

# CHAPTER 4

## THEORY OF MACHINES

YEAR 2012

ONE MARK

MCQ 4.1

The following are the data for two crossed helical gears used for speed reduction :

**Gear I** : Pitch circle diameter in the plane of rotation 80 mm and helix angle  $30^\circ$ .

**Gear II** : Pitch circle diameter in the plane of rotation 120 mm and helix angle  $22.5^\circ$ .

If the input speed is 1440 rpm, the output speed in rpm is

- (A) 1200 (B) 900  
(C) 875 (D) 720

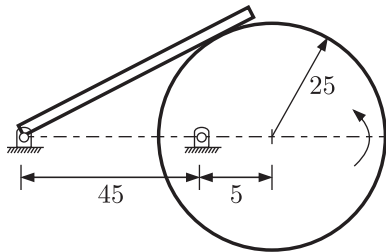
MCQ 4.2

A solid disc of radius  $r$  rolls without slipping on a horizontal floor with angular velocity  $\omega$  and angular acceleration  $\alpha$ . The magnitude of the acceleration of the point of contact on the disc is

- (A) zero (B)  $r\alpha$   
(C)  $\sqrt{(r\alpha)^2 + (r\omega^2)^2}$  (D)  $r\omega^2$

MCQ 4.3

In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (Note. All dimensions are in mm).



- (A) 0.05 (B) 0.1  
(C) 5.0 (D) 10.0

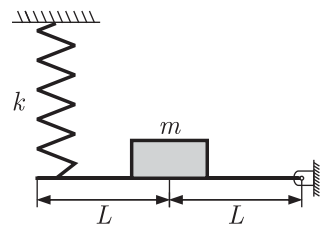
MCQ 4.4

A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in Joules is

- (A) 395  
(B) 790  
(C) 1580  
(D) 3160

**YEAR 2012****TWO MARKS****MCQ 4.5**

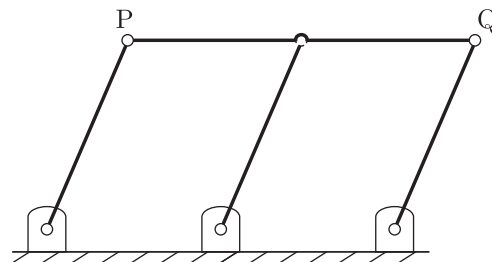
A concentrated mass  $m$  is attached at the centre of a rod of length  $2L$  as shown in the figure. The rod is kept in a horizontal equilibrium position by a spring of stiffness  $k$ . For very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is



- (A)  $\sqrt{\frac{k}{m}}$   
(B)  $\sqrt{\frac{2k}{m}}$   
(C)  $\sqrt{\frac{k}{2m}}$   
(D)  $\sqrt{\frac{4k}{m}}$

**YEAR 2011****ONE MARK****MCQ 4.6**

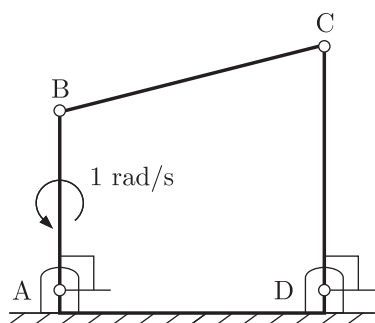
A double-parallelogram mechanism is shown in the figure. Note that PQ is a single link. The mobility of the mechanism is



- (A) -1  
(B) 0  
(C) 1  
(D) 2

**YEAR 2011****TWO MARKS****MCQ 4.7**

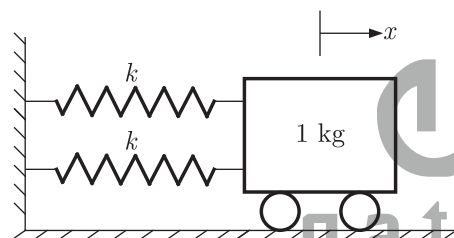
For the four-bar linkage shown in the figure, the angular velocity of link AB is 1 rad/s. The length of link CD is 1.5 times the length of link AB. In the configuration shown, the angular velocity of link CD in rad/s is



- (A) 3 (B)  $\frac{3}{2}$   
(C) 1 (D)  $\frac{2}{3}$

**MCQ 4.8**

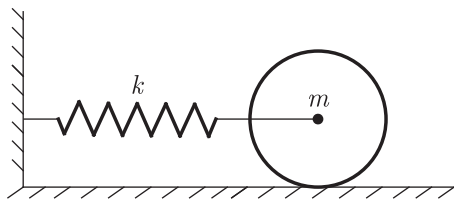
A mass of 1 kg is attached to two identical springs each with stiffness  $k = 20 \text{ kN/m}$  as shown in the figure. Under the frictionless conditions, the natural frequency of the system in Hz is close to



- (A) 32 (B) 23  
(C) 16 (D) 11

**MCQ 4.9**

A disc of mass  $m$  is attached to a spring of stiffness  $k$  as shown in the figure. The disc rolls without slipping on a horizontal surface. The natural frequency of vibration of the system is



- (A)  $\frac{1}{2\pi} \sqrt{\frac{k}{m}}$  (B)  $\frac{1}{2\pi} \sqrt{\frac{2k}{m}}$   
(C)  $\frac{1}{2\pi} \sqrt{\frac{2k}{3m}}$  (D)  $\frac{1}{2\pi} \sqrt{\frac{3k}{2m}}$

**YEAR 2010****ONE MARK****MCQ 4.10**

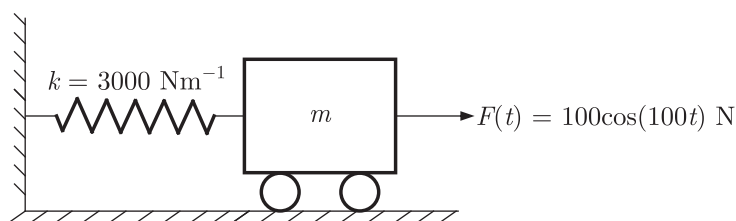
Mobility of a statically indeterminate structure is

- (A)  $\leq -1$  (B) 0  
(C) 1 (D)  $\geq 2$

- MCQ 4.11** There are two points P and Q on a planar rigid body. The relative velocity between the two points  
(A) should always be along PQ  
(B) can be oriented along any direction  
(C) should always be perpendicular to PQ  
(D) should be along QP when the body undergoes pure translation
- MCQ 4.12** Which of the following statements is INCORRECT ?  
(A) Grashof's rule states that for a planar crank-rocker four bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths  
(B) Inversions of a mechanism are created by fixing different links one at a time  
(C) Geneva mechanism is an intermittent motion device  
(D) Gruebler's criterion assumes mobility of a planar mechanism to be one
- MCQ 4.13** The natural frequency of a spring-mass system on earth is  $\omega_n$ . The natural frequency of this system on the moon ( $g_{\text{moon}} = g_{\text{earth}}/6$ ) is  
(A)  $\omega_n$   
(B)  $0.408\omega_n$   
(C)  $0.204\omega_n$   
(D)  $0.167\omega_n$
- MCQ 4.14** Tooth interference in an external involute spur gear pair can be reduced by  
(A) decreasing center distance between gear pair  
(B) decreasing module  
(C) decreasing pressure angle  
(D) increasing number of gear teeth

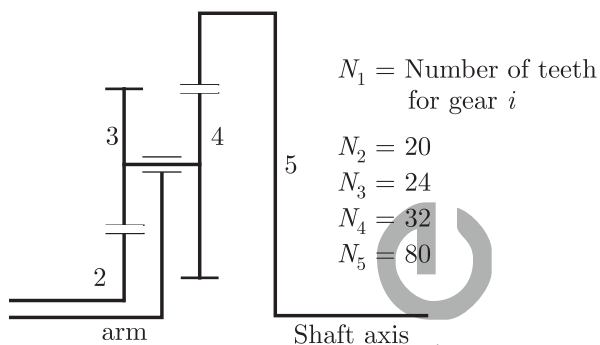
**YEAR 2010****TWO MARKS**

- MCQ 4.15** A mass  $m$  attached to a spring is subjected to a harmonic force as shown in figure. The amplitude of the forced motion is observed to be 50 mm. The value of  $m$  (in kg) is



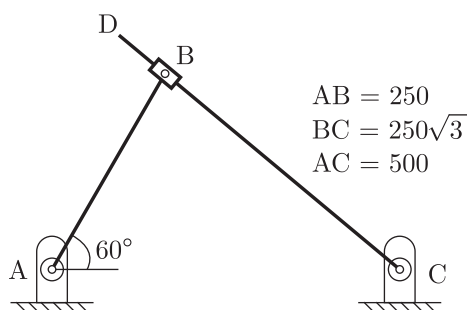
- (A) 0.1 (B) 1.0  
(C) 0.3 (D) 0.5

**MCQ 4.16** For the epicyclic gear arrangement shown in the figure  $\omega_2 = 100$  rad/s clockwise (CW) and  $\omega_{arm} = 80$  rad/s counter clockwise (CCW). The angular velocity  $\omega_5$  (in rad/s) is



- (A) 0 (B) 70 CW  
(C) 140 CCW (D) 140 CW

**MCQ 4.17** For the configuration shown, the angular velocity of link AB is 10 rad/s counterclockwise. The magnitude of the relative sliding velocity (in  $\text{ms}^{-1}$ ) of slider B with respect to rigid link CD is



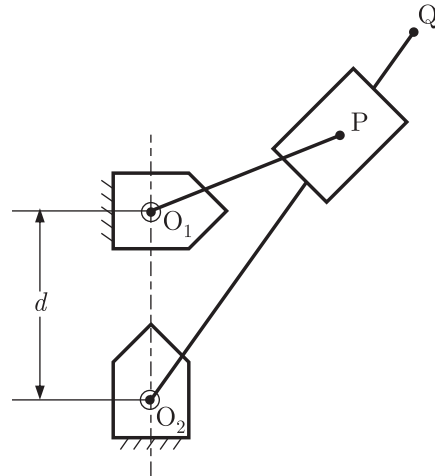
- (A) 0 (B) 0.86  
(C) 1.25 (D) 2.50

**YEAR 2009**

**ONE MARK**

**MCQ 4.18** A simple quick return mechanism is shown in the figure. The forward to

return ratio of the quick return mechanism is 2:1. If the radius of crank  $O_1P$  is 125 mm, then the distance ' $d$ ' (in mm) between the crank centre to lever pivot centre point should be



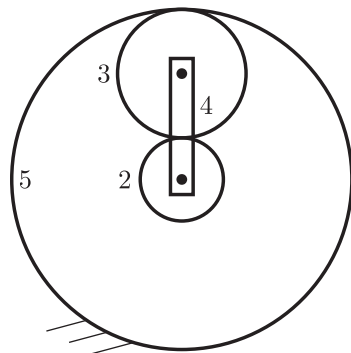
- (A) 144.3 (B) 216.5  
(C) 240.0 (D) 250.0

- MCQ 4.19** The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is
- (A) 350 (B) 705  
(C) 2810 (D) 4430

**YEAR 2009**

**TWO MARKS**

- MCQ 4.20** An epicyclic gear train is shown schematically in the given figure. The run gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm CCW (CCW=counter-clockwise and CW=clockwise).



The arm 4 attached to the output shaft will rotate at

- (A) 10 rpm CCW (B) 10 rpm CW  
(C) 12 rpm CW (D) 12 rpm CCW

**MCQ 4.21** An automotive engine weighing 240 kg is supported on four springs with linear characteristics. Each of the front two springs have a stiffness of 16 MN/m while the stiffness of each rear spring is 32 MN/m. The engine speed (in rpm), at which resonance is likely to occur, is

- (A) 6040 (B) 3020  
(C) 1424 (D) 955

**MCQ 4.22** A vehicle suspension system consists of a spring and a damper. The stiffness of the spring is 3.6 kN/m and the damping constant of the damper is 400 Ns/m. If the mass is 50 kg, then the damping factor ( $d$ ) and damped natural frequency ( $f_n$ ), respectively, are

- (A) 0.471 and 1.19 Hz (B) 0.471 and 7.48 Hz  
(C) 0.666 and 1.35 Hz (D) 0.666 and 8.50 Hz

**MCQ 4.23** Match the approaches given below to perform stated kinematics/dynamics analysis of machine.

Analysis	Approach
P. Continuous relative rotation	1. D' Alembert's principle
Q. Velocity and acceleration	2. Grubler's criterion
R. Mobility	3. Grashoff's law
S. Dynamic-static analysis	4. Kennedy's theorem
(A) P-1, Q-2, R-3, S-4	(B) P-3, Q-4, R-2, S-1
(C) P-2, Q-3, R-4, S-1	(D) P-4, Q-2, R-1, S-3

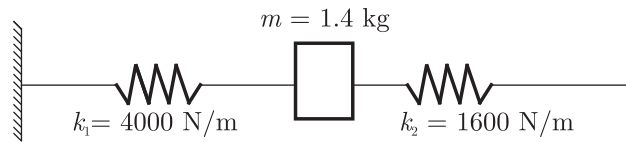
**YEAR 2008****ONE MARK**

**MCQ 4.24** A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism, using Gruebler's criterion, is

- (A) 0 (B) 1  
(C) 2 (D) 3

**YEAR 2008****TWO MARKS**

**MCQ 4.25** The natural frequency of the spring mass system shown in the figure is closest to



- (A) 8 Hz (B) 10 Hz  
(C) 12 Hz (D) 14 Hz

- MCQ 4.26** In a cam design, the rise motion is given by a simple harmonic motion (*SHM*)  $s = h/2(1 - \cos(\pi\theta/\beta))$  where  $h$  is total rise,  $\theta$  is camshaft angle,  $\beta$  is the total angle of the rise interval. The jerk is given by
- (A)  $\frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$  (B)  $\frac{\pi h}{\beta^2} \sin\left(\frac{\pi\theta}{\beta}\right)$   
(C)  $\frac{\pi^2 h}{\beta^2} \cos\left(\frac{\pi\theta}{\beta}\right)$  (D)  $-\frac{\pi^3 h}{\beta^3} \sin\left(\frac{\pi\theta}{\beta}\right)$

- MCQ 4.27** A uniform rigid rod of mass  $m = 1 \text{ kg}$  and length  $L = 1 \text{ m}$  is hinged at its centre and laterally supported at one end by a spring of spring constant  $k = 300 \text{ N/m}$ . The natural frequency  $\omega_n$  in rad/s is
- (A) 10 (B) 20  
(C) 30 (D) 40

**YEAR 2007****ONE MARK**

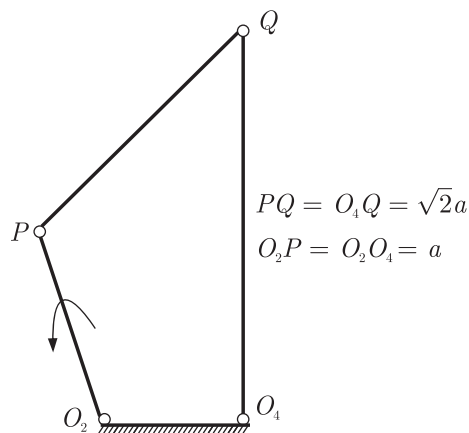
- MCQ 4.28** For an under damped harmonic oscillator, resonance
- (A) occurs when excitation frequency is greater than undamped natural frequency  
(B) occurs when excitation frequency is less than undamped natural frequency  
(C) occurs when excitation frequency is equal to undamped natural frequency  
(D) never occurs

**YEAR 2007****TWO MARKS**

- MCQ 4.29** The speed of an engine varies from 210 rad/s to 190 rad/s. During the cycle the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in  $\text{kg/m}^2$  is
- (A) 0.10 (B) 0.20  
(C) 0.30 (D) 0.40
- MCQ 4.30** The input link  $O_2P$  of a four bar linkage is rotated at 2 rad/s in counter clockwise direction as shown below. The angular velocity of the coupler PQ



in rad/s, at an instant when  $\angle O_4 O_2 P = 180^\circ$ , is



- (A) 4 (B)  $2\sqrt{2}$   
 (C) 1 (D)  $\frac{1}{\sqrt{2}}$

**MCQ 4.31** The natural frequency of the system shown below is



- (A)  $\sqrt{\frac{k}{2m}}$  (B)  $\sqrt{\frac{k}{m}}$   
 (C)  $\sqrt{\frac{2k}{m}}$  (D)  $\sqrt{\frac{3k}{m}}$

**MCQ 4.32** The equation of motion of a harmonic oscillator is given by

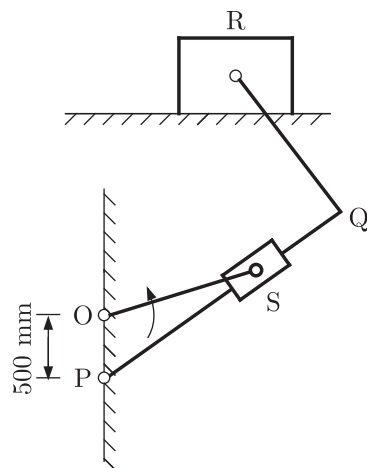
$$\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = 0$$

and the initial conditions at  $t = 0$  are  $x(0) = X, \frac{dx}{dt}(0) = 0$ . The amplitude of  $x(t)$  after  $n$  complete cycles is

- (A)  $Xe^{-2n\pi(\frac{\xi}{\sqrt{1-\xi^2}})}$  (B)  $Xe^{2n\pi(\frac{\xi}{\sqrt{1-\xi^2}})}$   
 (C)  $Xe^{-2n\pi(\frac{\sqrt{1-\xi^2}}{\xi})}$  (D)  $X$

### ● Common Data For Q.33 Q. 34

A quick return mechanism is shown below. The crank OS is driven at 2 rev/s in counter-clockwise direction.



