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ESE 2018 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test 5 : Flow of fluids, hydraulic machines and hydro power

Design of concrete and Masonry Structures-1 + Strength of Materials-2

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Roll No: C E 1 8 M T D L A 6 0 6

Test Centres					Student's Signature
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	55
Q.2	55
Q.3	45
Q.4	
Section B	
Q.5	45
Q.6	
Q.7	
Q.8	47
Total Marks Obtained	247

Signature of Evaluator

Cross Checked by

1) Good with speed & Efficiency
2) attempted/selection of questions very well
3) Practice other good level of Questions.

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Section A : Flow of fluids, hydraulic machines and hydro power

Q.1 (a) Explain the Newton's law of viscosity and distinguish between Newtonian and non-Newtonian fluids.

Sol(n)

(1) Newtons Law of viscosity states that shear stress is directly proportional to rate of shear strain ($\frac{d\theta}{dt}$).

$$\tau \propto \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$$\theta = \frac{x}{y}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

07

$$\boxed{\tau = \mu \frac{du}{dy}}$$

constant of propotionality (μ) is called dynamic viscosity of fluid

(2) General equation of shear stress is

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

Newtonian fluid	Non-newtonian Fluid
- shear stress $\propto$ shear strain	- shear stress not directly propotional to shear strain
- $B=0$, $A=\mu$, $n=1$	B can be 0 or not 0, $n \neq 1$
- ex:- water, air, Gasoline	ex:- toothpaste, ink, ketchup

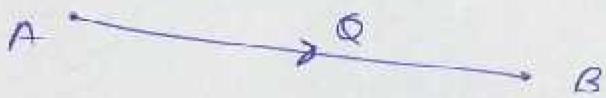
Q.1 (b) A 600 mm diameter pipe is used to pump water with a head loss of 20 m. The power consumption is to be reduced by laying another pipeline of appropriate diameter by the side of the existing pipeline with the same overall length and the same friction factor thereby reducing the loss to 10 m, but still delivering the same discharge jointly through both the pipes. What should be the diameter of this additional pipe?

[12 marks]

Soln

$D = 600\,mm \qquad h_{f_1} = 20\,m$

$h_{f_2} = 10\,m$



By Darcy Weischast equation

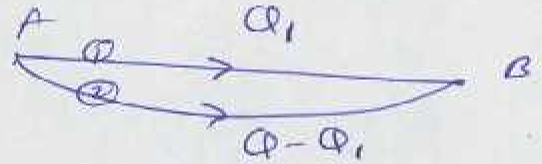
$$h_{f_1} = 20 = \frac{f l Q^2}{12.1 D^5}$$

$$\text{let } \frac{fl}{12.1} = K$$

12

$$20 = K\frac{Q^2}{D^5} \Rightarrow \boxed{\frac{1.297}{\sqrt{K}} = Q} \;\; \text{—}(1)$$

(II) Discharge remains same $= Q$



$$h_f = 10 = \frac{f l Q_1^2}{12.1 D_1^5} = \frac{f l (Q - Q_1)^2}{12.1 D_2^5}$$

$$\boxed{\frac{Q_1^2}{D_1^5} = \frac{(Q - Q_1)^2}{D_2^5}}$$

now

$$10 = \frac{K Q_1^2}{0.6^5}$$

$$Q_1 = \frac{0.882}{\sqrt{K}}$$

now

$$10 = K\frac{(Q - Q_1)^2}{D_2^5}$$

$$D_2^{2.5} = \sqrt{K}\frac{(Q - Q_1)}{\sqrt{10}}$$

$$D_2^{2.5} = \cancel{\sqrt{K}} \left(\frac{\frac{1.247}{\cancel{\sqrt{K}}} - \frac{0.882}{\cancel{\sqrt{K}}}}{\sqrt{10}} \right)$$

$$\boxed{D_2 = \frac{0.668\,m}{(\sqrt{10})^{\frac{1}{2.5}}}} = 0.4216\,m$$

$$\boxed{D_2 = 421.6\,mm \simeq 422\,mm}$$

Thus dia of new pipe = 422 mm

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Q.1 (c) The shear stress distribution in a laminar boundary layer is given as:

$$\tau = \tau_0\left(1 - \frac{y}{\delta}\right) \text{ where } \delta = \text{boundary layer thickness}$$

Determine the Shape factor for this boundary layer.

[12 marks]

Soln

$$\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$$

$$\mu \frac{du}{dy} = \tau_0\left(1 - \frac{y}{\delta}\right)$$

$$\frac{\mu}{\tau_0}\int_0^u du = \int_0^y dy - \int_0^y \frac{y\,dy}{\delta}$$

$$\boxed{\frac{\mu}{\tau_0} u = y - \frac{y^2}{2\delta}}$$

at $y = \delta$ $\frac{\mu u}{\tau_0} = \delta - \frac{\delta}{2} = \frac{\delta}{2}$

12

free stream u_0 $\rightarrow$ $\boxed{u_{max} = \frac{\delta \tau_0}{2\mu}}$

Displacement thickness $= \delta_*$

$$\delta_* = \int_0^\delta \left(1 - \frac{u}{u_0}\right) dy$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

$$\frac{u}{u_0} = \frac{\tau_0 2\mu}{\mu \delta \tau_0}\left(y - \frac{y^2}{2\delta}\right)$$

$$\boxed{\frac{u}{u_0} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}}$$

$$\delta_* = \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$\delta_* = \delta - \delta + \frac{\delta}{3} = \boxed{\frac{\delta}{3} = \delta_*}$$

Now momentum thickness $= \theta$

$$\theta = \int_0^\delta \frac{u}{u_0}\left(1-\frac{u}{u_0}\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left(1-\frac{2y}{\delta}+\frac{y^2}{\delta^2}\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right)$$

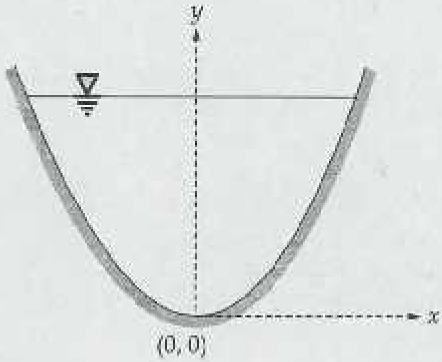
$$\theta = \delta - \frac{4}{3}\delta + \frac{2}{4}\delta - \frac{\delta}{3} + \frac{2\delta}{4} - \frac{\delta}{5}$$

$$\boxed{\theta = \frac{2\delta}{15}}$$

shape factor $= \frac{\delta^*}{\theta} = \frac{\delta}{3\times 2\delta}15 = 2.5$

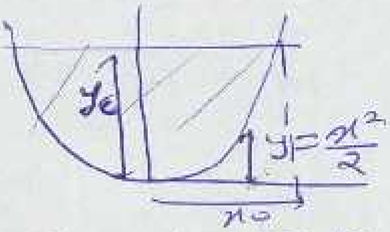
$$\boxed{S.F. = 2.5}$$

Q.1 (d) A parabolic channel is defined by the equation $x^2 = 2y$. Determine the critical depth for a discharge of 2.0 m^3/s.



