

# SOM

## Handwritten Notes for

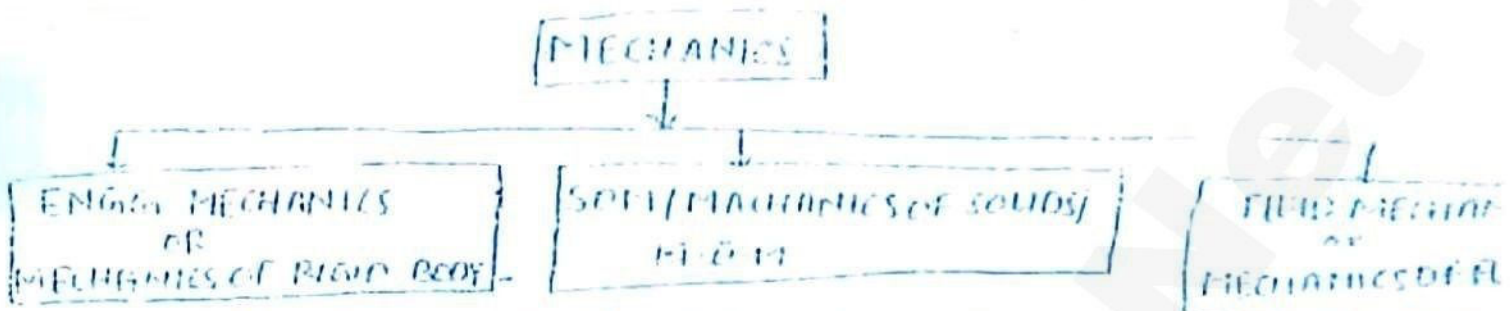
## Mechanical Engg.

### Topic wise Notes Free Download



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## Fundamental of SOM and Machine Design

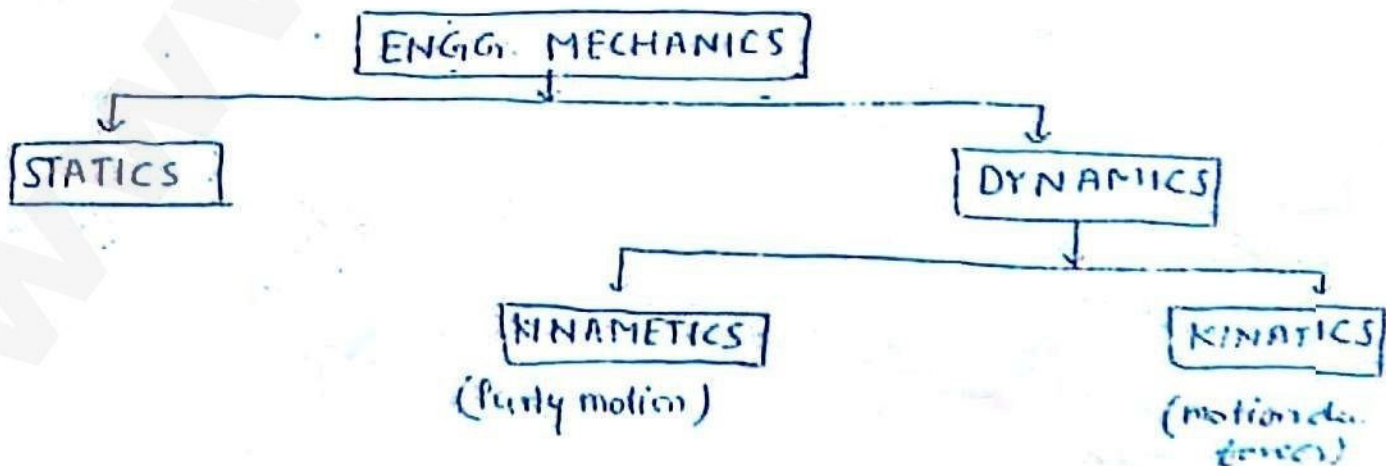
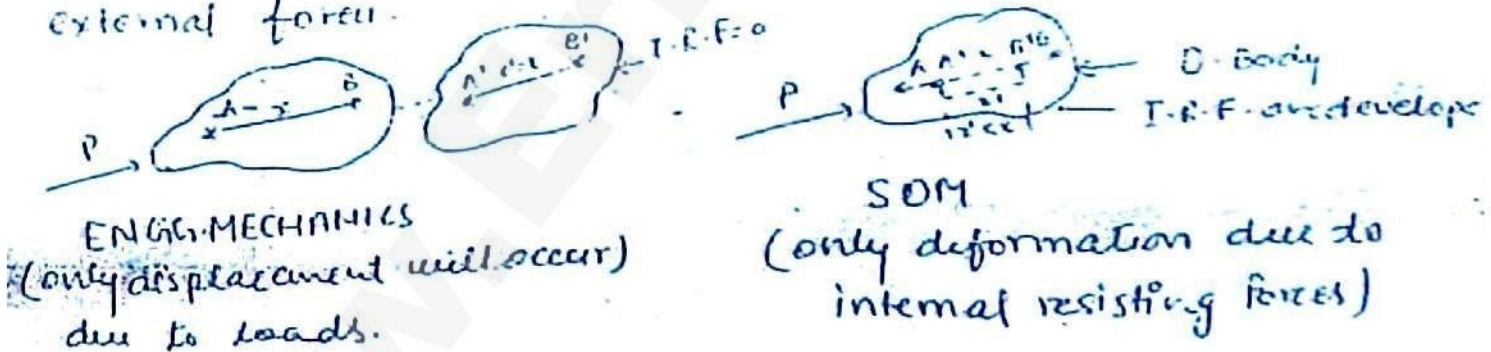


+ Branch of Science in which study about the force which is imparted on any body. Forces will be external & internal. External forces is known as load and internal force is known as stress. They affect on structures (beams, column, etc elements) and fluids.

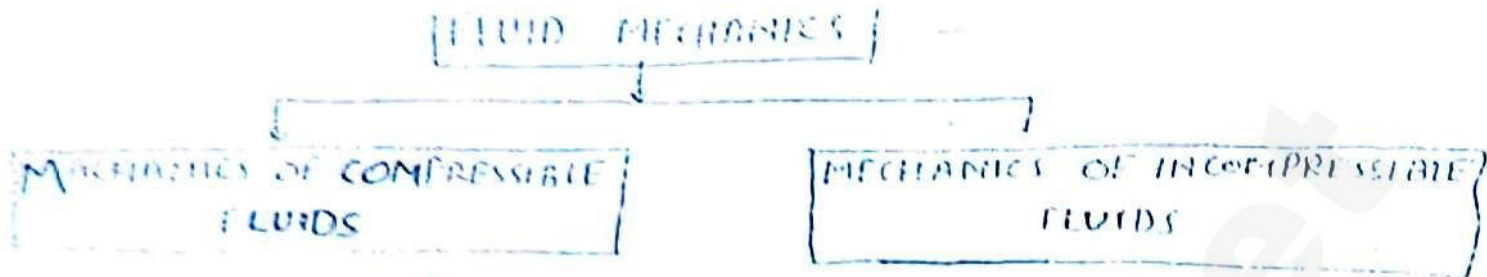
In Engg mechanics, we calculate external force applying on the rigid body.

In S.O.M, we calculate force applying on deformed body (internal resisting forces).

These force developed because of deformation due to some external force.

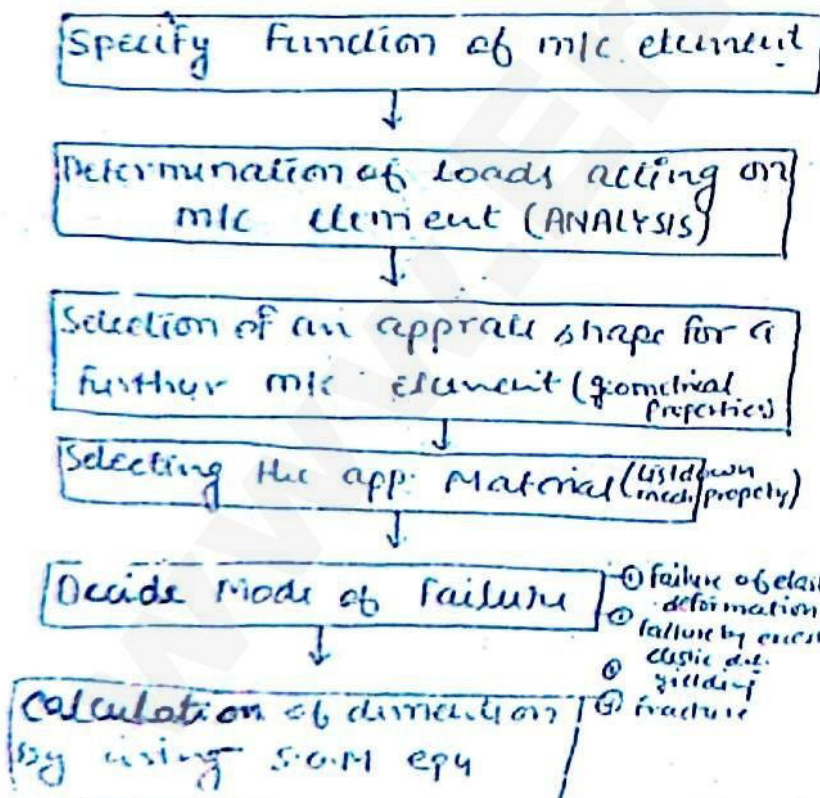






- Engg. design is defined as iterative decision making activity to produce a drawing or plan to convert resources optimally into a product or device or a system to satisfy human need.
- The ultimate aim of design is to select an appropriate shape, material, size, manufacturing process details in such a way that resulting machine component should perform its given objective or function satisfactorily. (4)
- A m/c component is said to be failure when it does not perform given objective satisfactorily.

### ○ STEPS USED IN DESIGN OF M/C ELEMENT →



#### Explanation

#### Shaft design →

1.  $P = x \text{ kW at } Y \text{ rpm}$

2. Twisting Moment =  $\frac{P \times 60 \times 10^6}{2 \pi N}$   $\frac{\text{N-m}}{\text{Nmm}}$

3. Circular C/S selected based on

On  $T = Z_p \tau_s$   $Z_{p \text{ circular}} > Z_{p \text{ non circular}}$

for diameter  $Z_{p \text{ solid}} > Z_{p \text{ hollow}}$

for weight (Space Utilization)  $Z_{p \text{ hollow}} > Z_{p \text{ solid}}$   $(\text{weight consideration})$

$A_s = \frac{\pi}{4} d^4$ ,  $A_h = \frac{\pi}{4} (D^4 - d^4)$   $\left. \begin{matrix} \text{geometrical} \\ \text{property} \end{matrix} \right\}$

$I = \frac{\pi}{64} d^4$ ,  $J = \frac{\pi}{32} d^4$

$Z_p = \frac{I}{r} = \frac{K}{16} d^3$ ,  $Z = \frac{J}{\rho}$

4. Shaft are always ductile mat.  
○ Mild steel (low  $\sigma_y$  &  $\sigma_u$ )  
(low  $\tau$ )

(2)



prepare the drawing (Part),  
Assy. drawing

(b) Alloy steel  $\rightarrow$  Nickel, Chromium, Vanadium (2)  
(Provide more strength) but expensive  
(c)  $T = 2000 = \frac{1}{1600} \frac{\text{Srs}}{\text{N}}$   
 $d = ?$

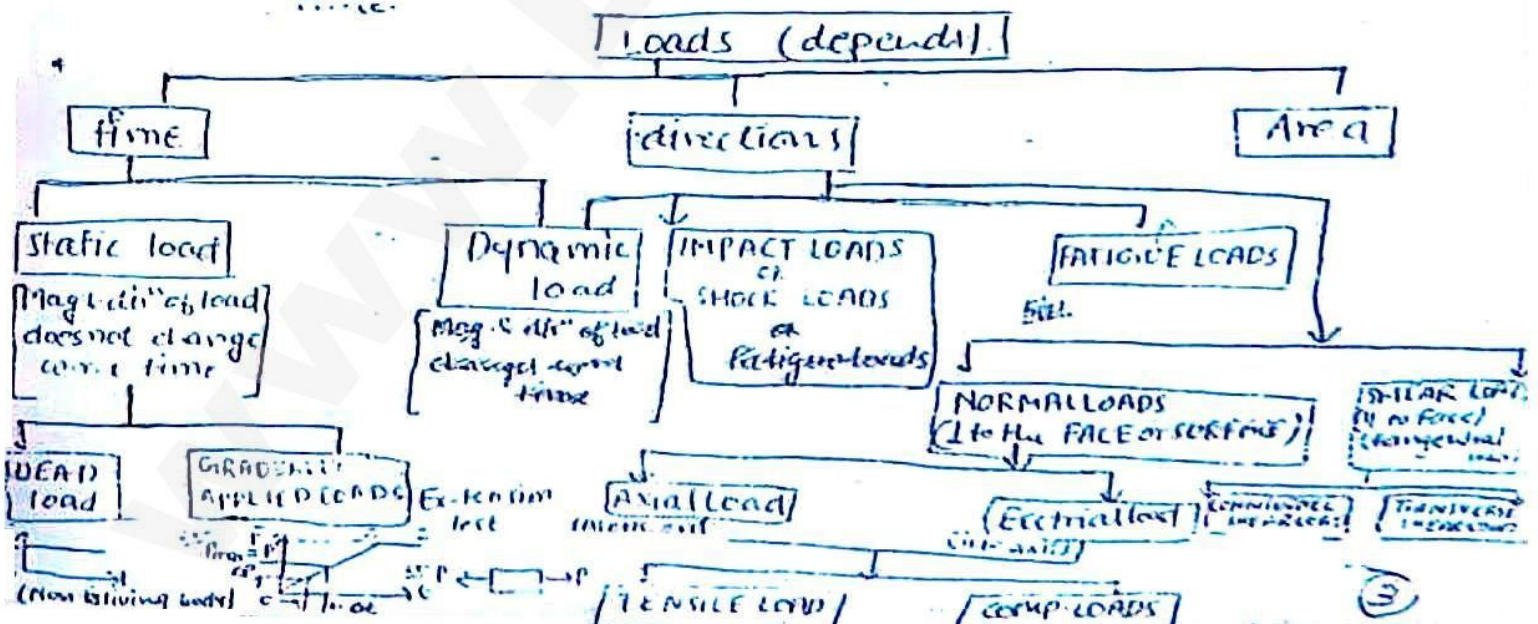
LOADS  $\rightarrow$  An external force / couple to which a component is subjected when it coming into service.

Example  $\rightarrow$  weight of a body, which is pressure, Twisting moment, Bending Moment, Gas loads, frictional forces, inertia forces, centrifugal force, wind pressure etc.  
\* All the forces are surface forces. Surface forces acting on area and the self weight (body force) acting on volume.  
( $\gamma \times \text{Volume}$ )

Surface force  $\rightarrow P = \sigma(A)$

Self force  $\rightarrow P = \gamma \times \text{Volume}$

- \* Loads are vector quantity (magnitude & direction) [1 order tensor]
- \* Tensor are quantities which have more directions.  
(stress) (2D), (vector) (strain).
- \* Scalar quantities have only magnitude [0 order tensor]
- \* Stress and strains are 2 order tensor.
- \* Loads are depend on direction, area where is loaded & time.



## 0 IMPACT LOADS

Impact loads are those loads which are acting for on a m/c component for a short interval of time.

Ex- Shock Absorbers, punching etc.

\* Dynamic loads considered vibration.

\* If  $\boxed{t < \frac{T}{2}}$  is satisfied then load will impact load.  
times load applied cycle time

If  $\boxed{t > 3T} \Rightarrow$  static load

If  $\boxed{t = 2T}$  then we will assume impact becoz, more stress is produced.

\* Whenever there is a doubt regarding static and impact load  $\boxed{\frac{T}{2} < t < 3T}$  (6)

It is better to assume impact load becoz it is worst load. ( $\sigma_{\text{impact}} > \sigma_{\text{static}}$ )

$$\boxed{\sigma_{\text{impact}} = \sigma_{\text{static load}} \times \text{impact factor}}$$

$$\boxed{\text{Impact Factor} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}}$$

$$\boxed{I.F. \geq 2}$$

deformation

$$\boxed{\sigma_{\text{impact}} \geq 2 \sigma_{\text{static}}}$$

\* When  $h=0$ , impact load assumed as suddenly applied loads ( $I.F.=2$ ) then  $\boxed{\sigma_{\text{suddenly}} = 2 \sigma_{\text{static}}}$

For the calculation of  $h$ , we will calculate as

(i)  $\delta_{st} = PL/AE$

(ii)  $I.F.$

(iii)  $\sigma_{st}$

(iv)  $\sigma_{\text{impact}}$

(4)



$$hI = vI = 1+1 = 2 \text{ impact } I = \text{chance of failure } I' \left[ h = \frac{V^2}{2g} \right]$$

$$\text{when } h=0 \Rightarrow v=0 \Rightarrow I \cdot F = 2$$

$$\text{Then } I \cdot F = 1 + \sqrt{1 + \frac{V^2}{g S_{SI}}}$$

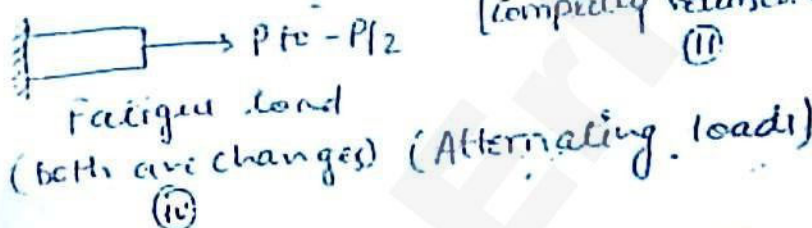
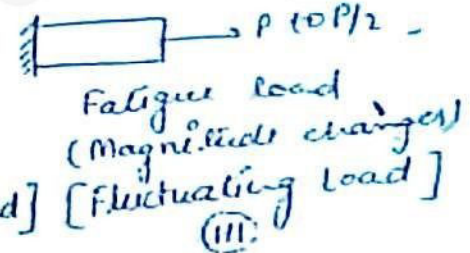
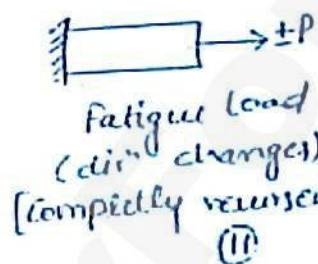
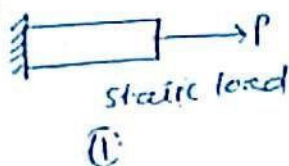
$$\left. \begin{aligned} \sigma_a &= P/A \\ \tau_b &= M/Z \\ \tau_s &= T/r \end{aligned} \right\} \sigma_{\text{static}}$$

$$S_{SI} = PL/AE$$

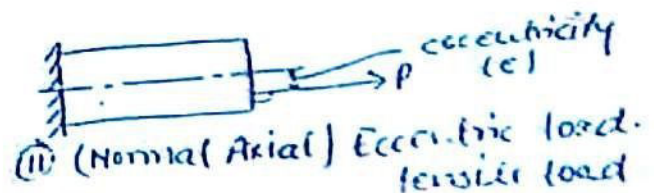
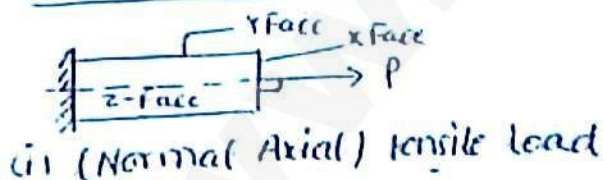
$$S_{SI} = \frac{PL^3}{3EI}, \frac{PL^3}{48EI}, \dots$$

### Q FATIGUE LOADS →

Are those loads whose magnitude or direction are both magnitude and direction are changes w.r.t time and these loads are repeatedly apply w.r.t time.



### LOADS - (Acc to direction) → ① NORMAL FORCE



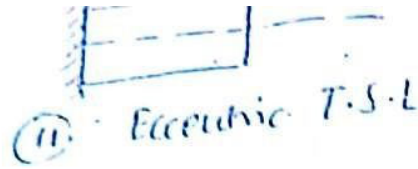
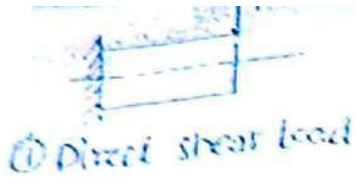
### ② SHEAR LOADS

LONGITUDINEAL SHEAR LOAD

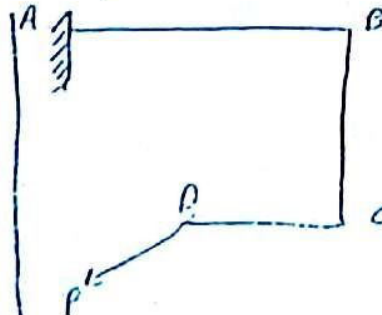
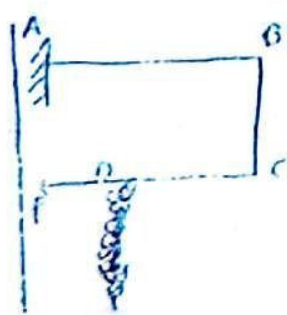
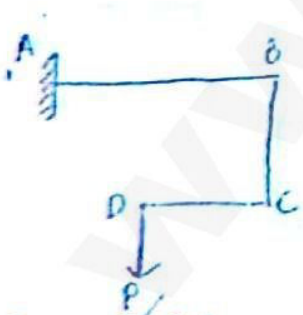
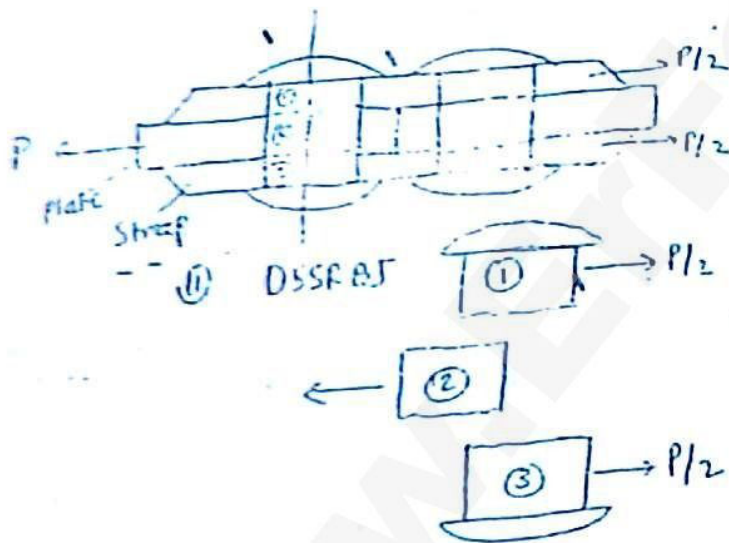
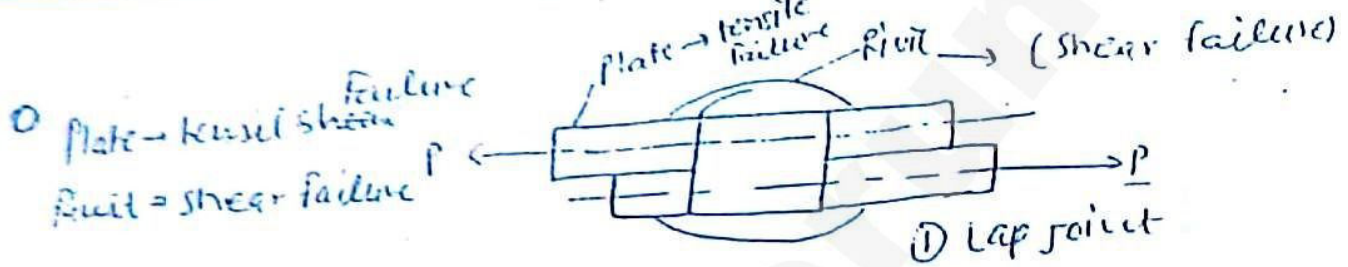
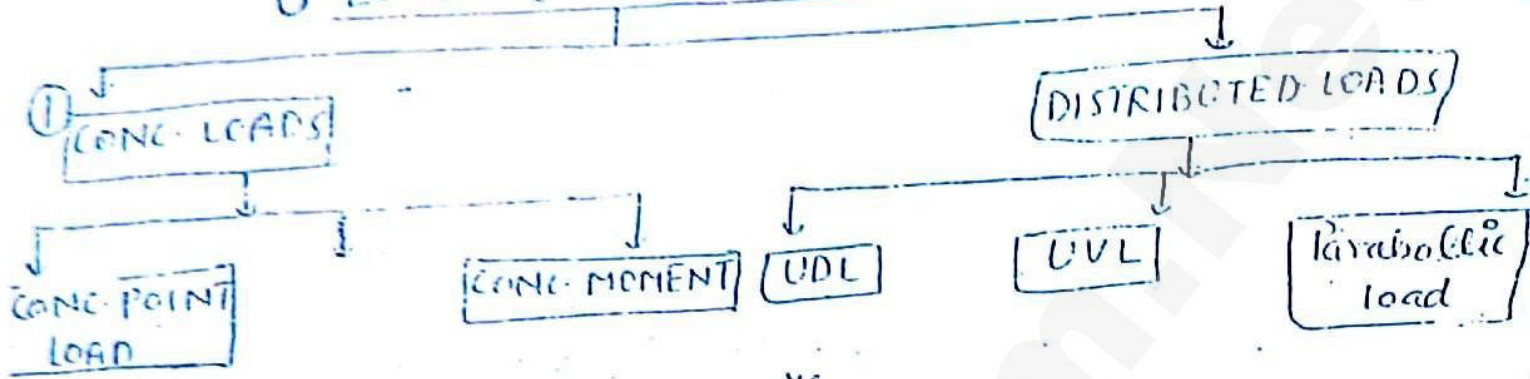
TRANSVERSE SHEAR LOAD

DIRECT SHEAR LOAD  
(L passing through axis)

ECCENTRIC T.S.L  
(L not passing axis)



## LOADS (ACC TO AREA)



DC - D.S.L  
BC - E-Axial tensile



Oblique plane is that plane which is  $\perp$  to the both vertical and horizontal plane. Couples are made in all 3 planes.

1. When a couple is a twisting couple, then plane of the couple  $\perp$  to the axis. (Twisting Moment)
2. When a couple is a bending couple, then plane of couple is passing through the axis.

### Twisting Moment or Couple or Torque $\rightarrow$

A couple is said to be twisting couple when plane of couple  $\perp$  to axis of member or parallel to plane of c/s.

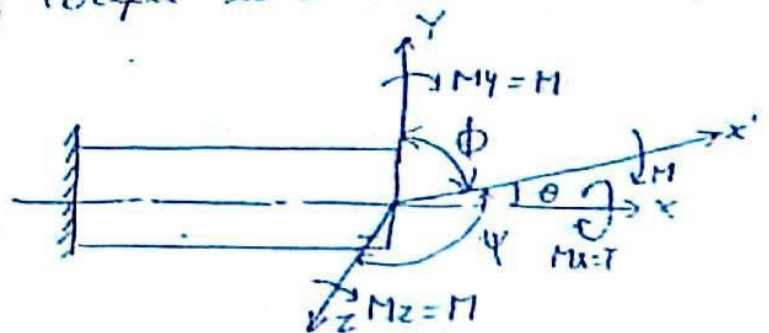
### Bending Couple or Bending Moment $\rightarrow$

A couple is said to be bending couple of B.M when plane of couple passes through the axis of the member or  $\perp$  to the plane of c/s.

A member is subjected to both bending moment and twisting moment when plane of couple is inclined to the axis of the member.

### Oblique Plane:-

3 couples are acting on the plane.



$M_x = M \cos \theta \rightarrow$  Twisting Couple  $\rightarrow M_t = T$  (Semiclosed ellipse) (Dot  $\rightarrow$ )

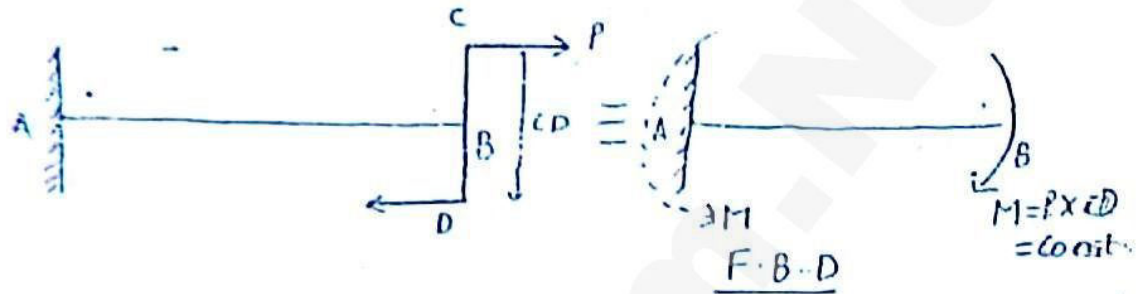
$M_y = M \cos \phi \rightarrow$  Bending couple  $\rightarrow M_b = M$  (Arc of circle) ( $\rightarrow \leftarrow$ )

$M_z = M \cos \psi \rightarrow$  Bending Couple  $\rightarrow M_b = M$  (Arc of circle)



## Pure Bending

A member is said to be under pure bending it is subjected to equal and opposite couples in a plane passing through the axis of member. OR.  $\perp$  to plane of cis.



Pure bending  
(Subjected only to ~~stress~~ stress)

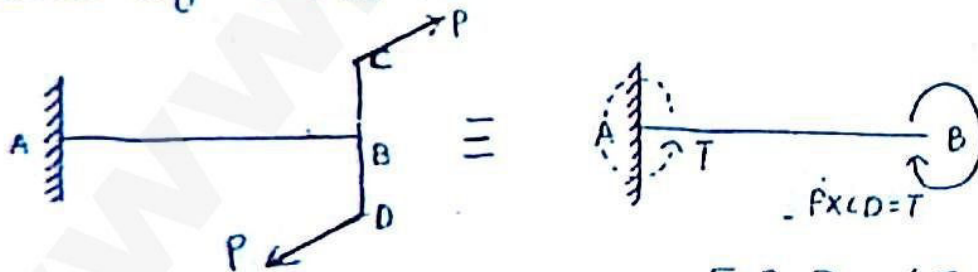
Note:-  $F = \frac{dM}{dx} = 0$

if  $(M = \text{const})$  then  $\boxed{F = 0}$

$$\sigma_b = \frac{M}{Z_{NA}} \quad Z_{NA} = \frac{I_{NA}}{y_{max}} = \frac{\pi}{32} D^3$$

## 0 PURE TORSION

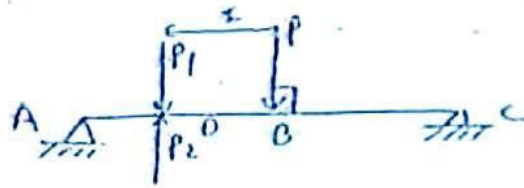
A member is said to be under pure torsion when it is subjected to equal and opposite plane couple in a plane  $\perp$  axis of the member or parallel to the plane of cross-section.



F.B.D ( $T = \text{const}$ )  
(Subjected only to torsion/shear)

$$\tau_s = \frac{T}{Z_p} \quad Z_p = \text{polar section Modulus} = \frac{J}{R} = \frac{\pi}{32} d^3$$

# TRANSVERSE SHEAR LOAD



TSL = Variable Beam

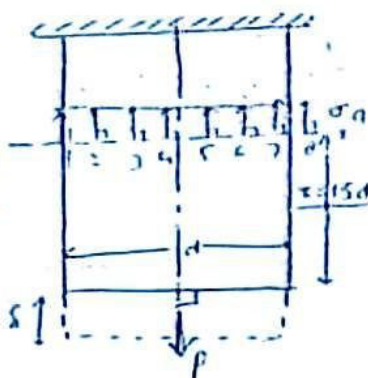
↳  $\sigma_b \times \text{shear stress}$   
" " " " " "  
Non zero

$P_1 = P_2 = P$   
Shear force Bending stress ( $M = Pl$ )

## 0 AXIAL LOAD

### Case-I-

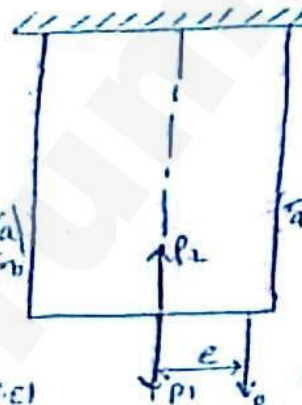
Uniformly distributed  
linear with  
not uniformly distributed



$$S = \frac{PL}{AE}$$

$\sigma_a = \pm P/A$  (Axial Load)

### Case-II-



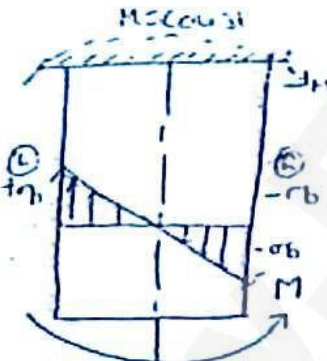
①  $P_1 = P_2 = P$   
ecc. axial load = P  
B.M = (P.e)

(Ecc. Axial Load)

$$\sigma_1 = \frac{P}{A} + \frac{M}{ZNA}$$

$$\sigma_r = \sigma_a \pm \sigma_b$$

### Case-III-

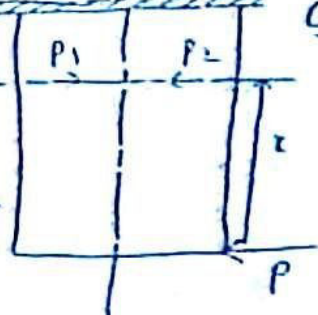


(Bending) pure

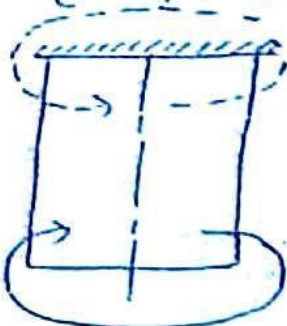
$$\sigma_b = \frac{M}{ZNA}$$

### Case-IV-

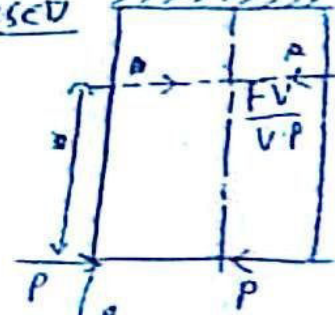
variable  
B.M  
SF  $\neq 0$



(TSL)



### Case-V-



$\frac{Tcl}{H \cdot \text{PIATE}}$   
(Pure torsion)

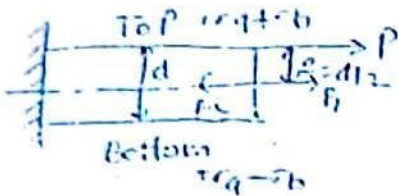
$$\tau_s = T/ZP$$

(TSV)  $P_1 = P_2 = P$   
 $M = P \cdot e = (P \cdot l)$



Note: E. axial loading, neutral axis will not coincide with the centroidal axis of member.

Ques: For a member as shown in figure. determine the max stress induced in a member?



$$\sigma_{\max} = \sigma_{\text{Top}} = \sigma_a + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{Z_{NA}}$$

(i)  $M = P \cdot e = P$

(ii)  $e = d/2$

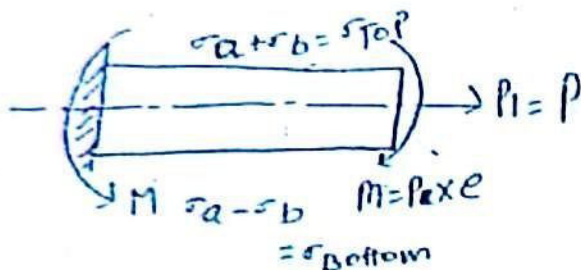
$$= \frac{4P}{\pi d^2} + \frac{P \cdot e}{\frac{\pi d^3}{32}}$$

$$= \frac{4P}{\pi d^2} + \frac{16P \cdot d}{\pi d^3}$$

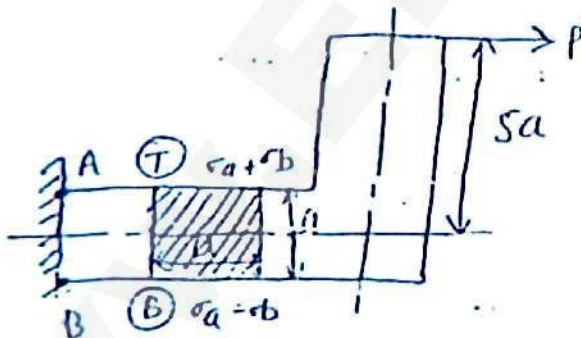
$$= \frac{4P}{\pi d^2} + \frac{16P}{\pi d^2}$$

(72)

$$\sigma_{\max} = \frac{20P}{\pi d^2}$$



Ques: For the member as shown in figure determine max. stress induced at point A.



$$(\sigma_{\max})_A = \sigma_{\text{Top}} = \sigma_a + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{Z}$$

$$= \frac{P}{a^2} + \frac{P \cdot 5a}{\frac{\pi a^3}{6}}$$

$$= \frac{31P}{a^2} \text{ (Tensile)}$$

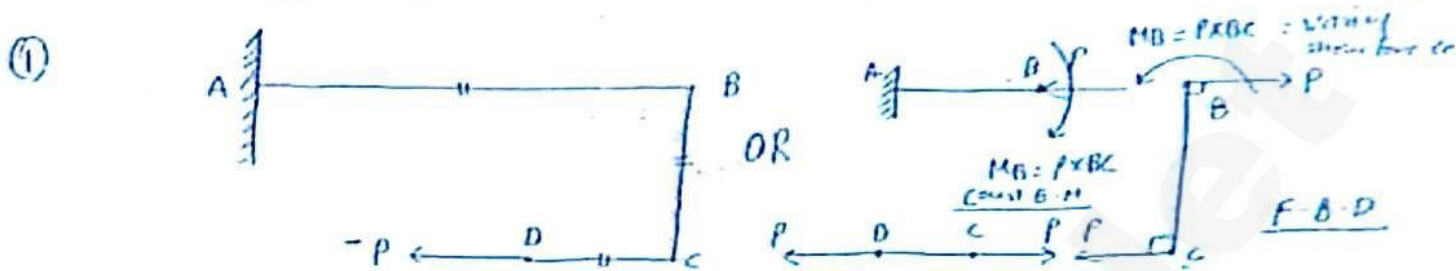
$$(\sigma_{\max})_B = \sigma_{\text{Bottom}} = \sigma_a - \sigma_b$$

$$= \frac{P}{A} - \frac{M}{Z}$$

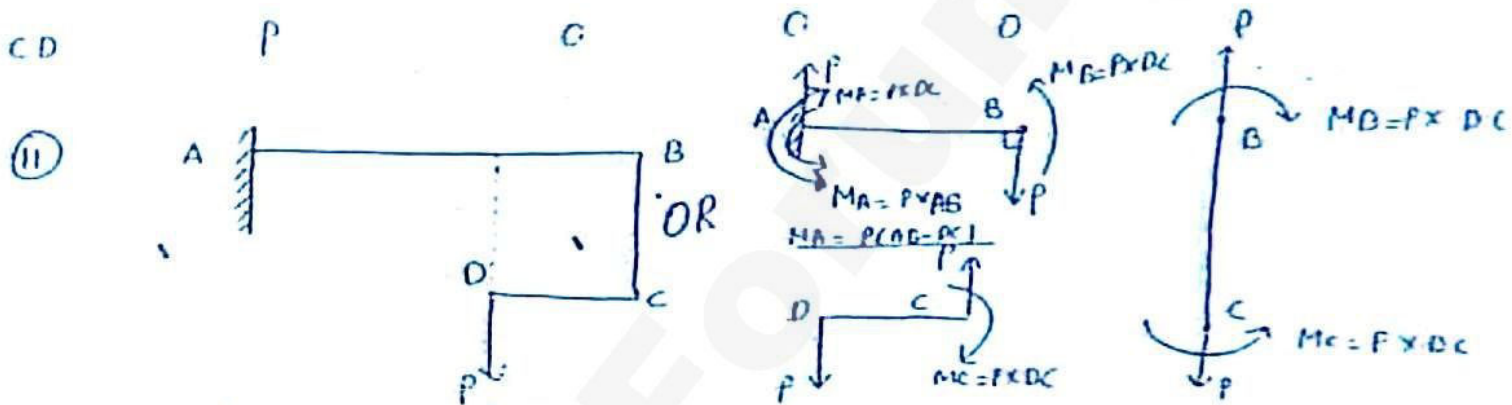
$$= \frac{P}{a^2} - \frac{P \cdot 5a}{\frac{\pi a^3}{6}} = -\frac{29P}{a^2} \text{ (Compression)}$$



Q. For the structural member ABCD as shown in figure determine the effect of load on AB, BC and CD.

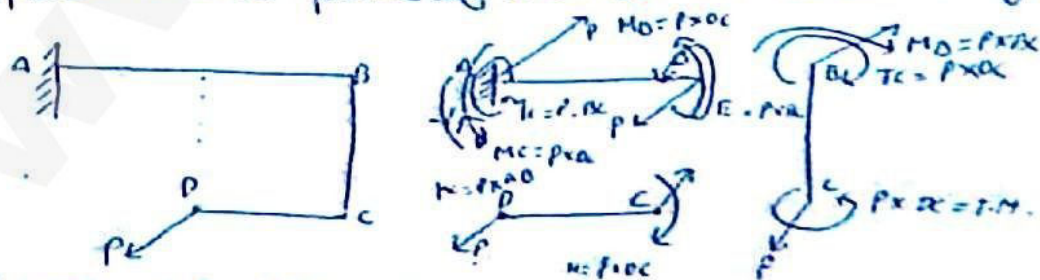


	Axial load	SF	B.M.	T.M.
AB	$-P$	0	$P \times BC$	0
BC	0	$P$	$P \times BC$	0
CD	$P$	0	0	0



	Axial load	S.F	B.M	T.M
AB	0	$P$	$P \cdot (AB - CD)$	0
BC	$P$	0	$P \times DC$	0
CD	0	$P$	$P \times DC$	0

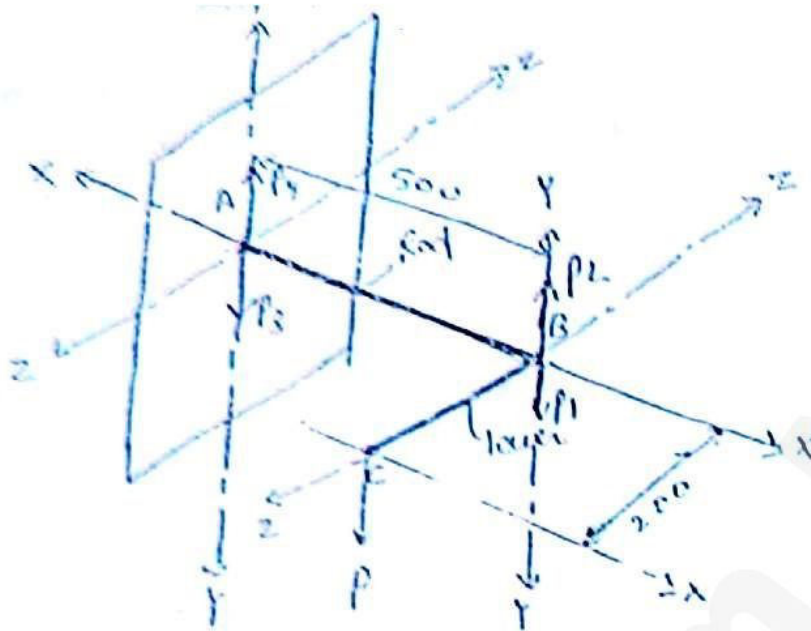
Q. Repeat the above question if load is applied at point D in a plane perpendicular to plane of member as shown in figure.



	Axial Load	S.F	BM	TM
AB	0	$P$	$P \cdot (AB - DC)$	$P \times BC$
BC	0	$P$	$P \times BC$	$P \times DC$
CD	0	$P$	$P \times DC$	0

Since the load is T.M. but essentially T.M. so for B.M. & SF calculation.

Q4:



System

LEVER  $\Rightarrow$  S.F = P & B.M =  $P \times 100$

ROD  $\Rightarrow$  S.F = P & B.M =  $P \times 500$ , T.M =  $P \times 200$

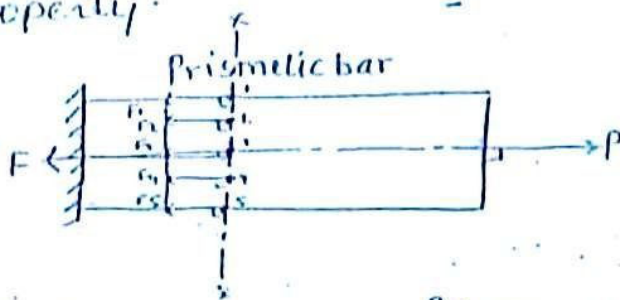
LEVER  $\Rightarrow$   $P_1 =$  S.F;  $P_2$  &  $P_4 \Rightarrow$  B.M =  $P \times 200$

ROD  $\Rightarrow$  T.M =  $P \times 200$ ,  $P_1$  &  $P_4 \Rightarrow$  B.M =  $P \times 500$ , S.F =  $P_1 = P$

## Stress and Strain

STRESS →

Defined as the magnitude of internal resisting force develop at a point against the deformation cause due to loads. It always variable parameter. Not properly.



$F = \text{total internal resisting offered by CLs}$

\* prismatic bar means uniform c/s area.

$$F = F_1 + F_2 + F_3 + F_4 + F_5$$

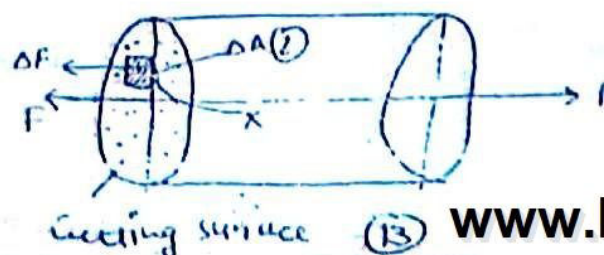
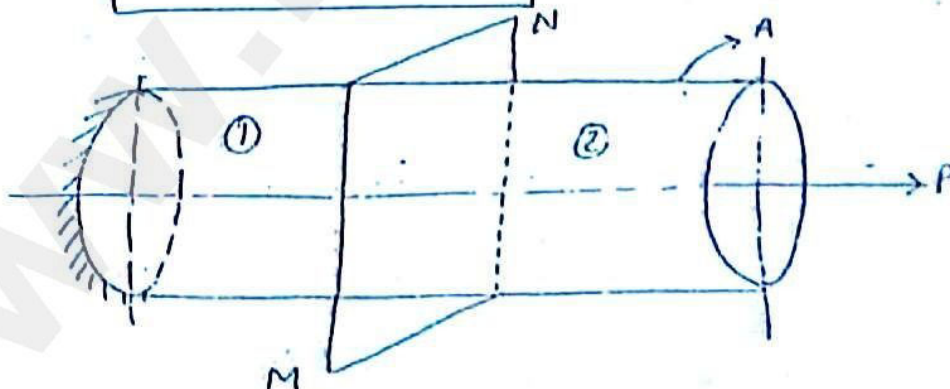
$$F = 5F_1 \Rightarrow F_1 = \frac{F}{5} = \frac{F}{A}$$

$$F_1 \Rightarrow \tau_{avg} = \frac{F}{A} = \frac{P}{A} = \frac{\text{Total internal resisting force}}{\text{Area}}$$

This  $\tau_{avg}$  is uniformly distributed.

\* If  $\tau_{avg}$  is not uniformly distributed, then

$$\sigma = \frac{L \epsilon}{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$



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$$\sigma_x = \frac{\Delta F}{\Delta A} \quad \text{introducing limit -}$$

$$\sigma_x = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

$$\int dF = \int \sigma_x dA$$

$$F = \sigma_x A$$

$$\boxed{\sigma_x = \frac{F}{A}}$$

Unit  $\rightarrow$  Pascal (Pa), MPa, GPa

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2 \text{ or } 1 \text{ N/mm}^2$$

$$1 \text{ GPa} = 10^3 \text{ MPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2 = 10^3 \text{ N/mm}^2$$

### ③ STRENGTH $\rightarrow$

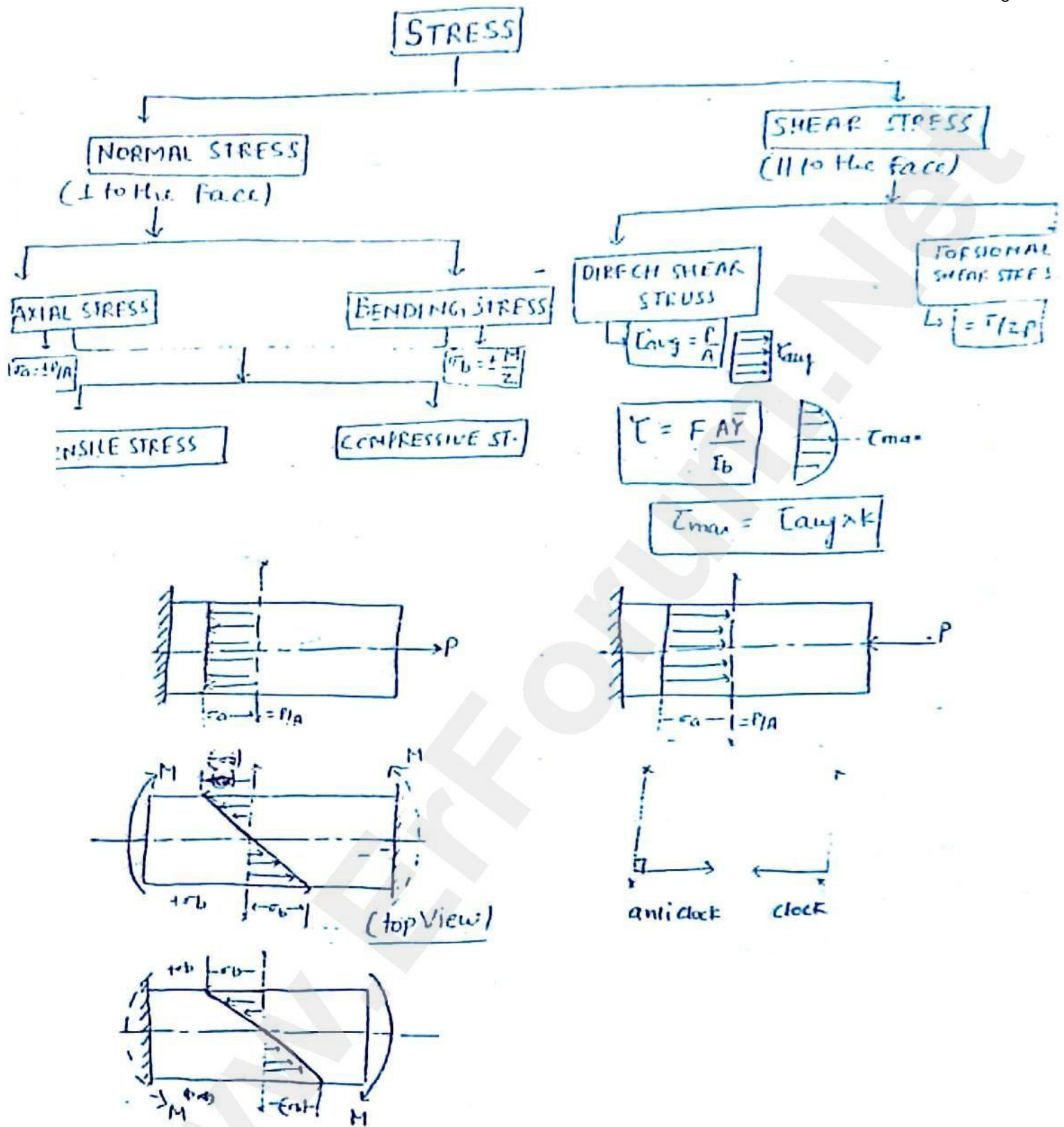
Define as the maximum or limiting stress that a material can withstand without any failure or fracture.

- $\sigma_{ind} \leq Y\text{-Strength} \Rightarrow$  [No yielding / Permanent deformation]  
No Failure
- $\sigma_{ind} \leq U.T. \text{ Strength} \Rightarrow$  No failure.
- Strength is a property. (constant for given material)

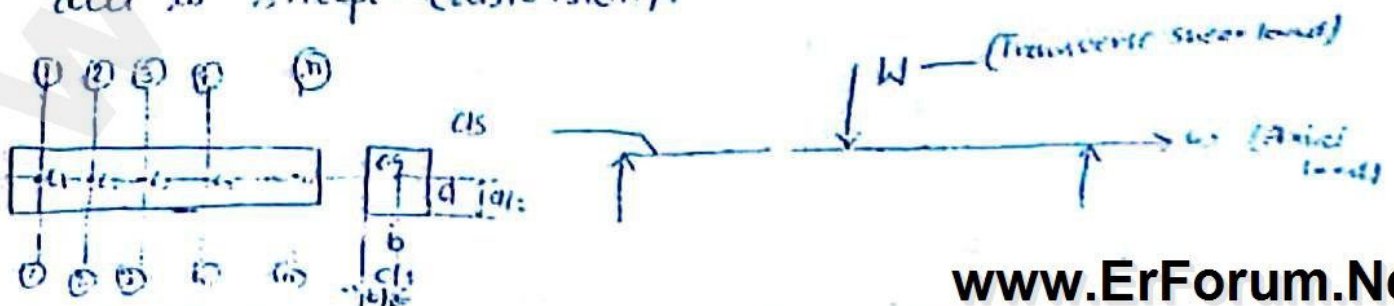
Unit  $\rightarrow$  same as stress.

### ③ TYPES OF STRESSES $\rightarrow$

Face on which acting direction of load  $\rightarrow$   $\sigma_{xx}$  or  $\sigma_x$   $\rightarrow$  Normal stress  
 $\tau_{xy}$   $\rightarrow$  Shear stress



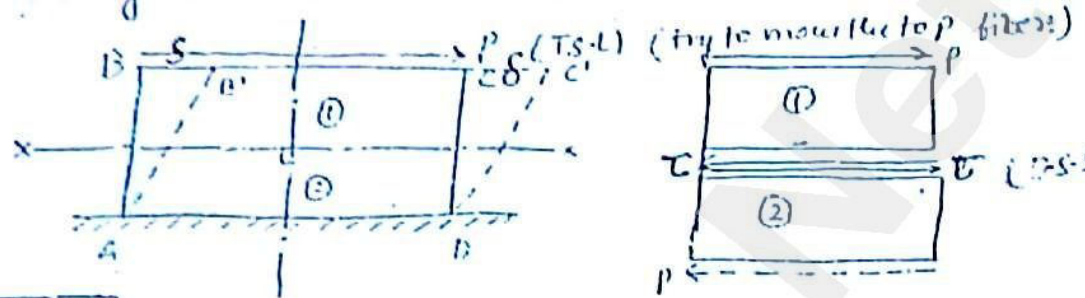
Normal stress is due to ~~direction~~ <sup>extension</sup> and shear stress is due to shape (distorsion).





$\bar{C}$  = Centroid of QS ( $C_1, C_2, C_3 \dots C_n$ )

- \* Axis is a imaginary line which passing through the each and every centroid of axis of each point (QS).



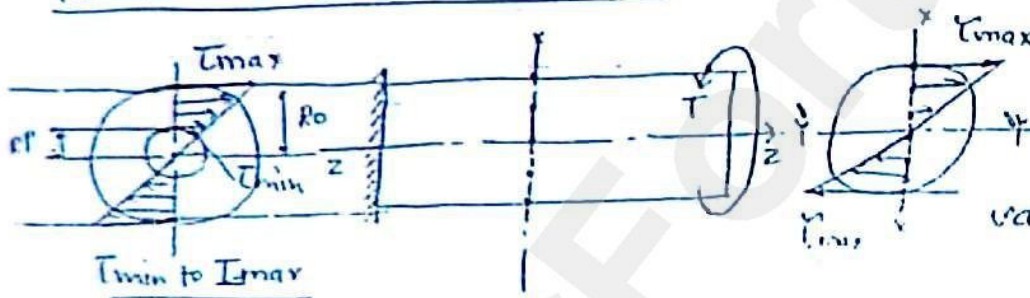
avg shear stress  
(shear stress)

$$\tau_{avg} = \frac{P}{A}$$

$$\tau_{max} = \tau_{avg} \times K$$

$$\left. \begin{aligned} K &= \text{circular} = \frac{4}{3} \\ K &= \text{rectangle} = \frac{3}{2} \end{aligned} \right\}$$

Torsional shear stress  $\rightarrow$  (side view)



$$\tau = \gamma$$

varies Q -  $\tau_{max}$

Torsional shear stress is view from Side View but B.M is view from top view.

$$\tau_{max} = \frac{I}{Z_p}$$

$$Z_p = \frac{I}{R}$$

$$\begin{aligned} R_o &\rightarrow \tau_{max} \\ R_i &\rightarrow \tau_{min} \end{aligned}$$

$$\tau_{min} = \frac{\tau_{max} R_i}{R_o}$$

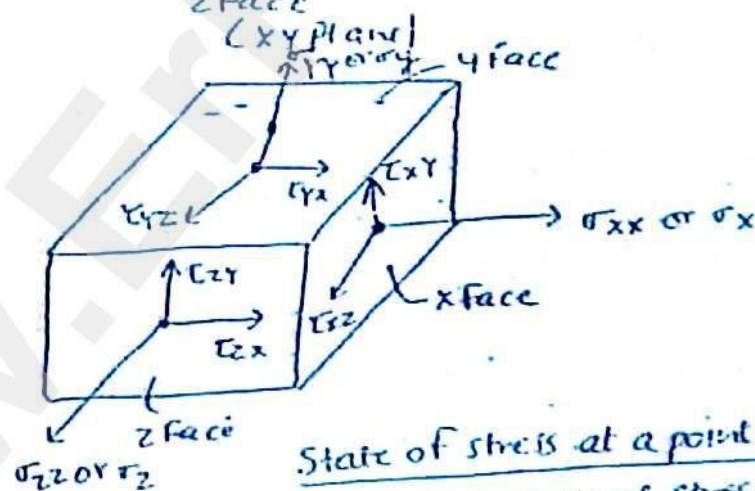
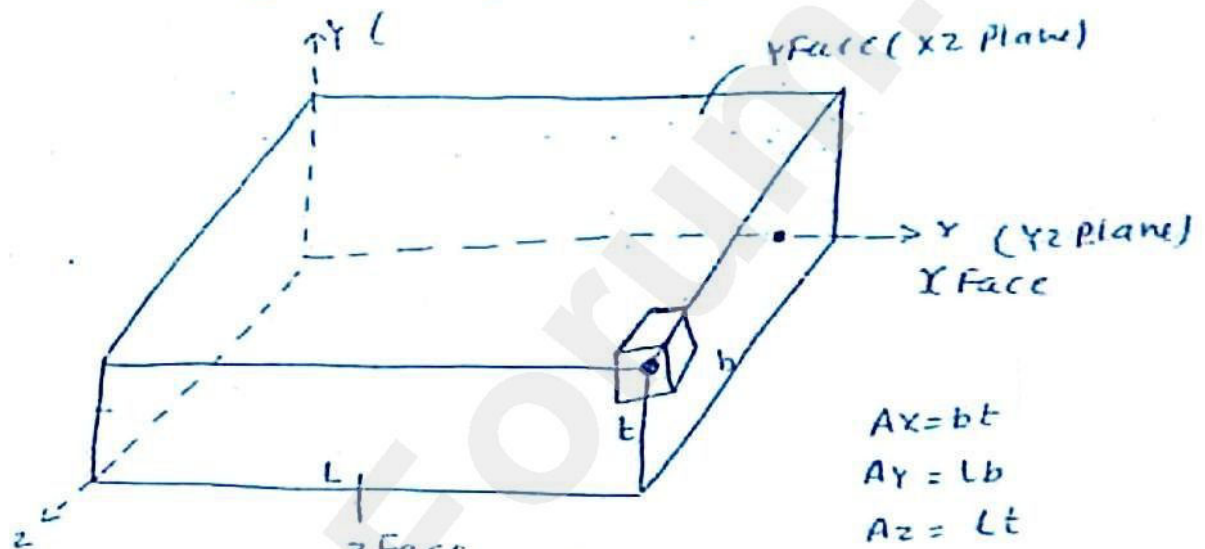
$$\frac{\tau_{max}}{\tau_{min}} = \frac{R_o}{R_i}$$

## STRESS TENSOR

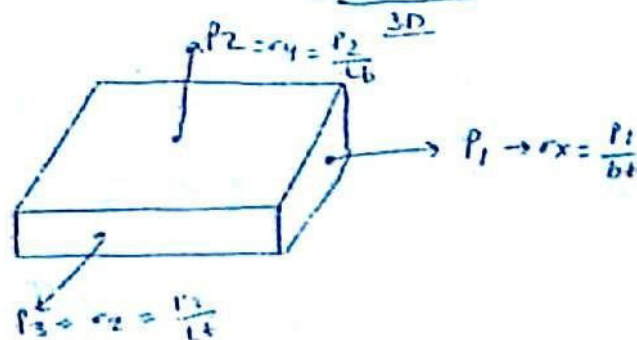
"Used to define the state of stress (How many stress <sup>compon.</sup> acting) at a point."

