ACE
Engineering Academy

Head Office : Sree Sindhi Guru Sangat Sabha Association, # 4-1-1236/1/A, King Koti, Abids, Hyderabad – 500001

HYDERABAD | DELHI | BHOPAL | PUNE | BHUBANESWAR | LUCKNOW | PATNA | BENGALURU | CHENNAI | VIJAYAWADA | VIZAG | TIRUPATHI | KUKATPALLY | KOLKATA

ESE-2018 MAINS TEST SERIES
Question Cum Answer Booklet (QCAB)

Electrical Engineering **Test-2** **Paper-II**

Control Systems + Systems and Signal Processing

Time Allowed: 3 Hours **Maximum Marks: 300**

ACE HALL TICKET No. :

HALL TICKET No. : (Issued by UPSC)

NAME OF THE CANDIDATE :

NAME OF THE CENTRE :

BRANCH : BATCH : ROLL No. : MOBILE No. :

TEST CODE : 202

DATE : 07-04-2018

INSTRUCTIONS TO CANDIDATES:

- This Question-cum- Answer (QCA) Booklet contains **68** pages. Immediately on receipt of booklet, please check that this QCA booklet does not have any misprint or torn or missing pages or items, etc. If so, get it replaced by a fresh QCA booklet.
- Candidates must read the instructions on this page and the following pages carefully before attempting the paper.
- Candidates should attempt all questions strictly in accordance with the specified instructions and in the space prescribed under each question in the booklet. Any answer written outside the space allotted may not be given credit.
- Question Paper in detachable form is available at the end of the QCA booklet and can be removed and taken by the candidates after conclusion of the exam

For filling by Examiners only

Question No.	Page No.	Marks
1	3	
2	11	
3	19	
4	27	
5	34	
6	42	
7	49	
8	56	
Grand Total		

Signature of the Invigilator

Signature of the Student

Marks Secured after Scrutiny

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions:

*There are **EIGHT** questions divided in **TWO** sections.*

*Candidate has to attempt **FIVE** questions in all.*

*Questions no. **1** and **5** are **compulsory** and out of the remaining, **THREE** are to be attempted choosing at least **ONE** question from each section.*

The number of marks carried by a question / part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-cum-Answer (QCA) Booklet in the space provided. No marks will be given for answers written in medium other than the authorized one.

Assume suitable data, if considered necessary and indicate the same authorized one.

Unless otherwise mentioned, symbols and notations carry their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

DONT'S:

1. Do not write your Name or Roll number or Sr. No. of Question-Cum-Answer-Booklet anywhere inside this Booklet.
   Do not sign the "Letter Writing" questions, if set in any paper by name, nor append your roll number to it.
2. Do not write anything other than the actual answers to the questions anywhere inside your Question-Cum-Answer-Booklet.
3. Do not tear off any leaves from your Question-Cum-Answer-Booklet. If you find any page missing, do not fail to notify the Supervisor/invigilator.
4. Do not write anything on the Question Paper available in detachable form or admission certificate and write answers at the specified space only.
5. Do not leave behind your Question-Cum-Answer-Booklet on your table unattended, it should be handed over to the Invigilator after conclusion of the exam.

DO'S :

1. Read the instructions on the cover page and the instructions specific to this Question Paper mentioned on the next page of this Booklet carefully and strictly follow them.
2. Write your Roll number and other particulars, in the space provided on the cover page of the Question-Cum-Answer-Booklet.
3. Write legibly and neatly. Do not write in bad/illegible handwriting.
4. For rough notes or calculations the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be valued.
6. Hand over your Question-Cum-Answer-Booklet personally to the invigilator before leaving the examination hall.
7. **Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.**

SECTION -A

Candidates must not write on this margin

1(a). The open loop transfer function of a unity feedback system is given by $G(s)=\frac{3e^{-2s}}{s(s+2)}$.

Find the gain and phase crossover frequencies in rad/sec.

(12M)

Sol

Given,

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-j2\omega}}{(j\omega)(2+j\omega)} \quad -①$$

At gain crossover frequency, ω_{gc},

$$|G(j\omega_{gc})| = 1$$

$$\left|\frac{3e^{-j2\omega_{gc}}}{(j\omega_{gc})(2+j\omega_{gc})}\right| = 1$$

$$\frac{3}{\omega_{gc}\sqrt{2^2+\omega_{gc}^2}} = 1$$

$$\omega_{gc}^4 + 4\omega_{gc}^2 - 9 = 0$$

$$\boxed{\omega_{gc} = 1.26 \text{ rad/s}}$$

∴ Gain crossover frequency $= 1.21$ rad/s

At phase crossover frequency ω_{pc},

$$\angle G(j\omega_{pc}) = -180^\circ$$

$$\angle e^{-j2\omega_{pc}} - \angle j\omega_{pc} - \angle 2+j\omega_{pc} = -180^\circ$$

$$-2\omega_{pc}\times\frac{180}{\pi} - 90^\circ - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = -180^\circ$$

Candidates must not write on this margin

$$\frac{360}{\pi}\omega_{pc} + \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = +90^\circ$$

$$\omega = 1\,rad/s \Rightarrow \frac{360}{\pi}\omega_{pc} + \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = 141.1^\circ$$

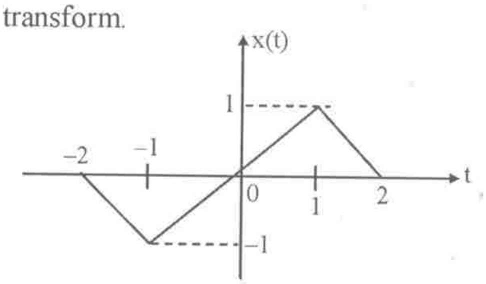
$$\omega = 0.5\,rad/s \Rightarrow \frac{360}{\pi}\omega_{pc} + \tan^{-1}\left(\frac{3\omega_{pc}}{2}\right) = 71.3^\circ$$

$$\omega = 0.6\,rad/s \Rightarrow \frac{360}{\pi}\omega_{pc} + \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = 85.45^\circ$$

$$\omega = 0.62\,rad/s \Rightarrow \frac{360}{\pi}\omega_{pc} + \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = 88.3^\circ$$

$\therefore$ By hit and trial phase crossover frequency $\omega_{pc} \simeq 0.63\,rad/s$

1(b). Find the Fourier transform of the signal x(t) as shown in figure using differentiation property of Fourier transform. **(12M)**



Sol

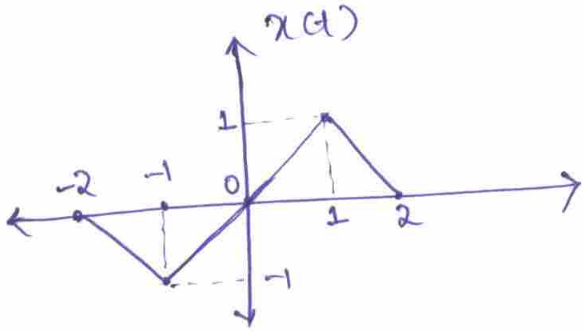
Differentiation property of Fourier Transform

$$x(t) \overset{FT}{\Longleftrightarrow} X(\omega)$$

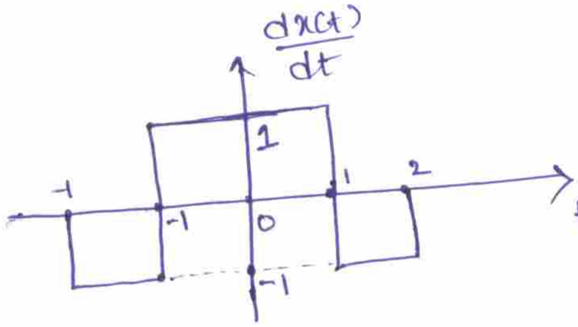
$$\frac{dx(t)}{dt} \overset{FT}{\Longleftrightarrow} j\omega X(\omega)$$

$$\frac{d^2x(t)}{dt} \overset{FT}{\Longleftrightarrow} (j\omega)^2 X(\omega)$$

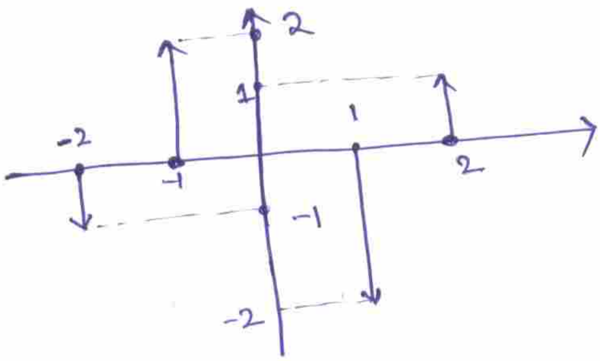
Candidates must not write on this margin



$\Leftarrow x(t)$



$\Leftarrow \frac{dx(t)}{dt}$



$\Leftarrow \frac{d^2x(t)}{dt}$

$$\therefore \frac{d^2x(t)}{dt} = -\delta(t+2) + 2\delta(t+1) - 2\delta(t-1) + \delta(t-2)$$

$$F\left(\frac{d^2x(t)}{dt}\right) = F\left[-\delta(t+2) + 2\delta(t+1) - 2\delta(t-1) + \delta(t-2)\right]$$

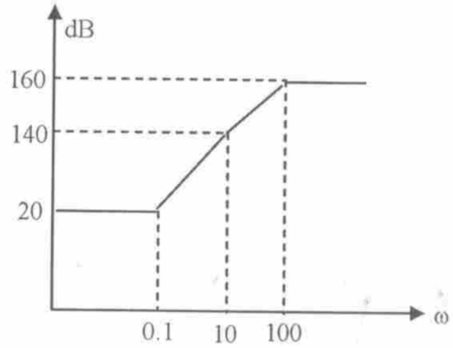
$$(j\omega)^2 X(\omega) = -e^{+j2\omega} + 2e^{j\omega} - 2e^{-j\omega} + e^{-j2\omega}$$

$$X(\omega) = \frac{2\left[e^{j\omega} - e^{-j\omega}\right] - \left[e^{j2\omega} - e^{-j2\omega}\right]}{-\omega^2}$$

$$\boxed{X(\omega) = \frac{2j}{\omega^2}\left[\sin 2\omega - 2\sin\omega\right]}$$

1.(c). The approximate Bode magnitude plot of a minimum phase system is shown in figure. Find the transfer function of the system. **(12M)**

Candidates must not write on this margin



Sol

For $\omega = 0.1$ to 10,

$$\text{Slope} = \frac{140-20}{\log 10 - \log 0.1} = \frac{120}{\log\frac{10}{0.1}} = 60\text{dB/dec}$$

For $\omega = 10$ to 100,

$$\text{slope} = \frac{160-140}{\log 100 - \log 10} = 20 \text{ dB/decade}$$

Initial slope $=0 \Rightarrow$ No poles or zeros at origin

At 0.1 rad/s,

change in slope $= +60$ dB/decade

$\Rightarrow$ 3 real zeros at 0.1 rad/s

At $\omega = 10$ rad/s,

change in slope $= 20-60 = -40$ dB/dec

$\Rightarrow$ 2 real poles at 10 rad/s.

At $\omega = 100$ rad/s,

change in slope $= 0-20 = -20$ dB/dec

$\Rightarrow$ 1 real pole at 100 rad/s.

