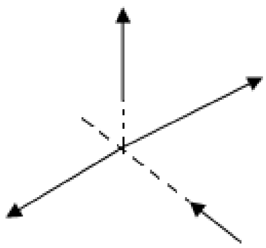
Review of the three laws of motion and vector algebra

1. Assertion (A): Moment and work both are vector quantities. **[IAS-2002]**
Reason (R): Both have the same unit of measurement - 'N-m'.
1. Ans. (d) *A is false.* Work is a scalar quantity.

2. Which one of the following statements is correct?
(a) Energy and work are scalars (b) Force and work are vectors
(c) Energy, momentum and velocity are vectors **[IAS-2000]**
(d) Force, momentum and velocity are scalar.
2. Ans. (a)

3. Consider the following statements:
1. Two couples in the same plane cane be added algebraically **[IAS-2000]**
2. Coplanar and concurrent forces are the ones which do neither lie in one plane nor meet at a point
3. Non-concurrent forces are the ones which do nut meet at a point.
4. A single forces may be replaced by a force and couple.
Which of these statements are correct?
(a) 1, 2 and 4 (b) 2, 3 and 4 (c) 1, 2 and 3 (d) 1, 3 and 4
3. Ans. (d)
Concurrent coplanar force: All line of action intersects at a common point.



4. Two forces A and B are acting at an angle θ.Their resultant 'R' will make an angle a with the force A, such that cos α is equal to **[IAS-1996]**

(a) $\dfrac{A+B\sin\theta}{\sqrt{A^2+B^2-2AB\cos\theta}}$ (b) $\dfrac{B\sin\theta}{\sqrt{A^2+B^2+2AB\cos\theta}}$

(c) $\dfrac{A+B\cos\theta}{\sqrt{A^2+B^2+2AB\cos\theta}}$ (d) $\dfrac{B\cos\theta}{\sqrt{A^2+B^2-2AB\cos\theta}}$

4. Ans. (c)

5. If the maximum and minimum resultant forces of two forces acting on a particle are 40 kN and 10 kN respectively, then the two forces in question would be **[IAS-2000]**
(a) 25kN and 15kN (b) 20kN and 20kN (c) 20 kN and 10 kN (d) 20 kN and 5kN
5. Ans. (a)

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R_{\max}^2 = P^2 + Q^2 + 2PQ = \left(P+Q\right)^2 = 40^2$$

$$R_{\min}^2 = P^2 + Q^2 - 2PQ = \left(P-Q\right)^2 = 10^2$$

$$\text{solving we get } P = 25\,kN \text{ and } Q = 15\,kN$$

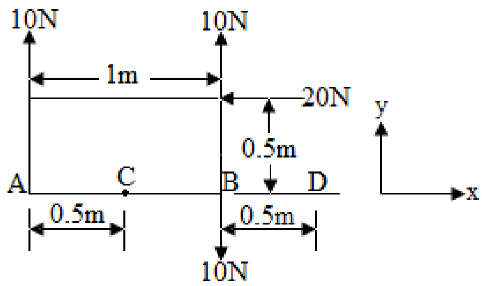
6. If two forces P and Q act at an angle θ, the resultant of these two forces would make an angle a with P such that **[IAS-1998]**

(a) tan $\alpha = \dfrac{Q\sin\theta}{P - Q\sin\theta}$ (b) tan $\alpha = \dfrac{P\sin\theta}{P + Q\sin\theta}$

(c) tan $\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$ (d) tan $\alpha = \dfrac{P\sin\theta}{P - P\cos\theta}$

6. Ans. (c)

7. A system of forces acting on a lamina is shown in the given figure. The resultant of the force system will meet AB at
(a) A (b) B (c) C (d) D



[IAS-1995]

7. Ans. (b)

8. Two forces act at a point. The first force has x and y components of 3N and -5 N respectively. The resultant of these forces falls on the x-axis and has a magnitude of -4 N. The x and y components of the second force is **[IAS-1997]**
(a) (-7,5) (b) (-7,-5) (c) (-7,0) (d) (+ 7, 0)

8. Ans. (a)

$P_x + Q_x = -4$ or $Q_x = -4 - P_x = -7N$

$P_y + Q_y = 0$ or $Q_y = -P_y = 5N$

9. The inertia force in a system is directed at **[IAS-1997]**
(a) zero degrees to the acceleration (b) 45 degrees to the acceleration
(c) 90 degrees to the acceleration (d) 180 degrees to the acceleration
9. Ans. (d) Inertia force = - ma

10. Assertion (A): Many times the weight of a body is treated as negligible while calculating total forces acting on it. **[IAS-1995]**
Reason(R): The body is weightless.
10. Ans. (c) A is true but R is false

Equilibrium of bodies

11. Assertion (A): The concept of equilibrium is derived from a balance of forces. Reason (R): The equilibrium requires that the resultant force acting over a body is zero. **[IAS-1998]**

11. Ans. (a)

12. Assertion (A): A particle under equilibrium moves with constant velocity.

Reason (R): If the resultant force acting over a particle is zero. Then the particle will be at rest or continue to move with the same velocity, if originally in motion. **[IAS-1996]**
12. Ans. (a)

13. Match List I with List II and select the correct answer using the codes below the lists:

List I	List II **[IAS-1995]**
A. Two parallel forces acting on a body, moving with uniform velocity in	1. Kinetic energy
B. A moving particle	2. Couple
C. Two coplanar parallel forces equal in magnitude and opposite in direction acting on a body	3. Forces in equilibrium
D. Two unequal forces acting on a body	4. Cause acceleration

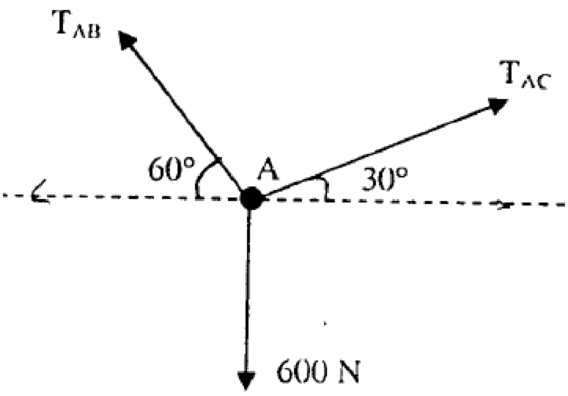
Codes:	A	B	C	D		A	B	C	D
(a)	1	3	4	2	(b)	3	2	1	4
(c)	3	1	2	4	(d)	3	4	1	2

13. Ans. (c)

14. Consider the following statements:
For a particle in plane in equilibrium **[IAS-1998]**
1. sum of the forces along X-direction is zero
2. sum of the forces along Y-direction is zero.
3. sum of the moments of all forces about any point is zero.

Of these statements:
(a) 1 and 3 are correct (b) 2 and 3 are correct
(c) 1 and 2 are correct (d) 1, 2 and 3 are correct.
14. Ans. (d)

15. If point A is in equilibrium under the action of the applied forces, the values of tension T_{AB} and T_{AC} are respectively
(a) 520 N and 300 N
(b) 300 N and 520 N
(c) 450 N and 150 N
(d) 150 N and 450 N

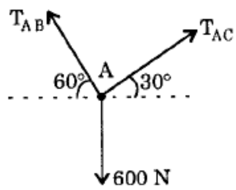


[GATE-2006]

15. Ans. (a)
variable must be zero.

$$T_{AB}\cos 60° = T_{AC}\cos 30° \quad ....(i)$$

$$T_{AB}\sin 60° + T_{AC}\sin 30° = 600 \quad ....(ii)$$

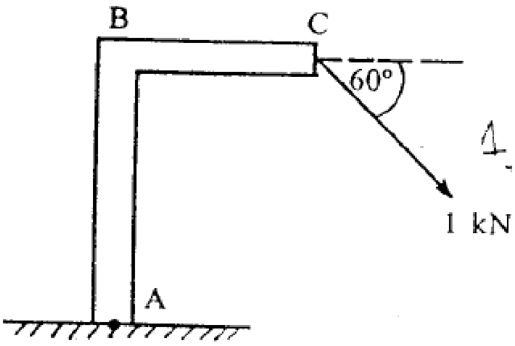


Solving equations (*i*) and (*ii*), we get

$$T_{AB} = 520N \quad \text{and} \quad T_{AC} = 300N$$

16. What is the thrust at the point A in lamp post shown in the figure?

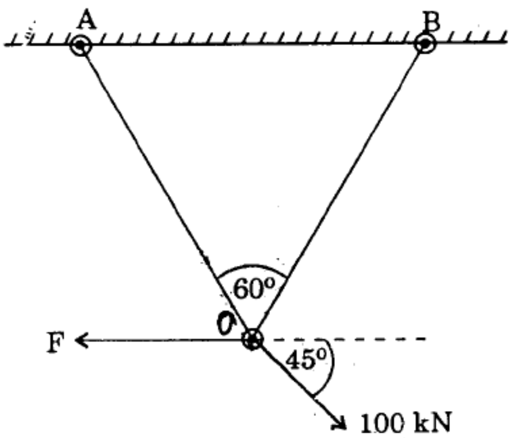
(a) 0.866 kN (b) 0.5 kN
(c) 1.388 kN (d) 1 kN



[IAS-2004]

16. Ans. (a) Axial thrust = vertical component
= 1x sin 60 kN
= 0.866 kN

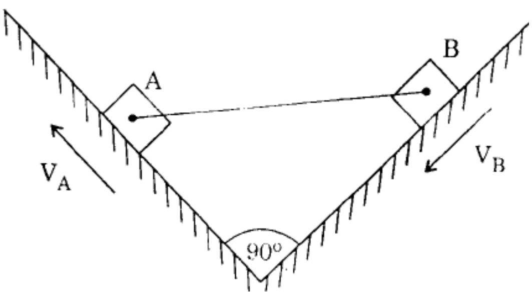
17. The force F such that both the bars AC and BC (AC and BC are equal in length) as shown in the figure are identically loaded, is
(a) 70.7 N
(b) 100 N
(c) 141.4 N
(d) 168 N



[IAS-2003]

17. Ans. (a) F = 100 cos 45 = 70.7 N.

18. In the given configuration of the mechanism as shown in the figure, V_A = 40 m/s and V_B = 80 m/s. The magnitude of velocity of slider B relative to the slider A is
(a) 30 m/s (b) 40 m/s
(c) 50 m/s (d) 30.5 m/s



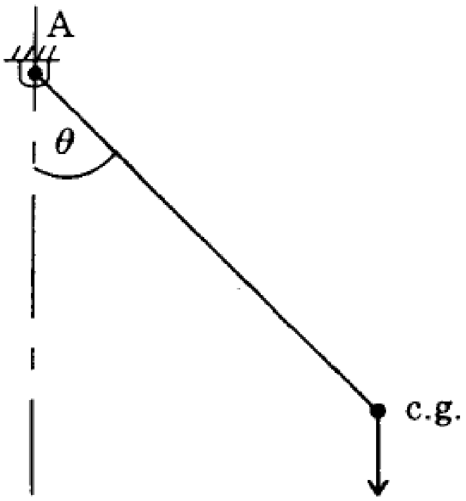
[IAS-2003]

18. Ans. (c) $V_{BA} = \sqrt{V_A^2 + V_B^2} = \sqrt{30^2 + 40^2} = 50\,m/s$

19. The figure shows a rigid body oscillating about the pivot A. If J is mass moment of inertia of the body about the axis of rotation, its natural frequency for small oscillations is proportional to

