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ESE 2018 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

Test 3 : Fluid Mechanics and Turbo Machinery

Thermodynamics-1 + Refrigeration and Air-conditioning-1

Heat Transfer-2 + Theory of Machines-2

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	45
Q.2	~~49~~ 40
Q.3	~~52~~ 51
Q.4	
Section-B	
Q.5	~~48~~ 49
Q.6	44
Q.7	
Q.8	
Total Marks Obtained	229

Signature of Evaluator   Cross Checked by

Excellent

Keep practicing
Good attempt

Section A : Fluid Mechanics and Turbo Machinery

Q.1 (a) (i) Classify following fluids in tabular form based on their density and viscosity: ideal fluid, incompressible fluid, Inviscid fluid, real fluid, Newtonian fluid, Non-Newtonian fluid, compressible fluid, perfect gas.

(ii) Draw the graph of shear stress versus shear strain rate for the following types of fluid/materials: Solid, plastic fluid, ideal plastic, non-newtonian fluid, newtonian fluid, dilatant fluid, inviscid and ideal fluid.

[6 + 6 marks]

Ans. (i) Based on density

compressible	Incompressible
	→ ideal fluid ~~→ Perfect gas~~

Ideal fluid → Incompressible fluid, no viscosity.

~~I~~ncompressible → No change in density

Inviscid fluid → Have no viscosity.

Real fluid → Have viscosity

Newtonian fluid → follows $\boxed{\tau = \mu \frac{du}{dy}}$

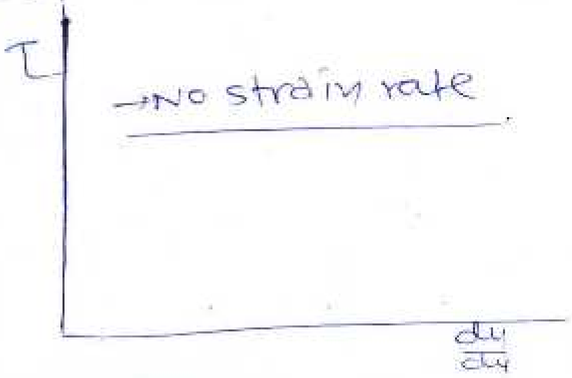
Non Newtonian fluid → Doesn't follow $\tau = \mu \frac{du}{dy}$
→ $\mu = f(t)$
→ $\mu = f\left(\frac{du}{dy}\right)$

Dilatant fluid → $\tau = \mu \left(\frac{du}{dy}\right)^n$ → $\boxed{n > 1}$

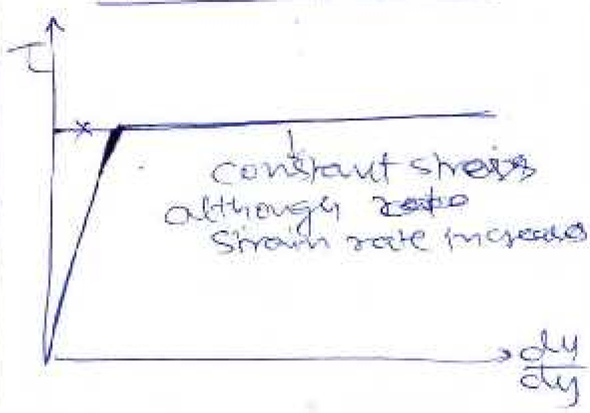
compressible fluid → Density changes are significant.

Perfect gas → follows $\boxed{PV = mRT}$ at all conditions

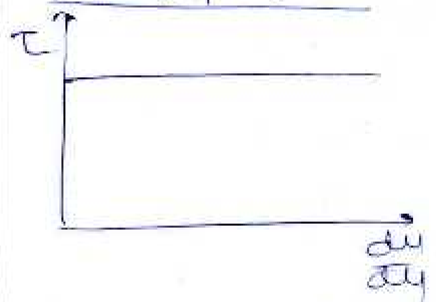
(ii) Solids



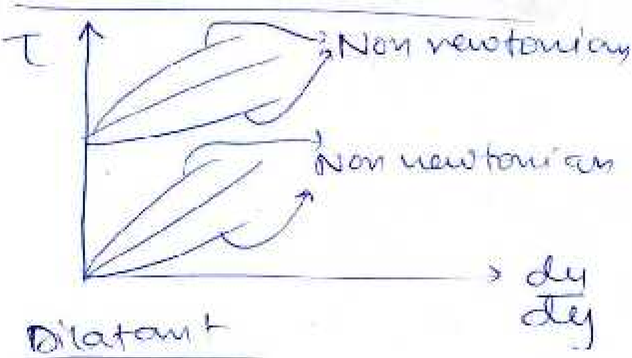
Plastic fluid



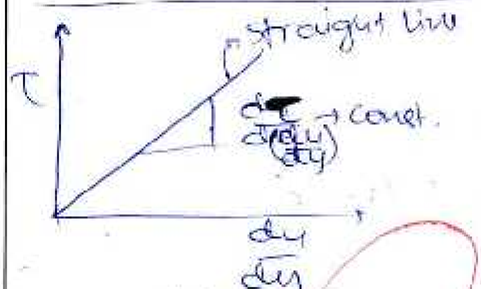
Ideal plastic



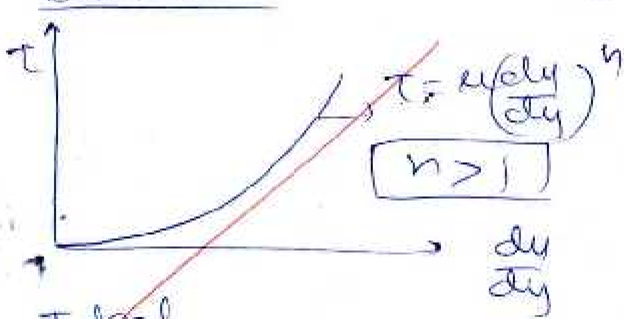
Non newtonial fluid



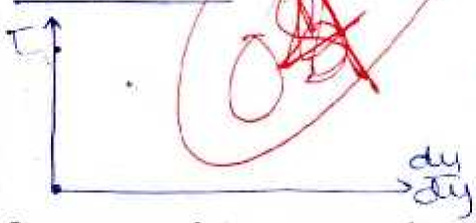
Newtonian fluid



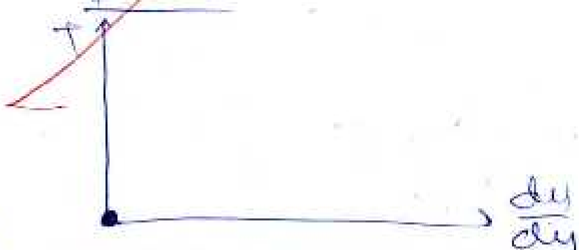
Dilatant



Inviscid



Ideal



Q.1 (b) Steam expands in a steam turbine isentropically from inlet to exhaust having an enthalpy drop = 15000 kJ/kg. Assuming optimum operating conditions, determine the mean diameter of the wheel if the turbine is

1. Single impulse stage
2. Single 50% reaction stage
3. Two-row curtis stage

Take the nozzle angle as 18° and blade speed as 4000 rpm.

[12 marks]

Soln

$\Delta h = 15000\ kJ/kg$ | $\alpha = 18^\circ$

$V_1 = 44.72\sqrt{\Delta h}$ | $N = 4000\ rpm$

$V_1 = 5477.0591\ m/s$

(1) Single impulse stage

At optimum conditions

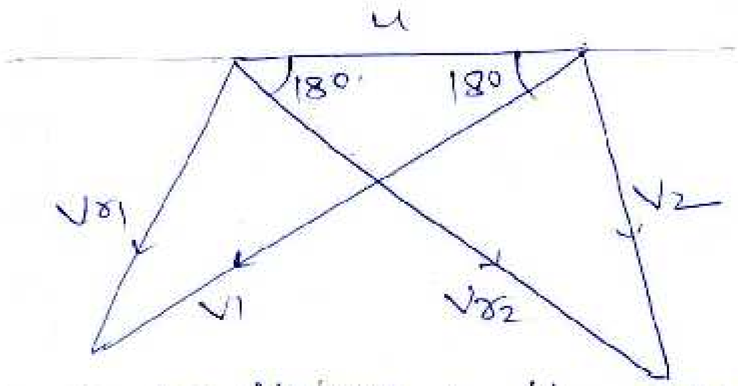
$$\frac{u}{V_1} = \frac{\cos\alpha}{2}$$

$$u = \frac{V_1 \cos\alpha}{2}$$

$$u = 2604.4964\ m/s$$

$$u = \frac{\pi D N}{60} \Rightarrow \boxed{D = 12.4356\ m}$$

(2) Single 50% reaction stage



optimum conditions → $\frac{u}{V_1} = \cos\alpha$

$u = V_1 \cos\alpha$

$u = 5208.9927$ m/s

$\frac{\pi D N}{60} = u$

$\boxed{D = 24.8711 \text{ m}}$

(3) Two row curtis stage

optimum conditions → $\frac{u}{V_1} = \frac{\cos\alpha}{4}$

$u = \frac{V_1 \cos\alpha}{4}$

$u = 1302.2482$ m/s

$\frac{\pi D N}{60} = u$

$\boxed{D = 6.2178 \text{ m}}$

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Q.1 (c) (i) The tangential component of velocity of incompressible fluid in 2-D flow is

$$v_\theta = -\frac{c\sin\theta}{r^2}$$

where c is a constant

1. Using continuity equation, determine the expression for radial velocity v_r.
2. Find the magnitude and direction of resultant velocity