[12 marks]

Soln



$\boxed{x^2 = 2y}$

For critical depth $y = y_c$, $x = x_o$

$F_r^2 = 1 = \frac{Q^2 T}{A^3 g} = 1$

Let $y = y_c$

$y_c = \frac{x_o^2}{2} \Rightarrow \boxed{x_o^2 = 2y_c}$

Area = A

$A = 2\int_0^{x_o} \left(y_c - \frac{x^2}{2}\right) dx$

12

$\frac{A}{2} = y_c \times x_o - \frac{x_o^3}{6}$

$A = 2\left[x_o \times \frac{x_o^2}{2} - \frac{x_o^3}{6}\right] = \frac{2x_o^3}{3}$

$\boxed{A = \frac{2x_o^3}{3}}$

$\frac{Q^2 (T)}{A^3 g} = 1$

$T = 2x_0 \quad A = \frac{2x_0^3}{3}$

$\frac{Q^2 T}{A^3 g} = 1$

$\frac{(2)^2 \times 2x_0}{\left(\frac{2x_0^3}{3}\right)^3} = g$

$\frac{4 \times 2 x_0}{\frac{8}{27} x_0^9} = g \Rightarrow x_0^8 = \frac{27}{g}$

$\boxed{x_0 = 1.135\,m}$

$y_c = \frac{x_0^2}{2} = 0.644\,m$

$\boxed{y_c = 0.644\,m}$

Q.1 (e) A model of scale 1 : 9 is constructed to study the performance of hydraulic structure in a power project. Neglecting the viscous and surface tension effects, determine

(i) rate of flow in the model for a discharge of 1000 m^3/s in prototype.

(ii) the energy dissipation in the prototype hydraulic jump, if the jump in the model dissipates 294.2 watts of energy.

[12 marks]

Soln

$L_r = \frac{1}{9}$ $\quad$ p → prototype, m → model

$Q_p = 1000\,m^3/s$

As per Froudes Law

$\boxed{\frac{v_m}{v_p} = v_r = \sqrt{L_r}}$

(1) For kinematic similarity

$\frac{Q_m}{Q_p} = \frac{A_m}{A_p} \times \frac{v_m}{v_p}$

$\frac{Q_m}{Q_p} = L_r^2 \times \sqrt{L_r}$

$\frac{Q_m}{Q_p} = L_r^{2.5}$

Do not write in this margin

$$Q_m = 1000 \times \frac{1}{9^{2.5}} = \cancel{0.559}\ 4.115\ m^3/s$$

$$\boxed{\text{Model discharge} = \cancel{0.559}\ 4.115\ m^3/s}$$

(ii) Now

$$\frac{\text{Energy}}{\text{time}} = \text{power}$$

$$\boxed{P_m = 294.2\,W}$$

$$\frac{P_m}{P_P} = F_r V_r$$

$$\frac{P_m}{P_P} = L_r^2 \times V_r^2 \times V_r$$

$$\frac{P_m}{P_P} = L_r^2 V_r^3 = L_r^2 \times L_r^{1.5}$$

$$\boxed{\frac{P_m}{P_P} = L_r^{3.5}}$$

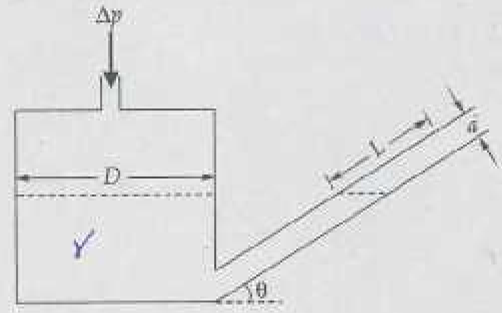
$$P_P = 294.2 \times \cancel{20}\,9^{3.5}$$

$$\boxed{P_P = \cancel{10.526\ MW}\ 643.415\ kW}$$

energy dissipation in prototype

$$\boxed{P_P = \cancel{10.526\ MW}}$$
643.415 kW

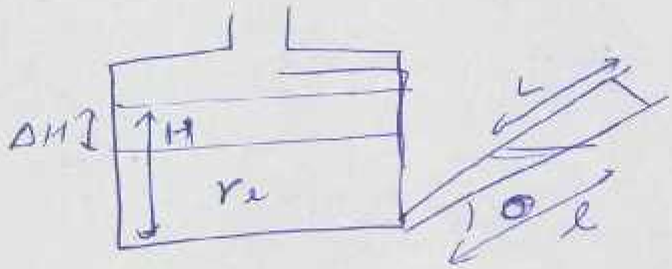
Q.2 (a) An inclined tube reservoir manometer is constructed as shown below. Analyze the manometer to obtain a general expression for the liquid deflection L, in the inclined tube in terms of applied pressure difference, Δp.



Also obtain a general expression for the manometer sensitivity and determine the parameter values that will give maximum sensitivity.

[20 marks]

Soln



$$L \times \frac{\pi d^2}{4} = \frac{\pi D^2}{4} \times \Delta H$$

$$\boxed{\Delta H = \frac{L d^2}{D^2}}$$

20

Now Before Δp application

$$\gamma \times H - \gamma l \sin\theta = 0 \quad \text{--- (1)}$$

$$\Delta p + \gamma (H - \Delta H) = \gamma (l + L) \sin\theta$$

$$\Delta p = \gamma (l + L) \sin\theta - \gamma (H - \Delta H)$$

By (1)

$$\Delta p = \gamma L \sin\theta + \gamma \Delta H$$

$$\boxed{\frac{\Delta p}{\gamma} = L \sin\theta + \frac{L d^2}{D^2}}$$

~~$\frac{\Delta p}{\gamma} - L\sin\theta$~~

~~$\frac{\Delta p}{\gamma} - L\frac{d^2}{D^2}$~~

$$\boxed{\frac{\frac{\Delta p}{\gamma}}{\sin\theta + \frac{d^2}{D^2}} = L}$$

To maximize sensitivity, 'θ' has to be decreased

~~θ = 0~~ & $\boxed{\theta = 0°}$

$\frac{d}{D}$ should decrease

$\boxed{D >>>> d}$ $\boxed{\theta \to 0°}$

Q.2 (b) A rectangular flume of 2 m width carries discharge at the rate of 2 m^3/sec. The bed slope of the flume is 4×10^{-4}. At a particular section, the depth of flow is 1 m. Assume rugosity coefficient as 0.014.