(a) J (b) J^2

(c) $\frac{1}{J}$ (d) $\frac{1}{\sqrt{J}}$

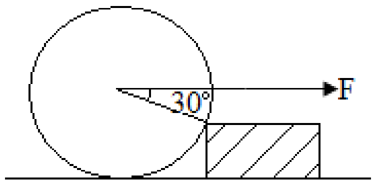


[IAS-2003]

19. Ans. (d)

20. A roller of weight W is rolled over the wooden block shown in the given figure. The pull F required to just cause the said motion is

(a) W/2 (b) W (c) $\sqrt{3}W$ (d) 2 W



[IAS-1995]

20. Ans. (c)

$$R \cos 60° = W$$

or, $R \times \frac{1}{2} = W,$

or, $R = 2\ W$

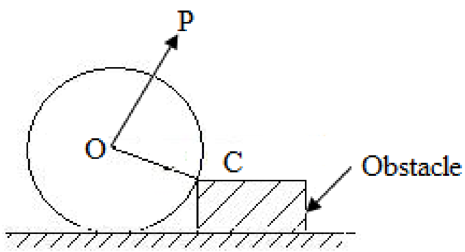
Also $F = R \cos 30° = 2W \times \frac{\sqrt{3}}{2} = \sqrt{3}\ W.$

21. The road roller shown in the given figure is being moved over an obstacle by a pull 'P'. The value of 'P' required will be the minimum when it is

(a) horizontal (b) vertical

(c) at 45° to the horizontal

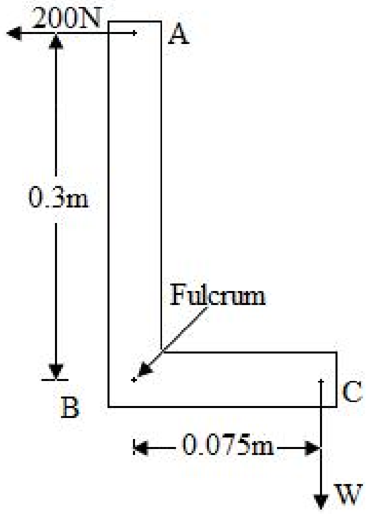
(d) perpendicular to the line CO



[IAS-1996]

21. Ans. (d)

22. A horizontal force of 200 N is applied at 'A' to lift the weight 'W' at C as shown in the given figure. The value of weight 'W' will be
(a) 200 N
(b) 400 N
(c) 600 N
(d) 800 N



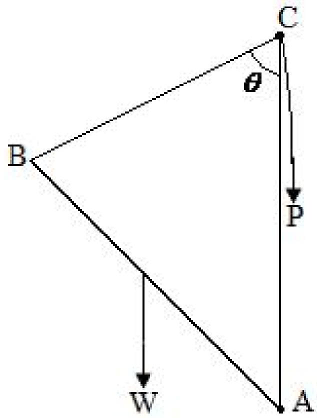
[IAS-1999]

22. Ans. (d) Taking moment about fulcrum 200 x 0.3 = W x 0.075 or W = 800 N

23. A uniform rod AB of weight W is movable in a vertical plane about a hinge at A, and is sustained in equilibrium by a weight P attached to a string BCP passing over a smooth peg C, as shown in the given figure, AC being vertical. If AC be equal to AB, then the weight P is

(a) $\dfrac{W}{\cos\theta}$ (b) $W\cos\theta$

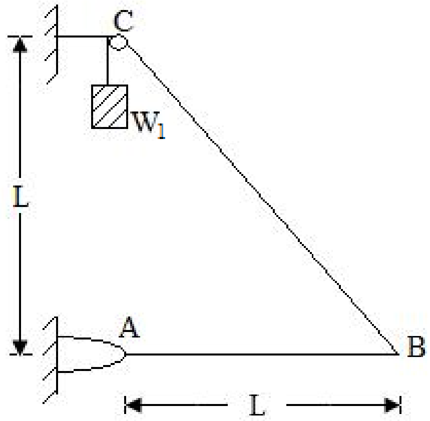
(c) $\dfrac{W}{\sin\theta}$ (d) $W\sin\theta$



[IAS-2001]

23. Ans. (a)

24. A uniform, heavy rod AB of length L and weight W is hinged at A and tied to a weight W_1 by a string at B.
The mass less string passes over a frictionless pulley (of negligible dimension) at C as shown in the figure. If the rod is in equilibrium at horizontal configuration, then
(a) $W_1 = W$ (b) $W_1 = W/2$
(c) $W_1 = \sqrt{2}\,W$ (d) $W_1 = W/\sqrt{2}$



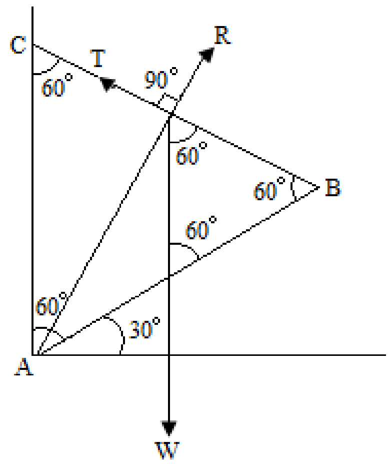
24. Ans. (d) If *T be* the tension in string *BC* and since it passes over smooth pulley C, therefore *T*= W_1

Reaction at B is $\frac{W}{2}$, $\frac{W}{2} = T\cos 45^o = W_1 \times \frac{1}{\sqrt{2}}$ or $W_1 = \frac{W\sqrt{2}}{2} = \frac{W}{2}$ **[IAS 1994]**

25. A uniform beam AB (see given figure) Pinned at A is held by the cable BC in the position shown If the tension in the cable is 200 kgf, then the weight of the beam and the reaction of the pin at A on the beam are respectively

(a) 300 kgf; 100$\sqrt{3}$ kgf, 30° (b) 400 kgf; 100$\sqrt{3}$ kgf, 60°

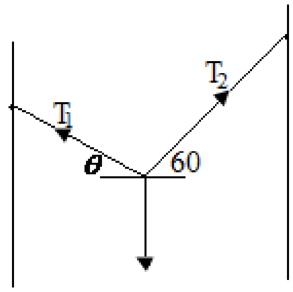
(c) 300 kgf; 200$\sqrt{3}$ kgf, 30° (d) 400 kgf; 200$\sqrt{3}$ kgf, 60°



25. Ans. (d) $\frac{W}{\sin 90} = \frac{T}{\sin(90+60)} = \frac{R}{\sin(90+30)}$ $or\ W = \frac{200\times 2}{1} = \frac{R\times 2}{\sqrt{3}}$ **[IAS 1994]**

26. A weight W is supported by two cables as shown in the given figure. The tension in the cable making angle θ will be the minimum when the value of θ is **[IAS 1994]**

(a) 0° (b) 30° (c) 45° (d) 60°

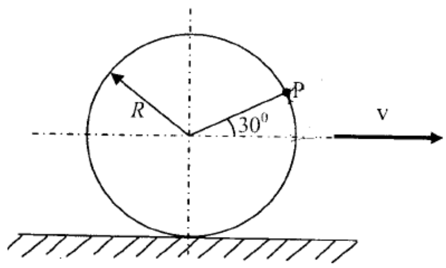


26. Ans. (b) $\dfrac{T_1}{\sin 150^o} = \dfrac{T_2}{\sin(90+\theta)} = \dfrac{W}{\sin[180-(60+\theta)]} = \dfrac{W}{\sin(90+30-\theta)}$

Since $T_1 \propto \dfrac{1}{\sin(90+\theta)}$, for T_1 to be least θ should be minimum.

And $T_1 \propto \dfrac{1}{\sin(90+30-\theta)}$, Again for minimum value of T_1; θ should be 30^0

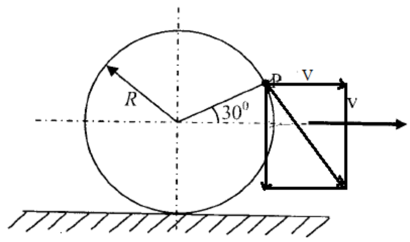
27. A circular disk of radius *R* rolls without slipping at a velocity v. The magnitude of the velocity at point P (see figure) is :



[GATE-2008]

27. Ans. (D)
Velocity in x – direction must be V.
Therefore $V_p \sin 60 = V$

Or $V_p = \dfrac{2}{\sqrt{3}}V$

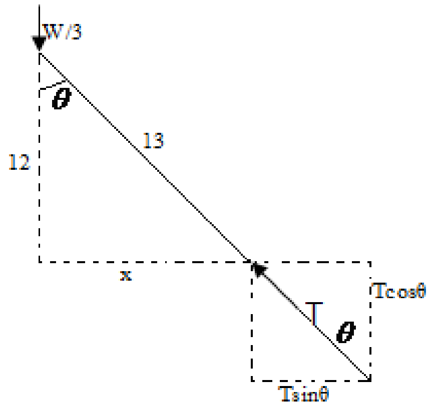


28. Weight of 120 kN is being supported by a tripod whose each leg is of the length of 13 m. If the vertical height of the point of attachment of the load is 12m, the force on the tripod leg would be

(a) 37.67 kN (b) 40 kN (c) 43.3 kN (d) 46.6 kN

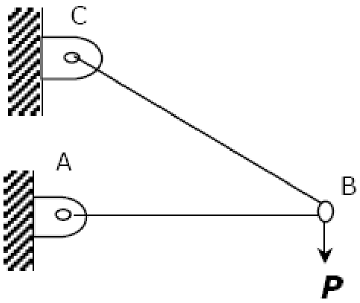
28. Ans. (c)
$T\cos\theta = W/3$

Or T = W/3cosθ = $\dfrac{120}{3\times\left(\dfrac{12}{13}\right)} = 43.3\,\text{kN}$



[IAS-2001]

.
29 Bars AB and BC, each of negligible mass, support load P as shown in the figure. In this arrangement,
(a) bar AB is subjected to bending but bar BC is not subjected to bending
(b) bar AB is not subjected to bending but bar BC is subjected to bending
(c) neither bar AB nor bar BC is subjected to bending
(d) both bars AB and BC are subjected to bending



[GATE-2001]

29 Ans. (c)

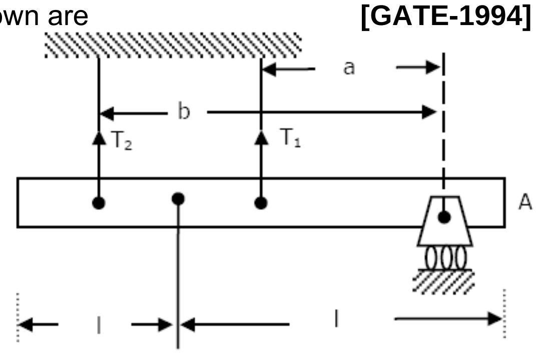
30. Figure shows a rigid bar hinged at A and supported in a horizontal by two vertical identical steel wires. Neglect the weight of the beam. The tension T_1 and T_2 induced in these wires by a vertical load P applied as shown are **[GATE-1994]**

(a) $T_1 = T_2 = \dfrac{P}{2}$

(b) $T_1 = T_2 = P$

(c) $T_1 = \dfrac{Pbl}{(a^2+b^2)}; T_2 = \dfrac{Pal}{(a^2+b^2)}$

(d) $T_1 = \dfrac{Pbl}{2(a^2+b^2)}; T_2 = \dfrac{Pal}{2(a^2+b^2)}$



30. Ans. (b)

Taking moment about A,

$$T_2.b + T_1.a = p.l \quad \ldots(i)$$

Since deflection, $\delta = \frac{TL}{AE}$,

$\therefore \quad \frac{\delta_2}{b} = \frac{\delta_1}{a}$ and $\frac{T_2}{b} = \frac{T_1}{a}$