[6 marks]

Soln

$$u_\theta = \frac{-c\sin\theta}{r^2}$$

1. continuity eqn

$$\frac{1}{r}\frac{\partial(r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0$$

→ Assuming fluid as incompressible and 2D flow.

$$\cancel{\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\left(\frac{2c\sin\theta}{r^3}\right) = 0}$$

$$\cancel{\frac{\partial}{\partial r}(r u_r)}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\left(\frac{-c\cos\theta}{r^2}\right) = 0$$

$$\frac{\partial}{\partial r}(r u_r) = \frac{c\cos\theta}{r^2}$$

$$r u_r = -\frac{c\cos\theta}{r} + K$$

$K \to$ const

$$\boxed{u_r = -\frac{c\cos\theta}{r^2} + \frac{K}{r}}$$

2. Magnitude

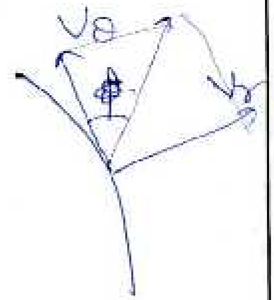
$$\vec{V} = u_r\hat{r} + u_\theta\hat{\theta}$$

$$|\vec{V}| = \sqrt{u_r^2 + u_\theta^2}$$

$$|\vec{V}| = \sqrt{\left(-\frac{c\cos\theta}{r^2} + \frac{K}{r}\right)^2 + \left(-\frac{c\sin\theta}{r^2}\right)^2}$$

$$\boxed{|\vec{V}| = \sqrt{\frac{c^2}{r^4} - \frac{2Kc\cos\theta}{r^3}}}$$

Direction



$$\tan\phi = \frac{V_r}{V_\theta}$$

$$\tan\phi = \frac{-\frac{c\cos\theta}{r^2} + \frac{K}{r}}{\frac{-c\sin\theta}{r^2}}$$

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$$\tan\phi = \frac{c\cos\theta - Kr}{c\sin\theta}$$

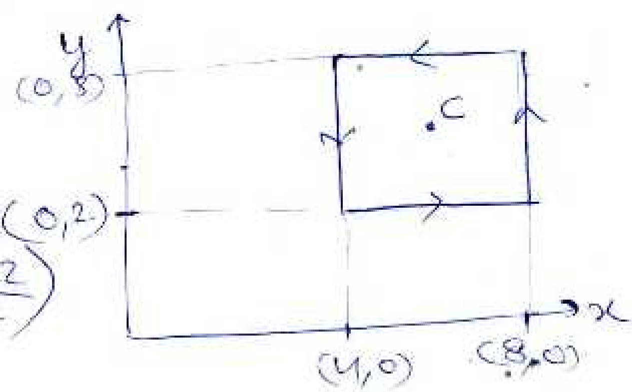
$$\boxed{\phi = \tan^{-1}\left(\cot\theta - \frac{Kr}{c\sin\theta}\right)}$$

Q.1 (c) (ii) If the velocity field is given by $u = (16y - 8x)$, $v = (8y - 7x)$, find the circulation around the closed curve defined by $x = 4$, $y = 2$, $x = 8$, $y = 8$.

[6 marks]

Sol^n

$u = 16y - 8x$
$v = 8y - 7x$



coordinates of C

$$C \equiv \left(4 + \frac{8-4}{2},\ 2 + \frac{8-2}{2}\right)$$

$$\boxed{C \equiv (6, 5)}$$

circulation = (vorticity) × A

$$\text{vorticity} = \vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 16y-8x & 8y-7x & 0 \end{vmatrix}$$

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$$\text{vorticity} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-7-16)$$

$$\text{vorticity} = -23\hat{k}$$

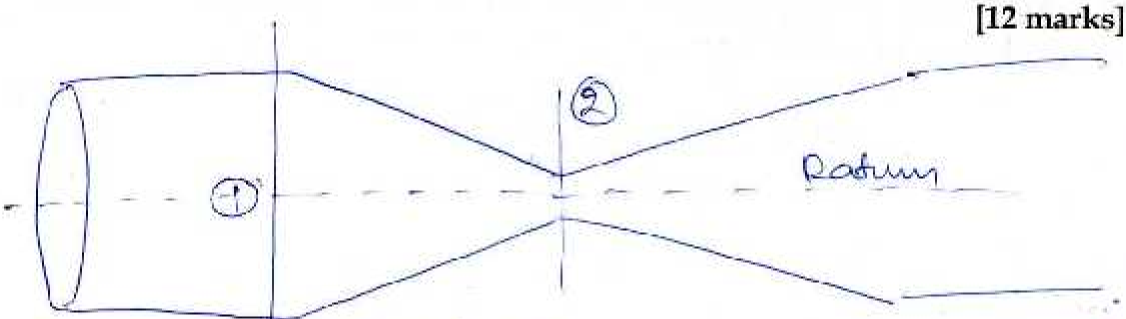
circulation $= -23 \times A\,\hat{k}$
$A = (8-4)(8-2) = 24$

$$\boxed{\text{Circulation} = -552\,\hat{k}\ m^2/s}$$

Q.1 (d) The inlet and throat diameters of a horizontal venturimeter are 36 cm and 12 cm respectively. The liquid flowing through the venturimeter is water. The gauge pressure at inlet is 13.734 N/cm² while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat.

[12 marks]

Solⁿ



$D_1 = 0.36$ m $\quad D_2 = 0.12$ m

$A_1 = \frac{\pi D_1^2}{4}$ $\quad A_2 = \frac{\pi D_2^2}{4}$

$\boxed{A_1 = 0.101787 \, m^2}$ $\quad \boxed{A_2 = 0.01131 \, m^2}$

Let Q be the flow rate

$V_1 = \frac{Q}{A_1} = 9.8244 \, Q$

$V_2 = \frac{Q}{A_2} = 88.4194 \, Q$

$P_1 = 13.734 \, N/cm^2 \Rightarrow h_1 = \frac{13.734 \times 10^4}{1000 \times 9.81}$

$h_1 = 14$ m of water

$P_2 = -37$ cm of Hg $\Rightarrow h_2 = \frac{-13600 \times 9.81 \times 0.37}{1000 \times 9.81}$

$h_2 = -5.032$ m of water.

Head at the inlet

$$H_1 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

$z_1 = 0$ (datum)

$$H_1 = 14 + \frac{(9.8244Q)^2}{2g} + 0$$

$$H_1 = 14 + 4.9194\,Q^2$$

Head at the throat

$$H_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$z_2 = 0$ (datum)

$$H_2 = -5.032 + \frac{(88.4194\,Q)^2}{2g}$$

$$H_2 = -5.032 + 398.4705\,Q^2$$

$h_L \rightarrow$ Head loss $\rightarrow h_L = 0.04\,(H_1)$

Applying modified Bernouli's b/w ① & ②

$$H_1 = H_2 + h_L$$

$$0.96\,H_1 = H_2$$

$$0.96(14 + 4.9194\,Q^2) = (-5.032 + 398.4705\,Q^2)$$

$$Q = 0.2165947\ m^3/s$$

$$\boxed{Q = 216.5947\ lit/s}$$

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Q.1 (e) (i) Explain rotodynamic and centrifugal pumps. Explain the following types of centrifugal pump with the help of figure/schematic:
- Centrifugal pump without diffuser
- Centrifugal pump with diffuser

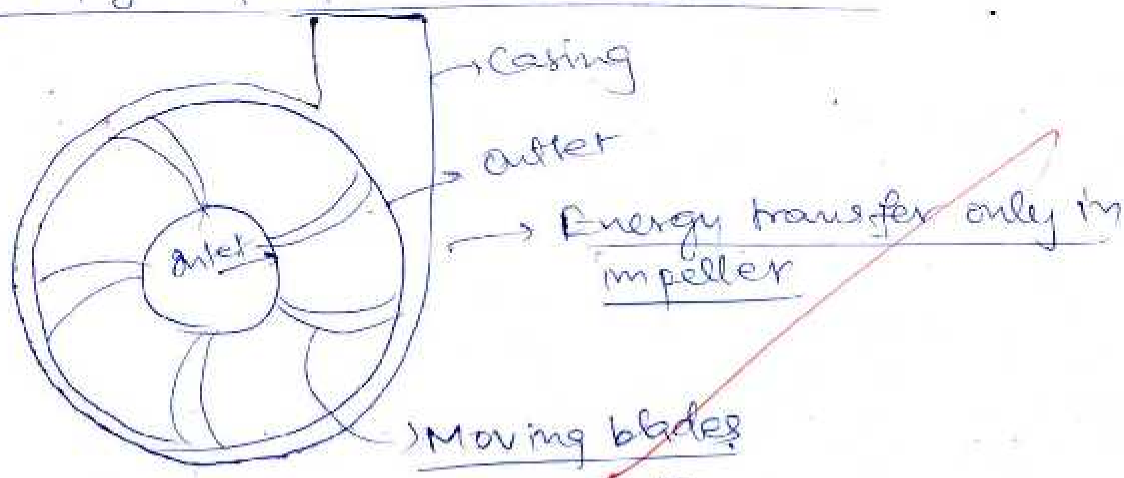
(ii) Explain the types of impellers used in a centrifugal pump with the help of schematics/sketches. What are the losses in a centrifugal pump encountered during its operation?