- MCQ 4.33** If the quick return ratio is 1 : 2, then the length of the crank in mm is  
 (A) 250 (B)  $250\sqrt{3}$   
 (C) 500 (D)  $500\sqrt{3}$

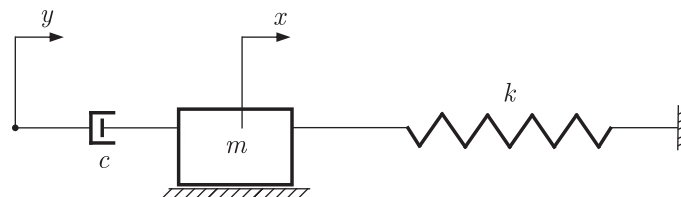
- MCQ 4.34** The angular speed of PQ in rev/s when the block R attains maximum speed during forward stroke (stroke with slower speed) is  
 (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$   
 (C) 2 (D) 3

YEAR 2006

ONE MARK

- MCQ 4.35** For a four-bar linkage in toggle position, the value of mechanical advantage is  
 (A) 0.0 (B) 0.5  
 (C) 1.0 (D)  $\infty$

- MCQ 4.36** The differential equation governing the vibrating system is



- (A)  $m\ddot{x} + c\dot{x} + k(x - y) = 0$   
 (B)  $m(\ddot{x} - \ddot{y}) + c(\dot{x} - \dot{y}) + kx = 0$   
 (C)  $m\ddot{x} + c(\dot{x} - \dot{y}) + kx = 0$   
 (D)  $m(\ddot{x} - \ddot{y}) + c(\dot{x} - \dot{y}) + k(x - y) = 0$

- MCQ 4.37** The number of inversion for a slider crank mechanism is  
 (A) 6 (B) 5  
 (C) 4 (D) 3

**YEAR 2006****TWO MARKS**

- MCQ 4.38** Match the item in columns I and II

Column I	Column II
P. Addendum	1. Cam
Q. Instantaneous centre of velocity	2. Beam
R. Section modulus	3. Linkage
S. Prime circle	4. Gear
(A) P-4, Q-2, R-3, S-1	(B) P-4, Q-3, R-2, S-1
(C) P-3, Q-2, R-1, S-4	(D) P-3, Q-4, R-1, S-2

- MCQ 4.39** If  $C_f$  is the coefficient of speed fluctuation of a flywheel then the ratio of  $\omega_{\max}/\omega_{\min}$  will be  
 (A)  $\frac{1-2C_f}{1+2C_f}$  (B)  $\frac{2-C_f}{2+C_f}$   
 (C)  $\frac{1+2C_f}{1-2C_f}$  (D)  $\frac{2+C_f}{2-C_f}$

- MCQ 4.40** Match the items in columns I and II

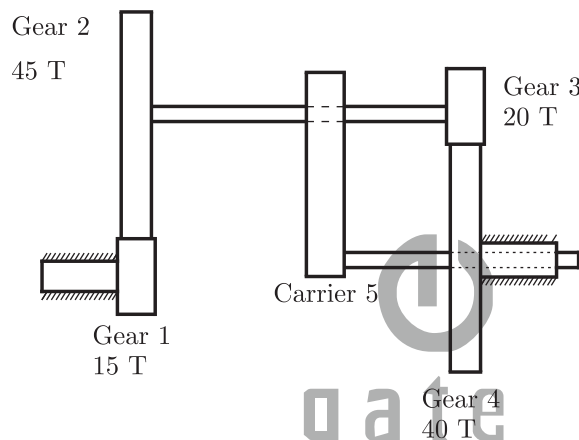
Column I	Column II
P. Higher Kinematic Pair	1. Grubler's Equation
Q. Lower Kinematic Pair	2. Line contact
R. Quick Return Mechanism	3. Euler's Equation
S. Mobility of a Linkage	4. Planar
	5. Shaper
	6. Surface contact
(A) P-2, Q-6, R-4, S-3	(B) P-6, Q-2, R-4, S-1
(C) P-6, Q-2, R-5, S-3	(D) P-2, Q-6, R-5, S-1

- MCQ 4.41** A machine of 250 kg mass is supported on springs of total stiffness 100 kN/m. Machine has an unbalanced rotating force of 350 N at speed of 3600 rpm. Assuming a damping factor of 0.15, the value of transmissibility ratio is  
 (A) 0.0531 (B) 0.9922  
 (C) 0.0162 (D) 0.0028

- MCQ 4.42** In a four-bar linkage,  $S$  denotes the shortest link length,  $L$  is the longest link length,  $P$  and  $Q$  are the lengths of other two links. At least one of the three moving links will rotate by  $360^\circ$  if
- (A)  $S + L \leq P + Q$  (B)  $S + L > P + Q$   
 (C)  $S + P \leq L + Q$  (D)  $S + P > L + Q$

● **Common Data For Q. 43 and Q. 44**

A planetary gear train has four gears and one carrier. Angular velocities of the gears are  $\omega_1, \omega_2, \omega_3$  and  $\omega_4$ , respectively. The carrier rotates with angular velocity  $\omega_5$ .



- MCQ 4.43** What is the relation between the angular velocities of Gear 1 and Gear 4 ?
- (A)  $\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$  (B)  $\frac{\omega_4 - \omega_5}{\omega_1 - \omega_5} = 6$   
 (C)  $\frac{\omega_1 - \omega_2}{\omega_4 - \omega_5} = -\left(\frac{2}{3}\right)$  (D)  $\frac{\omega_2 - \omega_5}{\omega_4 - \omega_5} = \frac{8}{9}$
- MCQ 4.44** For  $\omega_1 = 60$  rpm clockwise (CW) when looked from the left, what is the angular velocity of the carrier and its direction so that Gear 4 rotates in counterclockwise (CCW) direction at twice the angular velocity of Gear 1 when looked from the left ?
- (A) 130 rpm, CW (B) 223 rpm, CCW  
 (C) 256 rpm, CW (D) 156 rpm, CCW

● **Common Data For Q. 45 and Q. 46 :**

A vibratory system consists of a mass 12.5 kg, a spring of stiffness 1000 N/m, and a dash-pot with damping coefficient of 15 Ns/m.

- MCQ 4.45** The value of critical damping of the system is
- (A) 0.223 Ns/m (B) 17.88 Ns/m

(C) 71.4 Ns/m

(D) 223.6 Ns/m

**MCQ 4.46** The value of logarithmic decrement is

(A) 1.35

(B) 1.32

(C) 0.68

(D) 0.66

**YEAR 2005****ONE MARK****MCQ 4.47** The number of degrees of freedom of a planar linkage with 8 links and 9 simple revolute joints is

(A) 1

(B) 2

(C) 3

(D) 4

**MCQ 4.48** There are four samples P, Q, R and S, with natural frequencies 64, 96, 128 and 256 Hz, respectively. They are mounted on test setups for conducting vibration experiments. If a loud pure note of frequency 144 Hz is produced by some instrument, which of the samples will show the most perceptible induced vibration?

(A) P

(B) Q

(C) R

(D) S

**YEAR 2005****TWO MARKS****MCQ 4.49** In a cam-follower mechanism, the follower needs to rise through 20 mm during  $60^\circ$  of cam rotation, the first  $30^\circ$  with a constant acceleration and then with a deceleration of the same magnitude. The initial and final speeds of the follower are zero. The cam rotates at a uniform speed of 300 rpm. The maximum speed of the follower is

(A) 0.60 m/s

(B) 1.20 m/s

(C) 1.68 m/s

(D) 2.40 m/s

**MCQ 4.50** A rotating disc of 1 m diameter has two eccentric masses of 0.5 kg each at radii of 50 mm and 60 mm at angular positions of  $0^\circ$  and  $150^\circ$ , respectively. A balancing mass of 0.1 kg is to be used to balance the rotor. What is the radial position of the balancing mass ?

(A) 50 mm

(B) 120 mm

(C) 150 mm

(D) 280 mm

**MCQ 4.51** In a spring-mass system, the mass is 0.1 kg and the stiffness of the spring is 1 kN/m. By introducing a damper, the frequency of oscillation is found to be 90% of the original value. What is the damping coefficient of the damper ?

(A) 1.2 Ns/m

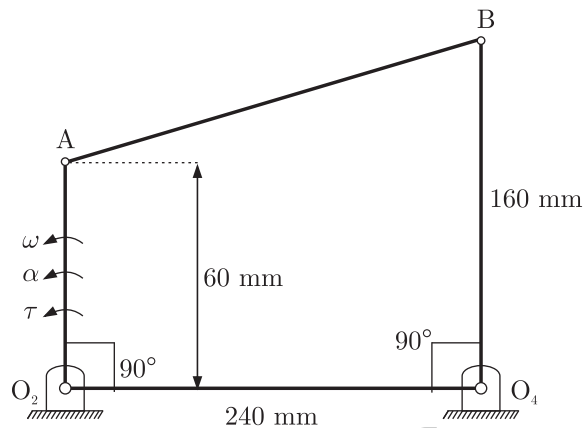
(B) 3.4 Ns/m

(C) 8.7 Ns/m

(D) 12.0 Ns/m

● Common Data For Q. 52, 53, and Q. 54

An instantaneous configuration of a four-bar mechanism, whose plane is horizontal is shown in the figure below. At this instant, the angular velocity and angular acceleration of link  $O_2A$  are  $\omega = 8 \text{ rad/s}$  and  $\alpha = 0$ , respectively, and the driving torque ( $\tau$ ) is zero. The link  $O_2A$  is balanced so that its centre of mass falls at  $O_2$ .

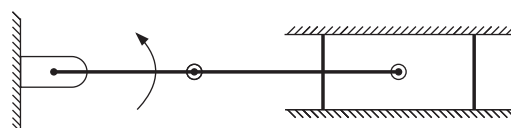


- MCQ 4.52** Which kind of 4-bar mechanism is  $O_2ABO_4$  ?  
 (A) Double-crank mechanism (B) Crank-rocker mechanism  
 (C) Double-rocker mechanism (D) Parallelogram mechanism
- MCQ 4.53** At the instant considered, what is the magnitude of the angular velocity of  $O_4B$ ?  
 (A) 1 rad/s (B) 3 rad/s  
 (C) 8 rad/s (D)  $\frac{64}{3}$  rad/s
- MCQ 4.54** At the same instant, if the component of the force at joint A along AB is 30 N, then the magnitude of the joint reaction at  $O_2$   
 (A) is zero (B) is 30 N  
 (C) is 78 N (D) cannot be determined from the given data

**YEAR 2004**

**ONE MARK**

- MCQ 4.55** For a mechanism shown below, the mechanical advantage for the given configuration is

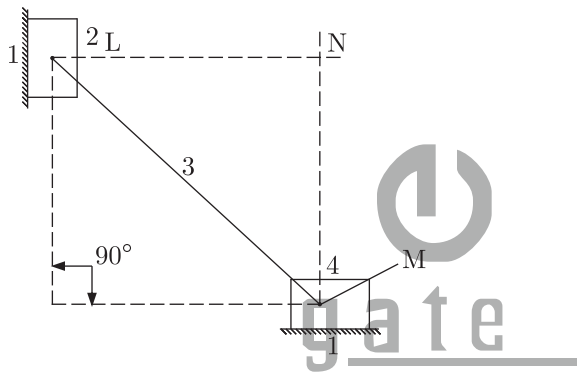


- (A) 0 (B) 0.5  
(C) 1.0 (D)  $\infty$

- MCQ 4.56** A vibrating machine is isolated from the floor using springs. If the ratio of excitation frequency of vibration of machine to the natural frequency of the isolation system is equal to 0.5, then transmissibility ratio of isolation is  
(A)  $1/2$  (B)  $3/4$   
(C)  $4/3$  (D) 2

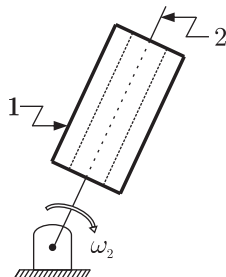
**YEAR 2004****TWO MARKS**

- MCQ 4.57** The figure below shows a planar mechanism with single degree of freedom. The instant centre 24 for the given configuration is located at a position



- (A) L (B) M  
(C) N (D)  $\infty$

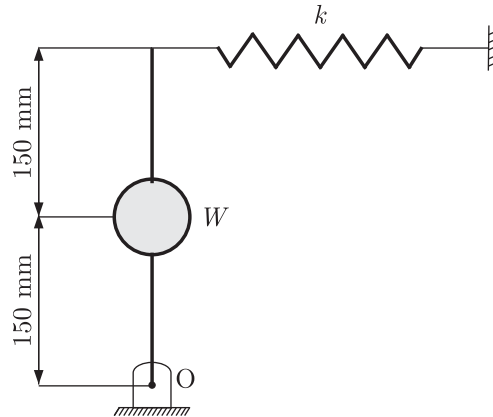
- MCQ 4.58** In the figure shown, the relative velocity of link 1 with respect to link 2 is 12 m/sec. Link 2 rotates at a constant speed of 120 rpm. The magnitude of Coriolis component of acceleration of link 1 is



- (A)  $302 \text{ m/s}^2$  (B)  $604 \text{ m/s}^2$   
(C)  $906 \text{ m/s}^2$  (D)  $1208 \text{ m/s}^2$

- MCQ 4.59** A uniform stiff rod of length 300 mm and having a weight of 300 N is pivoted at one end and connected to a spring at the other end. For keeping

the rod vertical in a stable position the minimum value of spring constant  $k$  needed is



- (A) 300 N/m (B) 400 N/m  
(C) 500 N/m (D) 1000 N/m

**MCQ 4.60** Match the following

Type of Mechanism	Motion achieved
P. Scott-Russel Mechanism	1. Intermittent Motion
Q. Geneva Mechanism	2. Quick return Motion
R. Off-set slider-crank Mechanism	3. Simple Harmonic Motion
S. Scotch Yoke Mechanism	4. Straight Line Motion
(A) P-2 Q-3 R-1 S-4	(B) P-3 Q-2 R-4 S-1
(C) P-4 Q-1 R-2 S-3	(D) P-4 Q-3 R-1 S-2

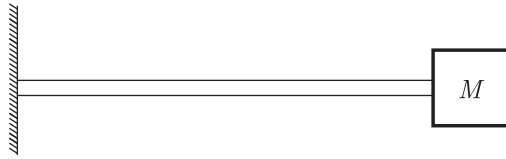
**MCQ 4.61** Match the following with respect to spatial mechanisms.