Thus $\quad T_1.b = T_2.a$

From equation (*i*) we have,

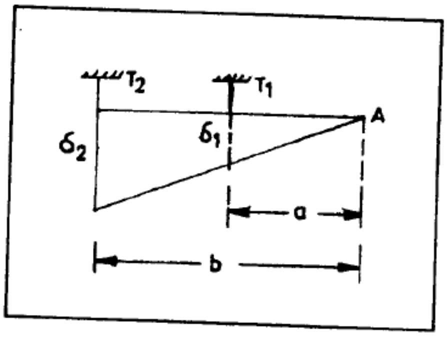
$T_1b^2 + T_1 ab = pbl$

and $\quad T_1ab - T_2 a^2 = 0$

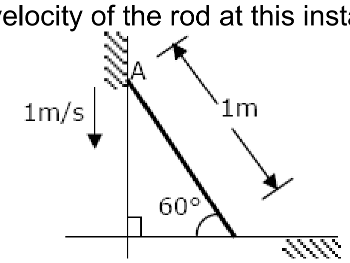
$\therefore \quad T_2 = \frac{Pbl}{a^2 + b^2}$

and $\quad T_1 = \frac{Pbl}{a^2 + b^2} \times \frac{a}{b}$

$$= \frac{\mathbf{Pal}}{\mathbf{a^2 + b^2}}$$

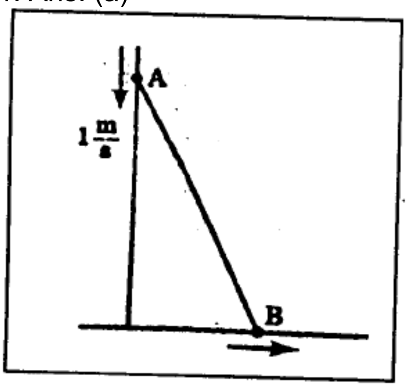


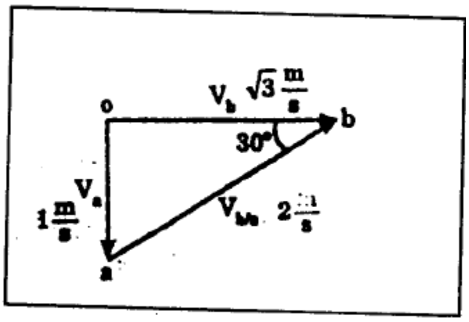
31. A rod of length 1 m is sliding in a corner as shown in figure. At an instant when the rod makes an angle of 60 degrees with the horizontal plane, the velocity of point A on the rod is 1m/s. The angular velocity of the rod at this instant **[GATE-1996]**



(a) 2 rad/s (b) 1.5 rad/s (c) 0.5 rad/s (d) 0.75 rad/s

31. Ans. (a)





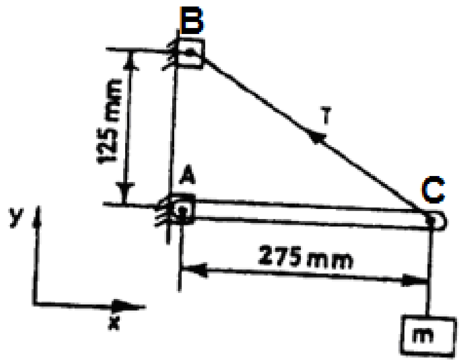
32. A mass of 35 kg is suspended from a weightless bar AC, which is supported by a cable CB and a pin at A as shown in the Fig. The pin reactions at A on the bar AB are

(a) $R_X = 343.4N$
$R_y = 755.4N$

(b) $R_X = 343.4N$
$R_y = 0$

(c) $R_X = 755.4N$
$R_y = 343.4N$

(d) $R_X = 755.4N$
$R_y = 0$



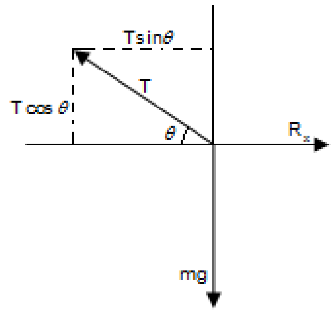
[GATE-1997]

.
32. Ans. (d)

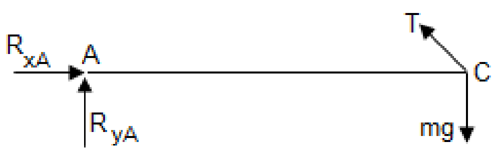
$R_y = 0$

$T\cos\theta = R_x$ and $T\sin\theta = mg$ or $T = \dfrac{mg}{\sin\theta}$

$$\therefore R_x = \frac{mg}{\sin\theta}\times\cos\theta = \frac{mg}{\tan\theta} = \frac{35\times 9.81}{\left(\frac{125}{275}\right)} = 755.4N$$



FBD of 'C'



$\therefore R_{xA} = R_x = 755.4$ N

$R_{yA} = R_y = 0$

FBD of bar AC

33. If a system is in equilibrium and the position of the system depends upon many independent variables, the principle of virtual work states that the partial derivatives of its total potential energy with respect to each of the independent variable must be

(a) -1.0 (b) 0 (c) 1.0 (d) ∞ **[GATE-2006]**

33. Ans. (b)

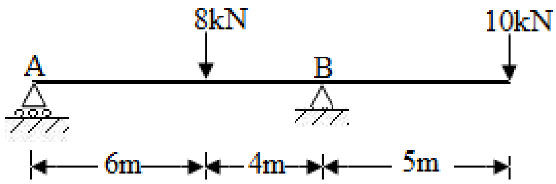
Total potential energy =j(independent variable)

Hence for a system in equilibrium,

Total potential energy = Constant

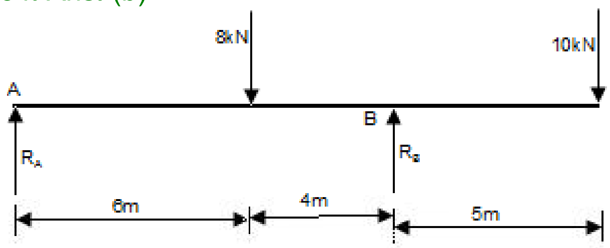
Thus, partial derivatives of its total potential energy with respect to each of independent variable must be zero.

34. The reaction (in kN) at the support 'A' for the beam shown in the given figure is
(a) 18 (b) 1.8 (c) 1.8 (d) 0.8



[IAS-1997]

34. Ans. (b)



Taking moment about B

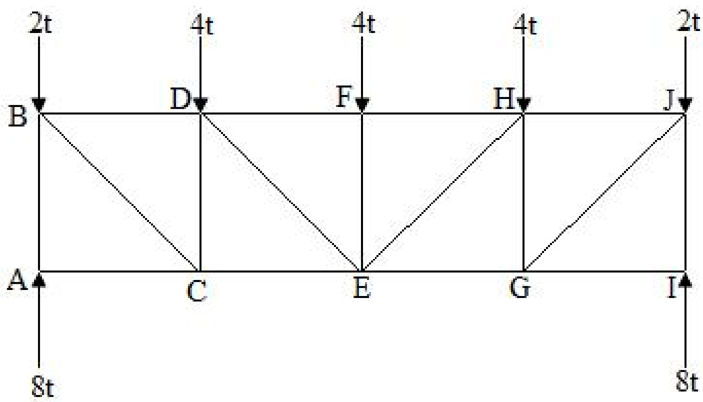
$-R_A \times 10 + 8 \times 4 - 10 \times 5 = 0$

R_A = -1.8

That means it is downward $R_A = 1.8 \downarrow$

Trusses

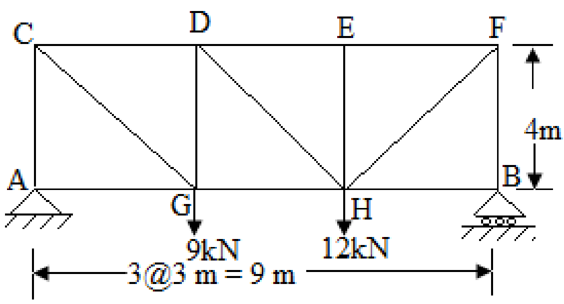
35. The given figure shows the loading pattern on a truss. The force in the member AC is
(a) zero
(b) 2 t
(c) 8 t
(d) statically indeterminate



[IAS-2001]

35. Ans. (a)

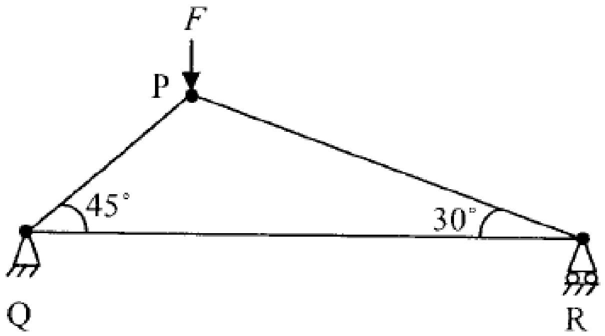
36. A truss of span 9 m is loaded as shown in the given figure. The number of two-force members carrying zero force is
(a) one (b) two (c) three (d) four



[IAS-1995]

36. Ans. (d) The vertical and horizontal reactions at any point have to be zero. Considering the points at *A, B,* and *E,* it will be noted that there are no horizontal force acting on these points. Thus there are no stresses in members AG, HB, ED and EF. Thus number of zero force members are four.

37. Consider a truss PQR loaded at P with a force F as shown in the figure.

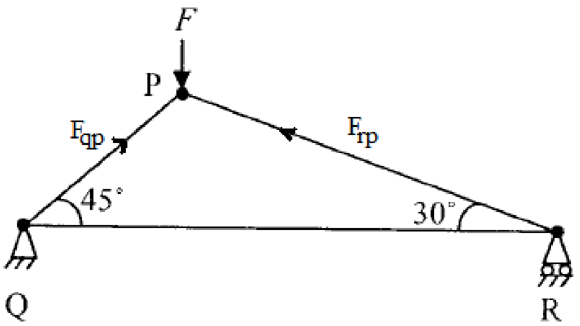


[GATE-2008]

The tension in the member QR is

(A) 0.5 F (B) 0.63 F (C) 0.73 F (D) 0.87 F

37. Ans. (B)



From Force Triangle $\dfrac{F}{\sin(180-45-30)} = \dfrac{F_{qp}}{\sin(90+30)} = \dfrac{F_{rp}}{\sin(90+45)}$

The tension in the member QR is $F_{qr}=F_{rp}\cos30=0.63F$

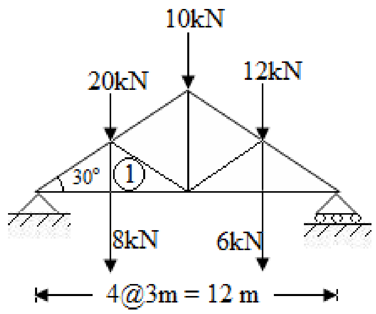
38. The force in the member 1 of the truss shown in the figure is

(a) 12 kN compressive

(b) 28 kN tensile

(c) 8 kN tensile

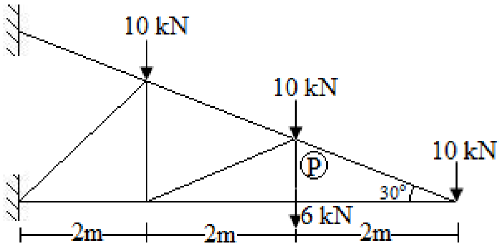
(d) 20 kN compressive



[IAS-1998]