[12 marks]

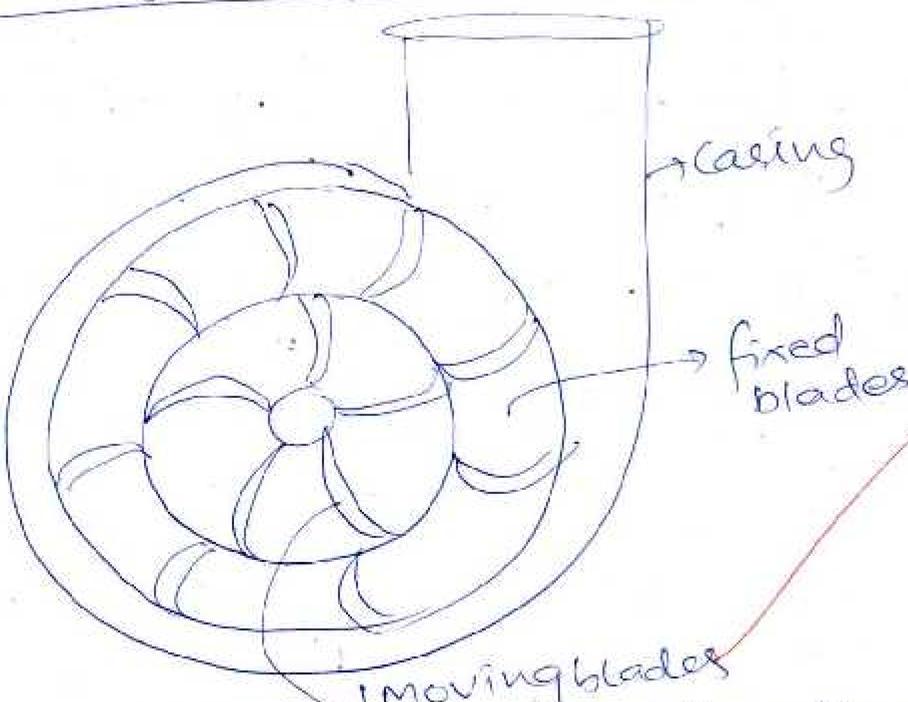
Soln (i) Rotodynamic & centrifugal pumps

↳ The increase in pressure is due to the centrifugal action of rotating impellar.

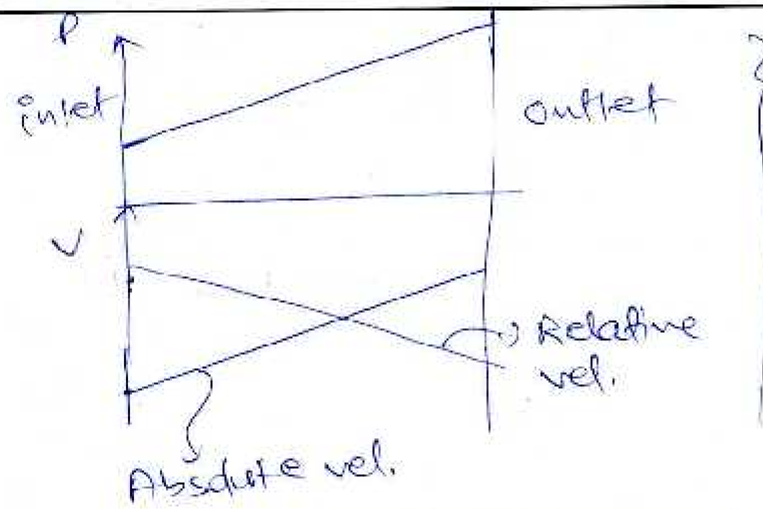
Centrifugal pump without diffuser



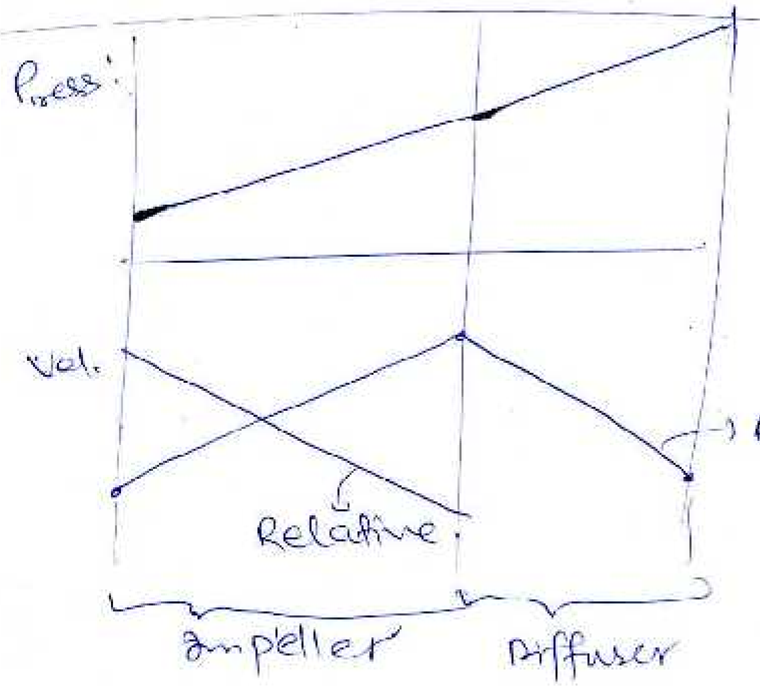
Centrifugal pump with diffuser



→ Energy transfer in impeller then the energy transformation in fixed blades.



→ for centrifugal pump without diffusers



→ for centrifugal compressor with diffusers

07

(ii) <u>Losses</u>

- → Leakage losses
- → frictional losses
- → Losses due to cavitation & surging
- → Separation losses in the exit tube casing.

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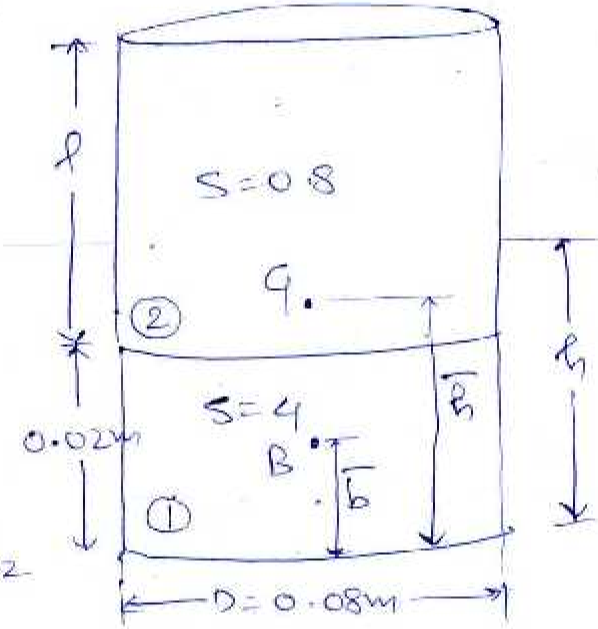
Q.2 (a) A composite solid cylinder has 80 mm diameter consists of an 80 mm diameter and 20 mm thick metallic plate having specific gravity 4.0 attached at the lower end of composite solid cylinder of specific gravity 0.8. Find the limits of the length of the wooden portion so that the composite cylinder can float in stable equilibrium in water with its axis vertical.

[20 marks]

Soln

$D = 80\,mm$.



Calculating $\bar{h}$

$$\bar{h} = \frac{\bar{h}_1 V_1 S_1 + \bar{h}_2 V_2 S_2}{S_1 V_1 + V_2 S_2}$$

$$\bar{h} = \frac{\frac{0.02}{2} \times A h_1 S_1 + \bar{h}_2 A h_2 S_2}{S_1 A h_1 + A h_2 S_2}$$

$\bar{h}_2 = 0.02 + \frac{l}{2}$ $\quad h_1 = 0.02\,m \quad h_2 = l$.

$$\bar{h} = \frac{4 \times 0.01 \times 0.02 + \left(0.02 + \frac{l}{2}\right) \times l \times 0.8}{4 \times 0.02 + \cancel{0.02 + \frac{l}{2}}\, l \times 0.8}$$

$$\boxed{\bar{h} = \frac{0.0008 + (0.02l + 0.5l^2) \times 0.8}{0.08 + l \times 0.8}} \quad \text{— ①}$$