(i) Is the slope of the channel mild or steep? How is this type of surface profile classified?

(ii) Using single step method, calculate the distance of the section downstream where the depth of flow is 0.90 m.

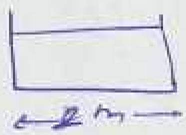
[20 marks]

Soln

(1) $Q = 2 m^3/s$

$S_0 = 4 \times 10^{-4}$

$n = 0.014$



Lets calculate slope for critical depth i.e. critical slope

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\left(\frac{2}{2}\right)^2 \times \frac{1}{9.81}\right)^{\frac{1}{3}}$$

$$\boxed{y_c = 0.467 m}$$

15

$S_{cr} = ?$

By Manning

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S_{cr}}$$

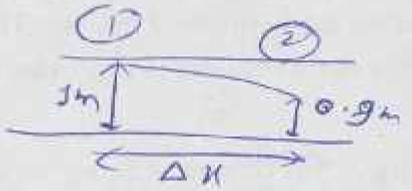
$$2 = \frac{1}{0.014} \times (2 \times 0.467) \left(\frac{0.934}{2.934}\right)^{\frac{2}{3}} \times \sqrt{S_{cr}}$$

critical slope $\boxed{S_{cr} = 41.346 \times 10^{-4}}$

$\boxed{S_{cr} > S_0} = 4 \times 10^{-4}$ thus

$\boxed{\text{channel is mild sloped}}$

Profile?



$$\frac{\Delta E}{\Delta x} = S_0 - S_f$$

At 1

$$E = y + \frac{v^2}{2g} = 1 + \frac{\left(\frac{2}{2\times1}\right)^2}{19.62}$$

$$E_1 = 1.051$$

$$v = \frac{1}{n} R^{2/3} \sqrt{S_f}$$

$$S_{f_1} = \frac{v^2 n^2}{R^{4/3}}$$

$$S_{f_1} = \frac{0.019^2 \times 1}{\left(\frac{2}{4}\right)^{4/3}}$$

$$S_{f_1} = 0.000454$$

At 2 $y = 0.9$

$$E = 0.9 + \frac{\left(\frac{2}{0.9\times2}\right)^2}{19.62}$$

$$= 0.963$$

$$S_{f_2} = \frac{1.111^2 \times 0.019^2}{\left(\frac{1.8}{3.8}\right)^{4/3}}$$

$$S_{f_2} = 0.000552$$

$$\frac{E_2 - E_1}{\Delta x} = S_0 - S_f$$

$$\frac{1.051 - 0.963}{\Delta x} = 4 \times 10^{-4} - \left(\frac{0.000579}{2} \right.$$

$$\frac{0.088}{\Delta x} = 0.0001127$$

$$\boxed{\Delta x = 780.834\,m}$$

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Q.2 (c) A reaction turbine operates at a speed of 450 rpm under a head of 120 m. Its inlet diameter is 1.2 m and the flow area is 0.40 m^2. The angle made by the absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Calculate

(i) the volume flow rate

(ii) the power developed

(iii) the hydraulic efficiency

Assume whirl at the outlet to be zero.

[20 marks]

Soln

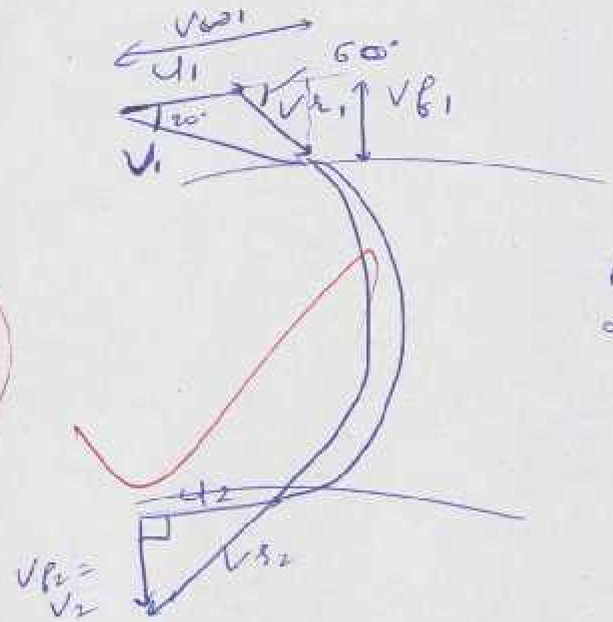
$N = 450\ rpm$

$H = 120m$

Assumption: $\boxed{V_{w_2} = 0}$

$D_1 = 1.2m$

$A f = 0.40m^2$



$\beta = 120°$

$\alpha = 20°$

20

$u_1 = \frac{\pi D_1 N}{60} = 28.274\ m/s$

From Inlet Δ

$\tan 20° = \frac{V_{f1}}{V_{w1}}$

$\tan 60° = \frac{V_{f1}}{V_{w1} - u_1}$

$\frac{\tan 20°}{\tan 60°} = \frac{V_{w1} - u_1}{V_{w1}}$

$$0.2101 = \frac{V_{w_1} - 28.274}{V_{w_1}}$$

$$\boxed{V_{w_1} = 35.794 \text{ m/s}}$$

Now

$$V_{f_1} = \tan 20^\circ \times V_{w_1}$$

$$\boxed{V_{f_1} = 13.027 \text{ m/s}}$$

Discharge

① $Q = A_f V_f$

$= 13.027 \times 0.4$

$$\boxed{Q = 5.211 \text{ m}^3/\text{s}}$$

Power

② $P = \rho Q V_{w_1} u_1$

$= 10^3 \times 5.211 \times 35.794 \times 28.274$

$$\boxed{P = 5273.738 \text{ kW}}$$

Hydraulic efficiency

③ $\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.794 \times 28.274}{120 \times 9.81}$

$$\boxed{\eta_h = 85.97\%}$$

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Q.3 (a) (i) In a rectangular laboratory flume of width 0.6 m and having adjustable bottom slope, a flow of 100 litres/second takes place. Assuming Chezy's constant, C = 56, determine the bottom slope necessary for uniform flow with depth of flow of 0.30 m. Also, calculate the conveyance and the state of flow (i.e. tranquil or rapid).

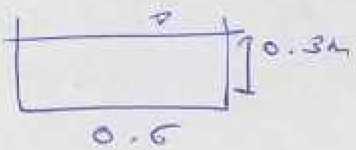
(ii) A wooden cone weighing 88 N floats with its apex downwards in a liquid of specific gravity 0.80. The specific gravity of wood is 0.50. What weight of steel piece, having specific gravity 7.8, suspended from apex of the cone by a chord will just suffice to submerge the cone? Also calculate tension in the chord.