Matrix

$$[\tau]_{3D} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \begin{matrix} \leftarrow \text{x-Face} \\ \leftarrow \text{y-Face} \\ \leftarrow \text{z-Face} \end{matrix}$$



State of stress at a point in 3D  
(Tri-axial state of stress at a point)





41 - only Normal stress then

$$[\sigma]_{3D} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

Q  $[\sigma]_{3D} = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 4 & 2 \\ 0 & 2 & 3 \end{bmatrix} \text{ MPa}$

What is shear stress on Y plane along Z dir.

$\tau_{yz} = 0 \text{ MPa} = \tau_{xz}$

\* Complementary shear stress are equal and opposite.

\*  $\left. \begin{array}{l} \tau_{xy} = -\tau_{yx} \\ \tau_{xz} = -\tau_{zx} \\ \tau_{yz} = -\tau_{zy} \end{array} \right\}$  Complementary shear stress are equal and opp.

$[\tau]_{3D} = \begin{bmatrix} \tau_{xy} & \tau_{yx} & \tau_{xz} \\ -\tau_{yx} & \tau_{xy} & \tau_{yz} \\ \tau_{xz} & \tau_{zy} & \tau_{yz} \end{bmatrix}$  Symmetric along diagonal.

\* The no. of stress component in a stress tensor for a point in 3D are [9] (3 Normal & 6 shear).

\* The No. of stress components in a stress tensor to define the state of stress at a point in 3D are [6] (3 Normal & 3 shear).

### 0. PLANE STRESS PROBLEMS [2D Problems]

Plane stress problems are those problem in which stress acting in any one of the mutual  $\perp$  faces passing through a point are assumed to be zero.

$\sigma_z = \tau_{zx} = \tau_{zy} = 0$  but some are complementary shear stresses so  $\tau_{xz} = \tau_{yz} = 0$ . So the size of matrix

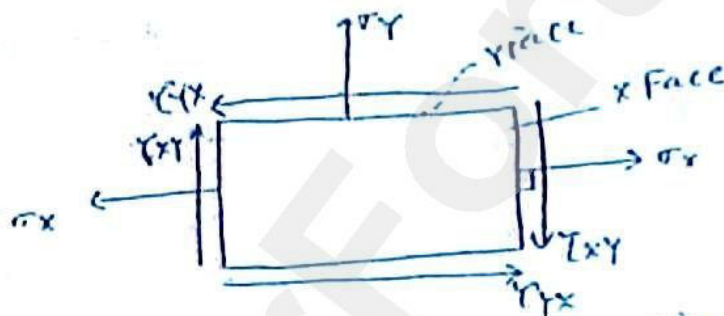
of 2D problem.

$$\begin{bmatrix} \sigma \\ \tau \end{bmatrix}_{2D} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

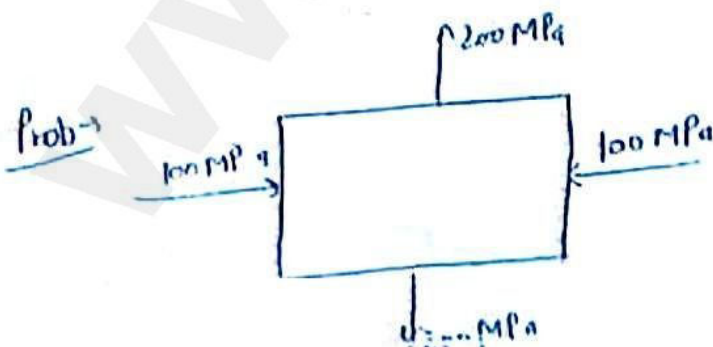
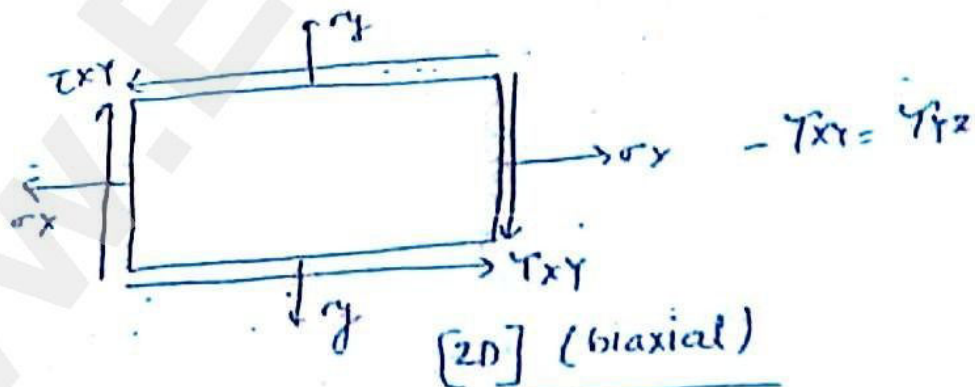
Stress tensor at a point in 2D.

- \* In the total no. of stress components in a stress tensor for a point in 2D are [4] (2 Normal, 2 shear)
- \* The No. of stress components in a stress tensor to define a state of stress at a point in 2D are [3] [2 Normal, 1 shear].

$$\left. \begin{aligned} \sum H &= 0 \\ \sum V &= 0 \\ \sum M &= 0 \end{aligned} \right\}$$

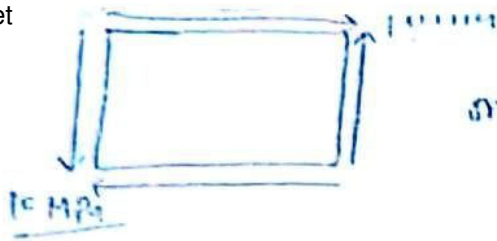


- \* Normal stress is produced due to change in dimensions  
Shear rotation



$$\left. \begin{aligned} \sigma_x &= -100 \text{ MPa} \\ \sigma_y &= +200 \text{ MPa} \\ \tau_{xy} &= 0 \end{aligned} \right\}$$



Prob

only shear stress

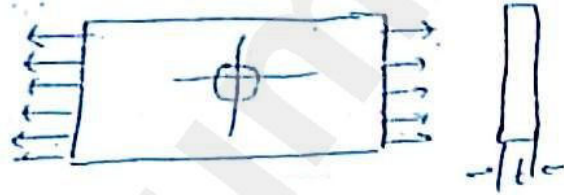
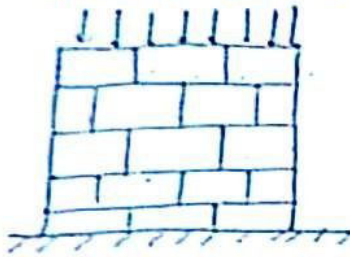
$$\tau_{xy} = -10 \text{ MPa}$$

$$\tau_{yx} = 10 \text{ MPa}$$

③ To convert the 3D into 2D →

\* One dimension should be very small as compared to other two.

\* load should be uniformly distributed.



## STRAIN TENSOR (E)

Three Normal strain and 6 shear strain

$$\left. \begin{array}{l} \sigma \rightarrow \epsilon_x, \epsilon_y, \epsilon_z \\ \tau \rightarrow \frac{\gamma_{xy}}{2}, \frac{\gamma_{yz}}{2}, \frac{\gamma_{zx}}{2} \\ \quad \frac{\gamma_{yx}}{2}, \frac{\gamma_{zy}}{2}, \frac{\gamma_{xz}}{2} \end{array} \right\}$$

$$\begin{array}{l} \text{--- /} \\ \text{state of strain} \\ \text{at a point in} \\ \text{3D} \end{array} \quad [E]_{3D} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_z \end{bmatrix} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \epsilon_z \end{bmatrix}$$

$$[\epsilon]_{2D} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_y \end{bmatrix}$$

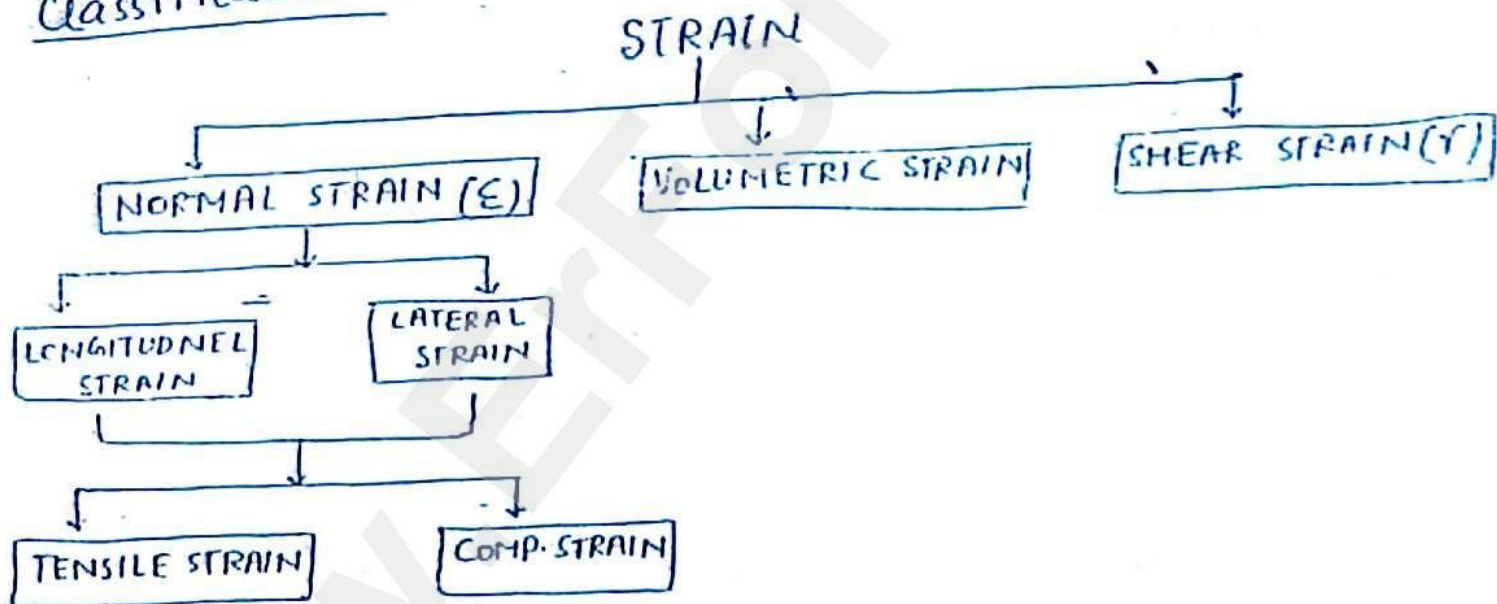
EXAMPLE →  $\sigma_{1,2} = \frac{1}{2} \left[ (\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2} \right]$

$$\epsilon_{1,2} = \frac{1}{2} \left[ (\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2} \right]$$

Definition →

"A body is said to be strained when relative positions of particles is altered due to applied loads."

Classification →

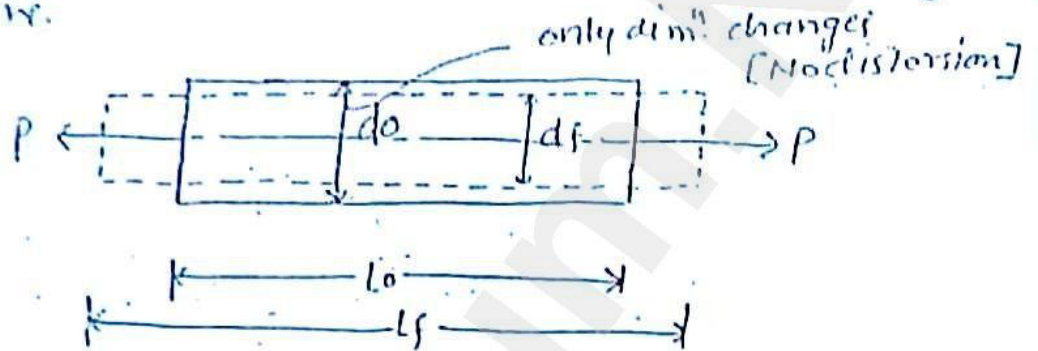


o Normal strain ( $\epsilon$ ) =  $\frac{\text{change in dim}^s}{\text{original dim}^s}$

o LONGITUDINAL STRAIN = GIVES strain in dim<sup>s</sup> which is in the dir<sup>n</sup> of applied load.



- o **LATERAL STRAIN** = Given strain in a dir<sup>n</sup> which is  $\perp$  to the dir<sup>n</sup> of applied load.
- o Every longitudinal strain is associated with 2 lateral strains.
- o Longitudinal strains and lateral strains are always in opposite nature.



$$\epsilon_x = \epsilon_{\text{long}} = \frac{\Delta L}{L_0} = \frac{L_f - L_0}{L_0} \rightarrow \text{+ve (tensile nature)}$$

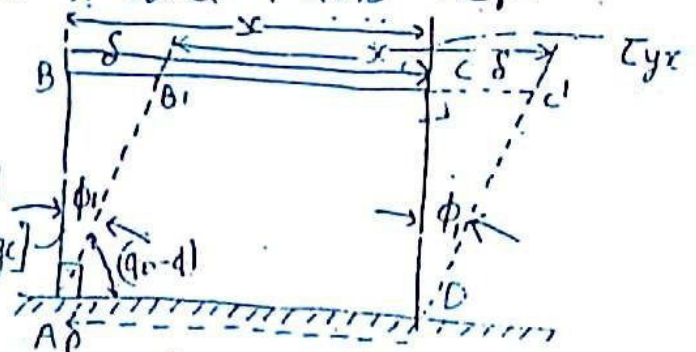
$$\epsilon_z = \epsilon_y = \epsilon_{\text{lateral}} = \frac{\Delta d}{d_0} = \frac{d_f - d_0}{d_0} \rightarrow \text{-ve (compressive nature)}$$

### ③ SHEAR STRAIN ( $\gamma$ )

Defined as a change in right angle between two line elements which are parallel to x and y axis resp.

$\phi$  = shear angle of y face

body gets disturbed  
[i.e. shape is change]



$$\gamma = \phi$$

$$\tan \phi = \frac{BB' \text{ or } CC'}{AB \text{ or } CD} = \frac{s}{AB}$$

$$\tau_{yx} \neq 0, \tau_{xy} = 0$$

(calculate  $\phi$  here)

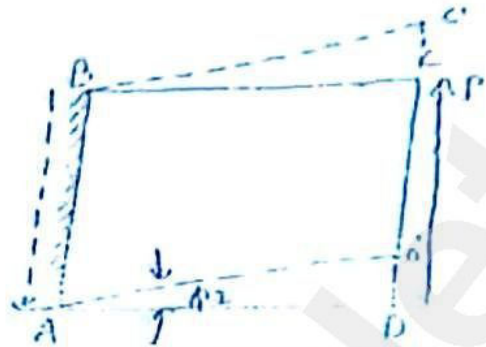
$$\sum M \neq 0$$

Case I  $\phi_1 = 0$

$\phi_2 =$  shear angle at x-faces

$$\boxed{\gamma = \phi_2}$$

$$\Sigma M \neq 0$$



$$\boxed{\tau_{yx} \neq 0, \tau_{xy} = 0}$$

both the cases are simultaneously.

$$\Sigma M = 0$$

$$\boxed{\tau_{xy} = -\tau_{yx}}$$

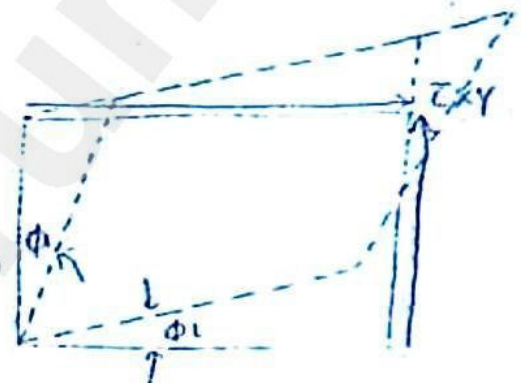
Case III  $\rightarrow$

$$\boxed{\gamma = \phi_1 + \phi_2}$$

If  $\phi_1 = \phi_2$  [for a square element]  $\rightarrow$

$$\gamma = 2\phi_1 \text{ or } 2\phi_2$$

$$\boxed{\phi_1 = \frac{\gamma}{2}}$$



$$\boxed{\tau_{xy} = -\tau_{yx}}$$

### ⊙ VOLUMETRIC STRAIN or DILATATION

Defined as change in direction to the normal stress.

$$\boxed{\epsilon_v = \frac{\delta V}{V}} \rightarrow \text{--- (i)}$$

\* It is type of normal stress.

\*  $\delta V =$  known when final dim's are known (experimentally)

$$\boxed{\delta V = \epsilon_v \cdot V} \rightarrow \text{--- (ii)}$$

$$\boxed{\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z}$$



\* Every longitudinal-strain subjected to 2 lateral strains.

Rectangle:

$$V = lbt \rightarrow (1)$$

$$\delta V = (\delta l)bt + (\delta b)lt + \delta t(lb) \rightarrow (2)$$

$$\frac{\delta V}{V} = \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta t}{t}$$

$$\boxed{\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z} \quad \text{proved}$$

\* to calculate the deflection Young's Modulus should be must.

\* Experimentally strain calculate not stress but theoretically stress calculate but not strain calculated.

Cylindrical body

$$V = \frac{\pi}{4} D^2 L$$

$$\delta V = \frac{\pi}{4} (\delta D^2) L + \frac{\pi}{4} D^2 (\delta L)$$

$$\frac{\delta V}{V} = \frac{\delta L}{L} + 2 \left( \frac{\delta D}{D} \right)$$

$$\boxed{\epsilon_V = \epsilon_x + 2(\epsilon_y \text{ or } \epsilon_z)}$$

\* There is only on lateral strain.

sphere:

$$V = \frac{4}{3} \pi R^3$$

Partial differentiation  $\rightarrow$

$$\frac{\delta V}{V} = 3 \left( \frac{\delta R}{R} \right)$$

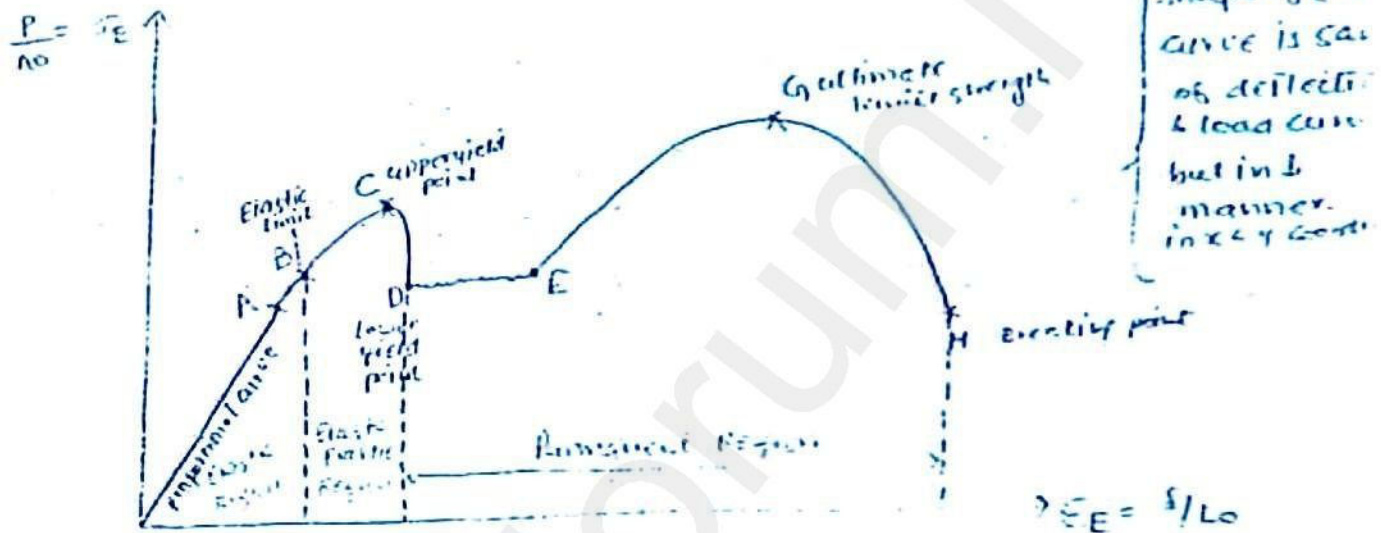
$$\boxed{\epsilon_V = 3 \left( \frac{\delta R}{R} \right)}$$

## Elastic Constants

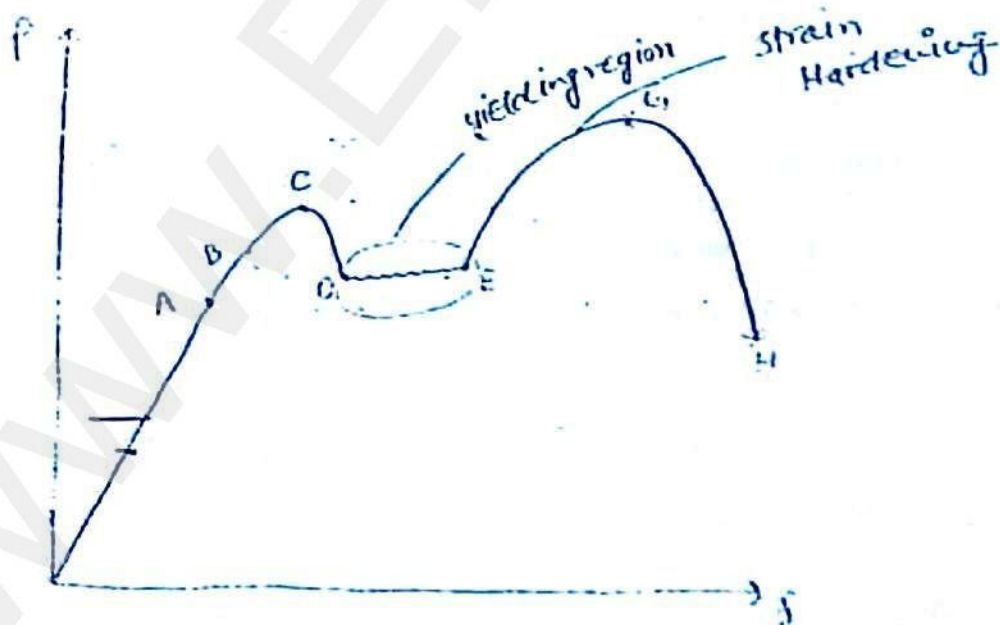
\* They represent elastic properties.

\* Three type of region -

- (i) elastic
- (ii) elastoplastic (Region b/w elastic and yield point)
- (iii) plastic.



\* In a tension test the deflection will be measured by elastometer. So we will plot load and deflection curve.





- C - Yield point (upper)
- D - Lower yield point
- D - E - yielding region
- G - Ultimate point
- H - Breaking point
- F - Strain Hardening starts.
- G - H - Necking Region

- \* Engg stress and Engg strain ( $\sigma_E$  &  $\epsilon_E$ ) are calculated by original dimensions. ( $P/A_0 \leftarrow S/L_0$ ).

P	S	d
$P_1$	$S_1$	$d_1$
$P_2$	$S_2$	$d_2$
$P_3$	$S_3$	$d_3$
$P_4$	$S_4$	$d_4$

$P \uparrow - L \uparrow - d \downarrow$

- \* Engg strain and stress are calculated based on original dimension.
- \* True stress and strain are calculated by using instantaneous dimensions.
- \* Theoretically we can determine engg. stress and strain only.
- \* But Actual stress and strain (true stress & strain) will give actual stress and strain induced in component. but theoretically it is not possible to determine true stress and true strain due to the unavailability of instantaneous dimensions.

$$\sigma_T = \sigma_E (1 + \epsilon_E)$$

$\sigma_T > \sigma_E$  (tensile test)

$\sigma_T < \sigma_E$  (compression test)

## TRUE STRAIN

Defined as sum of Engg. strain from instant to instant

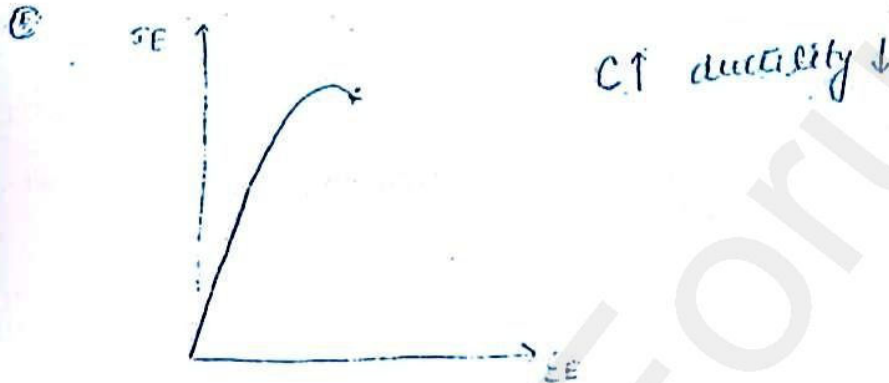
$$\epsilon_T = \epsilon_{E1} + \epsilon_{E2} + \epsilon_{E3} + \dots$$

$$\boxed{\epsilon_T = \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \dots}$$

$$\epsilon_T = \frac{L_f - L_0}{L_0}$$

Then

$$\boxed{\epsilon_T \neq \epsilon_E}$$



Stress-strain diagram of  
(brittle) cast iron (C-21.2)

Continued... Elastic constant →

- \* To obtain stress-strain Relationship.
- \* Theoretically stress can be determine but strain cannot be determined.  
So to determine strain (Theoretically), Elastic constant are required now.
- \* For a homogeneous and isotropic material, elastic constants are 4.
- \* [Young's Modulus (E), (G), (K),  $\mu$  or  $\frac{1}{m}$ ]
- \* But for a homogeneous and isotropic material the No. of independent elastic constant are 2.
- \* [Young's Modulus (E),  $\mu$  or  $\frac{1}{m}$ ]

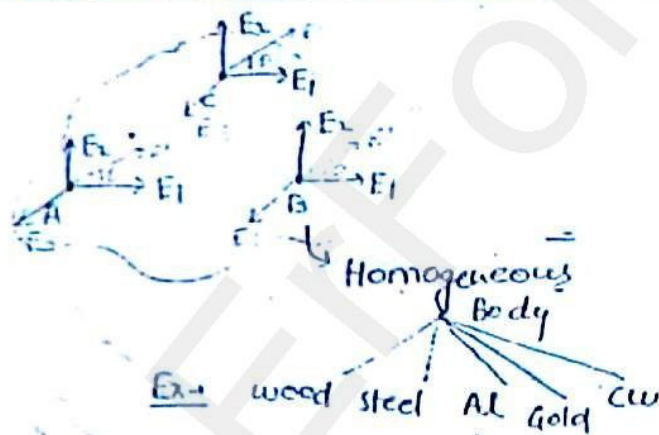


\* An isotropic material or anisotropic material, Independent Elastic constants are - 21.

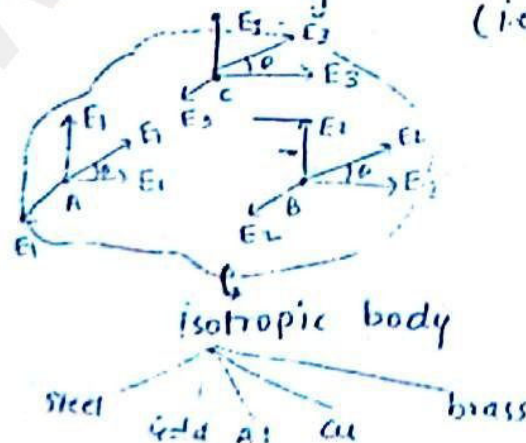
\* For an orthotropic material, No. of Independent Elastic constants are - 9.

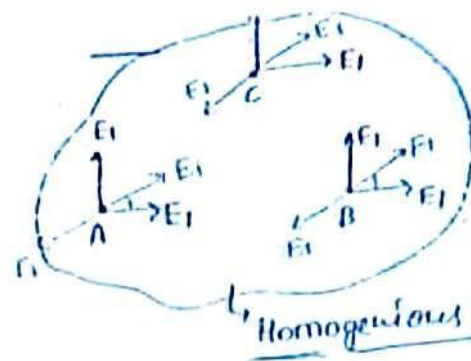
Material	IND. E. Constants.
(i) Isotropic	2
(ii) Orthotropic	9
(iii) Anisotropic	21

\* A material is said to be homogeneous when it exhibits same elastic properties at any point in a given direction. (i.e. the properties are independent of point)



\* A material is said to be isotropic when it exhibits same elastic properties in any directions at a given point. (i.e. properties are independent of directions)



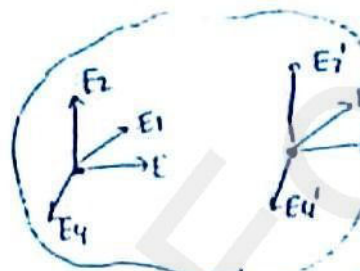


only one young's modulus

Homogeneous & Isotropic body

Steel  
Al  
Cu  
Gold

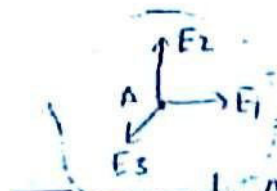
- \* A material is said to be Homogeneous and isotropic which it exhibits same elastic properties at any point and in any direction (i.e. properties are independent on point & direction)
- \* All homogeneous materials need not be isotropic & vice versa. but few materials are both homogeneous and isotropic.



Anisotropic - composite material.

### Advantages →

- \* Composite material are more than in strength and weight is less.
- \* Properties of composite material will change as desired
- \* A material is said to be orthotropic which it exhibits different elastic properties in 3- orthogonal directions.



Orthotropic Material (type of anisotropic)

plywood wood

$$E_x \neq E_y \neq E_z$$



## Relationship between Elastic Constant

$$\begin{aligned}
 \textcircled{i} \quad & \boxed{E = 2G(1+\mu)} \quad \rightarrow \textcircled{v} \quad \boxed{G = \frac{E}{2} \times \frac{1}{1+\mu}} \quad \rightarrow W \\
 \textcircled{ii} \quad & \boxed{E = 3K(1-2\mu)} \quad \rightarrow \textcircled{vi} \quad \boxed{K = \frac{E}{3} \times \frac{1}{1-2\mu}} \quad \rightarrow X \\
 \textcircled{iii} \quad & \boxed{E = \frac{9KG}{3K+G}} \quad \rightarrow \textcircled{vii}
 \end{aligned}$$

from eq. (vi) -  $K = +ve \Rightarrow \boxed{1-2\mu > 0}$   
 $\boxed{\mu < \frac{1}{2}}$  [for all engg. material]  
 $\mu < 0.5$

$$\boxed{0 \leq \mu \leq \frac{1}{2}} \quad \text{or} \quad \boxed{0 < \mu \leq \left| -\frac{1}{2} \right|}$$

from eq. (vii) -

$$\boxed{G \leq \frac{E}{2}}$$

if  $\mu = 0 \Rightarrow G = E/2$

if  $0 \leq \mu \leq \frac{1}{2} \Rightarrow G < E/2$

if  $\mu = \frac{1}{2} \Rightarrow G = \frac{E}{3}$

for Metal  $\Rightarrow \frac{1}{4} \leq \mu \leq \frac{1}{3}$

$$\boxed{G < \frac{E}{2}}$$

$$\boxed{\frac{E}{3} \leq G \leq \frac{E}{2}}$$

$$\boxed{0.33 \text{ to } 0.50}$$

By equating ① and ②

$$2G(1+\mu) = 3K(1-2\mu)$$

$$2G + 2G\mu = 3K - 6K\mu$$

$$2G\mu + 6K\mu = 3K - 2G$$

$$\boxed{\mu = \frac{3K - 2G}{2G + 6K}}$$

## YOUNG'S Modulus (E) or Modulus of elasticity

As per Hooke's law, (It is valid in uniaxial dir<sup>n</sup>)

$$\sigma \propto \epsilon$$

$$\boxed{\sigma = E \cdot \epsilon}$$

$$\boxed{E = \frac{\sigma}{\epsilon}}$$

\* Young's Modulus (E) is the ratio of Normal stress to Normal strain (or axial stress to normal strain).  
(Normal) (longitudinal)

\* It is used to determine longitudinal strain.

Units → MPa, GPa, Pa.

If  $E=1$  then

$$\boxed{E = \sigma \text{ (GPa)}}$$

Amount of stress applied on unit strain.

Young's Modulus can also be defined as amount of normal stress is required to cause a unit normal strain longitudinal strain (i.e. the final length is equal to double to its original length).

$$\boxed{E=1 \Rightarrow L_f = 2L_i}$$

$$L_f = 2L_i \Rightarrow (\sigma_{req})_{MS} = 200 \text{ GPa}$$

$$(\sigma_{req})_{CS} = 100 \text{ GPa}$$

$$(\sigma_{req})_{diamond} = 1200 \text{ GPa}$$

$$(\sigma_{req})_{rubber} = 10 \text{ GPa}$$

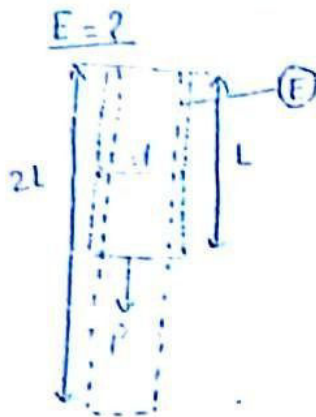
$$\boxed{E \uparrow \Rightarrow \sigma \uparrow = \text{accept linear elastic def} \uparrow \Rightarrow \delta \downarrow}$$



- \* A material is said to be more elastic than another material when it exhibits higher young's modulus than another material.

Examples Out of steel and Rubber, steel is more elastic than rubber. becoz of  $E_{\text{steel}} > E_{\text{rubber}}$

Problem

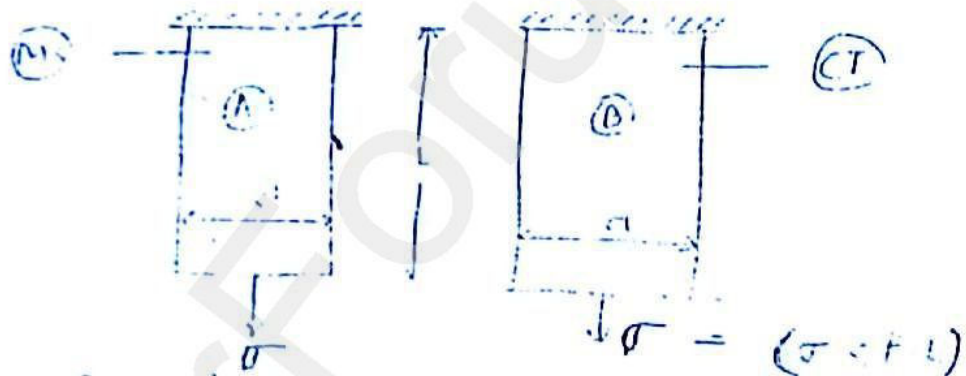


- ☒ (a)  $E$   
☐ (b)  $F/2$   
☐ (c)  $2E$   
☐ (d)  $El/4$

$$\delta = \frac{PL}{AE} = \frac{2PL}{AE}$$

but  $E$  doesn't change  
depend on dimension  
depends on material.

Gates Problem



which of following o/p are correct

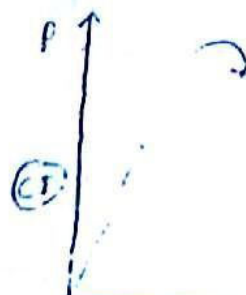
- ☐ (a)  $\delta_A = \delta_B$   
☒ (b)  $\delta_A < \delta_B$   
☐ (c)  $\delta_A > \delta_B$   
☐ (d)  $E_A = E_B$

(Higher the young's modulus of MS than CI so deflection of m.s will be less)

$$E_A > E_B \Rightarrow \delta_A < \delta_B$$

$$[\because \delta \propto \frac{1}{E}]$$

AND: deflection curve (MS & CI)



But  $\boxed{(\delta_{\text{Total}})_{MS} > (\delta_{\text{Total}})_{C.T.}} \quad \text{for total deflection}$

$$\boxed{(\delta_{\text{elastic}})_{MS} < (\delta_{\text{elastic}})_{CT}}$$

$$\boxed{(\delta_{\text{plastic}})_{MS} > (\delta_{\text{plastic}})_{CT}}$$

Problem → Intensity of stress require to cause unit linear strain → Young's Modulus.

### ① SHEAR MODULUS OR MODULUS OF RIGIDITY (G)

As per Hooke's Law -

$$\tau \propto \gamma \text{ or } \phi$$

$$\boxed{\tau = G\gamma}$$

$$\boxed{G = \frac{\tau}{\gamma} \text{ or } \frac{\tau}{\phi}}$$

When  $\gamma \text{ or } \phi = 1$

then  $\boxed{G = \tau}$

$$\boxed{G\tau \Rightarrow \tau \Rightarrow \text{acc to } \begin{matrix} \text{elastic} \\ \text{angular def.} \end{matrix} \uparrow \Rightarrow \phi_{\text{ind}} \downarrow}$$

different of E and G → Young's Modulus represent the longitudinal deformation. and Shear modulus used to determine  $\gamma$  and  $\phi$ .

### BULK MODULUS

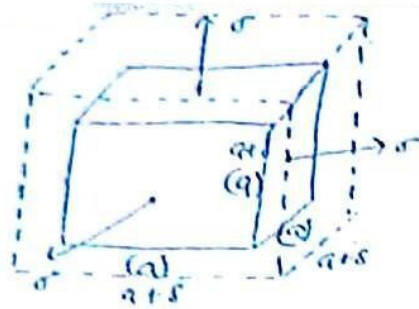
used to determine volumetric strain. and obtain a relationship b/w normal stress and volumetric strain.

(Hydrostatic or spherical stress) mag. &

used to determine —  $E_v$  and  $\delta_v$ .

nature of  
Normal stress in three  
mutual  $\perp$  dir. should  
be same.





- \* due to deformation shape will same but only change in volume when the stress forces will be change so shape will change. so that

$$\sigma = \frac{\delta V}{V} = \frac{\delta V}{a^3}$$

$$2\sigma = \frac{\delta V}{V} > \frac{\delta V}{a^3}$$

As per Hooke's Law  $\rightarrow$

$$\sigma \propto E_V$$

$$\sigma = K E_V$$

$$K = \frac{\sigma}{E_V} = \frac{\sigma V}{\delta V}$$

- \* Bulk modulus is defined as ratio of Normal stress to volumetric strain.

- \* Can also be defined as -

$$E_V = 1 \Rightarrow \boxed{K = \sigma \text{ MPa}} \\ (V_f = 2V_0)$$

- \* Bulk modulus is defined as amount of Normal stress required to cause unit volumetric strain.

$$\boxed{K \uparrow = \sigma \uparrow = -2ce \text{ to } E_V \uparrow = \delta V \downarrow}$$

- \* Bulk modulus is used to determine change in volume or volumetric strain when the stress applied on 3 faces will be same are called hydrostatic stress.

## POISSON'S RATIO ( $\mu$ )

used to determine lateral strain.

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$\epsilon_x, \epsilon_y, \epsilon_z$  → Lateral strain (11)  
 $\epsilon_x$  → Normal strain

+ lateral strain is used to determine  $\epsilon_v$  or  $\delta v$ .

As per hook's law -

Lateral strain  $\propto$  longitudinal strain

$$\text{Lateral strain} = \mu (\text{long. strain})$$

$$\mu = \left| - \frac{\text{Lateral strain}}{\text{long. strain}} \right| \quad \left[ \begin{array}{l} (-) \text{ sign means both lateral strain \& long-strain are in opposite to each other} \end{array} \right]$$

$$\boxed{\text{Lateral strain } (\epsilon_y \text{ or } \epsilon_z) = \mu \times \text{long. strain}} \quad \text{--- (E)}$$

As.  $\boxed{\mu \uparrow \Rightarrow \text{lateral strain} \uparrow \Rightarrow \epsilon_v \uparrow \Rightarrow \delta v \uparrow}$

If  $\mu = 0$  then, lateral strain ( $\epsilon_y \& \epsilon_z$ ) = 0

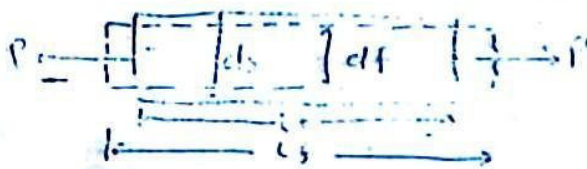
It means that that particular material will not exhibit any change in lateral dimension even in the presence of change in longitudinal dimensions.

Example → cork ( $\mu = 0$ ), concrete ( $\mu = -0.1$  to  $-0.2$ )

Metals ( $\mu = 0.25$  to  $0.33$ )

pure rubber ( $\mu = 0.5$ )

$$\hookrightarrow k = \infty \Rightarrow \epsilon_v = 0$$



$$\mu = \left| - \frac{\epsilon_y \text{ or } \epsilon_z}{\epsilon_x} \right|$$

$$\mu = \left| - \frac{\delta d / d_0}{\delta l / l_0} \right|$$



Problem

If  $E, G, K$  represent young's modulus, modulus of rigidity and bulk modulus resp. for an elastic material. Then

which one of following is possibly true.

- (A)  $G=2K$  (B)  $G=E$  (C)  $K=E$  (D)  $G=E=L$

$$\mu = \frac{3K-2G}{6K+2G}$$

30. In an experiment performed bulk modulus of a material is equal to its shear modulus then poisson's ratio is?

- (A) 0.25 (B) 0.33  
(C) 0.5 (D) 0.6

$$E = 2G(1+\mu)$$

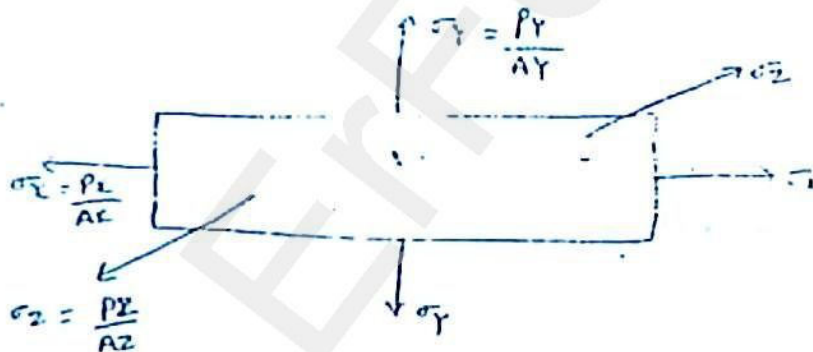
$$G = K$$

$$E = 3K(1-2\mu)$$

$$E/2(1+\mu) = E/3K(1-2\mu) \Rightarrow 2(1+\mu) = 3(1-2\mu)$$

$$\mu = 0.25$$

### 31. EXPRESSION FOR VOLUMETRIC STRAIN UNDER TRI-AXIAL LOADING $\rightarrow$



$$\epsilon_v = \frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\delta V = V[\epsilon_x + \epsilon_y + \epsilon_z]$$

$$\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} \\ \epsilon_y &= \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} \\ \epsilon_z &= \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \end{aligned}$$

for biaxial loading -  $\sigma_z = 0$

for uniaxial loading -  $\sigma_y = \sigma_z = 0$

Volumetric strain:  $\delta V = V [\epsilon_x + \epsilon_y + \epsilon_z]$

$$= \frac{V}{E} [\sigma_x (1-2\mu) + \sigma_y (1-2\mu) + \sigma_z (1-2\mu)]$$

$$\boxed{\delta V = V \left( \frac{1-2\mu}{E} \right) [\sigma_x + \sigma_y + \sigma_z]}$$

[if Material is rubber ( $\mu = 0.5$ ) then change in volume will be zero.]

\*  $\boxed{\delta V = 0} \Rightarrow$  if either  $\mu = 0.5$  or  $\sigma_x + \sigma_y + \sigma_z = 0$

for uniaxial loading:-

$$\sigma_y = \sigma_z = 0$$



$$\boxed{\delta V = V \left[ \frac{1-2\mu}{E} \right] \sigma_x}$$

$$\boxed{\epsilon_v = \left( \frac{1-2\mu}{E} \right) \sigma_x}$$

\*  $\epsilon_v = 0$  or  $\delta V = 0$  only when  $\mu = \frac{1}{2}$

\* for any metal other than rubber -

$$\delta V \neq 0 \quad [\because \mu \neq \frac{1}{2}] \quad [\sigma < 1.5]$$

\*  $\delta V = 0 \quad [\sigma > 1.5]$  plastic deformation.

Problem: An elastic body subjected to the direct compress stress ( $\sigma_x$ ) in the longitudinal direction. If the lateral strain in other two directions are prevented by applying ( $\sigma_y$  &  $\sigma_z$ ) in those dir's then  $\sigma_y = \sigma_z = ?$

$$\epsilon_x = -\frac{\sigma_x}{E} + \frac{\mu \sigma_y}{E} + \frac{\mu \sigma_z}{E}$$

$$\epsilon_x = -\frac{\sigma_x}{E} + \frac{\mu \sigma_y}{E} + \frac{\mu \sigma_y}{E}$$



$$\epsilon_v = -\frac{\sigma_x}{E} + \frac{2\mu\sigma_y}{E} = \frac{1}{E}[\sigma_x - 2\mu\sigma_y]$$



Solution

$$\sigma_y = \sigma_z = \sigma$$

$$\epsilon_y = \epsilon_z = 0$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_y)] = 0$$

$$\sigma_y - \mu(\sigma_x + \sigma_y) = 0$$

$$\mu(\sigma_x + \sigma_y) = \sigma_y$$

$$\mu\sigma_x + \mu\sigma_y = \sigma_y$$

$$\mu\sigma_x + \mu\sigma = \sigma$$

$$\mu\sigma = \sigma(1 - \mu)$$

$$\boxed{\sigma = \frac{\mu\sigma_x}{1 - \mu}} \quad \text{Ans}$$

problem

A material of balloon rubber has a  $\mu$  is 0.5. If uniform pressure is applied to grow the balloon the volumetric strain of material will be-

$$\mu = 0.5$$

$$\epsilon_v = \mu \left[ \frac{1 - 2\mu}{E} \right] [\sigma_x + \sigma_y + \sigma_z]$$

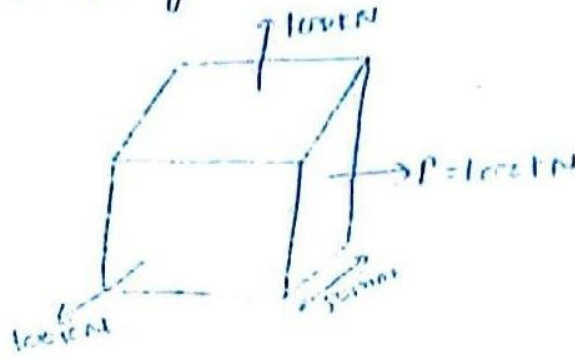
$$\boxed{\epsilon_v = 0}$$

$$\text{or } K = \frac{\sigma}{\epsilon_v} = \frac{-P}{\epsilon_v}$$

$$E = 3K(1 - 2\mu) = 0$$

$$\boxed{\epsilon_v = \frac{-P}{K} = \frac{-P}{\infty} = 0} \quad (\mu = 0, 1, \infty)$$

Problem For a cube as shown in figure determine change in volume of cube if young's modulus  $= (E) = 200 \text{ GPa}$  and  $\mu = 0.25$



$$\delta V = V \left( \frac{1-2\mu}{E} \right) (\sigma_x + \sigma_y + \sigma_z)$$

$$\frac{\delta V}{V} = \frac{1-0.50}{200 \times 10^6} [300 \text{ kN}]$$

$$\delta V = \frac{(50)^3 \cdot 0.5}{200 \times 10^6} [300 \times 10^3] \text{ kN}$$

$$\boxed{\delta V = 37.5}$$

$$\sigma_x = \sigma_y = \sigma_z = \frac{P_x}{A} = \frac{100 \times 10^3}{50 \times 50} = 40 \text{ MPa}$$

$$E = 3K(1-2\mu)$$

$$K = \frac{E}{3(1-2\mu)} = \frac{200}{3(1-2 \times 0.25)} = \frac{2000}{0.5} = \frac{400}{3} \text{ GPa}$$

$$K = \frac{\sigma_x}{\epsilon_x} \Rightarrow \frac{\sigma_x}{\frac{\delta V}{V}} = \frac{V \sigma_x}{\delta V}$$

$$\delta V = V \left[ \frac{\sigma_x}{K} \right] = \frac{50 \times 50 \times 50 \times 40}{\frac{400}{3} \times 10^3} = 37.5 \text{ mm}^3$$

Problem Repeat the above question if cube is replaced by rectangular block of  $200 \text{ mm} \times 50 \text{ mm} \times 10 \text{ mm}$ .

$$\delta V = \frac{V(1-2\mu)}{E} (\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{200 \times 50 \times 10}{2000} \left[ \frac{100}{100} + \frac{100}{50} + \frac{100}{200} \right]$$

$$= \frac{1}{2000} [0.1 + 0.2 + 0.05] [1-0.5]$$

$$\boxed{\delta V = 6.5 \times 10^{-5}} \text{ Ans}$$



## Strain Energy, Resilience, Toughness

### ① Strain Energy →

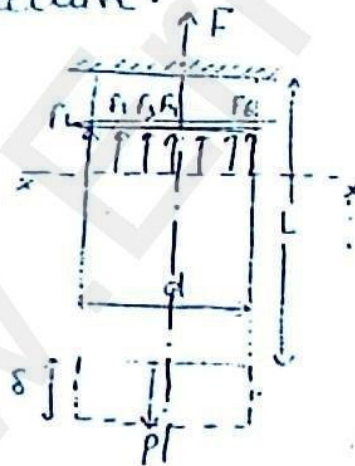
Defined as the ability of a material absorbed energy when it is strained.

### ② Resilience →

Defined as the ability of a material absorbed energy in the elastic region when it is strained.

### ③ Toughness →

Defined as the ability of a material to absorb energy prior to its fracture.



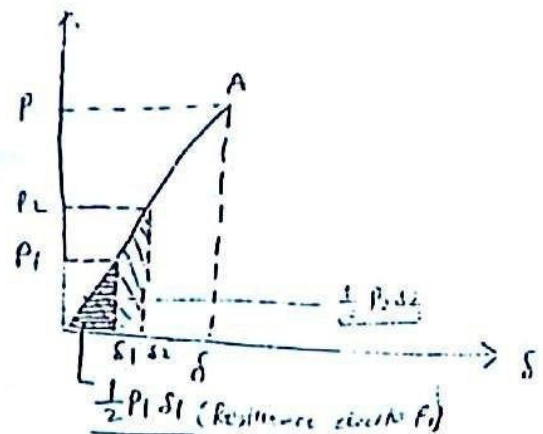
$$\text{w.d. by } P = \frac{1}{2} P \delta \rightarrow \textcircled{1}$$

Strain energy absorbed = w.d. by IRF (F) → ①

$$\textcircled{2} \quad S \cdot F = \text{w.d. by IRF (F)} = \text{w.d. by } P$$

$$S \cdot E = \text{w.d. by } P$$

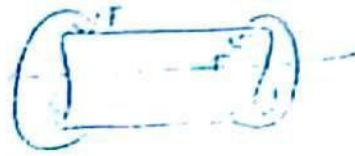
$$\boxed{S \cdot E = \frac{1}{2} P \cdot \delta} \quad (\text{axial loading})$$



\* Strain energy by torsion →

$$S.E = W.D \text{ by } T$$

$$S.E = \frac{1}{2} T \cdot \theta$$



\* Strain energy by bending →

$$S.E = W.D \text{ by } P$$

$$S.E = \frac{1}{2} P \times \frac{PL^3}{3EI}$$



\* In other words, it is equal to area under load-deflection curve within elastic region.

Proof Resilience →

Defined as the maximum energy absorption capacity of material in the elastic region.

$$\text{Resilience} = \frac{1}{2} P_1 \delta_1 \text{ or } \frac{1}{2} P_2 \delta_2$$

$$\text{Resilience} = \text{Area of } P \text{ vs } \delta \text{ curve within elastic region}$$

$$\text{Proof Resilience} = \text{Area of } P \text{ vs } \delta \text{ curve upto elastic limit}$$

Modulus of Resilience → (U.R)

$$U.R = \frac{\text{Proof Resilience}}{\text{Volume}}$$

$$U.R = P.R / V = \frac{1}{2} \frac{P_{EL}}{A_0} \cdot \frac{\delta_{EL}}{L_0}$$

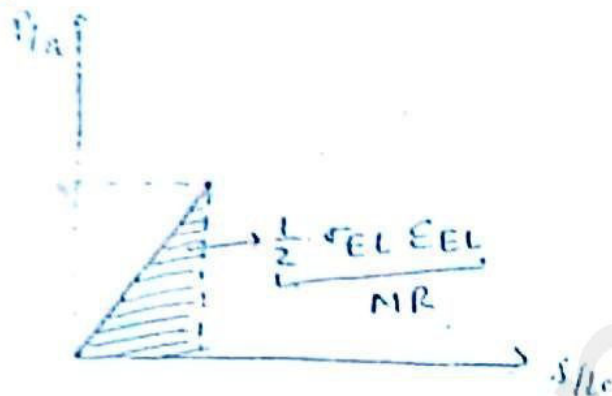
$$M.R (U.R) = \frac{1}{2} \sigma_{EL} \cdot \epsilon_{EL} = \frac{\sigma_{EL}^2}{2E}$$

$$\left[ \epsilon = \frac{\sigma}{E} \right]$$



\* defined as area of stress-strain (engg) curve upto elastic limit. or

defined as area of  $P$  vs  $\delta$  curve upto elastic limit per unit volume.



$$M.R = \frac{\sigma_{EL}^2}{2E}$$

$$\frac{P.R}{Vol} = \frac{\sigma_{EL}^2}{2E}$$

$$P.R = \frac{\sigma_{EL}^2}{2E} \times \text{Volume} \quad \text{or} \quad \frac{1}{2} P_{EL} \delta_{EL}$$

$$\text{Max S.E in elastic Region} = \frac{\sigma_{EL}^2}{2E} \times \text{Volume}$$

$$S.E \uparrow \Rightarrow \text{Volume} \uparrow \Rightarrow, \sigma_{EL} \uparrow \Rightarrow E \downarrow$$

(U)

\* The member subject to impact loading strain energy will be increase. S.E plays important role in design consideration.

$$S.E \uparrow \Rightarrow \tau_{\text{Impact}} \downarrow \Rightarrow \text{change of failure} \downarrow$$

$$\tau_{\text{Impact}} = \sigma_{st} \times I.F$$

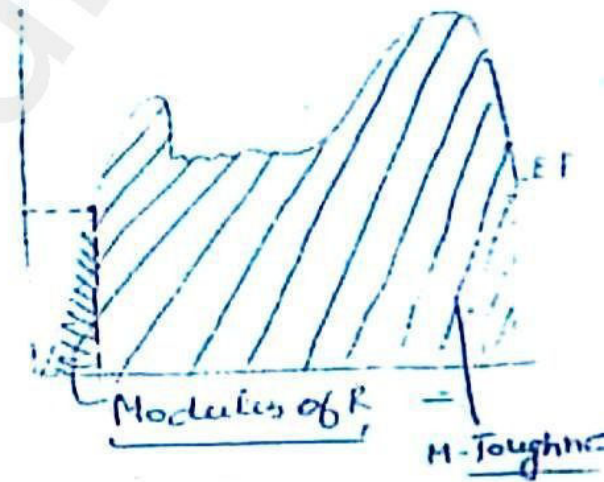
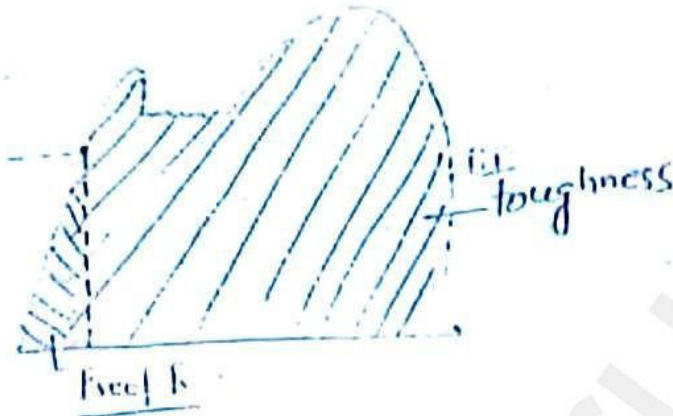
$$I.F = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$\delta_{st} = \frac{PL}{AE} + \frac{PL^3}{3EI}$$

$[L] \text{ or } E \downarrow \Rightarrow S_{uc} \uparrow \Rightarrow I.F. \downarrow \Rightarrow \sigma_{\text{Impact}} \downarrow \Rightarrow \text{chance of failure} \downarrow$

Toughness  
Continue

Total area of load vs deflection curve.  
(upto fracture point)



Modulus of toughness →

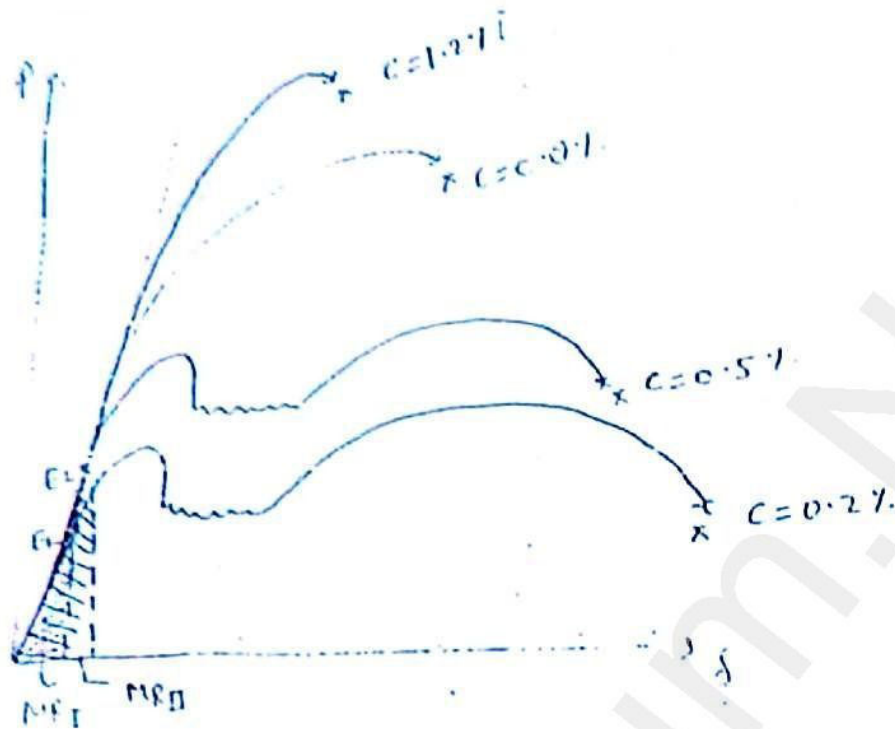
\* Resilience and toughness are the two important properties for a material of a component (m/c component) when it is subjected to impact loading.