$\Sigma F_V = 0: \quad W = F_B$

$$(\rho_1 V_1 + \rho_2 V_2)\,\cancel{g} = (\rho_w V_s \cancel{g})$$

$$\left(\frac{\rho_1}{\rho_w}\right) \cancel{A} \times h_1 + \left(\frac{\rho_2}{\rho_w}\right) \cancel{A} \times h_2 = \cancel{A} \times h$$

$$h = 4 \times 0.02 + 0.8 \times l.$$

$$\boxed{h = 0.08 + 0.8l}$$

$$\boxed{\bar{b} = \frac{h}{2} = 0.04 + 0.4l.}$$

$$BG = \bar{h} - \bar{b}$$

$$BG = \frac{0.0008 + (0.02l + 0.5l^2)\times 0.8}{0.08 + 0.8l} - (0.04 + 0.4l)$$

$$BG = \frac{0.01 + (0.2l + 5l^2)}{1 + 10l} - (0.04 + 0.4l)$$

$$BG = \frac{0.01 + 0.2l + 5l^2 - 0.04(1+10l)^2}{1+10l}$$

$$BG = \frac{0.01 + 0.2l + 5l^2 - 0.04 - 4l^2 - 0.8l}{1+10l}$$

$$\boxed{BG = \frac{l^2 - 0.6l - 0.03}{1+10l}}$$

Now,

$$I = \frac{\pi d^4}{64}$$

$$V_{sub.} = \frac{\pi d^2}{4} \times h$$

$$BM = \frac{I}{V} = \frac{d^2}{16h} = \frac{0.08^2}{16\times(0.08 + 0.8l)}$$

$$\boxed{BM = \frac{I}{V} = \frac{1}{200(1+10l)}}$$

Now, for stability

$$GM = BM - BG > 0$$

So, $BM - BG > 0$

$$BM > BG$$

$$\frac{1}{200(1+10l)} > \frac{l^2 - 0.6l - 0.03}{(1+10l)}$$

$$l^2 - 0.6l - 0.03 < 0.005$$

$$l^2 - 0.6l - 0.035 < 0$$

$$(l - 0.6536)(l + 0.0536) < 0$$

$$l \in (-0.0536, 0.6536)$$

$\because$ l can't be negative

so, $\boxed{l \in [0, 0.6536)}$ m.

$\boxed{l_{max} = 0.6536\,m}$

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Q.2 (b) (i) The velocity distribution for laminar flow in a round pipe of radius R is given by the equation

$$\frac{u}{u_{\max}} = 1-\left(\frac{r}{R}\right)^2$$

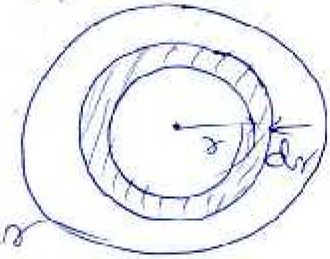
where u is the velocity at a radius r and $u_{\max}$ is the maximum velocity at the centreline. Determine the value of the kinetic energy and momentum correction factors if the average velocity is used to represent the flow. Explain the implication of these correction factors.

[10 marks]

Solⁿ

$$\frac{u}{u_{max}} = 1-\left(\frac{r}{R}\right)^2$$

c/s view



$$u_{avg} = \frac{\int u\,dA}{A}$$

$$u_{avg} = \frac{\int u_{max}\left(1-\frac{r^2}{R^2}\right)\times 2\cancel{\pi} r\,dr}{\cancel{\pi}R^2}$$

$$u_{avg} = \frac{2u_{max}}{R^2}\int_0^R\left(1-\frac{r^2}{R^2}\right)\cdot r\,dr$$

$$\boxed{u_{avg} = \frac{u_{max}}{2}}$$

Now,

Momentum correction factor

$$\beta = \frac{\int V^2\,dA}{V_{avg}^2 A}$$

$$\beta = \frac{\int \cancel{u_{max}^2}\left(1-\left(\frac{r}{R}\right)^2\right)^2\times 2\cancel{\pi} r\,dr}{\frac{\cancel{u_{max}^2}}{4}\times\cancel{\pi}R^2}$$

$$\beta = \frac{8}{R^2}\int_0^R\left(1-\left(\frac{r}{R}\right)^2\right)^2\times r\,dr$$

$$\boxed{\beta = \frac{8}{6} = 1.3333}$$

Kinetic energy correction factor

$$\alpha = \frac{\int v^3 dA}{V_{avg}^3 A}$$

$$\alpha = \frac{\int u_{max}^3 \left(1-\left(\frac{r}{R}\right)^2\right)^3 2\pi r\, dr}{\frac{u_{max}^3}{8} \times \pi R^2}$$

$$\alpha = \frac{16}{R^2}\int_0^R \left(1-\left(\frac{r}{R}\right)^2\right)^3 \cdot r\, dr$$

$$\alpha = \frac{16}{R^2} \times \frac{1}{8}R^2$$

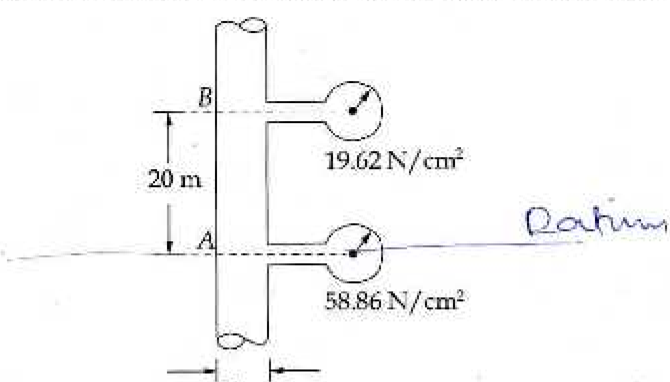
$$\boxed{\alpha = 2}$$

Amplication ??

8

~~8/6~~

Q.2 (b) (ii) Crude oil of viscosity 1.6 poise and relative density 0.9 flows through a 25 mm diameter vertical pipe. The pressure gauges fixed 20 m apart read 58.86 N/cm^2 and 19.62 N/cm^2 as shown in figure. Find the direction and rate of flow through the pipe.



[10 marks]

Soln

$$\boxed{\mu = 0.16 \text{ Pa-s}}$$

$$\boxed{\rho = 900 \text{ kg/m}^3}$$

$d = 25$ mm

Let the flow rate be Q m^3/s

$$V = \frac{Q}{A}$$

$$V = \frac{4Q}{\pi d^2}$$

$$\boxed{V = 2037.1833\,Q}$$

Let the head loss $\rightarrow h_L$

$$\boxed{h_L = \frac{fLQ^2}{12.1\,d^5}}$$

Assuming laminar flow

$$f = \frac{64}{Re} = \frac{64\mu}{\rho V d}$$

$$f = \frac{64 \times 0.16}{900 \times 2037.1833\,Q \times 0.025}$$

$$\boxed{f = \frac{0.0002234}{Q}}$$ Put in h_L

$$\boxed{h_L = 37812.19711\,Q}$$

Let the flow is from A to B

$P_A = 58.86 \times 10^4$ N/m² $\rightarrow h_A = \frac{P_A}{\rho g}$

$$\boxed{h_A = 66.67\text{ m}}$$

$P_B = 19.62 \times 10^4$ N/m² $\rightarrow h_B = \frac{P_B}{\rho g}$

$$\boxed{h_B = 22.22\text{ m}}$$

Applying modified Bernoulli's b/w A & B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

$V_A = V_B$ $\quad Z_A = 0$ (datum)

$Z_B = 20$ m

$66.67 = 22.22 + 20 + 37812.19711\,Q$

$$\boxed{Q = 0.00064466\ m^3/s}$$

since $Q \to$ positive $\to$ $\boxed{\text{flow is from A} \to \text{B}}$

Checking Re

$V = 2037.1833\,Q$

$V = 1.3172\ m/s$

$Re = \frac{\rho V d}{\mu} = 185.2373 < 2300$

Laminar

Q.2 (c) A rectangular draft tube is fitted at the exit of a large reaction turbine with a discharge rate of 100 m^3/s. Water leaves the draft tube with a velocity of 6.25 m/s. The area ratio of the draft tube is 1.8 and theoretical efficiency is 85%.

Determine:

1. area of cross-section at entry and exit,
2. head gained by the draft tube,
3. additional power generated,
4. head loss in the draft tube

Assume the potential head to be constant for flow through the draft tube.

[20 marks]

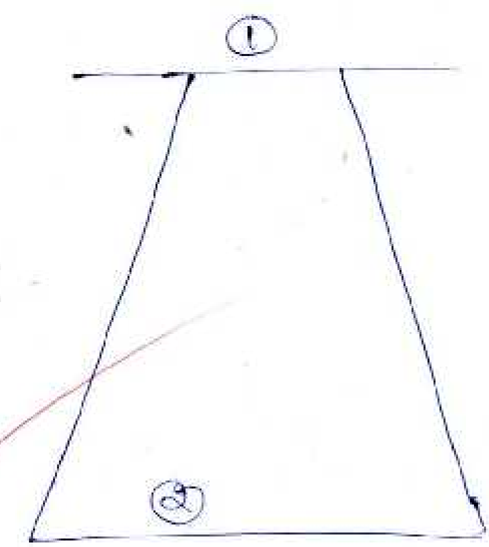
Soln

$Q = 100\ m^3/s$

$V_2 = 6.25\ m/s$

$$\boxed{\frac{A_2}{A_1} = 1.8}$$

$$\boxed{\eta_{th} = 0.85}$$



$Q = AV$

$Q = A_2 V_2$

$A_2 = \frac{Q}{V_2}$

$\boxed{A_2 = 16\ m^2} \rightarrow A_1 = \frac{A_2}{1.8}$

$\boxed{A_1 = 8.89\ m^2}$

If the potential head is same

$Z_1 = Z_2$

$$\eta_{th} = \frac{\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L}{\frac{V_1^2}{2g} - \frac{V_2^2}{2g}} \quad ; h_L \rightarrow \text{head loss}$$

$$V_1 = \frac{Q}{A_1} \Rightarrow \boxed{V_1 = 11.2486 \text{ m/s}}$$

$$\text{Gained head} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$\boxed{\text{Gained head} = 4.4581 \text{ m of water}}$$

$$\eta_{th} = \frac{\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L}{\frac{V_1^2}{2g} - \frac{V_2^2}{2g}}$$

$$h_L = \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - \eta_{th}\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_L = 4.4581 - 0.85 \times \left(\frac{11.2486^2}{2\times 9.81} - \frac{6.25^2}{2g}\right)$$