$\therefore$ Transfer function for the given system is

$$TF = \frac{K\left[1+\frac{s}{0.1}\right]^3}{\left[1+\frac{s}{10}\right]^2\left[1+\frac{s}{100}\right]^1}$$

Candidates must not write on this margin

Magnitude from the graph is 20dB for $\omega \leq 0.1$ rad/s

$$\therefore 20 = 20\log K$$

$$\Rightarrow \boxed{K = 10}$$

$$\therefore TF = \frac{10\left[1+\frac{s}{0.1}\right]^3}{\left[1+\frac{s}{10}\right]^2\left[1+\frac{s}{100}\right]}$$

$$TF = \frac{10\times 10^2 \times 100}{0.1^3}\left[\frac{(s+0.1)^3}{(s+10)^2(s+100)}\right]$$

$$\boxed{TF = \frac{10^8(s+0.1)^3}{(s+10)^2(s+100)}}$$

1(d). Write the properties of ROC of Laplace Transform. (12M)

Candidates must not write on this margin

Sol

1. Region of convergence (ROC) consists of strips parallel to the $j\omega$-axis in the s-plane.
2. For rational Laplace transforms, ROC doesnot contain any poles
3. ROC of a function which is of finite duration and which is absolutely integrable is the entire s-plane.
4. For a right sided signal $x(t)$, if $Re\{s\} = \sigma_0$ is in the ROC, then all values of s for which $Re\{s\} > \sigma_0$ will also be in ROC.
5. For a left sided signal $x(t)$, if $Re\{s\} = \sigma_0$ is in the ROC, then all values of s for which $Re\{s\} < \sigma_0$ will also be in ROC.
6. For a two sided signal $x(t)$, if $Re\{s\} = \sigma_0$ is in the ROC, then ROC will consist of a strip in s-plane that includes $Re\{s\} = \sigma_0$
7. If laplace transform $X(s)$ of $x(t)$ is rational, then its ROC is bounded by poles or extended to infinity
8. ROC of Laplace transform of sum of two or more signals is the intersection of ROCs of those signals.

Candidates must not write on this margin

1(e). What are the frequency domain specifications, explain them with few words. **(12M)**

Sol

performance and characteristics of a system in frequency domain is measured in terms of the following frequency domain specifications.

1. Resonant peak, (M_r) :-

It is the maximum value of impulse response of the system when frequency is changed from 0 to ∞.

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

[for a second order system with negative feed back]

ξ = damping factor

2. Resonant frequency. (ω_r) :-

It is the frequency at which resonant peak occurs. It is indicative of speed of transient response.

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$

[for a second order system with unity negative feedback

ω_n = Natural frequency of oscillation of system

3. Bandwidth (ω_b) :-

It is the range of frequencies for which the system's normalized gain is more than

-3dB. The frequency at which the normalized gain is -3dB is called cutoff frequency. A large bandwidth corresponds to a small rise time or fast response.

$$\boxed{\omega_b = \omega_n \sqrt{1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}}}$$

[For a unity feedback second order system]

4. Cut off rate :-

The slope of log-magnitude graph near the cutoff frequency is called cutoff rate. The cutoff rate indicates the system's ability to distinguish signal from noise.

5. Gain Margin (GM) :-

It is the increase in open-loop gain in dB required to drive the closed-loop system to the verge of instability.

→ Gain Margin is given by the reciprocal of magnitude ~~of~~ open loop transfer function at phase crossover frequency. The frequency at which ~~A~~ phase of open loop transfer function is 180° is called phase crossover frequency.

$$\text{Gain Margin (GM)} = \frac{1}{|G(j\omega_{pc})|}$$

$$|GM|_{dB} = 20\log(GM)$$

Candidates must not write on this margin

6. Phase Margin (PM):-

Phase margin is defined as the additional phase lag to be added at the gain crossover frequency in order to bring the system to the verge of instability. The gain crossover frequency (ω_{gc}) is the frequency at which the magnitude of open loop transfer function is unity.

Phase margin is obtained by adding 180° to the phase angle of open loop transfer function at gain crossover frequency.

Both Gain Margin and phase margins are measures of stability.

Candidates must not write on this margin

2(a). The block diagram of a feedback system is shown in figure (a) and its response in figure (b).

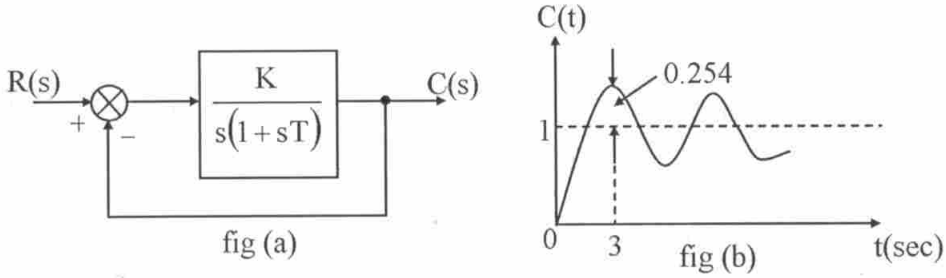


fig (a) fig (b)

(i) Find closed loop transfer function.

(ii) Determine the values of K and T from the response curve.

(iii) Time at which 2^{nd} over shoot will occur.

(iv) Time period of oscillations.

(v) Number of cycles output will perform before settling down with 2% tolerance. **(20M)**

Sol

Given,

$$G(s) = \frac{K}{s(1+sT)}$$

$$H(s) = 1$$

Candidates must not write on this margin

(i) closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{K}{s(1+sT)}}{1+\frac{K}{s(1+sT)}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K}} \rightarrow ①$$

(ii)

From ①,

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+\left(\frac{1}{T}\right)s+\left(\frac{K}{T}\right)} \rightarrow ②$$

From the graph shown in fig(b)

$$\left.\begin{aligned} t_p &= \text{peak time} = 3\,\text{sec} \\ M_p &= \text{peak overshoot} = 0.254 \\ \text{steady state value} &= 1 \end{aligned}\right\} ③$$

comparing ② with standard characteristic equation, we get

$$\omega_n^2 = \frac{K}{T} \quad \text{(and)} \quad 2\xi\omega_n = \frac{1}{T}$$

$$2\xi\sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\boxed{\omega_n = \sqrt{\frac{K}{T}}} \rightarrow ④ \qquad \boxed{\xi = \frac{1}{2\sqrt{KT}}} \rightarrow ⑤$$

$$\boxed{t_p \perp e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} =}$$

$t_p = \frac{\pi}{\omega_d} \Rightarrow 3 = \frac{\pi}{\omega_d} \Rightarrow \boxed{\omega_d = 1.0472 \text{ rad/s}}$

$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.254$

$\Rightarrow \frac{\xi}{\sqrt{1-\xi^2}} = 0.4362$

$\Rightarrow \frac{1-\xi^2}{\xi^2} = 5.255$

$\Rightarrow \boxed{\xi = 0.4}$

$\omega_n\sqrt{1-\xi^2} = 1.0472$

$\omega_n\sqrt{1-0.4^2} = 1.0472$

$\boxed{\omega_n = 1.142 \text{ rad/s}}$

From ④ and ⑤

$\omega_n \cdot \xi = \frac{1}{2T} = 0.4 \times 1.142$

$\Rightarrow \boxed{T = 1.095 \text{ sec}} \rightarrow ⑥$

substituting ⑥ in ④ we get,

$1.142 = \sqrt{\frac{K}{1.095}} \Rightarrow \boxed{K = 1.43}$

(iii) Time at which second overshoot occurs $= 3t_p$

$= 3\times3$

$= 9 \text{ sec}$

$\boxed{\therefore \text{ Time for second overshoot} = 9 \text{ sec}}$

(iv) Time period of oscillations $= 2t_p$

$= 2\times3$

$\boxed{\therefore \text{ Time period of oscillations} = 6 \text{ sec}}$

(v) 2% settling time $t_{s\,(2\%)} = \frac{4}{\xi\omega_n} = \frac{4}{0.4\times1.142}$

$= 8.756 \text{ sec}$

$\boxed{\therefore \text{ 2\% settling time} = 8.756 \text{ seconds}}$

Candidates must not write on this margin

2(b). Verify parseval's relation for the given sequence x(n), that is

$$\sum_{n=-\infty}^{\infty} x(n)x^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})X^*(e^{j\omega})d\omega \text{ for } x(n) = \left(\frac{1}{3}\right)^n u(n)$$ **(20M)**

Candidates must not write on this margin

Sol

Given sequence is $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$\Rightarrow x^*(n) = \left[\left(\frac{1}{3}\right)^n u(n)\right]^*$

$\Rightarrow x^*(n) = \left(\frac{1}{3}\right)^n u(n)$

$$LHS = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n) \left(\frac{1}{3}\right)^n u(n)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= 1 + \left(\frac{1}{9}\right)^1 + \left(\frac{1}{9}\right)^2 + \cdots$$

$$= \frac{1}{1-\frac{1}{9}}$$

$$L.H.S = \frac{9}{8} = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n) \quad -(1)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\cdot e^{-j\omega}\right)^n$$

$$X(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}} \quad \left[\because \left|\frac{1}{3}e^{-j\omega}\right| = \frac{1}{3} < 1\right]$$

Candidates must not write on this margin

$$X(e^{j\omega}) = \frac{1}{\left[1-\frac{\cos\omega}{3}\right] + j\left[\frac{\sin\omega}{3}\right]}$$

$$X^*(e^{j\omega}) = \frac{1}{\left[1-\frac{\cos\omega}{3}\right] - j\left[\frac{\sin\omega}{3}\right]}$$

$$X(e^{j\omega})\, X^*(e^{j\omega}) = \frac{1}{\left(1-\frac{\cos\omega}{3}\right)^2 + \frac{\sin^2\omega}{3^2}}$$

$$= \frac{1}{\frac{10}{9} - \frac{2\cos\omega}{3}}$$

$$X(e^{j\omega})\, X^*(e^{j\omega}) = \frac{9}{10-6\cos\omega}$$

$$R.H.S = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, X^*(e^{j\omega})\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{9}{10-6\cos\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{9}{10-6\left[\frac{1-\tan^2\omega/2}{1+\tan^2\omega/2}\right]}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{9\left[1+\tan^2\omega/2\right] d\omega}{4+16\tan^2\omega/2}$$

Assume $\tan\frac{\omega}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{\omega}{2}\,d\omega = dt$

$$RHS = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{9\sec^2\frac{\omega}{2}\,d\omega}{16\left[\frac{1}{4}+\tan^2\frac{\omega}{2}\right]}$$

$$= \frac{9}{32\pi}\int_{-\infty}^{\infty}\frac{2dt}{t^2+\frac{1}{4}} = \left[\frac{9}{16\pi}2\tan^{-1}(2t)\right]_{-\infty}^{\infty}$$

$$= \frac{9}{8\pi}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]$$

$$R.H.S = \frac{1}{2\pi}\int_{-\pi}^{\pi}X(j\omega)\overset{*}{X}(j\omega) = \frac{9}{8} \quad -②$$

From ① and ②

$$LHS = RHS \Rightarrow \boxed{\sum_{n=-\infty}^{\infty} x(n)\overset{*}{x}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})X^*(e^{j\omega})}$$

2(c). Given the following state-space description of a system:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \quad \text{and}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

Where X_1, X_2 are of the state-variable, u is the input and y is the output of the system:

(i) Find the state-transition matrix.