\* Resilience is an important property for a material of m/c component when it is subjected elastic deformation due to impact load.

Ex: Spring.

\* Toughness an imp. property of material of a m/c component when it is subjected to plastic or permanent deformation due to impact load.

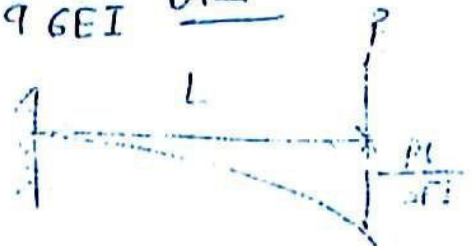
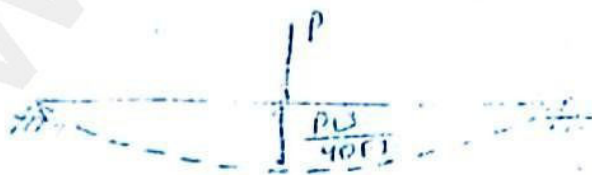




$C \uparrow = \text{Resilience} \uparrow = \text{Strength} \uparrow = \text{hardness} \uparrow = \text{wear resistant} \uparrow$   
 $= \text{yield point} \uparrow = \text{ultimate point} \uparrow = \text{cost} \uparrow$   
 $\text{toughness} \downarrow = \text{ductility} \downarrow = \text{Malleability} \downarrow = \text{formability} \downarrow$   
 $\text{Machinability} \downarrow$

Problems  $\rightarrow \frac{PL^3}{40EI}$  is the deflection under the load  $P$  when it is acting at mid span of a simply supported beam of length  $(L)$ , modulus of elasticity  $(E)$ , moment of inertia  $(I)$ , then strain energy due to bending is

$$SE = \frac{1}{2} \times P \times \delta \Rightarrow SE = \frac{1}{2} P \times \frac{PL^3}{40EI} = \frac{P^2 L^3}{80EI} \quad \underline{A_m}$$



Problem A square bar of side 4 cms and length 100 cms is subjected to an axial load. The same bar is then used as a cantilever beam and subjected to a end load P. The ratio of strain energies stored in the second case to that stored in 1st case is -

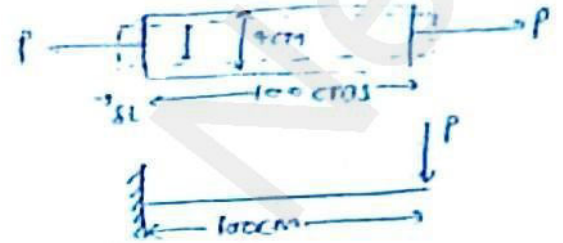
Sol. Ind Strained energy =  $\frac{1}{2} P \times \delta$

$$= \frac{1}{2} \times P \times \frac{PL^3}{3EI}$$

$$= \frac{P^2 L^3}{6EI} = \frac{P^2 \times 1^3 M}{6EI}$$

1st case Strained energy =  $\frac{1}{2} P \times \delta$

$$= \frac{1}{2} \times P \times \frac{PL}{AE} = \frac{P^2 \times 1}{2AE}$$



$$\frac{\text{Strained energy 2nd case}}{\text{Strained energy 1st case}} = \frac{\frac{P^2}{6EI}}{\frac{\frac{1}{2} P^2 \times 1}{AE}} = \frac{A}{3EI} = \frac{1 \times 0.4}{3 \times \frac{\pi}{4} \times \frac{1 \times 0.4^3}{12}} = \frac{16}{3\pi}$$

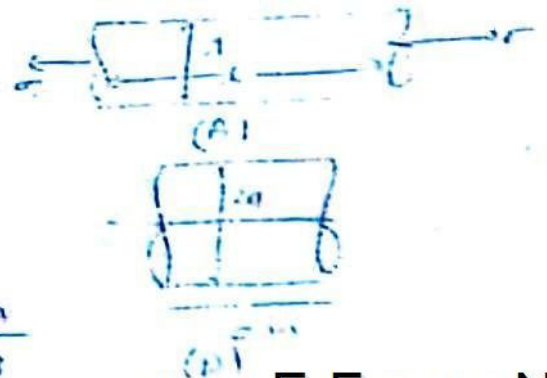
$$\boxed{\frac{U_2}{U_1} = \frac{\delta_2}{\delta_1} = 2500}$$

Problem A round bar (A) of length  $L_1$  and diameter  $\phi_1$  is subjected to an axial force producing stress  $\sigma_1$ . Another round bar (B) of length  $L_2$  and diameter  $\phi_2$  and of same material. It is also subjected to the same stress  $\sigma_1$ . The ratio of strain Energy in bar A to strain  $E_g$  in bar B is given by -

Soln  $\frac{(U \cdot E)_A}{(U \cdot E)_B} = \frac{\frac{1}{2} P \cdot \delta_A}{\frac{1}{2} P \cdot \delta_B} = \frac{\frac{\sigma_1^2 E_1^2}{2E_1} \cdot V_1}{\frac{\sigma_1^2 E_2^2}{2E_2} \cdot V_2}$

$$\frac{U_A}{U_B} = \frac{V_A}{V_B} = \frac{\frac{A_1 L_1}{L_2 A_2}}{\left(\frac{\phi_1}{\phi_2}\right)^2} = \frac{L_1}{L_2}$$

(45)





$$\frac{U_A}{U_B} = \frac{1}{4} \times 2 = \frac{0}{4} = 0.5 \text{ (Ans)}$$

Hint (Method 1)

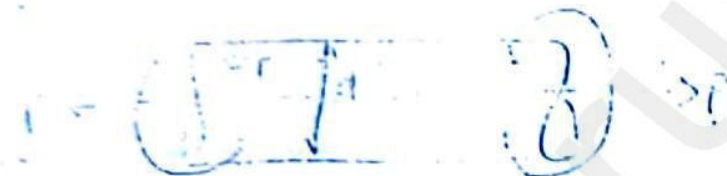
$$\frac{U_A}{U_B} = \frac{\frac{1}{2} P_A \delta_A}{\frac{1}{2} P_B \delta_B} = \frac{\frac{1}{2} \sigma_A \times A_A \times \frac{\sigma_A L_A}{E_A}}{\frac{1}{2} \sigma_B \times A_B \times \frac{\sigma_B L_B}{E_B}}$$

$$= \frac{\frac{1}{2} \times \sigma_A^2 \times A_A \times \frac{L_A}{E_A}}{\frac{1}{2} \times \sigma_B^2 \times A_B \times \frac{L_B}{E_B}}$$

$$= \frac{A_A \times L_A}{A_B \times L_B} = \left(\frac{d_A}{d_B}\right)^2 \times \frac{L_A}{L_B} = \frac{1}{4} \times 2 = 0.5$$

$$= 0.5$$

Problem 2



$U_1 = \delta$  S.E due to P.

$U_2 = \delta$  S.E due to T.

Total S.E =  $U_1 + U_2$

$$= \frac{1}{2} P \delta + \frac{1}{2} T \cdot \theta$$

$$= \frac{1}{2} P (\delta) + \frac{1}{2} T \theta$$

$$= \frac{1}{2} P \left[ \frac{P L^3}{A E} + \frac{T L}{G J} \right]$$

$$= \frac{1}{2} \frac{P^2 L}{A E} + \frac{1}{2} \frac{T^2 L}{G J}$$

$$= \frac{1}{2} \left[ \frac{P^2 L}{A E} + \frac{T^2 L}{G J} \right]$$

$$= \frac{1}{2} \left[ P \frac{P L}{A E} + T \times \frac{T L}{G J} \right]$$

$$= \frac{1}{2} \left[ \sigma^2 A^2 \frac{L}{A E} + \frac{\tau^2 \times 2 P^2 L}{E [(1+\mu) \pi d^3]} \right]$$

$$= \frac{1}{2} \left[ \frac{\sigma^2 A L}{E} + \frac{2 \tau^2 \cdot J^2 \cdot L}{P^2 \cdot G [(1+\mu) \pi d^3]} \right]$$

$$\frac{T}{J} = \frac{G \theta}{L}$$

$$\theta = \frac{T L}{G J}$$

$$\begin{cases} T = \frac{T}{2P} \\ T = \gamma \cdot 2P \\ = \gamma \cdot \frac{\pi}{16} d^3 \\ 2P = \frac{T}{P} \end{cases}$$

$$\left[ U_{\text{total}} S.E = \frac{1}{2} \frac{P^2}{E} \right] = \frac{1}{2} \frac{P^2 L}{EA} + \frac{2\pi \cdot \gamma \cdot (H \cdot D)}{R^2} \left| \right|$$

### ASSUMPTIONS →

- (i) Material should be homogeneous
- (ii) Material should be isotropic
- (iii) Material follows the hook's law
- (iv) CS should be prismatic
- (v) Residual stress (Pre-stress) and initial stress are neglected.
- (vi) Shear strain analysis carried out using stress and strain
- (vii) Saint Venant's principle is valid

### ① ALONGATION OF BAR DUE TO SELF WEIGHT

$\delta = PL/AE$ ,  $\sigma_a = P/A$ ,  $P = \sigma_a A$   
 self weight  $= P_L = \gamma \times \text{Volume}$   $\gamma = \text{sp. weight}$   
 $= \gamma \times A \times L$   
 Acting on the volume  
 self weight of body ( $P=0$ )  
 Mag. of  $P$  (Applied load) acting on surface

(A)  $P_{1-1} = P_{x-x} = P_{2-2} = P$

(B)  $\begin{cases} P_{1-1} = 0 \\ P_{x-x} = \gamma A x \\ P_{2-2} = \gamma A L \end{cases}$

$$P_{x-x} = \gamma A x$$

$$\sigma_{x-x} = \frac{P_{x-x}}{A} = \gamma x$$

$\sigma_{\text{Max}} = \gamma L$

$$\epsilon_{x-x} = \frac{\sigma_{x-x}}{E} = \frac{\gamma x}{E}$$

$$\frac{\delta_{\text{strip}}}{dx} = \frac{\gamma x}{E}$$

$$\delta(\text{strip}) = \frac{\gamma x}{E} dx$$

$$\delta(\text{total}) = \int_0^L \frac{\gamma x}{E} dx = \left( \frac{\gamma x^2}{2E} \right)_0^L = \frac{\gamma L^2}{2E}$$

$\delta(\text{total}) = \frac{PL^2}{2AE} = \frac{\gamma L^2}{2AE}$



Problem \* Elongation of a prismatic bar under its <sup>own weight</sup> is half of a elongation of a identical prismatic bar under an axial load which is equal to weight of bar.

Problem If all the dimensions of prismatic bar becomes double then what is elongation of a prismatic bar under its self weight in terms of original elongation.

Solution

$$\delta_1 = \frac{\gamma L^2}{2E} \propto \frac{PL}{2AE}$$

$$\delta_1 = \frac{\gamma (2L)^2}{2E}$$

$$\frac{\delta_2}{\delta_1} = \frac{\frac{\gamma L^2}{2E}}{\frac{\gamma (2L)^2}{2E}} \times \frac{2P}{\gamma 4L^2} = \frac{1}{4}$$

$$\boxed{\delta_1 \geq 4 \delta_2} \quad \underline{4 \text{ times}} \quad \underline{\text{Ans}}$$

Problem

\* Elongation of prismatic bar under self weight is directly proportional to  $L^2$  but independent of change in its dim

$$\boxed{\delta \propto L^2}$$

Problem

Repeat the above question under an axial load (P) instead of self weight -  $\left(\frac{1}{2}\right)$

Problem

If all the dimensions of a prismatic bar becomes double then strain energy of bar due to its self weight becomes 32 times the original stress energy.

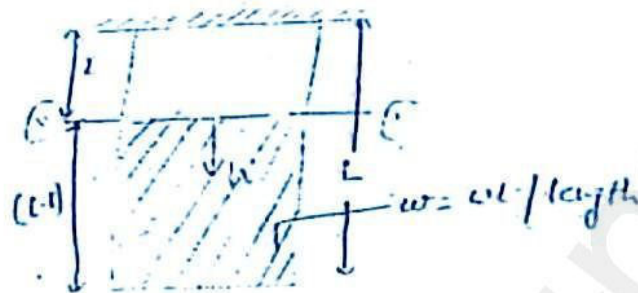
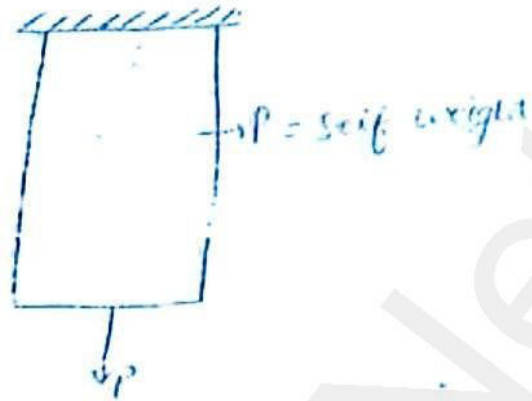
$$U = \frac{1}{2} P \delta = \frac{1}{2} \times 0.8P \times 4\delta_1 = 32 \left( \frac{1}{2} P \delta \right) \underline{\underline{\text{Ans}}}$$

$$P = \gamma A L = \gamma \pi 2d^2 \times L = 0.8 (\gamma A L) = 0.8P$$

$$\delta = \frac{\gamma L^2}{2E} = \underline{4\delta}$$

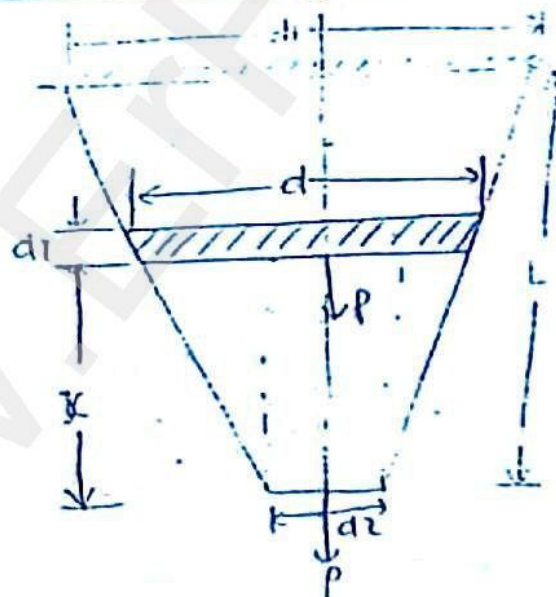
$$\delta_{total} = \frac{PL}{AE} + \frac{PL}{2AE}$$

$$= \frac{3}{2} \left[ \frac{PL}{AE} \right]$$



$$\text{total weight} = W + w(L-x)$$

### 0 ELONGATION OF A TAPERED BAR UNDER AXIAL LOAD



$$\delta_{strip} = \frac{PL}{AE} = \frac{P \times dL}{\frac{\pi}{4} d^3 \cdot E}$$

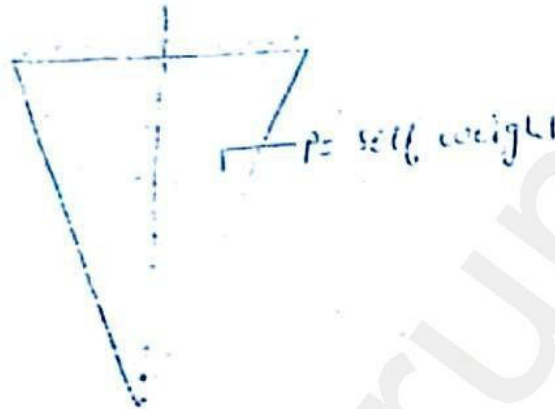
$$d = d_2 + (d_1 - d_2) \left( \frac{x}{L} \right)$$

$$\left[ \text{Total elongation} \right] \delta_{strip} = \frac{wL}{\pi d_1 d_2 E}$$



\* Eq<sup>n</sup> Elongation of tapered bar is equal to the elongation of prismatic bar which under axial load whose diameter of prismatic bar (d) is equal to geometric mean of dia. of tapered bar.

$$d = \sqrt{d_1 d_2}$$

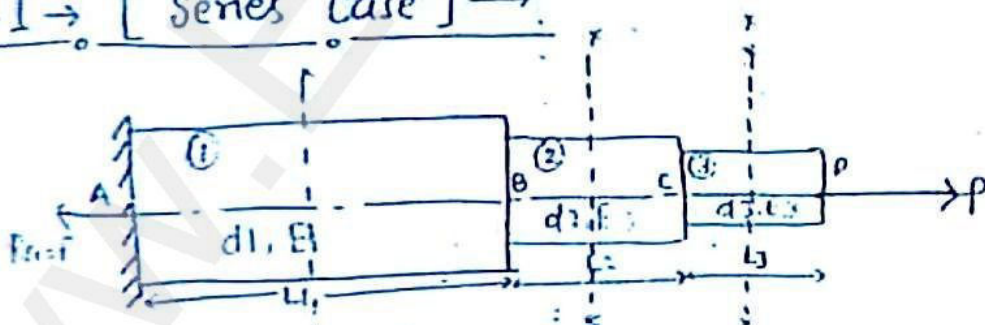


$$\delta = \frac{\gamma L^2}{6E}$$

### COMPOUND BARS

[Bars in Series and parallel]

\* Case-I → [Series Case] →



When no. of unknown <sup>or</sup> no. of reaction are less than no. of eq<sup>n</sup> is known as determinate statically.

No. of unknown or reaction  $\leq$  No. of useful st. eq<sup>n</sup>s

$$\left. \begin{array}{l} \sum H = 0 \\ \sum V = 0 \\ \sum M = 0 \end{array} \right\} \text{St. eq's}$$

(i) When bars are connected in series and single load is applied at free end.

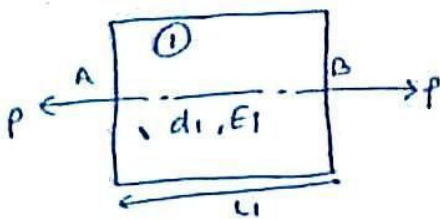
$$P_1 = P_2 = P_3 = P$$

Stress tensor will be  $= \begin{bmatrix} \sigma & 0 \\ 0 & 0 \end{bmatrix}$  (uniaxial)

\* Stress is developed when - only strain is restricted completely or partially.

(ii)  $\delta_{total} = \delta_1 + \delta_2 + \delta_3$

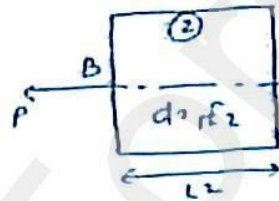
Methods  $\rightarrow$  (1) Free-body diagram  $\rightarrow$



$$P_1 = P$$

$$\delta_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{4 P L_1}{\pi d_1^2 E_1}$$

$$\sigma_1 = \frac{P_1}{A_1} = \frac{4 P}{\pi d_1^2}$$

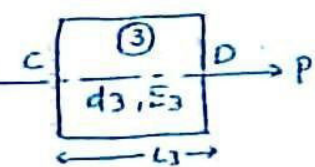


$$P_2 = P$$

$$\delta_2 = \frac{P_2 L_2}{A_2 E_2}$$

$$= \frac{4 P L_2}{\pi d_2^2 E_2}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{4 P}{\pi d_2^2}$$



$$P_3 = P$$

$$\delta_3 = \frac{P_3 L_3}{A_3 E_3} = \frac{4 P L_3}{\pi d_3^2 E_3}$$

$$\sigma_3 = \frac{P_3}{A_3} = \frac{4 P}{\pi d_3^2}$$

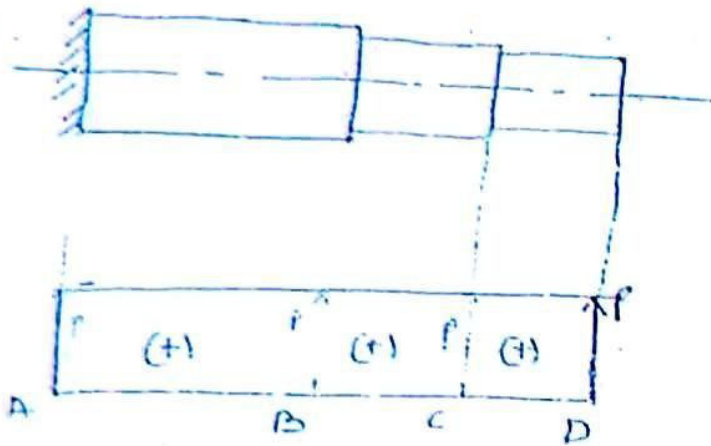
(2) Section diagram  $\rightarrow$

Algebraic No of sum is  $= P$   
 Forces applied only non-junction point

(3) Axial loading diagram  $\rightarrow$

$$\delta_{total} = \frac{4 P}{\pi} \left[ \sum_{i=1}^n \frac{L_i}{E_i d_i^2} \right]$$





\* diagram is closed that means all forces are in equilibrium.

\* Then the critical portion section is CD because it has max. shear stress (load) and least area

$$\sigma = \frac{P}{A} \quad \text{so if } A \downarrow \sigma \uparrow \text{ and } P \uparrow$$

$$\begin{aligned} \sigma_{\max} &= \sigma_3 \\ \sigma_{\min} &= \sigma_1 \end{aligned}$$

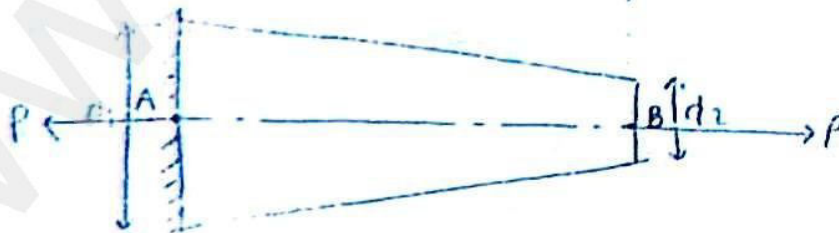
$$\frac{\sigma_{\max}}{\sigma_{\min}} = \frac{\sigma_3}{\sigma_1} = \left( \frac{d_1}{d_3} \right)^2 = \left( \frac{d_{\max}}{d_{\min}} \right)^2$$

\* Three type of questions are asked —

- ① Total deflection
- ② Critical Portion
- ③ Load on each section

### TAPERED BAR

CASE-I



Strength

$$\textcircled{5} \quad \left. \begin{aligned} (\sigma_{ind})_{max} &< Y\text{-Strength (failure occurs) (ductile)} \\ &< \text{Ultimate Tensile Strength (failure occurs) (brittle)} \end{aligned} \right\}$$

then we use factor of safety and decided by F.O.S (N).

Design stress

$$\boxed{(\sigma)_{max} \leq \text{Permissible st.}}$$

⑥ Yield strength and ultimate tensile stress is called static failure stress.

$$\textcircled{7} \quad \boxed{\sigma_{ind} \leq \sigma_{per}}$$

$$\boxed{\sigma_{per} = \frac{\text{Failure stress}}{N \text{ (F.O.S.)}}}$$

$$\textcircled{8} \quad \left. \begin{aligned} \text{Failure stress} &= YS \\ &= UTS \\ &= EL \end{aligned} \right\}$$

⑨ Endurance Limit →

Defined as the max value of completely reversed stress that a material can withstand for an infinite No. of cycle without a fatigue failure without crack initiation.

$$\boxed{\sigma_{ind} \leq \frac{EL}{N}}$$

← Permissible stress or allowable stress, design stress, working/safe stress.

$$* \frac{EL}{N} \rightarrow \sigma_{permissible}$$

$$* \text{But } EL, YS, UTS \rightarrow \sigma_{Failure}$$

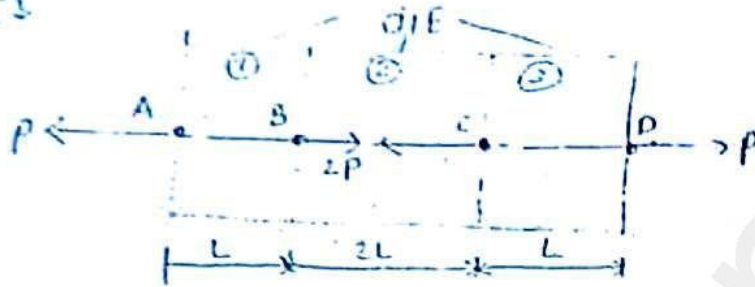


- \* A Taper bar are the assembly of infinite number of bars which are in series.

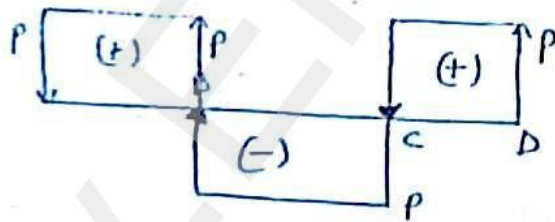
$$\frac{\sigma_{\max}}{\sigma_{\min}} = \left(\frac{d_1}{d_2}\right)^2$$

$$\delta_{\text{total}} = \delta_1 + \delta_2 + \dots + \delta_n$$

CASE-II-3



	$P_1$	$P_2$	$P_3$
a	$P$	$-P$	$P$
b			
c			
d			



$$\delta_{\text{total}} = \frac{1}{AE} [P_1 L_1 + P_2 L_2 + P_3 L_3]$$

$$= \frac{1}{AE} [P \times L - P \times 2L + P \times L]$$

$$\delta_{\text{total}} = 0$$

Problem

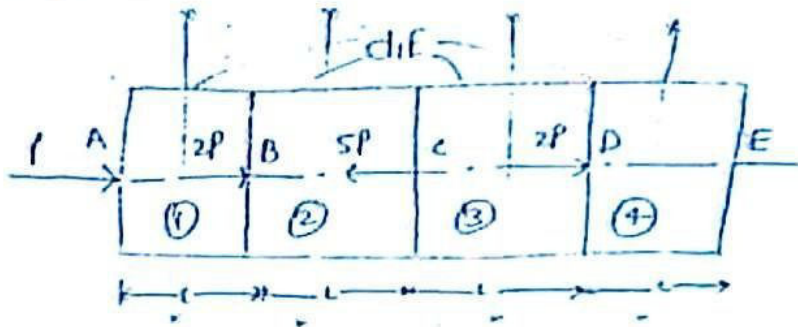
A compound bar as shown in fig. determine the following

- ① Load acting in each member.
- ② Critical portion of compound bar.



(ii) total deformation on compound bar.

Solution

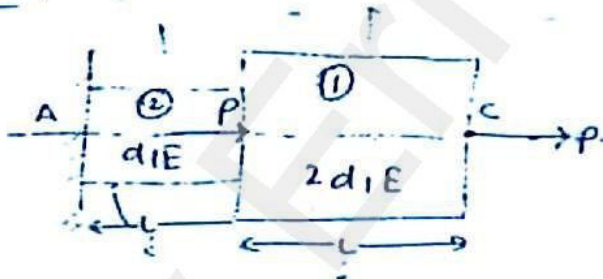


$$P_4 = 0, P_3 = 2P, P_2 = -3P, P_1 = -P$$

$P_3$  = critical portion because max. load

$$\begin{aligned} \text{total deformation} &= \frac{L}{AE} [P - 3P + 2P + 0] \\ &= \frac{-2PL}{AE} \end{aligned}$$

Problem: For a compound bar is shown in diagram determine the ratio of stresses in the 1st and second member ( $\sigma_1 / \sigma_2$ ).



$$P_1 = P, P_2 = 2P$$

$$\sigma_1 = \frac{P_1}{A_1}, \sigma_2 = \frac{P_2}{A_2}$$

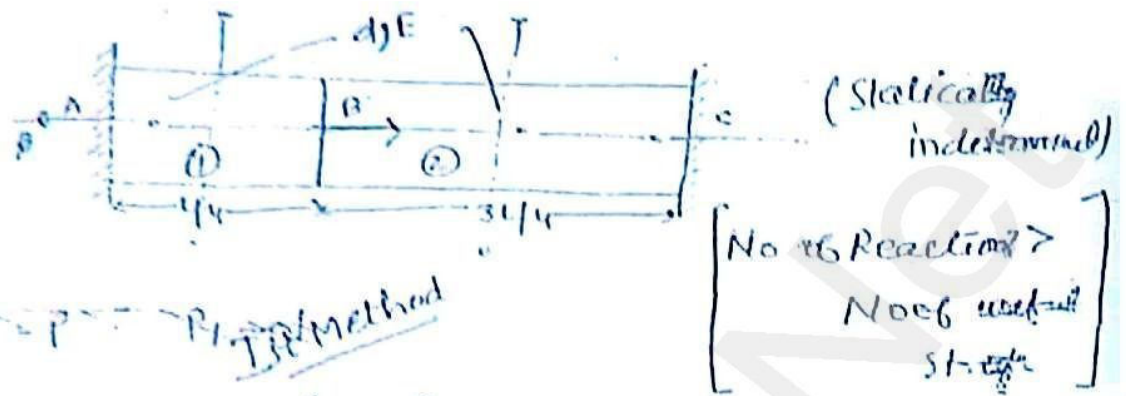
$$\sigma_1 = \frac{P}{A_1}, \sigma_2 = \frac{2P}{A_2}$$

$$\sigma_1 = \frac{4P}{\pi d_1^2}, \sigma_2 = \frac{4P}{\pi d_2^2}$$

$$\begin{aligned} \frac{\sigma_1}{\sigma_2} &= \frac{P_1/A_1}{P_2/A_2} = \frac{P_1}{P_2} \times \left(\frac{d_2}{d_1}\right)^2 \\ &= \frac{P}{2P} \left(\frac{d}{2d}\right)^2 \\ &= \frac{1}{8} \end{aligned}$$



CASE-III



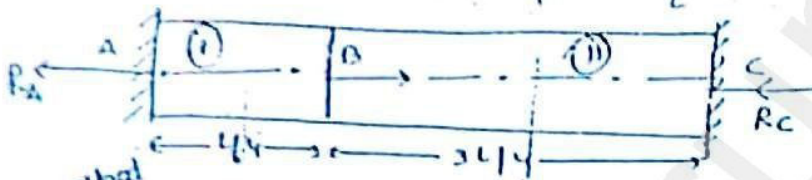
$P_1 = P$  (Method)

$$R_A = \frac{P \times 3L/4}{L/4}$$

$$R_A = \frac{P \times 3L/4}{L} = \frac{3P}{4}$$

$$R_C = \frac{P \times L/4}{L} = \frac{P}{4}$$

[application of eqn  
of deflection  
must should be  
same]



Ind Method

①  $P_1 = R_A$

②  $P_2 = -R_C$

③  $\sum u = 0$

$R_A + R_C = P$  ——— ④

④  $\delta_{total} = \delta_1 + \delta_2 = 0$

$\delta_1 = -\delta_2$

$$\frac{P_1 L_1}{A_1 E_1} = -\frac{P_2 L_2}{A_2 E_2}$$

$$R_A \times \frac{L}{4} = + R_C \times \frac{3L}{4}$$

$$R_A = 3R_C$$

from eq ④

$$R_A + R_C = P$$

$$3R_C + R_C = P$$

$$R_C = \frac{P}{4}$$

$$P_1 = \frac{3P}{4}$$

$$P_1 = R_A = \frac{3P}{4} \text{ (tension)}$$

$$P_2 = -R_C = -\frac{P}{4} \text{ (comp)}$$

\*Critical position - AB

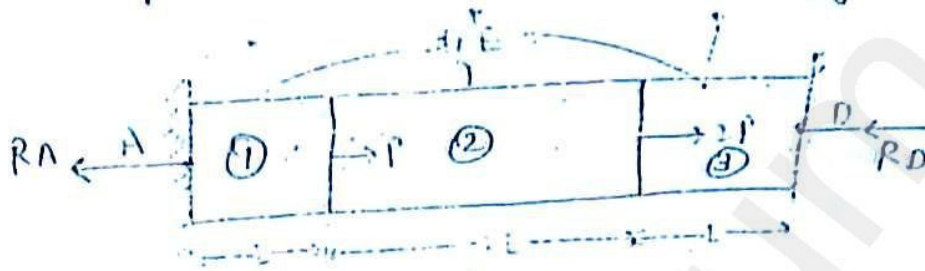
$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{P_{max}}{P_{min}}$$

$$= \frac{P_1}{P_2} = \underline{\underline{3}}$$

1st Member

Reaction at one end =  $\frac{\text{Applied load} \times \text{distance b/w applied load and another fixed end}}{\text{Distance b/w fixed ends.}}$

Problems determine the loads acting in each member for a compound bar as shown in figure.



Step-I

$$\begin{cases} P_1 = P_A = R_A \\ P_3 = P_D = -R_D = R_A - 3P \\ P_2 = R_A - P \end{cases}$$

Step-II

$$\begin{cases} R_A + R_D = 3P \\ R_D = 3P - R_A \end{cases}$$

Step-III

$$\begin{cases} R_A = \frac{P \times 3L + 2P \times L}{4L} \\ = \frac{5P}{4} \end{cases}$$

$$\begin{aligned} R_D &= 3P - R_A = 3P - \frac{5P}{4} = \frac{7P}{4} \\ \boxed{P_1 = P_A = \frac{5P}{4}} &\quad (\text{tension}) \\ P_2 &= R_A - P = \frac{5P}{4} - P = \frac{P}{4} \\ P_3 &= P_D = \frac{7P}{4} \end{aligned}$$

$$\boxed{P_3 = \frac{7P}{4}} \quad (\text{comp})$$

other

$$\delta_{total} = \delta_1 + \delta_2 + \delta_3 = 0$$

Step IV

$$= \frac{1}{AE} [P_1 L_1 + P_2 L_2 + P_3 L_3] = 0$$

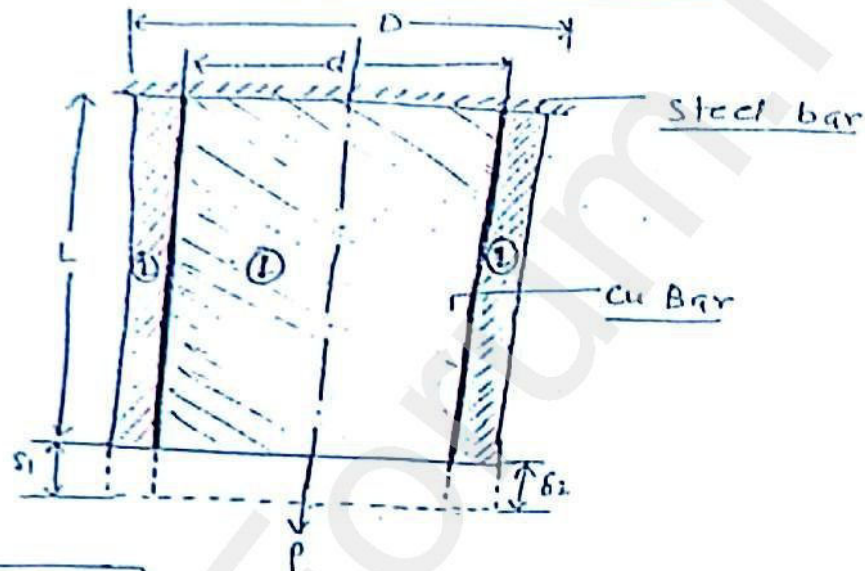
$$P_1 L_1 + P_2 L_2 + P_3 L_3 = 0 \Rightarrow R_A \times L + (R_A - P) \times L + (R_A - 3P) \times L = 0$$

$$R_A X L + 2 R_A L - 2 P L + R_A L - 3 P L = 0$$

$$4 R_A L = 5 P L$$

$$R_A = \frac{5P}{4} \quad \text{Ans}$$

### \* CASE - II - [ PARALLEL BARS ] →



$$\begin{aligned} \text{(i)} \quad & \delta_1 = \delta_2 \\ \text{(ii)} \quad & P = P_1 + P_2 \end{aligned}$$

[completing of  $P$  to the series combination method.]

$$\text{(iii)} \quad \textcircled{a} \quad \epsilon_1 = \epsilon_2 \quad [\text{only when } L_1 = L_2]$$

if lengths are not defined then assume same length

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

[Stress are directly proportional to  $E$ ]

$$\textcircled{b} \quad \text{when } L_1 \neq L_2$$

$$\delta_1 = \delta_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\frac{\sigma_1 L_1}{E_1} = \frac{\sigma_2 L_2}{E_2}$$



$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} \times \frac{L_2}{L_1}}$$

(C) Total load  $P = P_1 + P_2$

$$\boxed{P_1 = \sigma_1 A_1 + \sigma_2 A_2}$$

$$P_1 = A[\sigma_1 + \sigma_2] \Rightarrow \text{if } \underline{A_1 = A_2}$$

Problem A composite bar is made of steel and aluminium bar each having  $2\text{ cm}^2$  area of c/s. The composite bar is subjected to load  $P$ . If the stress in aluminium is  $10\text{ MPa}$ . And  $E$  of steel is 3 times of  $E$  of aluminium. The value of load  $P$  is -

Solution → Not given length  $= L_1 = L_2 = L$

$$E_s = 3 E_a, \quad \sigma_{aL} = 10\text{ MPa},$$

$$P = P_1 + P_2$$

$$= \sigma_1 A_1 + \sigma_2 A_2$$

$$P = A[\sigma_1 + \sigma_2] \quad \text{--- (1)}$$

$$\frac{\sigma_s}{\sigma_a} = \frac{E_s}{E_a} = 3$$

$$\boxed{\sigma_s = 3 \times \sigma_a = 30\text{ MPa}}$$

$$P = 2 \times [30 + 10]$$

$$\boxed{P = 80\text{ kN}} \quad \underline{\text{Ans}}$$

Problem A composite bar is made of bars of material 1 and material 2 having area  $100\text{ mm}^2$  each. Thus stress in material 1 is  $20\text{ MPa}$  ( $\sigma_1$ ) due to applied load ( $P = 6\text{ kN}$ ) if  $E_1 = 100\text{ kN/mm}^2$ , then  $E = ?$

$$L_1 = L_2 = L$$

$$P = P_1 + P_2$$

$$P = A[\sigma_1 + \sigma_2]$$

$$6\text{ kN} = 100 [20 + \sigma_2]$$

$$\sigma_2 = \frac{6000}{100}$$

$$\boxed{\sigma_2 = 60\text{ MPa}}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

$$\frac{20}{40} = \frac{100}{E_2}$$

$$E_2 = 200 \text{ kN/mm}^2$$

Problem:-

3 wires of equal  $\phi$ s and equal lengths (A same L same) but of different materials are fixed at a top to support a ring.  $E_1 = 2E_2 = 3E_3$  a load of 2.75 kN is applied on ring in such a way that the ring remains horizontal the load shared by wire 1 is  $(P_1)$ .

Sol

$$P_1 = ?, A = A_1 = A_2 = A_3, L = L_1 = L_2 = L_3$$

$$E_1 = 2E_2 = 3E_3, P = 2.75 \text{ kN}$$

$$P = P_1 + P_2 + P_3$$

$$2.75 = A(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = \frac{2E_2}{E_2} = 2$$

$$\sigma_1 = 2\sigma_2$$

$$\frac{\sigma_1}{\sigma_3} = \frac{E_1}{E_3} = \frac{3E_3}{E_3} = 3$$

$$\sigma_1 = 3\sigma_3$$

$$2.75 = A[\sigma_1 + 0.5\sigma_1 + 0.33\sigma_1]$$

$$2.75 = 1.03\sigma_1 \times A$$

$$\sigma_1 = \frac{2.75}{1.03 \times A}$$

$$\delta_1 = \delta_2 = \delta_3$$

$$\frac{P_1 L}{A E_1} = \frac{P_2 L}{A E_2} = \frac{P_3 L}{A E_3}$$

$$\frac{P_1}{E_1} = \frac{2P_2}{E_2} = \frac{3P_3}{E_3}$$

$$P_1 = 2P_2 = 3P_3$$

$$P = P_1 + P_2 + P_3$$

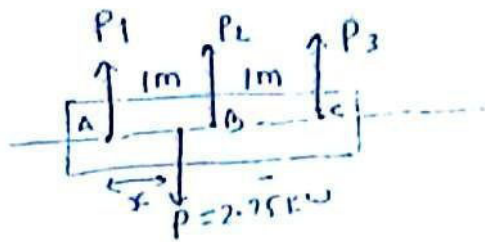
$$2.75 = P_1 + 0.5P_1 + 0.33P_1$$

$$2.75 = 1.03P_1$$

$$P_1 = \frac{2.75}{1.03} = 1.5 \text{ kN}$$

Problem Repeat the above question for the location of load on the ring. If the distance b/w wires is 1m and 1m.

FBD



$$\sum V = 0 \Rightarrow P = P_1 + P_2 + P_3$$

$$\sum M = 0$$

$$P \times x - P_2 \times 1 - P_3 \times 2 = 0$$

$$x = \frac{P_2 + 2P_3}{P}$$

given  $P_1 = 1.5 \text{ kN}$

$$P_1 = 2P_2$$

$$P_1 = 3P_3$$

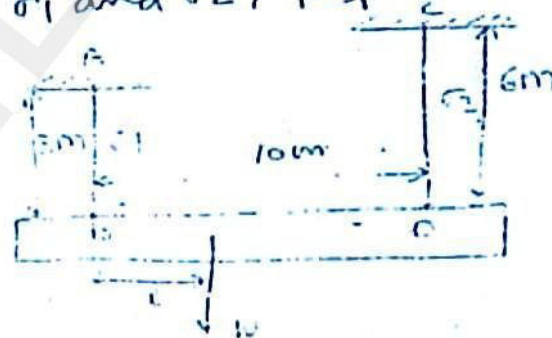
$$P_2 = 0.75 \text{ kN} \quad P_3 = 0.5$$

Then  $x = \frac{0.75 + 2 \times 0.5}{2.75}$

$$x = 0.636 \text{ m} \quad \text{Ans}$$

Problem In the given figure the wire AB and CD of the same material are used to suspend a rigid block to which graduated load (W) is applied in such a way that both the wires get stretched by the same amount. If the stress in wires AB and CD are  $\sigma_1$  and  $\sigma_2$ , resp. Then the ratio of  $\frac{\sigma_1}{\sigma_2} = ?$

Solution



$$\sum M = 0$$

$$\sigma_1 = \sigma_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{L_2}{L_1} \times \frac{E_2}{E_1} = \frac{6}{3} \times \frac{E}{E} = 2$$

Q1



- Problem: Repeat the above question, for the deter. of following
- location and magnitude of load (W).
  - loads acting in the both the wires. if areas of 1st and 2nd Bars are  $100 \text{ mm}^2$  and  $50 \text{ mm}^2$ .

$$P = [\sigma_1 A_1 + \sigma_2 A_2] -$$

$$P = \sigma_1 \times 100 + 0.5 \sigma_1 \times 50$$

$$P = 100\sigma_1 + 25.0\sigma_1$$

$$W = P = 125\sigma_1$$

$$\sigma_1 = \frac{W}{125}$$

$$P_1 = \sigma_1 A_1$$

$$= \frac{W}{125} \times 100$$

$$P_1 = \frac{4W}{5}$$

$$P_1 + P_2 = W$$

$$P_2 = \frac{W}{5}$$

F.B.D →

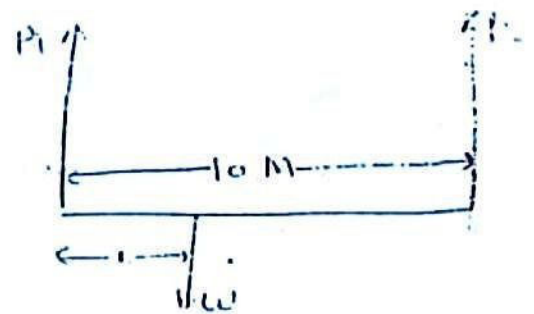
$$\Sigma M = 0$$

$$W \cdot x = P_2 \times 10 = 0$$

$$W \cdot x = \frac{W}{5} \times 10 = 0$$

$$W \cdot x = \frac{W}{5} \times 10$$

$$x = 2 \text{ m}$$



## THERMAL STRESS

The total stress in the system -

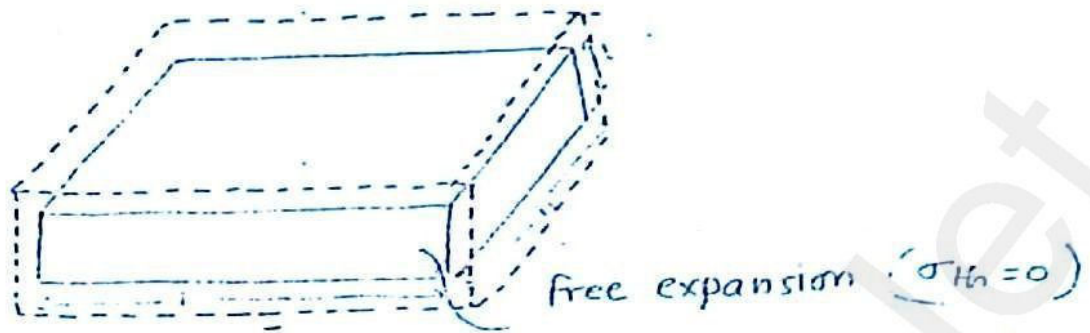
$$\sigma_{\text{total}} = \sigma_{\text{MECH}} + \sigma_{\text{THERMAL}}$$

(or)

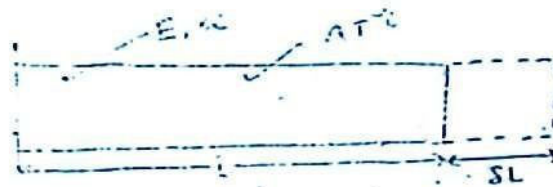
externally  
applied load [surface loads]

Temperature variation

(expansion & contraction)



CASE-I →  
[FREE EXP.]



$$\delta = \alpha \Delta T L$$

$$\sigma_{Thermal} = 0$$

CASE-II → A. [FIXED AT BOTH ENDS]

Completely prevented Case



\* Temp ↑, stress are in compressive nature.



$$L_0 = L + \delta$$

$$L_f = L$$

$$\epsilon_T = \frac{L_f - L_0}{L_0} = \frac{L - (L + \delta)}{L + \delta} = \frac{-\delta}{L + \delta} \approx \frac{-\delta}{L}$$

$$\epsilon_T = \frac{-\alpha \Delta T L}{L} = -\alpha \Delta T$$

$$\sigma_T = E \epsilon_T = -\alpha \Delta T E$$

$$\sigma_{\text{Thermal}} = \pm \alpha \Delta T E$$

'+'  $\rightarrow$  Temp decreases

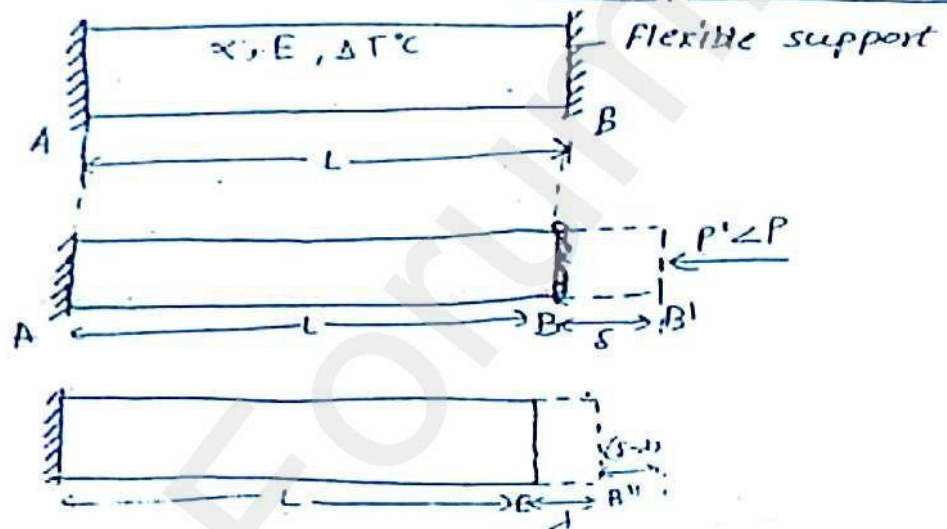
'-'  $\rightarrow$  Temp ↑

$$\epsilon_T = \pm \alpha \Delta T$$

\* Thermal stress independent on c/s area of member and length of the member.

\* But depend on 3 parameter  $\alpha, \Delta T, E$ .

CASE - III (Some amount of thermal exp./contr. is permitted)



Before Applying the Load -

$$L_0 = L + \delta$$

$$L_f = L + \delta$$

$$\Delta L = L_f - L_0 = \delta - \delta$$

$$\Delta L = -(\delta - \delta)$$

$$\epsilon_{Th} = \frac{-\Delta L}{L_0} = \frac{-(\delta - \delta)}{L + \delta} \approx \frac{-(\delta - \delta)}{L} \rightarrow$$

$$\sigma_{\text{Thermal}} = E \epsilon_{\text{Thermal}} = \frac{+(\delta - \delta)}{L} E \rightarrow (11)$$

$$\delta = \alpha \Delta T L$$



for completely prevented case (II), to determine the thermal stress and thermal strain, substitute in the above eq.

$$\Delta = 0$$

Problem.

A Copper of 25 cm length is fixed by means of supports at its ends. The supports can yield (permeable) by 0.01 cm. The temp. of bar is raised by  $100^\circ\text{C}$ . Then the stress induced in the bar if  $\alpha = 20 \times 10^{-6} / ^\circ\text{C}$ .  $E = 100 \text{ GPa}$ .

Solution

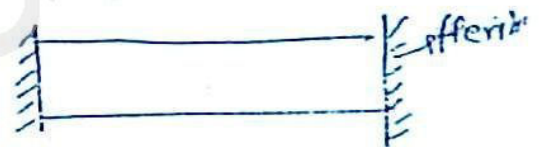
$$L = 25 \text{ cm}, \Delta = 0.01, E = 100 \text{ GPa}, \alpha = 20 \times 10^{-6}$$

$$T = 100^\circ\text{C}$$

$$\Delta T = \frac{E \Delta}{L} - \frac{(\delta - \Delta) E}{L}$$

$$E \Delta = \delta = \alpha \Delta T L$$

$$\delta = 20 \times 10^{-6} \times 100 \times 25$$



$$\sigma_{\text{Thermal}} = - \frac{(\delta - \Delta) E}{L}$$

$$= -160 \text{ MPa.}$$

Problem

A bar of length 2M is fixed at both ends. The initial tensile stress in the bar is 10 MPa. Then at a temp of  $10^\circ\text{C}$ , if the temp is raised to  $15^\circ\text{C}$ , the stress in the bar is  $E \alpha = 10 \times 10^{-6} / ^\circ\text{C}$ ,  $E = 200 \text{ GPa}$ ?

Sol.

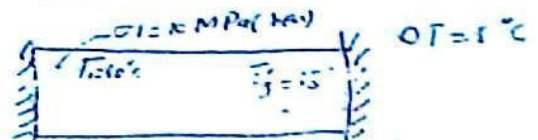
$$L = 2 \text{ M}, \Delta T = 5^\circ\text{C}, \alpha = 10 \times 10^{-6}, E = 200 \text{ GPa}$$

$$\sigma_{\text{Stress in bar}} = \sigma_{\text{Mech}} \pm \sigma_{\text{Thermal}}$$

$$\sigma_{\text{in bar}} = 10 - \alpha \Delta T E$$

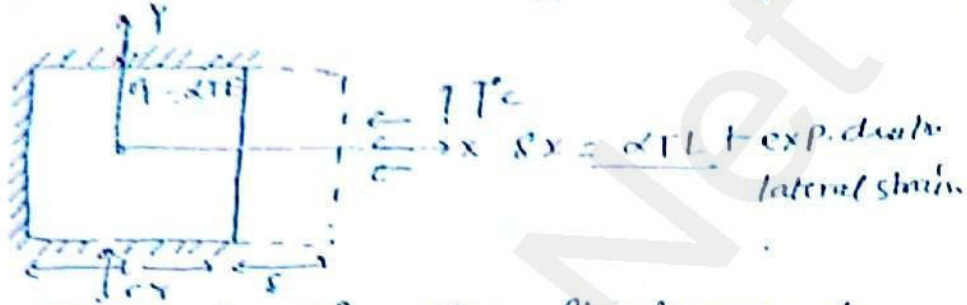
$$= 10 - 10 \times 10^{-6} \times 5 \times 200 \times 10^9$$

$$= 0 \text{ MPa}$$



Problem

A square plate of negligible thickness is fixed along 3 edges and free to expand along 4th edge. if



temp of plate is raised by  $T_c$ . The final expansion along the free edge will be -

$$\boxed{L\alpha T + \alpha T \cdot \mu L}$$

$$\boxed{L\alpha T (1 + \mu)}$$

$$\sigma_y = -\alpha T E$$

$$\epsilon_y = -\alpha T$$

$$\epsilon_x = -\mu \epsilon_y = \mu \alpha T$$

$$\frac{\delta}{L} = \mu \alpha T$$

$$\boxed{\delta = \mu \alpha T L}$$

Expansion along x-dir<sup>n</sup>

$$\delta x = \text{Free exp due to temp change} + \text{exp. due to lateral strain}$$

$$= \alpha T L + \mu \alpha T L$$

$$= \alpha T L (1 + \mu)$$

Problem

If cube of side (L) is fixed along two directions (Y and Z). Determine final expansion along 3rd dir<sup>n</sup> due to temp rise of T will be.

$$(\delta x)_{\text{total}} = \text{free expansion} + \text{exp. due to lateral strain} + \text{exp. due to lateral strain}$$

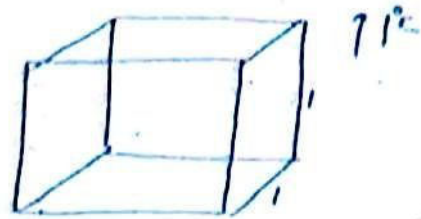
$$= \alpha T L + \mu \alpha T L + \mu \alpha T L$$

$$= \alpha T L (1 + 2\mu) \text{ Ans}$$



Problem

determine the change in volume.  
 $T = 1^\circ\text{C}$ ,  $L = 1\text{m}$



$$\Delta x = \alpha T L = \alpha$$

$$\Delta y = \alpha$$

$$\Delta z = \alpha$$

So total

$$L_f = (1+\alpha)$$

$$\text{final vol.} = (1+\alpha)^3$$

$$\frac{\Delta V}{V} = (1+\alpha)^3 - (\text{original volume})$$

$$\Delta V = (1+\alpha)^3 - 1^3$$

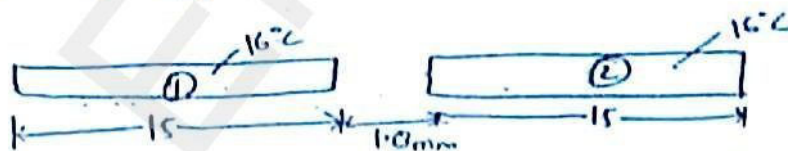
$$\boxed{\Delta V = 3\alpha} \text{ Ans}$$

Problem Rails of 15 m length were laid down on the track in the morning when temp were  $16^\circ\text{C}$ . A gap of 1.0 mm was kept b/w two consecutive rails. Upto what temp the rails will remain stress free

(a) if temp is raised further by  $15^\circ\text{C}$  then what will be magnitude and nature of stresses in the rails.  $\alpha = 16 \times 10^{-6}/^\circ\text{C}$

Sol.

$$L = 15\text{m}, T_1 = 16^\circ\text{C}, \alpha = 16 \times 10^{-6}/^\circ\text{C}, E = 200\text{GPa}$$



The condition for zero thermal stress in bar.

free exp should be  $\leq 1.0\text{ mm}$ .

$$\alpha \times \Delta T \times L \leq 1.0$$

$$16 \times 10^{-6} \times \Delta T \times 15000 \leq 1.0$$

$$\Delta T \leq 7.5^\circ\text{C} = T_f - T_i$$

$$7.5^\circ\text{C} = T_f - T_i$$

$$7.5 = T_f - 16$$

$$\boxed{T_f = 23.5^\circ\text{C}}$$



$$\sigma_{\text{thermal}} = \alpha \Delta T E$$

$$= 16 \times 10^{-6} \times \Delta T \times 200 \times 10^5$$

$$\delta_{\text{free}} = \alpha \Delta T L$$

$$= 16 \times 10^{-6} \times 15 \times 1500$$

$$= 3.6 \text{ mm}$$

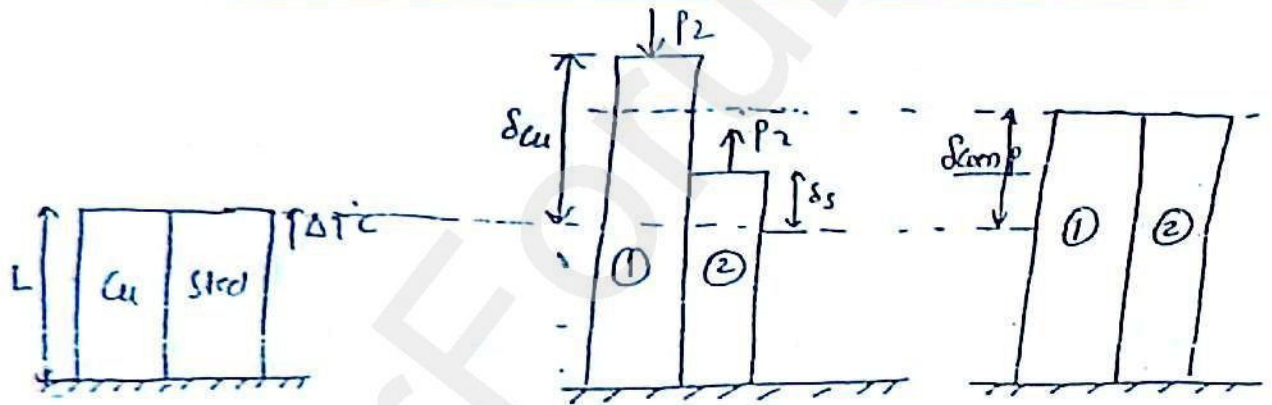
$$l = 1.0 \text{ mm}$$

$$\sigma_{\text{thermal}} = \frac{-(\delta - l) E}{L}$$

$$= \frac{-(1.0) \times 200 \times 10^5}{15000}$$

$$= -24 \text{ MPa} \quad \checkmark A_{22}$$

### THERMAL STRESSES IN COMPOUND BAR



$$\alpha_C > \alpha_{\text{steel}}$$

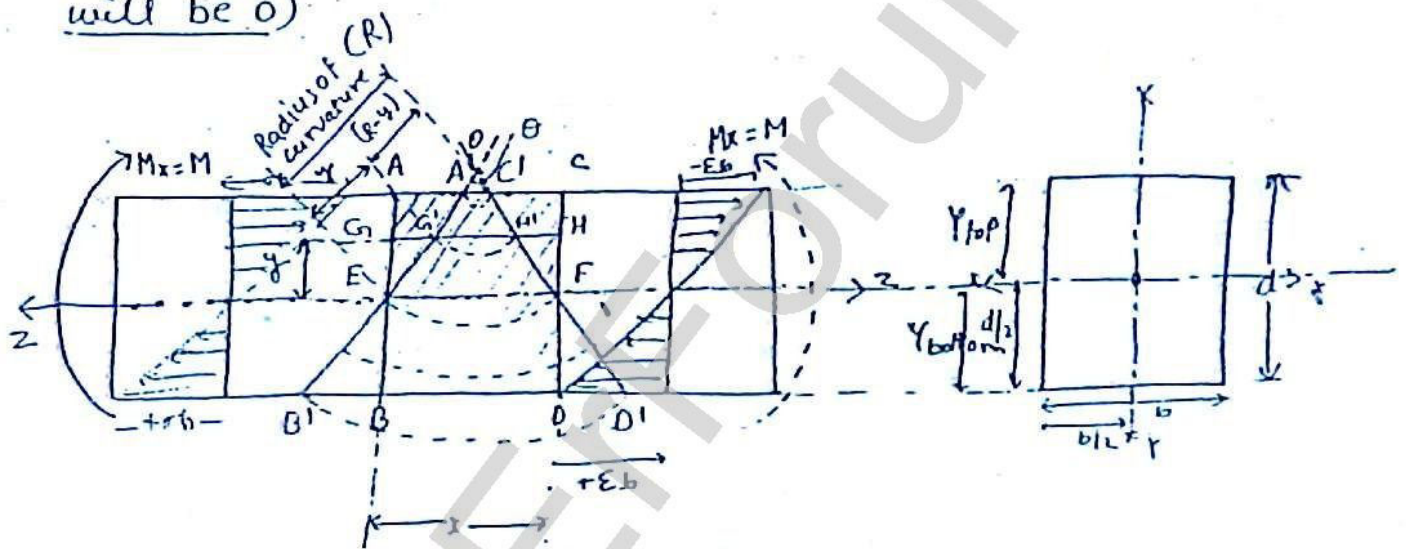
- \* Steel bar subjected to tensile thermal stresses.
- \* Copper bar subjected compressive " "

When compound bar is heated by  $\Delta T^\circ\text{C}$  then the bar with high coefficient of thermal exp. is subjected to compressive thermal stress where as bar with low co-efficient of thermal exp. is subjected to tensile thermal stress and vice versa.

