{V}$ $v = f\lambda$

$\omega = \frac{2\pi V}{\lambda}$

$\omega_{cr} = \omega_n = \frac{2\pi V}{\lambda}$

$V_{cr} = \frac{\lambda \times \omega_n}{2\pi}$

Eg:



Spring given an Excitation $y(t)$. Determine (a) amplitude of vibration of mass (m) (b) The force transmitting to support on left side. If force end of

(A)



$c\dot{x} + k(x-y) + m\ddot{x} = 0$

$m\ddot{x} + c\dot{x} + kx = ky$

$= ky\cos\omega t$

$\therefore X = \frac{KY}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$

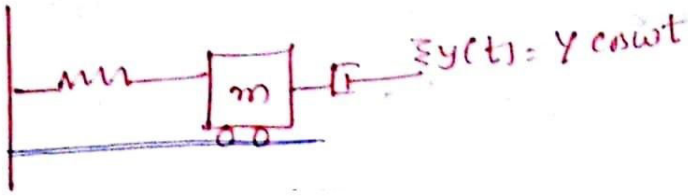
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only damping force is transmitting on to left side

$F_t = C\dot{x} = C\omega X$

$$\frac{C\omega ky}{\sqrt{(k-m\omega^2)^2 + (C\omega)^2}}$$

(2)



$m\ddot{x} + C\dot{x} + kx = C\dot{y}$

$= -CY\omega \sin\omega t$

(A)

$$X = \frac{-CY\omega}{\sqrt{(k-m\omega^2)^2 + (C\omega)^2}}$$

$F_T = KX$

$$F = \frac{KCY\omega}{\sqrt{(k-m\omega^2)^2 + (C\omega^2)^2}}$$

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(3) $T_n = 2\pi\sqrt{\frac{m}{k}} = 2$

$T_{n_1} = 2\pi\sqrt{\frac{m+2}{k}} = 3$

$m = 1.6$ kg

$\frac{m}{m+2} = \frac{4}{9}$

$9m = 4m + 8$

$5m = 8$

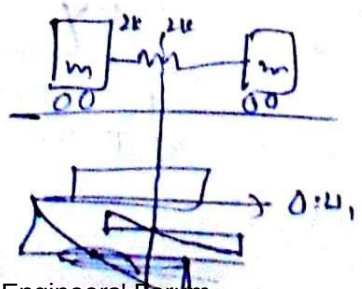
$m = \frac{8}{5} = 1.6$ kg

(5) k $\quad$ k $\quad$ $f = kx$

$2k$ $\quad$ $2k$ $\quad$ $4k$ $\quad$ $f = 4kx$

$\sqrt{\frac{k}{m}}$ $\quad$ $\sqrt{\frac{4k}{m}}$ $\quad$ $\sqrt{\frac{4k}{2}}$

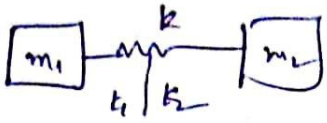
(7)



$\sqrt{\frac{2k}{m}}, 0$

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(a) Eg:-



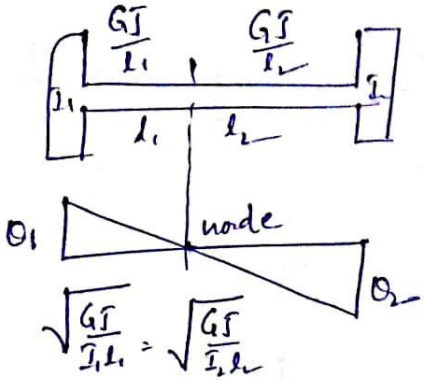
node not at center now

$$\sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k_2}{m_2}}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\omega_1 \sqrt{\frac{k}{\frac{m_1 m_2}{m_1 + m_2}}}$$