Types of Joint	Degree of constraints
P. Revolute	1. Three
Q. Cylindrical	2. Five
R. Spherical	3. Four
	4. Two
	5. Zero
(A) P-1 Q-3 R-3	(B) P-5 Q-4 R-3
(C) P-2 Q-3 R-1	(D) P-4 Q-5 R-3

**MCQ 4.62** A mass  $M$ , of 20 kg is attached to the free end of a steel cantilever beam of length 1000 mm having a cross-section of  $25 \times 25$  mm. Assume the mass of the cantilever to be negligible and  $E_{\text{steel}} = 200$  GPa. If the lateral vibration of this system is critically damped using a viscous damper, then damping



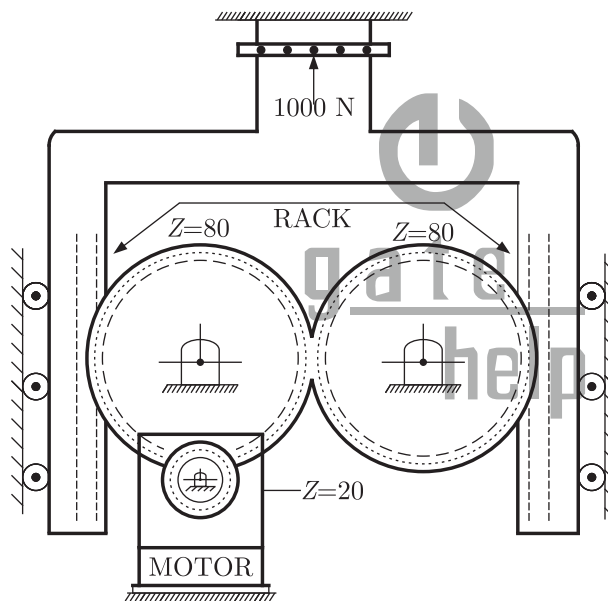
constant of the damper is



- (A) 1250 Ns/m  
(B) 625 Ns/m  
(C) 312.50 Ns/m  
(D) 156.25 Ns/m

● **Common Data For Q. 63 and Q. 64**

A compacting machine shown in the figure below is used to create a desired thrust force by using a rack and pinion arrangement. The input gear is mounted on the motor shaft. The gears have involute teeth of 2 mm module.



**MCQ 4.63** If the drive efficiency is 80%, the torque required on the input shaft to create 1000 N output thrust is

- (A) 20 Nm  
(B) 25 Nm  
(C) 32 Nm  
(D) 50 Nm

**MCQ 4.64** If the pressure angle of the rack is  $20^\circ$ , then force acting along the line of action between the rack and the gear teeth is

- (A) 250 N  
(B) 342 N  
(C) 532 N  
(D) 600 N

## YEAR 2003

## ONE MARK

- MCQ 4.65** The mechanism used in a shaping machine is  
 (A) a closed 4-bar chain having 4 revolute pairs  
 (B) a closed 6-bar chain having 6 revolute pairs  
 (C) a closed 4-bar chain having 2 revolute and 2 sliding pairs  
 (D) an inversion of the single slider-crank chain
- MCQ 4.66** The lengths of the links of a 4-bar linkage with revolute pairs are  $p, q, r$ , and  $s$  units. given that  $p < q < r < s$ . Which of these links should be the fixed one, for obtaining a “double crank” mechanism ?  
 (A) link of length  $p$  (B) link of length  $q$   
 (C) link of length  $r$  (D) link of length  $s$
- MCQ 4.67** When a cylinder is located in a Vee-block, the number of degrees of freedom which are arrested is  
 (A) 2 (B) 4  
 (C) 7 (D) 8

## YEAR 2003

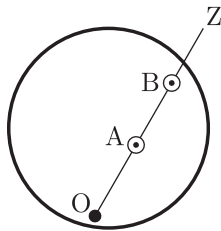
## TWO MARKS

- MCQ 4.68** For a certain engine having an average speed of 1200 rpm, a flywheel approximated as a solid disc, is required for keeping the fluctuation of speed within 2% about the average speed. The fluctuation of kinetic energy per cycle is found to be 2 kJ. What is the least possible mass of the flywheel if its diameter is not to exceed 1 m ?  
 (A) 40 kg (B) 51 kg  
 (C) 62 kg (D) 73 kg
- MCQ 4.69** A flexible rotor-shaft system comprises of a 10 kg rotor disc placed in the middle of a mass-less shaft of diameter 30 mm and length 500 mm between bearings (shaft is being taken mass-less as the equivalent mass of the shaft is included in the rotor mass) mounted at the ends. The bearings are assumed to simulate simply supported boundary conditions. The shaft is made of steel for which the value of  $E = 2.1 \times 10^{11}$  Pa. What is the critical speed of rotation of the shaft ?  
 (A) 60 Hz (B) 90 Hz  
 (C) 135 Hz (D) 180 Hz

● Common Data For Q. 70 and Q. 71 :

The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular

velocity  $\omega$ . Two other points A and B are located on the line OZ at distances  $r_A$  and  $r_B$  from O respectively.



- MCQ 4.70** The velocity of Point B with respect to point A is a vector of magnitude  
 (A) 0  
 (B)  $\omega(r_B - r_A)$  and direction opposite to the direction of motion of point B  
 (C)  $\omega(r_B - r_A)$  and direction same as the direction of motion of point B  
 (D)  $\omega(r_B - r_A)$  and direction being from O to Z
- MCQ 4.71** The acceleration of point B with respect to point A is a vector of magnitude  
 (A) 0  
 (B)  $\omega(r_B - r_A)$  and direction same as the direction of motion of point B  
 (C)  $\omega^2(r_B - r_A)$  and direction opposite to the direction of motion of point B  
 (D)  $\omega^2(r_B - r_A)$  and direction being from Z to O
- MCQ 4.72** The undamped natural frequency of oscillations of the bar about the hinge point is  
 (A) 42.43 rad/s  
 (B) 30 rad/s  
 (C) 17.32 rad/s  
 (D) 14.14 rad/s
- MCQ 4.73** The damping coefficient in the vibration equation is given by  
 (A) 500 Nms/rad  
 (B) 500 N/(m/s)  
 (C) 80 Nms/rad  
 (D) 80 N/(m/s)

**YEAR 2002****ONE MARK**

- MCQ 4.74** The minimum number of links in a single degree-of-freedom planar mechanism with both higher and lower kinematic pairs is  
 (A) 2  
 (B) 3  
 (C) 4  
 (D) 5
- MCQ 4.75** The Coriolis component of acceleration is present in  
 (A) 4 bar mechanisms with 4 turning pairs  
 (B) shape mechanism  
 (C) slider-crank mechanism  
 (D) scotch yoke mechanism

## YEAR 2002

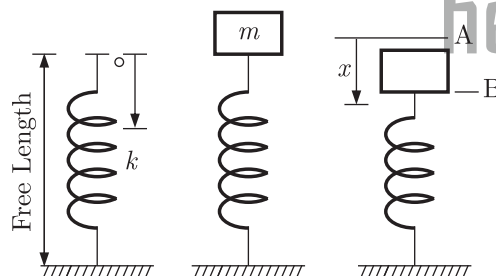
## TWO MARKS

- MCQ 4.76** If the length of the cantilever beam is halved, the natural frequency of the mass  $M$  at the end of this cantilever beam of negligible mass is increased by a factor of
- (A) 2 (B) 4  
(C)  $\sqrt{8}$  (D) 8

## YEAR 2001

## ONE MARK

- MCQ 4.77** For a spring-loaded roller follower driven with a disc cam,
- (A) the pressure angle should be larger during rise than that during return for ease of transmitting motion.  
(B) the pressure angle should be smaller during rise than that during return for ease of transmitting motion.  
(C) the pressure angle should be large during rise as well as during return for ease of transmitting motion.  
(D) the pressure angle does not affect the ease of transmitting motion.
- MCQ 4.78** In the figure shown, the spring deflects by  $\delta$  to position  $A$  (the equilibrium position) when a mass  $m$  is kept on it. During free vibration, the mass is at position  $B$  at some instant. The change in potential energy of the spring mass system from position  $A$  to position  $B$  is



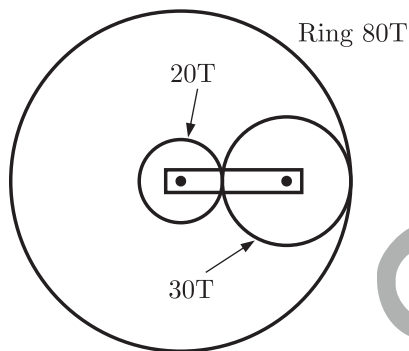
- (A)  $\frac{1}{2}kx^2$  (B)  $\frac{1}{2}kx^2 - mgx$   
(C)  $\frac{1}{2}k(x + \delta)^2$  (D)  $\frac{1}{2}kx^2 + mgx$

- MCQ 4.79** Which of the following statements is correct ?
- (A) Flywheel reduces speed fluctuations during a cycle for a constant load, but flywheel does not control the mean speed of the engine, if the load changes.  
(B) Flywheel does not reduce speed fluctuation during a cycle for a constant load, but flywheel does not control the mean speed of the engine, if the load changes.

- (C) Governor controls speed fluctuations during a cycle for a constant load, but governor does not control the mean speed of the engine, if the load changes.
- (D) Governor controls speed fluctuations during a cycle for a constant load, and governor also controls the mean speed of the engine, if the load changes.

**YEAR 2001****TWO MARKS****MCQ 4.80**

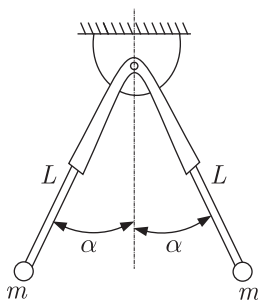
The sun gear in the figure is driven clockwise at 100 rpm. The ring gear is held stationary. For the number of teeth shown on the gears, the arm rotates at



- (A) zero (B) 20 rpm  
(C) 33.33 rpm (D) 66.67 rpm

**MCQ 4.81**

The assembly shown in the figure is composed of two massless rods of length  $L$  with two particles, each of mass  $m$ . The natural frequency of this assembly for small oscillations is



- (A)  $\sqrt{\frac{g}{L}}$  (B)  $\sqrt{\frac{2g}{(L \cos \alpha)}}$   
(C)  $\sqrt{\frac{g}{(L \cos \alpha)}}$  (D)  $\sqrt{\frac{(g \cos \alpha)}{L}}$

\*\*\*\*\*

## SOLUTION

**SOL 4.1** Option (B) is correct.

For helical gears, speed ratio is given by

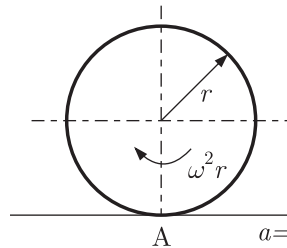
$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \times \frac{\cos \beta_2}{\cos \beta_1} \quad \dots(i)$$

$$N_1 = 1440 \text{ rpm}, D_1 = 80 \text{ mm}, D_2 = 120 \text{ mm}, \beta_1 = 30^\circ, \beta_2 = 22.5^\circ$$

Hence from Eq. (i)

$$\begin{aligned} N_2 &= \frac{D_1}{D_2} \times \frac{\cos \beta_1}{\cos \beta_2} \times N_1 = \frac{80}{120} \times \frac{\cos 30^\circ}{\cos 22.5^\circ} \times 1440 \\ &= 899.88 \simeq 900 \text{ rpm} \end{aligned}$$

**SOL 4.2** Option (D) is correct.



For A solid disc of radius ( $r$ ) as given in figure, rolls without slipping on a horizontal floor with angular velocity  $\omega$  and angular acceleration  $\alpha$ .

The magnitude of the acceleration of the point of contact ( $A$ ) on the disc is only by centripetal acceleration because of no slip condition.

$$v = \omega r \quad \dots(i)$$

Differentiating Eq. (1) w.r.t. ( $t$ )

$$\frac{dv}{dt} = r \frac{d\omega}{dt} = r \cdot \alpha \quad \left( \frac{d\omega}{dt} = \alpha, \frac{dv}{dt} = a \right)$$

or,

$$a = r \cdot \alpha$$

Instantaneous velocity of point  $A$  is zero

So at point  $A$ , Instantaneous tangential acceleration = zero

Therefore only centripetal acceleration is there at point  $A$ .

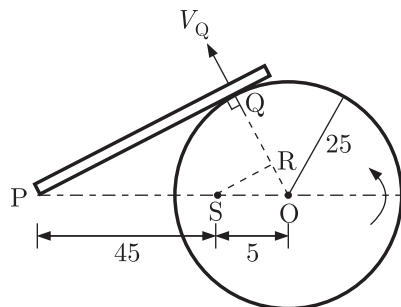
$$\text{Centripetal acceleration} = r\omega^2$$

**SOL 4.3** Option (B) is correct.

From similar  $\triangle PQO$  and  $\triangle SRO$

$$\frac{PQ}{SR} = \frac{PO}{SO} \quad \dots(i)$$

$$PQ = \sqrt{(50)^2 - (25)^2} = 43.3 \text{ mm}$$



From Eq. (i)  $\frac{43.3}{SR} = \frac{50}{5}$

$$SR = \frac{43.3 \times 5}{50} = 4.33 \text{ mm}$$

Velocity of  $Q$  = Velocity of  $R$  (situated at the same link)

$$V_Q = V_R = SR \times \omega = 4.33 \times 1 = 4.33 \text{ m/s}$$

Angular velocity of  $PQ$ .  $\omega_{PQ} = \frac{V_Q}{PQ} = \frac{4.33}{43.3} = 0.1 \text{ rad/s}$

**SOL 4.4**

Option (B) is correct.

For flywheel

$$K.E = \frac{1}{2} I \omega^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.83 \text{ rad/s}$$

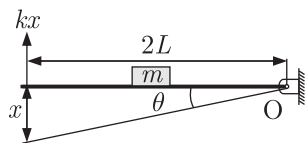
$$I \text{ (for solid circular disk)} = \frac{1}{2} m R^2 = \frac{1}{2} \times 20 \times (0.2)^2 = 0.4 \text{ kg} \cdot \text{m}^2$$

Hence,  $K.E = \frac{1}{2} \times (0.4) \times (62.83)^2 = 789.6 \simeq 790 \text{ Joules.}$

**SOL 4.5**

Option (D) is correct.

For a very small amplitude of vibration.



From above figure change in length of spring

$$x = 2L \sin \theta = 2L\theta \quad (\text{is very small so } \sin \theta \simeq \theta)$$

Mass moment of inertia of mass ( $m$ ) about  $O$  is

$$I = mL^2$$

As no internal force acting on the system. So governing equation of motion from Newton's law of motion is,

$$I\ddot{\theta} + kx \times 2L = 0$$

or,  $mL^2\ddot{\theta} + k2L\theta \times 2L = 0$

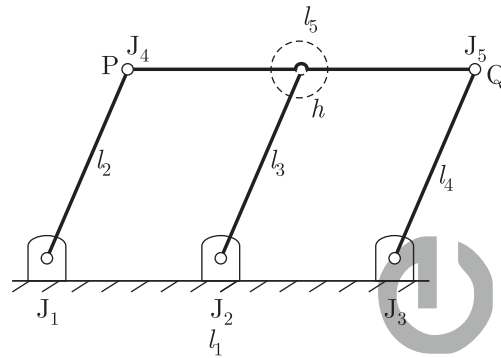
$$\ddot{\theta} + \frac{4kL^2\theta}{mL^2} = 0$$

or  $\ddot{\theta} + \frac{4k\theta}{m} = 0$

Comparing general equation  $\ddot{\theta} + \omega_n^2\theta = 0$  we have

$$\omega_n^2 = \frac{4k}{m} \Rightarrow \omega_n = \sqrt{\frac{4k}{m}}$$

**SOL 4.6** Option (C) is correct.



Given that PQ is a single link.