## Pure Bending

### [BENDING STRESSES IN A BEAM]

A member or a beam is said to be under pure bending when it is subjected to two equal and opposite couples in a plane passing through the longitudinal axis of member or in a plane  $\perp$  to plane of C/S. such that bending moment remains constant (shear force will be 0)



\* Shape not changes because no shear force is acting. Only orientation will change

⊙ Neutral Axis is defined as the line of intersection b/w neutral layer and plane of C/S.

$$\begin{aligned} \epsilon_{GH} &= \frac{G'H' - GH}{GH} \\ &= \frac{G'H' - EF}{EF} = \frac{G'H' - E'F'}{E'F'} \end{aligned}$$

$$\epsilon_{GH} = \frac{(R-y)\theta - R\theta}{R\theta} = \frac{+y}{R} \theta$$



- \*  $y$  is variable and  $R$  not variable.

$$\boxed{\epsilon \propto y}$$

- \* From the above eq we conclude that strain varies linearly neutral axis to extreme fibers and strain always max. at extreme fibers. ( $y_{max}$  at stream fibers)

- \* For symmetrical CS ( $y_{top} = y_{bottom}$ ) ( $E_x, \square, \sqsupset, \bigcirc, \underline{\square}$ ) the strains at the extreme fibers are equal but opposite nature.

- \* For unsymmetrical CS like ( $\Delta, T$ , etc) ( $y_{top} \neq y_{bottom}$ ) the strains at the extreme fibers are unequal and opposite nature.

$$\boxed{\epsilon_{top} \neq \epsilon_{bottom}}$$

- \* As per Hook's law

$$\sigma \propto \epsilon \propto y$$

$$\sigma = E \epsilon$$

$$\sigma = E (\pm y/R)$$

$$\boxed{\sigma_b = \sigma_z = \pm \frac{E y}{R}}$$

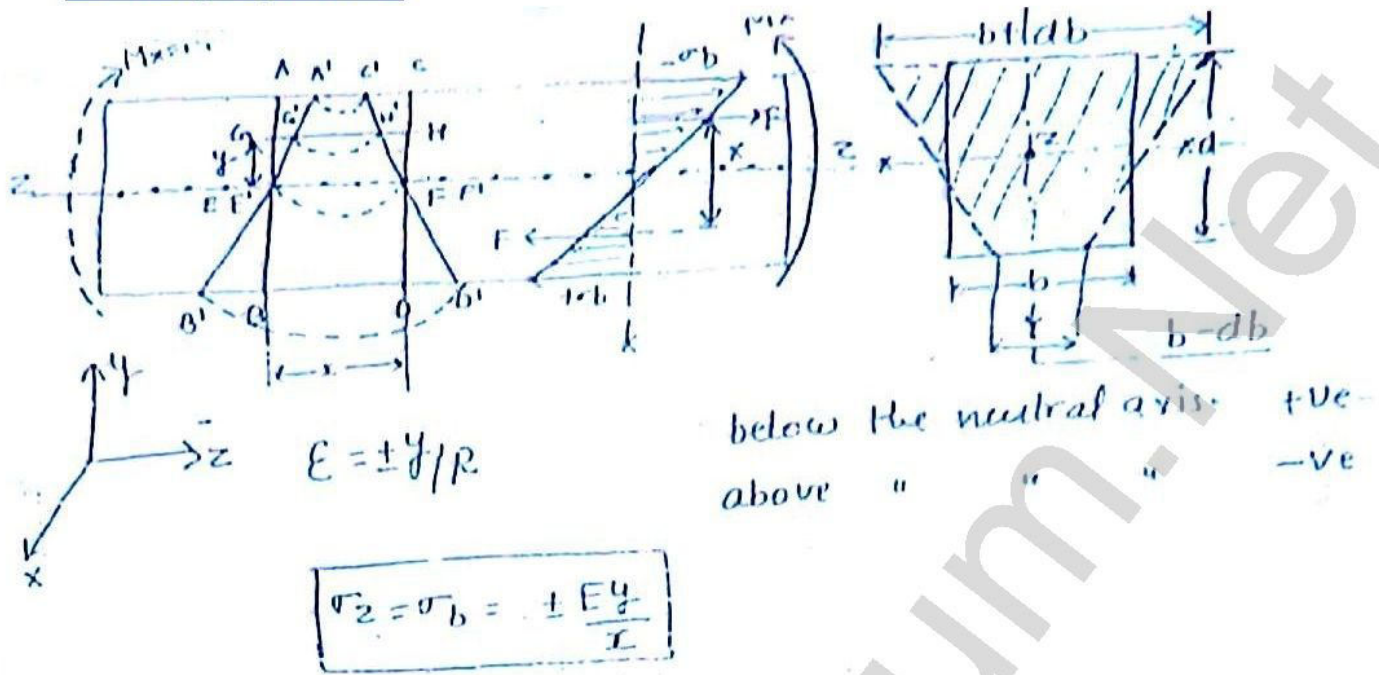
1st eq

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

—————→ ①



## Bending equations



State of stress at pure bending.

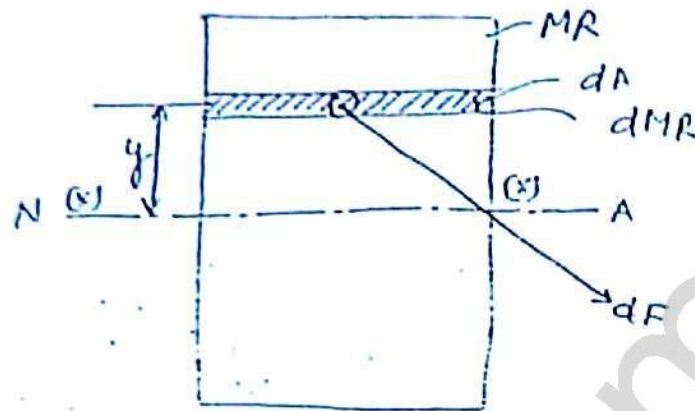


- \* lateral strains are opposite to longitudinal strain. Here longitudinal are in compressive ( $-\sigma_b$ ) so lateral strain will be tensile so that c/s become  $\uparrow$  and vice-versa. (and at bottom fibers long. strain is in tension so lateral strain will be compressive so that c/s at bottom will  $\downarrow$ )
- $$\left[ \begin{array}{l} \text{B. Moment} = \text{Constant} \\ \text{Shear Stress} = 0 \end{array} \right]$$

## \* MOMENT OF RESISTANCE [MR] →

When a body in the Bending then a internal resisting forces apply i.e; called moment of Resistance (MR). and produced a couple due to the resisting force. Of MR offers a Resisting bending moment.

"Defined as the resisting couple or <sup>resisting</sup> bending moment offered by the c/s of the member."



Moment of resistance offered by strip

$$dM_R = dF \times y$$

First moment of c/s

$$dM_R = \sigma \times dA \times y$$

$$= \frac{E y}{R} \times dA \times y$$

$$\int dM_R = \int \frac{E y^2}{R} dA$$

$$M_R = \frac{E}{R} \int y^2 dA$$

$\int y^2 dA$  = Second M.O. Area about an neutral axis in the plane of c/s.

[ In case of bending always take  $I_{xx}$ ,  $I_{yy}$  but in case of twisting take  $I_{zz}$  (becoz  $\perp$  to the axis). ]

$$\int y^2 dA = I_{xx} \text{ or } I_{yy} \text{ (Neutral)}$$

$$= \text{M.O. Area about N.A.}$$

So that

$$M_R = \frac{E}{R} \times I_{NA}$$



Ind Eq<sup>n</sup>  $\boxed{\frac{M_R}{I_{NA}} = \frac{E}{R}} \longrightarrow \textcircled{II}$

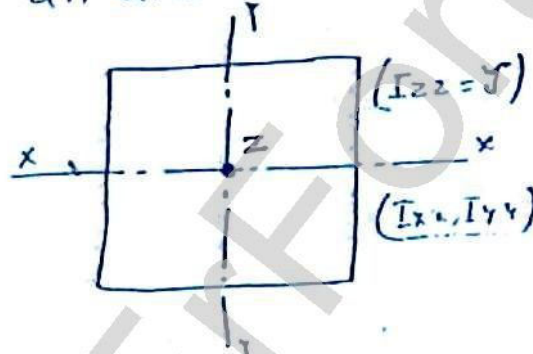
Conclude about  $\textcircled{I}$  and  $\textcircled{II}$

\*\*\*  $\boxed{\frac{M_R}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}}$

Equation of bending

Euler's Bernoulli's equation Radius of curvature at N-Axis

$\textcircled{F}$  Moment of Inertia - is defined as a second moment of area about an axis which is in the plane of c/s.



POLAR MOMENT OF INERTIA  $[I_{zz}]$  or  $[J] \rightarrow$

defined as the second moment of area about an axis which is  $\perp$  to the plane of c/s.

ASSUMPTIONS  $\rightarrow [B.Eq] \rightarrow$

- (i) Material is assumed to homogeneous and isotropic
- (ii) Obeys Hook's Law [Stress in the elastic limit]
- (iii) Member is assumed to be prismatic. (uniform c/s)
- (iv) Member is assumed to be pure bending.  
(shear force will be zero, B.M = const.)



(v) plane Transverse sections (C/s) remain plane even after Bending. ( $\epsilon \propto y$ )

problem

While pure bending, the plane section remain plane becoz (i)  $M \propto y$

$$(ii) \epsilon \propto y$$

$$(iii) \sigma \propto y$$

(vi) Young's Modulus (E) remains same in tension & comp.

$$E_{top} = \frac{\sigma_{top}}{\epsilon_{top}} = \frac{\sigma_{top}}{\epsilon_c}$$

$$E_{bottom} = \frac{\sigma_{bottom}}{\epsilon_{bottom}} = \frac{\sigma_{bottom}}{\epsilon_{ten}}$$

For ductile material -

$$E_{comp} = E_{tension} \quad \boxed{S_{rc} > S_{rs} > S_{rt}}$$

For brittle material -

$$\boxed{S_{uc} < S_{us} < S_{ut}}$$

(vii) The member is assumed to bend about one of its principal axis.

(viii) The member is assumed to bend in a form of circle.

### ANALYSIS OF BENDING EQUATION →

$$\frac{M}{I} = \frac{M_B}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$\downarrow$   
A

$\downarrow$   
B

$\downarrow$   
C

## Design Criteria →

- (a) Strength criterion  
 ↳ based upon  $\sigma_{per}$  or  
 $\tau_{allow}$ ,  $\sigma_{safe}$ ,  $\tau_{design}$

- (b) Rigidity criterion  
 ↳ based upon  $\delta_{per}$

## Safe design →

(a) Max. Stress induced  $\leq \sigma_{per}$

↳ SOM equation (For calculation)

$$\left. \begin{array}{l} \tau_a = P/A \\ \tau_b = \frac{M}{Z} \\ \tau_s = \frac{T}{Z_p} \end{array} \right\} \text{Strength criterion}$$

- (b)  $\text{Max. def}^n \text{ ind.} \leq \delta_{per}$   
 ↳ Calculated by SOM eq.

$$A = B$$

$$\frac{MR}{I_{NA}} = \frac{\sigma_b}{y} \Rightarrow \sigma_b = \frac{MR \cdot y}{I_{NA}}$$

$[(\sigma_b)_{max}]_{ind} = \frac{MR Y_{max}}{I_{NA}}$	$= \frac{MR}{Z} = \frac{M_{max}}{Z} \leq \sigma_{per} \rightarrow \textcircled{1}$
$[(\sigma_b)_{max}]_{ind} = \frac{MR}{I_{NA}/Y_{max}}$	$\Rightarrow Z \uparrow \Rightarrow [(\sigma_b)_{max}]_{ind} \downarrow$ $\Rightarrow \text{bending failure} \downarrow$

$\frac{I_{NA}}{Y_{max}} = Z = \text{section modulus (used for Bending Moment)}$

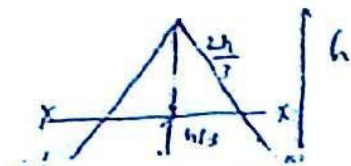
(i)   $= \frac{\pi d^3}{32}$

(ii)   $= \frac{bd^2}{6} \text{ or } \frac{db^2}{6}$

(iii)   $= \frac{a^3}{6}$

(iv)   $\Rightarrow I_{xy} = \frac{bh^3}{36}$

$$I_{xx} = \frac{bh^3}{12}$$





Best section  $\rightarrow$  a section which has higher 'Z'.

$$(Z)_{\text{circular}} < (Z)_{\text{non-circular}}$$

for comparison of beams/members w.r.t bending

$$MR = (\sigma_b)_{\text{per}} \cdot Z$$

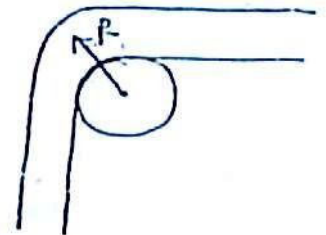
$$MR \uparrow \Rightarrow \sigma_{\text{per}} \uparrow \Rightarrow Z \uparrow = M \uparrow = \text{chance of failure} \downarrow$$

$$B = C$$

$$\frac{\sigma_b}{y} = \frac{E}{R} \Rightarrow \sigma_b = \frac{E y}{R} \rightarrow (II)$$

$$[(\sigma_b)_{\text{max}}]_{\text{ind}} = \frac{E y_{\text{max}}}{R} \rightarrow (III)$$

[used only radius of curvature.]



$$A = C$$

$$\frac{MR}{I_{NA}} = \frac{E}{R}$$

[used for both B.M and radius of curvature]

$I_{NA}$  = Calculated then

$d = ?$ ,  $a$ ,  $b$ ,  $d = ?$

$E \cdot I$  = Flexural Rigidity

$$E \cdot I = M \cdot R$$

$R = 1$

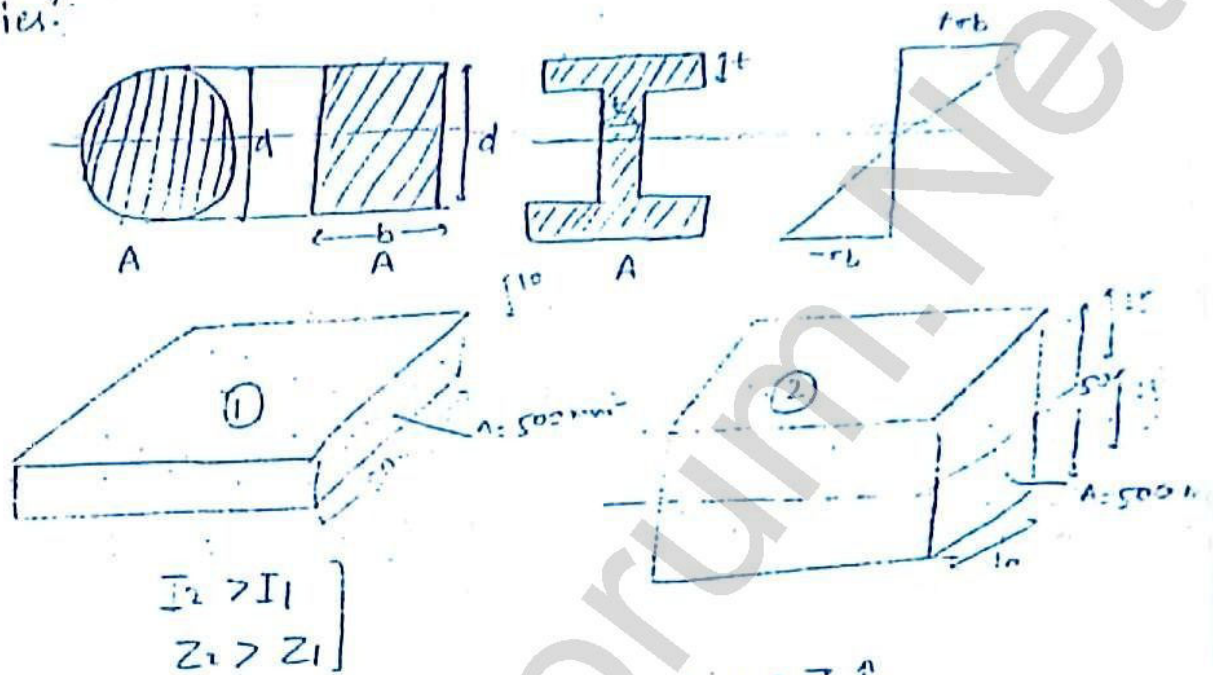
$$E \cdot I = M \cdot R$$

$-E \cdot I \uparrow \Rightarrow MR \uparrow \Rightarrow \text{res to bending} \Rightarrow \text{deflection slope}$



"E.I is also defined as the amount of moment of resistance offered by the C/S of a beam for unit radius of curvature of N. Axis?"

Problem -



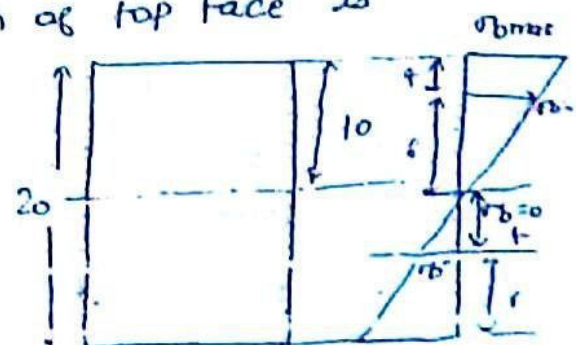
- \* Rails are made as I section. bcoz height  $Z \uparrow$
- \* Since the fibres near the top and bottom of the beam are severely stressed than the fibres near the neutral axis. Hence it is advantageous to place as much material as far away from the neutral axis.

$$\left. \begin{array}{l} Z_I < Z_{II} < Z_{III} \\ \sigma_{bI} > \sigma_{bII} > \sigma_{bIII} \\ M_{RI} < M_{RII} < M_{RIII} \end{array} \right\} \dots$$

Problem A beam of  $\square$  C/S of width 10 cms and depth 20cms is subjected to a bending moment of 20KNms. The stress developed at a distance of 10 cm of top face is

(i) zero because  $y = 0$

$$\sigma_b = \frac{M}{Z} = \frac{M y_{max}}{I_{max}}$$



Problem Repeat the above question for the max. bending stress induced in a beam and  $\sigma_b$  developed at a distance of 4cm from the top face is:-

Ist Method.  $(\sigma_b)_{\max} = \frac{My_{\max}}{I_{NN}} = \frac{20}{\frac{bd^3}{12} \times \frac{1}{y}} \left[ I = \frac{bd^3}{12} \right] = \frac{20}{\frac{10 \times 12}{12} \times \frac{1}{6}} = \frac{20 \times 6}{10} = 12 \text{ MPa}$

$(\sigma_b)_{4\text{cm}} = \frac{20 \times 4}{666.66} = 12 \text{ MPa}$

IIrd Method.  $\frac{(\sigma_b)_{\max}}{\sigma_b} = \frac{y_{\max}}{y}$

$$\frac{30}{\sigma_b} = \frac{10}{6}$$

$$\boxed{\sigma_b = 18 \text{ MPa}}$$

Problem Repeat the above question for the bending stress developed at a distance of 6cm from the bottom face is, if  $E = 200 \text{ GPa}$

$$E = \frac{\sigma_b}{\epsilon} \Rightarrow$$

$$\sigma_b = \frac{M}{Z} = \frac{20 \times 10^6 \times 10}{666.66 \times 10^4} = 12 \text{ MPa}$$

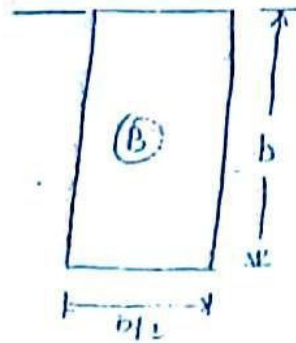
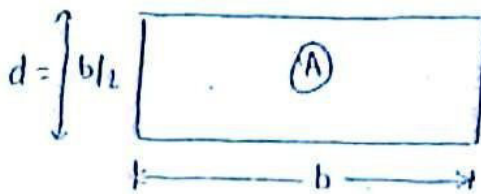
$\sigma = 12 \text{ MPa}$  then

$$\epsilon = \frac{12 \text{ MPa}}{200 \times 10^3 \text{ MPa}}$$

$$\boxed{\epsilon = 6 \times 10^{-5}}$$

Problem A beam C/I is used in two different orientations as shown below. Bending moment applied to the beam in the cases are same, the max  $\sigma_b$  induced in cases A & B are related as:-





(a)  $\sigma_A = \sigma_B$

(b)  $\sigma_A = \sigma_B/2$

(c)  $\sigma_A = 2\sigma_B$

(d)  $\sigma_A = \sigma_B/4$

$I \propto d^3$

$Z_A < Z_B$


$\sigma_A > \sigma_B \left[ \because \sigma \propto \frac{1}{Z} \right]$

$I = \int y^2 dA$

$$\frac{\sigma_A}{\sigma_B} = \frac{M_A/Z_A}{M_B/Z_B} \Rightarrow \frac{Z_B}{Z_A} = \frac{I_B/y_B}{I_A/y_A} = \frac{(b/2)(b)^2/6}{b(b/2)^2/6}$$

$$\frac{\sigma_A}{\sigma_B} = \frac{\frac{b}{2} \times b^2}{\frac{b \times b^2}{4}} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \times \frac{4}{1} = 2$$

$$\boxed{\sigma_A = 2\sigma_B} \quad \text{Ans}$$

Problem A beam of  of C/S (with sides of  $\square$  horizontal & vertical) is subjected to a bending Moment (M) &  $(\sigma_b)_{\max}$  is 100 MPa. If the diagonals of sections take vertical and horizontal dir<sup>n</sup>. B.M remain in same. Then max. stress developed in case is-

(a) 100 MPa  
 (b)  $100\sqrt{2}$  MPa  
 (c)  $100/\sqrt{2}$  MPa  
 (d) 50 MPa

$(\sigma_b)_{\max} = 100 \text{ MPa}$

As per question, area of  $\square$  is

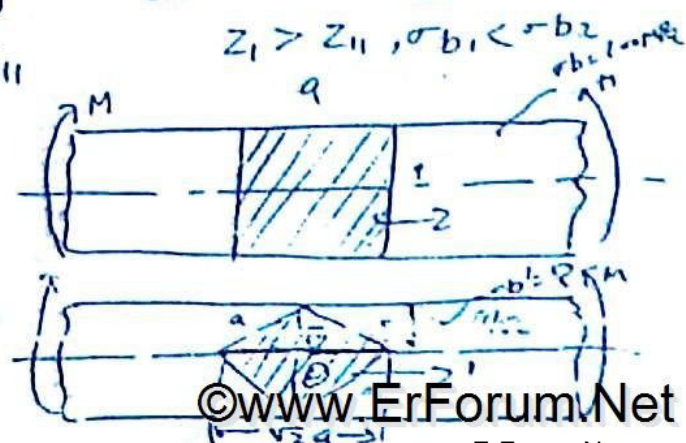
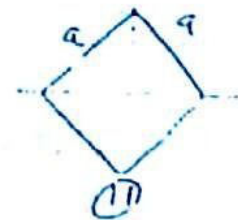
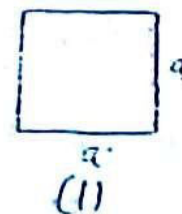
1 from neutral axis but d in (II) fig

so  $Z_{II} > Z_{I}$  so  $(\sigma_b)_I < (\sigma_b)_{II}$

$\square > \diamond$ . So Ans is  $100\sqrt{2}$

$Z = \frac{a^3}{6}$

$Z' = 2Z_1$

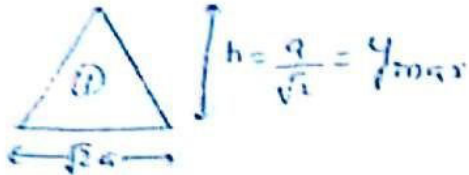
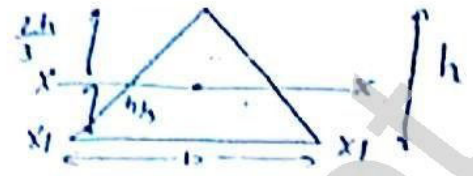




$$I_{xx} = \frac{bh^3}{36}$$

$$I_{xy} = \frac{bh^3}{12}$$

$$Z = \frac{I_{xx}}{y_{max}} = \frac{I_{xx}}{y_{top}} = \frac{bh^3/36}{\frac{2}{3}h} = \frac{bh^2}{24}$$



$$Z_1 = \frac{I_{yy}}{x_{max}} = \frac{\frac{1}{12} \times \sqrt{3} a \left(\frac{a}{\sqrt{3}}\right)^3}{\frac{a}{\sqrt{3}}}$$

$$Z_1 = \frac{\sqrt{3} a^3}{24} = \frac{a^3}{12\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$Z' = 2Z_1 = \frac{\sqrt{3} a^3}{12} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{a^3}{6\sqrt{3}}$$

$$\frac{\sigma_b'}{\sigma_b} = \frac{M/Z'}{M/Z} = \frac{Z}{Z'} = \sqrt{3}$$

$$\sigma_b' = 100\sqrt{3} \text{ MPa}$$

□ (Z) and (◇) (Z')

$$Z = 1.414 Z'$$

□ with side are vertical and horizontal is 1414% stronger than ◇ with diagonals are vertical & horizontal under Bending

Problem A 0.2 mm thick tape goes over a frictionless pulley of 25 mm dia.  $E = 100 \text{ GPa}$ . Then max.  $\sigma_b$  induced in a tape is

$$\left[ \gamma = 12.5 \text{ m} \right]$$

$$\sigma_b / \max = \frac{M_{max}}{Z}$$

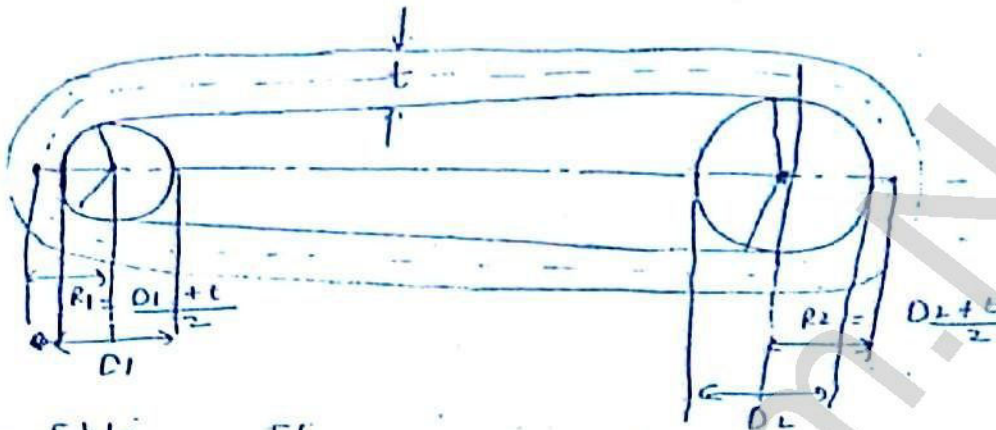
$$\left[ \frac{\sigma_b}{y} = \frac{E}{R} \right] \Rightarrow \sigma_b = \frac{E \cdot y}{R}$$

⑥

$$\sigma_b = \frac{E \cdot \frac{t}{2}}{\left( \frac{\pi t^3}{12} \right)} = \frac{100 \times 10^9 \times 0.1}{12.5 \times 10^{-9}} = \frac{100 \times 10^9}{12.6} = 800 \text{ MPa}$$

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bending stress  $\downarrow \Rightarrow D \uparrow$  and  $t \downarrow$



$$\sigma_{bi} = \frac{Et/2}{\frac{D_1+t}{2}} = \frac{Et}{D_1+t}$$

$$\sigma_{bi} = \frac{Et}{D_1+t}$$


$$(\sigma_b)_{max} = \sigma_{bi} = \frac{Et}{D_1+t} \quad \text{or} \quad \frac{Et}{D_1} = \frac{Et}{D_{min}}$$

$$\frac{(\sigma_b)_{max}}{(\sigma_b)_{min}} = \frac{D_{max}}{D_{min}} = \frac{D_2}{D_1}$$

$$(\sigma_b)_{max} \leq \sigma_{per}$$

$$\frac{Et}{D_1} \leq \sigma_{per}$$

$$\frac{D_1}{t} \geq \frac{E}{\sigma_{per}}$$

Problem Det. the ratio of sides of a strongest  beam which has been obtain from circular log of wood of a dia

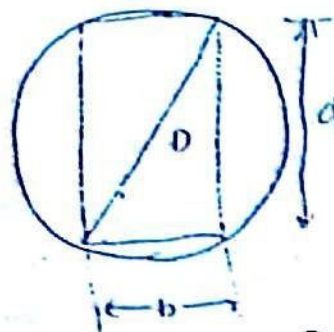
$$D. \quad z_{max} = I/y$$

$$\frac{bd^3}{6} = I/y$$

$$b^2 + d^2 = D^2$$

$$d^2 = D^2 - b^2$$

$$z = \frac{b(D^2 - b^2)}{6}$$



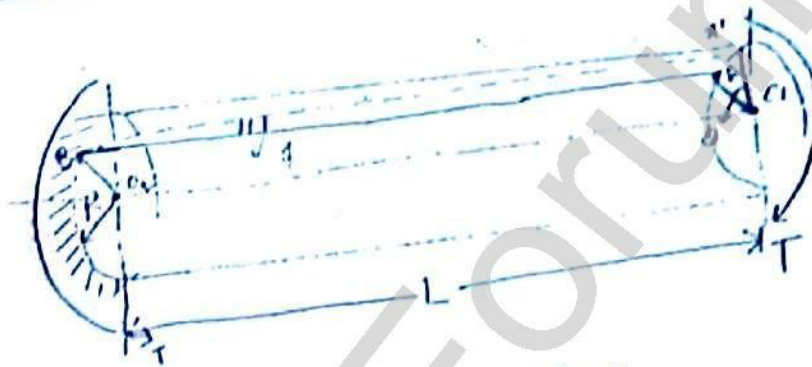




## Pure Torsion

A member or a shaft is said to be under pure torsion when it is subjected to two equal and opposite couples in a plane parallel to plane of C/S and  $\perp$  to longitudinal axis of a member such that magnitude of the couple remains constant throughout member.

### TORSION OF A CIRCULAR SHAFT $\rightarrow$



$O_1$  - free end where  $T$  is applied

$O_2$  - fixed end

$\theta$  - Angle b/w  $O_1A$  and  $O_1A'$  = angular displacement or angular twist

$\phi$  - shear angle = angle by which line present on surface is distorted [ $\angle ABA'$ ]

② Effect of  $r$  and  $L$  on  $\theta$  and  $\phi$   $\rightarrow$

① Effect of  $r$  on  $\theta$  and  $\phi$   $\rightarrow$

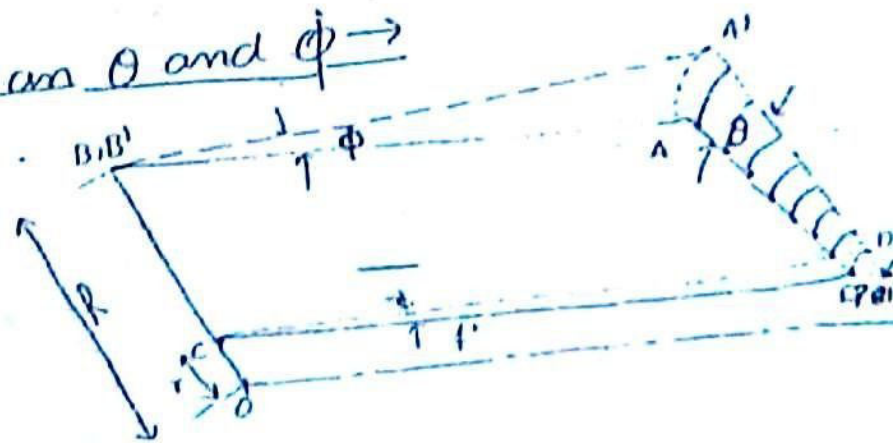
$\phi' < \phi$  but  $\theta' = \theta$

[ $\because r < R$ ]

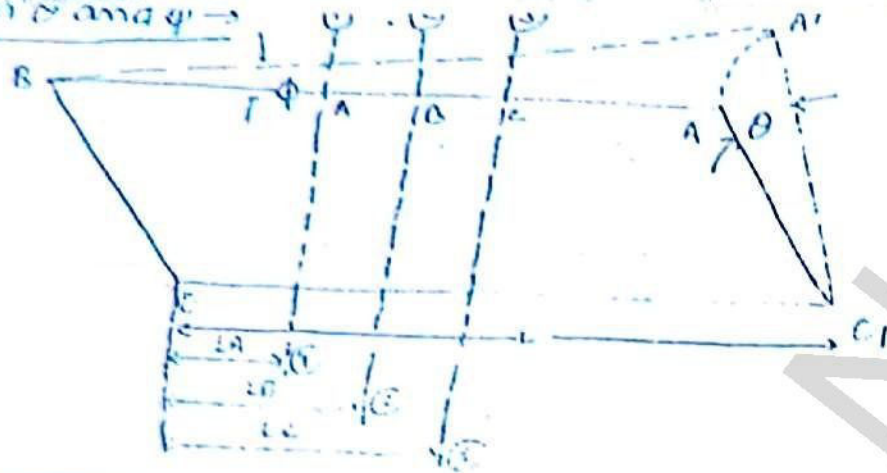
$\phi \propto r$

$$\boxed{\tau \propto \phi \propto r}$$

+  $\theta$  is independent of  $r$ .



2 Effect of  $l$  on  $\phi$  and  $\theta$  →



$$\boxed{\phi_A = \phi_B = \phi_C}$$

$$\left[ \begin{array}{l} \therefore \Delta A = \Delta B = \Delta C = R \\ l_A \neq l_B \neq l_C \end{array} \right]$$

\*  $\phi$  is independent of  $l$ .

$$\theta' < \theta \quad [l < L]$$

$$\boxed{\theta \propto l}$$

\* From torsion theory, we can conclude that w.r.t Fig ① and ②, the following conclusions will be made —

① Shear angle is directly proportional to  $(r)$  but independent of  $(l)$ . Hence we can conclude that shear angle or shear strain or shear stress is always maximum at any point of the surface of the shaft becoz  $(R)$  where  $(r)$  is maximum —

② Angular twist  $\theta \propto l$  but it is independent on  $(r)$ . Hence angular twist is always maximum on a C/S which is far away from the fixed end i.e. free end.

$$\Delta BAA'$$

$$\tan \phi = \frac{AA'}{BA}$$

$$\boxed{\phi = \frac{R\theta}{L}}$$

$$\boxed{\tau = G\gamma = \frac{GR\theta}{L}}$$

$$\boxed{\tau = G\phi}$$

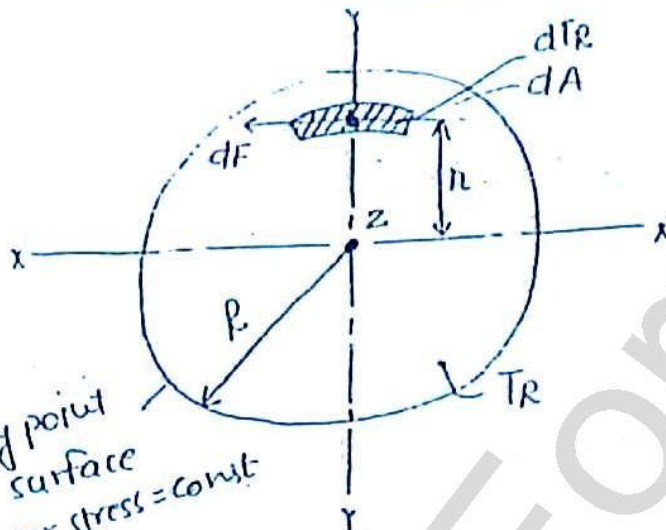
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1st eq of torsion  $\frac{\tau}{R} = \frac{G\theta}{L}$   $\rightarrow$  (I)

## MOMENT OF RESISTANCE [ $T_R$ ]

Max. Shear Stress  $T = \frac{G R \theta}{L}$



$R \rightarrow \tau$   
 $r \rightarrow \tau'$   
 $dF \rightarrow$  internal resisting tangential force

At any point on surface  
 Shear stress = const

$$\tau' = \frac{\tau r}{R}$$

$$\begin{aligned} dT_R &= dF \times r \\ &= \tau' \times dA \times r \\ &= \frac{\tau r}{R} \times r \times dA \end{aligned}$$

$$\int dT_R = \int \frac{\tau r^2}{R} dA$$

$$T_R = \frac{\tau}{R} \int r^2 dA$$

$$T_R = \frac{\tau}{R} \times J$$

2nd eq. of torsion  $\frac{\tau}{R} = \frac{T_R}{J}$   $\rightarrow$  (II)

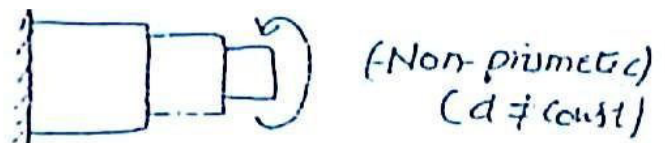
$$\therefore \int r^2 dA = J_{zz} = \text{M.O.I or } J \text{ (Polar M.O.I)}$$

Then  $\frac{T}{J} = \left[ \frac{T_R}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \right]$  eq<sup>n</sup> of torsion

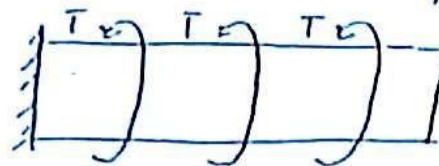
- $\tau$  - shear stress at a distance from radius ( $R$ )  
 $J$  - Polar M.O.I =  $\pi/32 \times d^4$  (Solid shaft),  $\frac{\pi}{32} (d_o^4 - d_i^4)$  (Hollow shaft)  
 $G$  - Shear Modulus  
 $T_R$  - Moment of Resistance  
 $\theta$  - Angular twist at a distance ( $L$ ).  
 $L$  - length of shaft

### ASSUMPTIONS →

- (i) Prismatic Member
- (ii) Material be homogeneous and isotropic.
- (iii) Shaft should be pure torsion (torque = constant)
- (iv) Assume Material obeys Hooke's Law (Elastic torsion)
- (v) The plane section remain plane after twisting. ( $\phi \propto \tau$ ,  $\tau \propto r$ )



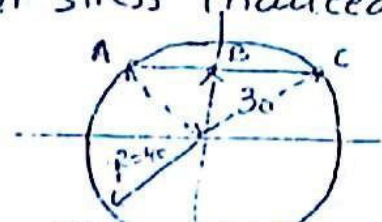
3 times torsion eq apply



Single application of torsion eq<sup>n</sup>.  
(Not Under Pure torsion)  
( $T \neq \text{const}$ )

Ques: Det the shear stress induced at point A BC for the  $\overline{C/H}$  of shaft as shown figure. If max. shear stress induced is 100 MPa due to applied torque ( $T$ ).

$T_A = \tau_c = \tau_{\text{max shear stress}}, T_B = ?$   
 $[\because r_c = r_o = R]$





$$\tau' = \frac{T r}{R} = \frac{100 \times 30}{40}$$

$$\tau_B' = 75 \text{ MPa} \quad \underline{\text{Ans}}$$

Problem

Repeat the above qu. for the shear angle develop at point ABC if shear modulus = 80 GPa

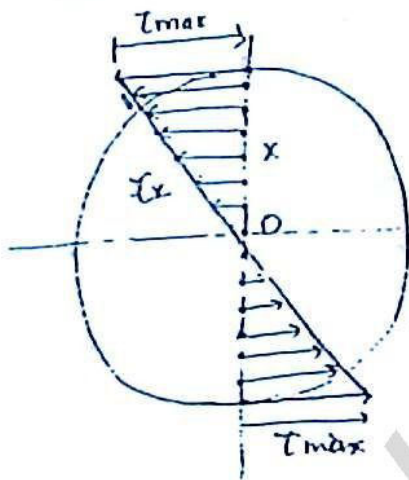
Solution

$$G = 80 \text{ GPa}$$

$$\phi_C = \phi_A = \gamma = \frac{\tau_{\text{max}} r_C}{G} = \frac{100 \text{ MPa}}{80 \text{ GPa}} = 1.25 \times 10^{-3} \text{ radians}$$

$$\phi_B = \frac{\tau_B'}{G} = \frac{75 \text{ MPa}}{80 \times 10^3 \text{ MPa}} = 0.94 \times 10^{-3} \text{ radians}$$

SHEAR STRESS DISTRIBUTION →

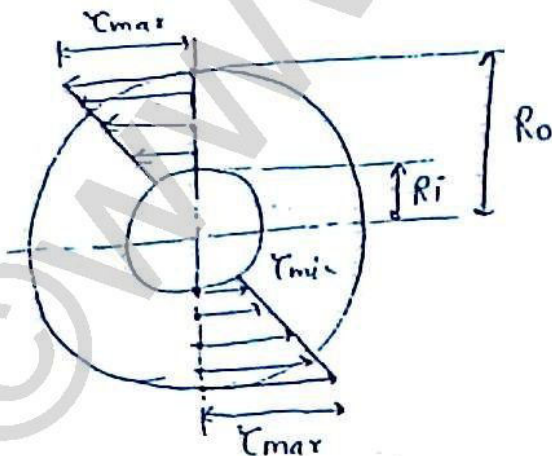


$$\tau_{\text{max}} = \frac{T}{Z_p}$$

$$\tau_x = \frac{\tau_{\text{max}} x}{R}$$

$$I = \frac{\pi}{32} d^4$$

① Solid Circular Shaft



$$\frac{\tau_{\text{max}}}{\tau_{\text{min}}} = \frac{R_o}{R_i}$$

$$\tau_{\text{max}} = \frac{T}{Z_p}$$

$$I = \frac{\pi}{32} (d_o^4 - d_i^4)$$

(ii) Hollow Shaft

$$\textcircled{1} \quad \underline{A = B} \quad \frac{T_R}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T_R \times R}{J}$$

$$\boxed{(\tau_{\max})_{\text{ind}} = \tau = \frac{T_R}{J/R} = \frac{T_R}{Z_P}}$$

$Z_P = J/R = \text{Polar Section Modulus}$

Safe design-

$$(\tau_{\max})_{\text{ind}} \leq \tau_{\text{per}}$$

$$\frac{T}{Z_P} \leq \frac{S_{YS}}{N(F.O.S)}$$

$$\text{Where } T = \frac{P \times 60}{2\pi N} \times 10^6 \quad \text{N mm}$$

Calculate -  $Z_P \geq \quad \text{mm}^3$

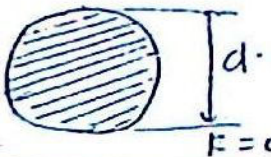

$$\boxed{Z_P = \frac{\pi}{16} d^3}$$

$d \geq \quad \text{mm}$  [if  $d=45$  then take 46].

$\tau_{\max} \uparrow \Rightarrow Z_P \downarrow \Rightarrow \text{More}$

$(\tau_{\max})_{\text{ind}} \downarrow \Rightarrow Z_P \uparrow \Rightarrow \text{Higher shaft design} \Rightarrow \text{less deformation}$

$$\boxed{(\tau_{\max})_{\text{ind}} = \frac{T}{Z_P}} \quad \text{Used when 'P' and 'T' is known}$$

Geometrical property	 $d$ $k=0$	 $d_i$ $d_o$ $k = d_i/d_o$ ( $k < 1$ )
Area	$\pi/4 d^2$	$\pi/4 d_o^2 [1 - k^2]$
$I$ ( $I_{xx}$ or $I_{yy}$ )	$\pi/64 d^4$	$\pi/64 d_o^4 [1 - k^4]$
$J = 2I$	$\pi/32 d^4$	$\pi/32 d_o^4 [1 - k^4]$
$Z = I/y_{\max}$	$\pi/32 d^3$	$\pi/32 d_o^3 [1 - k^4]$
$Z_P$	$\frac{\pi}{16} d^3$	$\frac{\pi}{16} d_o^3 [1 - k^4]$



## FOR COMPARISON OF SHAFT :-

$$\boxed{T_R = Z_p \tau_s} \quad T_{per} = \frac{S_f \cdot s}{N}$$

$$\boxed{P \uparrow = T \uparrow = T_R \uparrow = Z_p \uparrow \text{ and } \tau_s \uparrow}$$

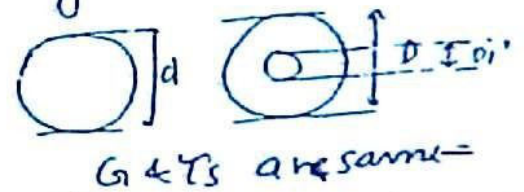
$$\boxed{T_R \propto Z_p} ; (\tau_{max})_{ind} \propto \frac{1}{Z_p}$$

The best ds for a shaft is that which has higher  $Z_p$  (Polar Section modulus).

- \* Solid shaft  $\rightarrow$  diameter constrained  
Hollow shaft  $\rightarrow$  weight constrained

Problem Det. the ratio of power transmission capacity of solid and hollow circular shaft when they are rotating at same rpm with same material and having same diameter.

$$\frac{\text{Power of solid shaft}}{\text{Power of Hollow shaft}} > 1$$



$$\tau_s \frac{P_s}{P_H} = \frac{(T_{cp})_s}{(T_{cp})_H} = \frac{T_s}{T_H} = \frac{(Z_p \times \tau_s)_s}{(Z_p \times \tau_s)_H}$$

$$\frac{P_s}{P_H} = \frac{(Z_p)_s}{(Z_p)_H} = \frac{\frac{\pi}{16} d^3}{\frac{\pi}{16} D^3 [1 - k^4]}$$

$$\boxed{\frac{P_s}{P_H} = \frac{1}{1 - k^4} > 1}$$

$$\text{II if } k = \frac{1}{2} \quad \frac{P_s}{P_H} = \frac{1}{1 - \frac{1}{16}} = \frac{16}{15} = 1.07 > 1$$

When radial space is constrained ( $d_{ia} = \text{const}$ ) (for Hollow & solid shaft)  
Solid shaft are best for power transmission in comparison to hollow shaft.

$$(ZP)_{\text{solid}} > (ZP)_{\text{hollow}}$$

Problem Repeat the above question, if weight and lengths of solid and hollow shaft is equal instead of equal diameter. Same material, same rpm.

$$\frac{P_s}{P_H} = \frac{(T\omega)_s}{(T\omega)_H} = \frac{T_s}{T_H} = \frac{(ZP \times T)_s}{(ZP \times T)_H} = \frac{\pi/16 d^3 \tau_s}{\pi/16 D^3 [1-k^4] \tau_s}$$

$$\frac{P_s}{P_H} = \frac{d^3}{D^3 [1-k^4]} = \left(\frac{d}{D}\right)^3 \times \frac{1}{(1-k^4)} \rightarrow \text{--- (1)}$$

$$\therefore W_s = W_H$$

$$\rho_s V_s = \rho_H V_H$$

$$A_s l_s = A_H l_H$$

$$A_s = A_H$$

$$\pi/4 d^2 = \pi/4 D^2 [1-k^2]$$

$$\left\{ \frac{d}{D} = \sqrt{1-k^2} \right\}$$

$$\text{Then } \left\{ \frac{P_s}{P_H} = \frac{T_s}{T_H} = \frac{\sqrt{1-k^2}}{1+k^2} \right\} < 1 \quad \text{proven}$$

$$\text{If } k = \frac{1}{2}$$

$$\left\{ \frac{P_s}{P_H} = \frac{\sqrt{1-(\frac{1}{2})^2}}{1+(\frac{1}{2})^2} = 0.7 \right\}$$

\* Power shaft of power transmission cap

\* If weight is constrained, hollow shafts are best for power transmission in comparison to solid shaft because  $(ZP)_H > (ZP)_S$

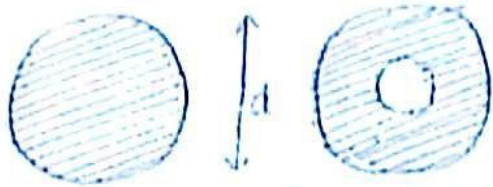
$$\begin{aligned} T_s &= \pi/16 d^3 \tau_s \\ T_H &= \pi/16 D^3 [1-k^4] \tau_s \end{aligned}$$

$$\therefore \left\{ \begin{aligned} P &\propto T \propto d^3 \propto ZP \\ (T_s)_{\text{hollow}} &\propto \frac{1}{d^3} \propto \frac{1}{ZP} \end{aligned} \right\} \quad \text{checked}$$



$$\left[ \frac{T_R}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \right]$$

equating  $\left[ \frac{\tau}{R} = \frac{G\theta}{L} \right] \Rightarrow \left[ \tau = \frac{G\theta R}{L} = \tau_{max} \right]$  used when  $\theta$  is given



Material,  $L$  &  $d$  same

- (a)  $\frac{\tau_s}{\tau_H} = ? (1)$  [if they are subjected to same T.M.]  
 $= (1-k^4)$
- (b)  $\frac{\tau_s}{\tau_H} = 1$  [if they are twisted by same angle of twist  $\theta$ ]

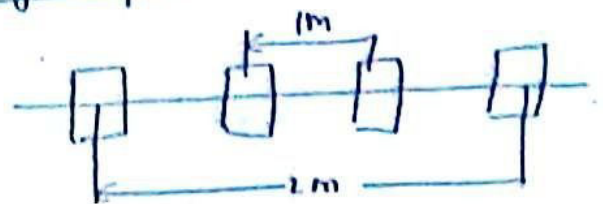
\* If  $T$  and  $\theta$  is given.

$$\left[ \frac{T_R}{J} = \frac{G\theta}{L} \right]$$

conventional  
 unit  $\left[ \theta_{ind} = \frac{T_R \cdot L}{G \cdot J} \leq \theta_{per} \right]$

Condition of shaft design under rigidity criterion  $\rightarrow$

take  $L = 1 \text{ m}$



(\*)  $GJ = \text{Torsional Rigidity}$

$GJ \uparrow = \theta_{ind} \downarrow \Rightarrow \phi \downarrow = \tau \downarrow = \text{chance of failure} \downarrow$

$$GJ = \frac{T_R \cdot L}{\theta}$$

$$\left[ \phi = \frac{R\theta}{L} \right]$$

if  $\frac{\theta}{L} = 1 \Rightarrow \left[ GJ = T_R \right]$

$$\left[ GJ = \frac{T_R}{\theta/L} \right]$$

## TORSIONAL STIFFNESS

$$\boxed{\theta = \frac{T}{GJ}} \quad * \text{ twist required per unit angular twist (rad)}$$

$$\boxed{\theta = \frac{T}{GJ} = \frac{G\theta}{L}} \quad * \frac{T}{J} = \frac{G\theta}{L} \Rightarrow \frac{T}{\theta} = \frac{GJ}{L}$$

Prob. A Hollow circular shaft inner radius 3cm and outer radius 5cm and  $L = 100$  cm is subjected to twisting moment so that angular twist is 0.01 radians. Max. shear angle in a shaft is:

Sol.  $\text{if } r_i = 3\text{cm}, r_o = 5\text{cm}, L = 100\text{cm}, \theta = 0.01$

$$\phi = \frac{R\theta}{L} = \frac{5 \times 0.01}{100} = 5 \times 10^{-3} \text{ rad}$$

Prob. Repeat the above question for the shear stress developed at a inner radius of shaft if  $E = 200 \text{ GPa}, L = 0.25$

Q  $\frac{\tau_{\max}}{\tau_{\min}} = \frac{R_o}{R_i} \quad \boxed{\phi = \gamma}$

$$\tau_{\min} = \tau_{\max} \frac{R_i}{R_o}$$

$$\phi_{\max} = \gamma = 0.0005 \text{ rad}$$

$$\begin{aligned} \tau_{\max} &= G \gamma_{\max} \\ &= \frac{F}{2(1+\mu)} \times 0.0005 \\ &= 24 \text{ MPa} \end{aligned}$$

$$\tau_{\min} = 24 \text{ MPa} \times \frac{30}{50}$$

$$\boxed{\tau_{\min} = 14.4 \text{ MPa}}$$



Problem A solid shaft of diameter ( $d$ ) is replaced by a hollow shaft of same material and length the outside dia. of hollow shaft ( $d_o$ ) is  $\left(\frac{2d}{\sqrt{3}}\right)$  while inside dia.  $\left(\frac{d}{\sqrt{3}}\right)$ . What is ratio of torsional stiffness. H/S

Sol-

$$\left[ \frac{q}{\theta} \right] = \frac{q_H}{q_S} = \frac{T_H / \theta_H}{T_S / \theta_S} = \frac{G J_H / L}{G J_S / L} = \frac{J_H}{J_S}$$

$$\frac{q_H}{q_S} = \frac{J_H}{J_S} = \frac{\pi d_o^4 / 32 (1 - k^4)}{\pi d^4 / 32}$$

$$\left[ \frac{q_H}{q_S} = \frac{5}{3} \right] \text{ Answer.}$$

Problem The max. and min shear stresses in a hollow circular shaft of outer diameter 20mm and thickness 2mm, Subjected to a torque of 92.7 Nmm will be -

Solution

$$d_o = 20 \text{ mm}, d = 18 \text{ mm}, r_o = 10, r_i = 8$$

$$\left( \frac{\tau_{\max}}{\tau_{\min}} \right)_{\text{Hollow}} = \frac{R_o}{R_i} = \frac{10}{8} = \frac{5}{4}$$

$$\begin{aligned} \tau_{\max} &= \frac{T}{Z_p} = \frac{92.7}{\frac{\pi D_o^3}{16} (1 - k^4)} = \frac{92.7}{\frac{\pi 20^3}{16} \left[ 1 - \left( \frac{18}{20} \right)^4 \right]} \\ &= \frac{1403.2 \times 1000}{25132.74 [0.59]} \\ &= \underline{100 \text{ MPa}} \quad (99.95 \text{ MPa}) \end{aligned}$$

$$\frac{\tau_{\max}}{\tau_{\min}} = \frac{5}{4}$$

$$\frac{100}{\tau_{\min}} = \frac{5}{4}$$

$$\tau_{\min} = \frac{400}{5} = \underline{80 \text{ MPa}} \text{ ans}$$

Problem: A steel shaft (A) of dia (d) and length (L) is subjected to torque (T) another shaft (B) made of (Al) of same dia. and length (0.5L) is also subjected to the same torque (T)  $G_{\text{of steel}} = 2.5 G_{\text{of Al}}$  ( $G_s = 2.5 G_{Al}$ ) The shear stress in a steel shaft = 100 MPa,  $\tau_{Al} = \dots$  MPa is

Soln:

$$\tau_{\max} = \frac{G \theta R}{L} = \frac{T}{Z_p}$$

$$\frac{(\tau_{\max})_{Al}}{(\tau_{\max})_{\text{steel}}} = \frac{T_{Al}/Z_{pAl}}{T/Z_{ps}} = \frac{Z_{ps}}{Z_{pAl}} = \frac{\pi/16 d^3}{\pi/16 d^3} = 1$$

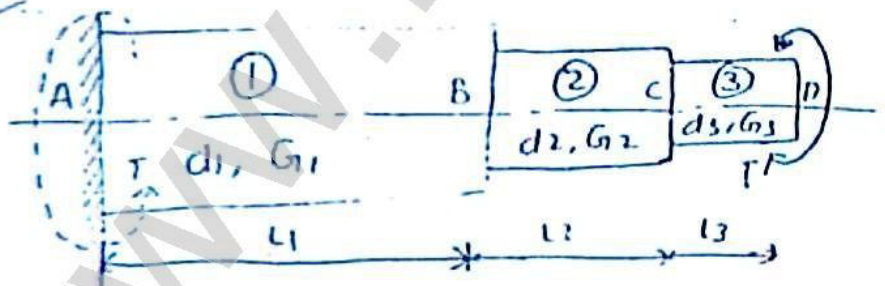
$$(\tau_{\max})_{Al} = (\tau_{\max})_s \times 1$$

$$= 100 \text{ MPa}$$

## CONNECTIONS OF SHAFTS

### ① SERIES COMBINATION $\rightarrow$ [COMPOUND SHAFTS]

CASE 1



(i)  $T_1 = T_2 = T$

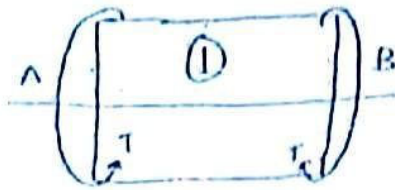
(ii)  $\theta_{\text{total}} = \theta_3 + \theta_2 + \theta_1$

$$\theta_{\text{total}} = \frac{TL_1}{G_1 J_1} + \frac{TL_2}{G_2 J_2} + \frac{TL_3}{G_3 J_3}$$

③  $\theta_{\text{total}} = T \left[ \frac{L_1}{G_1 J_1} + \frac{L_2}{G_2 J_2} + \frac{L_3}{G_3 J_3} \right]$



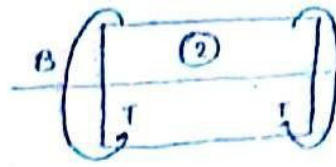
F.B.D →



$$T_1 = T$$

$$\theta_1 = \frac{TL_1}{G_1 J_1}$$

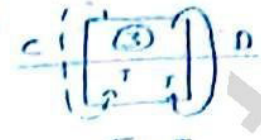
$$\tau_1 = \frac{16T}{\pi d_1^3}$$



$$T_2 = T$$

$$\theta_2 = \frac{TL_2}{G_2 J_2}$$

$$\tau_2 = \frac{16T}{\pi d_2^3}$$



$$T_3 = T$$

$$\theta_3 = \frac{TL_3}{G_3 J_3}$$

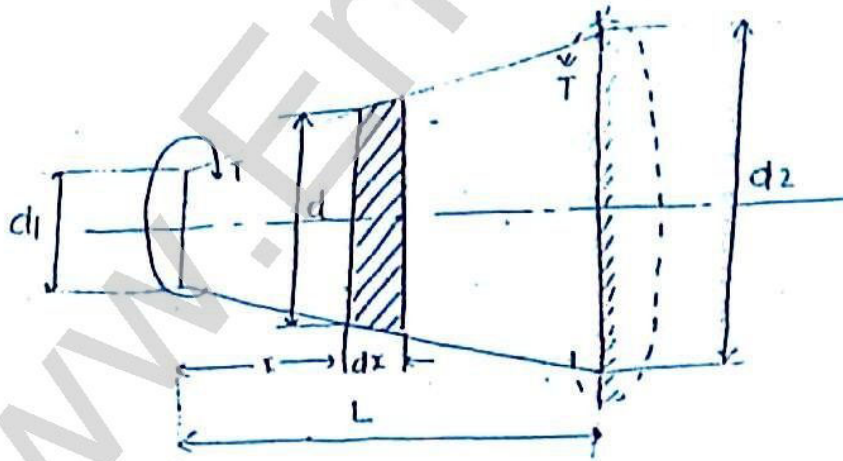
$$\tau_3 = \frac{16T}{\pi d_3^3}$$

\*\*\*

$$\frac{\tau_{\max}}{\tau_{\min}} = \frac{\tau_3}{\tau_1} = \left( \frac{d_1}{d_3} \right)^3 = \left( \frac{d_{\max}}{d_{\min}} \right)^3$$

$$\theta_{\text{total}} = T \left[ \sum_{i=1}^n \frac{L_i}{G_i J_i} \right]$$

Torsion of a tapered shaft → (UNDER PURE TORSION)



$$\tau_{\max} = \frac{16T}{\pi d_1^3}$$

$$\tau_{\min} = \frac{16T}{\pi d_2^3}$$

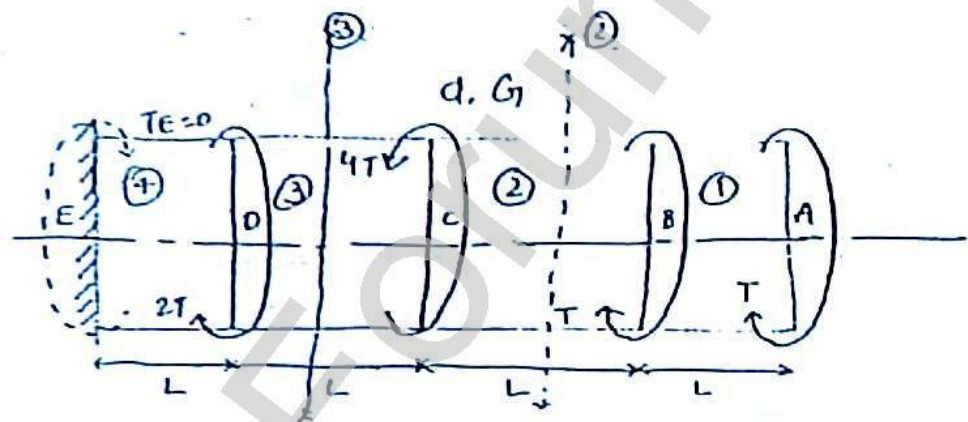
$$\frac{\tau_{\max}}{\tau_{\min}} = \left( \frac{d_2}{d_1} \right)^3 = \left( \frac{d_{\max}}{d_{\min}} \right)^3$$

$$\theta_{\text{ship}} = \frac{T x dx}{G \times \frac{\pi}{32} d^4}$$

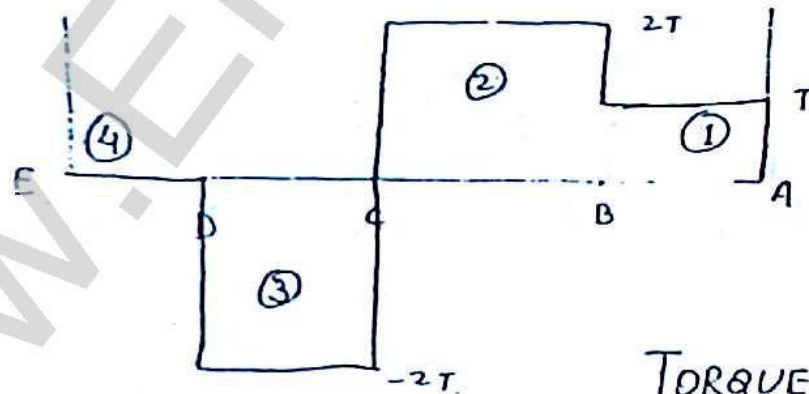
$$d = d_1 + (d_2 - d_1) \left( \frac{x}{L} \right)$$

$$\theta_{\text{total}} = \int \theta_{\text{ship}} = \frac{32TL}{\pi G} \left[ \frac{d_1^2 + d_1 d_2 + d_2^2}{3 d_1^3 d_2^3} \right]$$

CASE-II →



$$T_1 = T, T_2 = 2T, T_3 = -2T, T_4 = 0$$



TORQUE DIAGRAM

CRITICAL PORTION - BC & CD

$$\text{Max } \tau_{\text{max}} = \tau_2 = \tau_3 = \frac{16 \times (2T)}{\pi d^3}$$

$$\tau_{\text{max}} = \frac{32T}{\pi d^3}$$



determine total angular twist at free end.