(ii) Transfer function of the system,

(iii) Complete solutions of the system when input is the step function and having zero initial condition. **(5+5+10 =20M)**

Sol

Given,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$A = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix} \quad B = \begin{bmatrix}0\\ 1\end{bmatrix} \quad C = \begin{bmatrix}1 & 0\end{bmatrix} \quad D = [0]$$

Candidates must not write on this margin

(i) State transition matrix is given by,

$$\phi(t) = L^{-1}\left[[sI - A]^{-1} \right]$$

$$sI - A = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+3)s+2}\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2}\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{(s+3)}{(s+2)(s+1)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{s+1} + \frac{2}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{2}{s+1} - \frac{1}{s+2} \end{bmatrix}$$

$$\phi(t) = L^{-1}\left[(sI - A)^{-1} \right] = L^{-1}\begin{bmatrix} \frac{-1}{s+1} + \frac{2}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{2}{s+1} - \frac{1}{s+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} -e^{-t} + 2e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

Candidates must not write on this margin

$$\therefore \text{State transition matrix is}$$

$$\boxed{\phi(t) = \begin{bmatrix} -e^{-t}+2e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & 2e^{-t}-e^{-2t} \end{bmatrix}}$$

(ii) Transfer function $= C(sI-A)^{-1}B + D$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{(s^2+3s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} s & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+3s+2}$$

$$= \frac{1}{(s+2)(s+1)}$$

$$\boxed{\therefore \text{Transfer function} = \frac{1}{(s+2)(s+1)}}$$

(iii) State equations for a unit step input under zero initial conditions

$$x(t) = \underset{\text{response}}{\text{Zero input}} + \underset{\text{response}}{\text{Zero state}}$$

$$= \phi(t)x(0) + L^{-1}[\phi(s) B U(s)]$$

$[x(0) = 0 \text{ given}]$

$$\therefore\ x(t) = L^{-1}\left[\phi(s)\, B\, U(s)\right]$$

$\left[\because \text{Step input}\right]$ $$= L^{-1}\left[\begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\left[\frac{1}{s}\right]\right]$$

$$= L^{-1}\begin{bmatrix} \frac{1}{2s} - \frac{1}{s+1} + \frac{1}{2(s+2)} \\ \frac{3}{2s} - \frac{2}{s+1} + \frac{1}{2(s+2)} \end{bmatrix}$$

$$\boxed{x(t) = \begin{bmatrix} \frac{1}{2}u(t) - e^{-t} + \frac{1}{2}e^{-2t} \\ \frac{3}{2}u(t) - 2e^{-t} + \frac{1}{2}e^{-2t} \end{bmatrix}}$$

Candidates must not write on this margin

3(a). Obtain Direct Form–I, Direct Form-II, cascade and parallel form realization for $y(n) = -0.1y(n-1)+0.2y(n-2) +3x(n) +3.6x(n-1)+0.6x(n-2)$ **(20M)**

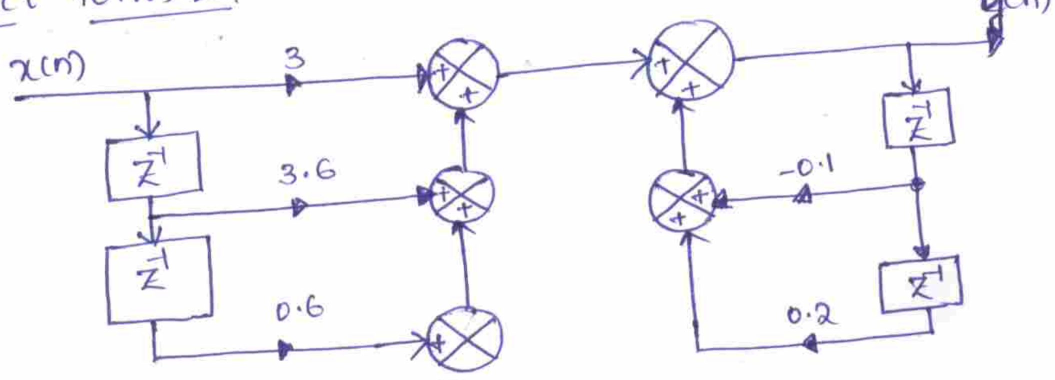
Sol

Given,

$$y(n) = 3x(n) + 3.6x(n-1) + 0.6x(n-2) - 0.1y(n-1) + 0.2y(n-2)$$

$$Y(z) = 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z) - 0.1z^{-1}Y(z) + 0.2z^{-2}Y(z) \quad \text{—(1)}$$

Direct form I :-



From ①,

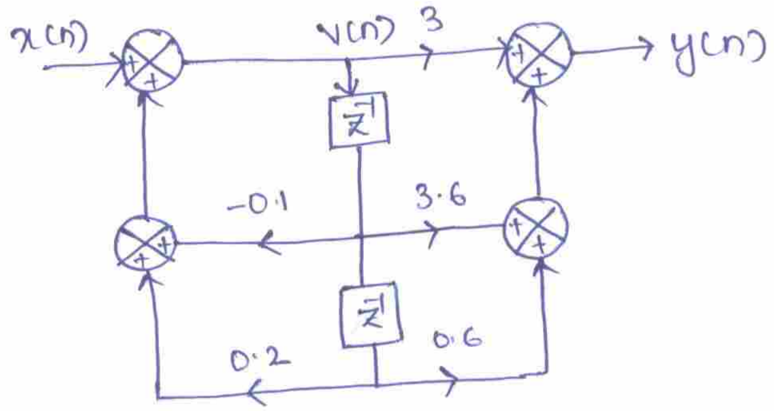
$$Y(z)\left[1+0.1z^{-1}-0.2z^{-2}\right] = \left[3+3.6z^{-1}+0.6z^{-2}\right]X(z)$$

Candidates must not write on this margin

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}} \rightarrow ②$$

$$\frac{Y(z)}{V(z)}\cdot\frac{V(z)}{X(z)} = \left(3+3.6z^{-1}+0.6z^{-2}\right)\frac{(1)}{\left(1+0.1z^{-1}-0.2z^{-2}\right)}$$

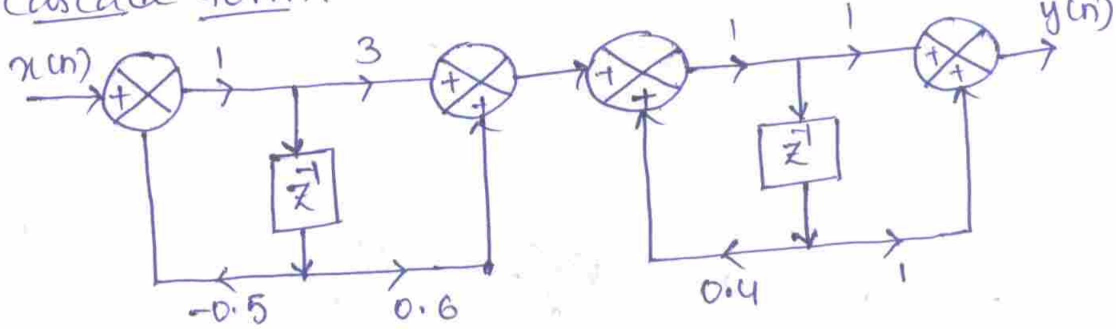
Direct form II:-



From ②,

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{\left(1+0.1z^{-1}-0.2z^{-2}\right)} = \left[\frac{3+0.6z^{-1}}{1+0.5z^{-1}}\right]\cdot\left[\frac{1+z^{-1}}{1-0.4z^{-1}}\right]$$

Cascade form:-

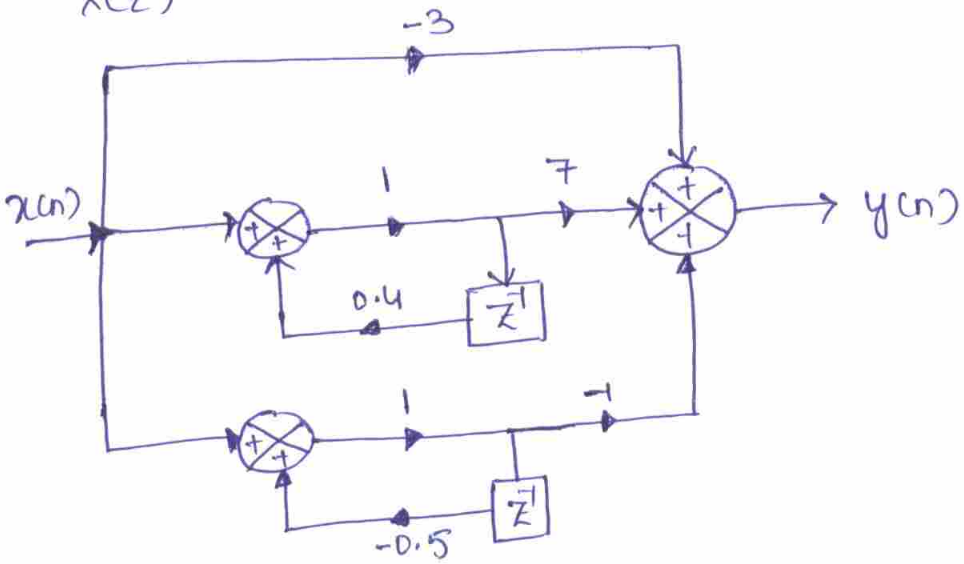


Parallel form :-

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{(1+0.5z^{-1})(1-0.4z^{-1})} = A + \frac{B}{1+0.5z^{-1}} + \frac{C}{(1-0.4z^{-1})}$$

Candidates must not write on this margin

$$\frac{Y(z)}{X(z)} = -3 + \frac{7}{1-0.4z^{-1}} + \frac{(-1)}{1+0.5z^{-1}}$$



3(b). Obtain the root locus diagram for a unity feedback system with open – loop transfer function. $G(s) = \frac{K}{s(s^2+6s+10)}$ **(20M)**

Sol

$$G(s) = \frac{K}{s(s^2+6s+10)}$$

For the given OLTF,

Number of poles, P = 3

Number of zeros, Z = 0

Number of root locus branches = 3

Number of asymptotes = P − Z = 3 − 0 = 3

Angle of asymptotes $= \frac{(2l+1)180^\circ}{P-Z}$ $l = 0,1,2 \ldots P-Z-1$

$= \frac{(2l+1)180^\circ}{3}$ $l = 0,1,2 \ldots P-Z-1$

$\therefore$ Angle of asymptotes $= 60^\circ, 180^\circ, 300^\circ$

Centroid of Asymptotes, $\sigma = \dfrac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeros}}{P-Z}$

$$\sigma = \frac{(0-3-3)-0}{3}$$

$$\boxed{\sigma = -2}$$

Breakaway point,

characteristic equation is

$$1 + K G_1(s) H_1(s) = 0$$

$$K = \frac{-1}{G_1(s) H_1(s)}$$

Roots of $\frac{dK}{ds} = 0$, which are on the root locus will be valid break away/break in points.

$$K = -s(s^2+6s+10)$$

$$K = -s^3 - 6s^2 - 10s$$

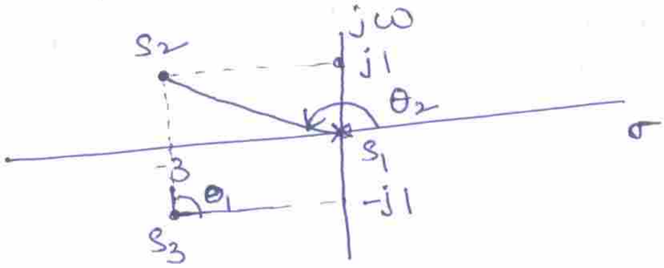
$$\frac{dK}{ds} = 0 \Rightarrow 3s^2 + 12s + 10 = 0$$

$$\Rightarrow \boxed{s = -1.18 \text{ rad/s}} \text{ and } \boxed{-2.81 \text{ rad/s}}$$

Both these points are valid as the entire negative real axis is a part of root locus characteristic equation is [for OLTF]

$s(s^2+6s+10) = 0$

$s_1 = 0$; $s_{2,3} = -3 \pm j1$



$\theta_1 = 90^\circ$ $\quad \theta_2 = 90^\circ + \tan^{-1} 3 = 161.6^\circ$

Angle of departure [at $-3+j1$] is

$\theta_{d_1} = 180 + \Sigma \angle \text{Angle from zeros} - \Sigma \angle \text{Angle from poles}$