[10 + 10 = 20 marks]

Soln

(1) $B = 0.6\,m$

$Q = 0.1\,m^3/s$



$C = 56$

S_0 = bottom slope = ?

$y_n = 0.3\,m$

As per chezy

$Q = C\sqrt{RS} \times A$

15

$A = 0.6 \times 0.3 = 0.18\,m^2$

$P = 0.6 + 2 \times 0.3 = 1.2\,m$

$R = A/P = 0.15\,m$

$0.1 = 56\sqrt{0.15} \times \sqrt{S_0} \times 0.18$

$$\boxed{S_0 = 0.00065601 = \frac{1}{1524.09}}$$

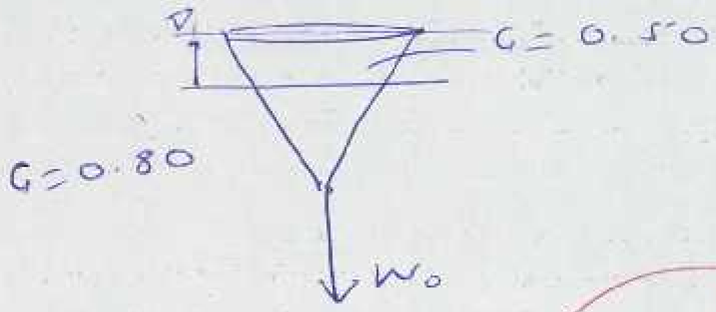
$v = \frac{Q}{A} = \frac{0.1}{0.18} = \frac{5}{9}\,m/s$

froudes # $\frac{V}{\sqrt{gy}} = \frac{5/9}{\sqrt{9.81 \times 0.3}} = 0.323$

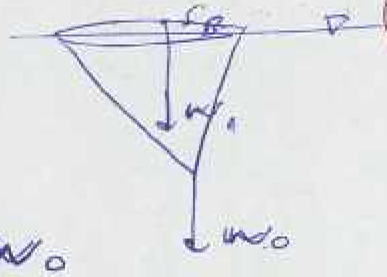
$F_r < 1 \Rightarrow$ Flow is

$\boxed{\text{Tranquil}}$

(ii)



$W_1 = 88 N$??



Let it be W_o

$F_B = W_1 + W_o$, $V \rightarrow$ volume of cone

$W_1 = 88 N = \gamma_w \times 0.5 \times V$

$$\boxed{V = \frac{88}{4905} m^3}$$

Now

F_B

$0.8 \times \gamma_w \times V = 88 + W_o$

$W_o = -88 + 0.8 \times 9810 \times \frac{88}{4905}$

$$\boxed{W_o = 52.8 N}$$??

$\boxed{\text{Tension in chord} = W_o = 52.8 N}$.

Q.3 (b) A centrifugal pump delivers 40 litres/second of water to a height of 20 m through a 150 mm diameter and 100 m long pipeline. The overall efficiency of pump is 70% and Darcy's friction factor, f = 0.06 for the pipeline. Assuming inlet losses in the suction pipe is equal to 0.33 m, determine the power required to drive the centrifugal pump.

[20 marks]

Soln

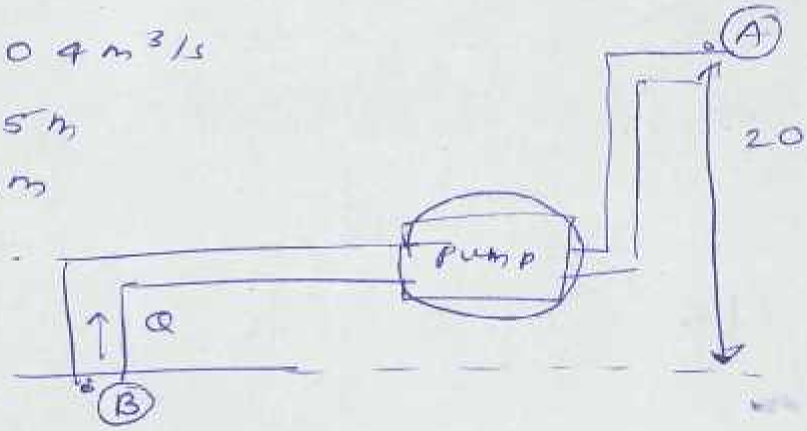
$Q = 0.04\, m^3/s$

$d = 0.15\, m$

$l = 100\, m$

$\eta = 0.70$

$f = 0.06$



Bernoulli's between A and B

Let pump adds effective head h_p

$h_p = 20 + (h_f) +$ entry loss + exit loss

As per darcy weisbach equation

$$h_f = \frac{f l Q^2}{12.1 D^5} = \frac{0.06 \times 100 \times 0.04^2}{12.1 \times 0.15^5}$$

$$\boxed{h_f = 10.448\, m}$$

20

entry loss = 0.33 m

$$\text{exit loss} = \frac{v^2}{2g} = \frac{Q^2}{2gA^2} = \frac{0.04^2}{19.62 \times \left(\frac{\pi}{4} \times 0.15^2\right)^2}$$

$$= 0.261\, m$$

$h_p = 20 + 10.448 + 0.33 + 0.261$

$$\boxed{h_p = 31.039\, m}$$

power pump generate $= \gamma_w Q h_p$

$= 9.81 \times 0.04 \times 31.0339$

$= 12.178$ kW

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power required $= \frac{P}{\eta}$

$= \frac{12.178}{0.70} = 17.39\,kW$

$\boxed{\text{Power needed} = 17.39\,kW}$

Q.3 (c) A velocity field is given by $\vec{V} = Ax\hat{i} - Ay\hat{j}$; the unit of velocity is m/s and x and y are in meters. $A = 0.3\text{s}^{-1}$.

(i) Obtain an equation for the streamlines in the xy-plane.

(ii) If the particle passing through point (x, y) is marked at time $t = 0$ then determine the location of the particle at time, $t = 6$ sec.

(iii) What is the velocity of the this particle at time, $t = 6$ sec?

(iv) Show that equation of the particle path (i.e. pathline) is the same as the equation of the streamline.