38. Ans. (c)

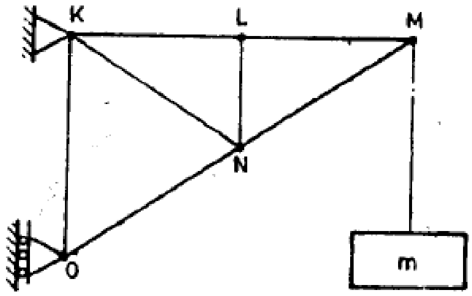
39. The force in the member 'p' of the truss shown in the given figure is
(a) 16kN tensile (b) 16 kN compressive
(c) 4 kN tensile (d) 6 kN tensile



[IAS-1997]

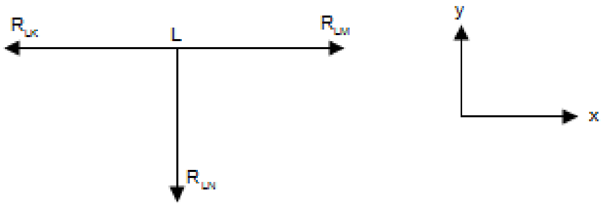
39. Ans. (d)

40. The figure shows a pin-jointed plane truss loaded at the point M by hanging a mass of 100 kg. The member LN of the truss is subjected to a load of
(a) 0 Newton
(b) 490 Newton in compression
(c) 981 Newton in compression
(d) 981 Newton in tension



[GATE-2004]

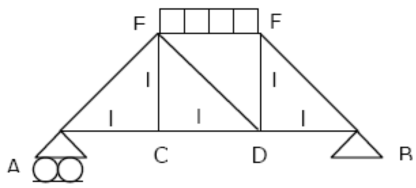
40. Ans. (a)



$\sum F_x = 0$ gives $R_{LK} = R_{LM}$

$\sum F_y = 0$ gives $R_{LN} = 0$

41. A truss consists of horizontal members (AC, CD DB and EF) and vertical members (CE and DE) having length l each. The members AE, DE and BF are inclined at 45° to the horizontal. For the uniformly distributed load "P" per unit length on the member EF of the truss shown in figure given below, the force in the member CD is



$(a)\frac{pl}{2}$ $(b)\ pl$ $(c)\ 0$ $(d)\frac{2pl}{3}$ **[GATE-2003]**

41. Ans. (a)

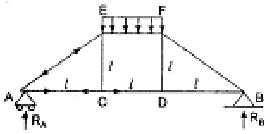
Total load on EF member = Pl

where l = length of EF

For horizontal equilibrium

$$R_A + R_B = Pl$$

Taking moment about A., we have



$$Pl \times \frac{3l}{2} = R_B \times 3l$$

$$\Rightarrow \qquad R_B = \frac{Pl}{2}$$

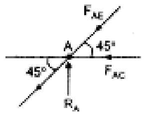
Considernig a point A.

For horizontal equilibrium

$$F_{AC} + F_{AE}\cos 45° = 0$$

For vertical equilibrium

$$R_A - F_{AE}\sin 45° = 0$$



$$\Rightarrow \qquad F_{AE} = \frac{R_A}{\sin 45°} = \frac{Pl}{2\times\frac{1}{\sqrt{2}}} = \frac{Pl}{\sqrt{2}}$$

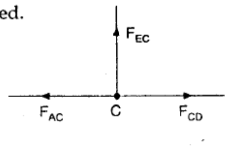
$$\therefore \qquad F_{AC} = -F_{AE}\cos 45°$$

$$= \frac{-Pl}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{-Pl}{2}$$

–ve sign shows that the force on member AC is opposite of assumed.

Now considering a point C

For horizontal equilibrium



$$F_{AC} = F_{CD} = \frac{Pl}{2}$$

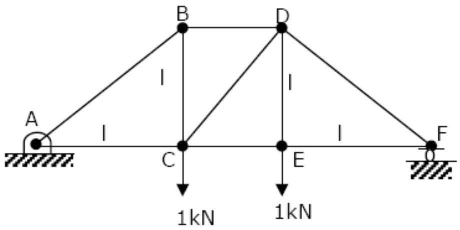
42. For the loading on truss shown in the figure, the force in member CD is

(a) 0 kN

(b) 1 kN

(c) $\sqrt{2}$

(d) $1/\sqrt{2}$



[GATE-2001]

.

42. Ans. (a)

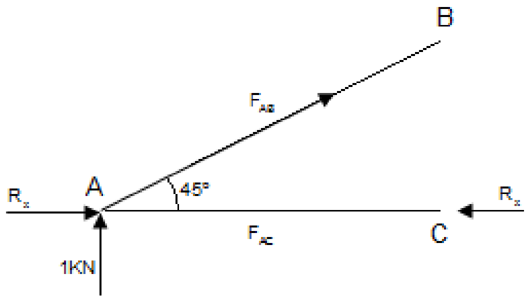
$F_{AB}\cos 45° = 1\text{kN}$

or $F_{AB} = \sqrt{2}\,\text{kN}$ and $F_{AC} = 0$

Similarly $F_{EF} = 0$ & $F_{CE} = 0$

And $F_{BC} + F_{FD}\cos 45° = 1\text{kN}$

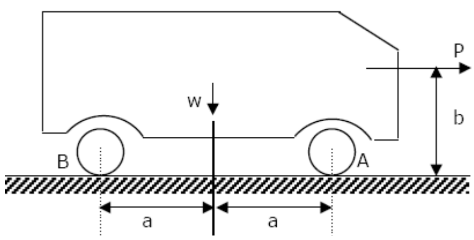
or $F_{CD}\sin 45° + 0 = 0$ or $F_{CD} = 0$



43. An automobile of weight W is shown in Figure. A pull 'P' is applied as shown. The reaction at the front wheels (location A) is

$(a) W/2 - pb/2a$ $\quad (b) W/2 + pb/2a$

$(c) W/2 - pa/2b$ $\quad (d) W/2$



[GATE-2000]

43. Ans. (d)

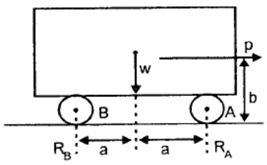
2.13 $R_A + R_B = W$

Taking moment about B, $\Sigma M_B = 0$

$\Rightarrow$ $R_A \times 2a = W \times a + P \times b$

$\Rightarrow$ $R_A = \dfrac{W \times a + P \times b}{2a}$

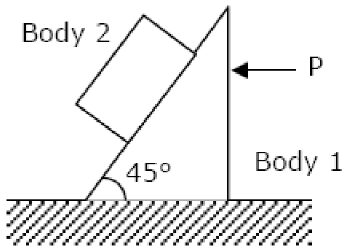
$= \dfrac{W}{2} + \dfrac{Pb}{2a}$



44. Bodies 1 and 2 shown in the 'figure have equal mass m. All surfaces are smooth. The value of force P required to prevent sliding of body 2 on body 1 is

(a) P = 2mg (b) P = $\sqrt{2}$ mg

(c) P = 2$\sqrt{2}$ mg (d) P = mg



[GATE-2001]

.

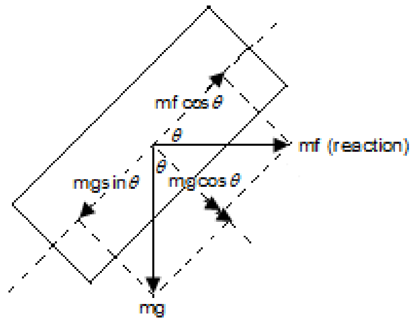
44. Ans. (a) FBD of body – 2 given in the figure

Let P cause an acceleration 'f'

To prevent sliding $mf\cos\theta = mg\sin\theta$

Or $f = g\tan\theta = g\tan 45^\circ = g$

Force 'P' cause acceleration on both body – 1 & body -2 so P = (2m) f = 2mg



Friction

45. Match List I with List .II and select the correct answer using the codes given below the Lists:

List I List II **[IAS-2002]**

A. Static Friction 1. Angle between the normal reaction and the resultant of force of friction and normal reaction

B. Dynamic Friction 2. The force of friction developed between two bodies at rest

C. Solid Friction 3. The force of friction between two bodies in motion

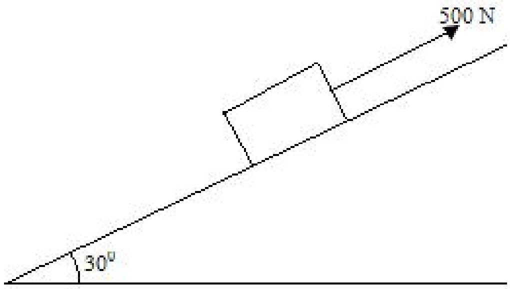
D. Angle of Friction 4. Friction between dry surfaces which are not lubricated

Codes:	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	4	1	2	3
(c)	2	1	4	3	(d)	4	3	2	1

45. Ans. (a)

46. The block shown in the given figure is kept in equilibrium and prevented from sliding down by applying a force of 500 N. The coefficient of friction is $\frac{\sqrt{3}}{5}$. The weight of the block would be

(a) 4000 N (b) 2500 N
(c) 1000 N (d) 500 N



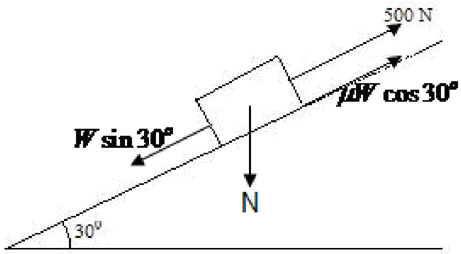
[IAS-2001]

46. Ans. (b)

$$W\sin 30^o = 500 + \mu W\cos 30^o$$

$$or\, W\left(\sin 30^o - \mu\right) = 500$$

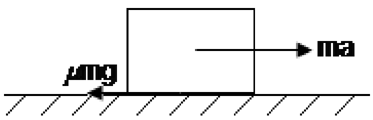
$$or\, W = 2500\,N$$



47. A box rests in the rear of a truck moving with a declaration of 2 m/s^2. To prevent the box from sliding, the approximate value of static coefficient of friction between the box and the bed of the truck should be **[IAS-1999]**

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

47. Ans. (b)



$$\text{or } \mu = \frac{ma}{mg} \text{ or } \mu = \frac{a}{g} = \frac{2}{9.81} \approx 0.20$$

48. An automobile having a mass of 2000 kg cruises at a speed of 20 m/s. If all the wheels are jammed suddenly, how far will the automobile skids before it comes to a halt, assuming that the vehicle does not spin or topple? The coefficient of friction between the ground and the tyres is 0·8. **[IAS-2007]**

(a) 10 m (b) 25 m (c) 30 m (d) None of the above

48. Ans. (b) Kinetic energy = Frictional loss of energy
= Frictional x Length of travel

$$or\, \frac{1}{2}mv^2 = \mu mgL \qquad or\, L = \frac{V^2}{2\mu g} = \frac{20^2}{2\times 0.8\times 10} = 25\,m$$

[Here use g = 10 m/s^2 for simplify calculation]

49. A train of weight 200 x 10^{4}N is running on a horizontal track at a constant speed of 10 m/s, overcoming a constant frictional force of 20 x 10^3 N. What is the power of the engine driving the train? **[IAS-2004]**