$\theta_{d1} = 180 + 0 - [\theta_2 + \theta_1]$

$\theta_{d1} = 180 - [90 + 161.6^\circ]$

$\boxed{\theta_{d1} = -72^\circ}$

Similarly Angle of departure at $-3-j1$ is 72°

Intersection of root locus with imaginary axis.

characteristic equation is

$s^3 + 6s^2 + 10s + K = 0$

Routh table

s^3	1	10
s^2	6	K
s^1	$\frac{60-K}{6}$	0
s^0	K	

For marginal stability,

$\frac{60-K}{6} = 0$

$\Rightarrow \boxed{K = 60}$

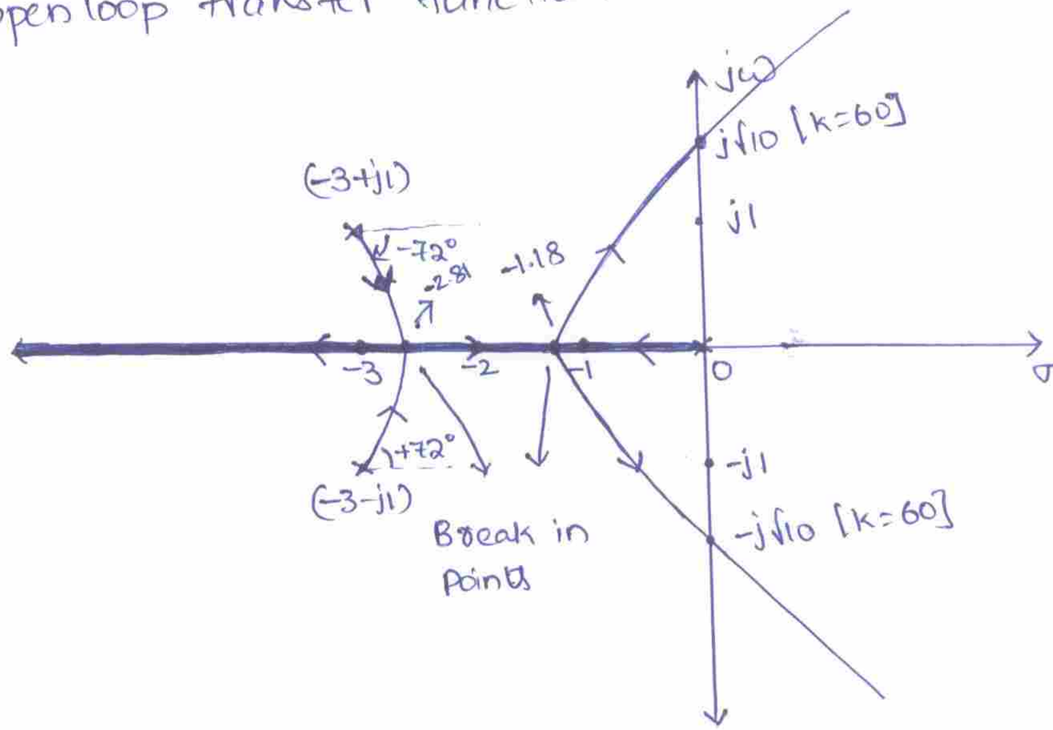
Auxilary equation is

$6s^2 + 60 = 0 \Rightarrow \boxed{\omega_n = \sqrt{10} \text{ rad/s}}$

Candidates must not write on this margin

By incorporating all the results the following is the root locus for given open loop transfer function.

Candidates must not write on this margin



3(c). The following facts are given about the signal x(t) with laplace transform X(s)

(i) x(t) is real and even.

(ii) X(s) has four poles, no zeros in a finite s-plane.

(iii) X(s) has a pole at $s = \frac{1}{2}e^{j\frac{\pi}{4}}$.

(iv) $\int_{-\infty}^{\infty} x(t)dt = 4$.

Determine X(s). **(20M)**

Sol (i) Given $x(t)$ is real and even

$x(t)$ is real $\Rightarrow$ $X^*(s^*) = X(s)$

$\Downarrow$

If $X(s)$ has a pole (or zero) at s_0, then it has a pole or zero at s_0^* also. ①

$x(t)$ is even $\Rightarrow$ $x(t) = x(-t)$, then

$\Rightarrow$ $\boxed{X(s) = X(-s)}$ ②

(ii) $X(s)$ has 4 poles and no zeros in finite s-plane.

let P_1, P_2, P_3, P_4 be four poles

$$\therefore X(s) = \frac{K}{(s-P_1)(s-P_2)(s-P_3)(s-P_4)}$$

(iii) Given one pole $P_1 = \frac{1}{2} e^{j\pi/4}$
(let)

$$\boxed{P_1 = \frac{1+j}{2\sqrt{2}}}$$

If $\frac{1+j}{2\sqrt{2}}$ is a pole then $\frac{1-j}{2\sqrt{2}}$ should also be a pole [from ①]

$$\therefore \boxed{P_2 = \frac{1-j}{2\sqrt{2}}}$$

$$\therefore X(s) = \frac{K}{\left[s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4}\right]\left[s^2 - (P_3+P_4)s + P_3P_4\right]}$$

But according to ② $X(s) = X(-s)$

$$\Rightarrow \frac{K}{\left(s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4}\right)\left(s^2 - (P_3+P_4)s + P_3P_4\right)} = \frac{K}{\left(s^2 + \frac{s}{\sqrt{2}} + \frac{1}{4}\right)\left[s^2 + (P_3+P_4)s + P_3P_4\right]}$$

$$\therefore \left.\begin{aligned} P_3P_4 &= \frac{1}{4} \\ P_3+P_4 &= \frac{-1}{\sqrt{2}} \end{aligned}\right\} \quad \boxed{\begin{aligned} P_3 &= \frac{-1+j}{2\sqrt{2}} \\ P_4 &= \frac{-1-j}{2\sqrt{2}} \end{aligned}}$$

Candidates must not write on this margin

$$\boxed{\therefore X(s) = \frac{k}{\left(s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4}\right)\left(s^2 + \frac{s}{\sqrt{2}} + \frac{1}{4}\right)}} \rightarrow (3)$$

Candidates must not write on this margin

(iv) Given $\int_{-\infty}^{\infty} x(t)\, dt = 4 \rightarrow (4)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, e^{-(0)t}\, dt$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{k}{\left(0 - 0 + \frac{1}{4}\right)\left(0 + 0 + \frac{1}{4}\right)} = 4 \qquad \text{[from (3) and (4)]}$$

$$16k = 4$$

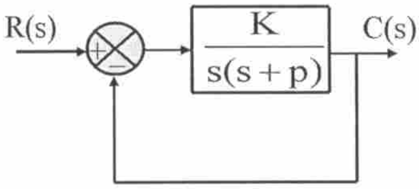
$$\boxed{k = \frac{1}{4}}$$

$$\therefore X(s) = \frac{\left(\frac{1}{4}\right)}{\left(s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4}\right)\left(s^2 + \frac{s}{\sqrt{2}} + \frac{1}{4}\right)}$$

$$= \frac{\left(\frac{1}{4}\right)}{s^4 + \frac{1}{16}} = \frac{4}{16s^4 + 1}$$

$$\boxed{\therefore X(s) = \frac{4}{16s^4 + 1}}$$

4(a). The block diagram of unity feedback control system is given in fig.



The Parameters K and p are both greater than Zero. Determine the Values of 'K' and 'p' to satisfy the following frequency domain specifications Resonant peak M_p = 1.04 and Reasonant frequency = 11.55 rad/sec. **(20M)**

Candidates must not write on this margin

Sol

Given,

$$G(s) = \frac{K}{s(s+p)}$$

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{K}{s^2+ps+K}$$

Characteristic equation is,

$$s^2+ps+K=0$$

comparing the above characteristic equation with the standard form,

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

we get,

$$\omega_n^2 = K \quad \text{(and)} \quad 2\xi\omega_n = P$$

$$\boxed{\omega_n = \sqrt{K}} \rightarrow (1) \qquad 2\xi\sqrt{K} = P$$

$$\boxed{\xi = \frac{P}{2\sqrt{K}}} \rightarrow (2)$$

Candidates must not write on this margin

Given Resonant peak, $M_p = 1.04$

$$M_p = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$$

$$\Rightarrow 2\xi\sqrt{1-\xi^2} = \frac{1}{1.04}$$

$$\Rightarrow 4\xi^2(1-\xi^2) = \left(\frac{1}{1.04}\right)^2 = 0.924$$

$$\Rightarrow \xi^2 - \xi^4 = 0.2311$$

$$\Rightarrow \xi^4 - \xi^2 + 0.2311 = 0$$

$$\Rightarrow \xi^2 = 0.637,\ 0.3625$$

$$\Rightarrow \boxed{\xi = 0.798,\ 0.6}$$

Since M_p is greater than unity, damping ratio (ξ) should be less than 0.707

$$\therefore \boxed{\xi = 0.6}$$

Given Resonant frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

(given) $11.55 = \omega_n\sqrt{1-2(0.6)^2}$

$$\Rightarrow \boxed{\omega_n = 21.83\ \text{rad/s}}$$

[from ①] $\Rightarrow \sqrt{K} = 21.83$

$$* \boxed{K = 476.44} *$$

From ② $\xi = \frac{P}{2\sqrt{K}}$

$$0.6 = \frac{P}{2\sqrt{476.44}} \Rightarrow \boxed{P = 26.2} *$$

4(b). The transfer function of a phase-lead controller is given as $G_C(s)=\frac{1+aTs}{1+Ts}$, $a>1$ and T is a constant depending on the circuit parameters. Derive the maximum value of the phase lead which can be obtained from this controller. **(20M)**

Candidates must not write on this margin

Sol

Given,

$$G_c(s) = \frac{1+aTs}{1+Ts} \quad a>1 \text{ and } T \text{ is constant}$$

$$G_c(j\omega) = \frac{1+j\omega aT}{1+j\omega T}$$

phase lead at any frequency is given by

$$\phi = \angle G_c(j\omega) = \tan^{-1}(\omega aT) - \tan^{-1}(\omega T) \rightarrow (1)$$

$$\frac{d\phi}{d\omega} = \frac{aT}{1+(\omega aT)^2} - \frac{T}{1+(\omega T)^2} = 0 \quad \left[\text{for max phase lead}\right]$$

$$\Rightarrow \frac{a\cancel{T}}{1+(\omega aT)^2} = \frac{\cancel{T}}{1+(\omega T)^2}$$

$$\Rightarrow a(1+2\omega T+\omega^2T^2) = 1+2\omega aT+\omega^2a^2T^2$$

$$\Rightarrow a+2a\cancel{\omega}T+a\omega^2T^2 = 1+2\cancel{\omega}aT+\omega^2a^2T^2$$

$$\Rightarrow (a-1) = a\omega^2T^2(a-1)$$

$$\Rightarrow \omega^2T^2a = 1$$

$$\Rightarrow \boxed{\omega = \frac{1}{T\sqrt{a}}} \rightarrow (2)$$

Substituting (2) in (1) we get ϕ_{max}

$$\phi_{max} = \tan^{-1}\left(\frac{1}{\cancel{T}\sqrt{a}}\cdot a\cancel{T}\right) - \tan^{-1}\left(\frac{1}{\cancel{T}\sqrt{a}}\cdot\cancel{T}\right)$$

$$\phi_{max} = \tan^{-1}(\sqrt{a}) - \tan^{-1}\left(\frac{1}{\sqrt{a}}\right)$$

$$= \tan^{-1}\left[\frac{\sqrt{a} - \frac{1}{\sqrt{a}}}{1 + \sqrt{a}\cdot\frac{1}{\sqrt{a}}}\right]$$

$$\boxed{\phi_{max} = \tan^{-1}\left[\frac{a-1}{2\sqrt{a}}\right] \quad \text{(or)} \quad \phi_{max} = \sin^{-1}\left(\frac{a-1}{a+1}\right)}$$