Head loss $\boxed{h_L = 0.6687 \text{ m of water}}$

Additional power generated

$$P_{add} = \rho g Q h_{gain}$$

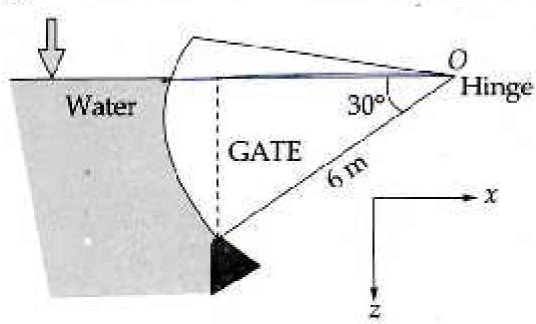
$P_{add} = 1000 \times 9.81 \times 100 \times 4.4581$

$P_{add} = 4.373396 \times 10^6 \ W$

$\boxed{P_{add} = 4.3734 \ MW}$

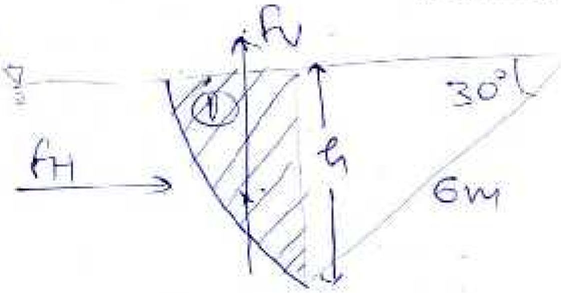
13

Q.3 (a) (i) A tainter gate is used as a control at the crest of a small spillway as shown below. If the width of gate is 2 m, find the magnitude of the force on the gate for the conditions shown. What angle will the resultant force make with the horizontal?



[5 marks]

Sol^n



F_H = force on the horizontal projection of gate

$F_H = \rho g \frac{h}{2} \cdot h \cdot B$

$B \rightarrow$ width of gate.

$h = r \sin 30 = 3 m$

$B = 2m$

$\boxed{F_H = 88.290 \ kN}$

F_V = weight of liquid supported upto water surface

$F_V = \rho v g = \rho g (B) A_1$

$A_1 = \frac{30}{360} \times \pi \times 6^2 - \frac{1}{2} \times 6 \times 6 \cancel{\cos 30} \sin 30$

$A_1 = 1.6305 \ m^2$

$F_V = 1000 \times 9.81 \times 2 \times 1.6305$

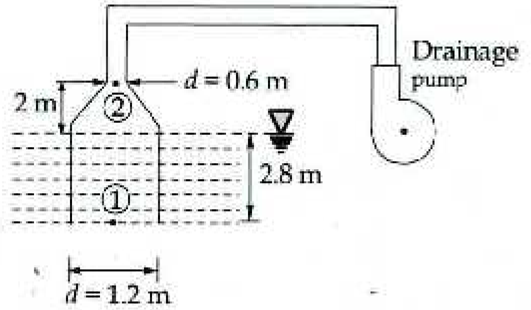
$\boxed{F_V = 31.9914 \ kN}$

5

$\theta = \tan^{-1} \frac{F_V}{F_H}$

$\boxed{\theta = 19.9177°}$

Q.3 (a) (ii) A drainage pump has inlet as a vertical pipe with a tapered section, which is filled with water as shown in the following figure. Diameter at ends of the tapered section are 1.2 m and 0.6 m respectively. The pipe is running full of water. The free water surface is 2.8 m above the inlet and 2 m below from 0.6 m diameter section. The pressure at the upper end of the pipe is 28 cm of Hg (abs) and the head loss between two sections is 1/10th of the velocity head at top section. Find the discharge of water in the pipe.



[15 marks]

Sol^n

Let the flow rate be $Q \ m^3/s$

$d_1 = 1.2 m \rightarrow A_1 = \frac{\pi}{4} d_1^2 \rightarrow A_1 = 1.13097 \ m^2$

$d_2 = 0.6 m \rightarrow A_2 = \frac{\pi}{4} d_2^2 \rightarrow A_2 = 0.2827 \ m^2$

$V_1 = \frac{Q}{A_1} \Rightarrow \boxed{V_1 = 0.8842 \ Q}$

$V_2 = \frac{Q}{A_2} \Rightarrow \boxed{V_2 = 3.5373 \ Q}$

let the $P_{atm} = 1.0135$ bar.

then $P_1 = P_{atm} + \rho g h$.

$P_1 = 1.0135 \times 100 + 9.81 \times 2.8$

$\boxed{P_1 = 128.793 \text{ kPa}}$

$P_2 = 28$ cm of Hg.

$P_2 = \frac{13600 \times 9.81 \times 0.28}{1000}$

$\boxed{P_2 = 37.3565 \text{ kPa}}$

$h_L \rightarrow$ head loss

$h_L = \frac{1}{10} \times \frac{V_2^2}{2g} = 0.1 \times \frac{V_2^2}{2g}.$

Modified Bernoulli's b/w ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$Z_1 = 0$ (datum)

$Z_2 = 4.8$ m.

$$\frac{128.793}{9.81} + \frac{0.8842^2 Q^2}{2 \times 9.81} + 0 = \frac{37.3565}{9.81} + \frac{1.1 \times 3.5373^2}{2 \times 9.81} Q^2 + 4.8$$

13

$\boxed{Q = 2.61387 \text{ m}^3/\text{s}}$

$\boxed{Q = 2613.87 \text{ lit/s}}$

Q.3 (b) **(i)** In a rough pipe of diameter 0.8 m and length 4500 m, water is flowing at the rate of $0.8\ m^3/s$. If the average height of roughness is 0.48 mm, find the power required to maintain this flow.

[10 marks]

Soln

$d = 0.8m$

$L = 4500m$

$Q = 0.8 m^3/s$ $\rightarrow V = \frac{4Q}{\pi d^2}$ $\therefore$ $\boxed{V = 1.5915\ m/s}$

$k = 0.48\ mm$

Assuming flow to be turbulent

$\frac{1}{\sqrt{f}} = 1.74 + 2\log\left(\frac{R}{k}\right)$ → for rough pipe.

$\frac{1}{\sqrt{f}} = 1.74 + 2\log\left(\frac{0.4}{0.48\times10^{-3}}\right)$

$\boxed{f = 0.017397}$

19

$h_L = \frac{fLQ^2}{12.1d^5}$

$\boxed{h_L = 12.6366\ m \text{ of water}}$

Power required

$\mapsto P = \rho g Q h_L$.

$P = 1000\times9.81\times0.8\times12.6366\ W$

$\boxed{P = 99.1720\ kW}$

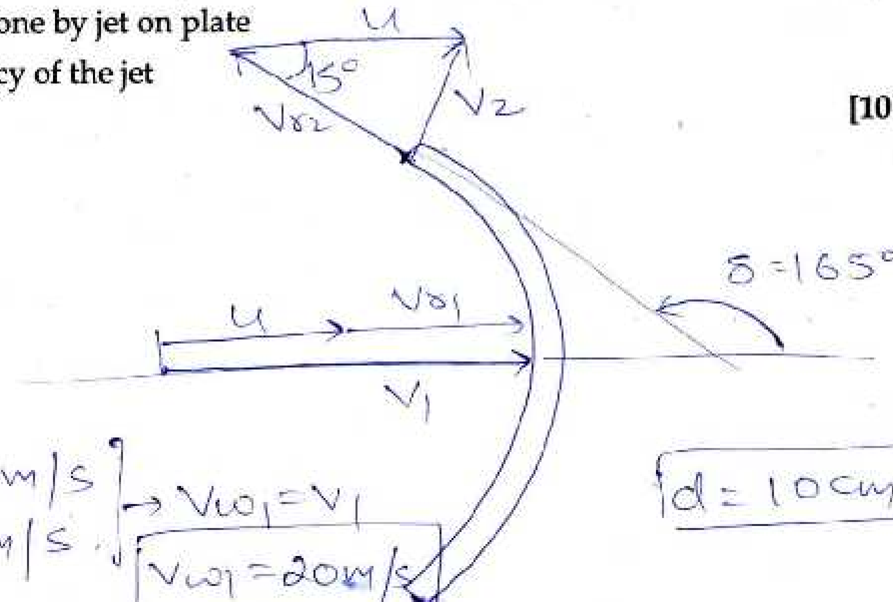
Do not write in this margin

Q.3 (b) **(ii)** A jet of water of diameter 10 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of jet. The jet is deflected through an angle of 165°. Assuming the plate is smooth then determine:

1. Force exerted on the plate in the direction of jet
2. Work done by jet on plate
3. Efficiency of the jet

[10 marks]

soln



$V_1 = 20\,m/s$
$u = 8\,m/s$
$\rightarrow V_{w1} = V_1$
$\boxed{V_{w1} = 20\,m/s}$

$\boxed{d = 10\,cm}$

$V_{r1} = V_1 - u$
$V_{r1} = 12\,m/s$

for smooth plate

$V_{r1} = V_{r2}$
$\boxed{V_{r2} = 12\,m/s}$

$V_{w2} = u - V_{r2}\cos 15$
$\boxed{V_{w2} = -3.5911\,m/s}$

$\Delta V_w = V_{w1} - V_{w2} \Rightarrow \boxed{\Delta V_w = 23.5911\,m/s}$

$\dot{m} = \rho A (V_1 - u)$
$\dot{m} = 1000 \times \frac{\pi}{4} \times 0.1^2 \times (20-8)$
$\boxed{\dot{m} = 94.2478\,kg/s}$

force exterted = $\dot{m}\,\Delta V_w$

$\boxed{F = 2.2234 \text{ kN}}$

6

$WD = F \cdot u$

$\boxed{WD = 17.7873 \text{ kW}}$

$$\eta = \frac{u \cdot \Delta V_w}{\frac{V_1^2}{2}} \times 100$$

$\boxed{\eta = 94.3644\ \%}$

5×7

Q.3 (c) An inward flow radial turbine has the following data:

Power : 160 kW

Speed : 32000 rpm

Outer diameter of the impeller : 20 cm

Inner diameter of the impeller : 8 cm

Absolute velocity of gas at entry : 387 m/s

Absolute velocity of gas at exit : 193 m/s (radial)

The gas enters the impeller radially. Draw the velocity triangles at the entry and exit of the impeller then determine:

1. the mass-flow rate and
2. the percentage energy transfer due to the change of radius

[20 marks]

Soln

$P = 160$ kW

$N = 32000$ rpm

$d_o = 0.2$ m $\rightarrow$ $u_1 = \frac{\pi d_o N}{60}$

$\boxed{u_1 = 33.5103 \text{ m/s}}$

$d_i = 0.08$ m $\rightarrow$ $u_2 = \frac{\pi d_i N}{60}$

$\boxed{u_2 = 13.4041 \text{ m/s}}$

$\boxed{V_1 = 387 \text{ m/s}}$

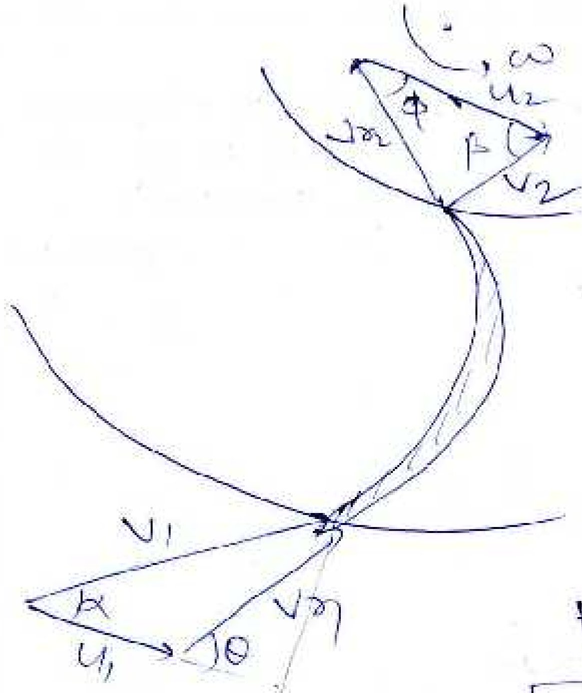
$\boxed{V_2 = 193 \text{ m/s}}$

Assuming radial inlet i.e.,

$\boxed{\theta = 90^\circ}$

& assuming no whirl at outlet

$\boxed{\beta = 90^\circ}$



$V_{w1} = u_1$

$\boxed{V_{w1} = 335.103 \text{ m/s}}$

$\boxed{V_{w2} = 0}$

$H = \frac{V_{w1} u_1}{g}$

$\boxed{H = 11446.8930 \text{ m}}$

$P = \dot{m} g H \rightarrow \dot{m} = \frac{P}{gH}$

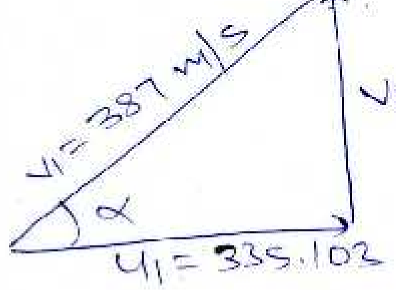
$\boxed{\dot{m} = 1.4248 \text{ kg/s}}$

18

% energy transfer due to change of radius

$= \frac{\frac{u_1^2 - u_2^2}{2}}{V_{w1} u_1} \times 100 = \boxed{42\ \%}$

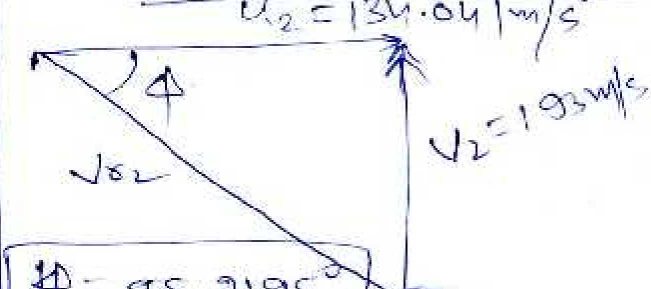
Inlet vel. triangle



$\alpha = 30.0144^\circ$

$V_{r1} = 193.5846$ m/s

Outlet vel. triangle



$\boxed{\phi = 55.2195^\circ}$

$\boxed{V_{r2} = 234.9808 \text{ m/s}}$

Do not write in this margin

Section B : Thermodynamics-1 + RAC-1, HMT-2 + TOM Machines-2

Q.5 (a) 1 kg of air is heated at constant volume until its temperature becomes 3.5 times original temperature, then it is expanded isothermally until it reaches its original pressure. The gas is then cooled at constant pressure until it is restored to the original state. Determine the net work done if the initial temperature is 310 K.

[12 marks]

Soln

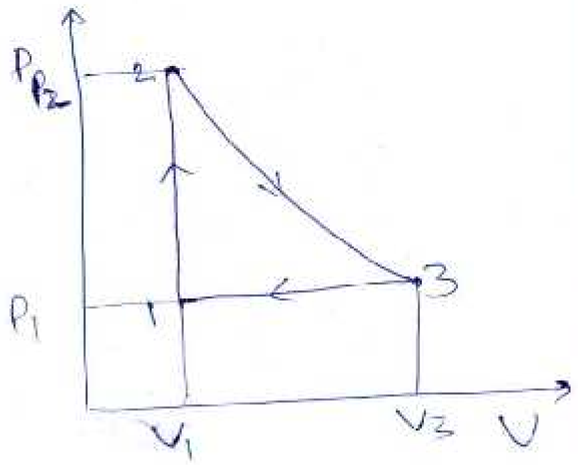
$m = 1\,kg$

Process → 1-2

Const. volume

$P \propto T$

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$



$T_1 = 310\,K$

$T_2 = T_1\left(\frac{P_2}{P_1}\right)$

$T_2 = 3.5\,T_1$ → $P_2 = 3.5\,P_1$

$\boxed{T_2 = 1085\,K}$

At 2 & 3 $\quad T_2 = T_3$ { Isothermal process)

$\boxed{T_3 = 1085\,K}$

$P_2 V_2 = P_3 V_3$ { " }

$V_3 = \frac{P_2}{P_3} \times V_2$

But $V_2 = V_1$ & $P_3 = P_1$

$V_3 = \frac{3.5\,P_1}{P_1} \times V_2$

$\boxed{V_3 = 3.5\,V_2}$

for 1-2 $\quad WD = 0$ → Const. ~~work~~ volume

for 2-3 $\quad WD = P_2 V_2 \ln \frac{P_2}{P_3}$ → Isothermal

for 3-1 $\quad WD = P_3(V_1 - V_3)$ → Const. pressure

$$(WD)_{net} = P_2 V_2 \ln \frac{P_2}{P_3} + P_3(V_1 - V_3)$$

$$(WD)_{net} = 3.5 P_1 V_1 \ln 3.5 + P_1(V_1 - 3.5 V_1)$$

$$(WD)_{net} = 1.8847 P_1 V_1$$

$$(WD)_{net} = 1.8847 mRT_1$$

$$\boxed{(WD)_{net} = 167.6791 \text{ kJ}}$$

11

Q.5 (b) The mass of the turbine rotor of a ship is 22 tonne and has a radius of gyration of 0.62 m. Its speed is 1800 rpm. The ship pitches 6° above and 6° below the horizontal position. A complete oscillation takes 30 seconds and the motion is simple harmonic. Determine the following:

1. Maximum gyroscopic couple
2. Maximum angular acceleration of the ship during pitching and
3. The direction in which the bow will tend to turn when rising, if the rotation of the rotor is clockwise when looked from the left.