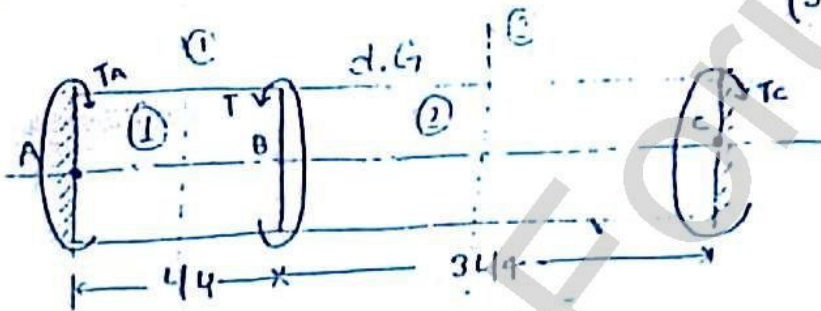
$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 + Q_4$$

$$= \frac{L}{GJ} [T_1 + T_2 + T_3 + T_4]$$

$$= \frac{L}{GJ} [T + 2T - 2T + 0]$$

$$Q_{\text{total}} = \frac{TL}{GJ} = \frac{32 TL}{\pi G d^4}$$

### CASE - III      SHAFTS FIXED AT THE BOTH ENDS (Statically Indeterminate)



$$T = AC \omega \quad (\text{RHS})$$

$$T_C = T_A = CW \quad (\text{RHS})$$

Step I.  $T_1 = -T_A$  ,  $T_2 = +T_C$  or  $T - T_A$

Step II -  $\sum T = 0$   
 $T_A + T_C = T \Rightarrow T_C = T - T_A$

Step III.  $\theta_{\text{total}} = \theta_1 + \theta_2 = 0$

$$\theta_1 = -\theta_2$$

$$\frac{T_1 L_1}{GJ_1} = -\frac{T_2 L_2}{GJ_2}$$

$$T_A \times \frac{44}{4} = T_C \times \frac{344}{4}$$

$$T_A = 3T_C$$

$$T_A = 3T/4$$

$$J_1 = -T_A = -\frac{3T}{4} = \frac{3T}{4} \quad (\text{ACW})$$

$$T_2 = T_C = T/4 \quad (\text{CW})$$

Stresses induced in a member  $\rightarrow$

$$\frac{\tau_1}{\tau_2} = \frac{16T_1/\pi d_1^3}{16T_2/\pi d_2^3}$$

$$\boxed{\frac{\tau_1}{\tau_2} = \frac{T_1}{T_2} = 3}$$

$$\theta_1 = \frac{T_1 L_1}{G J_1} = \frac{3T/4 \times 4/4}{G \times \frac{\pi d^4}{32}}$$

$$\boxed{\theta_1 = \frac{-6TL}{G\pi d^4}}$$

$$\theta_2 = -\theta_1$$

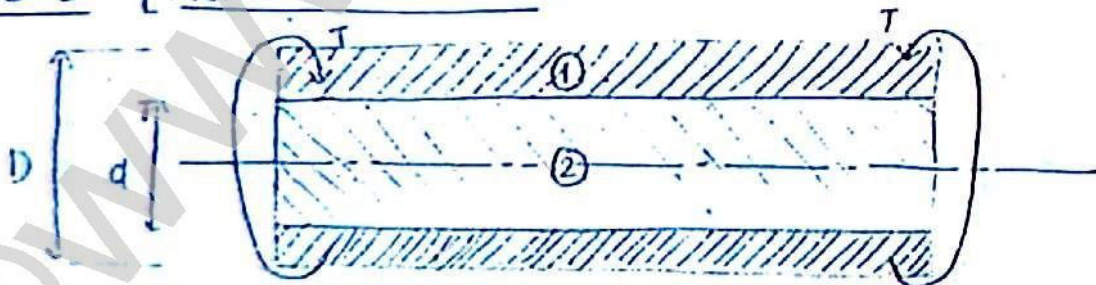
$$\boxed{\theta_2 = \frac{6TL}{G\pi d^4}}$$

Method II.

$$\left. \begin{aligned} T_A &= \frac{T \times 3L/4}{L} = 3T/4 \\ T_C &= -\frac{T \times L/4}{L} = -T/4 \end{aligned} \right\}$$

## 2) SHAFTS IN PARALLEL

CASE-I [FREE AT BOTH END]



i)

$$\theta = \theta_1 = \theta_2$$

ii)

$$T = T_1 + T_2$$

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow T = \frac{GJ\theta}{L}$$



$$\text{Then } T = \frac{G_1 J_1 \theta_1}{L_1} + \frac{G_2 J_2 \theta_2}{L_2}$$

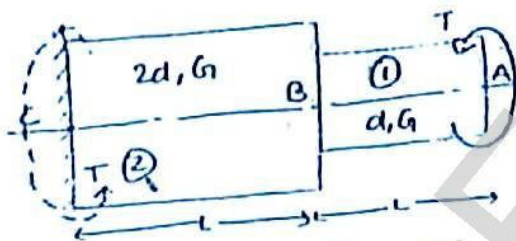
$$= \frac{\theta}{L} [G_1 J_1 + G_2 J_2]$$

$$\theta = \frac{T L}{G_1 J_1 + G_2 J_2}$$

$$\theta_1 = \frac{T_1 L_1}{G_1 J_1} = \theta \Rightarrow T_1 = ?$$

$$T = T_1 + T_2 \Rightarrow T_2 = ?$$

Problem for the compound shaft as shown in figure. determine angular twist at A (free end). if angular twist at B is  $\theta$



$$\theta_B = ?$$

$$\theta_A = ?$$

$$T_1 = T_2 = T$$

$$T_1 = T, T_2 = T$$

$$\theta_A = \frac{T_1 L_A}{G_1 J_1} = \frac{T \times L}{G \times J_1}$$

$$\theta_{\text{total}} = \theta_A + \theta_B$$

$$= \frac{T L}{G} \left[ \frac{1}{J_1} + \frac{1}{J_2} \right]$$

$$= \frac{T L}{G} \left[ \frac{1}{\frac{\pi \cdot d^4}{32}} + \frac{1}{\frac{\pi \cdot (2d)^4}{32}} \right]$$

$$= \frac{32 T L}{\pi \cdot G} \left[ \frac{1}{d^4} + \frac{1}{16 d^4} \right]$$

$$= \frac{2 \cdot 32 T L}{\pi \cdot G \cdot d^4} \left[ \frac{17}{16} \right]$$

$$= 2 \times 17 \times \frac{T L}{\pi G d^4}$$

$$\theta_{\text{total}} = 17 \theta$$

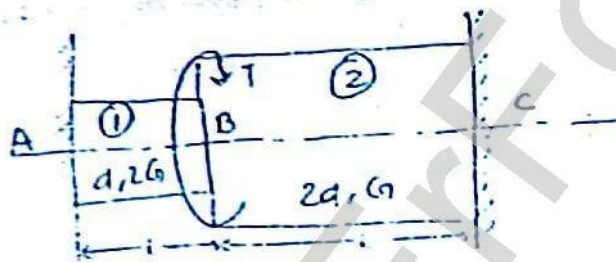
$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$

$$\frac{\theta_1}{\theta_2} = \frac{\frac{T_1 L_1}{G J_1}}{\frac{T_2 L_2}{G J_2}} = \frac{T_2}{T_1} = \left(\frac{d_2}{d_1}\right)^4 = 16$$

$$\boxed{\theta_1 = 16\theta_2}$$

$$\begin{aligned}\theta_{\text{total}} &= \theta_1 + \theta_2 \\ &= 16\theta_2 + \theta_2 \\ &= 17\theta_2 \\ &= 17\theta_2 A_2\end{aligned}$$

Problem → A compound shaft as shown in fig. Determine ratio of reactions at fixed ends.



$$\frac{T_A}{T_C} = ?$$

$$\theta_1 = -\theta_2$$

$$\frac{T_1 L_1}{G_1 J_1} = -\frac{T_2 L_2}{G_2 J_2}$$

$$\frac{T_A L}{\frac{2G \times \pi}{4} \times \frac{d^4}{4}} = \frac{T_C L}{\frac{2G \times \pi}{4} \times \frac{16d^4}{4}}$$

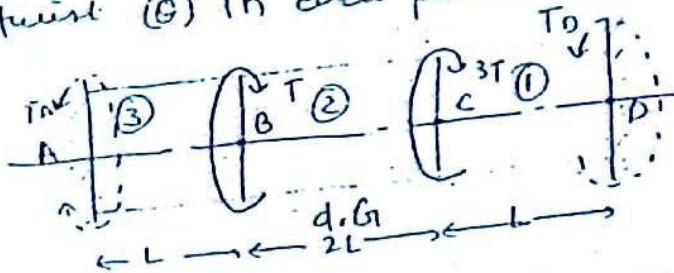
$$\boxed{\frac{T_A}{T_C} = \frac{1}{8}}$$

Problem → For the above question also determine max  $T_{\text{max}}$  induced &  $\theta_1$  in each portion of shaft.

$$\theta_A = \frac{T_A L_A}{G_A J_A}$$



Problem determine ( $\tau_{max}$  induced at each portion and angular twist ( $\theta$ ) in each portion.



$$T_1 = T_D, T_2 = T_D - 3T, T_3 = T_D - 4T = (-T_A)$$

$$T_A + T_D = 4T$$

$$T_A = 4T - T_D$$

$$\theta_{total} = \theta_1 + \theta_2 + \theta_3 = 0$$

$$\theta_{total} = \frac{1}{GJ} [T_1 L_1 + T_2 L_2 + T_3 L_3] = 0$$

$$T_1 L_1 + T_2 L_2 + T_3 L_3 = 0$$

$$T_D \cdot L + (T_D - 3T) \times 2L + (T_D - 4T) \times L = 0$$

$$T_D \cdot L + 2LT_D - 6TL + T_D L - 4TL = 0$$

$$4T_D \cdot L = 10T \cdot L$$

$$T_D = \frac{5T}{2}$$

$$T_A + T_D = 4T$$

$$T_A = 4T - T_D = 4T - \frac{5T}{2}$$

$$T_A = \frac{3T}{2}$$

$$T_1 = -2.5T, T_2 = \frac{5T}{2} - 3T = -\frac{T}{2} = -0.5T, T_3 = \frac{5T}{2} - 4T = -1.5T$$

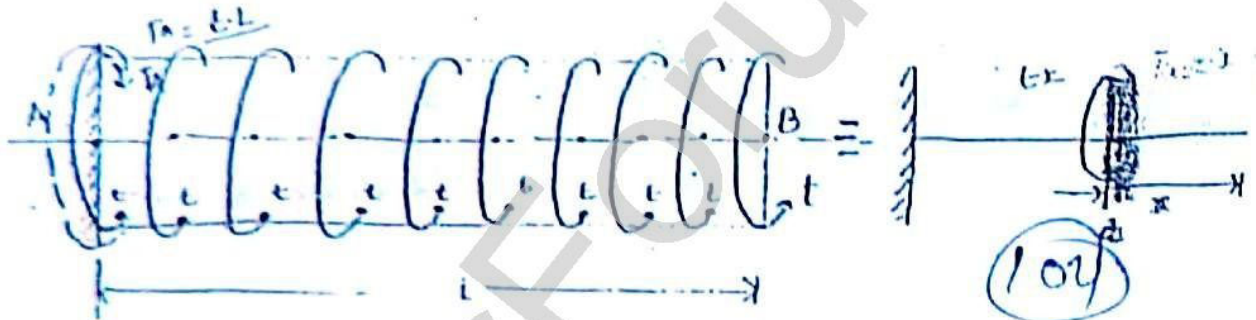
$$\tau_1 = \frac{16T_1}{\pi d_1^3}, \quad \tau_2 = \frac{16T_2}{\pi d_2^3}, \quad \tau_3 = \frac{16T_3}{\pi d_3^3}$$

$$\tau_1 = \frac{16 \times 2.5T}{\pi \times d^3}, \quad \tau_2 = \frac{16 \times 1.5T}{\pi d^3}, \quad \tau_3 = \frac{16 \times 1.5T}{\pi d^3}$$

$$\tau_1 = \frac{40.0T}{\pi d^3}, \quad \tau_2 = \frac{8T}{\pi d^3}, \quad \tau_3 = \frac{24.0T}{\pi d^3}$$

\*\*\*\*\*  
problem

Q.1. Max. shear stress induced and angular twist at the free end for a shaft as shown in figure. and the shaft is loaded on uniformly distributed torque. (10 marks)



$$\tau_A = \tau_{\max} = \frac{16 T_{\max}}{\pi d^3}$$

$$= \frac{16 t \cdot L}{\pi d^3}$$

$$\theta_{\text{strip}} = \frac{t x dx}{GJ}$$

$$\theta_{\text{total}} = \int \theta_{\text{strip}} = \int_0^L \frac{t x dx}{GJ}$$

$$\theta_{\text{total}} = \frac{t}{GJ} \left[ \frac{L^2}{2} \right]$$

$$\theta_{\text{total}} = \frac{t \cdot L^2}{2 GJ}$$

$$\boxed{\theta_{\text{total}} = \frac{16 t \cdot L^2}{G \pi d^4}}$$

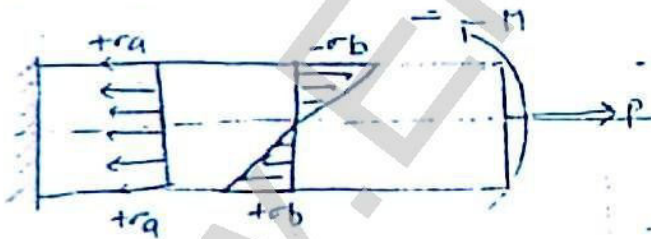
Q.2. Repeat a above question if the shaft is subjected to uniformly varying torque (0 to t)



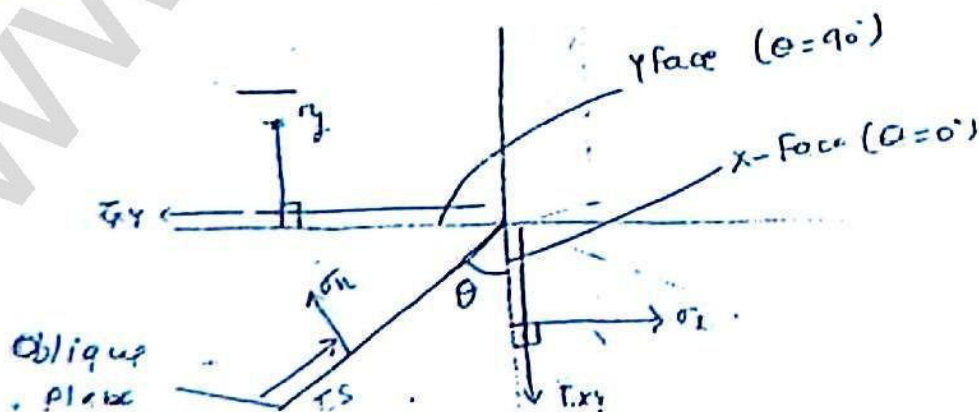
# Combined Stresses

## Principal stresses and Strains

- Combined stresses means both normal and shear stress is applied.
  - If only shear stress is applied then max. shear induced.
  - If only twist or torsion stress is applied along the axis, max. torsion stress is induced.
- ① A point is said to be under combined stress when it is subjected to both normal and shear stress due to externally applied loads.
- ② The aim of topic is to derive the expressions for max. normal stress and max. normal strain, max shear stress and max. shear strain when a point is under combined stresses.



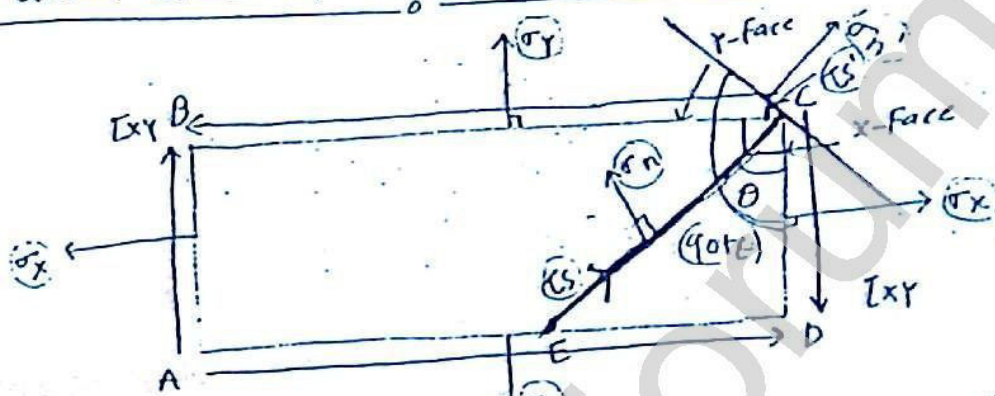
- Critical point is bottom fibers. It is not a combined stress because both are normal stress.



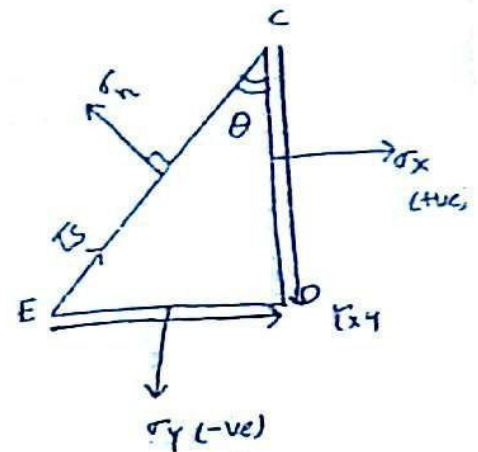
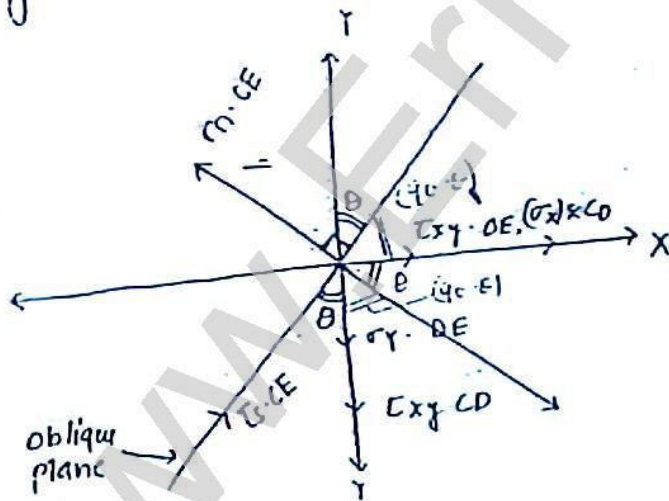
- Step Used —
- (I)  $\sigma_x = ?$
  - (II)  $\sigma_y = ?$
  - (III)  $\tau_{xy} = ?$
  - (IV)  $\sigma_n = ?$ ,  $\tau_s = ?$
  - (V)  $(\sigma_n)_{\max} = ?$ ,  $(\tau_s)_{\max} = ?$
  - (VI)  $(\sigma_n)_{\max} \leq \sigma_{\text{per}}$   
 $(\tau_s)_{\max} \leq \tau_{\text{per}}$  } safe design

## STATE OF STRESS AT A POINT IN 2D →

[Bi-axial state of stress at a point]



Taking COE in equilibrium →



$$\sin \theta = \frac{DE}{CE}$$

$$\cos \theta = \frac{CD}{CE}$$

$$\textcircled{I} \quad \sigma_n = \frac{1}{2} [\sigma_x + \sigma_y] + \frac{1}{2} [\sigma_x - \sigma_y] \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\textcircled{II} \quad \tau_s = -\frac{1}{2} [\sigma_x - \sigma_y] \sin 2\theta + \tau_{xy} \cos 2\theta$$

To check these two equations take  $\theta = 0^\circ$  get  $\sigma_x$   
 $\theta = 90^\circ$  get  $\sigma_y$



consider a perpendicular plane on the oblique plane.  
to get the  $(\sigma_n')$  and  $(\tau_s')$  substitute instead of  $\theta$ , take  
 $\theta = 90^\circ + \theta$  in above equations:-

$$(\sigma_n)' = (\sigma_n)_{\theta=90^\circ+\theta} = \frac{1}{2} [\sigma_x + \sigma_y] + \frac{1}{2} [\sigma_x - \sigma_y] (-\cos 2\theta) + \tau_{xy} (-\sin 2\theta)$$

eq (i) and (iii)

$$\sigma_n + \sigma_n' = \sigma_x + \sigma_y$$

"The sum of the complementary normal stress is always remains constant and equal to sum of  $\sigma_x$  and  $\sigma_y$ "

$$\begin{aligned} (\tau_s)' &= (\tau_s)_{\theta=90^\circ+\theta} = -\frac{1}{2} [\sigma_x - \sigma_y] (-\sin 2\theta) + \tau_{xy} [-\cos 2\theta] \\ &= -\left[ -\frac{1}{2} (\sigma_x - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta \right] \end{aligned}$$

$$\tau_s' = -\tau_s$$

$$\tau_s' + \tau_s = 0$$

"The sum of complementary shear stress is always zero and equal and opposite."

$$\sigma_x = 100 \text{ MPa}$$

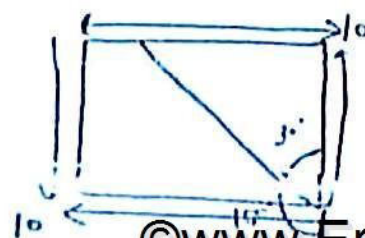
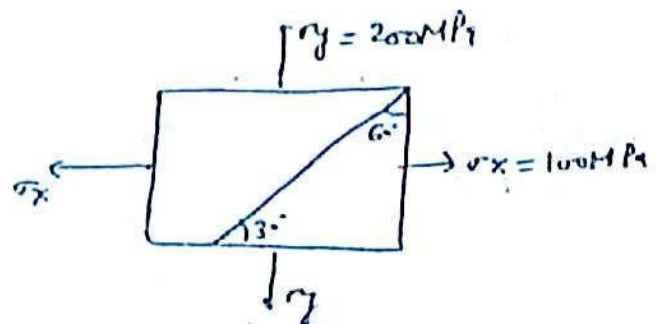
$$\sigma_y = 200 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\theta = 60^\circ$$

$$\sigma_x = \sigma_y = 0$$

$$\tau_{xy} = 0$$

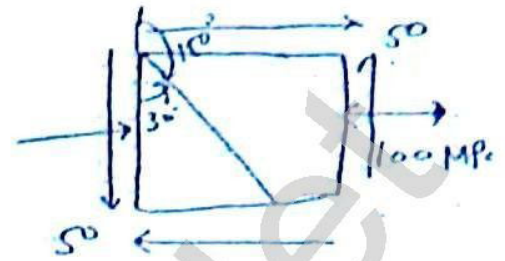




$$\sigma_x = -100 \text{ MPa} \quad , \sigma_y = 0$$

$$\tau_{xy} = -50 \text{ MPa}$$

$$\theta = -3^\circ \quad (150^\circ)$$



## PRINCIPAL PLANES AND PRINCIPAL STRESS.

### [ PLANES OF ZERO SHEAR STRESS ]

- \* The nature of principal stress is always normal stress.
- \* At the different  $\theta$  value equations gives max. normal stress and max. shear stress.

### 1. LOCATIONS OF PRINCIPAL PLANES -

$$\tau_s = 0 = -\frac{1}{2} [\sigma_x - \sigma_y] \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

Example -  $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1$

$$2\theta = 135^\circ, 315^\circ$$

$$\boxed{\theta_{1,2} = 67\frac{1}{2}^\circ, 157\frac{1}{2}^\circ}$$

two planes (2D) (Principal)

- \* Principal planes are always mutually  $\perp$  to each other.

$$\boxed{\theta_2 = \theta_1 \pm 90^\circ}$$

Principal planes  
or planes of zero  $\tau_n$

or planes of pure  $\sigma_n$

or planes of comp.  $\sigma_n$

or planes of max-min  $\sigma_n$

Normal stress

Complimentary stress

Max and Min. Normal stress

Magnitude of Principal stresses  $\rightarrow$

$$(\sigma_n)_{\theta=\theta_1} = x \text{ MPa}$$

$$[(\sigma_n)_{\theta=\theta_1} + (\sigma_n)_{\theta=\theta_2} = \sigma_x + \sigma_y]$$

$$(\sigma_n)_{\theta=\theta_2} = \sigma_x + \sigma_y - x \text{ MPa}$$

$$= -2x \text{ MPa [Consider only mag. not dir!]}$$

$\sigma_1$  = Major Principal stress

$\sigma_2$  = Max. Normal stress

= Large of  $[(\sigma_n)_{\theta=\theta_1} \text{ and } (\sigma_n)_{\theta=\theta_2}]$

$\sigma_1 = 2x \text{ MPa [Max. Normal stress induced at point]}$

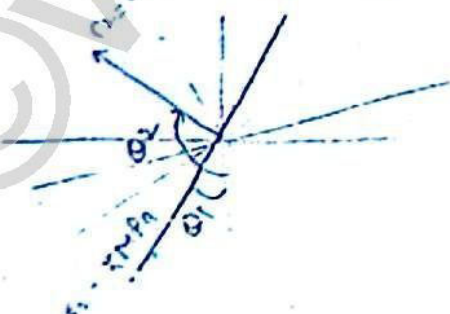
=  $< \sigma_{\text{per}} = (\text{safe})$  [Remaining  $(\omega-1)$  plane has less than  $2x \text{ MPa}$ ]

$\sigma_2$  = Minor Principal stress

= Minimum Normal stress

= Smaller of  $[(\sigma_n)_{\theta=\theta_1} \text{ or } (\sigma_n)_{\theta=\theta_2}]$

=  $(\sigma_{\theta=\theta_1})_n = x \text{ MPa [Min Normal stress induced at point]}$



[Remaining  $(\omega-2)$  planes have less than  $2x$  and more than  $x \text{ MPa}$ ]



## LOCATION OF MAX- AND MIN. NORMAL STRESS PLANES

$$\frac{d}{d\theta}(\sigma_n) = 0$$

$$\Rightarrow \boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \quad \theta_{1,2} = 67\frac{1}{2}^\circ, 157\frac{1}{2}^\circ$$

Now we can conclude that max and min  $\sigma_n$  planes are coincides with principal planes. Hence principal planes or oblique planes which carry zero shear stress and Max. and min normal stressers.

Ist Method:-

$$\textcircled{a} \quad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta_{1,2} = ?$$

$$\textcircled{b} \quad (\sigma_n)_{\theta=\theta_1} = \dots$$

$$\textcircled{c} \quad \sigma_n + \sigma_n' = \sigma_x + \sigma_y$$

$$\sigma_n' = ?$$

$$\textcircled{d} \quad \sigma_1 = \dots$$

$$\sigma_2 = \dots$$

IInd Method:- \*\*\*

(Simple)

$$\boxed{\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

This method gives only magnitude of principal stressers.

Example:-

$$\therefore \sigma_{1,2} = 10, -100 \text{ MPa}$$

$$\sigma_1 = -100 \text{ MPa}$$

$$\sigma_2 = 10 \text{ MPa}$$

= (Max. Compressive stress induced at point)

= (Minimum tensile Normal stress)



## Max. $\tau_s$ Planes $\rightarrow$ [Complimentary $\tau_{max}$ Plane]

### 1. LOCATION:-

$$\frac{d}{d\theta} (\tau_s) = 0$$

Max.  $\tau_s$  planes are those oblique planes which carry max. shear stress in addition to some amount of Normal stress.

$$\tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

$$\tan 2\theta = \frac{-1}{[2\tau_{xy} / \sigma_x - \sigma_y]}$$

### Example:

$$\tan 2\theta = \frac{-1}{-1} = 1 \quad (\text{1st and 3rd coordinate})$$

$$2\theta = 45^\circ, 225^\circ$$

$$\theta = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ$$

$$\theta_4 = \theta_3 \pm 90^\circ$$

\*  $\tau_{max}$  plane are also be mutually perpendicular

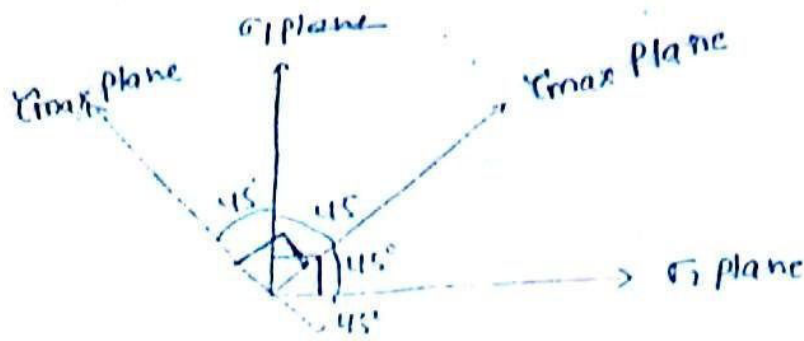
$$\theta_1 = 67\frac{1}{2}^\circ, \theta_2 = 157\frac{1}{2}^\circ$$

$$\theta_3 = \theta_1 \pm 45^\circ$$

$$\begin{aligned} '+' &= \theta_1 < 45^\circ \\ '-' &= \theta_1 > 45^\circ \end{aligned}$$

$$\theta_4 = \theta_2 \pm 45^\circ$$

$$\left. \begin{aligned} \theta_1 \\ \theta_2 &= \theta_1 \pm 90^\circ \\ \theta_3 &= \theta_1 \pm 45^\circ \\ \theta_4 &= \theta_3 \pm 90^\circ \\ &= \theta_2 \pm 45^\circ \end{aligned} \right\}$$



- \* Max.  $\tau_{max}$  planes are mutually  $\perp$  to each other and they are also inclined at  $45^\circ$  at the both the principal planes.

## 2. Magnitude of $\tau_{max}$ :-

$$\tau_{max} = (\tau_s)_{\theta=\theta_3 \text{ or } \theta_4} = \text{--- Y MPa} < \sigma_{perm} = \text{safe}$$

2nd Method :-

$$\text{Absolute } \tau_{max} = \text{larger of } \left[ \left| \frac{\sigma_1 - \sigma_2}{2} \right|, \left| \frac{\sigma_2 - \sigma_3}{2} \right|, \left| \frac{\sigma_3 - \sigma_1}{2} \right| \right]$$

for 2D cases,  $\sigma_3 = 0$

$$\text{Absolute } \tau_{max} = \text{larger of } \left[ \left| \frac{\sigma_1 - \sigma_2}{2} \right|, \left| \frac{\sigma_2}{2} \right|, \left| \frac{\sigma_1}{2} \right| \right]$$

$$4^{\text{th}} \quad \text{Absolute } \tau_{max} = \left| \frac{\sigma_1 - \sigma_2}{2} \right| \text{ or } \left| \frac{\sigma_1}{2} \right|$$

$\sigma_{1,2}$  are in opp. nature.  $\sigma_{1,2}$  are in same nature.

- \* Max. shear stress is developed at a particular point.

- \* In the design of a component always absolute max. shear stress should be taken into consideration.

\*\*\*  
To determine max. shear stress in a plane stress problem, always consider in plane max.  $\tau_{max}$  as the max. shear stress.

$$\tau_{max} = \left| \frac{\sigma_1 - \sigma_2}{2} \right|$$



## PLANES OF PURE SHEAR OR PLANES OF ZERO NORMAL STRESS

### 1. LOCATIONS:-

$$\sigma_n = 0 = \frac{1}{2} [\sigma_x + \sigma_y] + \frac{1}{2} [\sigma_x - \sigma_y] \cos 2\theta + xy \sin 2\theta$$

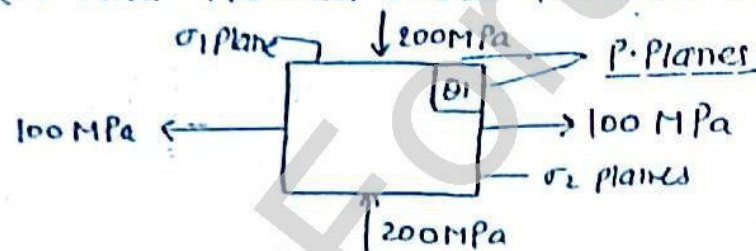
$$2\theta = \dots$$

$$\theta_{5.6} = \dots$$

$$\tau_s' = (\tau_s)_{\theta = \theta_5 \text{ or } \theta_6} = \dots < \tau_{\max.}$$

problem For the state of stress at a point as shown in fig. Determine (i) Max. Normal stress induced at that point.

solution



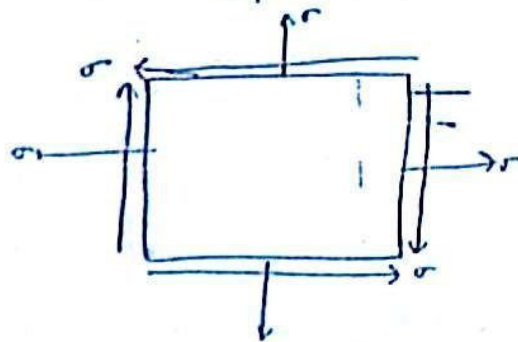
$$\sigma_n = \frac{1}{2} \quad \sigma_1 = -200 \text{ MPa (Max Normal stress)}$$

$$\sigma_2 = 100 \text{ MPa (Min Normal stress)}$$

$$\theta_1 = 90^\circ, \theta_2 = 0^\circ$$

When a point is subjected to only normal stress and then normal stresses itself becomes the principal stress.

oblem For: state of stress at a point as shown in figure



Determine max. Normal stress induced at that point



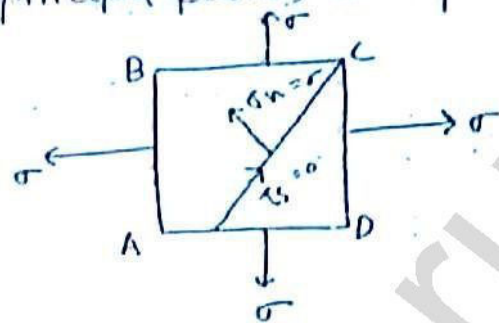
$$\sigma_x = \sigma_y = \tau_{xy} = \sigma$$

$$\sigma_{1,2} = \frac{1}{2} \left[ (\sigma + \sigma) \pm \sqrt{(\sigma - \sigma)^2 + 4\sigma^2} \right]$$

$$= 2\sigma, 0$$

$$\boxed{\sigma_1 = 2\sigma}$$

Problem For a state of stress at a point as shown in fig. determine location of principal planes and principal stresses.



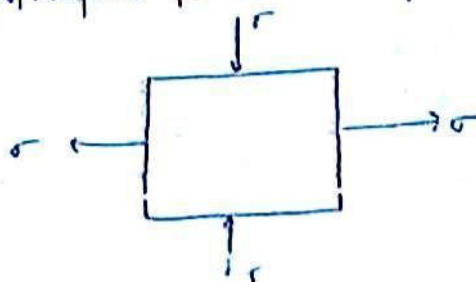
$$\sigma_n = \frac{1}{2} [\sigma + \sigma] + \frac{1}{2} [\sigma - \sigma] \cos 2\theta + 0 \sin 2\theta$$

$$\boxed{\sigma_n = \sigma}$$

$$\tau_s = \frac{1}{2} [\sigma - \sigma] \sin 2\theta + (0) \cos 2\theta = 0$$

- \* The magnitude of  $\sigma_n$  and  $\tau_s$  are independent on  $\theta$
- \* principal planes lie in all the direction.

\* When a point is subjected to equal and same nature of normal stresses only, then any oblique plane passing through the point carries zero shear stress and same amount of Normal stress. Hence in this particular case, all the planes passing through the point are known as principal plane. i.e. principal planes lie in all direction.



$$\left. \begin{aligned} \sigma_n &= \sigma \cos 2\theta \\ \tau_s &= \sigma \sin 2\theta \end{aligned} \right\}$$

problem: The Major and minor principal stress at a point are 120 MPa and 70 MPa resp. on a plane passing through a point, the normal stress is 115 MPa. The shear stress on this plane will be -

solution

$$\sigma_1 = 120 \text{ MPa}, \quad \sigma_2 = 70 \text{ MPa}, \quad \sigma_x = 115 \text{ Pa}, \quad \tau_{xy} = ?$$

$$\sigma_1 = \frac{1}{2} [\sigma_x + \sigma_y] \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

but

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$120 + 70 = 115 + \sigma_y$$

$$\boxed{\sigma_y = 75 \text{ MPa}}$$

then

$$120 = \frac{1}{2} [115 + 75] \pm \sqrt{(115 - 75)^2 + 4\tau_{xy}^2}$$

$$\boxed{\tau_{xy} = 15 \text{ MPa}}$$

problem

Repeat the above question for in plane max. shear stress and absolute shear stress.

$$\text{In plane max. shear stress} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{120 - 70}{2}$$

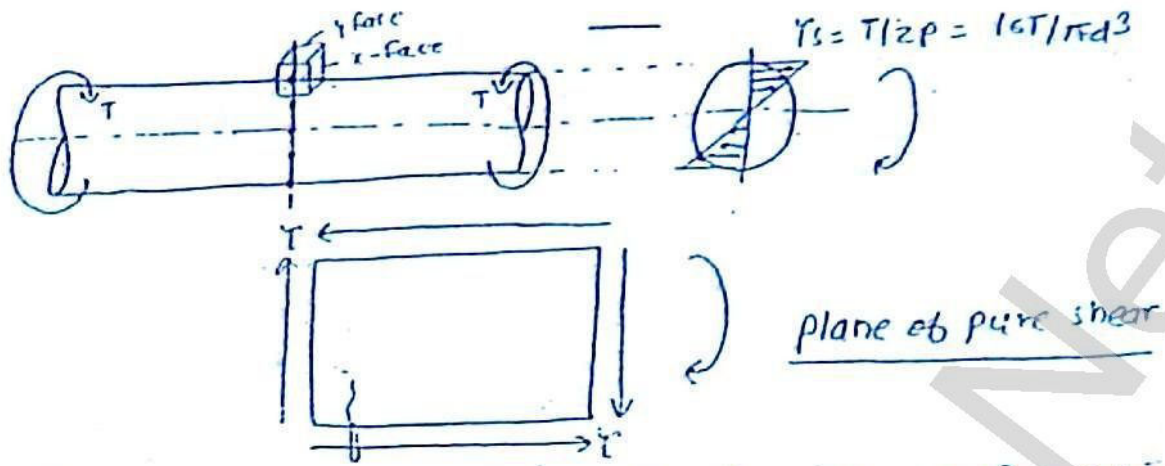
$$= \underline{25 \text{ MPa}} \quad \tau_{\text{per}} = \underline{50 \text{ MPa}}$$

$$\text{Absolute shear stress} = \left| \frac{\sigma_1}{2} \right| = 60 \text{ MPa}$$

PRINCIPAL PLANE & PRINCIPAL STRESS WHEN a point in PURE SHEAR:-

(shaft is subjected to pure torsion)  $\rightarrow$



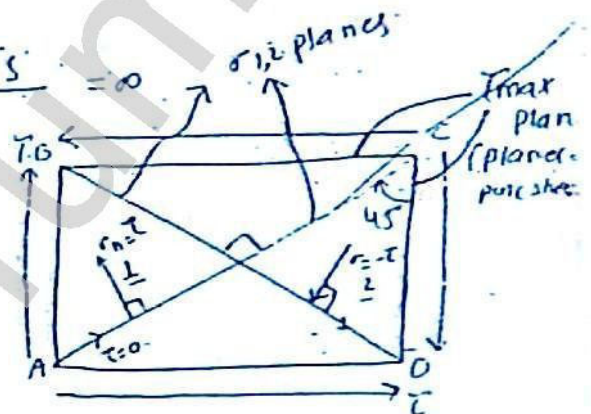


[State of stress at a point which is on the surface which is in pure shear]

$$\tan 2\theta = \frac{2\tau_s}{\sigma_x - \sigma_y} = \frac{2\tau_s}{0} = \infty$$

$$2\theta = 90^\circ, 270^\circ$$

$$\theta = 45^\circ, 135^\circ$$



$$(\sigma_n)_{\theta=01=45^\circ} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 90^\circ + \tau_s \sin 90^\circ$$

$$(\sigma_n)_{\theta=01=45^\circ} = +T \quad (\text{tensile})$$

$$\sigma_n + \sigma_{n'} = \sigma_x + \sigma_y = 0$$

$$\sigma_{n'} = -T \quad (\text{compressive})$$

$$\sigma_{1,2} = \pm T$$

\* When a point is under pure shear, principal planes are located at an angle of  $45^\circ$  and  $135^\circ$  from the ref plane ( $x$ -face).

\* The max. and min normal stress (Principal stresses) or (tensile and comp.) are max. tensile stress and max. comp. stresses induced at a point are equal.



to the maximum shear stress or applied shear stress.

$$\sigma_{1,2} = \pm \tau = \frac{16T}{\pi d^3}$$

$\gamma_{\max}$  planes -

$$\theta_3 = \theta_1 - 45^\circ = 45^\circ - 45^\circ = 0^\circ$$

$$\theta_4 = \theta_3 + 90^\circ = 0 + 90^\circ = 90^\circ$$

$$\gamma_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\tau - (-\tau)}{2} = \tau$$

- \*  $\gamma_{\max}$  planes are coincides with planes of pure shear.
- \* The strain analysis at a point under combined stress is similar to the stress analysis at a point under combined stress (i.e: Normal stressers should be replaced by corresponding normal strain) (and shear stress by half of shear strain).

$$\tau_x \rightarrow \epsilon_x, \quad \sigma_y \rightarrow \epsilon_y, \quad \tau_{xy} \rightarrow \frac{\gamma_{xy}}{2}$$

$\epsilon_n$  and  $\gamma_s$  on a oblique plane -

$$\epsilon_n = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y] \cos 2\theta + \left(\frac{\gamma_{xy}}{2}\right) \sin 2\theta$$

$$\frac{\gamma_s}{2} = -\frac{1}{2}[\epsilon_x - \epsilon_y] \sin 2\theta + \left(\frac{\gamma_{xy}}{2}\right) \cos 2\theta$$

$$\gamma_s = -[\epsilon_x - \epsilon_y] \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$\epsilon_n + \epsilon_{n'} = \epsilon_x + \epsilon_y$$

$$\gamma_s = -\gamma_{s'}$$

principal plane:-

$$\gamma_s = 0 \Rightarrow \tan 2\theta = \frac{2(\gamma_{xy}/2)}{\epsilon_x - \epsilon_y}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

⊕ - tensile strain  
⊖ - compressive strain

\*\*\*

$$\epsilon_{1,2} = \frac{1}{2} \left[ (\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4(\gamma_{xy}/2)^2} \right]$$

$$\text{Absolute } \frac{\gamma_{\max}}{2} = \text{larger of } \left[ \left| \frac{\epsilon_1 - \epsilon_2}{2} \right|, \left| \frac{\epsilon_2}{2} \right|, \left| \frac{\epsilon_1}{2} \right| \right]$$

\*\*\*

$$\text{Absolute } \gamma_{\max} = \text{larger of } [|\epsilon_1 - \epsilon_2|, |\epsilon_2|, |\epsilon_1|]$$

$$\gamma_{\max} = |\epsilon_1 - \epsilon_2|$$

$\epsilon_{1,2}$  are in opposite in nature

$$\gamma_{\max} = |\epsilon_1|$$

$\epsilon_{1,2}$  are in same nature

\* For the analysis of stress and strain under uniaxial state of stress substitute either  $\left[ \begin{matrix} \sigma_x \text{ or } \sigma_y = 0 \\ \gamma_{xy} = 0 \end{matrix} \right]$  and in the equations of biaxial state of stress or strain

$$\left[ \begin{matrix} \epsilon_x \text{ or } \epsilon_y = 0 \\ \gamma_{xy} = 0 \end{matrix} \right]$$



$$\sigma_n = \frac{1}{2} [\sigma_x + 0] + \frac{1}{2} [\sigma_x - 0] \cos 2\theta + 0$$

$$\sigma_n = \frac{\sigma_x}{2} (1 + \cos 2\theta) = \sigma \cos^2 \theta \quad \text{For uniaxial}$$

$$\gamma_s = \frac{1}{2} [\sigma - 0] \sin 2\theta + 0$$

$$\gamma_s = \left| \frac{\sigma}{2} \sin 2\theta \right|$$



$$\left[ \begin{array}{l} (\sigma_n)_{\max} = \sigma_1 = \sigma ; \text{ when } \theta = 0^\circ \text{ and } 180^\circ \\ (\sigma_n)_{\min} = \sigma_3 = \sigma ; \text{ when } \theta = 90^\circ \text{ and } 270^\circ \\ \tau_{\max} = |\sigma/2| \Rightarrow \theta = 45^\circ \end{array} \right]$$

### MOHR'S CIRCLE :- (GRAPHICAL METHOD)

CASE-I → State of stress having same nature and different in magnitude ( $\sigma_x > \sigma_y$ )

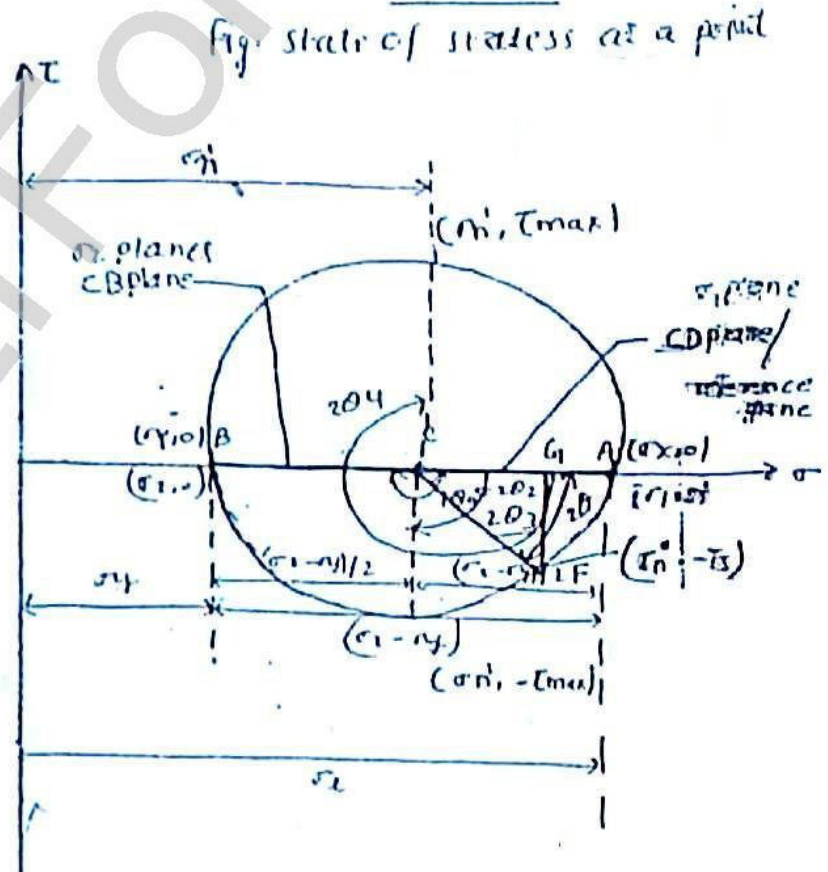
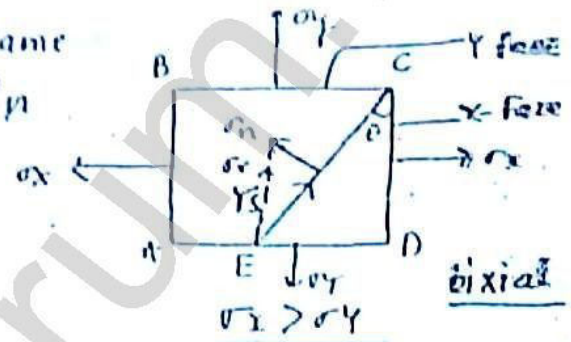
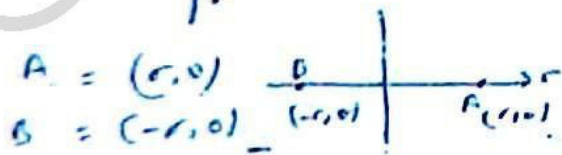
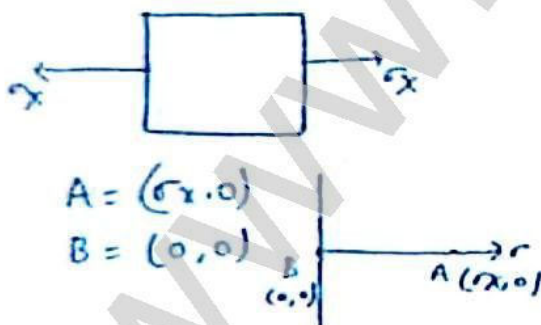
A = state of stress at x-face

B = state of stress at y-face

Step-I →

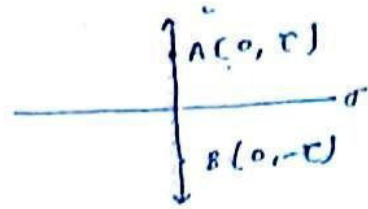
Mark two points A and B corresponding state of stress on x and y faces respectively.

Uniaxial →

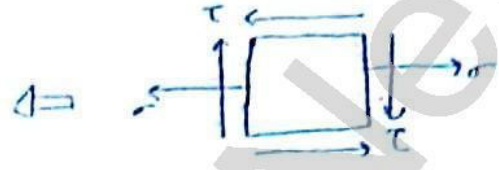
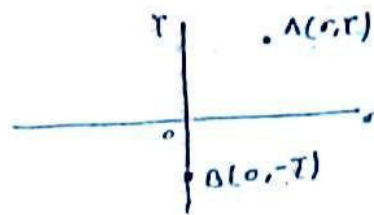


Mohr's Circle

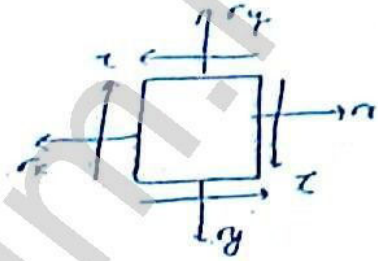
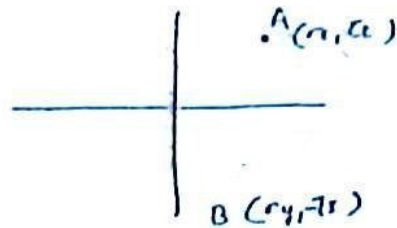
③  $A(0, \tau)$   
 $B(0, -\tau)$



④  $A(\sigma, \tau)$   
 $B(0, -\tau)$



⑤  $A(\sigma_x, \tau_{xy})$   
 $B(\sigma_y, -\tau_{xy})$



Step-II → After marking the points, join these points

Step-III → Take a bisection point (C) or centre and radius are,

$$\text{radius} = CA \text{ or } CB$$

$$= \frac{\sigma_x - \sigma_y}{2}$$

Then draw a circle.

CA = represent the CD plane.  
 CB = " " CB " " ]

\* Any circle in Mohr's circle represent the double angle to the actual angle.

Example →

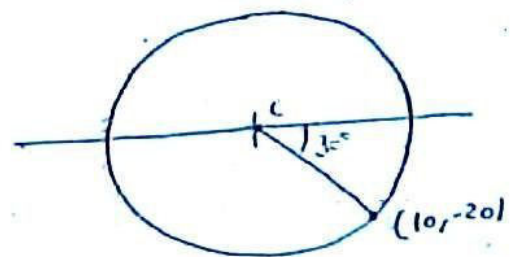
\* The coordinate point in any circle represent Normal stress, shear stress corresponding to plane.

\* All the principal planes lies on radius line on x-axis.

CA and CB are the principal planes.  
 (i) and (ii) planes.

$$\sigma_1 = \sigma_A = \sigma_x$$

$$\sigma_2 = \sigma_B = \sigma_y$$

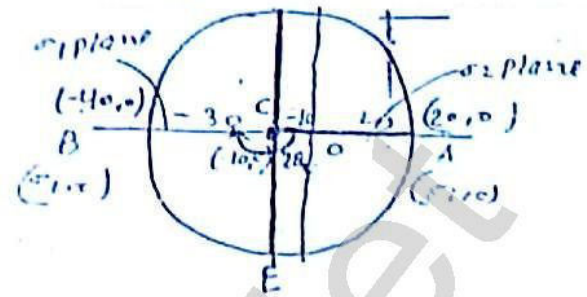




\* CA is  $\sigma_2$  (minor) plane because here stress magnitude is less.

\* CB is  $\sigma_1$  (Major) plane because stress at B is higher than stress at A magnitude wise.

\* Consider only magnitude when locating  $\sigma_1$  and  $\sigma_2$ .