[20 marks]

soln

$\vec{V} = 0.3x\hat{i} - 0.3y\hat{j} = u\hat{i} + v\hat{j}$

$u = 0.3x \qquad v = -0.3y$

(i) Streamline

$\frac{dx}{u} = \frac{dy}{v} \qquad \frac{dy}{dx} = \frac{v}{u}$

10

$\frac{dy}{dx} = -\frac{0.3y}{0.3x}$

$\frac{dy}{y} = -\frac{dx}{x}$

$\ln y = -\ln x + \ln k$

equation of streamline $\boxed{xy = k'} = \psi \qquad k' = \text{constant}$

(ii) $x = \int u\,dt \Rightarrow \frac{dx}{dt} = u$

$y = \int v\,dt \qquad \frac{dy}{dt} = v$

~~$x = 0.3x$~~ At ~~After~~ $\boxed{6 = t}$, location $= (x_0, y_0)$

$\frac{dx}{dt} = 0.3x$

$\frac{dx}{x} = 0.3t$

$\ln x \Big|_x^{x_0} = 0.3 \times 6$

$\ln \frac{x_0}{x} = 1.8$

$\boxed{x_0 = x e^{1.8}}$

$\frac{dy}{dt} = -0.3y$

$\ln y \Big|_y^{y_0} = -0.3t$

$\ln \frac{y_0}{y} = -1.8$

$\boxed{y_0 = y e^{-1.8}}$

location $= (xe^{1.8}, ye^{-1.8})$

Do not write in this margin

$\vec{V} = 0.3x\hat{i} - 0.3y\hat{j}$

at $t = 6 \Rightarrow (x_0, y_0)$

$V_{t=6} = 0.3\, x_0\hat{i} - 0.3 y_0\hat{j}$

$V_{t=6} = 0.3 \times x e^{1.8}\hat{i} - 0.3 y e^{-1.8}\hat{j}$

$\cancel{V_{t=6} = 0.3 e^{1.8}[x\hat{i} - y\hat{j}]}$

$\boxed{V_{t=6} = 0.3[x e^{1.8}\hat{i} - y e^{-1.8}\hat{j}]}$ x, y are location at t=0

(ii) path line equation

$x = \int u\,dt \qquad y = \int v\,dt$

$\frac{dx}{dt} = u \qquad \frac{dy}{dt} = v$

$\frac{dx}{dt} = 0.3x \qquad \frac{dy}{dt} = -0.3y$

$\ln(x/x_1) = 0.3t \qquad \ln(y/y_1) = -0.3t$

$\frac{x}{x_1} = e^{0.3t}$ — (1) $\qquad \frac{y}{y_1} = e^{-0.3t}$ — (2)

$(x_1, y_1) \rightarrow$ location at $t = 0$

multiply (1) × (2)

$\frac{x}{x_1} \times \frac{y}{y_1} = e^{0.3t} \times e^{-0.3t}$

$\boxed{xy = k}$

equation of streamline and path line same

Section B : Design of concrete and Masonry Structures-1 + Strength of Materials-2

Q.5 (a) (i) Explain Mises-Henky Theory.

(ii) Explain the limitations of Rankine's theory of failure.

[6 + 6 = 12 marks]

(i) Mises Henky Theory →

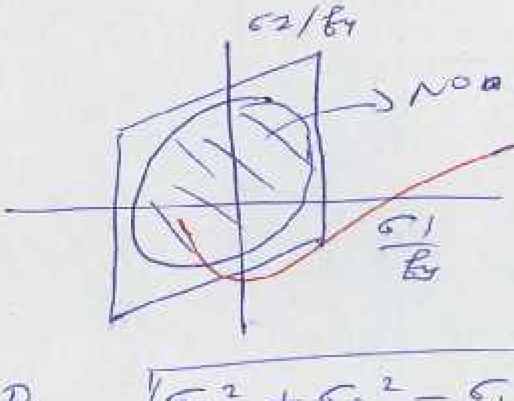
- Also called as maximum shear strain energy theory or distortion energy theory

- It states maximum shear strain energy theory stored in a material must not exceed maximum shear strain energy stored in a material under uniaxial tension.

If σ_1, σ_2, σ_3 represent principal stresses and f_y → yield strength of material and G → shear modulus then

As per this theory

$$\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le \frac{f_y^2}{6G}$$



6.5

In 2D octahedral theory → $\boxed{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 \le f_y^2}$

- Applicable for pure shear case
- Applicable for ductile material
- Not for brittle materials

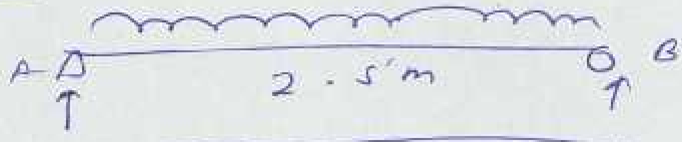
(ii) Rankines theory limitation

- Not for ductile materials
- Not applicable to pure shear case
- Not for hydrostatic loading
- only applicable to brittle materials

Q.5 (b) A laminated wooden beam 120 mm wide and 180 mm deep is made of three planks of size 120 mm × 60 mm each glued together to resist longitudinal shear. The beam is simply supported over a span of 2.5 m. If the allowable shearing stress in the glued joint is 0.5 N/mm², determine the safe UDL the beam can carry.

[12 marks]

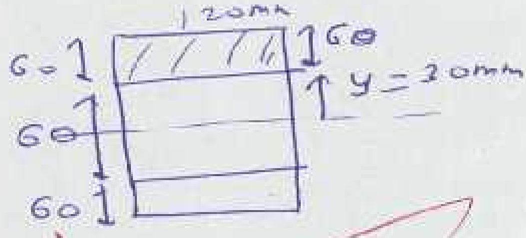
Sol



$$\boxed{\tau_{max} = 0.5\ N/mm^2}$$

Maximum shear stress will be at supports A and B as $V_{max} = \frac{wl}{2}$

$l = 2.5\ m$



$$\tau_{y=30mm} = \frac{VA\bar{y}}{Ib} = \frac{V(60\times120)(30+30)}{120\times\frac{180^3}{12}\times120}$$

$$\tau_0 = \tau_{y=30mm} = \frac{V}{16200}$$

12

Now

$$\tau_0 \le 0.5\ N/mm^2$$

$$\frac{V}{16200} \le 0.5\ N/mm^2$$

$$V \le 8100$$

$$\frac{w\times2500}{2} \le 8100$$

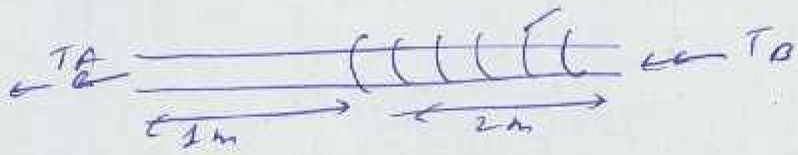
$$\boxed{w \le 6.48\ N/m}$$

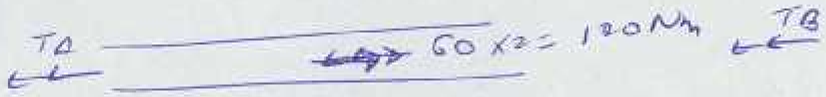
Loading $\boxed{w \le 6.48\ N/m}$

Q.5 (c) A solid steel bar of diameter 20 mm and length 3 m is fixed at both ends. It is loaded with uniformly distributed torsion of 60 Nm/m as shown below. Plot the variation of torsional load along the length and also calculate maximum angle of twist in the bar if modulus of elasticity of material is 200 GPa and Poisson's ratio is 0.25.