∴ Maximum phase lead that can be obtained by the given compensator is $\phi_{max} = \tan^{-1}\left[\frac{a-1}{2\sqrt{a}}\right] = \sin^{-1}\left[\frac{a-1}{a+1}\right]$

Candidates must not write on this margin

Candidates must not write on this margin

4(c). Derive the expressions for trigonometric Fourier series coefficients. **(20M)**

Sol Trignometric fourier series expansion of a signal $x(t)$ with time period 'T_0' is

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t} \rightarrow (1)$$

$$\omega_0 = \frac{2\pi}{T_0}$$

Calculation of a_0:-

Integrate (1) from t_a to $t_a + T_0$ i.e over one timeperiod.

$$\int_{t_0}^{t_0+T_0} x(t)\,dt = \int_{t_0}^{t_0+T_0} a_0\,dt + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T_0} \cos(n\omega_0 t)\,dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T_0} \sin(n\omega_0 t)\,dt$$

$$\int_{t_0}^{t_0+T_0} x(t)\,dt = a_0(T_0)$$

$$\boxed{a_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\,dt}$$

Calculation of a_n :-

multiply ① by $\cos m\omega t$ and integrate

from t_0 to t_0+T_0

$$\int_{t_0}^{t_0+T_0} x(t)\cos m\omega t = \int_{t_0}^{t_0+T_0} a_0 \cos m\omega t + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T_0} \cos n\omega t \cos m\omega t\,dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T_0} \cos m\omega t \sin n\omega t\,dt$$

we know that $\int_{t_0}^{t_0+T_0} \cos m\omega t \sin n\omega t\,dt = 0$ for all m & n

$$\int_{t_0}^{t_0+T_0} \cos m\omega t\,[\cos n\omega t] = 0 \text{ for } [m \neq n]$$
$$= \frac{T_0}{2} \text{ for } m = n$$

Candidates must not write on this margin

$$\therefore \int_{t_0}^{t_0+T_0} x(t) \cos n\omega t = 0 + a_n \left[\frac{T_0}{2}\right] + 0 \quad [\because m=n]$$

$$\boxed{\therefore a_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t) \cos n\omega t \, dt}$$

Calculation of b_n:-

multiply ① by $\sin n\omega t$ and integrate from 't_0' to 't_0+T_0'

$$\int_{t_0}^{t_0+T} x(t) \sin m\omega t = \int_{t_0}^{t_0+T_0} a_0 \sin \omega m t \, dt + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T_0} \cos n\omega t \sin m\omega t \, dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T_0} \sin n\omega t \sin m\omega t \, dt$$

we know,

$$\int_{t_0}^{t_0+T_0} \cos m\omega t \sin n\omega t \, dt = 0 \quad [\text{for all } n]$$

$$\int_{t_0}^{t_0+T_0} \sin n\omega t \sin m\omega t \, dt = 0 \quad [\text{for } m \neq n]$$

$$= \frac{T_0}{2} \quad [\text{for } m=n]$$

$$\therefore \int_{t_0}^{t_0+T_0} x(t) \sin n\omega t = b_n \left[\frac{T_0}{2}\right] \quad [\because m=n]$$

$$\boxed{\therefore b_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t) \sin n\omega t \, dt}$$

Candidates must not write on this margin

SECTION - B

5(a). Apply impulse invariant method and find H(z) for $H(s)=\frac{s+a}{(s+a)^2+b^2}$, T = 1sec. (12M)

Candidates must not write on this margin

Sol

Given, $H(s) = \frac{s+a}{(s+a)^2+b^2}$

$$\Downarrow$$

$$h(t) = e^{-at}\cos bt\, u(t)$$

$$\Downarrow$$

$$h(n) = e^{-anT}\cos bnT\, u(n)$$

$$\Downarrow$$

$$\boxed{h(n) = e^{-an}\cos bn\, u(n)} \quad [T = 1sec]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-1}$$

$$= \sum_{n=-\infty}^{\infty} e^{-an}\cos bn\, u(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} e^{-an}\left[\frac{e^{jbn}+e^{-jbn}}{2}\right] z^{-n}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\left[e^{-(a-jb)}z^{-1}\right]^n + \left[e^{-(a+jb)}z^{-1}\right]^n$$

$$= \frac{1}{2}\left[\frac{1}{1-e^{-(a-jb)}z^{-1}} + \frac{1}{1-e^{-(a+jb)}z^{-1}}\right]$$

$$\boxed{H(z) = \frac{1-e^{-a}\cos b\, z^{-1}}{1-2e^{-a}\cos b\, z^{-1} + e^{-2aT}z^{-2}}}$$

Candidates must not write on this margin

5(b). The open loop transfer function of a unity feedback control system is given as $G(s) = \frac{as+1}{s^2}$. Find the value of 'a' to give a phase margin of 45^0. **(12M)**

Sol

Given, $G(s) = \frac{as+1}{s^2}$

$H(s) = 1$ [$\because$ unity feed back]

$\therefore G(s)H(s) = \frac{as+1}{s^2}$

$G(j\omega)H(j\omega) = \frac{a(j\omega)+1}{(j\omega)^2}$

$G(j\omega)H(j\omega) = -j\frac{a}{\omega} + \left(\frac{-1}{\omega^2}\right) \rightarrow (1)$

$\angle G(j\omega)H(j\omega) = -180^0 + \tan^{-1}\left(\frac{\frac{a}{\omega}}{\frac{1}{\omega^2}}\right)$

$\boxed{\angle G(j\omega)H(j\omega = -180^0 + \tan^{-1}(a\omega)} \rightarrow (2)$

$$\text{Phase margin} = 180^\circ + \underline{\angle G(j\omega)H(j\omega)}\Big|_{\omega=\omega_{gc}}$$

Candidates must not write on this margin

$$|G(j\omega_{gc})H(j\omega_{gc})| = 1$$

$$\sqrt{\left(\frac{a}{\omega_{gc}}\right)^2 + \left(\frac{1}{\omega_{gc}^2}\right)^2} = 1 \quad [\text{from } 1]$$

$$\frac{a^2}{\omega_{gc}^2} + \frac{1}{\omega_{gc}^4} = 1$$

$$\boxed{\omega_{gc}^4 - a^2 \cdot \omega_{gc}^2 - 1 = 0} \rightarrow 3$$

$$\text{Phase margin} = 45^\circ = 180^\circ + (-180^\circ + \tan^{-1}(a\omega_{gc}))$$

$$\Rightarrow \boxed{a\omega_{gc} = 1}$$

substituting $a\omega_{gc} = 1$ in 3

$$\omega_{gc}^4 - 1^2 - 1 = 0$$

$$\omega_{gc}^4 = 2$$

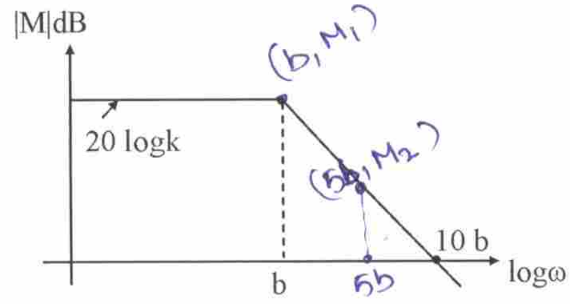
$$\boxed{\omega_{gc} = 2^{1/4}}$$

$$\therefore a\, 2^{1/4} = 1$$

$$a = \frac{1}{2^{1/4}} = 0.841$$

$$\therefore \boxed{a = 0.841}$$

5(c). For the transfer function $G(s)\,H(s) = \dfrac{K}{\left(1+\dfrac{s}{b}\right)}$ the error in diagram at $\omega = 5b$. **(12M)**



Candidates must not write on this margin

Sol

Given,

$$G(s)\,H(s) = \frac{K}{1+\frac{s}{b}}$$

At $\omega = 5b \Rightarrow |M| = \dfrac{K}{\sqrt{1+\left(\frac{5b}{b}\right)^2}} = \dfrac{K}{\sqrt{26}}$

$$M_{dB} = 20\log|M|$$

$$M_{dB} = 20\log\frac{K}{\sqrt{26}}$$

$$M_{dB} = 20\log K - 20\log\sqrt{26}$$

$$\boxed{M_{dB} = 20\log K - 14.14\,dB} \rightarrow ①$$

From the figure,

Slope $= -20dB/$ decade $[\because$ only 1 pole$]$

$M_1 = 20\log K$ @ $\log\omega_1$ [where $\omega_1 = b$]

$M_2 = ?$ @ $\log\omega_2$ [where $\omega_2 = 5b$]

$$\text{Slope} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

Candidates must not write on this margin

$$-20 = \frac{M_2 - 20\log K}{\log 5b - \log b}$$

$$\boxed{M_2 = 20\log K - 13.98} \rightarrow (2)$$

$$\text{Error} = \text{Exact value} - \text{Approximate value}$$

$$= (20\log K - 14.14) - (20\log K - 13.98)$$

$$= -0.161\,dB$$

$$\boxed{\therefore |\text{Error}| = 0.161\,dB}$$

5(d). Consider a LTI system with unit impulse response h(n) = $(\alpha)^n$ u(n), where 'α' is real and $0 < \alpha < 1$. If the input is x(n) = $(\beta)^n$ u(n), $0 < |\beta| < 1$, determine the output y(n) in the form y(n) = $[k_1(\alpha)^n + k_2(\beta)^n]$ u(n). **(12M)**

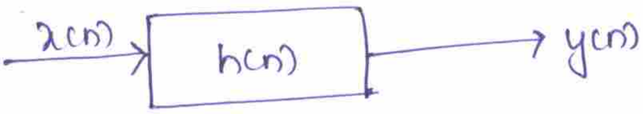
Candidates must not write on this margin

Sol

Given,

$$h(n) = \alpha^n u(n) \qquad 0<\alpha<1 \text{ \& } \alpha \text{ is real}$$

$$x(n) = \beta^n u(n) \qquad 0<\beta<1$$



$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}$$

$$X(z) = \sum_{n=\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} (\beta z^{-1})^n = \frac{1}{1-\beta z^{-1}} = \frac{z}{z-\beta}$$

$$Y(z) = H(z) X(z) = \left(\frac{z}{z-\alpha}\right)\left(\frac{z}{z-\beta}\right)$$

$$\frac{Y(z)}{z} = \frac{z}{(z-\alpha)(z-\beta)} = \frac{A}{(z-\alpha)} + \frac{B}{(z-\beta)}$$

$A+B = 1$ and $-A\beta - B\alpha = 0$

$$A = -\frac{B\alpha}{\beta}$$

$$-\frac{B\alpha}{\beta} + B = 1$$

$$\boxed{B = \frac{\beta}{\beta-\alpha}} \qquad \boxed{A = \frac{-\alpha}{\beta-\alpha}}$$

$$\frac{Y(z)}{z} = \frac{\left(\frac{-\alpha}{\beta-\alpha}\right)}{z-\alpha} + \frac{\left(\frac{\beta}{\beta-\alpha}\right)}{z-\beta}$$