(a) 800 kW (b) 1200 kW (c) 200 kW (d) 400 kW

49. Ans. (c) Power needed at constant speed to overcome frictional resistance only

$P = F_t \times V = 20 \times 10^3 \times 10 = 200$ kW

50. A car of mass 2000 kg is travelling at a constant speed of 50 m/s. The car has to resist a wind drag F_D = 10 v (N), where v is the velocity of the car in m/s. The power required by the engine is **[IAS-2002]**
(a) 5 kW (b) 10 kW (c) 25 kW (d) 50 kW
50. Ans. (c) Power required is to overcome drag only.

$$P = F_D \times v = 10v \times v = 10v^2 = 10 \times 50^2 \text{ W} = 25 \text{kW}$$

51. A train of weight 5000 kN is pulled on a level track with a constant speed of 10 m/s. The frictional force is 75 kN. The power of the engine is **[IAS-2000]**
(a) 375kW (b) 750kW (c) 1875kW (d) 3000kW
51. Ans. (b) for constant speed Energy needed to overcome frictional resistance
= Frictional force x velocity = 75 x 10 kW = 750 kW

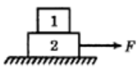
52. A wheel of mass m and radius r is in accelerated rolling motion without slip under a steady axle torque T. If the coefficient of kinetic friction is μ, the friction force from the ground on the wheel is **[GATE-1996]**
(a) μ mg (b) T/r (c) zero (d) none of the above
52. Ans. (a)

53. A block of mass 5 kg is thrust up a 30° inclined plane with an initial velocity of 4 m/sec. It travels a distance of 1.0 m before it comes to rest. The force of friction acting on it would be. **[GATE-1994]**
53. Ans. 15.5 N

54. Two books of mass 1 kg each are kept on a table, one over the other. The coefficient of friction on every pair of contacting surfaces is 0.3 the lower book is pulled with a horizontal force F. The minimum value of F for which slip occurs between the two books is **[GATE-2005]**
(a) zero (b) 1.06 N (c) 5.74 N (d) 8.83 N
54. Ans. (d)

μ for each surface is same = 0.3

Its free body diagram ($m_1 = m_2 = 1$ kg)



$$N = mg = g \text{ N}$$
$$N' = 2mg = 2g \text{ N}$$

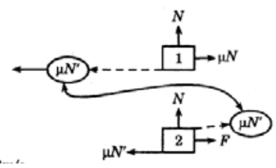
Equation of motion of 1

$$\mu N = 1 \times a_1$$
$$\Rightarrow a_1 = \mu g \quad \text{....(i) towards right}$$

Equation of motion of 2

$$F - (-\mu N + \mu N') = 1 \times a_2$$
$$\Rightarrow F - (3\mu g) = a_2 \quad \text{....(ii) towards right}$$

Now for relative motion between two blocks



$$a_2 > 0$$
$$F - 3\mu g > 0$$
$$F > 3\mu g$$
$$F > 3 \times 0.3 \times 9.8$$
$$F > 8.82 \text{ N}$$
$$\therefore F = 8.83 \text{ N}$$

55. Which one of the following principles cannot be used to solve problems involving friction?
(a) D' Alembert's principle (b) Equations of Motion **[IAS-2002]**
(c) Principle of work and energy (d) Conservation of energy
55. Ans. (d) As friction is involved there is energy loss so conservation of energy is not possible.

Properties of surfaces II: Second moment of area

Method of Virtual Work

Motion in a plane: Introduction to polar coordinates

56. Match List I with List II and select the correct answer using the codes given below the lists: **[IAS-1999]**

List I	List II
A. Average acceleration	1. a = constant
B. Instantaneous acceleration	2. a=0
C. Uniform motion	3. $\frac{\Delta v}{t}$
D. Uniformly accelerated motion	4. $\lim_{\delta t \to 0} \frac{\Delta v}{\Delta t}$

Codes:	A	B	C	D		A	B	C	D
(a)	2	4	3	1	(b)	4	3	2	1
(c)	3	4	1	2	(d)	3	4	2	1

56. Ans. (d)

57. When the system is given a constant angular velocity rather than an angular displacement, it is known as **[IAS-2002]**
(a) step function input (b) harmonic input
(c) unit step displacement (d) variable input
57. Ans. (b)

58. A straight rod of length L(t), hinged at one end and freely extensible at the other end, rotates through an angle $\theta(t)$ about the hinge. At time t, L(t) == 1 m, $\dot{L}(t) = 1 m/s$, $\theta(t) = \frac{\pi}{4} rad \text{ and } \dot{\theta}(t) = 1 rad/s.$ The magnitude of the velocity at the other end of the rod is **[GATE-2008]**
58. Ans. (B) Velocity in vertical direction (V_{vert}) = $\dot{L}(t) = 1 m/s$,
Velocity in radial direction (V_{radial}) = $r\omega$ = L (t)$\times\dot{\theta}(t)$ = 1×1 = 1 m/s
$\therefore V = \sqrt{V_{vert}^2 + V_{radial}^2} = \sqrt{2}$ m/s

59. The equation of motion of a body is given by s = $2t^3 + 3t^2 + 7$ (where s is in metre and t is in seconds). Starting from rest, it will travel in 2 seconds, a distance of
(a) 35 m (b) 28 m (c) 27 m (d) 20 m **[IAS-1998]**
59. Ans. (b)

$S = 2t^3 + 3t^2 + 7$

$S_2 - S_o = 2t^3 + 3t^2 + 7 - 7 = 2 \times 2^3 + 3 \times 2^2 = 28m$

60. A car moving with uniform acceleration covers 450 m in a 5 second interval, and covers 700 m in the next 5 second interval. The acceleration of the car is **[GATE-1998]**

(a) 7 m/s^2 (b) 50 m/s^2 (c) 25 m/s^2 (d) 10 m/s^2

60. Ans. (d)

Explanation. For t_1 = 5 secs, S_1 = 450 m and for t_2 = 5 secs S_2 = 700 m

From the relation $S = ut + \frac{1}{2}at^2$

$450 = 5u + \frac{1}{2} \times a \times 25$ [since initial velocity = u]

or $450 = 5u + \frac{25a}{2}$ ......(i)

Also after first 5 seconds, velocity of car,

$v = u + at$

$= u + 5a.$

After first 5 seconds, u + 5a will be the initial velocity for next 5 seconds, therefore

$700 = (4 + 5a)5 + \frac{1}{2}a \times 25 = 20 + 25a + \frac{25a}{2}$

From equation (i), we have

$700 = \left(5u + \frac{25a}{2}\right) + 25a = 450 + 25a,$

or $a =$ **10 m/sec².**

Alternately

Considering the v-t graph

Area ACDB = 450

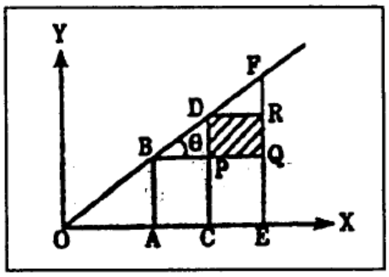
Area CEFD = 700

Difference is the shaded region

DPQR = 250

$\therefore PD = \frac{250}{5} = 50$

$a = \tan\theta = \frac{PD}{BP} = \frac{50}{5} = \mathbf{10}$



61. Consider the following statements: **[IAS-1999]**

If the distance's' covered by a car on a smooth road in time 't' is equal to [t^2 (t - 1) + 2], then the

1. Maximum velocity of the car will be at t = 1/3
2. Velocity of the car is ($3t^2$ - 2t)
3. Acceleration of the car is (6t - 2)
4. Car will travel a distance of 6 units starting from rest.

Which one of the following is correct?

(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

61. Ans. (a) S = t^2(t-2) + 2 = t^3-t^2+2

$\text{Velocity}(v) = \frac{dS}{dt} = 3t^2 - 2t$.

Acceleration (a) = $\frac{d^2S}{dt^2} = \frac{dV}{dt} = 6t - 2 \text{ for } V_{max}, a = 0, \text{or } t = 1/3 \text{units}$

62. The motion of a particle (distance in meters and time in seconds) is given by the equation s = $2t^3$+ 3t. Starting from t = 0, to attain a velocity of 9 m/s, the article will have to travel a distance of

(a) 5 m (b) 10 m (c) 15 m (d) 20 m **[IAS-1996]**

62. Ans. (a) $V = \frac{ds}{dt} = 6t^2 + 3 = 9$ gives $t = 1$ sec. or $s = 2\times 1^3 + 3\times 1 = 5$m

63. Consider the following statements:

A particle starting from rest is acceleration g along a straight line with an acceleration kt, where h is a constant and t is the time elapsed. After time, 't' **[IAS-1998]**

1. its velocity is given by kt^2.
2. its velocity is given by 1/2 kt^2.
3. the distance covered is given by 1/2 kt^3
4. the distance covered is given by 1/6 kt^3

Of these statements:

(a) 1 and 3 are correct (b) 2 and 4 are correct

(c) 1 and 4 are correct (d) 2 and 3 are correct.

63. Ans. (b) $\frac{d^2x}{dt^2} = kt \therefore \frac{dv}{dt} = kt$ or $\int_0^v dv = \int_0^t ktdt$ or $v = \frac{1}{2}kt^2$

$\therefore \frac{dx}{dt} = \frac{1}{2}kt^2$ or $\int_0^x dx = \int_0^t \frac{1}{2}kt^2 dt$ or $x = \frac{1}{6}kt^3$

64. The time variation of the position of a particle in rectilinear motion is given by x = $2t^3$ + t^2 + 2t. If v is the velocity and a the acceleration of the particle in consistent units, the motion started with

(a) v = 0, a = 0 (b) v = 0, a = 2 (c) v = 2, a = 0 (d) v = 2, a = 2

64. Ans. (d) **[GATE-2005]**

Given, $x = 2t^3 + t^2 + 2t$

$\therefore$ $v = \frac{dx}{dt} = 6t^2 + 2t + 2,$ $\Rightarrow v_t = 0 = 2$

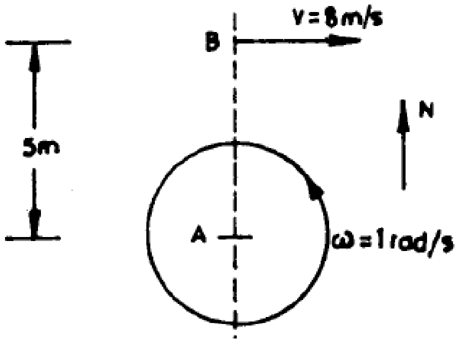
and $a = \frac{dv}{dt} = 12t + 2,$ $\Rightarrow a_t = 0 = 2$

65. If a body is moving with uniform acceleration "a", then the distance travelled by the body in the nth second is given by (where u is the initial velocity) **[IAS-1998]**

(a) $\frac{u+a}{2}(1-2n)$ (b) $\frac{u+a}{2}(n-2)$ (c) $u+\frac{a}{2}(2n-1)$ (d) $u+\frac{a}{2}(n-1)$

65. Ans. (c) $s = ut + \frac{1}{2}at^2$ calculate $S_n^{th} = s_n - s_{n-1}$

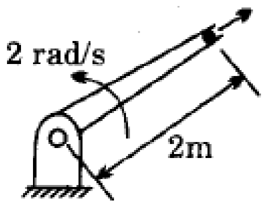
66. As shown in Figure. a person A is standing at the centre of a rotating platform facing person B who is riding a bicycle, heading East. The relevant speeds and distances are shown in given figure person a bicycle, heading East. At the instant under consideration, what is the apparent velocity of B as seen by A?
(a) 3 m/s heading East (b) 3 m/s heading West
(c) 8 m/s heading East (d) 13 m/s heading East



[GATE-1999]