Eg:-



$$\sqrt{\frac{GJ}{I_1 l_1}} = \sqrt{\frac{GJ}{I_2 l_2}}$$

$$l_1 I_1 = l_2 I_2$$

$$\frac{\theta_1}{\theta_2} = \frac{l_1}{l_2} = \frac{I_2}{I_1}$$ [Node is close to heavy rotor]

(8)

$$\frac{X}{Y} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{X}{Y} = \frac{1}{1-r^2}$$

$\omega = 2\pi 10\pi$

$$\frac{0.01}{0.2} = \frac{1}{1-r^2}$$

$\frac{\omega}{\omega_n} = 4.$

$1 - r^2 = -20$

$$1 - r^2 = \pm \frac{0.2}{0.01}$$

$0.01 \quad \omega_n^2 \quad \frac{k}{m}$

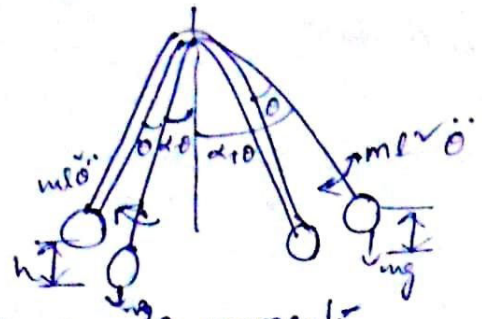
$\frac{X}{Y} = \frac{k}{k - m\omega^2}$

$= 0.2$

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(9)

To get Eqution of motion



about axis. moment

$$ml^2\ddot{\theta} + ml^2\ddot{\theta} + mgl\sin(\alpha+\theta) - mgl\sin(\alpha-\theta) = 0$$

$$2ml^2\ddot{\theta} + 2mgl\cos\alpha\sin\theta = 0$$

$$2ml^2\ddot{\theta} + 2mgl\cos\alpha\,\theta = 0 \qquad \sin\theta \simeq \theta$$

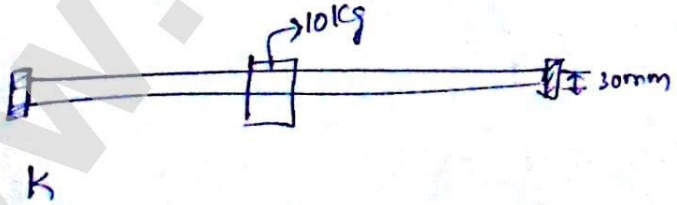
$$\omega_n = \sqrt{\frac{2mgl\cos\alpha}{2ml^2}} = \sqrt{\frac{g\cos\alpha}{l}}$$

By Energy method

$$K.E = \frac{1}{2}ml^2\dot{\theta}^2 + \frac{1}{2}ml^2\dot{\theta}^2$$

$$P.E = mg($$

(10)



Simply supported

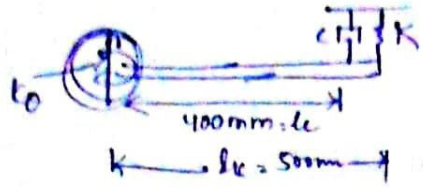
$$K = \frac{48EI}{l^3}$$

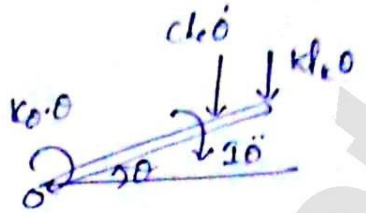
$$= \frac{48 \times 2.1 \times 10^{11} \times \pi\left[30\times10^{-3}\right]^4}{64 \times (0.5)^3}$$

$$f = 90\ Hz$$

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(11)





If bar is displaced by θ

$T = k_\theta \cdot \theta$

Take moments about 'O'.

$$k_\theta \cdot \theta + I\ddot\theta + c l_c \dot\theta l_c + (k l_k \theta) l_c = 0$$

$$\frac{m l^2}{3}\ddot\theta + c l_c^2 \dot\theta + k l_k^2 \theta + k_\theta \theta = 0$$

$$\frac{m l^2}{3}\ddot\theta + c l_c^2 \dot\theta + (k l_k^2 + k_\theta)\theta = 0$$

(12) $\frac{m l^2}{3} \rightarrow m_{eq}$; $c l_c^2 \rightarrow C_{eq}$; $k l_k^2 + k_\theta \rightarrow k_{eq}$

C_{eq} units $= \frac{N}{\frac{m}{s}} \times m^2$

$= N\text{-sec-m}$

(13) (k_1, k_2) series $\rightarrow \frac{k_1 k_2}{k_1 + k_2}$ + k_3 parallel.