[12 marks]

sol^n

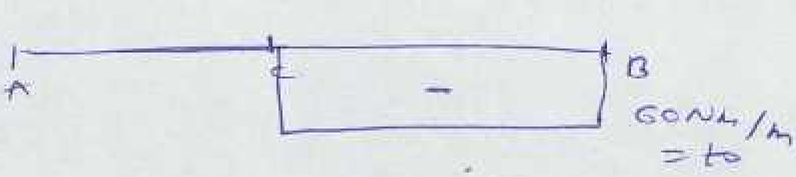




Torsional load between ~~C &~~ B = 60Nm/m

$\boxed{t_0 = 60 Nm/m}$ $\boxed{T_A + T_B = 120 Nm}$

Torsional load



12

In AC   CB

$\phi_{AB} = 0$

$$0 = \frac{+T_A \times 1}{GJ} + \int_0^2 \frac{(T_A - t_0 x)\,dx}{GJ}$$

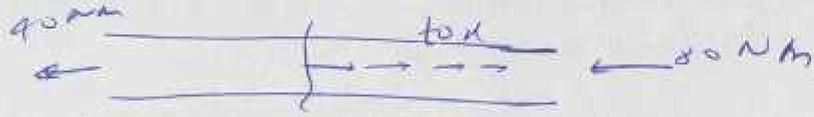
$$-T_A = T_A \times 2 - \frac{t_0 x^2}{2}\Big|_0^2$$

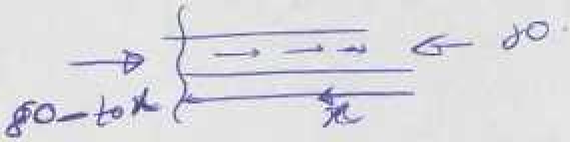
$$-3T_A = \frac{-t_0 \times 2 \times 2}{2} \Rightarrow 2t_0$$

$$-3T_A = -120$$

$$\boxed{T_A = 40 Nm}$$

$\boxed{T_B = 80 \text{ Nm}}$





$$\phi_{x/B} = \int_0^{x_0} \frac{(80-60x)\,dx}{GJ}$$

$$\phi_{x/B} = \left[80x_0 - \frac{60x_0^2}{2}\right] \times \frac{1}{GJ}$$

To maximise ϕ

$$\frac{d\phi}{dx} = 0 \qquad 80 - \frac{60}{\cancel{2}} \times \cancel{2} x_0 = 0$$

from B $\longrightarrow$ $\boxed{x_0 = \frac{4}{3}}$

$$\phi_{max} = \left[80 \times \frac{4}{3} - \frac{60}{2} \times \left(\frac{4}{3}\right)^2\right] \times \frac{1}{GJ}$$

$$\left(G = \frac{E}{2(1+\mu)} = \frac{200}{2(1+0.25)} = 80 \text{ GPa} \right.$$

$$\phi_{max} = \frac{\frac{160}{3}}{\frac{\pi}{32} \times 0.02^4 \times 80 \times 10^9}$$

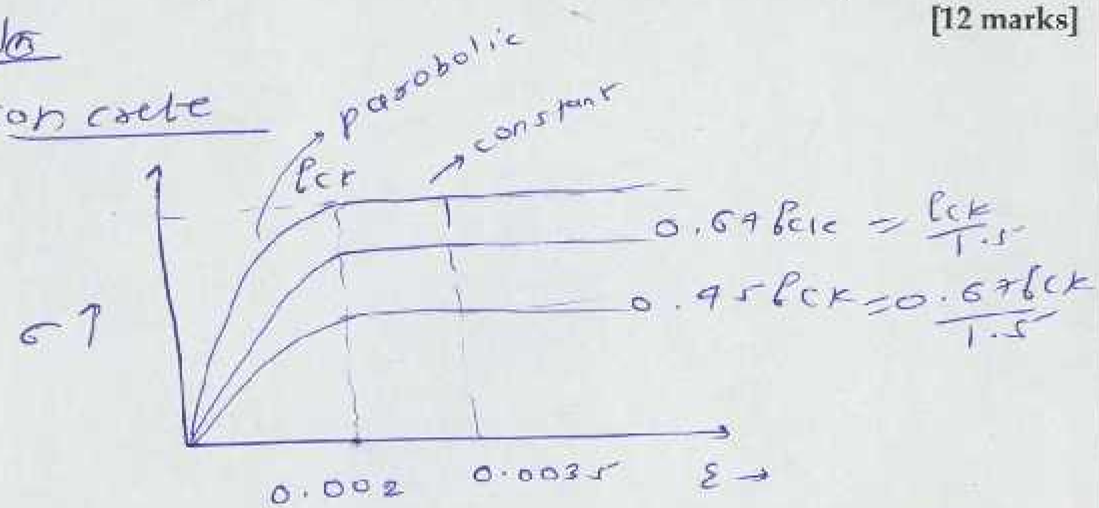
$$\boxed{\phi_{max} = 0.04244 \text{ radians}} = 2.432^\circ$$

Q.5 (d) Explain in brief the design stress-strain curve of concrete and steel respectively.

[12 marks]

Soln

Concrete



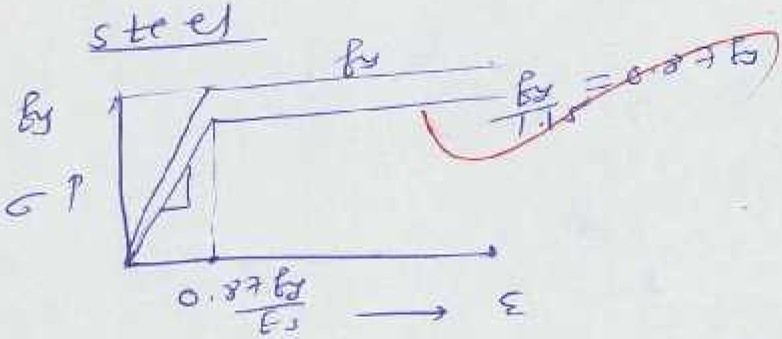
① stress strain curve of concrete is taken as parabolic rectangular, it validates test results

② curve is parabolic upto strain of 0.002 at which stress in concrete = 0.45 fck, after that stress remains constant, strain increases, At ε = 0.0035, concrete crushes in compression.

07

③ Two partial safety factors considered, first 1.5 because of size effect, another 1.5 for material strength

steel



① σ-ε curve is considered straight line upto σ = 0.87 fy, after that stress remains constant only strain increases. slope till 0.87fy = σ is Es

② partial factor of safety = 1.15 considered for steel, as there is better control over its quality.