66. Ans. (d)

67. A shell is fired from cannon. At the instant the shell is just about to leave the barrel, its velocity relative to the barrel is 3 m/ s, while the barrel is swinging upwards with a constant angular velocity of 2 rad/s. The magnitude of the absolute velocity of the shell is
(a) 3 m/s (b) 4m/s (c) 5 m/s (d) 7 m/s

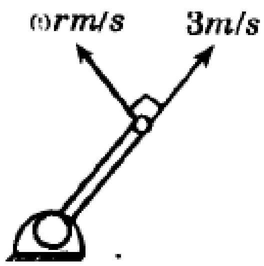


[GATE-2005]

67. Ans. (c)

$$\omega r = 2 \times 2 = 4 \text{m/sec}$$

$$v_{net} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$$

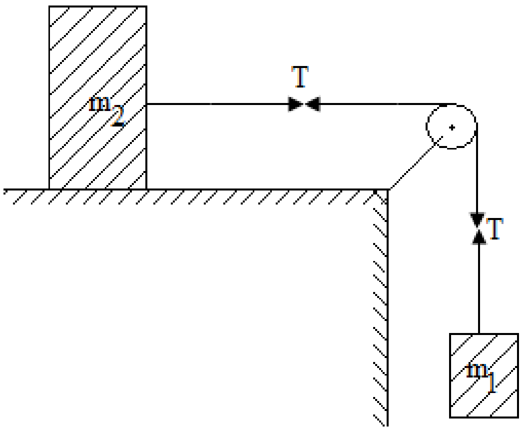


Motion with constraints

68. In the given figure, two bodies of masses m1 and m_1 and m_2 connected by a light inextensible string passing over a smooth pulley. Mass m_2 lies on a smooth horizontal plane. When mass m_1 moves downwards, the acceleration of the two bodies is equal to

(a) $\dfrac{m_1 g}{m_1 + m_2} m/s^2$ (b) $\dfrac{m_2 g}{m_1 - m_2} m/s^2$

(c) $\dfrac{m_2 g}{m_1 + m_2} m/s^2$ (d) $\dfrac{m_1 g}{m_1 - m_2} m/s^2$



[IAS-1995]

68. Ans. (a)

Also

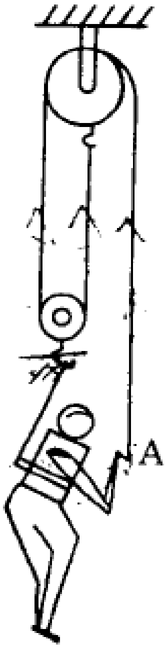
$$m_1 - T = m_1 \times a \quad \text{... }(i)$$

$$m_2 a = T - m_2 \quad \text{... }(ii)$$

From equations (*i*) and (*ii*), $a = \frac{m_1 g}{m_1 + m_2}$

69. For the arrangement shown in the figure what is the force with which a person weighing 500N pulls the rope downward at A to support himself without falling?

(a) 166.7 N
(b) 200 N
(c) 250 N
(d) 500 N



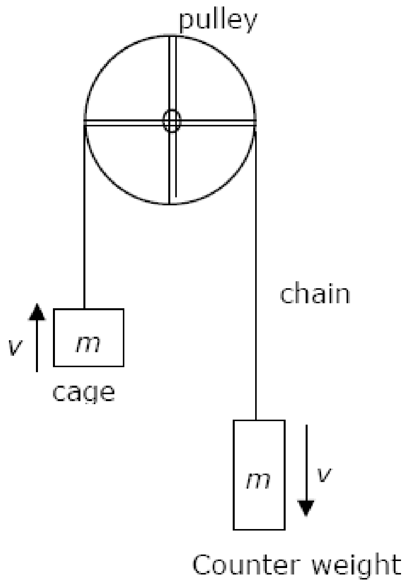
[IAS-2003]

69. Ans. (a)

70. An elevator (lift) consists of the elevator cage and a counter weight, of mass m each. The cage and the counterweight are connected by a chain that passes over a pulley. The pulley is coupled to a motor. It is desired that the elevator should have a maximum stopping time of t seconds from a peak speed v. If the inertias of the pulley and the chain are neglected, the minimum power that the motor must have is

$(a)\frac{1}{2}mv^2$ $(b)\frac{mv^2}{2t}$

$(c)\frac{mv^2}{t}$ $(d)\frac{2mv^2}{t}$



[GATE-2005]

70. Ans. (c)

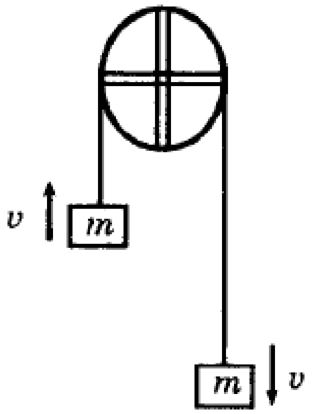
Power = $\dfrac{dW}{dt}$ (rate of doing work or rate of change of K.E. energy of the system)

Initial Kinetic energy of the system = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

Final Kinetic energy = 0

Time duration it occurs = t

$\Rightarrow$ Power = $\dfrac{mv^2}{t}$



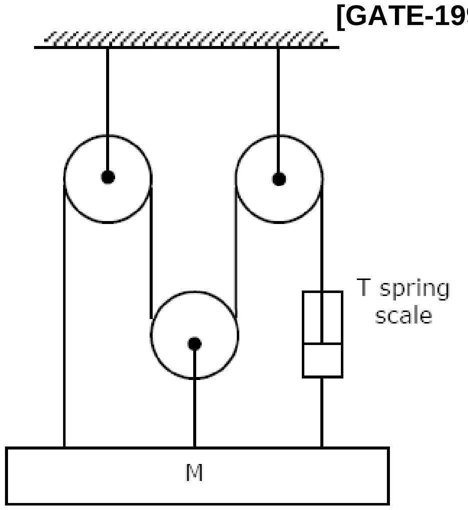
71. A spring scale indicates a tension T in the right hand cable of the pulley system shown in figure. Neglecting the mass of the pulleys and ignoring friction between the cable and pulley the mass m is **[GATE-1995]**

(a) $\dfrac{2T}{g}$

(b) $\dfrac{T\left(1+e^{4\pi}\right)}{g}$

(c) $\dfrac{4T}{g}$

(d) None of the above

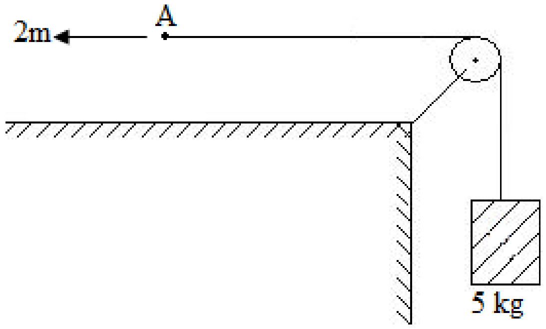


71. Ans. (c)

mg = 4T

$$m = \frac{4T}{g}$$

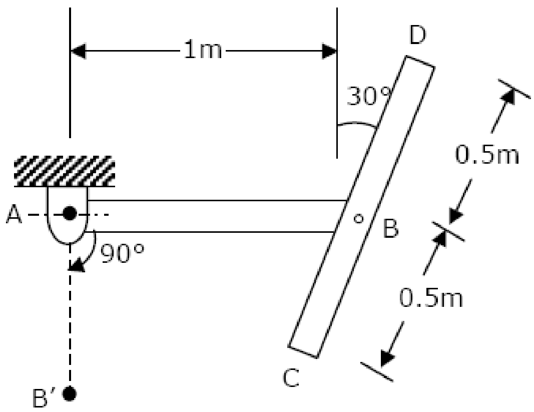
72. Consider the following statements: **[IAS-1997]**
A man holding the point A of the rope walks 2 m (see figure given). Assuming the pulley to be frictionless and g = 10 m/s^2 one can say that the



1. work done by the man is 100 J.
2. work done by the man is zero.
3. work done by the force of gravity is 100 J.
4. increase in potential energy of the 5 kg mass is 100 J.

Of these statements:
(a) 1 and 4 are correct (b) 2 and 4 are correct
(c) 1, 3 and 4 are correct (d) 2 and 3 are correct
72. Ans. (a)

73. AB and CD two uniform and identical bars of mass 10 kg each, as shown in figure. The hinges at A and Bare frictionless. The assembly is released from rest and motion occurs in the vertical plane. At the instant that the hinge B passes the point B, the angle between the two bars will be **[GATE-1996]**



(a) 60 degrees (b) 37.4 degrees (c) 30 degrees (d) 45 degrees
73. Ans. (c) As 'B' hinge frictionless no torque is applied to bar CD so no angle change occurred.

74. An elevator weighing 10,000 kgf attains an upward velocity of 4 m/s in two seconds with uniform acceleration. The tension in the cable will be approximately. **[IAS 1994]**
(a) 8,000 kgf (b) 10,000 kgf (c) 12,000 kgf (d) 20,000 kgf

74. Ans. (a) Elevator acceleration, a = $\frac{v-4}{t}=\frac{4}{2}=2\,m/s^2$

$\therefore$ Tension cable = W – ma = 10000 - $\frac{10000}{9.81}\times 2 \approx 8000\,kgf$

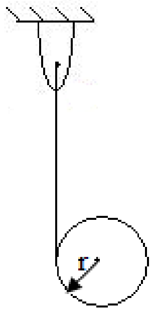
75. A cord is wrapped around a cylinder of radius 'r' and mass 'm' as shown in the given figure. If the cylinder is released from rest, the velocity of the cylinder, after it has moved through a distance 'h' will be

(a) $\sqrt{2gh}$

(b) $\sqrt{gh}$

(c) $\sqrt{4gh/3}$

(d) $\sqrt{gh/3}$



[IAS 1994]

75. Ans. (c)

76. A person, carrying on his head a jewellery box of weight 'W' jumped down from the third storey of a building. Before touching the ground, he would feel a load of magnitude

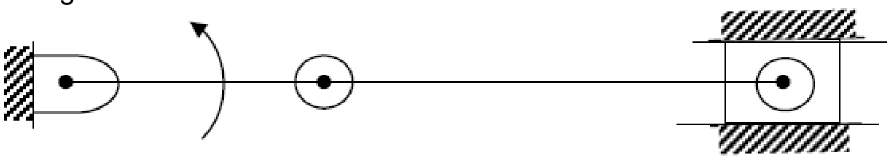
(a) zero (b) $\frac{W}{2}$ (c) W (d) infinity **[IAS-1999]**

76. Ans. (a) Weight of a freely falling body is

Motion with friction and drag

Mechanical Advantage

77. For a mechanism shown below, the mechanical advantage for the given configuration is



[GATE-2004]

(a) 0 (b) 0.5 (c) 1.0 (d) ∞

77. Ans. (a)

78. For a four-bar linkage in toggle position, the value of mechanical advantage is

(a) 0.0 (b) 0.5 (c) 1.0 (d) ∞ **[GATE-2006]**

78. Ans. (d)

$$\text{Mechanical advantage} = \frac{\text{Load (output force)}}{\text{Effort (input force)}}$$

For a four bar linkage in toggle position, effort = 0

$\therefore$ Mechanical Advantage $= \infty$

Momentum

Work and Energy

79. What is the work done if a bucket of water weighing 10 N is pulled up from a well 2 0 m deep by a rope weighing 1 N/m? **[IAS-2004]**