Step III-

$$\left. \begin{aligned} 2\theta_1 &= 0^\circ, \theta_1 = 0^\circ \\ 2\theta_2 &= 180^\circ, \theta_2 = 90^\circ \end{aligned} \right\}$$

\* CDE =  $\tau_{max}$  plane  $\rightarrow$  radius parallel to y axis

$$\boxed{\begin{aligned} \sigma_n &< \sigma_x \\ \sigma_n &> \sigma_y \end{aligned}}$$

$$\begin{aligned} \sigma_n' &= OC \\ &= OB + BC \\ &= \sigma_y + \frac{\sigma_x - \sigma_y}{2} \end{aligned}$$

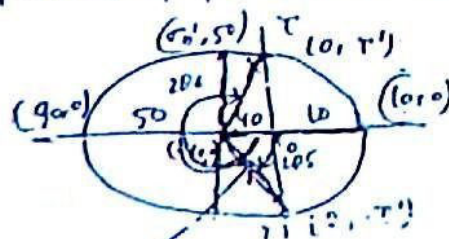
$$\boxed{\sigma_n' = \frac{\sigma_x + \sigma_y}{2}}$$

$$\begin{aligned} \text{Inplane } \tau_{max} &= \text{radius} \\ &= \frac{\sigma_x - \sigma_y}{2} \end{aligned}$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_x - \sigma_y}{2}}$$

$$\begin{aligned} 2\theta_3 &= 90^\circ \Rightarrow \theta_3 = 45^\circ \\ 2\theta_4 &= 270^\circ \Rightarrow \theta_4 = 135^\circ \end{aligned}$$

\* for this case planes of pure shear doesnot exist.



plane of pure shear

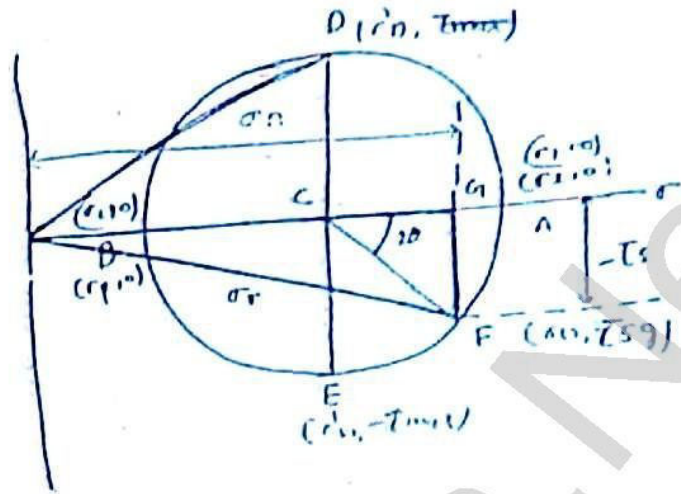
\* Angle:  $\tau' = \sigma_x = \sigma_y$

$$\left. \begin{aligned} \sigma_n &= OG \\ \tau_s &= FG \end{aligned} \right\}$$

OD - resultant stress in  $\tau_{max}$  plane

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_s^2}$$

$$\sigma_n + \sigma_{gn'} = \sigma_x + \sigma_y$$



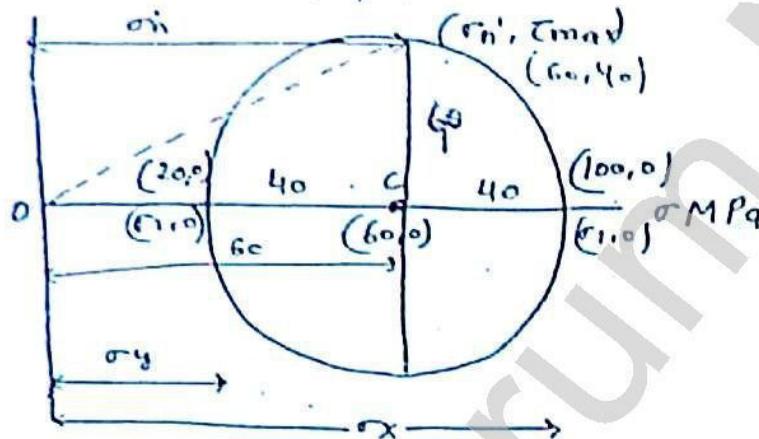
\* Following conclusion can be made →

1. Any plane passing through a point represented by radius of the Mohr's circle.
2. Any plane is represented by double its actual angle.
3. Co-ordinates of any points on the circle is representing normal stress or shear stress on the corresponding plane.
4. Principal planes are represented by radius (radius) line on x-axis.
5.  $\sigma_1$ -plane is represented by the radius line on x-axis and which has maximum or large x-co-ordinate.
6.  $\sigma_2$ -plane is also represented by the radius line on x-axis which is having smaller x-co-ordinate.
7.  $\tau_{max}$  planes are represented by radii parallel to y-axis.
8. In plane maximum shear stress is given by the radius of Mohr's circle.
9. Resultant stress on any plane is given by the line joining corresponding point on the circle with the origin.
10. Plane of pure shear is exist when circle intersects y-axis i.e. (when  $\sigma_x = \sigma_y$  are in opposite nature).



Q4: For the Mohr's circle as shown in Fig, determine the following.

- co-ordinates of centre of Mohr's circle
- maximum and minimum normal stress
- Normal stress on  $\tau_{max}$  plane.
- Resultant stress on  $\tau_{max}$  plane



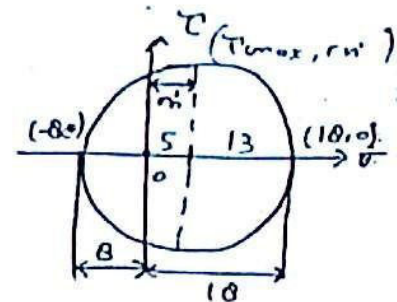
- co-ordinates of centre =  $(60, 0)$
- $\sigma_1 = 100 \text{ MPa}$        $\sigma_2 = 20 \text{ MPa}$   
 $\sigma_n = 60$        $\tau_{max} = 40$

$$\sigma_r = \sqrt{60^2 + 40^2}$$

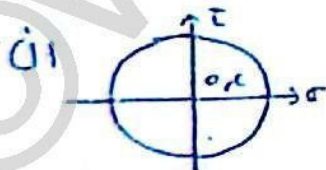
$$\sigma_r = 72.16 \text{ MPa}$$

W.B Page-29  
 Q4-19:-

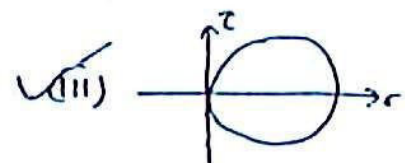
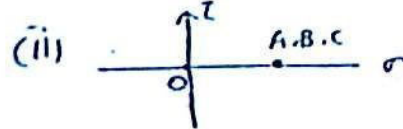
$$\left. \begin{array}{l} \sigma_1 = 18 \\ \sigma_2 = -8 \\ \tau_{max} = 13 \\ \sigma_n = 5 \end{array} \right\}$$

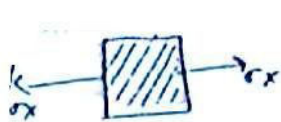


Select appropriate Mohr's circle when a point is subjected to uniaxial state of stress.



(i)





$$A = (\sigma_x, 0)$$

$$B = (0, 0)$$

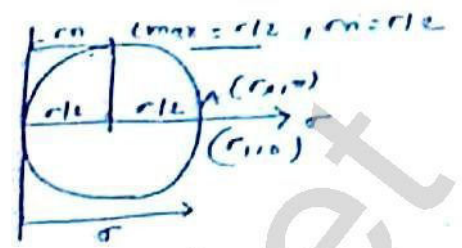
$$\sigma_1 = \sigma$$

$$\sigma_2 = 0$$

$$\sigma_{max} = \sigma/2$$

$$\sigma_{min} = \sigma/2$$

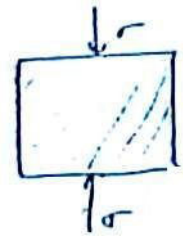
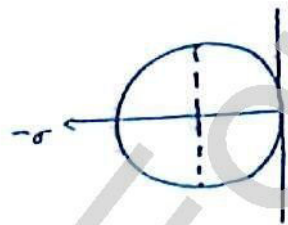
$$\sigma_r = \sqrt{\frac{\sigma^2}{4} + \frac{\sigma^2}{4}} = \frac{\sigma}{\sqrt{2}}$$



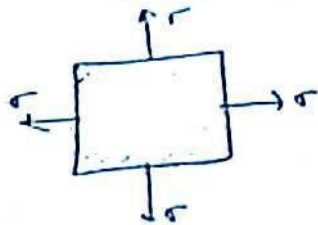
Ques: For uniaxial state of stress, Mohr's circle is a circle which is tangential to  $\sigma$  axis and whose radius is app. half of the applied stress.

$$A = (0, 0)$$

$$B = (\sigma, 0)$$



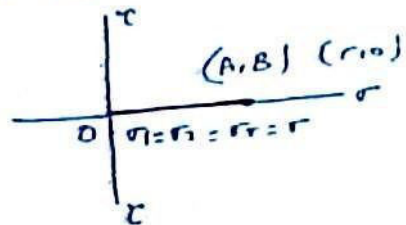
Ques: Select appropriate Mohr's circle when point is subjected to equal and same nature of normal stress in the mutual  $\perp$  direction.



$$A = (\sigma, 0)$$

$$B = (\sigma, 0)$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma$$



\* When point is subjected to equal and same nature of normal stress Mohr's circle becomes a point.

Practical Example  $\rightarrow$  when body immersed in fluid

In this case, max. normal stress, min normal stress and resultant stress on any plane is equal to applied stress.

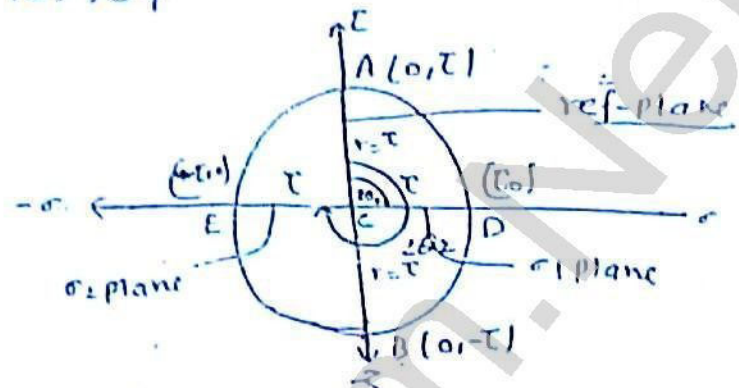
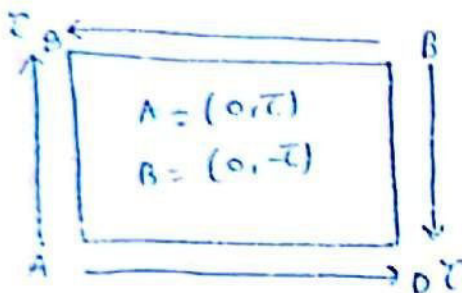
In this case, principal planes lie's in all directions.

For this case inplane max. shear stress is zero becoz circle is a point & absolute max. shear stress is  $(\sigma/2)$



Ques Draw the Mohr's circle when the point is under pure shear. Determine.

$\sigma_1, \sigma_2, \epsilon_1, \epsilon_2, \tau_{max}, \theta_1, \theta_2$



$$\left. \begin{array}{l} \sigma_1 = \tau \\ \sigma_2 = -\tau \end{array} \right\}$$

When point is under pure shear

$$2\theta_1 = 90^\circ \Rightarrow \boxed{\theta_1 = 45^\circ}$$

$$2\theta_2 = 270^\circ \Rightarrow \boxed{\theta_2 = 135^\circ}$$

OA and OB =  $\tau_{max}$  [planes of zero shear / pure shear] =  $\tau$

$$2\theta_3 = 0 \Rightarrow \boxed{\theta_3 = 0}$$

$$2\theta_4 = 180^\circ \Rightarrow \boxed{\theta_4 = 90^\circ}$$

### RELATIONSHIP BETWEEN PRINCIPAL STRAINS and PRINCIPAL STRESS

$$\sigma_{1,2} = \frac{1}{2} \left[ (\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right] \longrightarrow \textcircled{1}$$

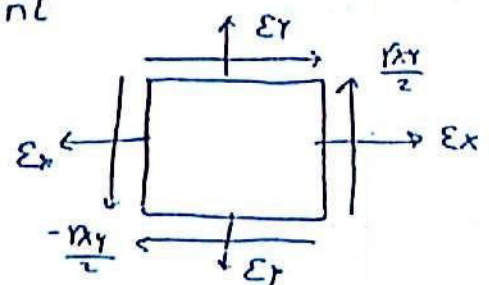
$$\epsilon_{1,2} = \frac{1}{2} \left[ (\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2} \right] \longrightarrow \textcircled{2}$$

$\sigma_x, \sigma_y$  = state of stress at a point

u:-  $\sigma_{1,2} = ?$

$\epsilon_{1,2}$  = calculated

$\sigma_{1,2}$  = calculated intervals of  $\epsilon_{1,2}$



Eq(1) is used to determine principal stresses when state of stress at a point is known.

\* Eq(2) is used to determine principal strain when state of strain at a point is known.

$$\left. \begin{aligned} \epsilon_1 &= \frac{1}{E} \left[ \sigma_1 - \mu (\sigma_2 + \sigma_3) \right] \\ \epsilon_2 &= \frac{1}{E} \left[ \sigma_2 - \mu (\sigma_1 + \sigma_3) \right] \\ \epsilon_3 &= \frac{1}{E} \left[ \sigma_3 - \mu (\sigma_1 + \sigma_2) \right] \end{aligned} \right\} \text{ (A)}$$

The above set of equation is used to determine principal stresses when state of stress at a point is known or further principal stress are known.

For 2D - Case  $\rightarrow$

$$\sigma_3 = 0$$

$$\epsilon_1 = \frac{1}{E} \left[ \sigma_1 - \mu \sigma_2 \right] \rightarrow \text{③}$$

$$\epsilon_2 = \frac{1}{E} \left[ \sigma_2 - \mu \sigma_1 \right] \rightarrow \text{④}$$

$$\epsilon_3 = \frac{1}{E} \left[ -\mu (\sigma_1 + \sigma_2) \right] \quad \text{lateral strain} \rightarrow \text{⑤}$$

\* [plane stress problems will not given plane strain.]

From eq (3)  $\rightarrow$

$$\sigma_1 = E \epsilon_1 + \mu \sigma_2 \rightarrow \text{⑥}$$

From eq (4)  $\rightarrow$

$$\sigma_2 = E \epsilon_2 + \mu \sigma_1 \rightarrow \text{⑦}$$

put  $\sigma_2$  value in eq (6)

$$\sigma_1 = E \epsilon_1 + \mu [E \epsilon_2 + \mu \sigma_1]$$

$$\sigma_1 = E \epsilon_1 + \mu E \epsilon_2 + \mu^2 \sigma_1$$



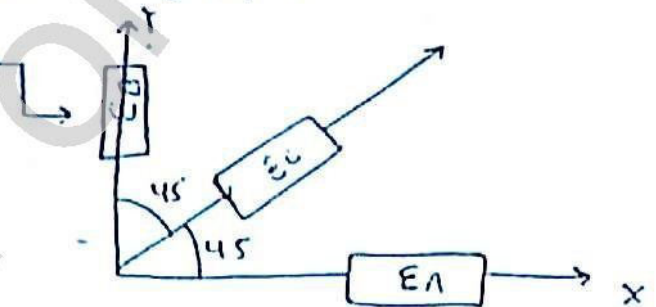
$$\left. \begin{aligned} \sigma_1 &= \frac{E}{1-\mu^2} [\epsilon_1 + \mu \epsilon_2] \\ \text{Similarly,} \quad \sigma_2 &= \frac{E}{1-\mu^2} [\epsilon_2 + \mu \epsilon_1] \end{aligned} \right\} \textcircled{B}$$

The above set of equation (B) are used to determine principal stresses when state of stress at a point is known or principal stress are known.

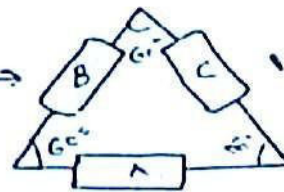
Ques. Page-28  $\epsilon_{90^\circ} = 400 \mu = \epsilon_x$ ,  $\epsilon_{45^\circ} = 375 \mu$ ,  $\epsilon_{90^\circ} = 200 \mu = \epsilon_y$   
 $\epsilon_1 = ?$

$$\epsilon_{1,2} = \frac{1}{2} [(\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4(\epsilon_{45^\circ})^2}]$$

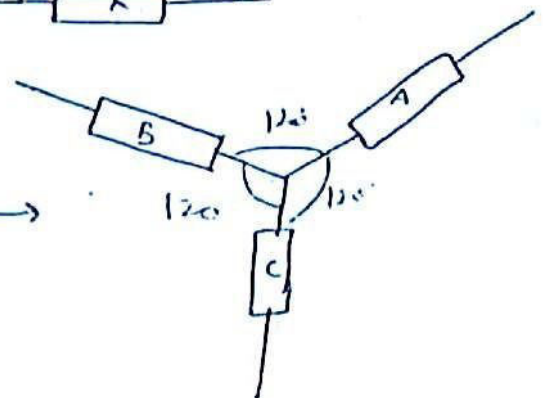
Rectangular strain gauge rosette



Delta strain gauge rosette



Star strain gauge rosette



Rosette is defined as the arrangement of three strain gauge which are used to measure the normal strain in three directions.

Formula:-

$$(\epsilon_n)_\theta = \frac{1}{2} [\epsilon_x + \epsilon_y] + \frac{1}{2} [\epsilon_x - \epsilon_y] \cos 2\theta + \frac{\gamma_{xy}}{2} (\sin 2\theta)$$

$$(\epsilon_n)_{\theta=0} = \frac{1}{2} [\epsilon_x + \epsilon_y] + \frac{1}{2} [\epsilon_x - \epsilon_y] \cos 0 + \frac{\gamma_{xy}}{2} \sin 0 = \epsilon_x$$

$$= \frac{1}{2} [\epsilon_x + \epsilon_y] + \frac{1}{2} [\epsilon_x - \epsilon_y] + 0 = \epsilon_x$$

$$(\epsilon_n)_{\theta=90^\circ} = \frac{1}{2} [\epsilon_x + \epsilon_y] + \frac{1}{2} [\epsilon_x - \epsilon_y] (-1) + 0 = \epsilon_y$$

$$(\epsilon_n)_{\theta=45^\circ} = \frac{1}{2} [\epsilon_x + \epsilon_y] + \frac{1}{2} [\epsilon_x - \epsilon_y] \cos 90^\circ + \frac{\gamma_{xy}}{2} \sin 90^\circ$$

$$375 \mu = \frac{1}{2} [400 \mu + 200 \mu] + \frac{\gamma_{xy}}{2} \times 1$$

$$750 \mu = 600 \mu + \gamma_{xy}$$

$$150 \mu = \gamma_{xy}$$

$$\text{or } \boxed{\frac{\gamma_{xy}}{2} = 75 \mu}$$

Then

$$\epsilon_{1,2} = \frac{1}{2} [400 \mu + 200 \mu \pm \sqrt{(400 \mu - 200 \mu)^2 + 4(75 \mu)^2}]$$

$$= \frac{1}{2} [600 \mu \pm \sqrt{40000 \mu^2}]$$

$$\epsilon_{1,2} = 425 \mu, 175 \mu$$

Ques:- Repeat the above question for principal stress if young's modulus  $E = 200$  and  $\mu = 0.25$

$$\sigma_1 = \frac{E}{1-\mu^2} [\epsilon_1 + \mu \epsilon_2] =$$

$$\sigma_2 = \frac{E}{1-\mu^2} [\epsilon_2 + \mu \epsilon_1] =$$

Ans:- A  $\epsilon_{60}$  strain rosette or delta rosette consist of 3 active and resistance strain gauge. obtain the expression for  $\epsilon_x, \epsilon_y$  and  $\gamma_{xy}$  if reading of strain gauges are given  $\epsilon_a, \epsilon_b, \epsilon_c$ .

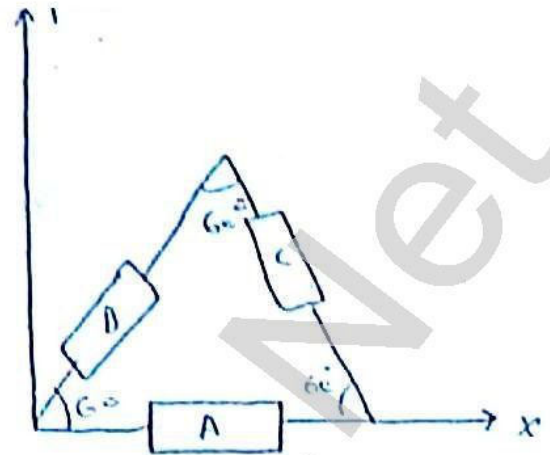


$$(\epsilon_n)_{\theta=0} = \epsilon_A$$

$$(\epsilon_n)_{\theta=60^\circ} = \epsilon_B$$

$$(\epsilon_n)_{120^\circ-\theta} = \epsilon_C$$

$$\epsilon_x, \epsilon_y, \gamma_{xy}$$



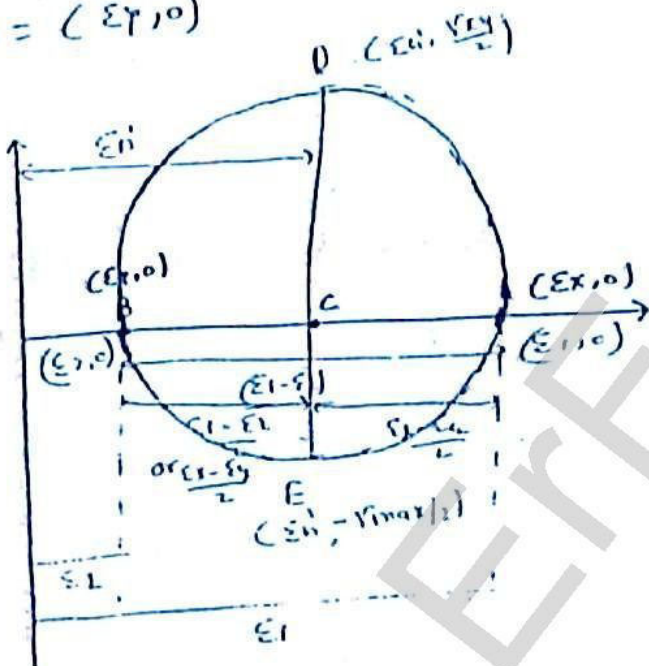
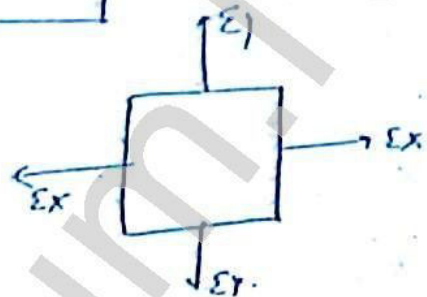
Ans:

- $\epsilon_x = \epsilon_A$
- $\epsilon_y = \frac{1}{3} [2\epsilon_B + 2\epsilon_C - \epsilon_A]$
- $\gamma_{xy} = \frac{2}{\sqrt{3}} [\epsilon_B - \epsilon_C]$

$$Y=30$$

$$\frac{v_{\max}}{2} = r \Rightarrow v_{\max} = 2r = \text{dia of circle}$$

$$B = (\varepsilon_T, 0)$$



$$\frac{r_{\max}}{2} = r = \frac{\epsilon_X - \epsilon_Y}{2} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\gamma_{\max} = 2r = \epsilon_X - \epsilon_Y = \epsilon_1 - \epsilon_2 \quad \text{Inplane}$$

$$T_{\max} = \left| \frac{\sigma_1}{2} \right| \text{ or } \left| \frac{\sigma_1 - \sigma_2}{2} \right|$$

$r_1, r_2$  are same nature

Ans.  $\frac{\gamma_{\max}}{2} = \left| \frac{\epsilon_1}{2} \right|$  or  $\left| \frac{\epsilon_1 - \epsilon_2}{2} \right|$

$\Delta \star$

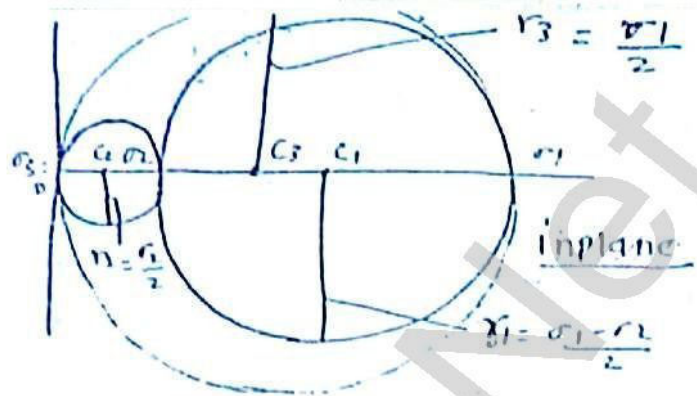
absolute max.

$y_{max} = |\epsilon_1|$  or  $|\epsilon_1 - \epsilon_2|$

same nature      opp nature

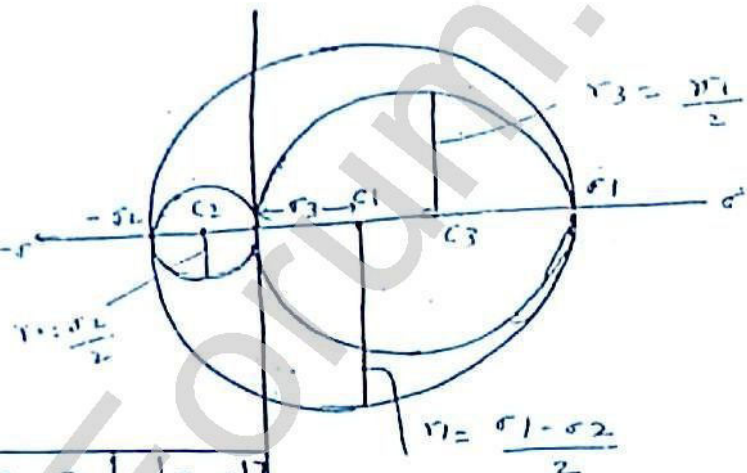


$$\text{Abs. } \tau_{\max} = \frac{\sigma_1}{2}$$

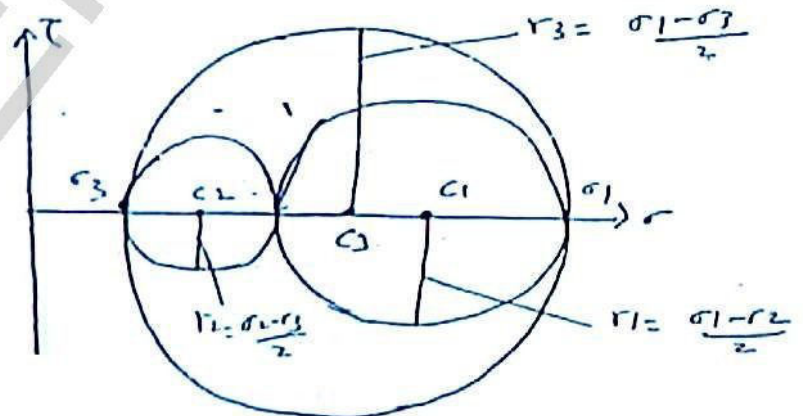


$$\text{Abs } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\text{inplane } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2}{2}, \frac{\sigma_1}{2}$$



$$\text{Abs. } \tau_{\max} = \text{larger of } \left| \frac{\sigma_1 - \sigma_2}{2} \right|, \left| \frac{\sigma_2 - \sigma_3}{2} \right|, \left| \frac{\sigma_3 - \sigma_1}{2} \right|$$



w.B page-26  
 $\frac{4-2}{2} \rightarrow \epsilon_1 = 1000 \times 10^{-6}, -600 \times 10^{-6} = \epsilon_2$

$$\begin{aligned} \tau_{\max} &= \epsilon_1 - \epsilon_2 \\ &= 1000 \times 10^{-6} + 600 \times 10^{-6} \\ &= 1600 \times 10^{-6} \text{ A.L.} \end{aligned}$$

when nothing gives find abs max strain.

Q4 2c)

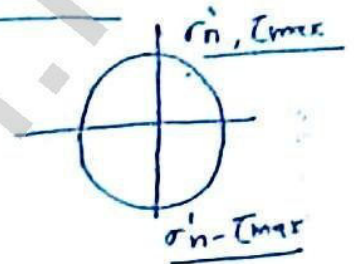
$$\sigma_1 = \sigma_x = 40 \text{ MPa}$$

$$\sigma_2 = \sigma_y = -20 \text{ MPa}$$

$$\begin{aligned} \text{in-plane-} \tau_{\max} &= \frac{\sigma_1 - \sigma_2}{2} \\ &= \frac{40 - (-20)}{2} \\ &= \underline{\underline{10 \text{ MPa}}} \end{aligned}$$

$$\begin{aligned} \text{Abs. } \tau_{\max} &= \frac{\sigma_1}{2} \\ &= \underline{\underline{20 \text{ MPa}}} \end{aligned}$$

$$(\pm 10, 20) \text{ MPa}$$





## Theories of failure

Theories of failure are used to determine the safe dimension of a component when it is subjected to combined stresses. [i.e. both normal and shear stresses due to load].

Safe design →

Max stress induced  $\leq$  permissible stress.

$$\sigma_1 \text{ or } \tau_{\max} \leq \frac{\text{Failure Stress}}{\text{F.O.S (N)}}$$

- \* Ductile material fails in shear.
- \*  $S_{yt}$ ,  $S_{ut}$  are the failure stresses under tensile loading for ductile and brittle material resp. while  $\sigma_1$  and  $\tau_{\max}$  are stresses under combined stresses loading conditions.
- \* So failure stresses are not known so we use the theories of failure.
- \* Theories of failures establishes relationship b/w combined stress and properties obtained from tension test ( $S_{yt}$  and  $S_{ut}$ ).

\* Theories of failure are used under combined stresses due to unavailability of failure stress.

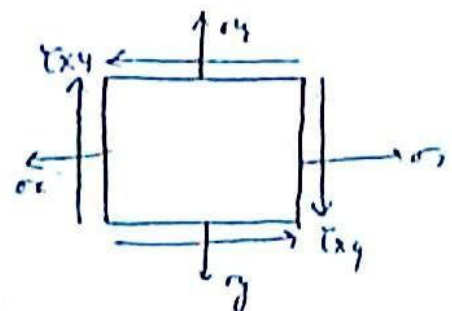
Maximum PRINCIPAL STRESS THEORY [RANKINE THEORY]  
or [M.P.S.T.]

MAXIMUM NORMALS THEORY

CONDITION FOR FAILURE →

$$\sigma_1 > S_{yt} \text{ or } S_{ut} \\ S_{yc} \text{ or } S_{uc}$$

for failure design F.O.S (N) = 1



Safe design →

$$\sigma_1 \leq \frac{S_{yt}}{N} \quad \text{or} \quad \frac{S_{ut}}{N}$$

FOS (N) > 1 (safe design)

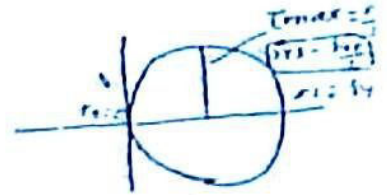
This theory is applicable for the design of components of brittle material because brittle materials are weak in normal stress (weak in tension) then shear.

This theory gives the safe design of component which are made of ductile material under biaxial state of stress when principal stress are in same nature.

MAX. SHEAR STRESS THEORY or [GUEST AND TRESCA'S] →

when max. shear stress theory under combined stresses exceeds, failure is likely occurs.

$$\text{Abs. } \tau_{\max} > S_{rs} \quad \text{or} \quad \frac{S_{yt}}{2}$$



Safe design →

$$\text{Abs. } \tau_{\max} \leq \frac{S_{rs}}{N} \quad \text{or} \quad \frac{S_{yt}}{2N}$$

Abs.  $\tau_{\max}$  - Larger of  $\left| \frac{\sigma_1 - \sigma_2}{2} \right|$ ,  $\left| \frac{\sigma_1 - \sigma_3}{2} \right|$ ,  $\left| \frac{\sigma_3 - \sigma_1}{2} \right|$

for 2D.

$$\text{Abs. } \tau_{\max} = \left| \frac{\sigma_1}{2} \right| \quad \text{or} \quad \left| \frac{\sigma_1 - \sigma_2}{2} \right|$$

for same nature -

$$\left| \frac{\sigma_1}{2} \right| \leq \frac{S_{yt}}{2N}$$

$$\sigma_1 \leq \frac{S_{yt}}{N}$$

for opp nature.

$$\left| \frac{\sigma_1 - \sigma_2}{2} \right| \leq \frac{S_{yt}}{2N}$$

$$\sigma_1 - \sigma_2 \leq \frac{S_{yt}}{N}$$



- \* MPST and MIST will give the same result under biaxial state of stress when principal stress in same nature.

## MAXIMUM PRINCIPAL STRAIN THEORY [MAX NORMAL STRAIN TH] [ST. VENANT'S STRAIN THEORY]

When maximum principal strain theory under combined stress exceeds, failure occurs.

$$\epsilon_1 > \epsilon_{y.p} \text{ or } \frac{S_{yt}}{E}$$

$$E = \frac{\sigma}{\epsilon} \Rightarrow \epsilon = \frac{\sigma}{E} \Rightarrow \epsilon_{yp} = \frac{S_{yt}}{E}$$

Safe design -

$$\epsilon_1 \leq \epsilon_{y.p} \text{ or } \frac{S_{yt}}{EN}$$

$$\frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)] \leq \frac{S_{yt}}{EN}$$

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \frac{S_{yt}}{N}$$

for 3D

$$\sigma_1 - \mu\sigma_2 \leq \frac{S_{yt}}{N}$$

for 2D

## TOTAL STRAIN ENERGY THEORY [HAIRY THEORY] →

When total strain energy per unit volume under combined stresses exceeds, total strain energy per unit volume at yield point, failure is likely to occur.

∴ condition for failure -

$$\text{Total SE/Vol} > [SE/Vol]_{y.p.}$$

Safe design -

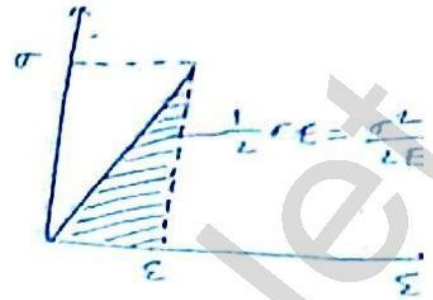
$$\text{Total SE/Vol} \leq [SE/Vol]_{y.p.}$$

$$\text{Total } S \cdot E / \text{Vol} = \frac{1}{2} \sigma_1 \epsilon_1 + \frac{1}{2} \sigma_2 \epsilon_2 + \frac{1}{2} \sigma_3 \epsilon_3$$

$$\epsilon_1 = \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)]$$

$$\epsilon_2 = \frac{1}{E} [\sigma_2 - \mu(\sigma_3 + \sigma_1)]$$

$$\epsilon_3 = \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)]$$



$$\text{Total } \frac{S \cdot E}{\text{Vol}} = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

\* To obtain  $(S \cdot E / \text{Vol})_{Y.P.}$  under tension test.

$$\sigma_1 = S_{Yt}, \sigma_2 = \sigma_3 = 0$$

$$\left( \frac{S \cdot E}{\text{Vol}} \right)_{Y.P.} = \frac{\sigma_1^2}{2E} = \frac{(S_{Yt})^2}{2E} \quad \text{for uniaxial}$$

Safe design → put value of total  $S \cdot E / \text{Vol} \rightarrow$

$$\frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \leq \frac{1}{2E} \left( \frac{S_{Yt}}{N} \right)^2$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \leq \left( \frac{S_{Yt}}{N} \right)^2 \quad \text{for 3D}$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \leq \left( \frac{S_{Yt}}{N} \right)^2 \quad \text{for 2D}$$

\* Graphical representation of strain energy theory is ellipses.

$$x^2 + y^2 - 2xy = a^2 \quad \text{equation of ellipses.}$$

### MAXIMUM DISTORSION ENERGY THEORY / VON-MISES THEORY

When maximum distortion energy / vol. exceeds under combined stresses, failure is likely occurs

Condition of failure →

$$M \cdot D \cdot E / \text{Vol} \geq (D \cdot E / \text{Vol})_{Y.P.}$$



Safe design  $\rightarrow$

$$M \cdot D \cdot E / Vol \leq (D \cdot E / Vol)_{Y.P}$$

$$\frac{\text{Total } S \cdot E}{Vol} = \frac{D \cdot E}{Vol} + \frac{Vol \cdot S \cdot E}{Vol}$$

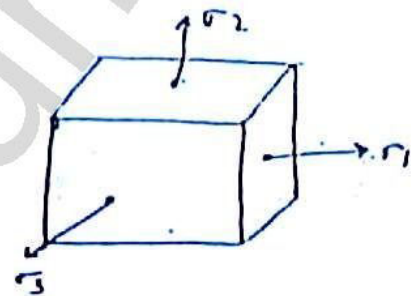
$$\frac{D \cdot E}{Vol} = \frac{\text{Total } S \cdot E}{Vol} - \frac{Vol \cdot S \cdot E}{Vol}$$

$$\frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$\frac{1}{3} [\sigma_1 + \sigma_2 + \sigma_3]^2 \left( \frac{1-2\mu}{2E} \right)$$

$$\frac{Vol \cdot S \cdot E}{Vol} = \frac{1}{2} (\text{Avg. stress}) (E \nu)$$

$$\frac{Vol \cdot S \cdot E}{Vol} = \frac{1}{2} \left[ \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right] [E \epsilon_1 + E \epsilon_2 + E \epsilon_3]$$



then,

$$M \cdot D \cdot E / Vol = \left( \frac{1+\mu}{6E} \right)^2 [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

To get  $\rightarrow (D \cdot E / Vol)_{Y.P} \rightarrow \sigma_1 = S_{yt}, \sigma_2 = \sigma_3 = 0$

$$\left( \frac{D \cdot E}{Vol} \right)_{Y.P} = \frac{2\sigma_1^2}{3E} \left( \frac{1+\mu}{6E} \right)^2$$

$$= 2 \left( \frac{S_{yt}}{N} \right)^2 \times \left( \frac{1+\mu}{6E} \right)^2$$

$$\left[ \frac{1+\mu}{6E} \right]^2 \cdot \dots = 2 \left( \frac{S_{yt}}{N} \right)^2 \left( \frac{1+\mu}{3E} \right)^2$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \leq 2 \left( \frac{S_{yt}}{N} \right)^2 \quad \text{for 3D}$$

For 2D,  $\sigma_3 = 0$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \leq \left( \frac{S_{yt}}{N} \right)^2 \quad \text{ellipses equation}$$

this theory is represent the ellipses equation.

M.P.S.T  $\rightarrow$  ductile under biaxial when  $\sigma_1, \sigma_2$  are in same nature and brittle material.

M.S.S.T  $\rightarrow$  ductile (oversafe design and uneconomic)

\* M.D.E.T  $\rightarrow$  ductile (best theory) because for a safe design and economic design compared then M.S.S.T.

$$MSST \rightarrow S_{ys} = \frac{S_{yt}}{2} = 0.5 S_{yt} = 50\% \text{ of } S_{yt}$$

$$MDET \rightarrow S_{ys} = \frac{S_{yt}}{\sqrt{3}} = 0.577 S_{yt} = 57.7\% \text{ of } S_{yt}$$

$$\sigma_{ind} \leq \frac{\text{Failure stress}}{N}$$

Propose  $\downarrow$

$$\frac{x}{d^2}, \frac{y}{d^3} \leq \frac{\text{failure stress}}{N}$$

$\text{dimension} \propto \frac{1}{\text{failure stress}}$

$$(\text{dim}^n)_{MDET} < (\text{dim}^n)_{MSST}$$

$\therefore (S_{ys})_{MDET} > (S_{yt})_{MSST}$

$$MPST \rightarrow S_{ys} = S_{yt} = 1 S_{yt}$$

$\downarrow$   
safe and economical design

values obtain for dimensions



# SUMMARY OF T.O.F.

T.O.F. COND. FOR FAILURE

DESIGN SUMMARY

Me & Te Eq.

$$\tau_e = \frac{S_{yt}}{N} \text{ EQNS. (either } \tau_x \text{ or } \tau_y \text{ or } \tau_{xy})$$

GRAPHICAL REPRESENT.

1. MIPST (Rankine's Theory)

$$\sigma_1 \leq \frac{S_{yt}}{N} \text{ or } \frac{S_{ut}}{N} \rightarrow \tau_1 \leq \frac{S_{yt}}{N} \text{ or } \frac{S_{ut}}{N}$$

2. MIPST (GUEST-TRESCA'S)

$$\tau_{max} \leq \frac{S_{yt}}{2N} \text{ or } \frac{S_{ut}}{2N} \rightarrow \tau_{max} = \text{larger of } \left[ \frac{\sigma_1 - \sigma_3}{2}, \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2} \right] \rightarrow \tau_1, \tau_2 \leq \frac{S_{yt}}{2N}$$

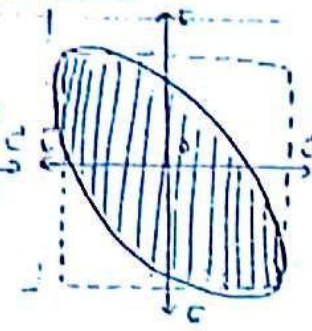
3. MIDET or MSSET or VON-MISES

$$\frac{M \cdot D \cdot E}{Vol.} = \left[ \frac{D \cdot E}{Vol.} \right] \cdot r.p.$$

$$(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 \leq 2 \left( \frac{S_{yt}}{N} \right)^2$$

$$M_e = \sqrt{\frac{M^2 + \frac{3}{4} T^2}{\left( \frac{S_{yt}}{N} \right)^2}} \rightarrow \tau_1 = \sigma_1 = \frac{32 M}{\pi d^3}, \tau_2 = \sigma_2 = \frac{16 T}{\pi d^3}$$

$$\sigma_e = \sqrt{\sigma_x^2 + 2 \tau_{xy}^2}$$

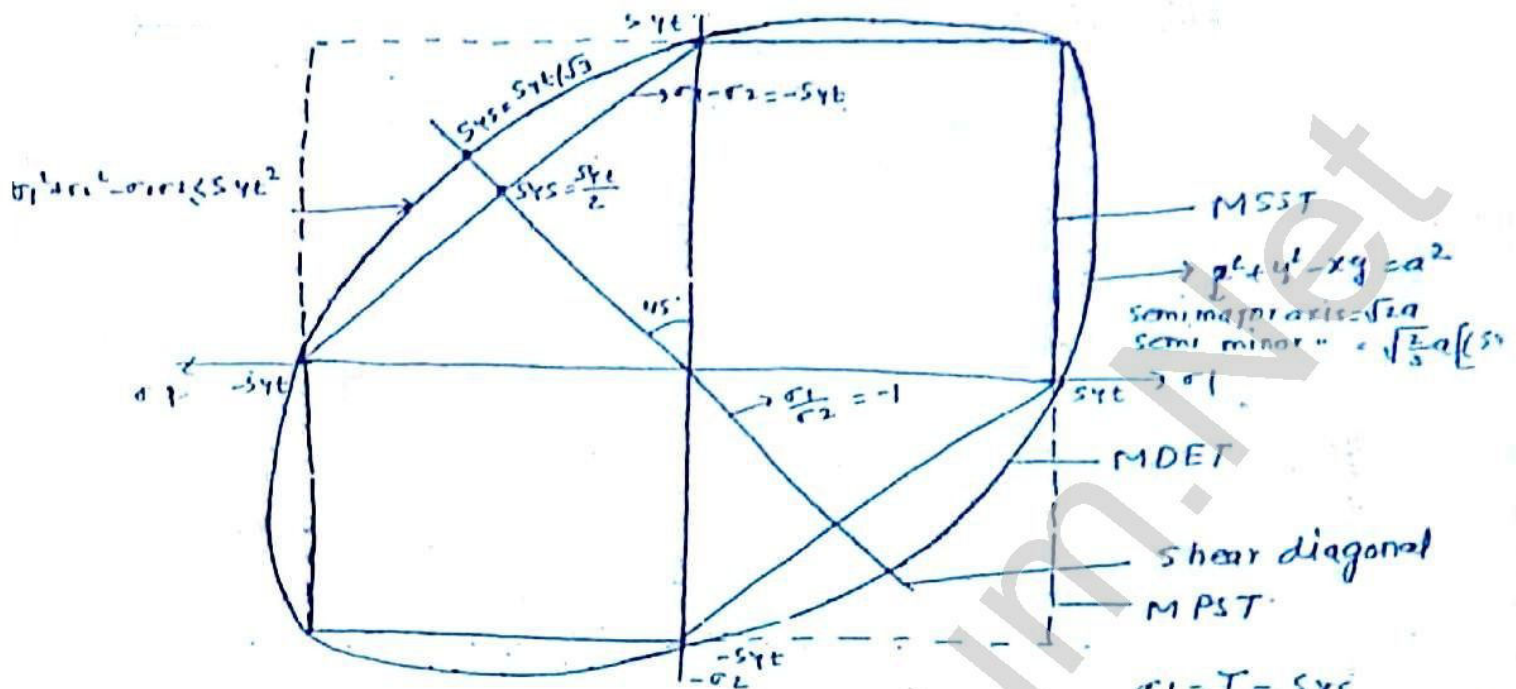


Note-1. Me and Te Equations should be used when component is subjected to both Bending moment and twisting moment simultaneously. OR either B.M and T.M also.

2. (per tensile stress)

(T) Equation should be used, when the component is subjected to normal stress in 1 dir's (i.e either  $\sigma_1$  or  $\sigma_2$  should be zero) and shear stress only.

3. Design Eq. can be used for any loading equations. (No constraint) but it's better to use the eq. if principal stress known directly.



$SYT \rightarrow$  tension test

$SYS \rightarrow$  tension test  $\Rightarrow \frac{\sigma_1}{\sigma_2} = -1$

Area bounded by  $\uparrow \Rightarrow SYS \uparrow$   
 the curve  $\Rightarrow dim^{ns} \downarrow$

$MPST \Rightarrow SYT = SYS$

$MPST \Rightarrow \frac{SYS}{SYT} = 1$

$MSST \Rightarrow \frac{SYS}{SYT} = \frac{1}{2}$

$MDET \Rightarrow \frac{SYS}{SYT} = \frac{1}{\sqrt{3}}$

TOF	$SYS/SYT$	
MPST	1	(smaller dim, unsafe) (uneconomic)
MSST	1/2	(safe, uneconomic)
MPST	$1/(1+\mu) = 0.77$	( $\mu = 0.33$ ) (unsafe)
MPST	$1/\sqrt{2(1+\mu)} = 0.62$	
MDET	$1/\sqrt{3} = 0.577$	(safe) economic



$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \leq (S_{yt})^2$$

$$\downarrow$$

$$\text{semi major axis} = \frac{S_{yt}}{\sqrt{1-\mu}}$$

$$\text{semi minor axis} = \frac{S_{yt}}{\sqrt{1+\mu}}$$

Example →  
MDET →

① Design Eq<sup>ns</sup>

$$\sigma_x = \sigma_b = \frac{32M}{\pi d^3} \rightarrow \textcircled{1}$$

$$\sigma_y = 0 \rightarrow \textcircled{3}$$

$$\tau_{xy} = \tau_s = \frac{16T}{\pi d^3} \rightarrow \textcircled{2}$$

$$\sigma_{1,2} = \frac{1}{2} \left[ \sigma_x \pm \sqrt{\sigma_x^2 + 4\tau_{xy}^2} \right]$$

$$\sigma_{1,2} = \frac{1}{2} \left[ \frac{32M}{\pi d^3} \pm \sqrt{\left( \frac{32M}{\pi d^3} \right)^2 + 4 \left( \frac{16T}{\pi d^3} \right)^2} \right]$$

$$\sigma_{1,2} = \frac{1}{2} \left[ \frac{32M}{\pi d^3} \pm \frac{32}{\pi d^3} \sqrt{M^2 + T^2} \right]$$

$$= \frac{1}{2} \times \frac{32}{\pi d^3} \left[ M \pm \sqrt{M^2 + T^2} \right]$$

$$\sigma_{1,2} = \frac{16}{\pi d^3} \left[ M \pm \sqrt{M^2 + T^2} \right]$$

Designs MDET  $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \leq \left( \frac{S_{yt}}{N} \right)^2$

Calculate (unknown)  $d \geq \text{--- mm}$

② Method  $M_e$  and  $T_e$  eq<sup>ns</sup> →

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2} = \frac{\pi d^3}{32} \sigma_b \sqrt{\left( \frac{S_{yt}}{N} \right)^2}$$

$d = \text{calculated}$

$$\left[ \begin{aligned} M_e &= r_b z \\ &= r_b \times \frac{\pi}{32} d^3 \end{aligned} \right]$$

③ Method

eq<sup>ns</sup>

$$\sigma_1 = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_x = \sigma_b = \frac{32M}{\pi d^3}$$

$$\tau_{xy} = \frac{16T}{\pi d^3}$$

Ex- Solve by only Landolt method.

Page-37

10.  $\sigma_1 = 360 \text{ MPa}, \sigma_2 = 140 \text{ MPa}$

M.O.E.T,

Maximum working stress or permissible tensile stress ( $\sigma_t$ ) →

$$\sigma_t = \frac{1}{2} \left[ \sqrt{\sigma_1^2 + 4\tau_{xy}^2} \right]$$

$$\sigma_t = \frac{1}{2} \left[ \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \right] \leq \left( \frac{S_{yt}}{N} \right)^2$$

$$\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \leq \sigma_{\text{working}} \text{ or } \sigma_{\text{per}}$$

$$\sqrt{360^2 + 140^2 - 360 \times 140}$$

W.B 37  
Q415

$$\sigma_b = 80 \text{ MPa}, \tau_s = 30 \text{ MPa}$$

$$S_{yt} = 280 \text{ MPa}, N = ?$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_b^2 + 4\tau_s^2}$$

$$\frac{280}{N} = \sqrt{80^2 + 4 \times 30^2}$$

$$\boxed{N = 2.8}$$

W.B  
Q16

$$M_b = 400 \text{ kN-m}, T = 300 \text{ kN-m}$$

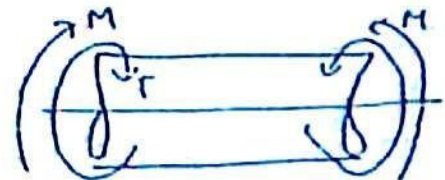
MPST →  $\sigma$

MSST →  $\tau$

$$\frac{\sigma}{\tau} = ? \text{ or } \frac{M_e}{T_e} = \frac{\frac{1}{2} [M + \sqrt{M^2 + T^2}]}{\sqrt{M^2 + T^2}} = \frac{\frac{\pi}{32} d^3 \sigma_b}{\frac{\pi}{16} d^3 \tau_s} = \frac{\sigma_b}{2\tau_s}$$

$$\frac{\sigma}{\tau} = \frac{1}{2} \frac{400 + \sqrt{400^2 + 300^2}}{\sqrt{400^2 + 300^2}}$$

$$\boxed{\frac{\sigma}{\tau} =}$$





Q412

$$MSS T \Rightarrow T_{per} = T$$

$$MPST \Rightarrow T_{per} = T_1 = ?$$

$$\frac{M_c}{T_c} = \frac{\frac{1}{2} [M + \sqrt{M^2 + T_1^2}]}{[\sqrt{M^2 + T_1^2}]} = \frac{\frac{\pi}{32} d^2 \sigma_b}{\frac{\pi}{16} d^3 \tau_s}$$

When  $M=0$  then

$$\frac{M_c}{T_c} = \frac{\frac{1}{2} [0 + \sqrt{0 + T_1^2}]}{\sqrt{0 + T_1^2}} = \frac{\sigma_b}{2\tau_s} \Rightarrow \frac{S_{yt}/H}{\frac{2 S_{ys}}{N}} = 1 \quad (S_{ys} = \frac{S_{yt}}{2})$$

$$\frac{T_1}{2T} = 1$$

$$T_1 = 2T$$

W.B.  
Q414 36

$$MPST \rightarrow M_{per} = M$$

$$MSS T \rightarrow M_{per} = M_1$$

$$\frac{M_1}{M} = 1 \Rightarrow M_1 = M$$

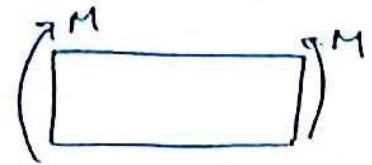
$$\frac{M_c}{T_c} = \frac{\frac{1}{2} [M_1 + \sqrt{M_1^2 + 0}]}{\sqrt{M_1^2}} = \frac{\frac{\pi}{32} d^3 \sigma_b}{\frac{\pi}{16} d^3 \tau_s} = \frac{\sigma_b}{2\tau_s} = \frac{S_{yt}/H}{\frac{2 S_{yt}/H}{N}} = 1$$

$$\Rightarrow \frac{M_1}{M} = 1$$

$$M_1 = M \quad \underline{Ans}$$

W.B. 36  
Q413

A-2, B-4, C-3, D-1

W.B. 40  
Q427

## Pressure Vessels

In addition to strength, there should be no leakage and in pressure vessels and they should withstand the pressure force.

pressure vessels are used to store fluid at various pressures.

$$\boxed{\frac{D}{t} = 20}$$

$$\frac{D}{t} \gg 20$$

Thin pressure vessels → gun barrels & hydraulic cylinder.

$$\frac{D}{t} < 20$$

Thick pressure vessels

where D — inner diameter

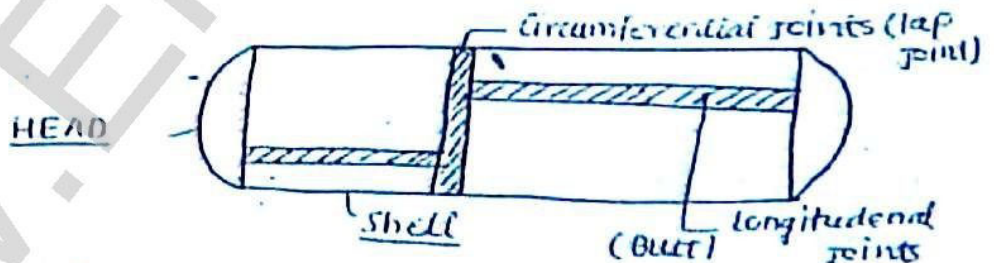
t — thickness of cylinder.

TYPES →

- (i) cylindrical Pressure Vessels
- (ii) spherical Pressure vessels.

Cylindrical P.V → (Thin) —

Boiler shells, pipes, tubes, storage tanks, pressure cookers, automobile tubes.



Now, they are made by joined the sheet by welding (lap and butt joint).

Circumferential joints is called lap joint is used required length of shell.

Longitudinal joints is called butt joint is used required dia. of shell.

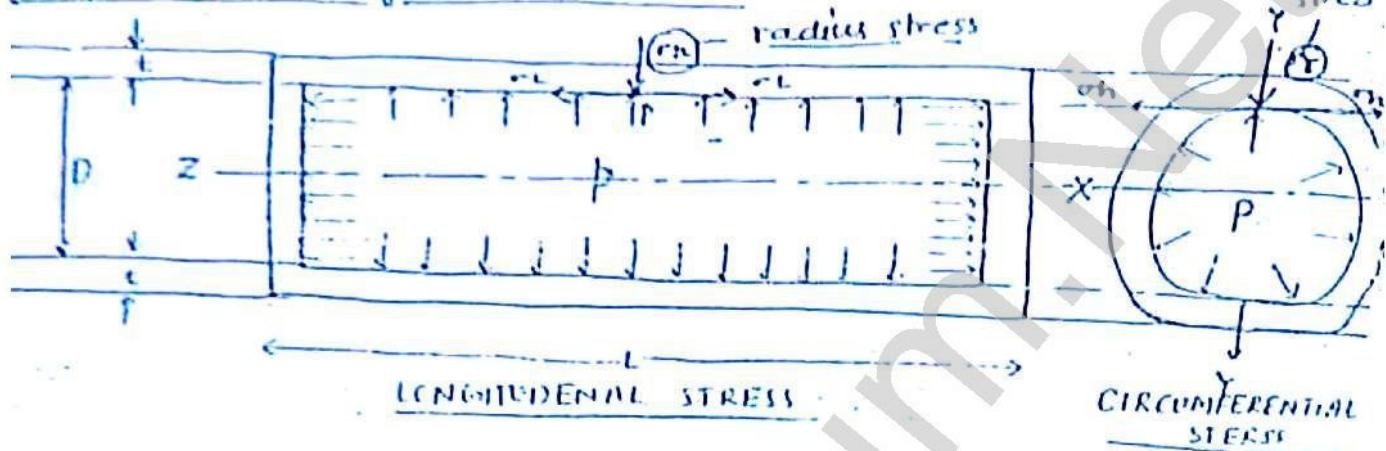
Butt joint is stronger to the lap joint

Due to failure of longitudinal joint, we are getting hoop stress.

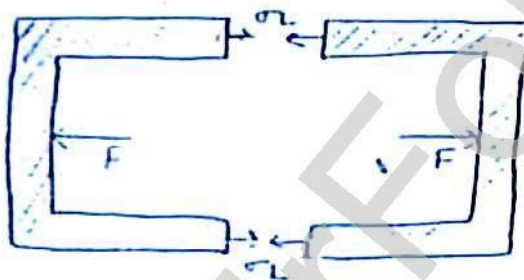


Due to failure of circumferential joint, longitudinal stress will produced.

Hoop stress > Longitudinal stress

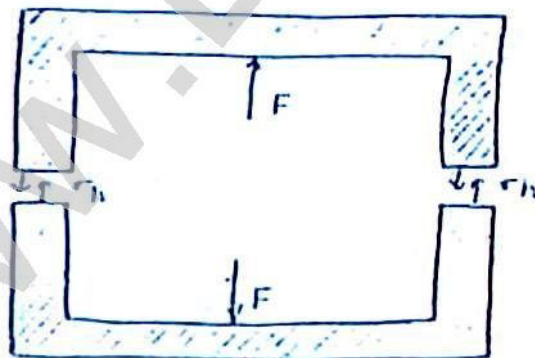


\* Pressure vessels always fails in circumferentially.

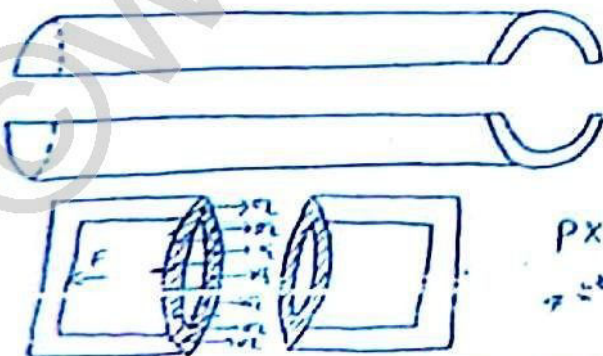


failure along circumferentially

\* When pressure vessels fail, longitudinal stress will produced.