Q.5 (e) State the assumptions made while analyzing the reinforced concrete beam using Limit State Method as per **IS 456:2000 Code.**

[12 marks]

soln

Assumptions

① plane section normal to axis remain plane even after bending. Linear strain distribution across depth of section

② Tensile strength of concrete ignored

③ strain in concrete at outermost compressed fibre = 0.0035 at which concrete crushes in compression and fails. Failure always occur by concrete crushing

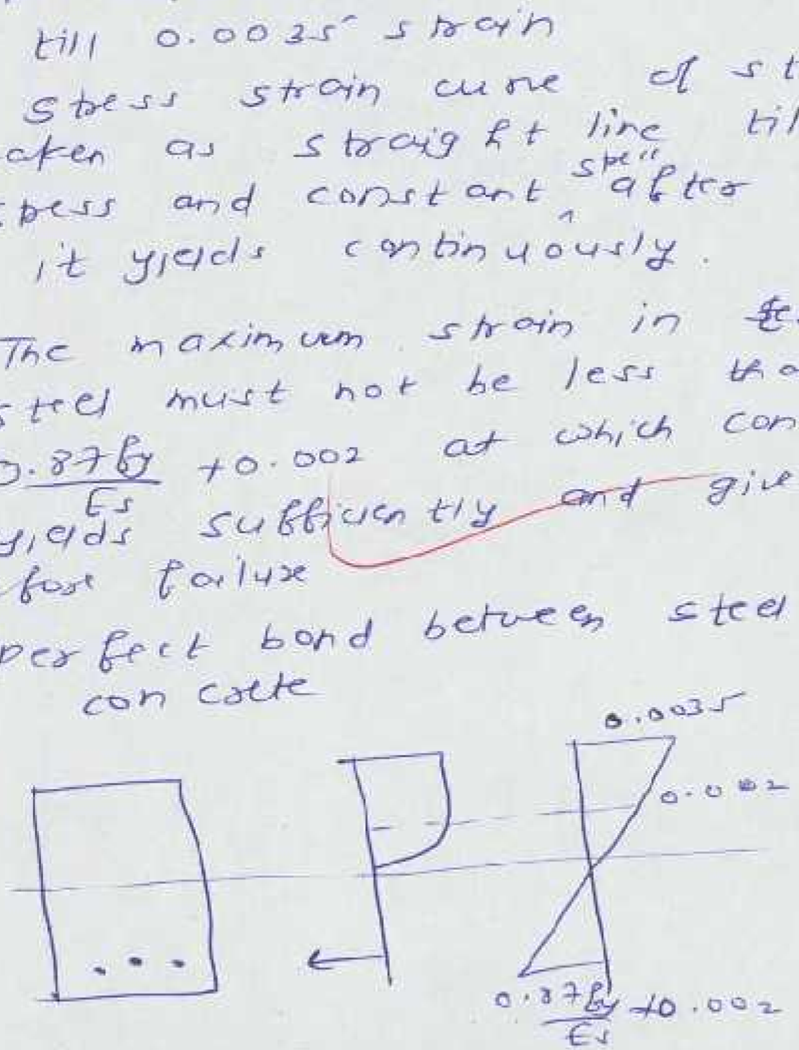
④ stress strain curve of concrete is taken as parabolic rectangular. parabolic till strain of 0.002 and stress reaches 0.45 fck, and rectangular till 0.0035 strain

08

⑤ stress strain curve of steel is taken as straight line till 0.87fy stress and constant after that, it yields continuously.

⑥ The maximum strain in tensile steel must not be less than $\frac{0.87f_y}{E_s}+0.002$ at which concrete yields sufficiently and gives warning before failure

⑦ perfect bond between steel and concrete



Q.8 (a) A steel shaft of diameter 100 mm rotates at 300 rpm. This steel shaft has a 20 mm thick bronze bushing over its entire length of 2 m. If the maximum shearing stress in steel shaft is not to exceed 40 N/mm^2, then find

(i) power of the engine

(ii) torsional rigidity of the shaft

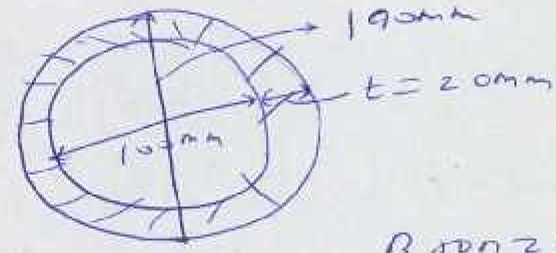
[Take G_{steel} = 84 kN/mm^2 and G_{bronze} = 42 kN/mm^2]

[20 marks]

Soln

ω = 300 rpm

l = 2 m



For steel

$D = 100$ mm

$\tau_{max} = 40$ N/mm^2

Bronze

$D_i = 100$ mm

$D_o = 140$ mm

(i) Now Let total torque = T

$$\frac{16 T_s}{\pi D^3} \le 40 \qquad T_s \rightarrow \text{torque in steel.}$$

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$$T_s \le \frac{40 \times \pi \times 100^3}{16}$$

$$\boxed{T_s \le 7.854 \text{ kNm}}$$

Now

$$\phi_{steel} = \phi_{Bronze}$$

$$\frac{T_s L}{(GJ)_s} = \frac{T_B L}{(GJ)_B}$$

$$\frac{7.854}{84 \times \frac{\pi}{32} \times 100^4} = \frac{T_B}{42 \times \frac{\pi}{32} \times (140^4 - 100^4)}$$

$$\boxed{T_B = 11.159 \text{ kNm}}$$

$$T = T_B + T_s = 19.013 \text{ kNm}$$

(i) power of engine = $T\omega$

$$= 19.013\ kNm \times \frac{300 \times 2\pi}{60}$$

$$\boxed{Power = 597.311\ kW}$$

(ii) Equivalent torsional rigidity

$(GJ)_{eq} = ?$

$\phi_{overall} = \phi_{shaft}$

$$\frac{T_{eq} L}{(GJ)_{eq}} = \frac{T_s L}{G_{st} J_{st}}$$

$$\frac{19.013 \times L}{(GJ)_{eq}} = \frac{7.854 \times L}{84 \times \frac{\pi}{32} \times 100^4}$$