Hence :  $l = 5$ ,  $j = 5$ ,  $h = 1$

It has been assumed that slipping is possible between the link  $l_5$  &  $l_1$ . From the kutzbach criterion for a plane mechanism, Numbers of degree of freedom or movability.

$$n = 3(l - 1) - 2j - h = 3(5 - 1) - 2 \times 5 - 1 = 1$$

**SOL 4.7** Option (D) is correct.

Given  $\omega_{AB} = 1 \text{ rad/sec}$ ,  $l_{CD} = 1.5l_{AB} \Rightarrow \frac{l_{CD}}{l_{AB}} = 1.5$

Let angular velocity of link CD is  $\omega_{CD}$

From angular velocity ratio theorem,

$$\frac{\omega_{AB}}{\omega_{CD}} = \frac{l_{CD}}{l_{AB}}$$

$$\omega_{CD} = \omega_{AB} \times \frac{l_{AB}}{l_{CD}} = 1 \times \frac{1}{1.5} = \frac{2}{3} \text{ rad/sec}$$

**SOL 4.8** Option (A) is correct.

Given  $k = 20 \text{ kN/m}$ ,  $m = 1 \text{ kg}$

From the Given spring mass system, springs are in parallel combination. So,

$$k_{eq} = k + k = 2k$$

Natural Frequency of spring mass system is,



$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

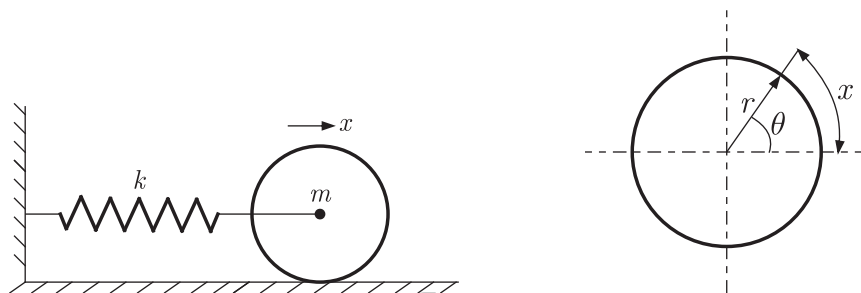
$$2\pi f_n = \sqrt{\frac{k_{eq}}{m}} \quad f_n = \text{Natural Frequency in Hz.}$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi} \sqrt{\frac{2k}{m}} = \frac{1}{2 \times 3.14} \sqrt{\frac{2 \times 20 \times 1000}{1}}$$

$$= \frac{200}{6.28} = 31.84 \text{ Hz} \simeq 32 \text{ Hz}$$

**SOL 4.9**

Option (C) is correct.



$$\theta = \frac{x}{r} \Rightarrow x = r\theta \quad \dots(i)$$

Total energy of the system remains constant.

So,

T.E. = K.E. due to translatory motion  
+ K.E. due to rotary motion + P.E. of spring

$$\begin{aligned} \text{T.E.} &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k x^2 \\ &= \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2 \quad \text{From equation (i) } \dot{x} = r \dot{\theta} \\ &= \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} \times \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2 \quad \text{For a disc } I = \frac{m r^2}{2} \\ &= \frac{3}{4} m r^2 \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2 = \text{Constant} \end{aligned}$$

On differentiating above equation w.r.t.  $t$ , we get

$$\frac{3}{4} m r^2 \times (2\dot{\theta}\ddot{\theta}) + \frac{1}{2} k r^2 (2\theta\dot{\theta}) = 0$$

$$\frac{3}{2} m r^2 \ddot{\theta} + k r^2 \theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m} \theta = 0$$

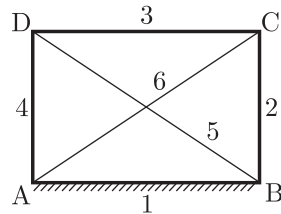
$$\omega_n^2 = \frac{2k}{3m} \Rightarrow \omega_n = \sqrt{\frac{2k}{3m}}$$

Therefore, natural frequency of vibration of the system is,

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2k}{3m}}$$

**SOL 4.10** Option (A) is correct.

Given figure shows the six bar mechanism.



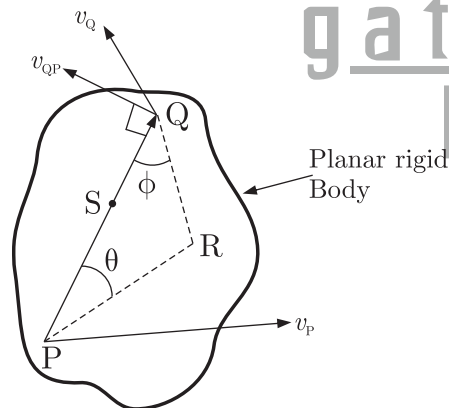
We know movability or degree of freedom is  $n = 3(l - 1) - 2j - h$

The mechanism shown in figure has six links and eight binary joints (because there are four ternary joints  $A, B, C$  &  $D$ , i.e.  $l = 6$ ,  $j = 8$   $h = 0$

So,  $n = 3(6 - 1) - 2 \times 8 = -1$

Therefore, when  $n = -1$  or less, then there are redundant constraints in the chain, and it forms a statically indeterminate structure. So, From the Given options (A) satisfy the statically indeterminate structure  $n \leq -1$

**SOL 4.11** Option (C) is correct.



Velocity of any point on a link with respect to another point (relative velocity) on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

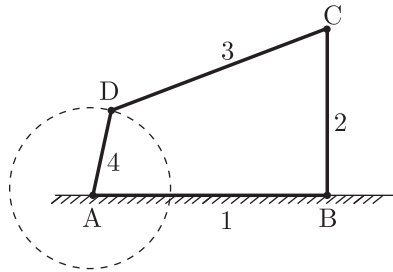
$$\begin{aligned} v_{QP} &= \text{Relative velocity between P \& Q} \\ &= v_P - v_Q \text{ always perpendicular to PQ.} \end{aligned}$$

**SOL 4.12** Option (A) is correct.

According to Grashof's law "For a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of remaining two link lengths if there is to be continuous relative motion

between the two links.

$$l_4 + l_2 \not> l_1 + l_3$$



**SOL 4.13** Option (A) is correct.

We know natural frequency of a spring mass system is,

$$\omega_n = \sqrt{\frac{k}{m}} \quad \dots(i)$$

This equation (i) does not depend on the  $g$  and weight ( $W = mg$ )

So, the natural frequency of a spring mass system is unchanged on the moon.

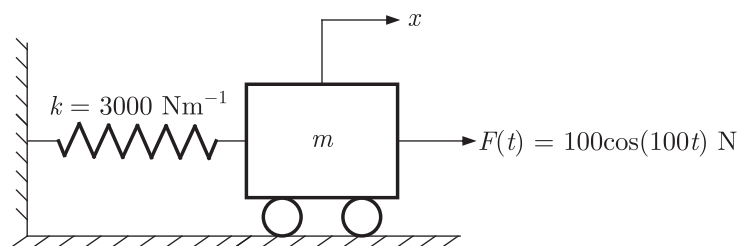
Hence, it will remain  $\omega_n$ , i.e.  $\omega_{moon} = \omega_n$

**SOL 4.14** Option (D) is correct.

When gear teeth are produced by a generating process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called undercutting. By undercutting the undercut tooth can be considerably weakened.

So, interference can be reduced by using more teeth on the gear. However, if the gears are to transmit a given amount of power, more teeth can be used only by increasing the pitch diameter.

**SOL 4.15** Option (A) is correct.



Given  $k = 3000 \text{ N/m}$ ,  $c = 0$ ,  $A = 50 \text{ mm}$ ,  $F(t) = 100 \cos(100t) \text{ N}$

$$\omega t = 100t$$

$$\omega = 100$$

It is a forced vibratory system.

From the Newton's law,

$$m\ddot{x} + kx = F \quad \dots(i)$$

And its general solution will be,

$$x = A \cos \omega t$$

$$\frac{dx}{dt} = \dot{x} = -A\omega \sin \omega t \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$$\frac{d^2x}{dt^2} = \ddot{x} = -A\omega^2 \cos \omega t$$

Substitute these values in equation (i), we get

$$\begin{aligned} -mA\omega^2 \cos \omega t + kA \cos \omega t &= 100 \cos(\omega t) \\ -mA\omega^2 + kA &= 100 \end{aligned}$$

Now substitute  $k = 3000 \text{ N/m}$ ,  $A = 0.05 \text{ m}$ , in above equation, we get

$$\begin{aligned} -m \times 0.05 \times (100)^2 + 3000 \times 0.05 &= 100 \\ -5m + 1.5 &= 1 \\ m &= 0.1 \text{ kg} \end{aligned}$$

#### Alternate Method:

We know that, in forced vibration amplitude is given by :

$$A = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad \dots(i)$$

Here,  $F(t) = 100 \cos(100t)$ ,  $F_o = 100 \text{ N}$ ,  $A = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$   
 $\omega = 100 \text{ rad/sec}$ ,  $k = 3000 \text{ Nm}^{-1}$ ,  $c = 0$

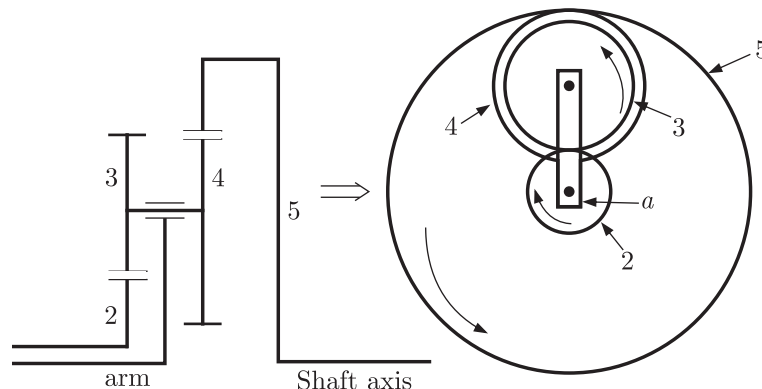
So, from equation (i), we get

$$\begin{aligned} A &= \frac{F_o}{k - m\omega^2} \\ k - m\omega^2 &= \frac{F_o}{A} \end{aligned}$$

$$3000 - m \times (100)^2 = \frac{100}{50 \times 10^{-3}}$$

$$10000m = 1000 \quad \Rightarrow m = 0.1 \text{ kg}$$

**SOL 4.16** Option (C) is correct.



Given  $N_i$  = No. of teeth for gear  $i$ ,

$N_2 = 20$ ,  $N_3 = 24$ ,  $N_4 = 32$ ,  $N_5 = 80$ ,  $\omega_2 = 100$  rad/sec (CW)

$\omega_{arm} = 80$  rad/sec (CCW) =  $-80$  rad/sec

The table of the motion given below :

Take CCW = - ve and CW = + ve

S. No.	Condition of Motion	Revolution of elements			
		Arm	Gear 2 $\omega_2$	Compound Gear 3 - 4, $\omega_3 = \omega_4$	Gear 5 $\omega_5$
1.	Arm 'a' is fixed & Gear 2 rotates through +1 revolution (CW)	0	+1	$-\frac{N_2}{N_3}$	$-\frac{N_2}{N_3} \times \frac{N_4}{N_5}$
2.	Gear 2 rotates through +x revolution (CW)	0	+x	$-x\frac{N_2}{N_3}$	$-x\frac{N_2}{N_3} \times \frac{N_4}{N_5}$
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	Total motion.	+y	x+y	$y - x\frac{N_2}{N_3}$	$y - x\frac{N_2}{N_3} \times \frac{N_4}{N_5}$

Note. Speed ratio =  $\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$

i.e.  $\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}$

Gear 3 & 4 mounted on same shaft, So  $\omega_3 = \omega_4$

And  $\omega_{arm} = y$  From the table

$$y = -80 \text{ rad/sec (CCW)}$$

$x + y = \omega_2 = 100$  From the table

$$x = 100 - (-80) = 180 \text{ rad/sec (CW)}$$

And  $\omega_5 = y - x \times \frac{N_2}{N_3} \times \frac{N_4}{N_5}$  From the table

$$= -80 - 180 \times \frac{20}{24} \times \frac{32}{80} = -140 \text{ rad/sec}$$

Negative sign shows the counter clockwise direction.

**SOL 4.17** Option (D) is correct.

Let,  $v_B$  is the velocity of slider B relative to link CD

The crank length  $AB = 250$  mm and velocity of slider B with respect to rigid link CD is simply velocity of B (because C is a fixed point).

Hence,  $v_B = (AB) \times \omega_{AB} = 250 \times 10^{-3} \times 10 = 2.5$  m/sec

**Alternate Method :**

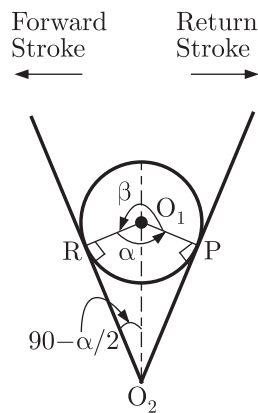
From the given figure, direction of velocity of CD is perpendicular to link AB & direction of velocity of AB is parallel to link CD.

So, direction of relative velocity of slider B with respect to C is in line with link BC.

Hence  $v_C = 0$

Or  $v_{BC} = v_B - v_C = AB \times \omega_{AB} - 0 = 0.025 \times 10 = 2.5 \text{ m/sec}$

**SOL 4.18** Option (D) is correct.



Given  $O_1P = r = 125 \text{ mm}$

Forward to return ratio = 2:1

We know that,  $\frac{\text{Time of cutting (forward) stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360 - \alpha}{\alpha}$

Substitute the value of Forward to return ratio, we have

$$\frac{2}{1} = \frac{360 - \alpha}{\alpha}$$

$$2\alpha = 360 - \alpha \Rightarrow \alpha = 120^\circ$$

And angle  $\angle RO_1O_2 = \frac{\alpha}{2} = \frac{120^\circ}{2} = 60^\circ$

Now we are to find the distance 'd' between the crank centre to lever pivot centre point ( $O_1O_2$ ). From the  $\triangle RO_2O_1$

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{O_1R}{O_1O_2} = \frac{r}{O_1O_2}$$

$$\sin(90^\circ - 60^\circ) = \frac{r}{O_1O_2}$$

$$O_1O_2 = \frac{r}{\sin 30^\circ} = \frac{125}{1/2} = 250 \text{ mm}$$

**SOL 4.19** Option (B) is correct.

Given  $\delta = 1.8 \text{ mm} = 0.0018 \text{ m}$

The critical or whirling speed is given by,

$$\omega_c = \sqrt{\frac{g}{\delta}}$$

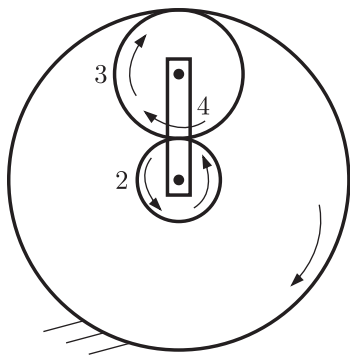
$$\frac{2\pi N_c}{60} = \sqrt{\frac{g}{\delta}} \quad N_c = \text{Critical speed in rpm}$$

$$N_c = \frac{60}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{60}{2 \times 3.14} \sqrt{\frac{9.81}{0.0018}}$$

$$= 9.55 \sqrt{5450} = 704.981 \approx 705 \text{ rpm}$$

**SOL 4.20** Option (A) is correct.

Given  $Z_2 = 20$  Teeth,  $Z_3 = 40$  Teeth,  $Z_5 = 100$  Teeth,  $N_5 = 0$ ,  
 $N_2 = 60$  rpm (CCW)



If gear 2 rotates in the CCW direction, then gear 3 rotates in the clockwise direction. Let, Arm 4 will rotate at  $N_4$  rpm. The table of motions is given below. Take CCW = +ve, CW = -ve

S. No.	Condition of Motion	Revolution of elements			
		Sun Gear 2	Planet Gear 3	Arm 4	Ring Gear 5
		$N_2$	$N_3$	$N_4$	$N_5$
1.	Arm fixed and sun gear 2 rotates +1 rpm (CCW)	+1	$-\frac{Z_2}{Z_3}$	0	$-\frac{Z_2}{Z_3} \times \frac{Z_3}{Z_5}$
2.	Give +x rpm to gear 2 (CCW)	+x	$-\frac{Z_2}{Z_3}x$	0	$-x\frac{Z_2}{Z_5}$
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	Total motion.	$y + x$	$y - x\frac{Z_2}{Z_3}$	+y	$y - x\frac{Z_2}{Z_5}$

Note : Speed ratio =  $\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$

Ring gear 5 is fixed. So,

$$N_5 = 0$$

$$y - x \frac{Z_2}{Z_5} = 0$$

From the table

$$y = \frac{Z_2}{Z_5} x = \frac{20}{100} x = \frac{x}{5} \quad \dots(i)$$

Given,

$$N_2 = 60 \text{ rpm (CCW)}$$

$$y + x = 60$$

From table

$$\frac{x}{5} + x = 60$$

$$x = 10 \times 5 = 50 \text{ rpm}$$

And from equation (i),

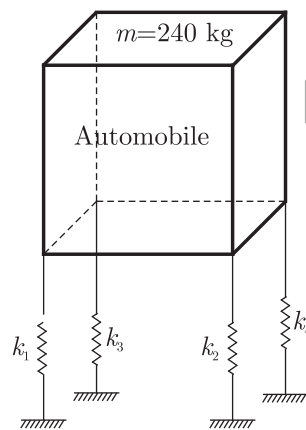
$$y = \frac{50}{5} = 10 \text{ rpm (CCW)}$$

From the table the arm will rotate at

$$N_4 = y = 10 \text{ rpm (CCW)}$$

#### SOL 4.21

Option (A) is correct.