[12 marks]

Sol^n

$m_{rotor} = 22000$ kg

$k_{rotor} = 0.62$ m

$I_{rotor} = mk^2$

$$\boxed{I_{rotor} = 8456.8 \text{ kgm}^2}$$

$N = 1800$ rpm $\boxed{\omega = 188.4956 \text{ rad/s}}$

① Amplitude of pitching $\Rightarrow \theta_0 = 6 \times \frac{\pi}{180}$

$$\boxed{\theta_0 = 0.1047 \text{ rad}}$$

ω of pitching $= \frac{2\pi}{T}$

$$\omega_p = \frac{2\pi}{30}$$

$$\boxed{\omega_p = 0.2094 \text{ rad/s}}$$

$(\omega_p)_{max} = \theta_0 \omega_p$

$(\omega_p)_{max} = 0.1047 \times 0.20944$

$\boxed{(\omega_p)_{max} = 0.02193 \text{ rad/s}}$

$(T_{gyro})_{max} = I\,\omega(\omega_p)_{max}$

$\boxed{(T_{gyro})_{max} = 34955.2628 \text{ Nm}}$

(2) Equation of pitching assuming motion to be simple harmonic

$\theta = \theta_0 \sin(\omega_p t)$

$\omega = \frac{d\theta}{dt} = \theta_0 \omega_p \cos(\omega_p t)$

11

$\alpha = \frac{d^2\theta}{dt^2} = -\theta_0 \omega_p^2 \sin(\omega_p t)$

$|(\alpha)_{max}| = |-\theta_0 \omega_p^2|$

$\boxed{|(\alpha)_{max}| = 0.004591 \text{ rad/s}^2}$

(3)

→ The bow will tend to turn towards the starboard side and the stern will tend to turn towards port side.

Q.5 (c) **(i)** Explain the concept of absolute entropy.

(ii) A 50 kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg-K for iron.

Determine:

1. the entropy change of the iron block
2. entropy change of the lake water
3. entropy generated during the process.

[4 + 8 Marks]

Sol^n

Entropy

→ Measure of Randomness.

→ At absolute 0K, the randomness of any substance is taken as zero.

→ As the temperature increases, molecular activity increases, so the entropy increases.

→ Every process is working with generation in entropy.

→ Entropy of the universe is increasing.

(ii) $m_{block} = 50\,kg$ $T_H = 500\,K$.

$T_{lake} = 285\,K$.

$(C_p)_{iron} = 0.45\ kJ/kgK$

$(T_f)_{iron} = 285\,K$.

Entropy change of the iron block

$$\Delta s_{iron} = (m\,C_p)_{iron} \ln \frac{T_f}{T_i}$$

$$\Delta s_{iron} = 50 \times 0.45 \ln \frac{285}{500}$$

$$\boxed{\Delta s_{iron} = -12.6476\ kJ/K}$$

$Q_{transferred\ to\ lake} = (mC_p)_{iron}(T_i^\circ - T_f)$

$Q_t = 50 \times 0.45 \times (500 - 285)$

$\boxed{Q_t = 4837.5\ kJ}$

(2) $\Delta S_{lake} = \dfrac{Q_t}{T_{lake}} \Rightarrow \boxed{\Delta S_{lake} = 16.9737\ kJ/K}$

(3) $S_{gen} = \Delta S_{iron} + \Delta S_{lake}$

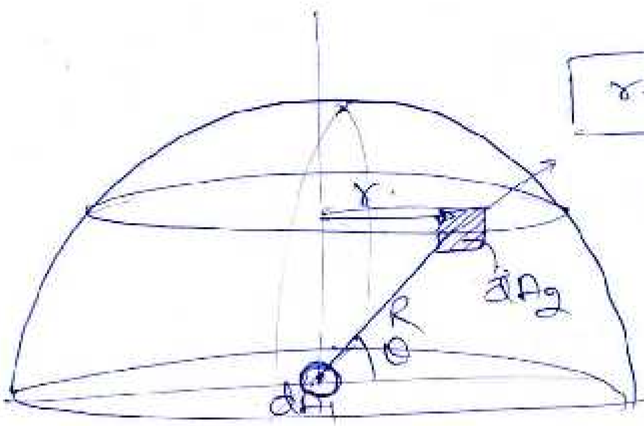
$\boxed{S_{gen} = 4.3261\ kJ/K}$

11

Q.5 (d) Show that the solid angle associated with the entire hemisphere is 2π Sr.

[12 marks]

Sol^n



$\boxed{r = R\cos\theta}$

$dA_2 = R^2\cos\theta\ \cancel{\ }\ d\theta\, d\phi$

Solid angle → cone angle.

$d\Omega = \dfrac{dA_2}{dA_1}$

10

$d\Omega = \dfrac{\int_0^{2\pi}\int_0^{\pi/2} R^2\cos\theta\, d\theta\, d\phi}{R^2}$

$\boxed{\Omega = 2\pi}$ Sr.

Q.5 (e) **(i)** Discuss the drawback of single effect absorption system.

(ii) What are the drawbacks of:

1. NH_3 and H_2O pair
2. H_2O and LiBr pair

For absorption system?

[6 + 6 marks]

(i) Drawbacks of VARS

→ Very low COP → around (1.1 – 1.3)

→ Mixing of refrigerant & absorbent occurs which might be taken to the condenser which can result in scaling.

→ Requirement of more no. of components can make the system bulky.

(ii) (a) NH_3 and H_2O → NH_3 → Refrigerant; H_2O → Absorbent

Drawbacks

→ NH_3 → Toxic in nature

↓

If gets leaked can cause hazard.

(b) LiBr & H_2O. → H_2O → Refrigerant; LiBr → Absorbent.

→ Drawbacks

→ H_2O → very high freezing temp.

↓

Only used for solar cooling

↓

Applications above 0°C

06

Q.6 (a) A system has a heat capacity at constant volume,

$$C_V = AT^2$$

where, $A = 0.042 \text{ J/K}^3$

The system is originally at 200 K, and a thermal reservoir at 100 K is available. What is the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir?

[20 marks]

Sol^n

$C_V = AT^2$

$A = 0.042 \text{ J/K}^3$

$T_1 = 200 \text{ K}$.

$\boxed{T_R = 100 \text{ K}.}$

$Q_{extracted} = \int_{T_R}^{T_1} C_V \, dT$

$Q_{ext.} = \int_{100}^{200} 0.042\, T^2 \, dT$.

$\boxed{Q_{ext.} = 98000 \text{ J}}$

$\Delta S_{sys} = \int_{T_R}^{T_1} \frac{C_V \, dT}{T}$

$\Delta S_{sys} = \int_{100}^{200} \frac{0.042\, T^{\cancel{2}} \, dT}{\cancel{T}}$

$\boxed{\Delta S_{sys} = -630 \text{ J/K}}$

$W_{max} = Q_{ext} - T_0(\Delta S_{sys} + \Delta S_{surr})$.

$\Delta S_{surr} = \frac{Q_{ext}}{T_R}$

$\boxed{\Delta S_{surr} = 980 \text{ J/K}}$

08

$W_{max} = Q_{ext} - T_0 (S_{gen})$

$= 98000 - 100 \times (980 - 630)$

$W_{max} = 63000\ J$

$\boxed{W_{max} = 63\ kJ}$

Ans.

$(\Delta S)_{sys} + (\Delta S)_{surr} \geq 0$

Q.6 (b) An insulated piston cylinder assembly has an initial volume V_1 and contains air at P_1 and T_1. Air is supplied to the cylinder from a pipeline, maintained to the constant P_P and T_P, through a valve fitted into the cylinder. This piston is restrained in such a manner that the pressure of the air in the cylinder remains constant at P_1 during the process of filling. The filling is terminated when the final volume V_2 is twice the initial volume V_1. Show that the final temperature of the air T_2 in the cylinder is given by

$$T_2 = \frac{2}{\dfrac{1}{T_1} + \dfrac{1}{T_P}}$$

If $P_1 = 400$ kPa, $T_1 = 200°C$, $T_P = 150°C$ and $V_1 = 1.2\ m^3$, estimate the mass of air that has entered the cylinder through valve.

[20 marks]

Sol^n

Insulated

State 1 → P_1 & V_1, T_1

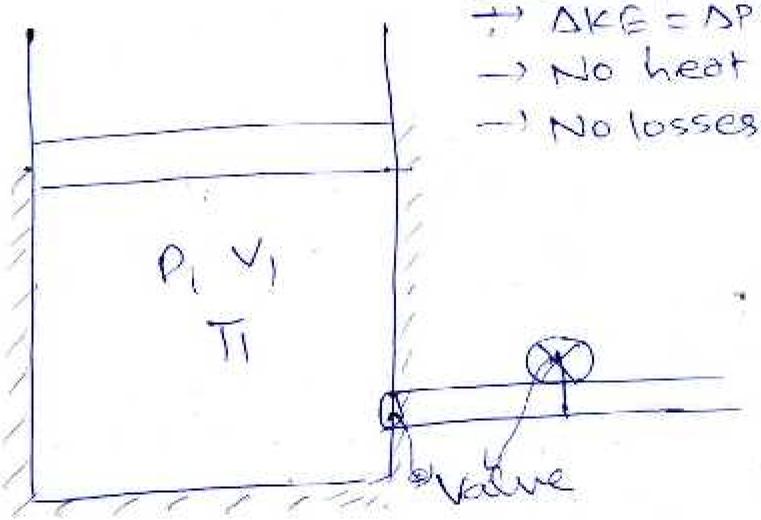
Supply condition

P_P & T_P.