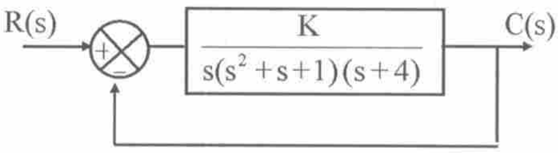
Candidates must not write on this margin

$$Y(z) = \left(\frac{-\alpha}{\beta-\alpha}\right)\frac{z}{z-\alpha} + \left(\frac{\beta}{\beta-\alpha}\right)\frac{z}{z-\beta}$$

$$y(n) = \left(\frac{-\alpha}{\beta-\alpha}\right)\alpha^n u(n) + \left(\frac{\beta}{\beta-\alpha}\right)\beta^n u(n)$$

$$\boxed{\therefore\ y(n) = \left[\left(\frac{-\alpha}{\beta-\alpha}\right)\alpha^n + \left(\frac{\beta}{\beta-\alpha}\right)\beta^n\right]u(n)}$$

5(e). Consider the closed loop feedback system shown in fig. Determine the range of 'K' for which the system is stable. **(12M)**



Sol

Given,

$$G(s) = \frac{K}{s(s^2+s+1)(s+4)}$$

$$H(s) = 1$$

$$1+G(s)H(s) = 1 + \frac{K}{s(s^2+s+1)(s+4)} = 0$$

$$\Rightarrow s\left[(s^2+s+1)(s+4)\right] + K = 0$$

$$\Rightarrow s\left[s^3+s^2+s+4s^2+4s+4\right] + K = 0$$

$$\Rightarrow \boxed{s^4+5s^3+5s^2+4s+K=0}$$

Characteristic equation for the given transfer function is

$$s^4 + 5s^3 + 5s^2 + 4s + K = 0$$

Applying Routh - Hurwitz criterion,

$$\begin{array}{c|ccc} s^4 & 1 & 5 & K \\ s^3 & 5 & 4 & 0 \\ s^2 & \frac{21}{5} & K & 0 \\ s^1 & \dfrac{\frac{81}{5} - 5K}{\frac{21}{5}} & 0 & \\ s^0 & K & & \end{array}$$

For stability, elements of first column should be of same sign.

$\Rightarrow \boxed{K > 0}$ and $\dfrac{\frac{81}{5} - 5K}{\frac{21}{5}} > 0$

$\Rightarrow \frac{81}{5} - 5K > 0$

$\Rightarrow 5K < \frac{81}{5}$

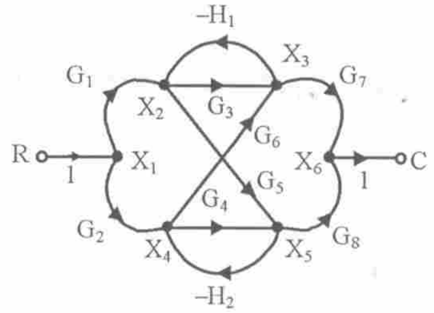
$\Rightarrow \boxed{K < \frac{81}{25}}$

$\boxed{\therefore \text{ Range of 'K' for stability is } 0 < K < \frac{81}{25}}$

Candidates must not write on this margin

6(a). Write the properties of signal flow graphs. Obtain the overall transfer function C/R from the signal flow graph shown in figure.

Candidates must not write on this margin



(20M)

Sol

properties of signal flow graphs (sfg)

1. signal flow graphs are applicable only to linear systems.

2. The equations for which a sfg is drawn must be algebraic equations written in the form of cause and effect

3. Nodes are used to represent variables. Normally the nodes are arranged from left to right, from the input to output following a succession of cause and effect relations through the system.

4. Signal travel along branches only in the direction described by the arrows of the branches.

5. A branch directing from node x_k to x_j represents the dependance of x_j upon x_k but not vice versa.

6. A signal x_k travelling along a branch between x_k and x_j is multiplied by the $\boxed{gra}$ gain of the branch a_{kj} so that a signal $a_{kj} x_k$ is delivered to x_j

7. For a given system, a sfg is not unique.

(ii)

Given signal flow graphs has the following Loops and Forward paths

Forward paths

$M_1 = G_1 G_3 G_7$

$M_2 = G_1 G_5 G_8$

$M_3 = G_2 G_4 G_8$

$M_4 = G_2 G_6 G_7$

$M_5 = -G_1 G_5 H_2 G_6 G_7$

$M_6 = -G_2 G_6 H_1 G_5 G_8$

Loops

$L_1 = -G_3 H_1$

$L_2 = -G_4 H_2$

$L_3 = G_5 H_2 G_6 H_1$

Non touching loops taken two at a time $= L_1 L_2$

$= G_3 G_4 H_1 H_2$

There are no non touching loops taken 3 or more at a time.

$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$

$$\boxed{\Delta = 1 + G_3 H_1 + G_4 H_2 - G_5 G_6 H_1 H_2 + G_3 G_4 H_1 H_2}$$

Candidates must not write on this margin

$\Delta_1 = 1 + G_4H_2 \qquad \Delta_2 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$\Delta_3 = 1 + G_3H_1$

Candidates must not write on this margin

According to Mason's gain formula,

$$\frac{C}{R} = \frac{\sum M_K \Delta_K}{\Delta}$$

$$\boxed{\frac{C}{R} = \frac{G_1G_3G_7[1+G_4H_2] + G_1G_5G_8 + G_2G_4G_8[1+G_3H_1] + G_2G_6G_7 - G_1G_5G_6G_7H_2 - G_2G_6G_5G_8H_1}{1 + G_3H_1 + G_4H_2 - G_5G_6H_1H_2 + G_3G_4H_1H_2}}$$

6(b). (i) Find the inverse z-transform of $X(z) = \dfrac{z(z^2-4z+5)}{(z-1)(z-2)(z-3)}$ for the following Roc's

(1) $2 < |z| < 3$ (2) $|z| > 3$ (3) $|z| < 1$

(ii) Find the inverse z-transform of $X(z) = \dfrac{z^2}{\left(z-\frac{1}{4}\right)^2}$ using convolution method.

(10+10 = 20M)

Sol

$$X(z) = \frac{z(z^2-4z+5)}{(z-1)(z-2)(z-3)}$$

$$\frac{X(z)}{z} = \frac{z^2-4z+5}{(z-1)(z-2)(z-3)}$$

$$= \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$= \frac{(A+B+C)z^2 - (5A+4B+3C)z + (6A+3B+2C)}{(z-B)(z-2)(z-3)}$$

Solving we get $A=1 \quad B=-1 \quad C=1$

$$\frac{X(z)}{z} = \frac{1}{z-1} - \frac{1}{z-2} + \frac{1}{z-3}$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-2} + \frac{z}{z-3}$$

(i) $2<|z|<3 = (|z|>2) \cap (|z|<3) \cap (|z|>1)$

$$x(n) = u(n) - 2^n u(n) - 3^n u(-n-1)$$

(ii) $|z|>3 = (|z|>1) \cap (|z|>2) \cap (|z|>3)$

$$x(n) = u(n) - 2^n u(n) + 3^n u(n)$$

(iii) $|z|<1 = (|z|<1) \cap (|z|<2) \cap (|z|<3)$

$$x(n) = -u(-n-1) + 2^n u(-n-1) - 3^n u(-n-1)$$

Candidates must not write on this margin

Candidates must not write on this margin

6(c). The open loop transfer function of a unity feedback system is given by $G(s) = \frac{5}{s(s+1)}$

Find the rise time, percentage overshoot, time of peak overshoot and settling time for a step input of 10 units. Also determine the value of peak overshoot. **(20M)**

Candidates must not write on this margin

Sol

Given,

$$G(s) = \frac{5}{s(s+1)}$$

$H(s) = 1$ [unity feedback]

Input, $r(t) = 10\,u(t)$ [step input of 10 units]

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH}$$

$$= \frac{\frac{5}{s(s+1)}}{1+\frac{5}{s(s+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{5}{s^2+s+5}$$

From the above transfer function,

characteristic equation is,

$$s^2+s+5=0 \quad -(1)$$

comparing eq (1) with standard characteristic equation: $s^2+2\xi\omega_n s+\omega_n^2=0$

we get,

$$\omega_n^2 = 5 \Rightarrow \boxed{\omega_n = \sqrt{5}\ \text{rad/s}}$$

$$2\xi\omega_n = 1 \Rightarrow 2\xi.\sqrt{5} = 1 \Rightarrow \boxed{\xi = \frac{1}{2\sqrt{5}}}$$

(i) Rise time, $\boxed{t_r = \dfrac{\pi - \theta}{\omega_d}}$

Candidates must not write on this margin

where, $\theta = \cos^{-1}(\xi)$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\theta = \cos^{-1}\left(\frac{1}{2\sqrt{5}}\right) = \underline{1.3453 \text{ rad}}$$

$$\omega_d = \sqrt{5}\sqrt{1-\left(\frac{1}{2\sqrt{5}}\right)^2} = \underline{2.18 \text{ rad/s}}$$

$$\therefore t_r = \frac{\pi - 1.3453}{2.18} = 0.824 \text{ sec}$$

$\boxed{\therefore \text{ Rise time, } t_r = 0.824 \text{ sec}}$

(ii) peak time, $\boxed{t_p = \dfrac{\pi}{\omega_d}}$

$$t_p = \frac{\pi}{2.18}$$

$\boxed{\text{peak, time, } t_p = 1.441 \text{ sec}}$

(iii) percentage peak over shoopt, $\boxed{\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$

$$\% M_p = e^{\frac{-\pi[1/2\sqrt{5}]}{\sqrt{1-(1/2\sqrt{5})^2}}} \times 100$$

$\boxed{\% M_p = 48.6\%}$

$$\Rightarrow \frac{\text{Peak value} - \text{Steady state value}}{\text{Steady state value}} = \frac{48.6}{100}$$

$$\Rightarrow \frac{\text{peak value} - 10}{10} = \frac{48.6}{100}$$

$$\Rightarrow \boxed{\text{peak value} = 14.86}$$

Candidates must not write on this margin

peak over shoot = peak value − steady state value

$$= 14.86 - 10$$

$$\boxed{\text{peak over shoot value} = 4.86}$$

(iv) Settling time, $\boxed{t_s\ [2\%\ \text{tolerance}] = \frac{4}{\xi \omega_n}}$

$$\Rightarrow t_s = \frac{4}{\left(\frac{1}{2\sqrt{5}}\right)(\sqrt{5})}$$

$$\Rightarrow \boxed{t_s = 8\,sec}$$

$$\boxed{\therefore 2\%\ \text{settling time} = 8\,sec}$$

7(a). The open-loop transfer function of a feedback control system is given by

$$G(s)H(s) = \frac{50}{s(1+0.1s)(1+0.2s)}$$

Determine stability using Nyquist plot. **(20M)**

Sol Given,

$$G(s)\,H(s) = \frac{50}{s(1+0.1s)(1+0.2s)} \rightarrow (1)$$

$$G(j\omega)\,H(j\omega) = \frac{50}{(j\omega)(1+0.1j\omega)(1+0.2j\omega)}$$

$$|G(j\omega)\,H(j\omega)| = \frac{50}{\omega\sqrt{1+(0.1\omega)^2}\,\sqrt{1+(0.2\omega)^2}}$$

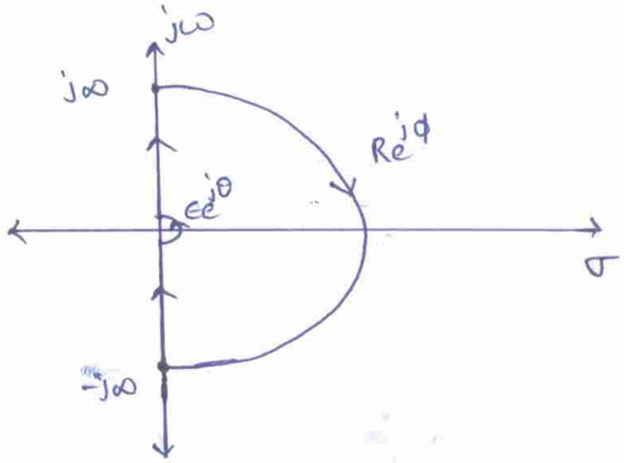
$$\angle G(j\omega)\,H(j\omega) = -90 - \tan^{-1}(0.1\omega) - \tan^{-1}(0.2\omega)$$

$$G(j\omega)\,H(j\omega) = \frac{-50j\,[1-j0.1\omega]\,[1-j0.2\omega]}{\omega\,[1+(0.1\omega)^2]\,[1+(0.2\omega)^2]}$$