$$GJ_{eq} = 84 \times \frac{\pi}{32} \times 100^4 \times \frac{19.013}{7.854}$$

$$\boxed{(GJ)_{eq} = 19.964 \times 10^8\ kNmm^2}$$

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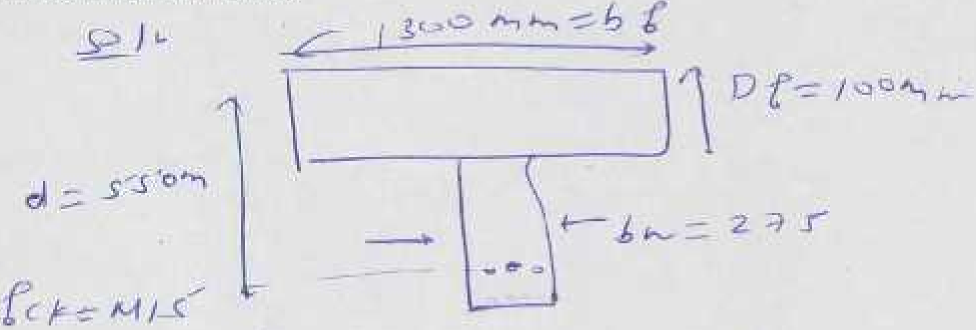
Q.8 (b) A T-beam of effective flange width 1300 mm, flange thickness 100 mm, rib width 275 mm has an effective depth of 550 mm. The beam is reinforced with 5 bars of 25 mm diameter. Find the ultimate moment of resistance by the limit-state method. Use M15 grade of concrete and Fe415 steel.

[20 marks]

Sol



fck = M15

$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$

(i) Checking location of neutral axis

Assume $x_u = D_f = 100$ mm

$C = 0.36 f_{ck} b_f D_f = 702000$

20

$T = 0.87 f_y A_{st} = 886150$

$\boxed{T > C}$ N.A. $x_u > D_f$

(ii) $C = T$

$0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w)(0.65 D_f + 0.15 x_u)$

$886150.29 = 1485 x_u + 6918.75 (65 + 0.15 x_u)$

$436931.54 = 2522.8125 x_u$

$\boxed{x_u = 173 \text{ mm}} < \frac{7 D_f}{3}$

$\frac{7 D_f}{3} = 233.33$

$\boxed{x_u < x_{u\,lim} = 0.48 d = 264 \text{ mm}}$

OK

MOR ⇒
effective flange depth = y_f

$$y_f = 0.15 x_u + 0.65 D_f$$

$$y_f = 65 + 0.15 \times 173 = 90.95 \text{ mm} < D_f$$

$$\boxed{MOR = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) y_f (d - y_f/2)}$$

$$MOR = 0.36 \times 15 \times 275 \times 173 (550 - 0.42 \times 173) + 0.45 \times 15 (1025) \times 90.95 \times (550 - \frac{90.95}{2})$$

$$\boxed{MOR = 440.108 \text{ kNm}}$$

Q.8 (c) Design a short circular column using M25 grade of concrete and Fe415 steel which is subjected to an axial load of 1000 kN under service load conditions. The column is to be provided with spiral ties. Also provide clear cover of 40 mm for reinforcement.

[20 marks]

Soln

① $\lambda = \frac{l_{eff}}{D} \leq 12 \rightarrow$ short column

Take $D = 400$ mm of column

?

② ~~e_{min} check~~

e_{min} $\left[\frac{D}{500} \right.$

M25 Fe 415 steel

$P = 1000$ kN $\quad P_u = 1.5P$
$= 1500$ kN

As per IS code, it will be designed for 5% overload

$P_u = 1.05 \left[0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc} \right]$

$\frac{1500 \times 10^3}{1.05} = 0.4 \times 25 \left[\frac{\pi}{4} \times 400^2 - A_{sc} \right] + 278.05\, A_{sc}$

7

$171934.3571 = A_{sc} \times 268.05$

$\boxed{A_{sc} = 641.43 \text{ mm}^2}$

$A_{sc\,min} = 0.8\% \frac{\pi}{4} D^2 = 1005 \text{ mm}^2$

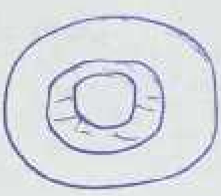
provide $(A_{sc})_{min}$

provide 6 # 16mm ϕ

A_{sc} provided $= 1206 \text{ mm}^2$

spiral reinforcement design

CC = 40 mm



$D_c = 400 - 2 \times 40$

$D_c = 320\ mm$

let lateral spiral reinforcement dia = ϕ_h = 6 mm

$D_{helical} = D_h = 320 - \phi_h = 314\ mm$

Now

$$0.36 \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \le \frac{V_h}{V_c}$$

$$\boxed{V_c = 1000 \times \frac{\pi}{4} \times D_c^2}$$

$$V_h = \frac{1000}{\text{spacing }(p)} \times \pi D_h \times \frac{\pi}{4} \times \phi^2$$

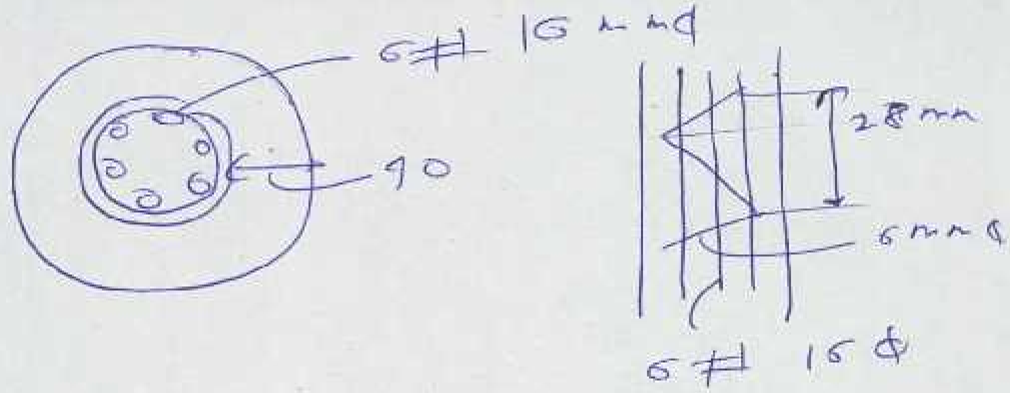
$$0.36 \times \frac{25}{415}\left(\frac{400^2}{320^2} - 1\right) \le \frac{1000 \times \pi D_h \times \frac{\pi}{4} \phi^2}{p \times 1000 \times \frac{\pi}{4} \times D_c^2}$$

$$0.01219 \le \frac{\pi \times 314 \times \frac{\pi}{4} \times 6^2}{p \times \frac{\pi}{4} \times 320^2}$$

$$\boxed{p \le 28.495\ mm}$$

$p \le \left[\begin{array}{l} 75\ mm \\ \frac{D_c}{6} = 53.33\ mm \end{array}\right.$ $\quad p \ge \left]\begin{array}{l} 3\phi_t = 18\ mm \\ 25\ mm \end{array}\right.$

provide 28 mm centre to centre (pitch) with 6 mm reinf


6# 16 mmɸ
40
28 mm
6 mmɸ
6# 16 ɸ