Given  $k_1 = k_2 = 16 \text{ MN/m}$ ,  $k_3 = k_4 = 32 \text{ MN/m}$ ,  $m = 240 \text{ kg}$

Here,  $k_1$  &  $k_2$  are the front two springs or  $k_3$  and  $k_4$  are the rear two springs.

These 4 springs are parallel, So equivalent stiffness

$$k_{eq} = k_1 + k_2 + k_3 + k_4 = 16 + 16 + 32 + 32 = 96 \text{ MN/m}^2$$

We know at resonance

$$\omega = \omega_n = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi N}{60} = \sqrt{\frac{k_{eq}}{m}}$$

$N$  = Engine speed in rpm

$$N = \frac{60}{2\pi} \sqrt{\frac{k_{eq}}{m}} = \frac{60}{2\pi} \sqrt{\frac{96 \times 10^6}{240}}$$



$$= \frac{60}{2\pi} \times 10^2 \times \sqrt{40} = 6042.03 \simeq 6040 \text{ rpm}$$

**SOL 4.22** Option (A) is correct.

Given  $k = 3.6 \text{ kN/m}$ ,  $c = 400 \text{ Ns/m}$ ,  $m = 50 \text{ kg}$

We know that, Natural Frequency

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.6 \times 1000}{50}} = 8.485 \text{ rad/sec} \quad \dots(i)$$

And damping factor is given by,

$$d \text{ or } \varepsilon = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{400}{2 \times \sqrt{3.6 \times 1000 \times 50}} \\ = \frac{400}{2 \times 424.26} = 0.471$$

Damping Natural frequency,

$$\omega_d = \sqrt{1 - \varepsilon^2} \omega_n \\ 2\pi f_d = \sqrt{1 - \varepsilon^2} \omega_n \quad f_d = \frac{\omega_d}{2\pi} \\ f_d = \frac{\omega_n}{2\pi} \times \sqrt{1 - \varepsilon^2} = \frac{8.485}{2 \times 3.14} \times \sqrt{1 - (0.471)^2} = 1.19 \text{ Hz}$$

**SOL 4.23** Option (B) is correct.

#### Analysis

- P.** Continuous relative rotation  
**Q.** Velocity and Acceleration  
**R.** Mobility  
**S.** Dynamic-static Analysis

#### Approach

- 3.** Grashoff law  
**4.** Kennedy's Theorem  
**2.** Grubler's Criterion  
**1.** D'Alembert's Principle

So, correct pairs are P-3, Q-4, R-2, S-1

**SOL 4.24** Option (B) is correct.

From Gruebler's criterion, the equation for degree of freedom is given by,

$$n = 3(l - 1) - 2j - h \quad \dots(i)$$

Given  $l = 8$  and  $j = 10$ ,  $h = 0$

$$n = 3(8 - 1) - 2 \times 10 = 1 \quad \text{from equation(i)}$$

**SOL 4.25** Option (B) is correct.

Given  $m = 1.4 \text{ kg}$ ,  $k_1 = 4000 \text{ N/m}$ ,  $k_2 = 1600 \text{ N/m}$

In the given system  $k_1$  &  $k_2$  are in parallel combination

So,  $k_{eq} = k_1 + k_2 = 4000 + 1600 = 5600 \text{ N/m}$

Natural frequency of spring mass system is given by,

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi} \sqrt{\frac{5600}{1.4}} = \frac{1}{2\pi} \times 63.245 = 10.07 \simeq 10 \text{ Hz}$$

**SOL 4.26** Option (D) is correct.

Jerk is given by triple differentiation of  $s$  w.r.t.  $t$ ,

$$\text{Jerk} = \frac{d^3 s}{dt^3}$$

$$\text{Given} \quad s = \frac{h}{2} \left( 1 - \cos \frac{\pi \theta}{\beta} \right) = \frac{h}{2} \left[ 1 - \cos \frac{\pi (\omega t)}{\beta} \right] \quad \theta = \omega t$$

Differentiating above equation w.r.t.  $t$ , we get

$$\frac{ds}{dt} = \frac{h}{2} \left[ -\frac{\pi \omega}{\beta} \left\{ -\sin \frac{\pi (\omega t)}{\beta} \right\} \right]$$

Again Differentiating w.r.t.  $t$ ,

$$\frac{d^2 s}{dt^2} = \frac{h}{2} \frac{\pi^2 \omega^2}{\beta^2} \left[ \cos \frac{\pi (\omega t)}{\beta} \right]$$

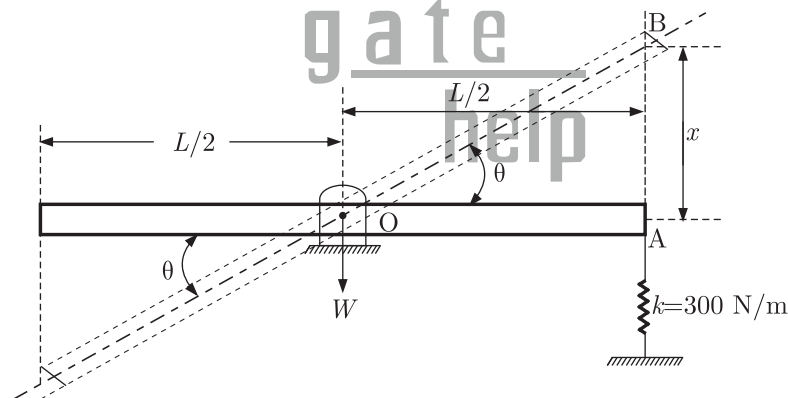
Again Differentiating w.r.t.  $t$ ,

$$\frac{d^3 s}{dt^3} = -\frac{h}{2} \frac{\pi^3 \omega^3}{\beta^3} \sin \frac{\pi \theta}{\beta}$$

Let  $\omega = 1 \text{ rad/sec}$

$$\frac{d^3 s}{dt^3} = -\frac{h}{2} \frac{\pi^3}{\beta^3} \sin \left( \frac{\pi \theta}{\beta} \right)$$

**SOL 4.27** Option (C) is correct.



Given  $m = 1 \text{ kg}$ ,  $L = 1 \text{ m}$ ,  $k = 300 \text{ N/m}$

We have to turn the rigid rod at an angle  $\theta$  about its hinged point, then rod moves upward at a distance  $x$  and also deflect in the opposite direction with the same amount. Let  $\theta$  is very very small and take  $\tan \theta \simeq \theta$

$$\text{From } \triangle AOB, \quad \theta = \frac{x}{L/2} \Rightarrow x = \frac{L}{2} \theta \quad \dots(i)$$

$$\text{and} \quad \theta = \omega t \Rightarrow \dot{\theta} = \omega \quad \dots(ii)$$

By using the principal of energy conservation,

$$\frac{1}{2} I \omega^2 + \frac{1}{2} k x^2 = \text{Constant}$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k\left(\frac{L}{2}\theta\right)^2 = c$$

From equation (i) and (ii)

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{8}L^2k\theta^2 = c$$

On differentiating w.r.t.  $t$ , we get

$$\frac{1}{2}I \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{8} \times 2\theta\dot{\theta} = 0 \quad \dots(\text{iii})$$

For a rigid rod of length  $L$  & mass  $m$ , hinged at its centre, the moment of inertia,

$$I = \frac{mL^2}{12}$$

Substitute  $I$  in equation (iii), we get

$$\frac{1}{2} \times \frac{mL^2}{12} \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{4}\theta\dot{\theta} = 0$$

$$\ddot{\theta} + \frac{3k}{m}\theta = 0 \quad \dots(\text{iv})$$

Compare equation (iv) with the general equation,

$$\ddot{\theta} + \omega_n^2\theta = 0$$

So, we have

$$\omega_n^2 = \frac{3k}{m}$$

$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3 \times 300}{1}} = 30 \text{ rad/sec}$$

**SOL 4.28** Option (C) is correct.

For an under damped harmonic oscillator resonance occurs when excitation frequency is equal to the undamped natural frequency

$$\omega_d = \omega_n$$

**SOL 4.29** Option (A) is correct.

Given  $\omega_1 = 210 \text{ rad/sec}$ ,  $\omega_2 = 190 \text{ rad/sec}$ ,  $\Delta E = 400 \text{ Nm}$

As the speed of flywheel changes from  $\omega_1$  to  $\omega_2$ , the maximum fluctuation of energy,

$$\Delta E = \frac{1}{2}I[(\omega_1)^2 - (\omega_2)^2]$$

$$I = \frac{2\Delta E}{[(\omega_1)^2 - (\omega_2)^2]} = \frac{2 \times 400}{[(210)^2 - (190)^2]} = \frac{800}{400 \times 20} = 0.10 \text{ kgm}^2$$

**SOL 4.30** Option (C) is correct.

Given,  $\angle O_4 O_2 P = 180^\circ$ ,  $\omega_{O_2 P} = 2 \text{ rad/sec}$

The instantaneous centre diagram is given below,

Let, velocity of point  $P$  on link  $O_2 P$  is  $V_P$ ,

$$V_P = \omega_{O_2 P} \times O_2 P = \omega_{O_2 P} \times (I_{12} I_{23}) = 2a \quad \dots(\text{i})$$

And  $P$  is also a point on link  $QP$ ,

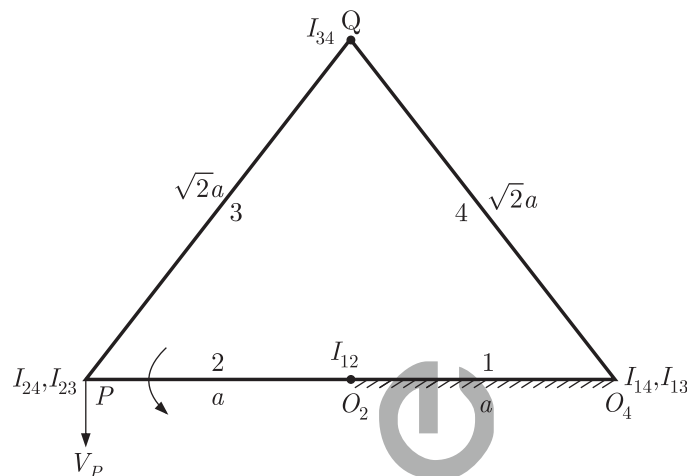
$$\begin{aligned} \text{So, } V_P &= \omega_{PQ} \times O_4P = \omega_{PQ} \times (I_{13}I_{23}) \\ &= \omega_{PQ} \times 2a \end{aligned} \quad \dots(ii)$$

Both the links  $O_2P$  and  $QP$  are runs at the same speed

From equation (i) and (ii), we get

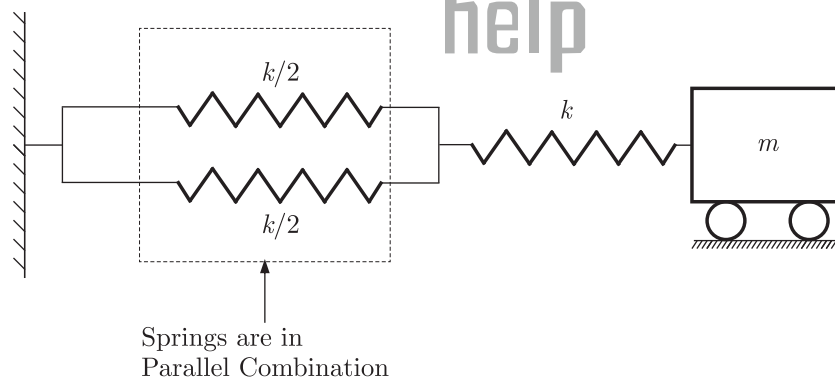
$$2a = \omega_{PQ} \times 2a$$

$$\text{or, } \omega_{PQ} = 1 \text{ rad/sec}$$



**SOL 4.31**

Option (A) is correct.



The springs, with stiffness  $\frac{k}{2}$  &  $\frac{k}{2}$  are in parallel combination. So their resultant stiffness will be,

$$k_1 = \frac{k}{2} + \frac{k}{2} = k$$

As  $k_1$  &  $k$  are in series, so the resultant stiffness will be,

$$k_{eq} = \frac{k \times k}{k + k} = \frac{k^2}{2k} = \frac{k}{2}$$

The general equation of motion for undamped free vibration is given as,

$$m\ddot{x} + k_{eq}x = 0$$

$$m\ddot{x} + \frac{k}{2}x = 0$$

$$\ddot{x} + \frac{k}{2m}x = 0$$

Compare above equation with general equation  $\ddot{x} + \omega_n^2 x = 0$ , we get  
Natural frequency of the system is,

$$\omega_n^2 = \frac{k}{2m} \Rightarrow \omega_n = \sqrt{\frac{k}{2m}}$$

**Alternative :**

$$k_{eq} = \frac{k}{2}$$

We know, for a spring mass system,

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k/2}{m}} = \sqrt{\frac{k}{2m}}$$

#### SOL 4.32

Option (A) is correct.

Given The equation of motion of a harmonic oscillator is

$$\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = 0 \quad \dots(i)$$

$$\ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = 0$$

Compare equation (i) with the general equation,

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

We get,

$$\frac{c}{m} = 2\xi\omega_n \quad \dots(ii)$$

$$\frac{k}{m} = \omega_n^2, \quad \Rightarrow \quad \omega_n = \sqrt{\frac{k}{m}} \quad \dots(iii)$$

$$\text{From equation (ii) \& (iii), } \xi = \frac{c}{2m \times \sqrt{\frac{k}{m}}} = \frac{c}{2\sqrt{km}}$$

Logarithmic decrement,

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi c}{\sqrt{c_c^2 - c^2}} \quad \dots(iv)$$

$$= \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi \times 2\xi\sqrt{km}}{(2\sqrt{km})^2 - (2\xi\sqrt{km})^2} = \frac{4\pi\xi\sqrt{km}}{\sqrt{4km - 4\xi^2 km}}$$

$$= \frac{2\pi\xi}{\sqrt{1 - \xi^2}}$$

$$\frac{x_1}{x_2} = e^{\frac{2\pi\xi}{\sqrt{1 - \xi^2}}}$$

If system executes  $n$  cycles, the logarithmic decrement  $\delta$  can be written as



From the  $\triangle TOP$ ,  $\cos \frac{\alpha}{2} = \frac{OT}{OP} = \frac{r}{500}$   $OT = r$

$$\cos 60^\circ = \frac{r}{500}$$

$$r = 500 \times \frac{1}{2} = 250 \text{ mm}$$

**SOL 4.34** Option (B) is correct.

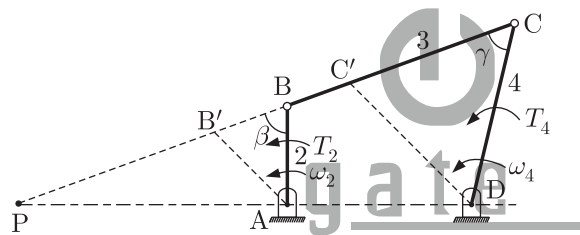
We know that maximum speed during forward stroke occur when  $QR$  &  $QP$  are perpendicular.