Failure along the longitudinally



$$p \times \frac{\pi D^2}{4} = \sigma_L \times \pi D t$$

$$\sigma_L = \frac{p D}{4 t}$$

$$\frac{\pi}{4} (D_o^2 - D_i^2)$$

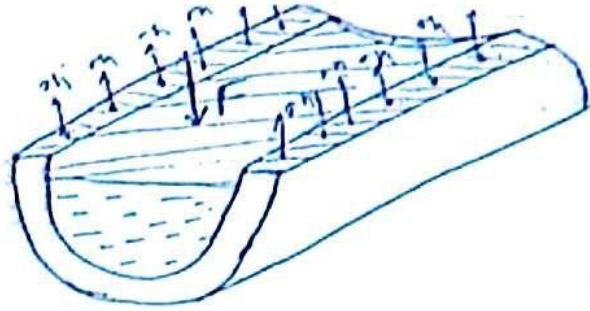
but in vessels

$$\frac{\pi}{4} ((D_o + t) - D_i)^2$$

$$\frac{\pi D t}{4}$$

$$P \times L \times D = \sigma_h \times 2 \times L \times t$$

$$\sigma_h = \frac{PD}{2t}$$



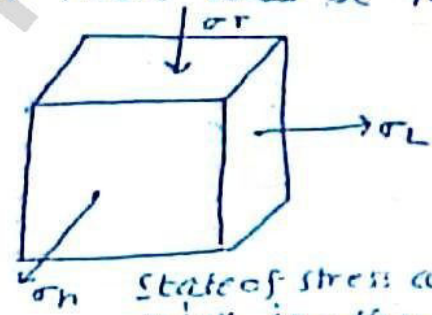
When vessels are failed circumferentially, longitudinal stresses produced.

When vessels are failed longitudinally, circumferential stress or hoop stresses produced.

Circumferential stress and longitudinal stress are always tangential (tensile) but radial stress is normal compressive stress.

At any point on the circumference of PV there will be three stress will developed  $\sigma_L, \sigma_h, \sigma_r$ .

There is no shear stress. These stresses ( $\sigma_L, \sigma_h, \sigma_r$ ) will itself become the principal stresses.

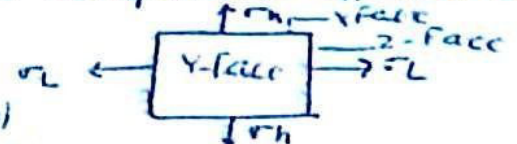
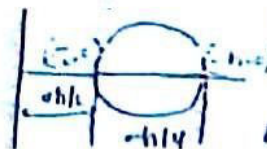


State of stress at a point in a thick cylinder.

$$\begin{cases} \sigma_1 = \sigma_h = PD/2t \\ \sigma_2 = \sigma_L = PD/4t = \sigma_h/2 = \sigma_1/2 \\ \sigma_3 = \sigma_r = -p \end{cases}$$

In a plane stress problem, 3rd stress becomes neglected as compared to other two. So ( $\sigma_r$ ) will be neglected as compared to  $\sigma_h$  and  $\sigma_L$  and it is condition for 2D.

$$\sigma_r \ll \sigma_h \text{ and } \sigma_L$$



Mohr's circle

In plane max.  $\tau_{max} = \frac{Pd}{4t} = \frac{\sigma_1 - \sigma_2}{2}$

Abs.  $\tau_{max} = \frac{Pd}{4t} = \frac{\sigma_1}{2}$



## PRINCIPAL STRAIN →

Strain produced in cylindrical vessels are always normal strain.

$$\boxed{\epsilon_h = \frac{\delta D}{D}} \quad , \quad \boxed{\epsilon_L = \frac{\delta L}{L}} \quad , \quad \epsilon_v = \frac{\delta V}{V} = 2\left(\frac{\delta D}{D}\right) + \frac{\delta L}{L}$$

$$\epsilon_1 = \epsilon_h \quad , \quad \epsilon_2 = \epsilon_L$$

$$\boxed{\epsilon_v = 2\epsilon_h + \epsilon_L} \quad \text{for objective}$$

$$\epsilon_1 = \epsilon_h = \frac{\delta D}{D} = \frac{1}{E} [\sigma_1 - \mu \sigma_2]$$

$$\epsilon_2 = \epsilon_L = \frac{\delta L}{L} = \frac{1}{E} [\sigma_2 - \mu \sigma_1]$$

$$\boxed{\epsilon_h = \frac{\delta D}{D} = \frac{\sigma_1}{2E} [2 - \mu]}$$

OR

$$\boxed{\epsilon_L = \frac{\delta L}{L} = \frac{\sigma_1}{2E} [1 - 2\mu]} \quad \text{OR}$$

$$\boxed{\epsilon_h = \frac{\delta D}{D} = \frac{pD}{4tE} [2 - \mu]}$$

OR

$$\boxed{\epsilon_L = \frac{\delta L}{L} = \frac{pD}{4tE} [1 - 2\mu]} \quad \text{OR}$$

$$\boxed{\delta D = \frac{pD^2}{4tE} [2 - \mu]}$$

$$\boxed{\delta L = \frac{pDL}{4tE} [1 - 2\mu]}$$

$$\epsilon_v = 2\left(\frac{\delta D}{D}\right) + \left(\frac{\delta L}{L}\right) \Rightarrow \epsilon_v = \frac{\delta V}{V} = 2\left[\frac{pD}{4tE} [2 - \mu] + \frac{pD}{4tE} [1 - 2\mu]\right]$$

$$\frac{\delta V}{V} = \frac{pD}{4tE} [4 - 2\mu + 1 - 2\mu]$$

$$\boxed{\frac{\delta V}{V} = \frac{pD}{4tE} [5 - 4\mu]} \quad \text{change in vol. of shell} \\ \text{[Fluid is Incomp.]}$$

Additional Vol. of fluid i.e. can be pumped inside the shell is equal to = change in vol. of shell + change in vol. of fluid

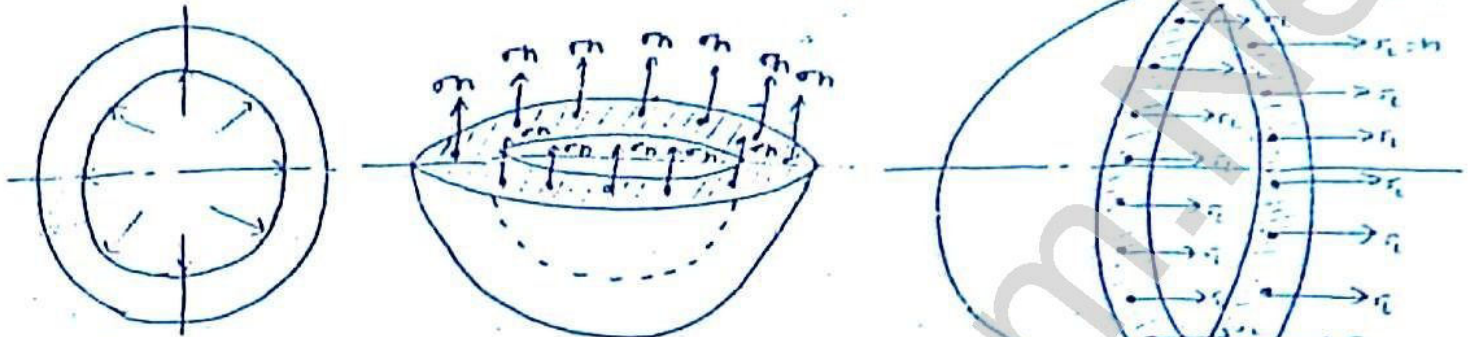
$$\text{Add. vol. of fluid pumped inside} = \frac{pD}{4tE} V [5 - 4\mu] + \frac{pV}{K} \quad \text{for compressible fluid}$$

Fluid is incompressible [K = ∞]

$$\boxed{\text{Add. vol. of fluid pumped inside} = \text{change in vol. of fluid} \left[ \frac{pD}{4tE} V [5 - 4\mu] \right]}$$

## ② Thin Spherical P.V.:

\* There is only one type of dimension will be considered so only only one type of stress (circumferential stress / hoop stress) will produced.



$$p \times \frac{\pi}{4} D^2 = \sigma_h \times \pi D t$$

$$\sigma_h = \frac{pD}{4t}$$

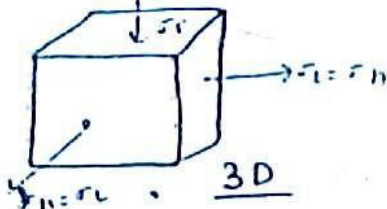
$$p \times \frac{\pi}{4} D^2 = \sigma_L \times \pi D t$$

$$\sigma_L = \frac{pD}{4t}$$

$$\sigma_h = \sigma_L = \frac{pD}{4t}$$

$$\sigma_r = -p$$

$\sigma_r$  can be neglected

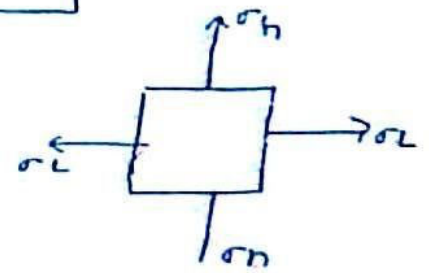


$$\sigma_1 = \sigma_2 = \sigma_h = \frac{pD}{4t} = \sigma_L$$

$$\sigma_3 = \sigma_r = -p$$

For 2D →

$$\sigma_1 = \sigma_2 = \sigma_h = \frac{pD}{4t}$$



$$\text{Abs. } \tau_{\max} = \frac{\sigma_1}{2} = \frac{pD}{8t}$$

$$\tau_{\max} \text{ in plane } = 0$$

Mohr's Circle:

$$\epsilon_h = \epsilon_L = \frac{\delta D}{D} = \frac{1}{E} [\sigma_1 - \mu \sigma_2]$$

$$\epsilon_h = \frac{\delta D}{D} = \frac{\sigma_1}{E} [1 - \mu]$$

$$\epsilon_h = \frac{\delta D}{D} = \frac{pD}{4tE} [1 - \mu]$$

$$\Delta B.C. (\sigma_h, \sigma) = \sigma$$

$$\epsilon_V = 3 \left( \frac{\delta D}{D} \right) = \frac{3pD}{4tE} (1 - \mu)$$

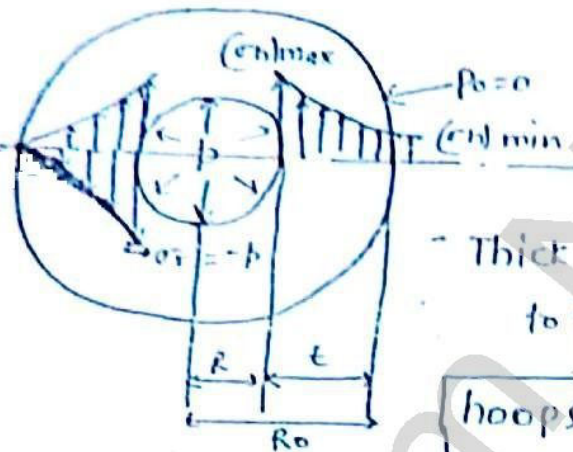
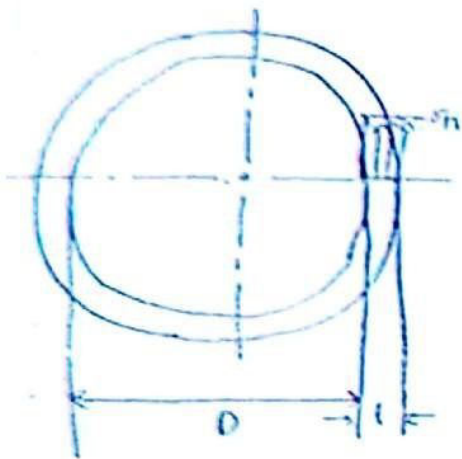
ASSUMPTIONS → (i) hoop stress is uniformly distributed  
(ii) radial stress will be neglected

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# THICK CYLINDERS

$$[D/t < 20]$$



Thick cylinder subjected to Internal pr. only.

hoop stress is tensile

Lami's Equation →

$$\left. \begin{aligned} p_r &= -a + \frac{b}{r^2} \\ p_h &= a + \frac{b}{r^2} \end{aligned} \right\}$$

a and b are Lami's constants

Boundary Conditions →

$$r = R_i \Rightarrow p_i = p$$

$$r = R_o \Rightarrow p_o = 0$$

$$p = -a + \frac{b}{R_i^2} \rightarrow (1)$$

$$0 = -a + \frac{b}{R_o^2} \rightarrow (2)$$

$$p = b \left[ \frac{1}{R_i^2} - \frac{1}{R_o^2} \right]$$

$$b = \frac{p R_i^2 R_o^2}{R_o^2 - R_i^2}$$

$$a = \frac{b}{R_o^2} = \frac{p R_i^2}{R_o^2 - R_i^2}$$

$$r = R_i \Rightarrow \sigma_h = a + \frac{b}{R_i^2}$$

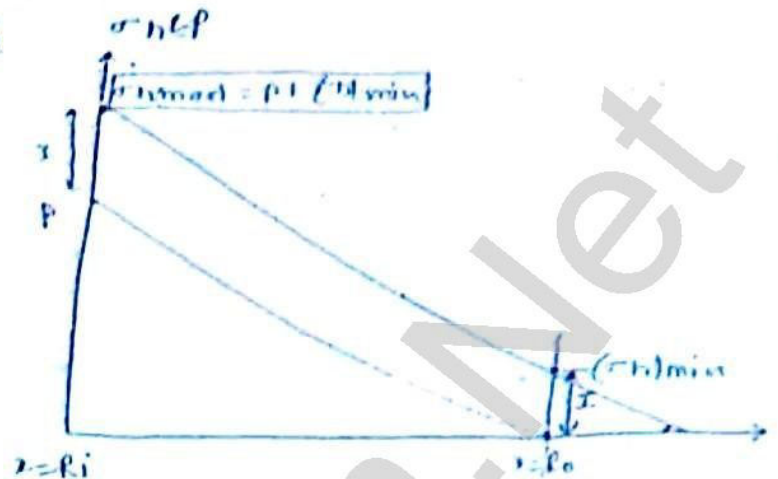
$$\sigma_h = \frac{p R_i^2}{R_o^2 - R_i^2} + \frac{p R_i^2 R_o^2}{(R_o^2 - R_i^2) R_i^2}$$

$$\left[ (\sigma_h)_{r=R_i} = p \left[ \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} \right] \right]$$

$$R_o \Rightarrow \sigma_h = \frac{p R_i^2}{R_o^2 - R_i^2} + \frac{p R_i^4}{R_o^4 - R_i^4}$$

$$(\sigma_h)_{x=R_o} = \frac{2 p R_i^2}{R_o^2 - R_i^2}$$

$$(\sigma_h)_{\max} = p + (\sigma_h)_{\min}$$



### THICK CYLINDER SUBJECTED TO EXTERNAL PRESSURE ONLY:-

Boundary Condition →

$$x = R_i \Rightarrow p_i = 0$$

$$x = R_o \Rightarrow p_o = p$$

$$0 = -a + \frac{b}{R_i^2}$$

$$p = -a + \frac{b}{R_o^2}$$

$$-p = b \left[ \frac{1}{R_i^2} - \frac{1}{R_o^2} \right]$$

$$b = \frac{-p R_i^2 R_o^2}{R_o^2 - R_i^2}$$

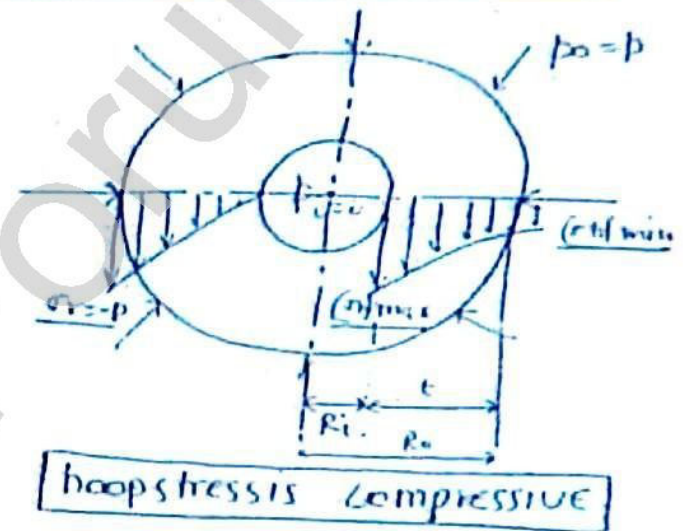
and

$$a = \frac{b}{R_i^2} = \frac{-p R_o^2}{R_o^2 - R_i^2}$$

$$\text{then } (\sigma_h)_{x=R_i} = \frac{-p R_o^2}{R_o^2 - R_i^2} + \frac{-p R_i^2 R_o^2}{(R_o^2 - R_i^2) R_i^2}$$

$$(\sigma_h)_{\max} = (\sigma_h)_{x=R_i} = \frac{-2 p R_o^2}{R_o^2 - R_i^2}$$

$$(\sigma_h)_{\min} = (\sigma_h)_{x=R_o} = \frac{-p [R_o^2 + R_i^2]}{[R_o^2 - R_i^2]}$$



Both internal and External pressure hoop stress is always maximum at inner radius and minimum at outer radius.



W.B.1.2  $p = 60 \text{ MPa}$ ,  $(\sigma_r)_{r=R_o} = 150 \text{ MPa}$

$$(\sigma_r)_{r=R_i} = p + (\sigma_r)_{r=R_o} \\ = 60 + 150 = 210 \text{ MPa}$$

W.B.2.2  $\epsilon_r = \frac{\delta D}{D} = \frac{1}{E} [\sigma_1 - \mu \sigma_2] = \frac{\sigma_1}{2E} [2 - \mu] = \frac{pD}{4tE} [2 - \mu]$   
 $= \frac{1}{E} [800 - 0.28 \times 400] = \frac{1}{2 \times 10^5} [800 - 1120] = -3.44 \times 10^{-4}$

W.B.3.1 (a) W.B.11. in pure shear  $\sigma_1 = \bar{\sigma}$ ,  $\sigma_2 = -\bar{\sigma}$

$$\sigma_1 = \tau = \frac{PD}{4t}, \quad \sigma_2 = -\tau = -\frac{PD}{4t} \quad f = \zeta$$

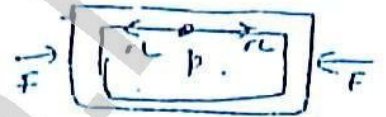
$$\sigma_1 = -F/A = -F/(\pi/4 D^2) = f/\pi^2$$

then  $\bar{\sigma} = p/2t = F/(\pi r^2 2t) = p/2$

$$F = \frac{3\pi \bar{\sigma} r^3}{2t}$$

$$\sigma_x = -\sigma_z = \frac{p r}{t} = \frac{F}{\pi r^2} - \frac{p r}{2t}$$

$$\frac{p r}{2t} + \frac{p r}{t} = \frac{F}{\pi r^2} \Rightarrow \frac{p r}{t} \left[ \frac{3}{2} \right] \pi r^2 = F$$



Q: A cast iron pipe  $D_i = 600 \text{ mm}$  and  $t = 100 \text{ mm}$  ( $\frac{D_i}{t} = 6$  so thick) carries water under a  $p = 10 \text{ MPa}$ . Sketch the radial pr. distribution and hoop stress dist<sup>n</sup> along the thickness of cylinder.

ms  $D_i = 600 \text{ mm}$ ,  $t = 100 \text{ mm}$   $\frac{D}{t} = 6 < 20$  thick cylinder  
 $p = 10 \text{ MPa}$

$$r = R_i = 300 \text{ mm} \Rightarrow 10 = -a + \frac{b}{300^2}$$

$$r = R_o = 400 \text{ mm} \Rightarrow 0 = -a + \frac{b}{400^2}$$

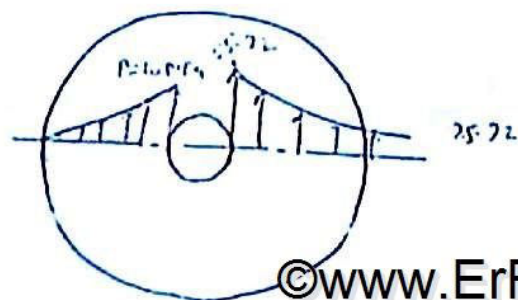
$$10 = b \left[ \frac{1}{300^2} - \frac{1}{400^2} \right]$$

$$b = 2.057 \times 10^6$$

$$a = \frac{b}{400^2} = 12.85$$

$$r = R_i = (\sigma_r)_{\max} = a + \frac{b}{300^2} = 35.72 \text{ MPa}$$

$$r = R_o = (\sigma_r)_{\min} = a + \frac{b}{400^2} = 25.72 \text{ MPa}$$



Q. A thick 50 cm outer dia and  $DI = 40\text{cm}$  is subjected to both internal and external pr. simultaneously. If  $I.P = 25\text{MPa}$   $(\sigma_h)_{ri} = 45\text{MPa}$ . Det the intensity of E.P.

$$r = R_i \Rightarrow p = 25\text{MPa}$$

$$r = R_o \Rightarrow p = p_o = ?$$

$$r = R_i \Rightarrow \sigma_h = 45\text{MPa}$$

### AUTO - FRETAGE :-

This is the method used to prestressing the cylinder. By using this, we ↑ the pressure carrying capacity of cylinder.

It increase the endurance limit.

Methods:-

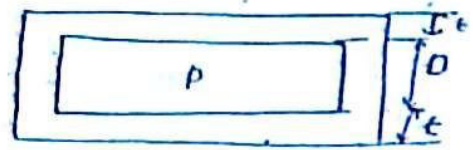
- I. Wire winding
- II. Compound cylinders.

$$\sigma_h \leq \sigma_t$$

$$\frac{pd}{2t} \leq \sigma_t$$

$$p \leq \frac{2t\sigma_t}{d}$$

↑ the pressure carrying capacity without change the dimensions.



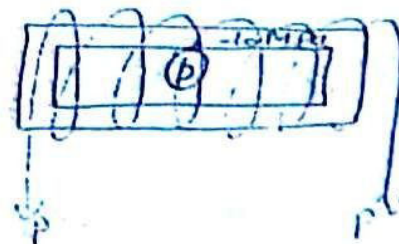
Wire Winding →

Assume  $\sigma_i = -10\text{MPa}$

Assume  $\sigma_h = \frac{pd}{2t} = 100\text{MPa}$

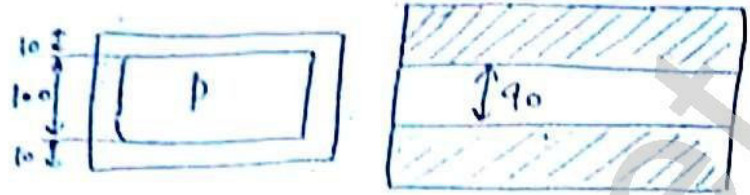
$$\sigma_{\text{total}} = \sigma_i + \sigma_h$$

$$\sigma_{\text{total}} = 90\text{MPa}$$





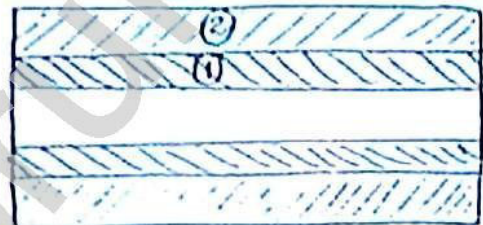
## ② Compound Cylinders :-



As residual (internal) compressive stress ( $\uparrow$ ), total stress ( $\downarrow$ ) hence pressure carrying capacity of the cylinder ( $\uparrow$ )

only compressive internal stress ( $\sigma_r$ ) is useful. If it is tensile then  $\sigma_{total}$  will ( $\uparrow$ ) due to  $\sigma_r$  hence pressure carrying capacity ( $\downarrow$ )

\* shaft compressed by hub ~~and~~ then the Mohr's circle will be a point.



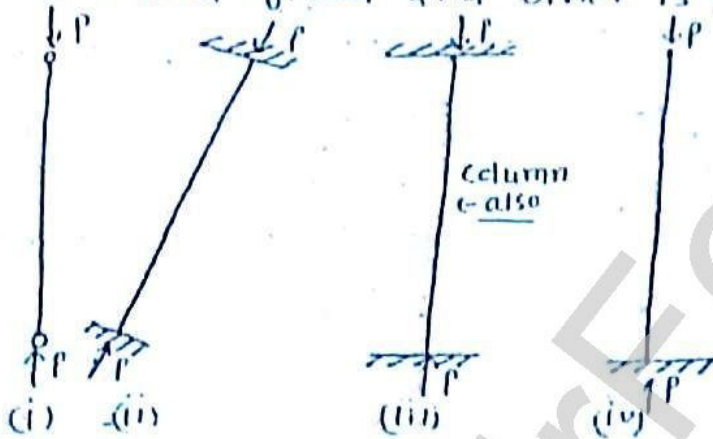
## Column and Struts

Structural Member subjected to axial compressive load.

Structural Member subjected to axial tensile load is called tie rod.

Column is vertical structural member which is fixed at both the ends and subjected to axial compressive load.

Strut is subjected to axial compressive load and it should be vertically or horizontally and may be fixed at both ends or one end fixed and other is free or both are free.



All are struts, but only (iii) is column.

All column are struts but all struts may not be column.

Ex: Wooden column

Ex: Steel column

Ex: Massive dimensioned column.

Load carrying capacity of column is depend upon 3 parameters

1) Materials

2) End conditions.

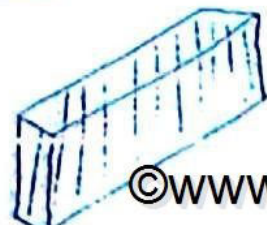
3) Its dimensions or

$$\text{Slenderness ratio} = \frac{Le}{K}$$

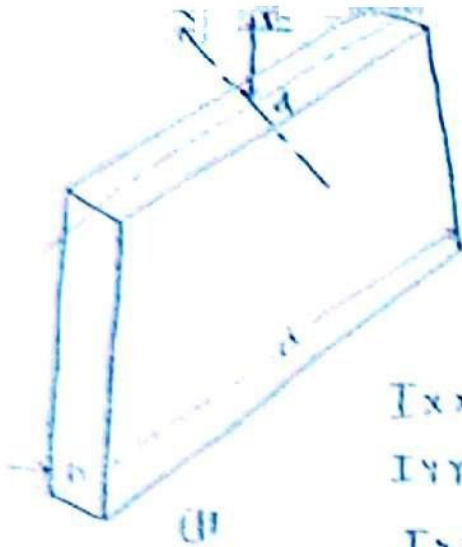
$$K = \sqrt{I/A}$$

$$P_{\text{WOODEN}} < P_{\text{CONCRETE}} < P_{\text{STEEL}}$$

when other conditions are same.



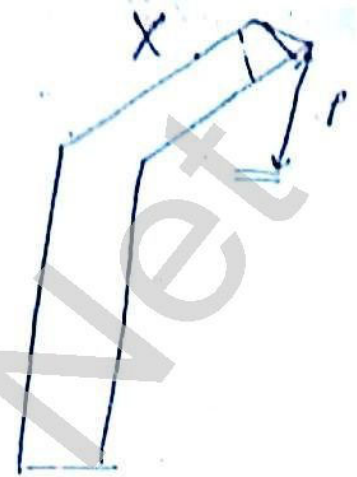
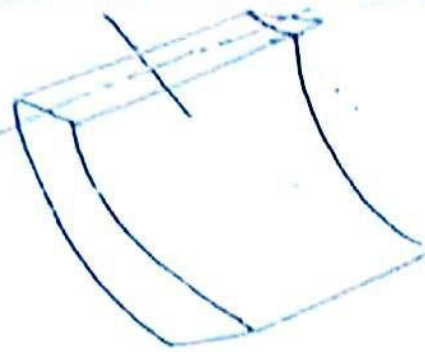




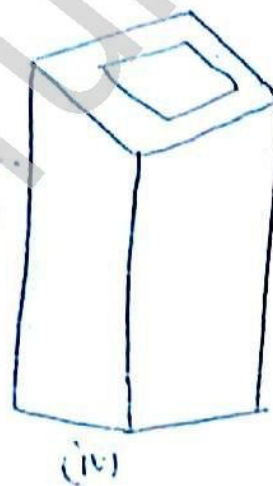
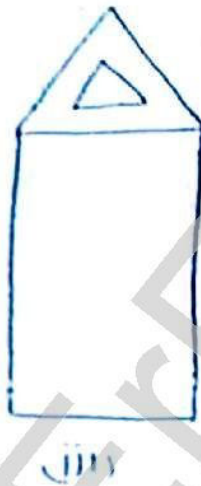
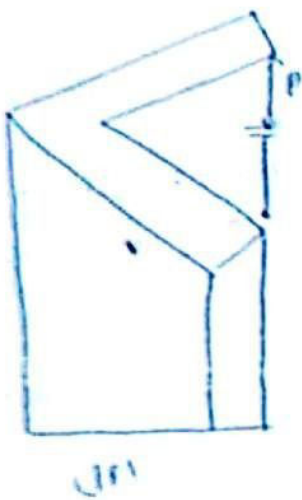
$$I_{xx} = \frac{1}{12} ab^3$$

$$I_{yy} = \frac{1}{12} ba^3$$

$$I_{yy} >> I_{xx}$$



\* A column is buckle where moment of inertia is less.



buckling is more in  $iv > iii > ii > i$

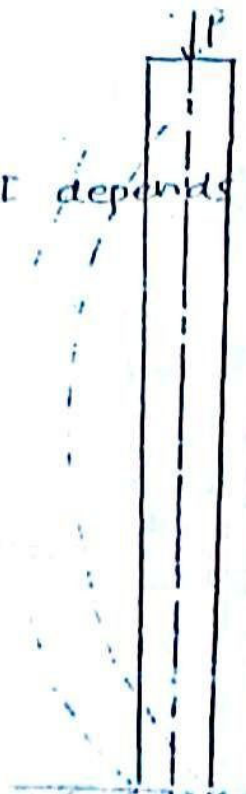
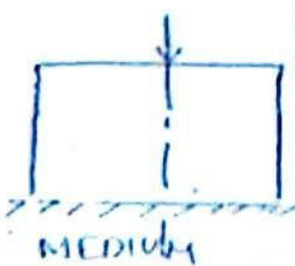
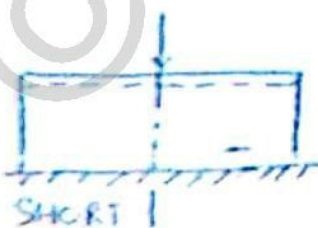
Buckling load depends on the shape as MOI depends on shape

MOI  $\uparrow$  = Buckling load  $\uparrow$

(MOI)<sub>hollow member</sub> = (MOI)<sub>Solid Member</sub>

### TYPES OF COLUMNS :-

- i. Short column
- ii. Medium column
- iii. Long column



→ Show. column is fails by crushing and strong in buckling

(ii) Medium column buckling load  $(P_e) \gg$  crushing load  $(P_c)$

Max stress induced  $\leq \sigma_{per}$

so medium column fails in both.

$$\sigma_{crush} \leq \frac{\sigma_{yc}}{N}$$

$$\left[ \frac{P}{A_c} \leq \frac{\sigma_{yc}}{N} \right] \Rightarrow \left[ P \leq A_c \frac{\sigma_{yc}}{N} \right]$$

(iii) long column is fails in buckling and strong in crushing.

$(\sigma_{max})_{ind} \leq \sigma_{per}$

$$(\sigma_{buckling}) \leq \sigma_{per}$$

We donot know buckling load. so Euler's give the eq for the buckling load.

\* Rankine give the formula for medium column but it used in all cases because medium column fails both crushing (short column) and buckling (long column). Ex. Connecting Rod.

\* Euler's formula is used for long column.

③ EULER'S FORMULA :-

$$P_e = -\frac{\pi^2 EI_{min}}{L_e^2} \text{ or } \frac{n\pi^2 EI_{min}}{L^2}$$

$$L_e = \alpha L \quad \alpha = \text{length fixity coefficient}$$

$$n = \frac{1}{\alpha^2}$$

then

End cond <sup>ns</sup> Values of $\alpha$ and $n$	(B-H)	(B-F)	(F-H)	(F-F)
$\alpha = 1$	1	$1/2$	$1/\sqrt{2}$	2
$n = \frac{1}{\alpha^2}$	1	4	2	$\frac{1}{4} = .25$
$(P_e)_{B-F} > (P_e)_{F-H} > (P_e)_{B-H} > (P_e)_{F-F}$				
$(P_e)_{B-F} = 4(P_e)_{B-H}$ $= 2(P_e)_{F-H}$ $= 16(P_e)_{F-F}$				



\* Values of  $(n)$  are used to compare the buckling load of the various columns which are made of same material, same  $q_s$  and same length but diff. end conditions.

Euler's buckling stress:-

$$\sigma_{\text{buckling}} = \sigma_e = \frac{P_e}{A} = \frac{\pi^2 E (I_{\min})}{L_e^2 A}$$

$$\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E k^2}{L_e^2}$$

$$\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E}{S^2}$$

$$k = \sqrt{I/A}$$

$$S = \frac{L_e}{k} = \text{Slenderness ratio.}$$

\* Buckling load  $\propto \frac{1}{S^2}$

So  $S \uparrow \Rightarrow$  then buckling load  $\downarrow \Rightarrow$  buckling tendency  $\uparrow$

$$(S)_{\text{Short Column}} < (S)_{\text{Medium Column}} < (S)_{\text{Long Column}}$$

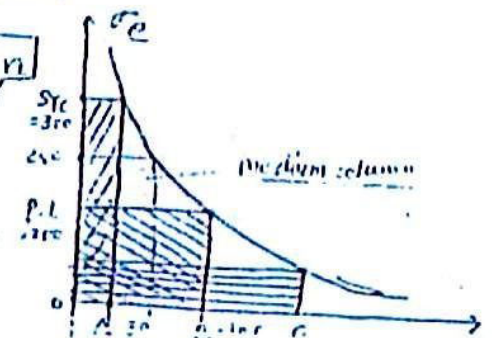
$$(P_e)_{\text{Short C.}} > (P_e)_{\text{Medium C.}} > (P_e)_{\text{Long C.}}$$

$$(\text{Buckling tendency})_{S.C.} < (B.T)_{M.C.} < (B.T)_{L.C.}$$

Mild Steel  $\Rightarrow S \leq 30 = \text{short column}$

$S > 100 = \text{Long column}$

$$\begin{aligned} \sigma_e &\leq \text{proportional limit} \Rightarrow \text{long column} \\ P.L < \sigma_e \leq S_{yc} &\Rightarrow \text{Medium} \\ \sigma_e > S_{yc} &\Rightarrow \text{Short Column.} \end{aligned}$$



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RANKINE'S FORMULA →

$$\boxed{\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}}$$

This formula valid for short and medium column & long column.

\* for short column,  $P_e \gg \gg P_c$

$$\frac{1}{P_R} = \frac{1}{P_c} + 0$$

$$\boxed{P_R \triangleq P_c = A \sigma_c}$$

\* for long column →  $P_c \gg \gg P_e$

$$\frac{1}{P_R} = 0 + \frac{1}{P_e}$$

$$\boxed{P_R \triangleq P_e = \frac{\pi^2 EI}{L_e^2}}$$

then finally,

$$\boxed{P_R = \frac{A \sigma_c}{1 + c \left( \frac{L_e}{k} \right)^2}} \quad \text{Medium column}$$

where,  $c$  = Rankine's Constant  $= \frac{\sigma_c}{\pi^2 E} = (a) \text{ or } (c)$

Assumptions:

- i. Length is more compared to cross-sectional dimensions.
- ii. effects of friction is neglected at the hinge
- iii. uniform c/s.
- iv. Self weights are neglected.
- v. Material is assumed to be homogeneous and isotropic.
- vi. Load is assumed to be truly axial.
- vii. Prismatic c/s.

Euler's and Rankine formula gives 5 to 10% error (or approx. values are come) so Johnson formula gives actual buckling load

CORE OR KERN OF A CROSS-SECTION:-

It is a portion of c/s such that load should act.

Core of c/s is defined as area with in which the load should act such that there is no tensile stress induced in the column i.e., all the fibres are in compressive loading only.

The condition such that there is no tensile stress induced.

The stress in left side  $\boxed{\sigma_L \leq 0}$



$$-\sigma_a + \sigma_b \leq 0$$

$$\sigma_b \leq \sigma_a$$

$$\frac{M_y}{Z_y} \leq \frac{P}{A} \Rightarrow \frac{Pe}{I_y / y_{max}} = \frac{P}{A y_{max}}$$

$$\frac{Pe}{\pi d^3 / 32} = \frac{P}{\pi d^2 / 4}$$

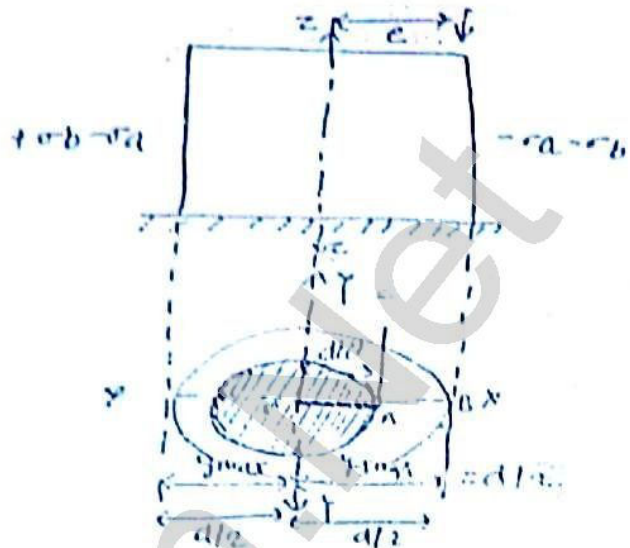
$$e \leq d/8$$

OA → all the fibres are in compression  
AB → " " " " " " tensile.

$$A_c = \frac{\pi}{4} (d/4)^2$$

$$A_c = \frac{A}{16}$$

$(\frac{1}{4} \text{ Rule})$  To get  $e_{max}$ .

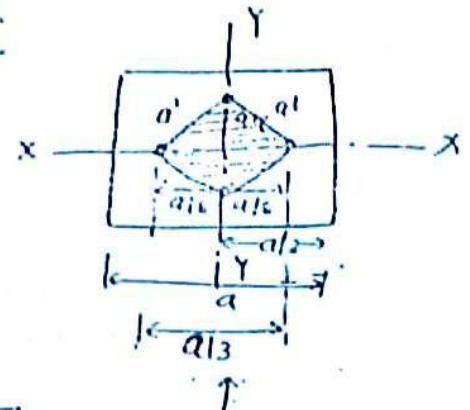


The core of a circular c/s is a circular section with diameter  $d/4$  and its area is  $\frac{1}{16}$  of actual c/s area.

$$\sigma_b \leq \sigma_a$$

$$\frac{M_{yy}}{I_{yy} / y_{max}} \leq \frac{P}{A}$$

$$\frac{Pe_x}{a^3 / 6} \leq \frac{P}{a^2} \Rightarrow e_x = \frac{a}{6} \quad \left[ \frac{1}{3} \text{ Rule} \right]_{\text{middle}}$$



$$e' = \sqrt{(a/6)^2 + (a/6)^2} = \frac{a}{6} \sqrt{2} = \frac{a}{3\sqrt{2}} \quad \leftarrow \text{CORE OF SQUARE side of square}$$

Area of Core:

$$A_c = \left( \frac{a}{3\sqrt{2}} \right)^2 = \frac{a^2}{18}$$

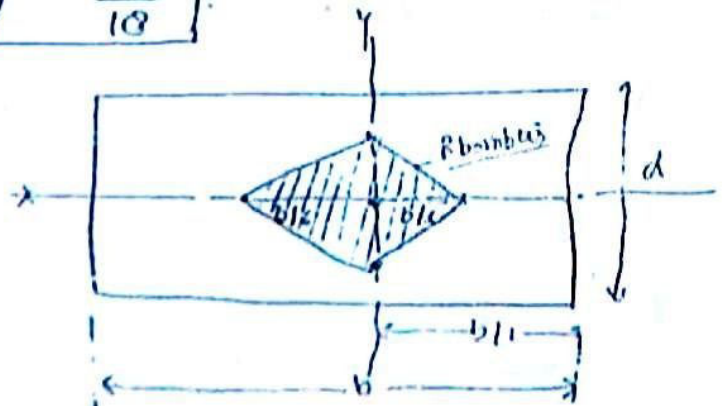
lies on x-axis.

$$\sigma_b \leq \sigma_a$$

$$\frac{M_{yy}}{Z_{yy}} \leq \frac{P}{A}$$

$$\frac{Pe_x}{I_{yy} / y_{max}} \leq \frac{P}{b d}$$

$$\frac{Pe_x y_{max}}{I_{yy}} \leq \frac{P}{b d}$$



$$\frac{P e_x b/2}{\frac{1}{12} d b^3} \leq \frac{P}{b d}$$

$$\boxed{e_x \leq b/6}$$

on Y-axis  $\rightarrow \sigma_{b_{yy}} \leq \sigma_a \Rightarrow \frac{M_{yy}}{I_{yy}} y_{max} = \frac{P}{A}$

$$\frac{P e_y y_{max}}{I_{yy}} \leq \frac{P}{b d} \Rightarrow \frac{P e_y \cdot d/2}{\frac{1}{12} b d^3} \leq \frac{P}{b d}$$

$$\boxed{e_y \leq d/6}$$

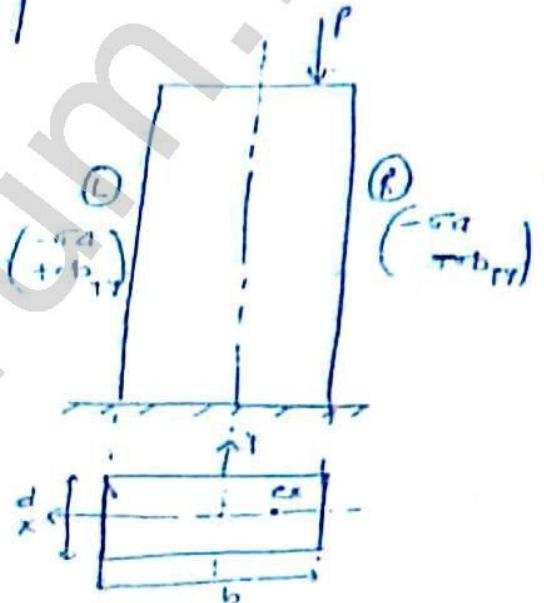
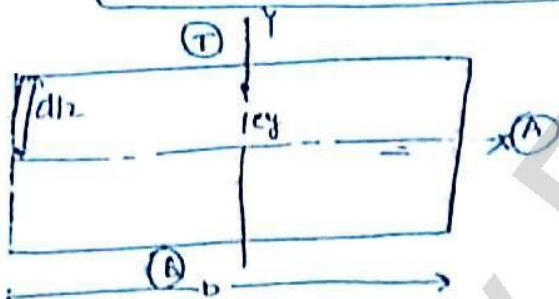
Square and rectangle obey  $\frac{1}{3}$  middle rule

$$\begin{aligned} a' &= \sqrt{(b/c)^2 + (b/c)^2} \\ a' &= \sqrt{2} (b/c) \\ \boxed{a' = \frac{b \sqrt{2}}{c}} \end{aligned}$$

Determine a type of hollow circular section

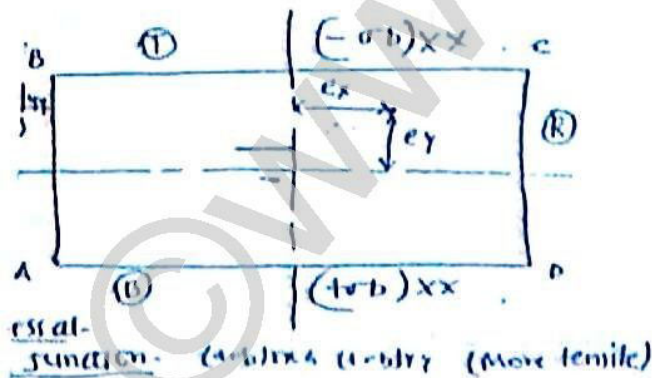
$$\begin{aligned} \sigma_{max} &= -\sigma_a - (\sigma_b)_{yy} \\ &= -\left[ \frac{P}{b d} + \frac{M_{yy} y_{max}}{I_{yy}} \right] \\ &= -\left[ \frac{P}{b d} + \frac{P e_x b/2}{\frac{1}{12} d b^3} \right] \end{aligned}$$

$$\boxed{\sigma_{max} = -\left[ \frac{P}{b d} + \frac{6 P e_x}{d b^2} \right]}$$



$$\begin{aligned} \sigma_{max} = \sigma_{Top} &= -\sigma_a - (\sigma_b)_{xx} \\ &= (-) \left[ \frac{P}{A} + \frac{M_{xx}}{I_{xx}} \right] \\ &= -\left[ \frac{P}{b d} + \frac{P e_y y_{max}}{I_{xx}} \right] \\ &= -\left[ \frac{P}{b d} + \frac{P e_y d/2}{\frac{1}{12} b d^3} \right] \end{aligned}$$

$$\boxed{\sigma_{max} = -\left[ \frac{P}{b d} + \frac{6 P e_y}{b d^2} \right]}$$



$$\begin{aligned} (\sigma_b)_{yy} \left\{ \begin{aligned} \sigma_A &= -\sigma_a + (\sigma_b)_{xx} + (\sigma_b)_{yy} \\ \sigma_B &= -\sigma_a + (\sigma_b)_{xx} + (\sigma_b)_{yy} \\ \sigma_C &= -\sigma_a - (\sigma_b)_{xx} + (\sigma_b)_{yy} \\ \sigma_D &= -\sigma_a + (\sigma_b)_{xx} - (\sigma_b)_{yy} \end{aligned} \right. \end{aligned}$$

$$\frac{P}{A} = \frac{P}{b d} \quad , \quad (\sigma_b)_{xx} = \frac{M_{xx}}{I_{xx}} = \frac{P e_y y_{max}}{\frac{1}{12} b d^3}$$



$$(r_b)_{yy} = \frac{M_{yy}}{z_{yy}} = \frac{P e_x \times b/2}{I_{xx}} = \frac{P e_x \times b/2}{\frac{1}{2} d b^3}$$

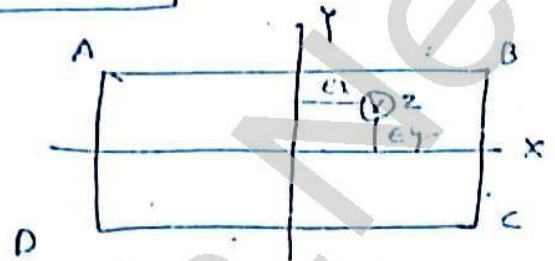
Then

$$\sigma_A = -\frac{P}{bd} + \frac{P e_y \cdot d/2}{\frac{1}{12} b d^3} + \frac{P e_x \times b/2}{\frac{1}{2} d b^3}$$

As all the stresses are calculated.

W.B.10] (a) Similar about example

W.B.10] (c)



W.B.11]

$$P e \propto I \propto d^4$$

$$d' = 0.9d$$

$$P e \propto d^4$$

$$P e \propto (0.9)^4 = 0.66$$

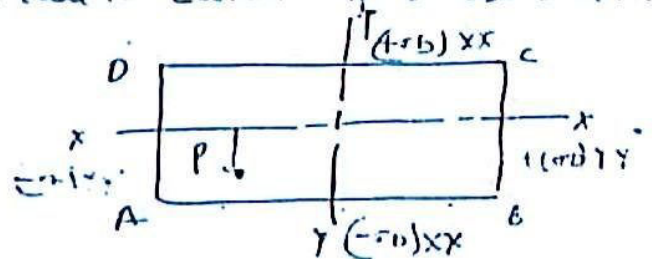
$$[P e \propto 66\%] \text{ so more than } 30\% \quad (c)$$

2. for a column as shown in fig. determine resultant stress at corner B will be if direct stress is 10 kPa (compressive) and max bending stresses of 10 kPa and 15 kPa due to eccentricities about x-x and y-y

$$\sigma_B = -\sigma_a - (\sigma_b)_{xx} + (\sigma_b)_{yy}$$

$$= -10 - 10 + 15$$

$$\sigma_B = -13 \text{ kPa} \quad \underline{\underline{Ans}}$$



## SFD and BMD

Beam = Any structural member is designed by two criterion.

Strength criterion.

Rigidity

Beam is defined as a structural member (whose longitudinal dimensions are larger in comparison to its dimensions.) which is subjected to transverse shear load (TSL) is a load  $\perp$  to axis. TSL causes variable shear force and variable bending moment. Hence to know the variation of shear force and bending moment over a length of beam, maximum values of shear force and Bending moment, you have to draw SFD and BMD. The maximum value of shear force and bending moment is used in the design of beam under Strength criterion. Deflection and slopes are used in the design of beam in the rigidity or stiffness criterion.

Strength criterion →

$$(\sigma_{\max})_{ind} \leq \sigma_{per}$$

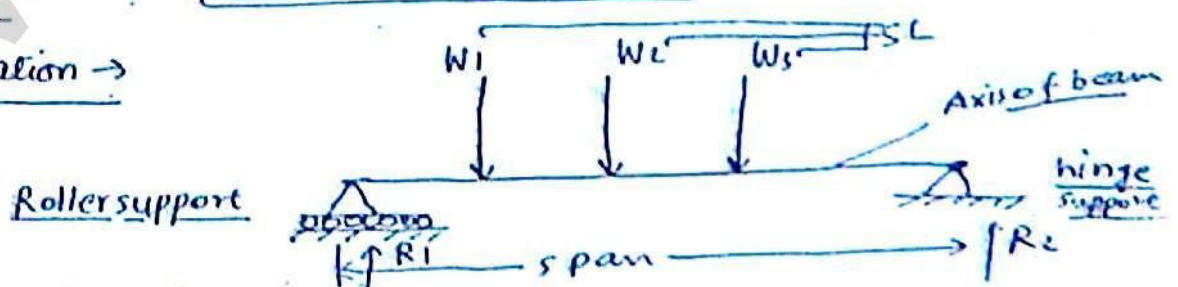
$$\frac{M_{\max}}{Z} \leq \sigma_{per}$$

$$Z \geq \frac{M_{\max}}{\sigma_{per}} \quad \text{--- BMD}$$

Rigidity criterion →

$$(\delta_{\max})_{ind} \leq \delta_{per}$$

General Representation →



Supports → keeps the beam in equilibrium condition.  

$$[\sum H = 0 ; \sum V = 0 ; \sum M = 0]$$

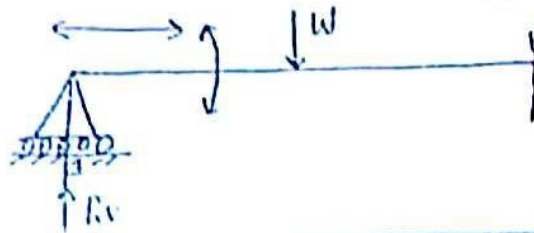
©www.ErForum.Net



## Types of Support:

- i. Roller Support
  - ii. Hinge support (Pin support)
  - iii. fixed support (Clamped or Built in)
- } Simple support

### 1) Roller Support:

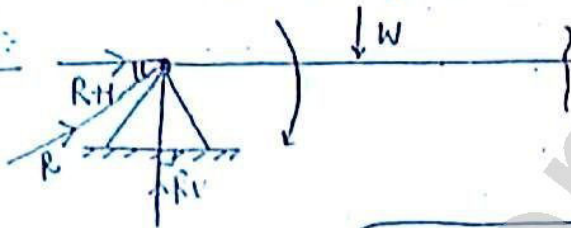


Horizontal Reaction = 0

$M=0$  permitting rotation.

No. of Reactions = No. of constrained Motions  
= 1 (Vertical Reaction)

### 2) Hinge Support:



$M=0$

because permitting the rotation

Reaction of hinge support:  $R = \sqrt{R_H^2 + R_V^2}$

$$\tan \theta = \frac{R_V}{R_H}$$

4f:

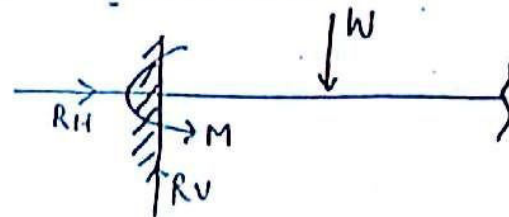


Horizontal Reaction will also be considered.

### 3) fixed Support →

$$R = \sqrt{R_H^2 + R_V^2}$$

$M \neq 0$  because restrict the rotation



## BEAMS

STATICALLY DETERMINATE BEAM

No. of Reaction ≤ No. of useful static eqn.

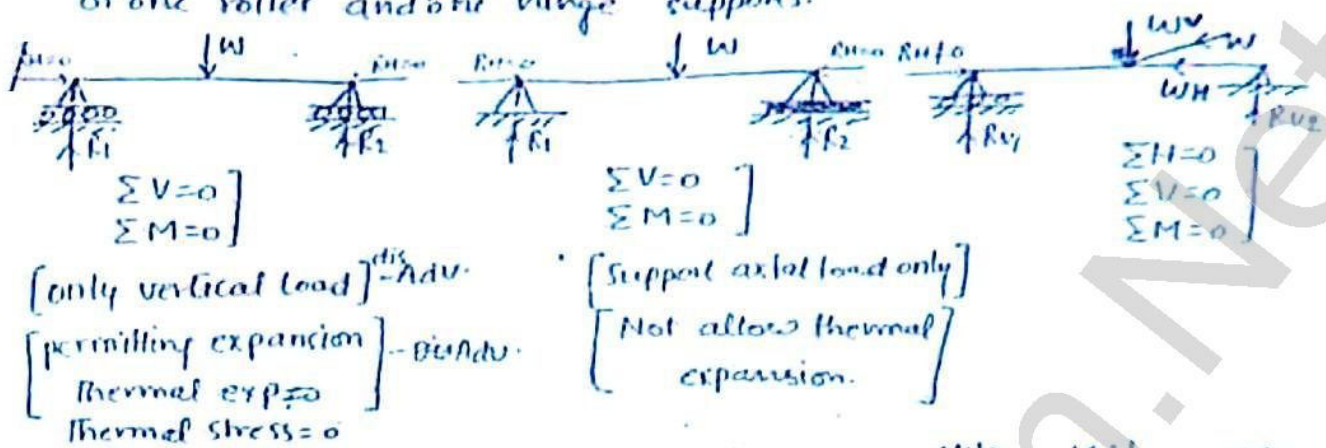


STATICALLY INDETERMINATE BEAM

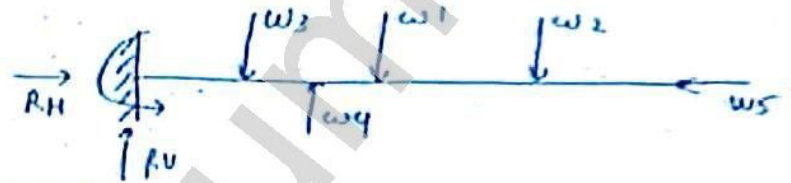
No. of Reaction > Useful equation



① S.S.B is a beam whose ends are supported by two roller, two hinge or one roller and one hinge supports.



## ② Cantilever Beam -



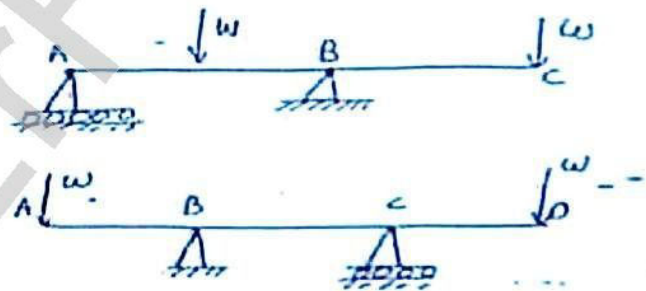
$R_V$  = algebraic sum of vertical loads

$$R_V = W_3 + W_1 + W_2 - W_4$$

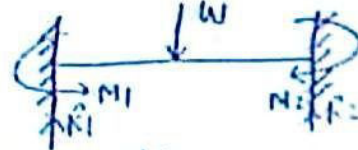
$R_H$  = algebraic sum of Horizontal loads =  $W_5$

$M_A$  = " " " Moments due to loads. ( $W_5 = 0$ )

## overhanging beams:-



## Fixed beam:-



## Continuous Beam:-

Same as S.S.B but span

is so large so some supports

btw the end supports are called intermediate supports. which restrict the deflection.

Ex shaft.

No. of Reaction = 3

No. of REs = 2

Size

intermediate beam.



### ⑤ Propped Cantilever Beam →

propped is a simple support (maybe roller & hinge) near the free end.

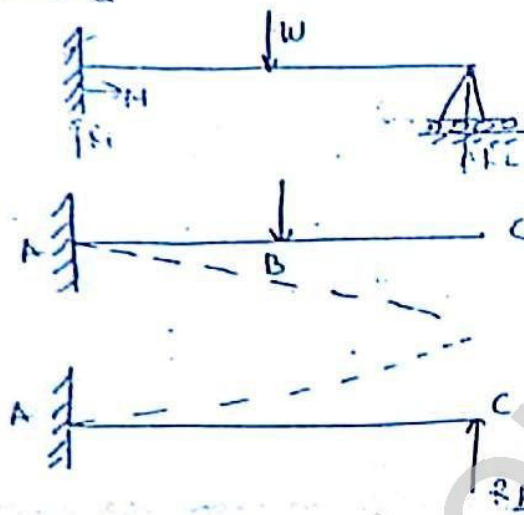
unknowns = 3  
equation = 2

then

$$3 > 2$$

statically indeterminate

Example:



$$R_1 + R_2 = W$$

$$\sum M_A = 0$$

$$-M + W \cdot a - R_2 \cdot L = 0$$

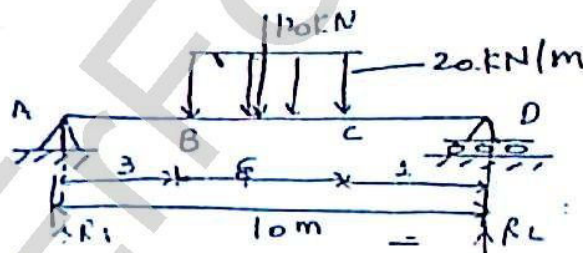
$$S_{C1} = S_{C2}$$

Calculate  $R_2$

then calculate  $R_1$

so calculate  $M$

Reactions:- U.D.L →



I total weight of distributed load = Area of plane figure  
 $= 20 \times 6$   
 $= 120 \text{ kN}$

II Represent total weight through C.G of plane figure

$$\sum V = 0$$

$$R_1 + R_2 = 120 \text{ kN}$$

$$\sum M_A = 0$$

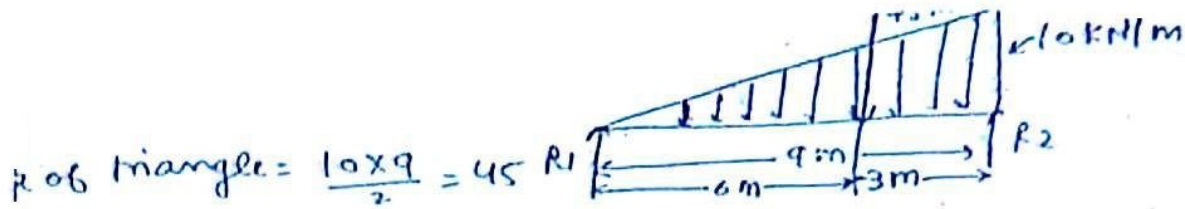
$$120 \times 6 - R_2 \times 10 = 0$$

$$120 \times 6 = 10 R_2$$

$$R_2 = 72$$

then

$$R_1 = 48 \text{ kN}$$



Weight through C.G. of plane of weight

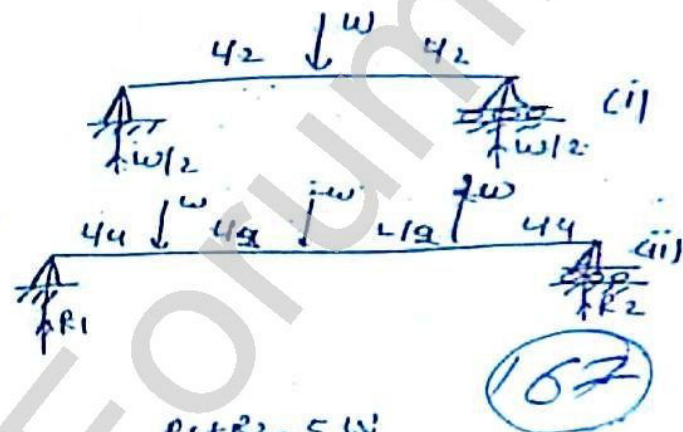
$$= \frac{1}{3} \times 9 = 3 \text{ m}$$

$$R_1 = \frac{45 \times 3}{9} = 15 \text{ kN}$$

or  $\sum V = 0$   
 $\sum M = 0$  then calculate  $R_1$  &  $R_2$

$$R_2 = \frac{45 \times 6}{9} = 30 \text{ kN}$$

B → ① Symmetric loading  
So  $R_1 = W/2$   
 $R_2 = W/2$



Symmetric. So

$$\frac{W \times 1.5L + 2W \times 3L/4 + 2W \times 4L/4}{1.5L}$$

$$R_1 + R_2 = 5W$$

$$R_1 = R_2$$

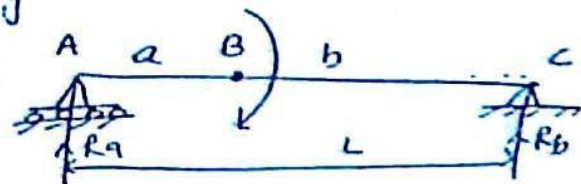
Let the reaction when simply supported beam is subjected to a concentrated load moment at any point of beam

$$R_A + R_B = 0 \quad (\text{No load acting})$$

$$\sum M_A = 0$$

$$M - R_B \times L = 0$$

$$R_B = \frac{M}{L} \quad \text{then} \quad R_A = -\frac{M}{L} \quad (b)$$



When beam is subjected to concentrated moment at any point on axis, reaction will be equal and opposite and they are equal to applied moment divided by length of beam

$$R_A = R_B = \frac{M}{L}$$



Ques:  $R_A = \frac{M}{4L}$   
 $R_B = -\frac{M}{4L}$

total Moment = M (anticlockwise)  
 $= 4m - 3m = M$

So  $R_A = \frac{\text{total Moment}}{\text{length}}$

(+ve)  $\rightarrow$  when R and M are equal  
 opposite direction  
 (-ve)  $\rightarrow$  when R and M are in same  
 direction.