Candidates must not write on this margin

$$= \frac{-15}{[1+(0.1\omega)^2]\,[1+(0.2\omega)^2]} + j\,\frac{-50\,[1-0.02\omega^2]}{\omega\,[1+(0.1\omega)^2]\,[1+(0.2\omega)^2]}$$

contour for given $G(s)\,H(s)$ is,



$R \to \infty$
$\epsilon \to 0$
$\phi \to \frac{\pi}{2}$ to $-\frac{\pi}{2}$
$\theta \to -\frac{\pi}{2}$ to $\frac{\pi}{2}$

At $s = -j\infty$,

$$G(j\omega)\,H(j\omega) = -0 + j(-0)$$

At $s = -j0$,

$$G(j\omega)\,H(j\omega) = -15 + j(+\infty)$$

At $s = +j0$

$$G(j\omega)\,H(j\omega) = -15 + j(-\infty)$$

At $s = +j\infty$

$$G(j\omega)\,H(j\omega) = -0 + j(+0)$$

From ①, substituting $s = \epsilon e^{j\theta}$ $[\epsilon \to 0]$

$$G(s)\,H(s) = \lim_{\epsilon \to 0} \frac{50}{\epsilon e^{j\theta}\,[1+\epsilon e^{j\theta}\,0.1]\,[1+0.2\,\epsilon e^{j\theta}]}$$

$$= \lim_{\epsilon \to 0} \frac{50}{\epsilon}\,e^{-j\theta}$$

$$= \infty\angle{-\theta} = \infty\angle{\pi/2} \text{ to } \infty\angle{-\pi/2}$$

[$\because$ θ varies from $-\pi/2$ to $\pi/2$]

From ①, substituting $s = Re^{j\phi}$

$$G(s)H(s) = \lim_{R\to\infty} \frac{50}{(Re^{j\phi})(1+0.1Re^{j\phi})(1+0.2Re^{j\phi})}$$

$$= \lim_{R\to\infty} \frac{50}{0.02R^3 e^{j3\phi}}$$

$$= \lim_{R\to\infty} \frac{2500}{R^3} e^{-j3\phi}$$

$$= 0\angle{-3\phi} = 0\angle{-270^\circ} \text{ to } 0\angle{+270^\circ}$$

[ϕ varies from $\pi/2$ to $-\pi/2$]

For real axis crossing points,

Imaginary term $= 0$

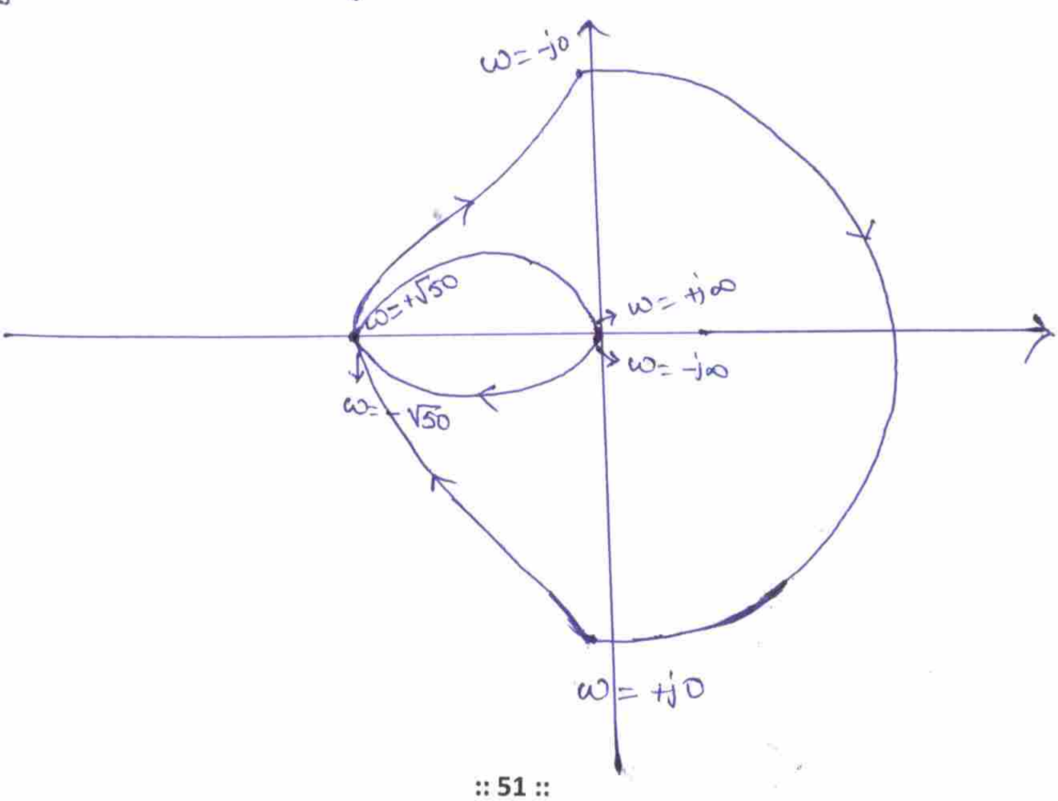
$\Rightarrow 1 - 0.02\omega^2 = 0$

$\Rightarrow \omega^2 = 50$

$\Rightarrow \boxed{\omega = \pm\sqrt{50} \text{ rad/s}}$

$$G(j\omega)H(j\omega)\Big|_{\omega=\sqrt{50}} = \frac{-15}{(1.5)(3)} = -\frac{10}{3} + j0$$

Nyquist plot by incorporating above features



Candidates must not write on this margin

7(b). Compute the IDFT of sequence

X(k) = {7, –0.707–j0.707, –j, 0.707–j0.707, 1, 0.707+j0.707,j,–0.707+j0.707}

Using DIT FFT algorithm. **(20M)**

Candidates must not write on this margin

Sol

Given, $X(0) = 7$

$X(1) = -0.707 - j0.707 = -e^{+j\frac{2\pi}{8}}$

$X(2) = -j = -e^{j\frac{2\pi}{8}\cdot 2}$

$X(3) = 0.707 - j0.707 = e^{j\frac{7\pi}{8}\times 2}$

$X(4) = 1$

$X(5) = 0.707 + j0.707$

$X(6) = j$

$X(7) = -0.707 + j0.707$

let, $W_8 = e^{-j\frac{2\pi}{8}}$

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

We know;

$$\boxed{\cancel{X(K)} = \frac{1}{N}\sum_{n=0}^{N-1} x(k)\, e^{j\frac{2\pi}{N}nk}}$$

We know,

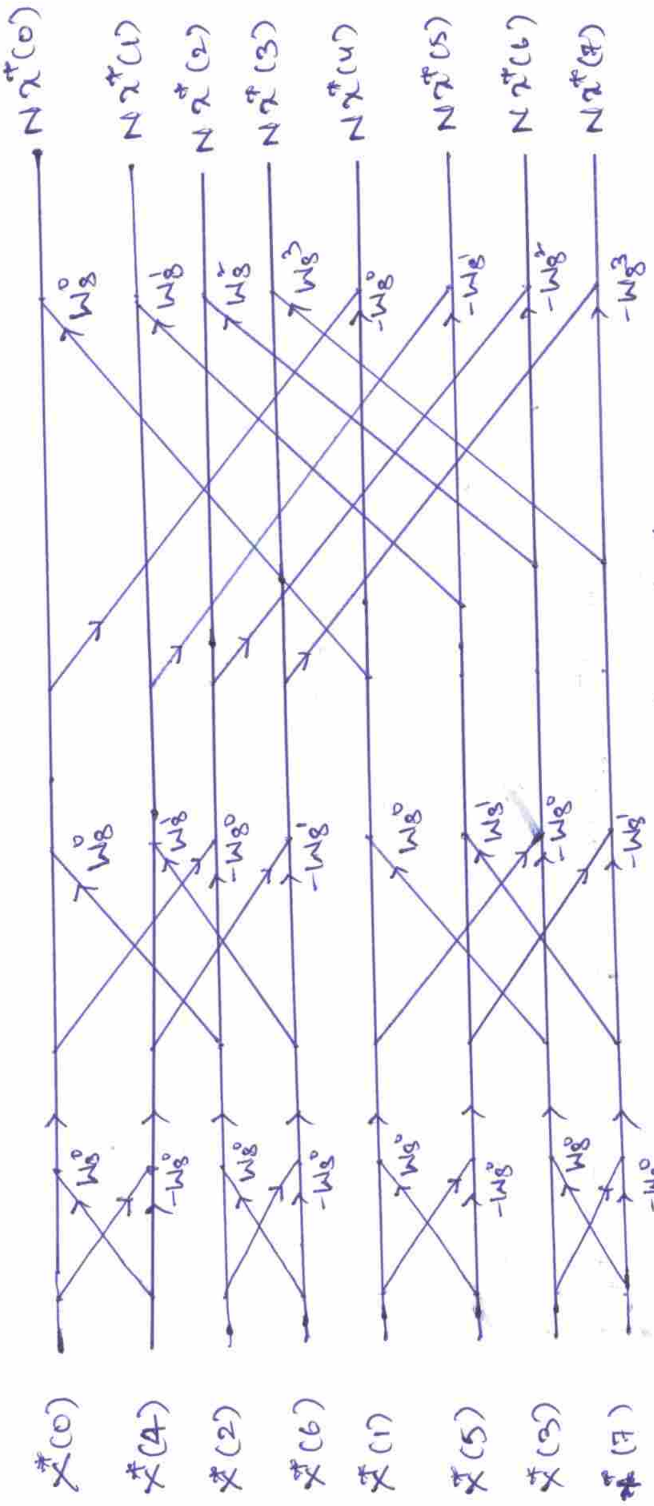
$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}nk} \quad n=0,1\ldots N-1$$

$$N x(n) = \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk}$$

$$\boxed{N x^*(n) = \sum_{k=0}^{N-1} X^*(k)\, e^{-j\frac{2\pi}{N}nk}}$$

$n = 0,1,2 \ldots N-1$

Candidates must not write on this margin



Input

$X^*(0)$	7
$X^*(4)$	1
$X^*(2)$	j
$X^*(6)$	-j
$X^*(1)$	-0.707+j0.707
$X^*(5)$	0.707-j0.707
$X^*(3)$	0.707+j0.707
$X^*(7)$	-0.707-j0.707

Stage 1 output

7+1 = 8
7-1 = 6
j-j = 0
j-(-j) = 2j
0
-1.414+j1.414
0
1.414+j1.414

Stage 2 output

8+0 = 8
6+(-j)(2j) = 8
8-0 = 8
6-(-j)(2j) = 4
0
0
0
-2.828+j2.828

Stage 3 output

8 8 8 8 8 8 8 0

$\therefore\ 8x^*(n) = \{8,8,8,8,8,8,8,0\}$

$x^*(n) = \{1,1,1,1,1,1,1,0\}$

$\boxed{x(n) = \{1,1,1,1,1,1,1,0\}}$

7(c). A circuit has the following transfer function $\frac{C(s)}{R(s)} = \frac{s^2+3s+4}{s^2+4s+4}$

Find c(t) when r(t) is a unit step.

State if the circuit is undamped, under damped, critically damped or over-damped.