So,  $V = OS \times \omega_{OS} = PQ \times \omega_{PQ}$   $V = r\omega$

$$250 \times 2 = 750 \times \omega_{PQ}$$

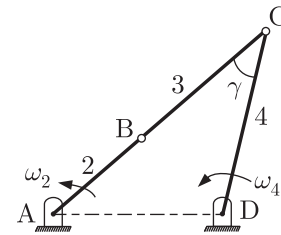
$$\omega_{PQ} = \frac{500}{750} = \frac{2}{3} \text{ rad/sec}$$

**SOL 4.35** Option (D) is correct.



$$M.A = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{R_{PD}}{R_{PA}}$$

ratio theorem



from angular velocity

Construct  $B'A$  and  $C'D$  perpendicular to the line  $PBC$ . Also, assign labels  $\beta$  and  $\gamma$  to the acute angles made by the coupler.

$$\frac{R_{PD}}{R_{PA}} = \frac{R_{C'D}}{R_{B'A}} = \frac{R_{CD} \sin \gamma}{R_{BA} \sin \beta}$$

So,  $M.A. = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{R_{CD} \sin \gamma}{R_{BA} \sin \beta}$

When the mechanism is toggle, then  $\beta = 0^\circ$  and  $180^\circ$ .

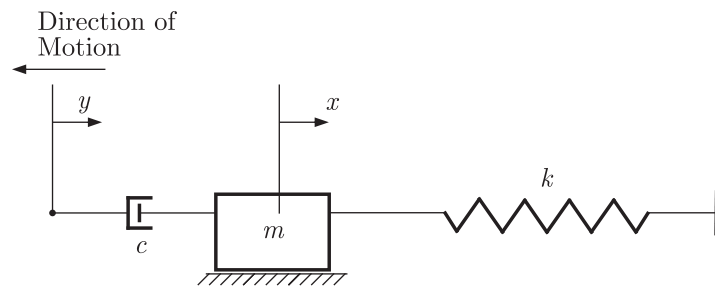
So  $M.A = \infty$

**SOL 4.36** Option (C) is correct.

Assume any arbitrary relationship between the coordinates and their first derivatives, say  $x > y$  and  $\dot{x} > \dot{y}$ . Also assume  $x > 0$  and  $\dot{x} > 0$ .

A small displacement gives to the system towards the left direction. Mass  $m$  is fixed, so only damper moves for both the variable  $x$  and  $y$ .

Note that these forces are acting in the negative direction.



Differential equation governing the above system is,

$$\sum F = -m \frac{d^2 x}{dt^2} - c \left( \frac{dx}{dt} - \frac{dy}{dt} \right) - kx = 0$$

$$m\ddot{x} + c(\dot{x} - \dot{y}) + kx = 0$$

**SOL 4.37** Option (C) is correct.

For a 4 bar slider crank mechanism, there are the number of links or inversions are 4. These different inversions are obtained by fixing different links once at a time for one inversion. Hence, the number of inversions for a slider crank mechanism is 4.

**SOL 4.38** Option (B) is correct.

Column I	Column II
P. Addendum	4. Gear
Q. Instantaneous centre of velocity	3. Linkage
R. Section modulus	2. Beam
S. Prime circle	1. Cam

So correct pairs are, P-4, Q-3, R-2, S-1

**SOL 4.39** Option (D) is correct.

The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed ( $C_f$ ).

Let,  $N_1$  &  $N_2$  = Maximum & Minimum speeds in r.p.m. during the cycle

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2} \quad \dots(i)$$

$$\text{Therefore, } C_f = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} \quad \text{from equation (i)}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$C_f = \frac{2(\omega_{\max} - \omega_{\min})}{\omega_{\max} + \omega_{\min}} \quad \omega_1 = \omega_{\max},$$



$$\omega_2 = \omega_{\min}$$

$$C_f \omega_{\max} + C_f \omega_{\min} = 2\omega_{\max} - 2\omega_{\min}$$

$$\omega_{\max}(C_f - 2) = \omega_{\min}(-2 - C_f)$$

Hence, 
$$\frac{\omega_{\max}}{\omega_{\min}} = -\frac{(2 + C_f)}{C_f - 2} = \frac{2 + C_f}{2 - C_f}$$

**SOL 4.40**

Option (D) is correct.

In this question pair or mechanism is related to contact & machine related to it.

**Column I**

- P.** Higher Kinematic Pair  
**Q.** Lower Kinematic Pair  
**R.** Quick Return Mechanism  
**S.** Mobility of a Linkage

**Column II**

- 2.** Line Contact  
**6.** Surface Contact  
**5.** Shaper  
**1.** Grubler's Equation

So correct pairs are, P-2, Q-6, R-5, S-1

**SOL 4.41**

Option (C) is correct.

Given  $m = 250$  kg,  $k = 100$  kN/m,  $N = 3600$  rpm,  $\varepsilon = \frac{c}{c_c} = 0.15$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 3600}{60} = 376.8 \text{ rad/sec}$$

Natural frequency of spring mass system,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \times 1000}{250}} = 20 \text{ rad/sec}$$

So, 
$$\frac{\omega}{\omega_n} = \frac{376.8}{20} = 18.84$$

$$T.R. = \frac{F_T}{F} = \sqrt{\frac{1 + \left(2\varepsilon \frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varepsilon \frac{\omega}{\omega_n}\right]^2}}$$

$$= \sqrt{\frac{1 + (2 \times 0.15 \times 18.84)^2}{[1 - (18.84)^2]^2 + [2 \times 0.15 \times 18.84]^2}}$$

$$= \sqrt{\frac{1 + 31.945}{[1 - 354.945]^2 + 31.945}} = \sqrt{\frac{32.945}{125309}} = 0.0162$$

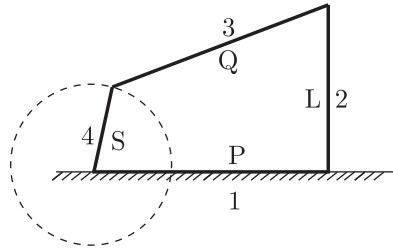
**SOL 4.42**

Option (A) is correct.

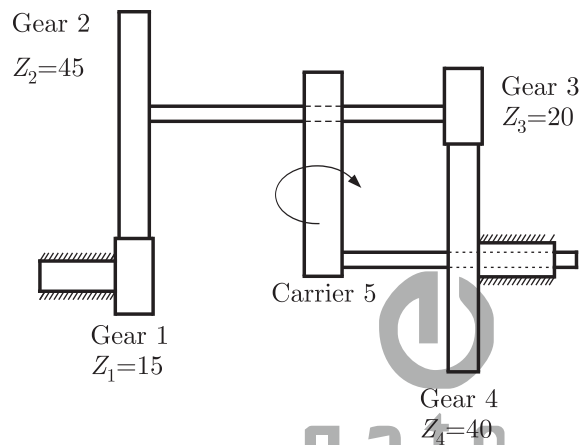
Here  $P$ ,  $Q$ ,  $R$ , &  $S$  are the lengths of the links.

According to Grashof's law : "For a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of remaining two link lengths, if there is to be continuous relative motion between the two links

$$S + L \leq P + Q$$



**SOL 4.43** Option (A) is correct.



The table of motions is given below. Take CW = +ve, CCW = -ve

S. No.	Condition of Motion	Revolution of elements			
		Gear 1 $N_1$	Compound Gear 2-3, $N_2 = N_3$	Gear 4 $N_4$	Carrier $N_5$
1.	Carrier 5 is fixed & Gear 1 rotates +1 rpm (CW)	+1	$-\frac{Z_1}{Z_2}$	$\frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4}$	0
2.	Gear 1 rotates through +x rpm (CW)	+x	$-x \frac{Z_1}{Z_2}$	$x \frac{Z_1 Z_3}{Z_2 Z_4}$	0
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	Total motion.	$x + y$	$y - x \frac{Z_1}{Z_2}$	$y + x \times \frac{Z_1 Z_3}{Z_2 Z_4}$	+y

Note

(i) Speed ratio =  $\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$

i.e. 
$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$

CCW = Counter clock wise direction (– ve)

CW = Clock wise direction (+ ve)

(ii) Gear 2 & Gear 3 mounted on the same shaft (Compound Gears)

So, 
$$N_2 = N_3$$

We know, 
$$\omega = \frac{2\pi N}{60}, \Rightarrow \omega \propto N$$

Hence, 
$$\frac{N_1 - N_5}{N_4 - N_5} = \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{(x + y) - y}{y + x \times \frac{Z_1 Z_3}{Z_2 Z_4} - y}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{x}{x \times \frac{Z_1 Z_3}{Z_2 Z_4}} = \frac{Z_2 Z_4}{Z_1 Z_3}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{45 \times 40}{15 \times 20} = 3 \times 2 = 6$$

**SOL 4.44** Option (D) is correct.

Given  $\omega_1 = 60$  rpm (CW),  $\omega_4 = -2 \times 60$  (CCW) = – 120 rpm

From the previous part,

$$\begin{aligned} \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} &= 6 \\ \frac{60 - \omega_5}{-120 - \omega_5} &= 6 \\ 60 - \omega_5 &= -720 - 6\omega_5 \\ \omega_5 &= -\frac{780}{5} = -156 \text{ rpm} \end{aligned}$$

Negative sign show the counter clock wise direction.

So,  $\omega_5 = 156$  rpm, CCW

**SOL 4.45** Option (D) is correct.

Given  $m = 12.5$  kg,  $k = 1000$  N/m,  $c = 15$  Ns/m

Critical Damping,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km}$$

On substituting the values, we get

$$c_c = 2\sqrt{1000 \times 12.5} = 223.6 \text{ Ns/m}$$

**SOL 4.46** None of these

We know logarithmic decrement,

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1 - \varepsilon^2}} \quad \dots(i)$$

And  $\varepsilon = \frac{c}{c_c} = \frac{15}{223.6} = 0.0671$   $c_c = 223.6 \text{ Ns/m}$

Now, from equation (i), we get

$$\delta = \frac{2 \times 3.14 \times 0.0671}{\sqrt{1 - (0.0671)^2}} = 0.422$$

**SOL 4.47** Option (C) is correct.

Given  $l = 8$ ,  $j = 9$

We know that, Degree of freedom,

$$n = 3(l - 1) - 2j = 3(8 - 1) - 2 \times 9 = 3$$

**SOL 4.48** Option (C) is correct.

The speed of sound in air = 332 m/s

For frequency of instrument of 144 Hz, length of sound wave

$$L_I = \frac{332}{144} = 2.30 \text{ m}$$

For sample  $P$  of 64 Hz,

$$L_P = \frac{332}{64} = 5.1875 \text{ m}$$

$Q$  of 96 Hz  $L_Q = \frac{332}{96} = 3.458 \text{ m}$

$R$  of 128 Hz  $L_R = \frac{332}{128} = 2.593 \text{ m}$

$S$  of 250 Hz  $L_S = \frac{332}{256} = 1.2968 \text{ m}$

Here, the length of sound wave of sample  $R$  ( $L_R = 2.593 \text{ m}$ ) is most close to the length of sound wave of Instrument ( $L_I = 2.30 \text{ m}$ ). Hence, sample  $R$  produce most perceptible induced vibration.

**SOL 4.49** Option (B) is correct.

Given  $N = 300 \text{ r.p.m}$

Angular velocity of cam,

$$\omega = \frac{2\pi N}{60} = 10\pi \text{ rad/sec}$$

Time taken to move  $30^\circ$  is,

$$t = \frac{\frac{\pi}{180} \times 30}{10\pi} = \frac{1}{6} = \frac{1}{60} \text{ sec}$$

Now, Cam moves  $30^\circ$  with a constant acceleration & then with a deceleration, so maximum speed of the follower is at the end of first  $30^\circ$  rotation of the cam and during this  $30^\circ$  rotation the distance covered is 10 mm, with initial velocity  $u = 0$ .

From Newton's second law of motion,

$$S = ut + \frac{1}{2}at^2$$

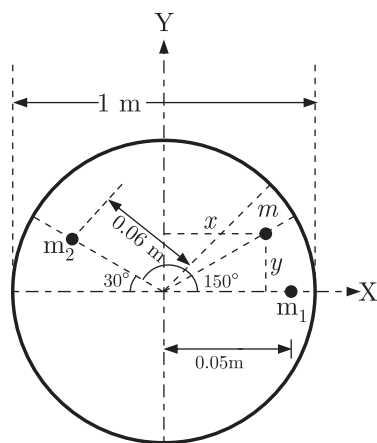
$$0.01 = 0 + \frac{1}{2} \times a \times \left(\frac{1}{60}\right)^2$$

$$a = 0.01 \times 2 \times (60)^2 = 72 \text{ m/sec}^2$$

Maximum velocity,

$$v_{\max} = u + at = 72 \times \frac{1}{60} = 1.2 \text{ m/sec}$$

**SOL 4.50** Option (C) is correct.



Given  $m_1 = m_2 = 0.5 \text{ kg}$ ,  $r_1 = 0.05 \text{ m}$ ,  $r_2 = 0.06 \text{ m}$

Balancing mass  $m = 0.1 \text{ kg}$

Let disc rotates with uniform angular velocity  $\omega$  and  $x$  &  $y$  is the position of balancing mass along  $X$  &  $Y$  axis.

Resolving the forces in the  $x$ -direction, we get

$$\Sigma F_x = 0$$

$$0.5[-0.06 \cos 30^\circ + 0.05 \cos 0^\circ]\omega^2 = 0.1 \times x \times \omega^2$$

$$0.5 \times (-0.00196) = 0.1x \quad F_c = mr\omega^2$$

$$x = -0.0098 \text{ m} = -9.8 \text{ mm}$$

Similarly in  $y$ -direction,

$$\Sigma F_y = 0$$

$$0.5(0.06 \times \sin 30^\circ + 0.05 \times \sin 0^\circ)\omega^2 = 0.1 \times y \times \omega^2$$

$$0.5 \times 0.03 = 0.1 \times y$$

$$y = 0.15 \text{ m} = 150 \text{ mm}$$

Position of balancing mass is given by,  $r = \sqrt{x^2 + y^2} = \sqrt{(-9.8)^2 + (150)^2}$   
 $= 150.31 \text{ mm} \simeq 150 \text{ mm}$

**SOL 4.51** Option (C) is correct.

Given  $m = 0.1 \text{ kg}$ ,  $k = 1 \text{ kN/m}$

Let,  $\omega_d$  be the frequency of damped vibration &  $\omega_n$  be the natural frequency of spring mass system.