(14) $k = \dfrac{k_{22}^3 \times 200 \times 10^9 \times \frac{(2\cdot5\times10^{-3})^4}{12}}{(1)^3}$. $\frac{3 k_{22} EI}{l^3}$

$k = 3$ =

$C_c = 2\sqrt{km}$

Rectangle $\frac{bd^3}{12}$

for square $= \frac{b^4}{12}$

(17) $m\ddot x + kx + c[\dot x - \dot y] = 0$

$m\ddot x + kx + c\dot x - c\dot y = 0$

(18) $m = 210$ kg, $K = 100000$ N/m

mean 350 N

$\omega_n = \sqrt{400000} = 20$

$\omega = \frac{3600 \times 2\pi}{60}$

$= 120\pi$

$\xi = 0.15$

$$\frac{F_T}{F_0} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-(6\pi)^2)^2 + (2\xi 6\pi)^2}}$$

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(23) $\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$

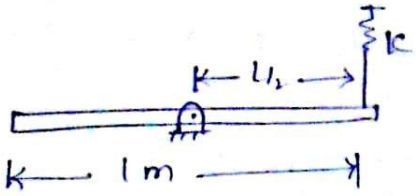
$x(t) = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$

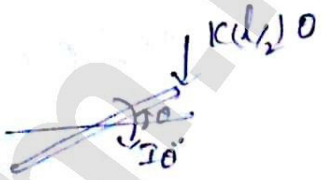
$t = nT_d$

$-\xi\omega_n t = -\xi\cdot\omega_n\cdot T_d\, n$

$= -\xi\cdot\omega_n\cdot\frac{2\pi}{\omega_n\sqrt{1-\xi^2}}\cdot n$

(24)





$I\ddot{\theta} + K\cdot\left(\frac{l}{2}\right)\theta\cdot\left(\frac{l}{2}\right) = 0$

about C.G

$I = \frac{ml^2}{12}$

$\frac{ml^2}{12}\cdot\ddot{\theta} + \frac{kl^2}{4}\cdot\theta = 0$

$\omega_n = \sqrt{\frac{\frac{kl^2}{4}}{\frac{ml^2}{k3}}} = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3\times300}{1}} =$

If displacement of mass, bolt the springs same.

Parallel.

(26) $\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.8}{1.8\times10^{-3}}}$ $\omega_n = \frac{2\pi N}{60}$

(27) $K_{eq} = 32 + 32 + 16 + 16$

$\omega_n = \sqrt{\frac{96\times10^6}{240}}$

(28) $K = 3600$ N/M, $C = 4100\ \frac{NS}{m}$ $m = 50$ kg

$C_c = 2\sqrt{km}$ $\xi = 0.471$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

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(29)

$T = 2\pi\sqrt{\frac{I}{g}}$

$T \propto \frac{1}{\sqrt{g}}$

$T_1\sqrt{g_1} = T_2\sqrt{g_2}$

$\omega_n \propto \sqrt{g}$

$\frac{\omega_{n_1}}{\sqrt{g_1}} = \frac{\omega_{n_2}}{\sqrt{g_2}}$

$\frac{\omega_{n_2}}{\omega_{n_1}} = \sqrt{\frac{g_{eq}/6}{g}}$

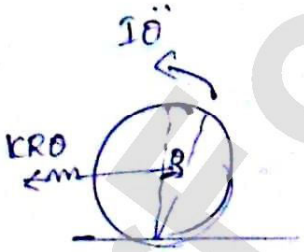
$\omega_n = \frac{2\pi}{T}$

$= \frac{2\pi}{T_1\sqrt{g_1}}\sqrt{g_1}$

(30) $X = \frac{F}{k - m\omega^2}$ $\omega = 100$

$50\times10^{-3} = \frac{100}{3000\,m - m[100]^2}$

(31)