(20M)

Candidates must not write on this margin

Sol

Given, $\frac{C(s)}{R(s)} = \frac{s^2+3s+4}{s^2+4s+4}$

r(t) is a step input

$\Rightarrow R(s) = \frac{1}{s}$

$\therefore \frac{C(s)}{(1/s)} = \frac{s^2+3s+4}{s^2+4s+4}$

$C(s) = \frac{1}{s} \cdot \frac{(s^2+3s+4)}{(s+2)^2}$

$C(s) = \frac{(s^2+3s+4)}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$\Rightarrow \frac{s^2+3s+4}{s(s+2)^2} = \frac{A(s+2)^2 + Bs(s+2) + Cs}{s(s+2)^2}$

$\Rightarrow A+B=1$; $4A+2B+C=3$; $4A=4$

$1+B=1$; $4(1)+2(0)+C=3$; $A=4/4$

$\boxed{B=0}$; $\boxed{C=-1}$; $\boxed{A=1}$

$\therefore C(s) = \frac{1}{s} - \frac{1}{(s+2)^2}$

$c(t) = \mathcal{L}^{-1}(C(s)) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{(s+2)^2}\right]$

$$\boxed{c(t) = (1 - te^{-t})u(t)}$$

characteristic equation is

$$s^2 + 4s + 4 = 0$$

Comparing it with the standard form of

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$\Rightarrow \omega_n^2 = 4$ and $2\xi\omega_n = 4$

$\Rightarrow \omega_n = 2$ $\quad 2\xi(2) = 4$

$$\boxed{\xi = 1}$$

$\boxed{\because \xi = 1, \text{ the system is critically damped}}$

Candidates must not write on this margin

8(a). State and prove circular shift in time domain property and circular convolution in time domain property of DFT **(20M)**

Candidates must not write on this margin

circular shift of DFT in time domain:-

If $X(K)$ is the DFT of a signal $x(n)$ and if we shift $x(n)$ cyclically by m samples i.e. $x_1(n) = x(n-m)_N$, then the DFT of shifted signal $x_1(n)$ is equal to

$$X_1(K) = e^{-j\frac{2\pi}{N}mK} X(K) \quad [\text{Where } x(n) \Leftrightarrow X(K)]$$

Proof:-

$$DFT[x(n-m)_N] = \sum_{n=0}^{N-1} x(n-m)_N \, e^{-j\frac{2\pi}{N}nK}$$

$$= \underbrace{\sum_{n=0}^{m-1} x(n-m)_N \, e^{-j\frac{2\pi}{N}nK}}_{1} + \underbrace{\sum_{n=m}^{N-1} x(n-m)_N \, e^{-j\frac{2\pi}{N}nK}}_{2}$$

$$1 \Rightarrow \sum_{n=0}^{m-1} x(n-m)_N \, e^{-j\frac{2\pi}{N}nK} = \sum_{n=0}^{m-1} x(N+n-m) \, e^{-j\frac{2\pi}{N}nK}$$

$$[l = N+n-m] \qquad = \sum_{l=N-m}^{N-1} x(l) \, e^{-j\frac{2\pi}{N}[l+m-N]K}$$

$$= \sum_{l=N-m}^{N-1} x(l) \, e^{-j\frac{2\pi}{N}(l+m)K} \, e^{j2\pi K}$$

$$\therefore 1 = \sum_{l=N-m}^{N-1} x(l) \, e^{-j\frac{2\pi}{N}lK} \, e^{-j\frac{2\pi}{N}mK}$$

$$2 \Rightarrow \sum_{n=m}^{N-1} x(n-m)_N \, e^{-j\frac{2\pi}{N}nK} = \sum_{n=m}^{N-1} x(n-m) \, e^{-j\frac{2\pi}{N}nK}$$

$$[l = n-m] \qquad = \sum_{l=0}^{N-m-1} x(l)\, e^{-j\frac{2\pi}{N}[m+l]k}$$

$$② \qquad = \underline{\sum_{l=0}^{N-m-1} x(l)\, e^{-j\frac{2\pi}{N}lk}\, e^{-j\frac{2\pi}{N}mk}}$$

$$\therefore \mathrm{DFT}[x(n-m)_N] = \sum_{l=N-m}^{N-1} x(l)\, e^{-j\frac{2\pi}{N}lk}\, e^{-j\frac{2\pi}{N}mk} + \sum_{l=0}^{N-m-1} x(l)\, e^{-j\frac{2\pi}{N}lk}\, e^{-j\frac{2\pi}{N}mk}$$

$$= e^{-j\frac{2\pi}{N}mk} \underbrace{\left[\sum_{l=0}^{N-1} x(l)\, e^{-j\frac{2\pi}{N}lk}\right]}_{X(k)}$$

$$\boxed{\mathrm{DFT}(x(n-m)_N) = e^{-j\frac{2\pi}{N}mk} X(k)}$$

<u>Circular convolution of DFT in time domain</u>:-

$$\left.\begin{array}{l} x_1(n) \overset{DFT}{\Leftrightarrow} X_1(k) \\ x_2(n) \overset{DFT}{\Leftrightarrow} X_2(k) \end{array}\right\} \Rightarrow x_1(n) \, Ⓝ \, x_2(n) \overset{DFT}{\Leftrightarrow} X_1(k) X_2(k)$$

<u>Proof</u>:-

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi}{N}nk}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j\frac{2\pi}{N}nk}$$

$$\mathrm{DFT}[x_1(n) \, Ⓝ \, x_2(n)] = \sum_{n=0}^{N-1} [x_1(n) \, Ⓝ \, x_2(n)]\, e^{-j\frac{2\pi}{N}nk}$$

$$= \sum_{n=0}^{N-1} \left[\sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)_N\right] e^{-j\frac{2\pi}{N}nk}$$

Candidates must not write on this margin

$$= \sum_{m=0}^{N-1} x_1(m) \left[\sum_{n=0}^{N-1} x_2(n-m)_N \, e^{-j\frac{2\pi}{N}nk} \right]$$

$$= \sum_{m=0}^{N-1} x_1(m) \left[e^{-j\frac{2\pi}{N}mk} X_2(k) \right] \quad \left[\text{From circular time shift property} \right]$$

$$= X_2(K) \sum_{m=0}^{N-1} x_1(m) \, e^{-j\frac{2\pi}{N}mk}$$

$$\boxed{DFT\left[x_1(n) \circledN x_2(n)\right] = X_2(K)\, X_1(K)}$$

Candidates must not write on this margin

8(b). Determine the transfer function of a compensator that will provide a phase lead of 45° and gain of 10dB at ω =8 rad/sec, without using graphical approach. **(20M)**

Sol

Given,

[Maximum] phase lead, $\phi_m = 45°$

Frequency at ϕ_m, $\omega_m = 8$ rad/s

Gain at ϕ_m, $K_m = 10$ dB

Transfer function of a standard phase lead compensator is,

$$G_c(s) = K \frac{1+\tau s}{1+\alpha\tau s}, \quad \alpha < 1$$

We know that, Maximum phase lead,

$$\boxed{\phi_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right]}$$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\sin 45^\circ = \frac{1-\alpha}{1+\alpha}$$

$$\frac{1}{\sqrt{2}} = \frac{1-\alpha}{1+\alpha}$$

$$\frac{1+\sqrt{2}}{1-\sqrt{2}} = \frac{2}{-2\alpha} \Rightarrow \alpha = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$\boxed{\alpha = 0.1716}$$

Candidates must not write on this margin

Frequency at which maximum phase lead occurs, $\boxed{\omega_n = \frac{1}{\tau\sqrt{\alpha}}}$

$$8 = \frac{1}{\tau\sqrt{0.1716}}$$

$$\boxed{\therefore \tau = 0.302 \text{ sec}}$$

Gain offered by the compensator (in dB) at maximum phase lead is given by,

$$|G_c(s)|_{s=8\text{rad/s}} = 20\log\left|K\frac{(1+\tau s)}{1+\alpha\tau s}\right|$$

10 dB (given) $= 20\log K + 20\log\sqrt{1^2+\tau^2 s^2} - 20\log\sqrt{1^2+\alpha^2\tau^2 s^2}$

Substituting $s = 8$ rad/s, $\alpha = 0.1716$, $\tau = 0.302$

$$10\text{dB} = 20\log K + 20\log\sqrt{1+(2.416)^2} - 20\log\sqrt{1+(0.4146)^2}$$

$$10 = 20\log K + [\log 2.615 - \log 1.17]20$$

$$\Rightarrow \boxed{K = 1.31}$$

$\therefore$ The transfer function of the compensator is $G_c(s) = 1.31 \dfrac{[1+(0.302)s]}{[1+(0.052)s]}$

Candidates must not write on this margin

8(c). Design a chebyshev low pass filter with pass band attenuation α_p = 1dB for $0 \le \omega \le 0.2\pi$ and stop band attenuation α_s = 15dB for $0.3\pi \le \omega_s \le \pi$, using bilinear transformation method. **(20M)**

Candidates must not write on this margin

Sol Given,

$\alpha_p = 1dB$

$\omega_p = 0.2\pi$

$\alpha_s = 15dB$

$\omega_s = 0.3\pi$

prewrapped frequencies are,

$\Omega_p = \frac{2}{T}\tan\left[\frac{\omega_p}{2}\right] = 0.65$ [T = 1sec]

$\Omega_s = \frac{2}{T}\tan\left[\frac{\omega_s}{2}\right] = 1.02$

order $N \geq \dfrac{\cosh^{-1}\left[\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}\right]}{\cosh^{-1}\left[\dfrac{\Omega_s}{\Omega_p}\right]} = 3.01$

$\therefore N = 4$ [rounded of to next higher integer]

$\xi = \sqrt{10^{0.1\alpha_p}-1} = 0.508$

$\mu = \xi^{-1} + \sqrt{1+\xi^{-2}} = 4.17$

$a = \Omega_p\left[\dfrac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.237$

$b = \Omega_p\left[\dfrac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.6918$

$\phi_k = \dfrac{\pi}{2} + \dfrac{(2k-1)\pi}{2N}$ $k = 1,2,3,4$

$\phi_1 = 112.5$, $\phi_2 = 157.5$, $\phi_3 = 202.5$, $\phi_4 = 247.5$

$$s_k = a\cos\phi_k + j\,b\sin\phi_k$$

$$s_1 = -0.0907 + j\,0.639$$

$$s_2 = -0.2189 + j\,0.2647$$

$$s_3 = -0.2189 - j\,0.2647$$

$$s_4 = -0.0907 - j\,0.639$$

$$\text{Denominator} = (s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.118)$$

$$\text{Numerator} = \frac{0.4165 \times 0.0118}{\sqrt{1+\epsilon^2}} = 0.04381$$

$$H(s) = \frac{0.04381}{(s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.118)}$$

Given bilinear transformation.

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z}\right]$$

$$H(z) = \frac{0.001836\left[1+z^{-1}\right]^4}{\left[1 - 1.499z^{-1} + 0.8482z^{-2}\right]\left[1 - 1.5542z^{-1} + 0.6492z^{-2}\right]}$$

Candidates must not write on this margin

Candidates must not write on this margin

Candidates must not write on this margin

FEEDBACK SHEET

(To be filled by examiner)

Rating on a scale of 1 to 10

(1 being the least and 10 being the highest)

1. Competency in understanding the question.

2. Competency in approach / methodology while answering

3. Competency in space management

4. Competency in drawing figures / circuits / black diagrams / graphs etc.

5. Competency in presentation and hand writing.

SPACE FOR ROUGH WORK

Candidates must not write on this margin

Candidates must not write on this margin

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

Candidates must not write on this margin

SPACE FOR ROUGH WORK

Candidates must not write on this margin

3b

~~5a~~

7a

8c