Hence,  $\omega_d = 90\%$  of  $\omega_n = 0.9\omega_n$  (Given) ... (i)

Frequency of damped vibration

$$\omega_d = \sqrt{(1 - \varepsilon^2)} \omega_n \quad \dots (ii)$$

From equation (i) and equation (ii), we get

$$\sqrt{(1 - \varepsilon^2)} \omega_n = 0.9\omega_n$$

On squaring both the sides, we get

$$1 - \varepsilon^2 = (0.9)^2 = 0.81$$

$$\varepsilon^2 = 1 - 0.81 = 0.19$$

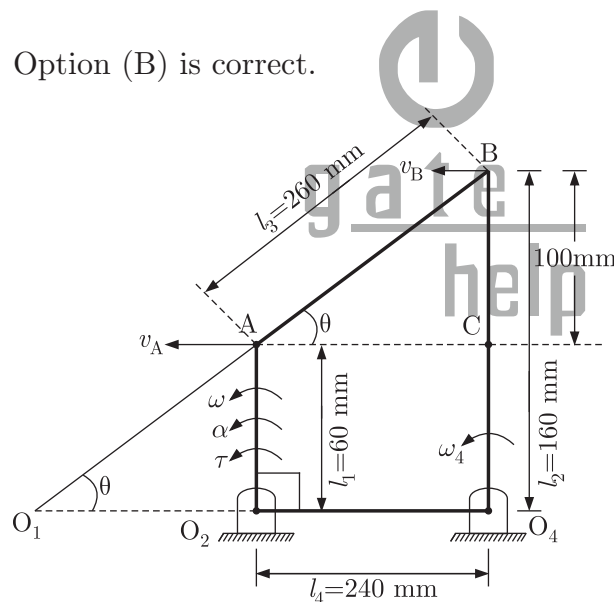
$$\varepsilon = \sqrt{0.19} = 0.436$$

And Damping ratio is given by,

$$\varepsilon = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$$

$$c = 2\sqrt{km} \times \varepsilon = 2\sqrt{1000 \times 0.1} \times 0.436 = 8.72 \text{ Ns/m} \simeq 8.7 \text{ Ns/m}$$

**SOL 4.52** Option (B) is correct.



From Triangle  $ABC$ ,

$$AB = \sqrt{(100)^2 + (240)^2} = \sqrt{67600} = 260 \text{ mm}$$

Length of shortest link  $l_1 = 60 \text{ mm}$

Length of longest link  $l_3 = 260 \text{ mm}$

From the Grashof's law,

$$l_1 + l_3 \ngtr l_2 + l_4$$

$$60 + 260 \ngtr 160 + 240$$

$$320 \ngtr 400$$

So,  $l_1 + l_3 < l_2 + l_4$

Also, when the shortest link  $O_2A$  will make a complete revolution relative to other three links, if it satisfies the Grashof's law. Such a link is known as crank. The link  $O_4B$  which makes a partial rotation or oscillates is known as rocker. So, crank rocker mechanism is obtained.

Here,  $O_2A = l_1 = 60 \text{ mm}$  is crank (fixed link)

Adjacent link,  $O_2O_4 = 240 \text{ mm}$  is fixed

So, crank rocker mechanism will be obtained.

**SOL 4.53** Option (B) is correct.

Let,  $\omega_4$  is the angular velocity of link  $O_4B$

From the triangle  $ABC$ ,

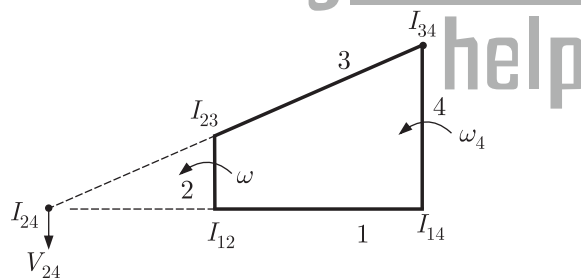
$$\tan \theta = \frac{100}{240} = \frac{5}{12} \quad \dots(i)$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

Also from the triangle  $O_1O_2A$ ,

$$\tan \theta = \frac{O_2A}{O_1O_2}$$

$$O_1O_2 = \frac{O_2A}{\tan \theta} = \frac{60}{\frac{5}{12}} = 144 \text{ mm}$$



From the angular velocity ratio theorem.

$$V_{24} = \omega_4 \times I_{24}I_{14} = \omega \times I_{24}I_{12}$$

$$\omega_4 = \frac{I_{24}I_{12}}{I_{24}I_{14}} \times \omega = \frac{144}{(240 + 144)} \times 8 = \frac{144}{384} \times 8 = 3 \text{ rad/sec}$$

**SOL 4.54** Option (D) is correct.

From the given data the component of force at joint A along  $AO_2$  is necessary to find the joint reaction at  $O_2$ . So, it is not possible to find the magnitude of the joint reaction at  $O_2$ .

**SOL 4.55** Option (D) is correct.

Mechanical advantage in the form of torque is given by,

$$M.A. = \frac{T_{output}}{T_{input}} = \frac{\omega_{input}}{\omega_{output}}$$

Here output link is a slider, So,  $\omega_{output} = 0$

Therefore,  $M.A. = \infty$

**SOL 4.56** Option (C) is correct.

Given  $\frac{\omega}{\omega_n} = r = 0.5$

And due to isolation damping ratio,

$$\varepsilon = \frac{c}{c_c} = 0 \quad \text{For isolation } c = 0$$

We know the transmissibility ratio of isolation is given by,

$$T.R. = \frac{\sqrt{1 + \left(2\varepsilon \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varepsilon \frac{\omega}{\omega_n}\right]^2}} = \frac{\sqrt{1 + 0}}{\sqrt{[1 - (0.5)^2]^2 + 0}} = \frac{1}{0.75} = \frac{4}{3}$$

**SOL 4.57** Option (D) is correct.

Given planar mechanism has degree of freedom,  $N = 1$  and two infinite parallel lines meet at infinity. So, the instantaneous centre  $I_{24}$  will be at  $N$ , but for single degree of freedom, system moves only in one direction.

Hence,  $I_{24}$  is located at infinity( $\infty$ ).

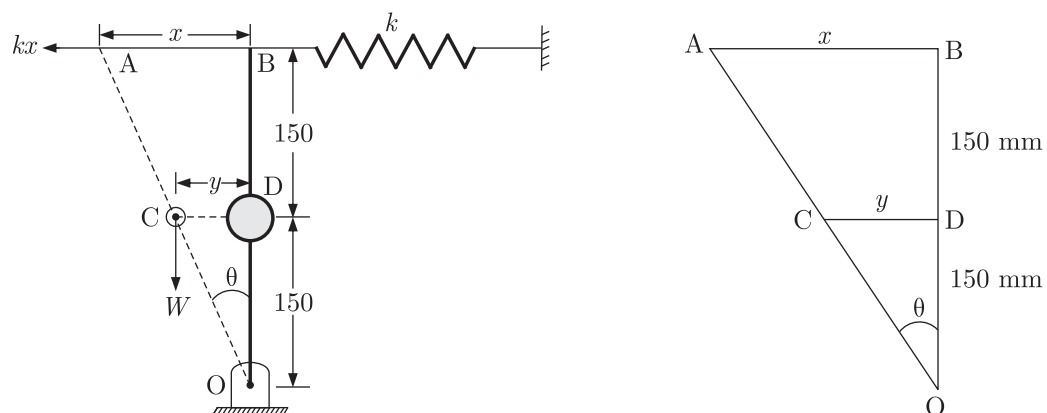
**SOL 4.58** Option (A) is correct.

Given  $N_2 = 120 \text{ rpm}$ ,  $v_1 = 12 \text{ m/sec}$

So, coriolis component of the acceleration of link 1 is,

$$a_{12}^c = 2\omega_2 v_1 = 2 \times \frac{2\pi \times 120}{60} \times 12 = 301.44 \text{ m/s}^2 \simeq 302 \text{ m/s}^2$$

**SOL 4.59** Option (C) is correct.





Given  $l = 300 \text{ mm} = 0.3 \text{ m}$ ,  $W = 300 \text{ N}$

Let, rod is twisted to the left, through an angle  $\theta$ .

From the similar triangle  $OCD$  &  $OAB$ ,

$$\tan \theta = \frac{y}{0.15} = \frac{x}{0.30}$$

If  $\theta$  is very very small, then  $\tan \theta \simeq \theta = \frac{y}{0.15} = \frac{x}{0.30}$

$$x = 0.30\theta \text{ and } y = 0.15\theta \quad \text{..(i)}$$

On taking moment about the hinged point  $O$

$$kx \times 300 + W \times y = 0$$

$$k = -\frac{Wy}{300x} = -\frac{300}{300} \times \left(\frac{y}{x}\right) = -\frac{1}{2} = -0.5 \text{ N/mm}$$

From equation (i)  $\frac{y}{x} = \frac{0.15\theta}{0.30\theta} = -500 \text{ N/m}$

Negative sign shows that the spring tends to move to the point B.

In magnitude,  $k = 500 \text{ N/m}$

**SOL 4.60** Option (C) is correct.

#### Types of Mechanisms

#### Motion Achieved

<b>P.</b>	Scott-Russel Mechanism	<b>4.</b>	Straight Line Motion
<b>Q.</b>	Geneva Mechanism	<b>1.</b>	Intermittent Motion
<b>R.</b>	Off-set slider-crank Mechanism	<b>2.</b>	Quick Return Mechanism
<b>S.</b>	Scotch Yoke Mechanism	<b>3.</b>	Simple Harmonic Motion

So, correct pairs are, P-4, Q-1, R-2, S-3

**SOL 4.61** Option (C) is correct.

#### Types of Joint

#### Degree of constraints

<b>P.</b>	Revolute	<b>2.</b>	Five
<b>Q.</b>	Cylindrical	<b>3.</b>	Four
<b>R.</b>	Spherical	<b>1.</b>	Three

So, correct pairs are P-2, Q-3, R-1

**SOL 4.62** Option (A) is correct.

Given  $M = 20 \text{ kg}$ ,  $l = 1000 \text{ mm} = 1 \text{ m}$ ,  $A = 25 \times 25 \text{ mm}^2$

$E_{\text{steel}} = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$

Mass moment of inertia of  $a$  square section is given by,

$$I = \frac{b^4}{12} = \frac{(25 \times 10^{-3})^4}{12} = 3.25 \times 10^{-8} \text{ m}^4$$

Deflection of a cantilever, Loaded with a point load placed at the free end is,

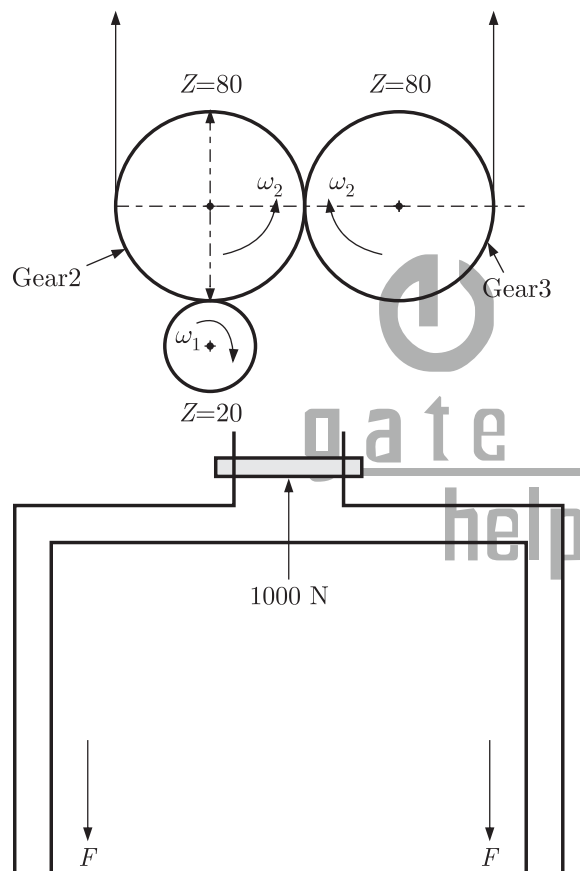
$$\delta = \frac{Wl^3}{3EI} = \frac{mgl^3}{3EI} = \frac{20 \times 9.81 \times (1)^3}{3 \times 200 \times 10^9 \times 3.25 \times 10^{-8}} = \frac{196.2}{19500} = 0.01 \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.01}} = 31.32 \text{ rad/sec}$$

Therefore, critical damping constant

$$\begin{aligned} c_c &= 2M\omega_n = 2 \times 20 \times 31.32 \\ &= 1252.8 \text{ Ns/m} \simeq 1250 \text{ Ns/m} \end{aligned}$$

**SOL 4.63** Option (B) is correct.



Let,  $Z$  is the number of teeth and motor rotates with an angular velocity  $\omega_1$  in clockwise direction & develops a torque  $T_1$ .

Due to the rotation of motor, the gear 2 rotates in anti-clockwise direction & gear 3 rotates in clockwise direction with the same angular speed.

Let,  $T_2$  is the torque developed by gear.

Now, for two equal size big gears,

$$\text{Module } m = \frac{D}{Z} = \frac{(\text{Pitch circle diameter})}{(\text{No. of teeth})}$$

$$D = mZ = 2 \times 80 = 160 \text{ mm}$$

(Due to rotation of gear 2 & gear 3 an equal force ( $F$ ) is generated in the downward direction because teeth are same for both the gears)

For equilibrium condition, we have

Downward force = upward force

$$F + F = 1000$$

$$F = 500 \text{ N}$$

And 
$$\eta = \frac{\text{Power Output}}{\text{Power Input}} = \frac{2 \times T_2 \omega_2}{T_1 \omega_1}$$

Output power is generated by the two gears

$$= \frac{2 \times \left(F \times \frac{D}{2}\right) \omega_2}{T_1 \omega_1} \quad \dots(i)$$

We know velocity ratio is given by

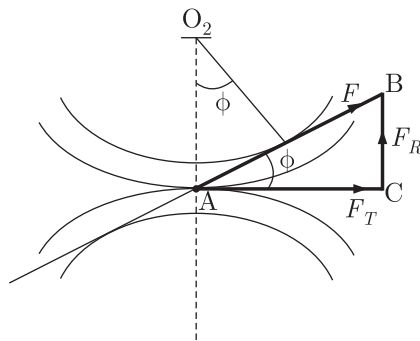
$$\frac{N_1}{N_2} = \frac{\omega_1}{\omega_2} = \frac{Z_2}{Z_1} \quad \omega = \frac{2\pi N}{60}$$

From equation (i), 
$$\eta = \frac{2 \times \left(F \times \frac{D}{2}\right)}{T_1} \times \frac{Z_1}{Z_2}$$

$$T_1 = \frac{F \times D}{\eta} \times \left(\frac{Z_1}{Z_2}\right) = \frac{500 \times 0.160}{0.8} \times \frac{20}{80} = 25 \text{ N-m}$$

**SOL 4.64**

Option (C) is correct.



Given pressure angle  $\phi = 20^\circ$ ,  $F_T = 500 \text{ N}$  from previous question.

From the given figure we easily see that force action along the line of action is  $F$ .

From the triangle  $ABC$ ,

$$\cos \phi = \frac{F_T}{F}$$

$$F = \frac{F_T}{\cos \phi} = \frac{500}{\cos 20^\circ} = 532 \text{ N}$$

**SOL 4.65**

Option (D) is correct.

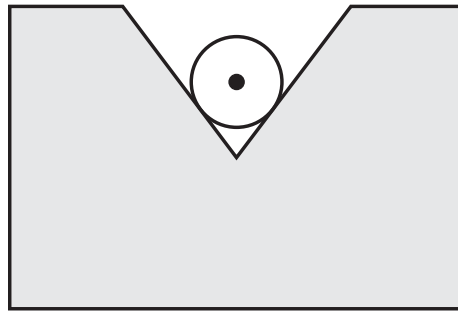
A single slider crank chain is a modification of the basic four bar chain. It is found, that four inversions of a single slider crank chain are possible. From these four inversions, crank and slotted lever quick return motion mechanism is used in shaping machines, slotting machines and in rotary internal combustion engines.

**SOL 4.66** Option (A) is correct.

Given  $p < q < r < s$

“Double crank” mechanism occurs, when the shortest link is fixed. From the given pairs  $p$  is the shortest link. So, link of length  $p$  should be fixed.

**SOL 4.67** Option (B) is correct.



We clearly see from the figure that cylinder can either revolve about  $x$ -axis or slide along  $x$ -axis & all the motions are restricted.

Hence, Number of degrees of freedom = 2 & movability includes the six degrees of freedom of the device as a whole, as the ground link were not fixed. So, 4 degrees of freedom are constrained or arrested.