Process → Isobaric (P = const.)

Final state $V_2 = 2V_1$

$P_2 = P_1$



Assumptions
→ $\Delta KE = \Delta PE = 0$.
→ No heat loss.
→ No losses during filling.

Let the initial mass was m_1

$$\boxed{m_1 = \frac{P_1 V_1}{R T_1}}$$

Let m kg. mass enters the arrangement.

$$\boxed{\underbrace{m_2}_{\text{final mass}} = m + m_1}$$

Now,
Unsteady Equation of energy exchange.

$$\frac{dU}{dt} = \dot{m}_i h_i - \dot{m}_e h_e + \dot{Q} - \dot{W}$$

$\dot{Q} = 0$. $\boxed{W = P\Delta V}$ → $W = P_1(V_2 - V_1)$

$\boxed{W = P_1 V_1}$ {∵ P_1 → const. & $V_2 = 2V_1$

$\boxed{\dot{m}_e = 0}$ → No exit

$$\frac{d}{dt}(m c_v T) = \dot{m}_i c_p T_p - \dot{W}$$

$$\int d(m c_v T) = \int (\dot{m}_i dt) c_p T_p - \int \dot{W} dt$$

$$m_2 c_v T_2 - m_1 c_v T_1 = m c_p T_p - P_1 V_1 \quad \text{—(1)}$$

At final condition → $m_2 R T_2 = P_2 V_2$

$\boxed{m_2 T_2 = \frac{2 P_1 V_1}{R}}$ $\boxed{m_1 T_1 = \frac{P_1 V_1}{R}}$

Put in (1).

$$C_v\left(\frac{2P_1V_1}{R}\right) - C_v\frac{P_1V_1}{R} = (m_2-m_1)C_pT_p - P_1V_1$$

$$\frac{C_v\frac{P_1V_1}{R} + P_1V_1}{C_pT_p} = m_2 - m_1$$

$$\boxed{m = \frac{P_1V_1}{T_p}\left(\frac{C_v}{C_pR} + \frac{1}{C_p}\right)}$$

$$m = \frac{P_1V_1}{T_p}\left(\frac{1}{R\gamma} + \frac{1}{\frac{R\gamma}{\gamma-1}}\right)$$

$$\boxed{m = \frac{P_1V_1}{RT_p}} \quad — (2)$$

$$m_2 - m_1 = \frac{P_1V_1}{RT_p}$$

$$\frac{2P_1V_1}{RT_2} - \frac{P_1V_1}{RT_1} = \frac{P_1V_1}{RT_p}$$

$$\frac{2}{T_2} = \frac{1}{T_1} + \frac{1}{T_p} \Rightarrow \boxed{T_2 = \frac{2}{\frac{1}{T_1}+\frac{1}{T_p}}}$$ #

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putting
$P_1 = 400$ kPa
$T_1 = 473$ K $\quad T_p = 423$ K
$V_1 = 1.2\ m^3$
$R = 0.287$ kJ/kgK
in (2)

$$m = \frac{400 \times 1.2}{0.287 \times 423}$$

$$\boxed{m = 3.9538\text{ kg}}$$ Ans

Q.6 (c) The following data refers to a 20 TR ice plant using ammonia as refrigerant:

The temperature of aqueous ammonia solution (liquid) leaving the condenser is 20°C and condensation of ammonia takes place at 25°C. Temperature of brine in the evaporator is –15°C. Before entering the expansion valve, ammonia is cooled to 20°C and the ammonia enters the compressor dry saturated. If the maximum allowable temperature rise for water is 5°C.

Calculate for one tonne of refrigeration the power required, the coefficient of performance of the plant and the amount of cooling water in the condenser. Use the properties given below in the table:

Saturation temp. °C	Enthalpy, kJ/kg		Entropy, kJ/kgK		Specific heat, kJ/kgK	
	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
–15	112.34	1426.54	0.4572	5.5490	4.396	2.303
25	298.90	1465.84	1.1242	5.0391	4.606	2.805

[20 marks]

Sol

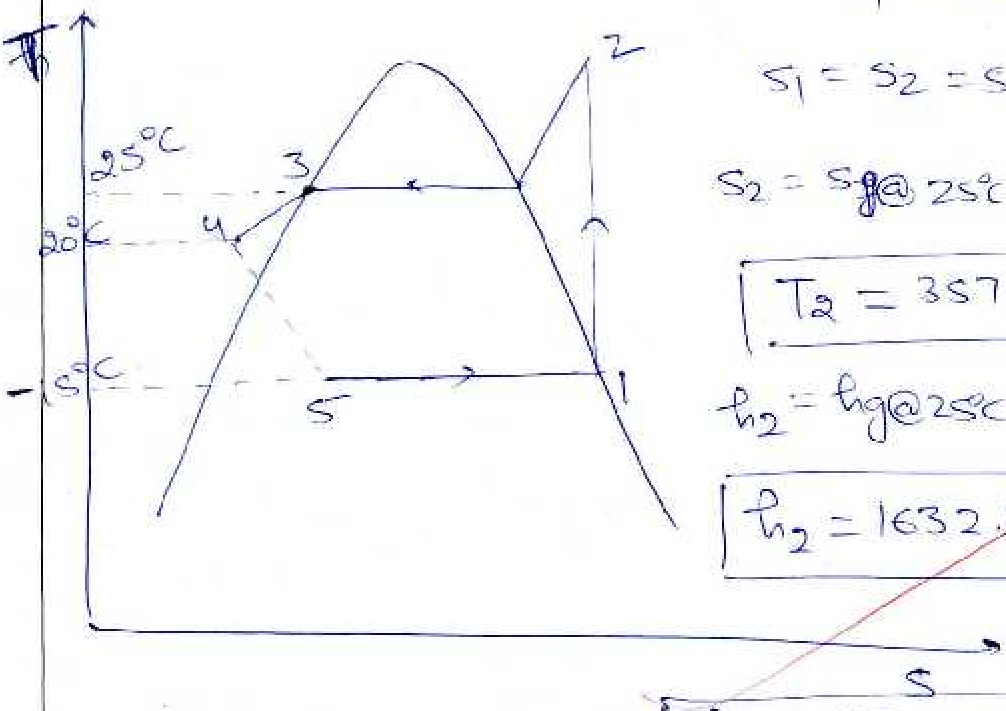
$RC = 20 \times 3.5167$ kW

$\boxed{RC = 70.334 \text{ kW}}$

At state 1

$h_1 = 1426.54$ kJ/kg

$s_1 = 5.5490$ kJ/kgK



$s_1 = s_2 = 5.5490$ kJ/kgK

$s_2 = s_{g@25°C} + C_{pv} \ln \frac{T_2}{T_{sat}}$

$\boxed{T_2 = 357.4073 \text{ K}}$

$h_2 = h_{g@25°C} + C_{pv} \Delta T$

$\boxed{h_2 = 1632.4775 \text{ kJ/kg}}$

$\boxed{h_3 = 298.90 \text{ kJ/kg}}$

$h_4 = h_3 - C_{pl@25°C} \Delta T$

$\boxed{h_4 = 275.87 \text{ kJ/kg}}$

$\boxed{h_4 = h_5 = 275.87 \text{ kJ/kg}}$

$RE = h_1 - h_5$

$RE = 1150.67$ kJ/kg

for $RC = 1$ TR $= 3.5167$ kW.

$\dot{m}_{NH_3} = \frac{RC}{RE}$

$\dot{m}_{NH_3} = 0.003108$ kg/s

$P_{req} = \dot{m}_{NH_3}(h_2 - h_1)$.

$\boxed{P_{req} = 0.6401 \text{ kW}}$

$COP = \frac{h_1 - h_5}{h_2 - h_1}$

$\boxed{COP = 5.5875}$

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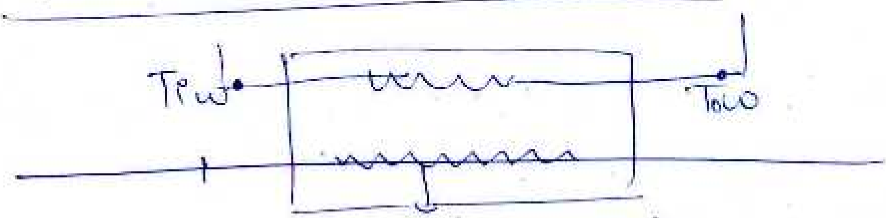
for $RC = 20$ TR $= 70.334$ kW

$\dot{m}_{NH_3} = \frac{RC}{RE}$

$\boxed{\dot{m}_{NH_3} = 0.0612 \text{ kg/s}}$



Condensing NH_3

$\dot{m}_w C_{pw} \Delta T_w = \dot{m}_{NH_3}(h_2 - h_4)$

$\dot{m}_w = \frac{\dot{m}_{NH_3}(h_2 - h_4)}{C_{pw} \Delta T_w}$

$\boxed{\dot{m}_w = 3.9609 \text{ kg/s}}$