(a) 200 N-m (b) 400 N-m (c) 500 N-m (d) 600 N-m

79. Ans. (b) Total work = work to pulled up water+ work to pull up rope

$$= 10\times 20+1\times 20\times\left(\frac{20}{2}\right) \left[\because change\,of\;C.G=\frac{20}{2}=10\,m\right]$$

$$= 400 \text{ Nm}$$

80. A man is pulling a bucket of water up to the roof of a building of 6 m height. The total weight of the rope is 20 N and the weight of the bucket with water is 100 N. The work done by the man is

(a) 720 N-m (b) 420 N-m (c) 660 N-m (d) 600 N-m **[IAS-2000]**

80. Ans. (c) C.G. of roof move up only 3 m & bucket up by 6m.

Therefore W = 20 x 3 +100 x 6 = 660 Nm

81. A man is drawing water from a well with the help of a bucket which leaks uniformly. The bucket weighs 200 N when full while 100 N of water leaks out by the time is arrives at the top. Water is available in the well at a depth of 10m. The work done by the man in drawing the water is **[IAS-1997]**

(a) 1000 J (b) 1500 J (c) 2000 J (d) 3000 J

81. Ans. (b)

$$\text{Work} = 100\times 10+\text{leakage}\times\text{average height of travel}$$

$$=1000+(200-100)\times\left(\frac{10}{2}\right)=1500\text{J}$$

82. A circular disc rolls down without slip on an inclined plane. The ratio of its rotational kinetic energy to the total energy is **[IAS-2003]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

82. Ans. (c)

$$\text{Rotational kinetic energy} = \frac{Mr^2\omega^2}{4}$$

$$\text{Total kinetic energy} = \frac{1}{2}mv^2+\frac{1}{2}I\omega^2 = \frac{1}{2}\left[mr^2\omega^2+\frac{mr^2\omega^2}{2}\right]$$

$$= \frac{1}{2}\cdot\frac{mr^2\times\omega^2\times 3}{2}$$

$$\therefore \quad \text{Ratio of these two} = \frac{\frac{1}{4}mr^2\omega^2}{\frac{1}{4}\cdot 3mr^2\omega^2} = \frac{1}{3}$$

83. If the momentum of a given particle is doubled, then its kinetic energy will be

(a) doubled (b) quadrupled (c) halved (d) unaffected **[IAS-1996]**

83. Ans. (b) Momentum = mv

If momentum is doubled v_2 must be 2v

$$\therefore \text{Kinetic energy}\,(E_2)=\frac{1}{2}mv_2^2+\frac{1}{2}m\times(2v)^2 = 4\times\frac{1}{2}mv^2$$

84. The power required by a machine having an efficiency of 80% for raising a load of 24 N through a distance of 36m in 1 minute is **[IAS-1998]**

(a) 12 W (b) 18 W (c) 50 W (d) 450 W

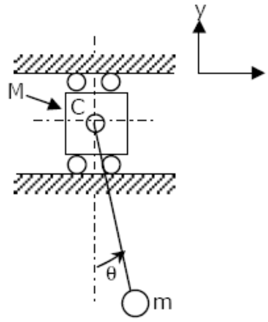
84. Ans. (b) Ideal power required $(P_i) = \dfrac{24 \times 36}{60} = 14.4\,W$

Actual power required $(P) = \dfrac{P_i}{\eta} = \dfrac{14.4}{0.8} = 18W$

Collisions

85. A mechanical system can be said to be conservative if the
(a) potential energy of the -system remains constant
(b) kinetic energy of the system remains constant **[IAS-1998]**
(c) sum of the kinetic and potential energies remains constant
(d) linear momentum remains constant
85. Ans. (c) conservative system is one that has total energy is constant.

86. Mass M slides in a frictionless slot in the horizontal direction and the bob of mass m is hinged to mass M at C, through a rigid mass less rod. This system is released from rest with θ = 30°. At the instant when θ = 0, the velocities of m and M can be determined using the fact that, for the system (i.e., m and M together),



[GATE-2001]

(a) the linear momentum in x and y directions are conserved but the energy is not conserved
(b) the linear momentum in x and y directions are conserved and the energy is also conserved
(c) the linear momentum in x direction is conserved and the energy is also conserved
(d) the linear momentum in y direction is conserved and the energy is also conserved
86. Ans. (c)

87. A ball falls from a height of 1 m, hits the ground and rebounds with half its velocity just before impact. Then after rising it falls and hits the ground and again rebounds with half its velocity just before impact, and so on. The total distance travelled by the ball till it comes to rest on the ground is **[GATE-1996]**

(a) 2 m (b) 5/3 m (c) 5/3 m (d) 5/4 m
87. Ans. (b)

Explanation. Total distance travelled $= h + \frac{e^2 \times 2gh}{2g} \times 2 + e^4 \times h \times 2 + ...$

$= h\,[1 + 2e^2 + 2e^4 + ...]$

$= h + 2e^2 h \left[1 + \frac{1}{4} + \frac{1}{16} + \dots\right]$

$= h\left[1 + 2e^2\left(\frac{1}{1-\frac{1}{4}}\right)\right] = 1\left[1 + 2\times\left(\frac{1}{2}\right)^2 \frac{4}{3}\right] = \frac{5}{3}$

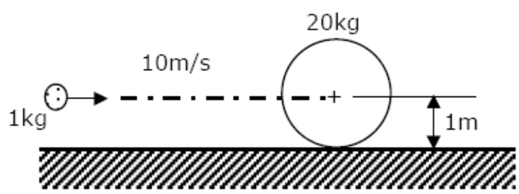
88. A hammer of mass 600 kg falls on to the top of a pile of mass 150 kg. What is the ratio of kinetic energy before this impact and kinetic energy after impact?
(a) 1·25 (b) 1·00 (c) 0·80 (d) 0 **[IAS-2007]**
88. Ans. (a) Momentum conservation gives us MV = (M+m)v Or $600\times V=(600+150).v$
Or V = 1.25 v

$$\frac{\text{Kinetic energy before impact}\left(\frac{1}{2}MV^2\right)}{\text{Kinetic energy after impact}\ \frac{1}{2}(M+m)V^2} = \frac{600\times V^2}{750\times v^2} = 1.25$$

89. A bullet is fired vertically upwards from a rifle, with a velocity of 110 m/s, from the top of a 115 m high tower. If g = 10 m/s^2, the velocity with which the bullet will strike the ground is **[IAS-1996]**
(a) 220 m/s (b) 175 m/s (c) 120 m/s (d) 115 m/s
89. Ans. (c)
By energy balance
Potential energy + kinetic energy at the top or tower = only kinetic energy at bottom of the tower.

Or $mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$ or $v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{110^2 + 2\times 10\times 115} = 120\,m/s$

90. A 1 kg mass of clay, moving with a velocity of 10 m/s, strikes a stationary wheel and sticks to it. The solid wheel has a mass of 20 kg and a radius of 1m. Assuming that the wheel and the ground are both rigid and that the wheel is set into pure rolling motion, the angular velocity of the wheel immediately after the impact is approximately



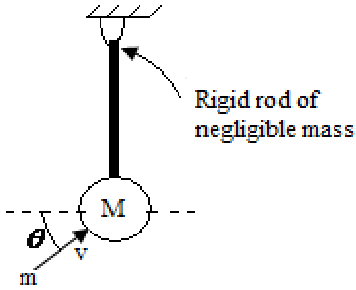
[GATE-2005]

(a) zero (b) $\frac{1}{3}$ rad/s (c) $\sqrt{\frac{10}{3}}$ rad/s (d) $\frac{10}{3}$ rad/s

90. Ans. (b)

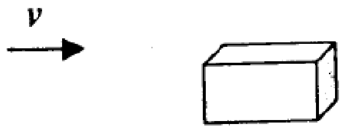
91. As shown in the given figure, a bullet of mass m and initial velocity v hits M and gets embedded into M.
Assertion (A): Just before and after collision, the total linear momentum of m and M together is conserved only in the horizontal direction and not in the vertical direction.

Reason (R): The total kinetic of m and M together is not conserved.



91. Ans. (b) **[IAS 1994]**

92. A bullet of mass "m" travels at a very high velocity v (as shown in the figure) and gets embedded inside the block of mass "M" initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance "s" along the floor. Assuming μ to be the coefficient of kinetic friction between the block and the floor and "g" the acceleration due to gravity what is the velocity v of the bullet? **[GATE-2003]**



$(a)\dfrac{M+m}{m}\sqrt{2\mu gs}$ $(b)\dfrac{M-m}{m}\sqrt{2\mu gs}$ $(c)\dfrac{\mu(M+m)}{m}\sqrt{2gs}$ $(d)\dfrac{M}{m}\sqrt{2\mu gs}$

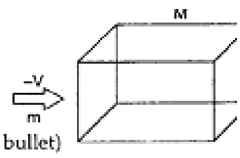
92. Ans. (a)

Let, m = mass of the bullet

v = velocity of bullet

M = mass of the block

u = final velocity of system (block + bullet)



Form conservation of linear momentum

$$mv = (M+m)\,u$$

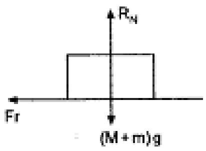
$$\Rightarrow \quad u = \frac{mv}{m+M} \quad \ldots$$

For horizontal equilibrium of body

$$(M+m)\,g = N$$

$$F_r = \text{Frictional force} = \mu N = u(M+m)\,g$$

$$\text{Frictional retardation} = \frac{\mu\,(M+m)g}{(M+m)} = -\mu g$$



(–ve sign shows that acceleration is opposite to motion)

Let v_f = final velocity of system = 0

$$v_f^2 = u^2 + 2as$$

$$u^2 + 2as = 0$$

$$u^2 - 2\mu gs = 0$$

From equation (*i*)

$$\left(\frac{mv}{M+m}\right)^2 = 2\,\mu gs$$

$$\frac{mv}{M+m} = \sqrt{2\,\mu gs}$$

$$\Rightarrow \quad v = \frac{M+m}{m}\sqrt{2\mu gs}$$

93. A bullet enters a plank of 30 mm thickness with a velocity of 100 m/s and emerges out from the plank with a velocity of 50 m/s. What is the minimum thickness of the plank so that the bullet remains embedded in the plank? **[IAS-2004]**
(a) 100 mm (b) 80 mm (c) 60 mm (d)40 mm
93. Ans. (d) Kinetic energy=Frictional Energy loss

$$\frac{1}{2}m\left(100^2-50^2\right)=R\times 0.03---(i)$$

$$and\ \frac{1}{2}m\left(100^2-o^2\right)=R\times x---(ii)$$

$$\therefore (ii)\div(i)\ \ gives$$

$$\frac{100^2}{100^2-50^2}=\frac{Rx}{R\times 0.03}\ \ or\ \ x=0.04m=40mm$$

94. Two masses, 2 kg and 8 kg, are moving with equal kinetic energy. The ratio of magnitudes of their momentum is **[IAS-2001]**
(a) 0.25 (b) 0.50 (c) 0.625 (d) 1.00
94. Ans. (b)

$$\frac{1}{2}mV^2=\frac{1}{2}Mv^2\ \ or\ 2\times V^2=8v^2\ \ or\frac{V}{u}=2$$

$$\text{Therefore}\frac{mV}{Mv}=\frac{2}{8}\times 2=0.5$$

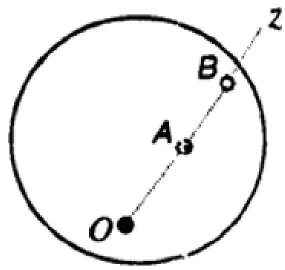
95. A bullet of mass 1 kg if fired with a velocity of u m/s from a gun of mass 10 kg. The ratio of kinetic energies of bullet and gun is **[IAS-2000]**
(a) 10 (b) 11 (c) 1.1 (d) 0.1
95. Ans. (a)

$$mu=MV\ \ or V=\frac{m}{M}u=\frac{u}{10}$$

$$ratio=\frac{\frac{1}{2}mu^2}{\frac{1}{2}MV^2}=\frac{m}{M}\times(10)^2=10$$

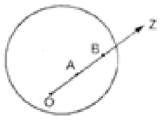
Data for Q. 96 - 97 are given below. Solve the problems and choose correct answers.