**SOL 4.68** Option (B) is correct.

Given  $N = 1200$  rpm,  $\Delta E = 2$  kJ = 2000 J,  $D = 1$  m,  $C_s = 0.02$

Mean angular speed of engine,

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 1200}{60} = 125.66 \text{ rad/sec}$$

Fluctuation of energy of the flywheel is given by,

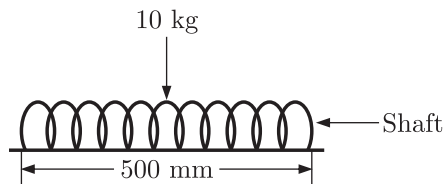
$$\Delta E = I\omega^2 C_s = \frac{1}{2} m R^2 \omega^2 C_s \quad \text{For solid disc } I = \frac{m R^2}{2}$$

$$\begin{aligned} m &= \frac{2\Delta E}{R^2 \omega^2 C_s} = \frac{2 \times 2000}{\left(\frac{1}{2}\right)^2 \times (125.66)^2 \times 0.02} \\ &= \frac{4 \times 2 \times 2000}{(125.66)^2 \times 0.02} = 50.66 \text{ kg} \simeq 51 \text{ kg} \end{aligned}$$

**SOL 4.69** Option (B) is correct.

Given  $m = 10$  kg,  $d = 30$  mm = 0.03 m,  $l = 500$  mm = 0.5 m,

$$E_{\text{shaft}} = 2.1 \times 10^{11} \text{ Pa}$$



We know that, static deflection due to 10 kg of Mass at the centre is given by,

$$\delta = \frac{Wl^3}{48EI} = \frac{mgl^3}{48EI} \quad \dots(i)$$

The moment of inertia of the shaft,

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.03)^4 = 3.974 \times 10^{-8} \text{ m}^4 \quad \dots(ii)$$

Substitute values in equation (i), we get

$$\begin{aligned} \delta &= \frac{10 \times 9.81 \times (0.5)^3}{48 \times 2.1 \times 10^{11} \times 3.974 \times 10^{-8}} \\ &= \frac{12.2625}{400.58 \times 10^3} = 3.06 \times 10^{-5} \text{ m} \end{aligned}$$

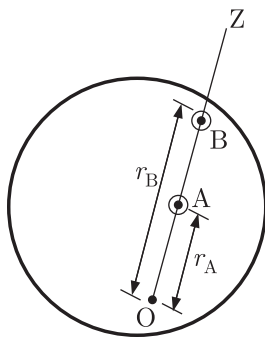
If  $\omega_c$  is the critical or whirling speed in r.p.s. then,

$$\begin{aligned} \omega_c &= \sqrt{\frac{g}{\delta}} \Rightarrow 2\pi f_c = \sqrt{\frac{g}{\delta}} \\ f_c &= \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{1}{2 \times 3.14} \sqrt{\frac{9.81}{3.06 \times 10^{-5}}} \\ &= \frac{1}{6.28} \sqrt{\frac{9.81}{30.6 \times 10^{-6}}} = 90.16 \text{ Hz} \simeq 90 \text{ Hz} \end{aligned}$$

#### SOL 4.70

Option (C) is correct.

Given, the circular disc rotates about the point O at a uniform angular velocity  $\omega$ .



Let  $v_A$  is the linear velocity of point A &  $v_B$  is the linear velocity of point B.

$$v_A = \omega r_A \text{ and } v_B = \omega r_B.$$

Velocity of point B with respect to point A is given by,

$$v_{BA} = v_B - v_A = \omega r_B - \omega r_A = \omega(r_B - r_A)$$

From the given figure,

$$r_B > r_A$$

So,  $\omega r_B > \omega r_A$

$$v_B > v_A$$

Therefore, relative velocity  $\omega(r_B - r_A)$  in the direction of point B.

**SOL 4.71** Option (D) is correct.

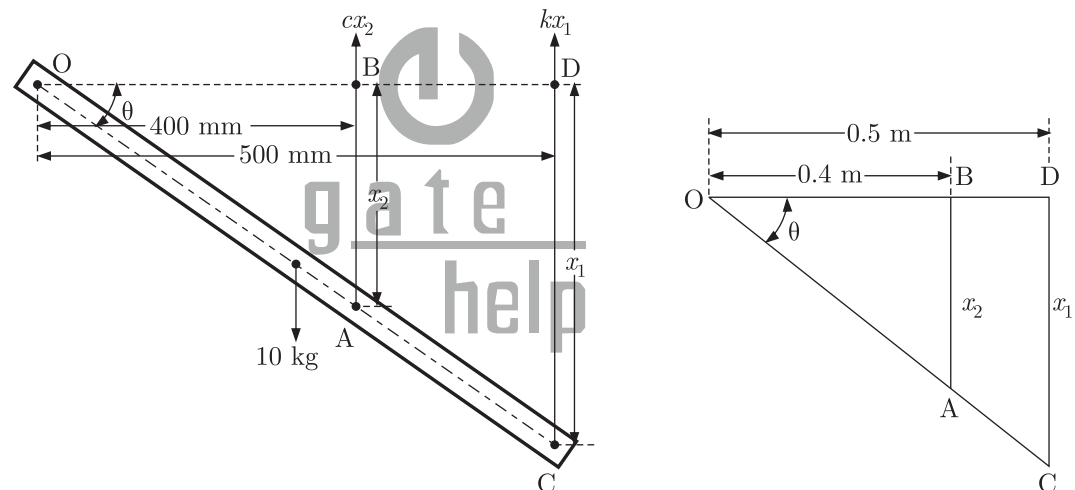
Acceleration of point B with respect to point A is given by,

$$a_{BA} = \omega v_{BA} = \omega \times \omega(r_B - r_A) = \omega^2(r_B - r_A) \quad \dots(i)$$

This equation (i) gives the value of centripetal acceleration which acts always towards the centre of rotation.

So,  $a_{BA}$  acts towards to  $O$  i.e. its direction from  $Z$  to  $O$

**SOL 4.72** Option (A) is correct.



Given  $m = 10 \text{ kg}$ ,  $k = 2 \text{ kN/m}$ ,  $c = 500 \text{ Ns/m}$ ,  $k_\theta = 1 \text{ kN/m/rad}$

$l_1 = 0.5 \text{ m}$ ,  $l_2 = 0.4 \text{ m}$

Let, the rigid slender bar twist downward at the angle  $\theta$ . Now spring & damper exert a force  $kx_1$  &  $cx_2$  on the rigid bar in the upward direction.

From similar triangle  $OAB$  &  $OCD$ ,

$$\tan \theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

Let  $\theta$  be very very small, then  $\tan \theta \simeq \theta$ ,

$$\theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

$$x_2 = 0.4\theta \text{ or } x_1 = 0.5\theta \quad \dots(i)$$

On differentiating the above equation, we get

$$\dot{x}_2 = 0.4\dot{\theta} \text{ or } \dot{x}_1 = 0.5\dot{\theta} \quad \dots(\text{ii})$$

We know, the moment of inertia of the bar hinged at the one end is,

$$I = \frac{ml_1^2}{3} = \frac{10 \times (0.5)^2}{3} = 0.833 \text{ kg} \cdot \text{m}^2$$

As no external force acting on the system. So, governing equation of motion from the Newton's law of motion is,

$$I\ddot{\theta} + c\dot{x}_2 l_2 + kx_1 l_1 + k_\theta \theta = 0$$

$$0.833\ddot{\theta} + 500 \times 0.4\dot{x}_2 + 2000 \times (0.5)x_1 + 1000\theta = 0$$

$$0.833\ddot{\theta} + 200\dot{x}_2 + 1000x_1 + 1000\theta = 0 \quad \dots(\text{iii})$$

$$0.833\ddot{\theta} + 200 \times 0.4\dot{\theta} + 1000 \times 0.5\theta + 1000\theta = 0$$

$$0.833\ddot{\theta} + 80\dot{\theta} + 1500\theta = 0 \quad \dots(\text{iv})$$

On comparing equation (iv) with its general equation,

$$I\ddot{\theta} + c\dot{\theta} + k\theta = 0$$

We get,  $I = 0.833$ ,  $c = 80$ ,  $k = 1500$

So, undamped natural frequency of oscillations is given by

$$\omega_n = \sqrt{\frac{k}{I}} = \sqrt{\frac{1500}{0.833}} = \sqrt{1800.72} = 42.43 \text{ rad/sec}$$

**SOL 4.73** Option (C) is correct.

From the previous part of the question

Damping coefficient,  $c = 80 \text{ Nms/rad}$

**SOL 4.74** Option (C) is correct.

From the Kutzbach criterion the degree of freedom,

$$n = 3(l - 1) - 2j - h$$

For single degree of Freedom ( $n = 1$ ),

$$1 = 3(l - 1) - 2j - h$$

$$3l - 2j - 4 - h = 0 \quad \dots(\text{i})$$

The simplest possible mechanisms of single degree of freedom is four-bar mechanism. For this mechanism  $j = 4$ ,  $h = 0$

From equation (i), we have

$$3l - 2 \times 4 - 4 - 0 = 0 \Rightarrow l = 4$$

**SOL 4.75** Option (B) is correct.

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated. Quick return motion mechanism is used in shaping machines, slotting machines and in rotary internal combustion engines.

**SOL 4.76** Option (C) is correct.

The deflection of a cantilever beam loaded at the free end is given by,

$$\delta = \frac{MgL^3}{3EI}$$

And natural frequency,

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{3EI}{ML^3}} \quad \dots(i)$$

If the length of the cantilever beam is halved, then

$$\omega_n' = \sqrt{\frac{3EI}{M \times \left(\frac{L}{2}\right)^3}} = \sqrt{8 \left(\frac{3EI}{ML^3}\right)}$$

From equation (i)

$$\omega_n' = \sqrt{8} \omega_n$$

So, natural frequency is increased by a factor  $\sqrt{8}$ .

**SOL 4.77** Option (C) is correct.

For a spring loaded roller follower driven with a disc cam, the pressure angle should be large during rise as well as during return for ease of transmitting motion.

If pressure angle is large, then side thrust will be minimum. Pressure angles of up to about  $30^\circ$  to  $35^\circ$  are about the largest that can be used without causing difficulties.

**SOL 4.78** Option (B) is correct.

Let initial length of the spring =  $L$

Potential energy at  $A$ ,

$$PE_A = mg(L - \delta)$$

and at  $B$ , 
$$PE_B = mg[L - (\delta + x)] + \frac{1}{2}kx^2$$

So, change in potential energy from position  $A$  to position  $B$  is

$$\begin{aligned} \Delta PE_{AB} &= PE_B - PE_A \\ &= mgL - mg\delta - mgx + \frac{1}{2}kx^2 - mgL + mg\delta \end{aligned}$$

$$\Delta PE_{AB} = \frac{1}{2}kx^2 - mgx$$

**SOL 4.79** Option (A) is correct.

The mean speed of the engine is controlled by the governor. If load increases then fluid supply increases by the governor and vice-versa.

Flywheel stores the extra energy and delivers it when needed. So, Flywheel reduces speed fluctuations.



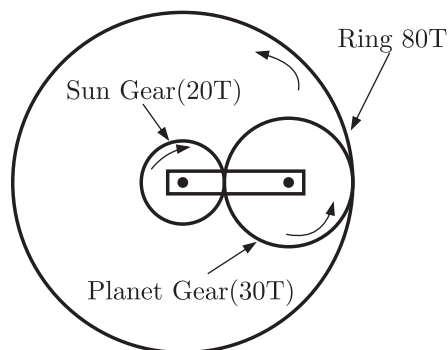
Flywheel reduce speed fluctuations during a cycle for a constant load, but Flywheel does not control the mean speed  $\left(N = \frac{N_1 + N_2}{2}\right)$  of the engine.

**SOL 4.80**

Option (B) is correct.

First make the table for the motion of the gears.

Take CW = +ve, CCW = -ve



S. No.	Condition of Motion	Arm	Sun Gear $N_S$	Planet Gear $N_P$	Ring Gear $N_G$
(i)	Arm is fixed & sun gear rotates +1 rpm (CW)	0	+1	$-\frac{Z_S}{Z_P}$	$-\frac{Z_S}{Z_R}$
(ii)	Sun Gear rotates through +x rpm (CW)	0	+x	$-x\frac{Z_S}{Z_P}$	$-x\frac{Z_S}{Z_R}$
(iii)	Add +y revolution to all elements	+y	+y	+y	+y
(iv)	Total Motion	+y	x + y	$y - x\frac{Z_S}{Z_P}$	$y - x\frac{Z_S}{Z_R}$

Let Teethes and speed of the sum gear, planet gear and ring gear is represented by  $Z_G$ ,  $Z_P$ ,  $Z_R$  and  $N_G$ ,  $N_P$ ,  $N_R$  respectively.

Given sun gear is driven clockwise at 100 rpm. So, From the table

$$x + y = 100 \quad \dots(i)$$

Ring gear is held stationary. From the table

$$y - x\frac{Z_S}{Z_P} = 0$$

$$y = x \times \frac{20}{80}$$

$$y = \frac{x}{4} \Rightarrow x = 4y \quad \dots(ii)$$

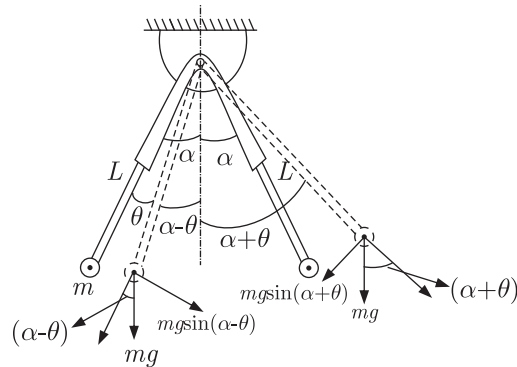
From equation on (i) and (ii)

$$4y + y = 100$$

$$y = 20 \text{ rpm}$$

**SOL 4.81** Option (D) is correct.

Give a small displacement  $\theta$  to the assembly. So assembly oscillates about its mean position.



From this a restoring torque is acts along the line of oscillation.

Net restoring torque,

$$T = mg \sin(\alpha + \theta) \times L - mg \sin(\alpha - \theta) \times L$$

$$T = mgL[\sin \alpha \cos \theta + \cos \alpha \sin \theta - \sin \alpha \cos \theta + \cos \alpha \sin \theta]$$

$$T = 2mgL \cos \alpha \sin \theta$$

For very small deflection  $\theta$ ,

$$\sin \theta \cong \theta$$

$$T = 2mgL\theta \cos \alpha$$

Now from newton's law,

$$I\ddot{\theta} + T = 0$$

$$I\ddot{\theta} + 2mgL\theta \cos \alpha = 0$$

$$2mL^2 \frac{d^2\theta}{dt^2} + (2mgL \cos \alpha) \theta = 0$$

$$I = mL^2 + mL^2$$

$$\frac{d^2\theta}{dt^2} + \frac{g \cos \alpha}{L} \theta = 0$$

On comparing with  $\ddot{\theta} + \omega_n^2 \theta = 0$ , we get

$$\omega_n^2 = \frac{g \cos \alpha}{L}$$

$$\omega_n = \sqrt{\frac{g \cos \alpha}{L}}$$

\*\*\*\*\*

# **GATE Multiple Choice Questions For Mechanical Engineering**

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## **UNIT 9. Metal Casting:**

Design of patterns, moulds and cores; solidification and cooling; riser and gating design, design considerations.

## **UNIT 10. Forming:**

Plastic deformation and yield criteria; fundamentals of hot and cold working processes; load estimation for bulk (forging, rolling, extrusion, drawing) and sheet (shearing, deep drawing, bending) metal forming processes; principles of powder metallurgy.

**UNIT 11. Joining:**

Physics of welding, brazing and soldering; adhesive bonding; design considerations in welding.

**UNIT 12. Machining and Machine Tool Operations:**

Mechanics of machining, single and multi-point cutting tools, tool geometry and materials, tool life and wear; economics of machining; principles of non-traditional machining processes; principles of work holding, principles of design of jigs and fixtures

**UNIT 13. Metrology and Inspection:**

Limits, fits and tolerances; linear and angular measurements; comparators; gauge design; interferometry; form and finish measurement; alignment and testing methods; tolerance analysis in manufacturing and assembly.

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Forecasting models, aggregate production planning, scheduling, materials requirement planning

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