I about contact pt

$= \frac{mr^2}{2} + mr^2$

$= \frac{3}{2}mr^2$

$\frac{3}{2}mR^2\ddot{\theta} + kR^2\theta = 0$

$\omega_n = \sqrt{\frac{kR^2}{\frac{3}{2}MR^2}} = \sqrt{\frac{2k}{3m}}$

Energy

$PE = \frac{1}{2}(k)x^2$

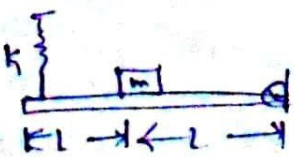
$K.E = \frac{1}{2}mx^2 + \frac{1}{2}\underbrace{\frac{mR^2}{2}}_{I_{CG}}\left(\frac{\dot{x}}{R}\right)^2$

$= \frac{1}{2}\left(\frac{3m}{2}\right)\dot{x}^2$

$x = R\theta$

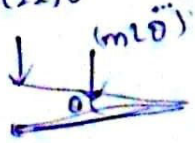
$\theta = \frac{x}{R}$

(34)



$I\ddot{\theta} + k(2l\theta)(2l) = 0$ (05)

moments



$a = L\ddot{\theta}$

$k(2l)\theta(2l) + (ml\ddot{\theta})(l) = 0$

$\omega_n = \sqrt{\frac{k\cdot 4l^2}{ml^2}} = \sqrt{\frac{4k}{m}}$

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(32) $x(0) = 0$

$\int f \cdot mv$ $\int x \cdot f$

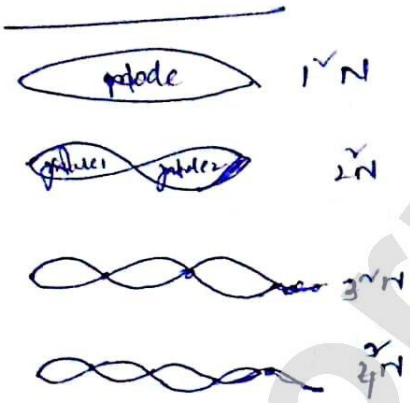
$10^{-4} \times 5000 = 1 \times V_0$

$V_0 = 0.5$ m/s

$X = \sqrt{x_0^2 + \left(\frac{V_0}{\omega_n}\right)^2} = \left\{ \frac{V_0}{\omega_n} = \frac{0.5}{1000} = 0.005\text{m} \right.$

$= 5$ mm

(33)



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(4) $M = 100$ kg

$m = 20$ kg. $e = 0.5$ mm $c =$

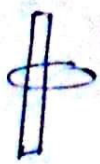
$k = 85000 \frac{N}{m}$

$\omega = 20\pi$

$X = \frac{m e \omega^2}{(k - M\omega^2)^2 + (c\omega)^2} = \frac{20 \times 0.5 \times 10^{-3} \times (20\pi)^2}{\left[85000 - (100 \times 400\pi^2 \times 100)\right]^2}$

=

(6)



$e = 2$mm

$\omega_n = 10$ rad/sec $\gamma = \frac{1}{11}\pi$

$\omega = \frac{2\pi \times 300}{60} = 10\pi$

$X = \frac{e(\gamma^2)}{1 - \gamma^2} = \frac{e(\pi^2)}{1 - \pi^2}$

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(7) $\frac{1}{2} I_2' \omega_A^2 = \frac{1}{2} I_2 \omega_B^2$

$$I_2' = I_2 \cdot \left(\frac{\omega_B}{\omega_A}\right)^2 = I_2 \cdot n^2$$

(8) * — for long bearings more critical speeds

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