The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω. Two other points A and B are located on the line OZ at distances r_A and r_B from O respectively.



96. The velocity of point B with respect to point A is a vector of magnitude **[GATE-2003]**
(a) 0
(b) ω (r_B - r_A) and direction opposite to the direction of motion of point B

(c) ω (r_B - r_A) and direction same as the direction of motion of point B
(d) ω (r_B - r_A) and direction being from O to Z.
96. Ans. (c)



B has the linear velocity $\omega_B r_B$
A has the linear velocity $\omega_A r_A$

Obviously, $\omega_B r_B > \omega_A r_A$

because $\omega_B = \omega_B = \omega$ and $r_B > r_A$

$\therefore$ $\omega r_B > \omega r_A$

So relative velocity $= \omega\,(r_B - r_A)$ in the direction of motion of point B
and, it will experience a centripetal acceleration of $\omega^2\,(r_B - r_A)$ towards "O"

97. The acceleration of point B with respect to point A is a vector of magnitude
(a) 0 **[GATE-2003]**
(b) $\omega(r_B$ - $r_A)$ and direction same as the direction of motion of point B
(c) $\omega^2(r_B$ - $r_A)$ and direction opposite to be direction of motion of point B
(d) $\omega^2(r_B$ - $r_A)$ and direction being from Z to O.
97. Ans. (d)

Accleration of piont B with respect to point A

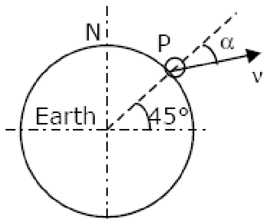
$$
\begin{aligned}
a_{B/A} &= \bar{\omega} \times \bar{v}_{B/A} \\
&= \omega . \omega\,(r_B - r_A) \\
&= \omega^2\,(r_B - r_A)
\end{aligned}
$$

and direction from z to O.

Rotational dynamics I: Angular momentum

Rotational dynamics II: Rotation about a fixed axis

98. A particle P is projected from the earth surface at latitude 45° with escape velocity v = 11.19 km/s. The velocity direction makes an angle α with the local vertical. The particle will escape the earth's gravitational field



[GATE-2001]

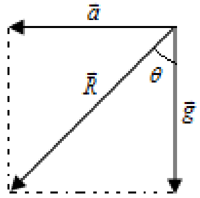
(a) only when α = 0 (b) only when α = 45°
(c) only when α = 90° (d) irrespective of the value of α
98. Ans. (d) Any body on earth has –ive potential energy=-mgR with respect to Universe. If we give same amount of energy by means that mass will escape from the earth. So

$$\frac{1}{2}m\upsilon^2 = mgR \quad \text{or } \upsilon = \sqrt{2gR} \text{ irrespective of the value of } \alpha$$

99. A stone of mass m at the end of a string of length 'l' is whirled in a vertical circle at a constant speed. The tension in the string will be maximum when the stone is
(a) at the top of the circle (b) half-way down from the top **[GATE-1994]**
(c) quarter-was down from the top (d) at the bottom of the circle
99. Ans. (d)

100. A simple pendulum of length 'l' hangs from the roof of a train moving with uniform acceleration a, then in the equilibrium position the pendulum **[IAS-2001]**
(a) leans back from the vertical by an angle $\tan^{-1}(g/a)$
(b) leans back from the vertical by an angle $\tan^{-1}(a/g)$
(c) moves forward from the vertical by an angle $\tan^{-1}(a/g)$
(d) moves forward from the vertical by an angle $\tan^{-1}(g/a)$
100. Ans. (b)

$$\tan\theta = \frac{a}{g} \quad or\ \theta = \tan^{-1}\left(\frac{a}{g}\right)$$



101. Moment of inertia of a solid cone about an axis passing through its centre of gravity and parallel to base (M is the mass of the cone, r is the radius of the base and h is the attitude of the cone) is **[IAS-2001]**
(a) $\frac{3M}{20}\left(r^2+\frac{h^2}{4}\right)$ (b) $\frac{3M}{10}\left(r^2+\frac{h^2}{4}\right)$ (c) $\frac{10}{3}Mr^2$ (d) $\frac{3}{5}Mr^2$
101. Ans. (a)

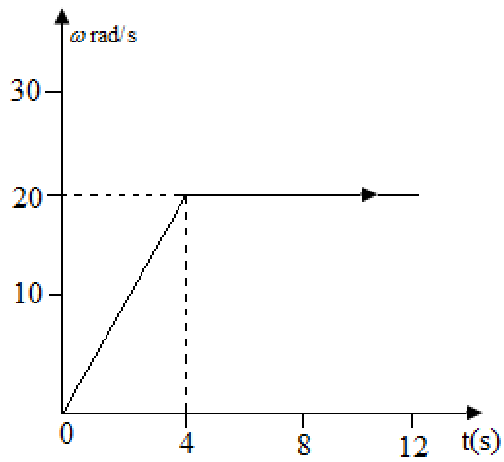
Rigid body dynamics III: Rotation and Translation

Rotational dynamics IV: Angular velocity and angular momentum

Rotational dynamics V: Kinetic energy, angular momentum and torque in 3-D

102. The escape velocity from the surface of the earth is approximately equal to
(a) 9.81 km/s (b) 11.2 km/s (c) 14.0 km/s (d) 22.0 km/s **[IAS-1996]**
102. Ans. (b)

103. The figure given above shows the angular velocity variation of a rotating disc, having a mass of 1 kg and radius 1 m. What is the value of the torque required so that the disc reaches its final speed from rest?
(a) 10 Nm
(b) 5 Nm
(c) 8 Nm
(d) 2 Nm



[IAS-2007]

103. Ans. (b) $\text{Torque(T)} = I.\alpha = mk^2\alpha = mk^2\dfrac{(\omega_2 - \omega_1)}{t} = 1\times 1^2 \times \left(\dfrac{20-0}{4}\right) = 5\,Nm$

104. The earth can be assumed as a uniform sphere. Suppose the earth shrinks by 1% in diameter, the new day period **[GATE-1998]**

(a) will not change from 24 hrs.
(b) will reduce by about 2%
(c) will reduce by about 1%
(d) will increase by about 1%

104. Ans. (d)

Explanation. Kinetic energy of earth, $E = \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

$$= \frac{1}{2}\left[\frac{2}{5}M\left(\frac{D}{2}\right)^2\right]\omega^2$$

$$E = \frac{MD^2\omega^2}{20}$$

$$\therefore \quad dE = \frac{M}{20}[\omega^2 .2D\, d(D) + D^2 2\omega .d(\omega)]$$

$$\frac{dE}{E} = \frac{2d(D)}{D} + \frac{2d(\omega)}{\omega}$$

As there is no change in energy (conservation of energy)

$$\frac{dD}{D} = \frac{d\omega}{\omega}$$

When diameter shrinks by 1% speed will increase by 1%.

105. A solid cylinder of mass m and radius r starts rolling from rest along an inclined plane. If it rolls without slipping from a vertical height h, the velocity of its centre of mass when it reaches the bottom is..... **[GATE-1994]**

105. Ans. $\dfrac{2}{3}\sqrt{2gh}$

106. Assertion (A): The density of an object on the moon is about one-sixth of that on the earth.

Reason (R): The gravitational acceleration on moon is about one-sixth of that on the earth. **[IAS-2002]**

106. Ans. (d) A is false. Mass will never change, $\text{density} = \dfrac{\text{mass}}{\text{volume}}$ as mass and volume both are constant. Density does not change but weight will be one-sixth due to gravitational acceleration.

107. Two pieces of steel and brass weighing 20 N and 10 N respectively fall freely under the action of gravity from a tower. For the two pieces which one of the following will be equal after falling an equal distance? **[IAS-2001]**

(a) Acceleration (b) Momentum (c) Potential energy (d) Kinetic energy

107. Ans. (a)

Harmonic oscillator I: Introduction

Harmonic oscillator II: damped oscillator

Harmonic oscillator III: Forced oscillations

Projectile motion

108. For a given velocity of the projectile, the range will be the maximum when the angle of projection with the horizontal is **[IAS-1997]**
(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

108. Ans. (d) $\text{Range}, R = \frac{V^2 \sin 2\theta}{g}$ it will be maximum when $\sin 2\theta = 1$ or $\theta = \frac{\pi}{4}$

109. A ball is thrown vertically upwards in air which offers resistance to motion. The ball takes t_1 seconds while going up a t_2 seconds while coming down.
Assertion (A): t_2 is more than t_1. **[IAS-1997]**
Reason (R): The total energy of the ball continually decreases. Hence average speed of the ball while coming down is less than that while going up.
109. Ans. (a)

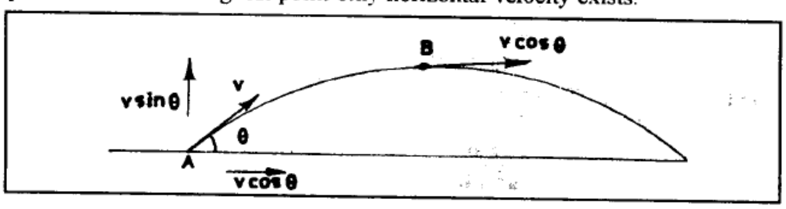
110. A stone is projected upwards with a certain velocity from the ground. It takes 'T' seconds to reach the maximum height. There is no air resistance. The time taken by the stone to reach the ground from its maximum height is **[IAS-1997]**
(a) 2 T (b) 1.5 T (c) 1.25 T (d) T
110. Ans. (d)

111. A ball is projected vertically upward with a certain velocity. It takes 40 seconds for its upward journey. The time taken for its downward journey is **[IAS-1995]**
(a) 10 s (b) 20 s (c) 30 s (d) 40 s.
111. Ans. (d) Time in upward journey is same as in downward journey.

112. A shell is fired from a cannon with a speed v at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon. The speed of other piece immediately after explosion is **[GATE-1994]**

(a) $3v\cos\theta$ (b) $2v\cos\theta$ (c) $\frac{3}{2}v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

112. Ans. (a)
Explanation. At the highest point only horizontal velocity exists.



Momentum at 'B' before explosion = $mv \cos\theta$.

After explosion half the mass traces back the path and hits the canon. Hence its velocity should be same as $v \cos\theta$ but in opposite direction.

$$\therefore \text{ Momentum after explosion } = \left(\frac{m}{2}\right)(-v\cos\theta) + \frac{m}{2} \times V$$

where 'V' = velocity of the second piece.

Equating momentum before impact and after impact, we get

$$mv\cos\theta = \frac{m}{2}(-v\cos\theta) + \frac{m}{2} \times V$$

or

$$V = 3\,v\cos\theta$$

113. Two halls of mass m and 2 m are projected with identical velocities from the same point making angles 30° and 60° with the vertical axis, respectively. The heights attained by the halls will be identical **[GATE-1994]**

113. Ans. False

Height attained $= \dfrac{u^2}{2g}\cos^2\alpha, \quad \cos^2 30° \neq \cos^2 60°$