OR

$R_A + R_B = 0 \Rightarrow R_A = -R_B$

$\Sigma M_A = 0 \Rightarrow M + 2M - 4M - R_D \times 4L = 0$

$R_D = -\frac{M}{4L}$  then  $R_A = \frac{M}{4L}$

Ques. S.S.B. as shown in Fig. Det. the reaction at support.

$R_A + R_B = P$

$\Sigma M_A = 0$

$2P + 4P - R_B \times 10 = 0$

$6P = R_B \times 10$

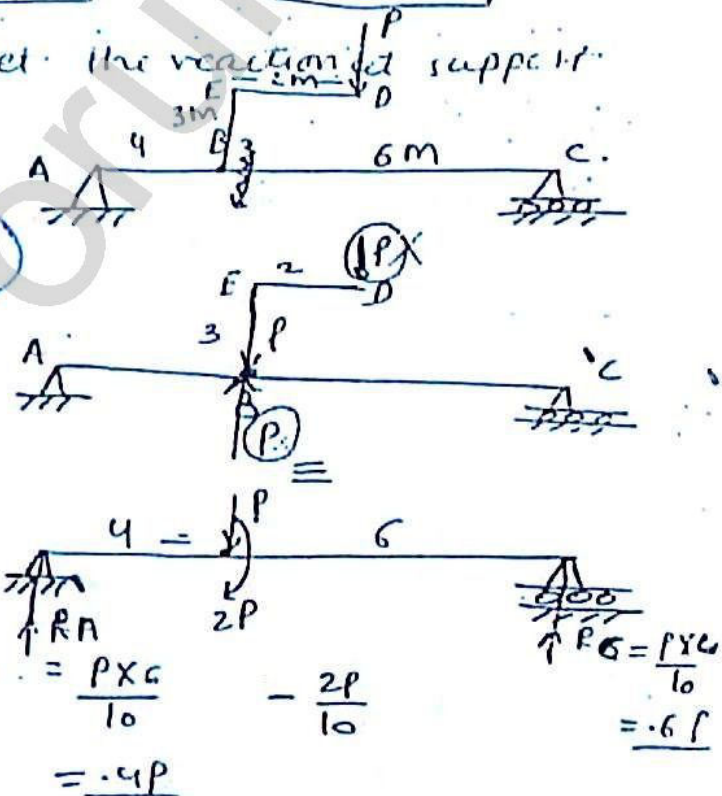
$R_B = 0.6P$

$R_A + 0.6P = P$

$R_A = 0.4P$

OR

168



Ques:

$R_A + R_C = 0$

$R_H = P$

$\Sigma M_A = 0$

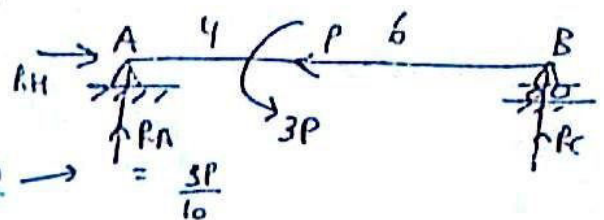
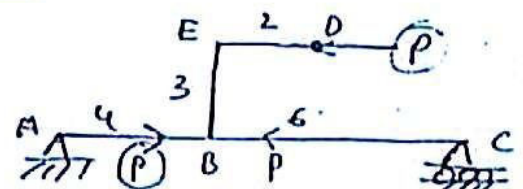
$-M + R_C \times 10 = 0$

$R_C = \frac{M}{10}$

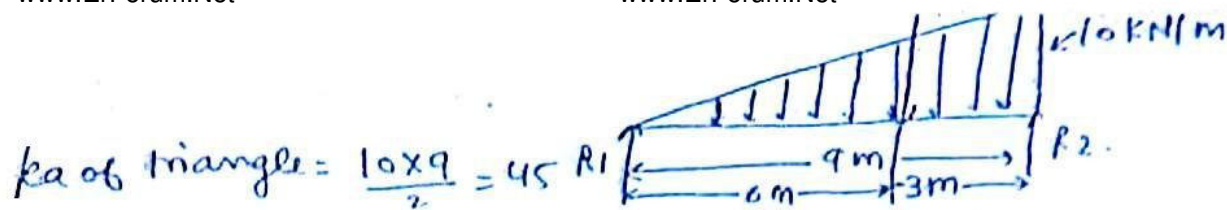
$R_C = 3P/10$

$R_A = -\frac{3P}{10}$

OR  $\rightarrow$



$R_A = \sqrt{P^2 + (-3P/10)^2} = \sqrt{P^2 + 0.09P^2} = P\sqrt{1.09}$



$$k_a \text{ of triangle} = \frac{10 \times 9}{2} = 45 \text{ kN}$$

Let weight through C.G. of plane of weight

$$= \frac{1}{3} \times 9 = 3 \text{ m}$$

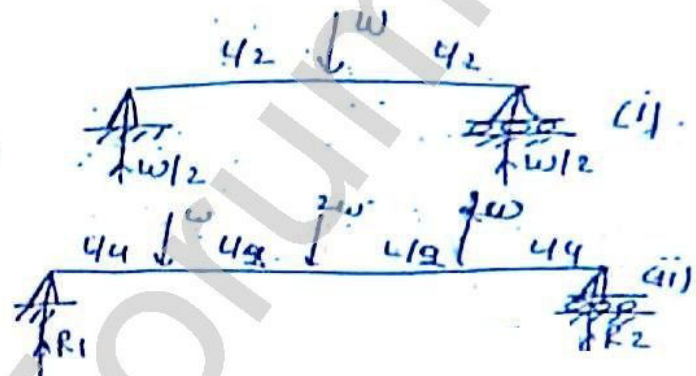
$$R_1 = \frac{45 \times 3}{9} = 15 \text{ kN}$$

$$R_2 = \frac{45 \times 6}{9} = 30 \text{ kN}$$

or  $\sum V = 0$   
 $\sum M = 0$  then calculate  $R_1$  &  $R_2$  :

S.B → ① Symmetric loading

$$\left. \begin{aligned} \text{So } R_1 &= W/2 \\ R_2 &= W/2 \end{aligned} \right\}$$



Not Symmetric. So

$$R_1 = \frac{W \times 1.75L + 2W \times 3L/4 + 2W \times L/4}{1.5L}$$

$$R_1 + R_2 = 5W$$

$$R_1 = ?$$

Q: Find the reaction when simply supported beam is subjected to a concentrated load moment at any point of beam

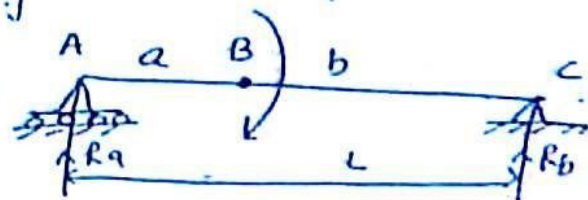
$$R_a + R_b = 0 \quad (\text{No load acting})$$

$$\sum M_A = 0$$

$$M - R_2 \times L = 0$$

$$\boxed{R_2 = +M/L} \quad \text{then} \quad \boxed{R_1 = +M/L} \quad (b)$$

(↑)



when beam is subjected to concentrated moment at any point on beam, reaction will be equal and opposite and they are equal to applied moment divided by length of beam

$$\boxed{R_A = R_B = M/L}$$



$$\left. \begin{aligned} R_A &= M/4L \\ R_B &= -M/4L \end{aligned} \right\}$$

$$\begin{aligned} \text{total Moment} &= M (\text{anticlockwise}) \\ &= 4m - 3m = M \end{aligned}$$

$$\text{So } R_A = \frac{\text{total Moment}}{\text{length}}$$

(+ve)  $\rightarrow$  when  $R$  and  $M$  are equal & opposite direction  
(-ve)  $\rightarrow$  when  $R$  and  $M$  are in same direction.

$$\text{OR } R_A + R_B = 0 \Rightarrow R_A = -R_B$$

$$\Sigma M_A = 0 \Rightarrow M + 2M - 4M - R_D \times 4L = 0$$

$$R_D = -M/4L \text{ Then } R_A = M/4L$$

Q. S.S.B as shown in Fig. Del. the reaction at support.

$$R_A + R_B = P$$

$$\Sigma M_A = 0$$

$$2P + 4P - R_B \times 10 = 0$$

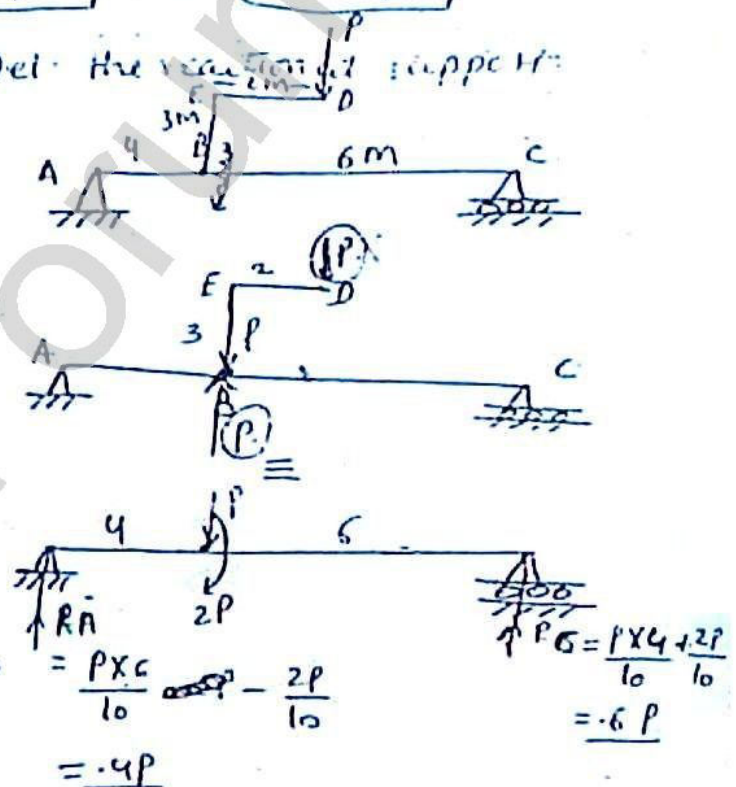
$$6P = R_B \times 10$$

$$R_B = -0.6P$$

$$R_A + 0.6P = P$$

$$R_A = 0.4P$$

OR



for:

$$R_A + R_C = 0$$

$$R_H = P$$

$$\Sigma M_A = 0$$

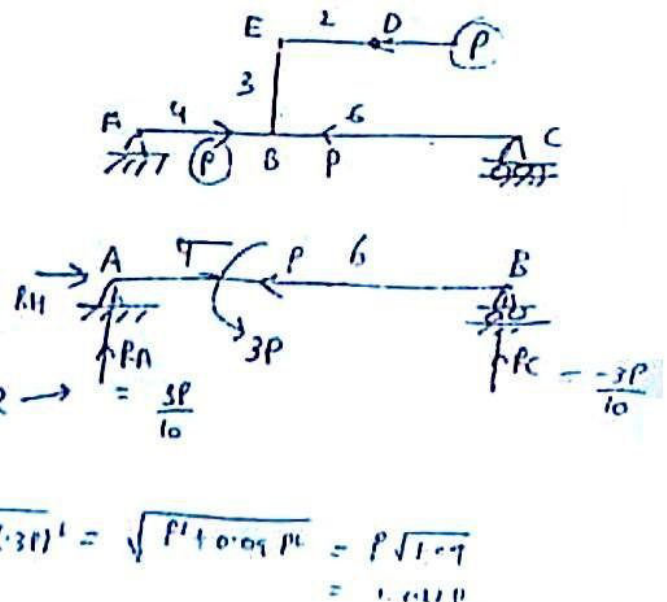
$$-M + R_C \times 10 = 0$$

$$R_C = M/10$$

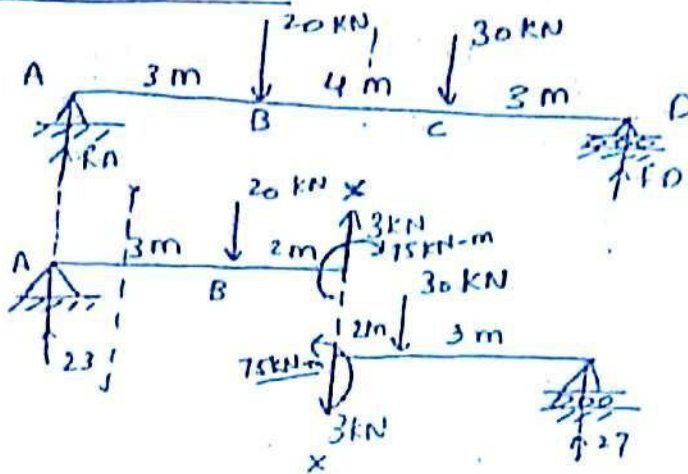
$$R_C = 3P/10$$

$$R_A = -\frac{3P}{10}$$

OR



$$R_A = \sqrt{P^2 + (-3P/10)^2} = \sqrt{P^2 + 0.09P^2} = P\sqrt{1.09} = 1.044P$$



$$R_A = \frac{20 \times 7}{10} + \frac{30 \times 3}{10}$$

$$R_A = 14 + 9 = 23 \text{ kN}$$

$$R_D = \frac{30 \times 7}{10} + \frac{20 \times 3}{10}$$

$$= 21 + 6 = 27 \text{ kN}$$

$$M_{xx} = 23 \times 5 - 20 \times 2$$

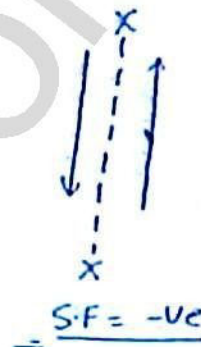
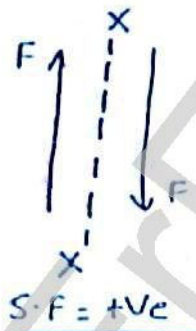
$$= 75 \text{ kNm-m}$$

$$M_{xx} = 27 \times 5 - 30 \times 2$$

$$= 75 \text{ kNm-m}$$

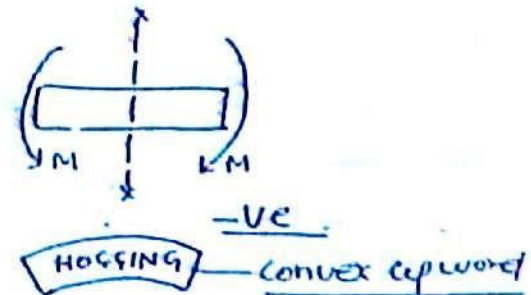
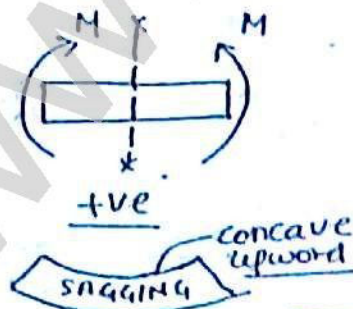
\* SF at a given section is defined as the algebraic sum of vertical forces (included vertical reactions) either left hand of the section or right hand of the section

S.F. Sign Conv<sup>n</sup> →



B.M at a given section is defined as the algebraic sum of moments either on the left hand side of section or right hand side of section

B.M Sign Conv<sup>n</sup> →



② Relationship b/w Load, shear force and Bending Moment :

$$W' = \frac{dF}{dx} \rightarrow \textcircled{1}$$

$$F' = \frac{dM}{dx} \rightarrow \textcircled{2}$$

$$\int_A^B dF = \int_A^B w dx$$

$$F_B - F_A = \int_A^B w dx$$

$(F_B - F_A) = \text{area of loading diagram b/w A \& B}$



$$\int_A^B dM = \int_A^B F \cdot dx$$

$$(M_B - M_A) = \int_A^B F \cdot dx \Rightarrow (M_B - M_A) = \text{area of S.F.D b/w B \& A}$$

\* Above two equation can be interpreted by two diff ways  $\rightarrow$

- (i) The slope of shear force diagram at a section = mag. of load at that section.
- (ii) The slope of bending moment diagram at a section = shear force at that section.
- (iii) 1st differential of shear force equation gives load acting at that section.
- (iv) 1st differential of Bending moment equation gives shear force at that section.
- (v) The area of loading diagram b/w B and A  $\neq$  difference of shear force b/w B and A.
- (vi) The area of S.F.D b/w B & A = difference of B.M b/w B & A
- (vii) Shear force variation is one order more than load variation whereas bending moment variation is one order more than shear force variation or two order more than load variation.

Type of load Load Variation	S.F Variation	B.M Variation
Zero (b/w point load)	<u>Constant</u> (horizontal line)	<u>Linear Variation</u> (Inclined line) ( $M \propto x$ )
Constant (U.D.L)	<u>Linear Variation</u> (Inclined lines) ( $F \propto x$ )	<u>Parabolic Variation</u> ( $M \propto x^2$ )
Linear (U.V.L)	<u>Parabolic Variation</u> ( $F \propto x^2$ )	<u>Cubic Parabolic Variation</u> ( $M \propto x^3$ )

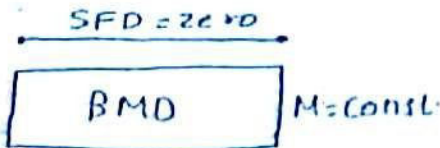
$$F = qM/dx \rightarrow (1)$$

$$w = dF/dx \rightarrow (2)$$

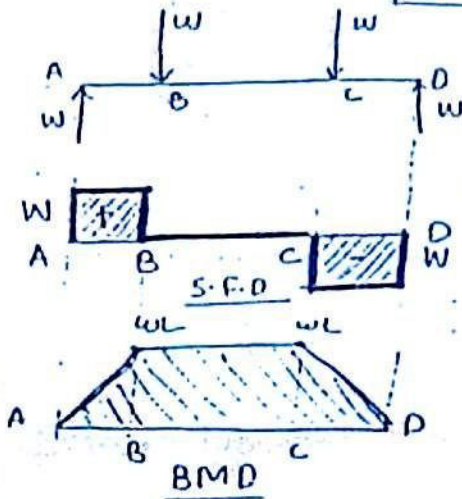
$$F = 0 = \frac{dM}{dx} \Rightarrow \text{Moment} = \text{Constant}$$

$$F = 0 = \frac{dM}{dx} \Rightarrow \text{Moment} = \text{Maximum}$$

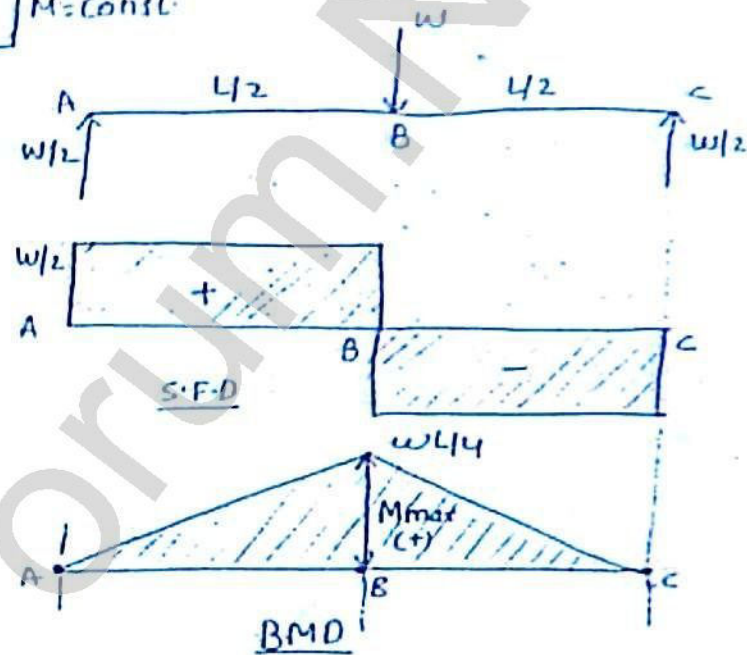
② pure bending  $\Rightarrow$



Ex-



Ex-



③ Point of  $M_{\max}$  @ zero S.F.:-

$$(SF)_{x-x} = \dots = 0$$

$$x = x^*$$

$(M)_{\max} = (M)_{x=x^*} = \dots$

④ Point of Contraflexure:

Point of contraflexure is a point where B.M. changes, its sign is; the nature of bending on either side of point of contraflexure is opposite in nature.

\* Point of contraflexure is located by equating B.M. Eq. to zero.

⑤ Cantilever Beam:-

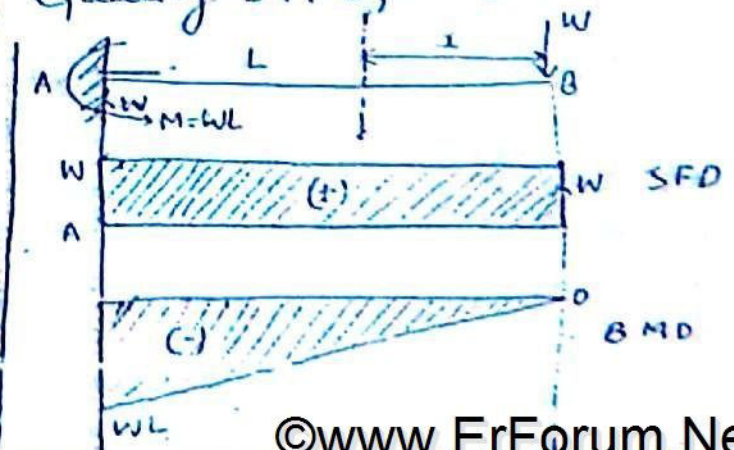
Case-I  $\rightarrow$

$$(SF)_{xx} = Wx$$

$$(M)_{xx} = -Wx$$

$$x=0 \Rightarrow (SF)_0 = W \Rightarrow M_B = 0$$

$$x=L \Rightarrow (SF)_L = W \Rightarrow M_D = -WL$$





Case-II → In cantilever beam, section (U)

always take from the free end

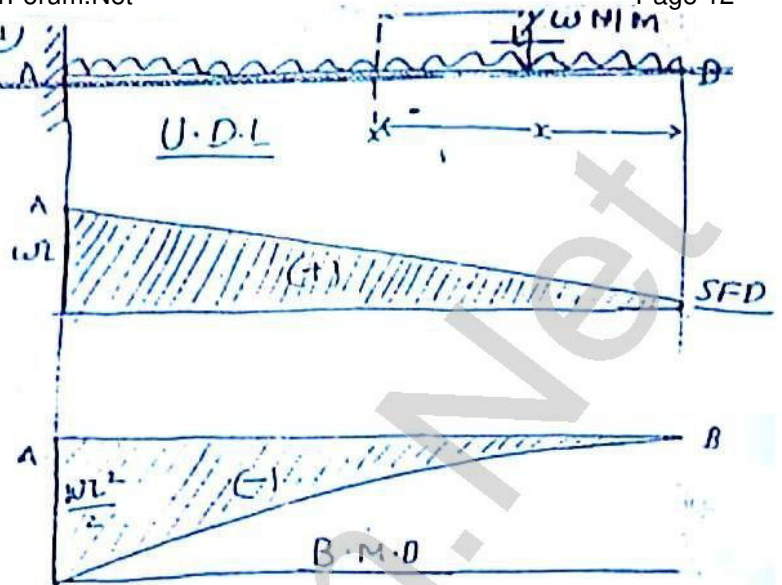
$$(SF)_{xx} = wx$$

$$M_{xx} = -\frac{wx^2}{2}$$

So  $M_{xx}$  is always one order more than the (SF)

$$x=0 \Rightarrow (SF)_B = 0 \Rightarrow (M)_{xx} = 0$$

$$x=L \Rightarrow (SF)_A = WL \Rightarrow (M_{xx})_A = -\frac{WL^2}{2}$$



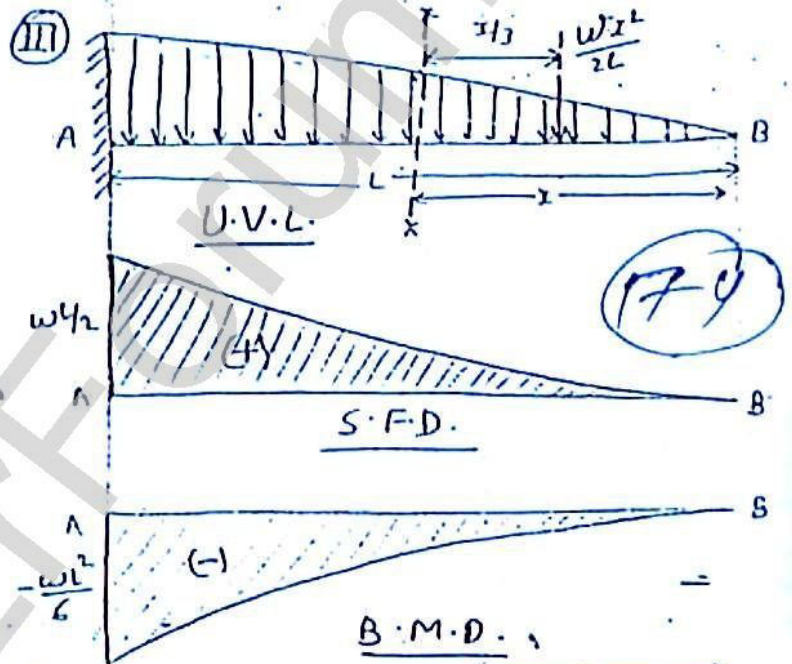
Case-III →  $(SF)_{xx} = \frac{wL^2}{2L}$

$$(M)_{xx} = -\frac{wL^2}{2L} \times \frac{x^3}{3} = -\frac{wL^3}{6L}$$

$$x=0 \Rightarrow (SF)_B = 0 \Rightarrow (M)_B = 0$$

$$x=L \Rightarrow (SF)_A = \frac{wL^2}{2L} = \frac{wL}{2}$$

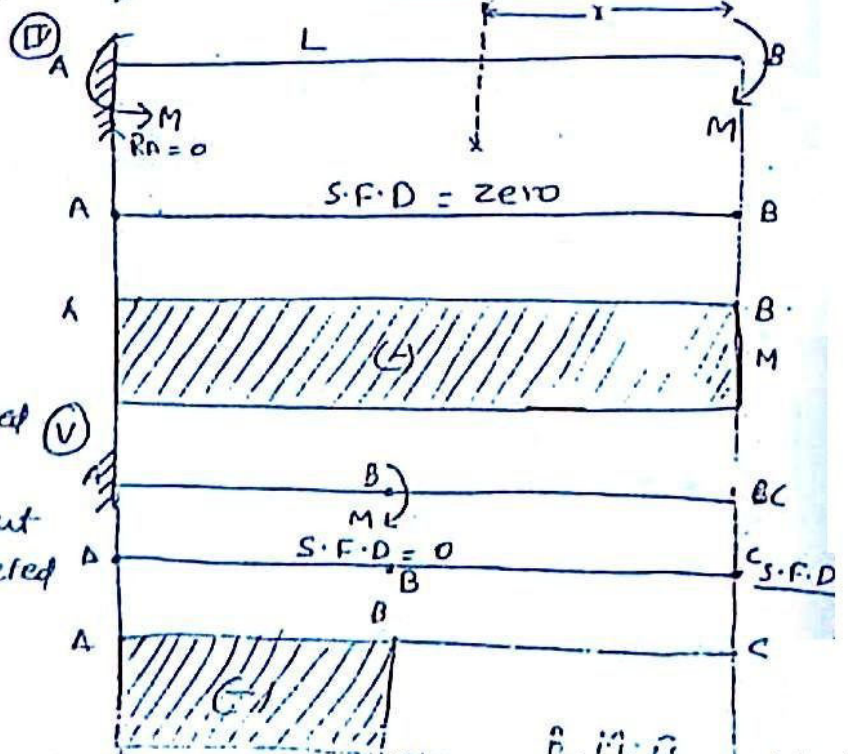
$$(M)_A = -\frac{wL^3}{6L} = -\frac{wL^2}{6}$$



Case-IV → Pure Bending:

$$(SF)_{xx} = 0, (M) = -M$$

This condition is valid for all the cases.



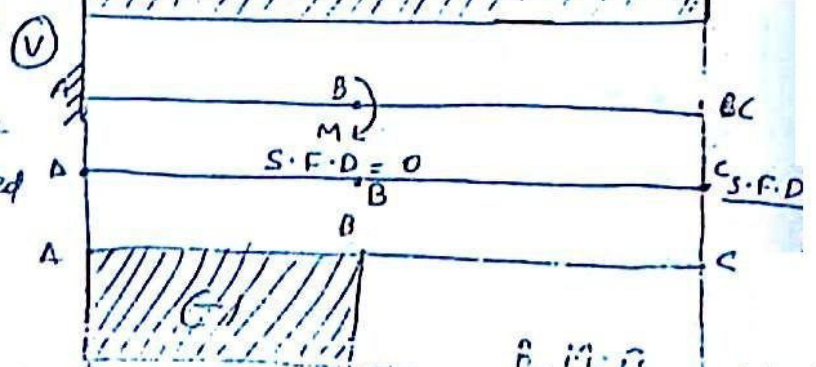
Case-V

$$M_C = 0$$

$$i_B = 0, M_B = -M$$

At Moment,  
critical Reaction  
always zero.

When concentrated  
M given take  
moment at point  
twice. Is it neglected  
and accepted



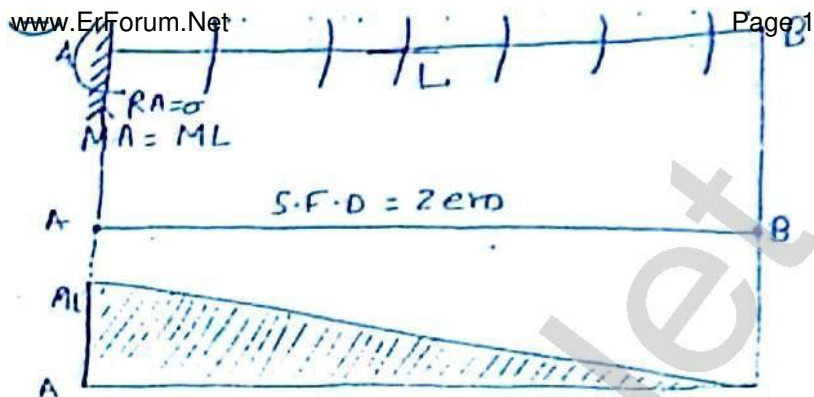


Case VI

$$M_B = 0$$

$$M_A = M_L$$

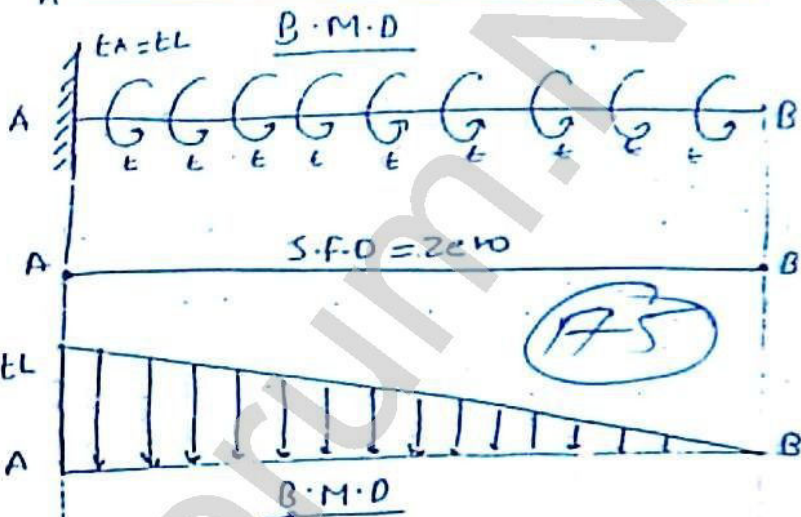
$$M_A = M_L - M_L = 0$$

Case VII

$$t_B = 0$$

$$t_A = t_L$$

$$t_A = t_L - t_L = 0$$



### Simply Supported Beam:-

Case-I

$$S_A = M/L$$

$$R_A + R_B = 0$$

$$R_A = -R_B$$

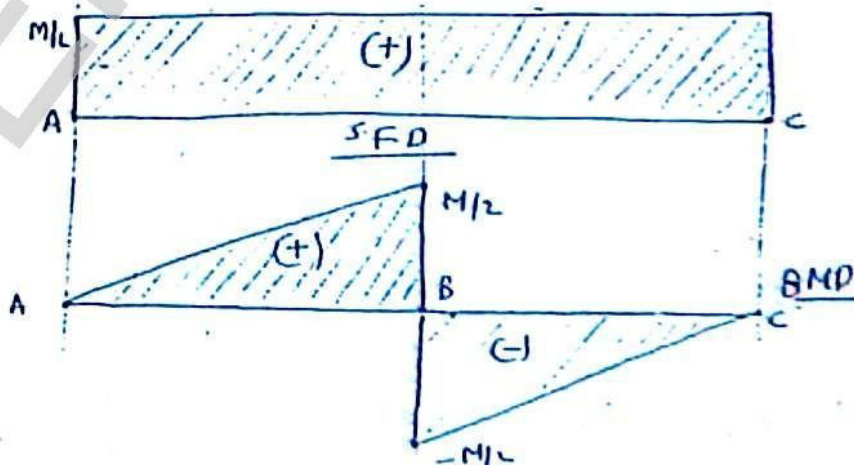
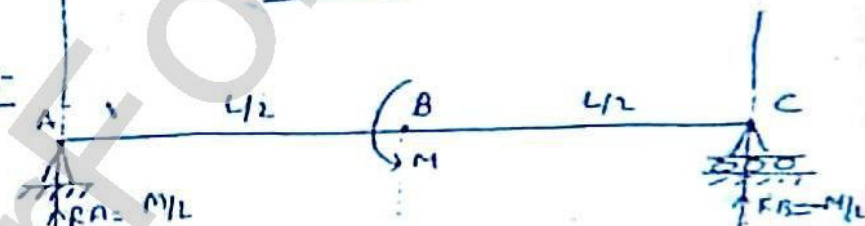
First neglected  
Concentrated  
Moment. Second  
time consider.

$$M_A = 0, M_B = 0$$

$$M_B = M/2 = \frac{M \times L}{2}$$

$$M_B = \frac{M \times L}{2} = \frac{M}{2}$$

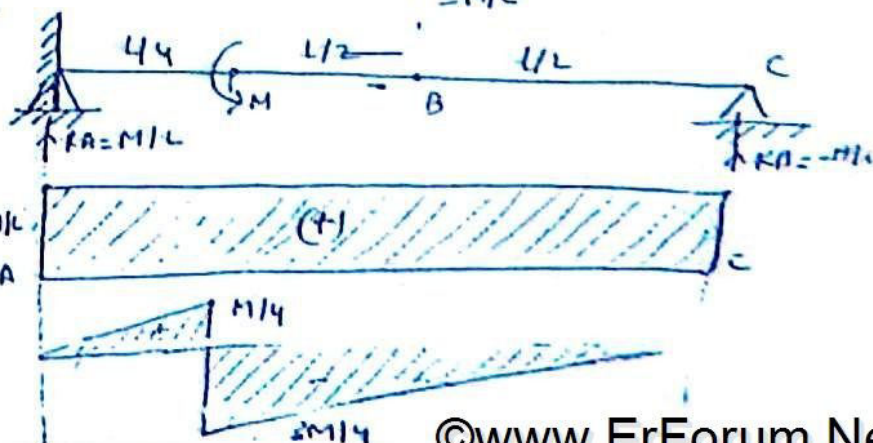
$$= \frac{M}{2} - M = -\frac{M}{2}$$



\* when simply s.b on concentrated moment →

(i) The Reaction are equal and opposite and they are equal to applied moment divided by length of beam.


The S.F.D is L with height is equal to Reaction.





(ii) BMD consist two triangle of opposite nature

iv) Point of contraflexure is point where concentrated moment is applied.



The diagram shows a horizontal beam with a concentrated moment  $M$  applied at a point. The beam is divided into two segments by the point of application of the moment. The point of contraflexure is indicated at the point where the moment is applied.

Case-II-

$$M_{X-X} = m_1 - m_2 = 0$$

Case III,



Case IV  $\rightarrow$

Q.5c V →

$$(SF)_{x_1} = \frac{\omega_1}{2} - \omega_2$$

$$M_{xx} = \frac{w_0 L}{2} (y) - \frac{w_0 y^2}{2}$$

$$x=0, \quad (SF)_A = \omega L/2$$

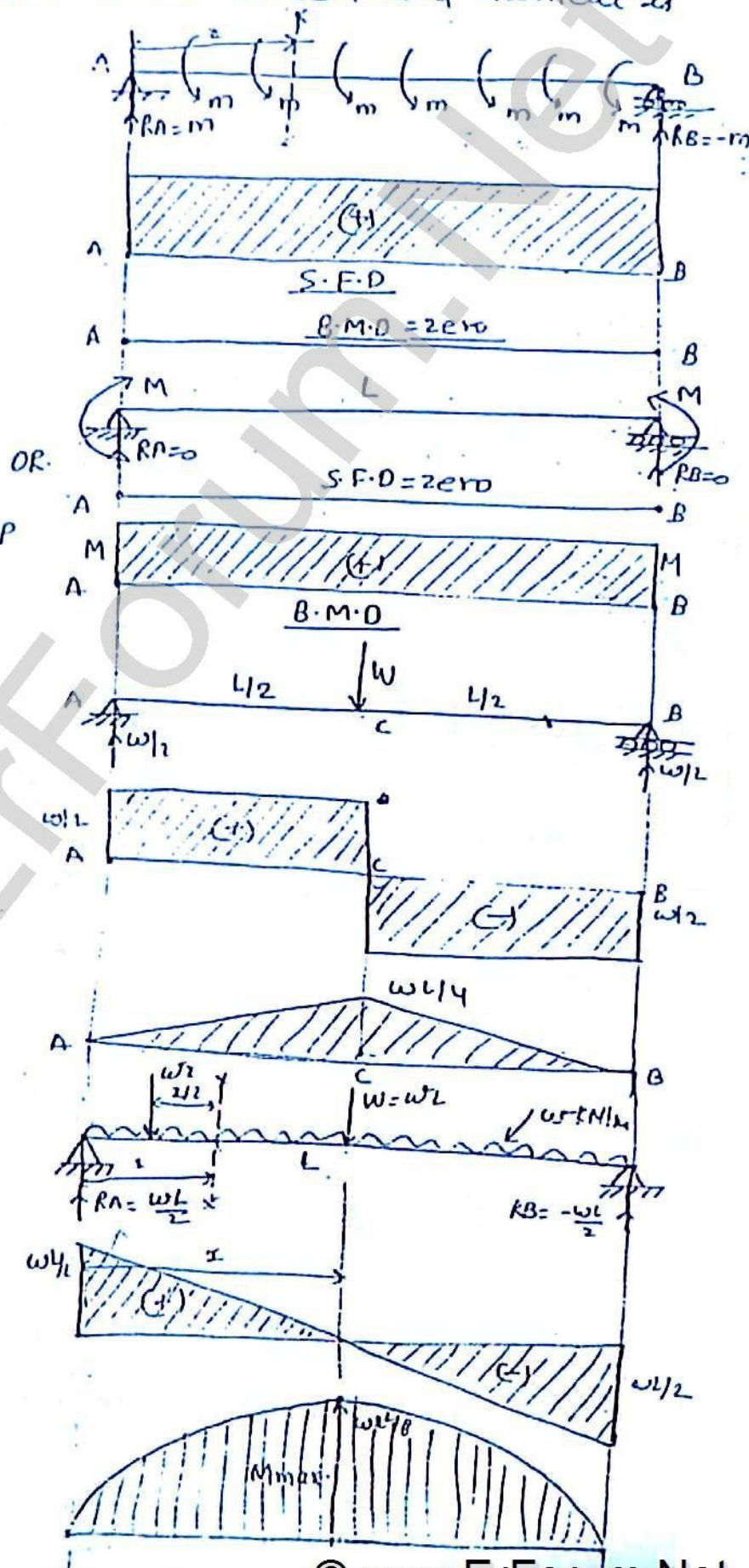
$$(M)_A = 0$$

$$X = L \begin{pmatrix} S \\ M \end{pmatrix}_B = - \begin{pmatrix} 104 \\ 2 \end{pmatrix}$$

$$(S.F)_1 = \frac{W_L}{L} - W_2 = 0$$

put  $x = 1/2$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2 \nabla^2 \psi}{2m}$$



Q4 when the SFD and BMD for simply supported beam over a entire portion.

Following Conclusion can be used in S.F.D and B.M.D.

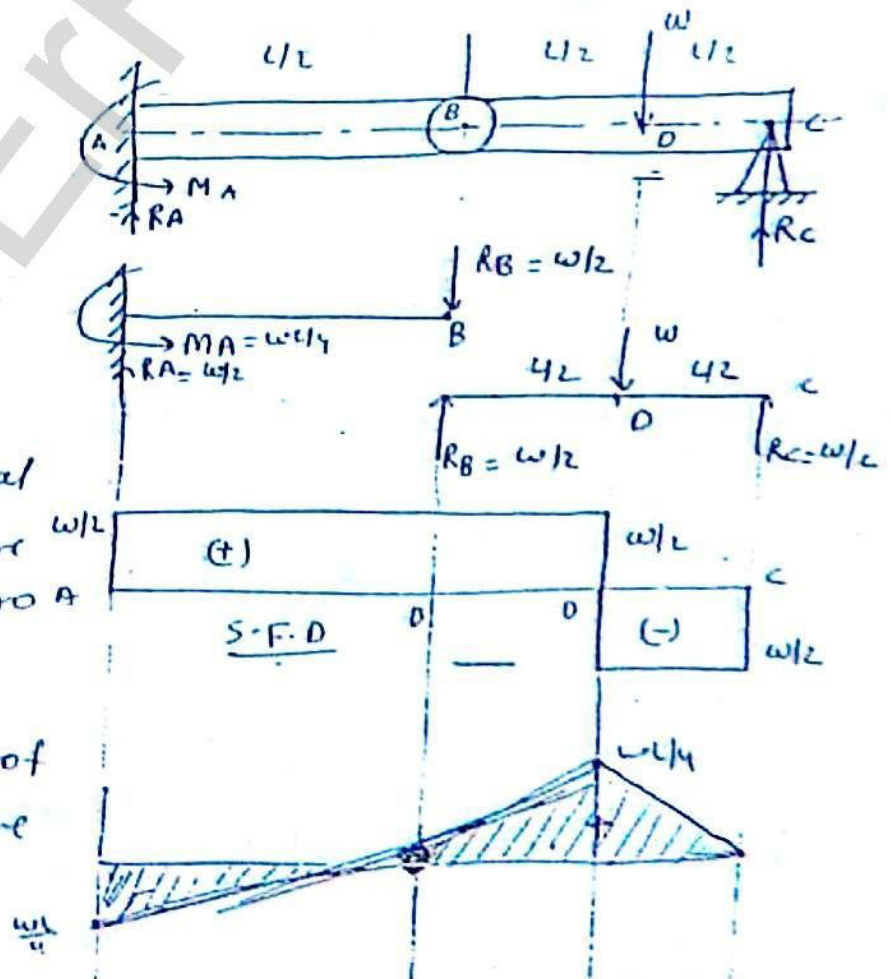
- i. Vertical reaction is given by height and the SFD at the support.
- ii. Moment Reaction at the support is given by the height of the BMD.
- iii. In presence of point load, there is a sudden change in shear force take
- iv. In presence of concentrated moment, there is sudden change in B.M.D.
- v. For pure bending S.F.D is a line considering is zero & B.M.D is a rectangle with an height is equal to applied moment.

### COMPOUND BEAMS →

$$\begin{aligned}
 M_A &= \frac{w}{2} \times \frac{3L}{2} - wL \\
 &= \frac{3wL}{4} - wL \\
 &= -\frac{wL}{4}
 \end{aligned}$$

In compound beam at internal hinge, action and reactions are equal and opposite & B.M is zero A because of free rotation.

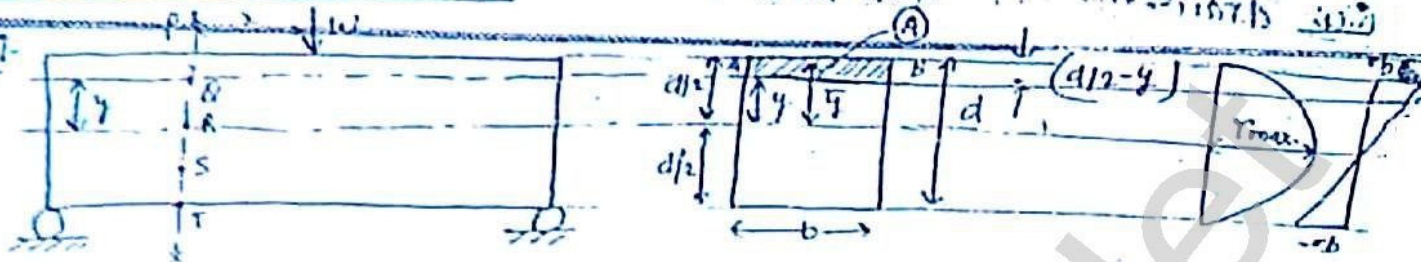
In compound beam, the point of contra flexure in a point where internal hinge is provided.





# SHEAR STRESSES IN BEAMS

Case I-



$$\sigma_b = \frac{M}{Z}$$

$$(\sigma_b)_{\max} = \frac{M_{\max}}{Z} \quad \text{--- (BMD)}$$

due to Transverse Shear load,  
S.F & B.M.

$$\tau_{AB} = \frac{F A \bar{y}}{I_b}$$

$$\bar{y} = y + \left( \frac{d}{2} - y \right)$$

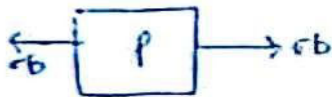
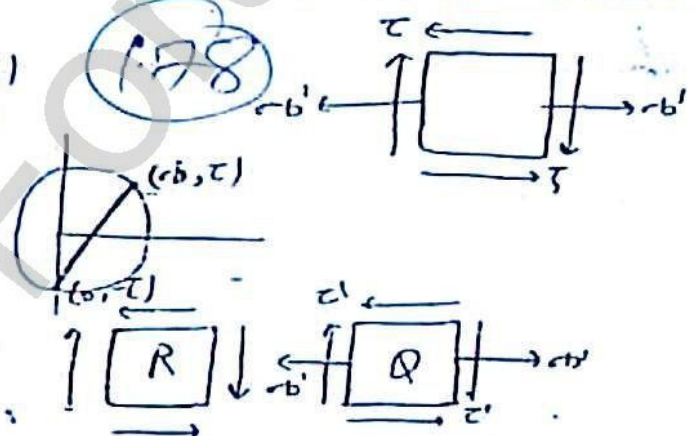
$$A = \left( \frac{d}{2} - y \right) b$$

- \* Shear force at extreme fibers will be zero and max at neutral axis.
- \* B.M at the extreme fibers will be max. and zero at neutral axis.

Mohr's Circle:

$$A = (\sigma_b', \tau)$$

$$B = (0, -\tau)$$



$$\tau_{\max} = \tau_{\text{avg}} \times k$$

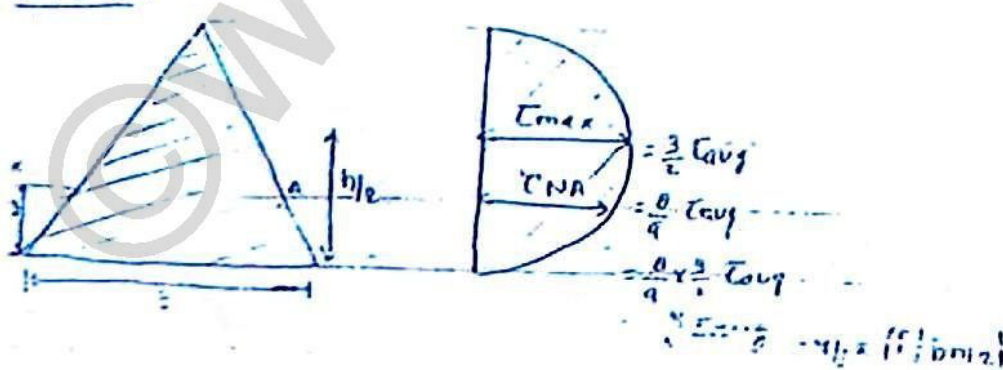
$$\tau_{\text{avg}} = \frac{F}{A} \quad \text{--- Shear force}$$

$$k = \text{Constant} \rightarrow k_{\text{rect}} = \frac{3}{2} \rightarrow \tau_{\text{avg}} = F/bd$$

$$k_{\text{circle}} = \frac{4}{3} \rightarrow \tau_{\text{avg}} = F/\pi d^2$$

$$k_{\text{triangle}} = \frac{3}{2} \rightarrow \tau_{\text{avg}} = F/(bh/2)$$

CASE-II



# Spring

Helical Spring  $\Rightarrow$  (Ref the Ref books for derivation)

$$\boxed{(\tau_{\max})_{\text{ind}} = \frac{8 P_{\max} \cdot D}{\pi d^3} \times k_w} \quad \left| \quad \tau_{\max} = \frac{T}{Z_p} = \frac{P d / 2}{\frac{\pi d^3}{16}} = \frac{8 P D}{\pi d^3} \right.$$

$D$  = Mean coil dia of spring

$d$  = dia of spring wire

$C$  = Spring Index =  $D/d$

$$\boxed{k = \frac{P}{\delta} \text{ N/mm}}$$

$$\boxed{k_w = \text{Wahl's factor} = k_{sh} \times k_c}$$

$k_{sh}$  = Shear stress correction factor

$k_c$  = curvature factor

Wahl's factor is consider the effect of direct shear stresses  
(load/Area) and curvature affect

$$[(\tau_{\max})_{\text{ind}}] = \frac{8 P_{\max} C}{\pi d^2} k_w \leq \tau_{\text{per}}$$

$$\frac{8 P_{\max} \cdot C \cdot k_w}{\pi d^2} \leq \frac{S_{ys}}{N} \text{ or } \frac{S_{yt}}{2N}$$

$$d \geq \text{--- mm}$$

$$\left. \begin{aligned} D &= C d \\ \tau_{\max} &= \frac{8 P D}{\pi d^3} \\ D &= D - d \end{aligned} \right\}$$



Max. deflection induced.

$$(Y_{\max})_{\text{ind}} = \frac{8PD^3n}{Gd^4}$$

$n = \text{no. of active coils}$

$n = ?$

$$k = P/s = P/Y_{\max}$$

$$D = cd \text{ put}$$

$$(Y_{\max})_{\text{ind}} = \frac{8Pc^3n}{Gd}$$

$$\frac{Gd}{8c^3n} = \frac{P}{Y_{\max}} = k$$

$$k = \frac{Gd}{8c^3n}$$

$$k \propto \frac{1}{n}$$

$$L_{\text{free}} = L_s + 1.15 Y_{\max}$$

$$L_{\text{free}} = nd + 1.15 Y_{\max}$$

$L_s = \text{solid length} = nd$

$$\frac{L_f}{D} \leq 3.5 \quad \text{No Buckling}$$



$$s = s_1 + s_2$$

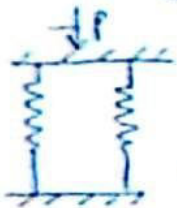
$$P_1 = P_2 = P$$

$$\frac{P}{k} = \frac{P_1}{k_1} + \frac{P_2}{k_2}$$

or

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

series spring



$$s_1 = s_2$$

$$P = P_1 + P_2$$

$$k = k_1 + k_2$$

parallel spring

## Deflection Of Beam

\* Equation of  $\gamma_{\max}$  and  $\theta_{\max}$ .  
 Linear displacement angular displacement

at the simply supported ends

Slope ( $\theta$ ) = max  
 Deflection ( $\gamma$ ) = 0

5-5 Beam,  $\gamma = \max$   
 $\theta = 0$

Ex-  $\theta \rightarrow$  Slope equation = 0  
 Calculate value of  $x = x^r$

$$\gamma_{\max} = \{\gamma(x)\}_{x=x^r} = \dots$$

Ex-  $\theta = 0$ ,  $\gamma = \max$  at center (D)

Ex- at fixed end A, in cantilever  
 deflection = 0  
 (Slope) $_A = 0$

Ex- fixed ends -  $\gamma = \theta = 0$   
 free ends -  $\gamma = \theta = \text{maximum}$

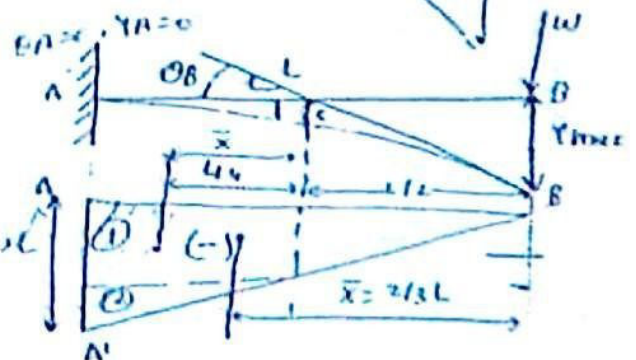
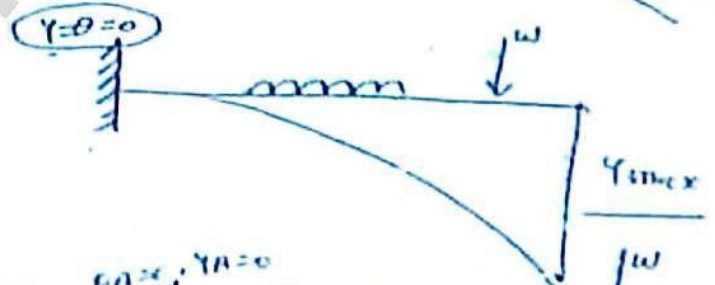
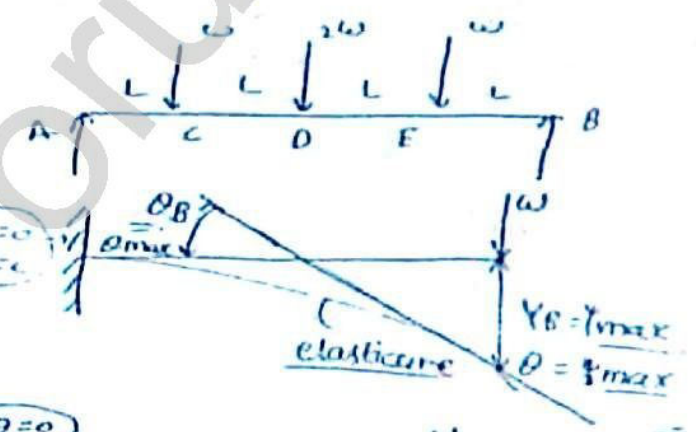
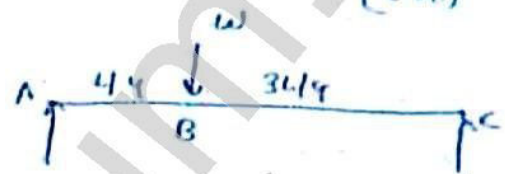
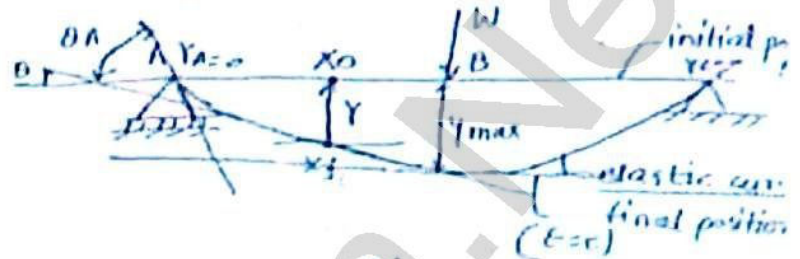
### 0 MOMENT AREA METHOD :- Case-I

I theorem  $\rightarrow$

$$\theta_B - \theta_A = \frac{1}{EI} \times \left[ \text{Area of B.M. Diagram} \right] \omega L$$

$$\theta_{\max} - 0 = \frac{1}{EI} \times \left[ \frac{1}{2} \times L \times \omega L \right]$$

$$\theta_{\max} = \frac{\omega L^2}{2EI} \quad (C-W)$$



Ref point  $\theta = 0$

Origin point  $\theta = \text{some value}$



II theorem

$$Y_B - Y_A = \frac{1}{EI} [\text{Moment of area of B.M.D. b/w } A \text{ to } B]$$

$$Y_B - Y_A = \frac{1}{EI} [A \bar{x}]$$

$$= \frac{1}{EI} \left[ \frac{WL^2}{2} \times \frac{2L}{3} \right]$$

$\bar{x}$  = distance of centroid of the area of B.M.D. from origin.

$$Y_{\max} = + \frac{WL^3}{3EI}$$

$$\theta_C = \theta_A = \frac{1}{EI} [A_1 + A_2]$$

$$Y_C - Y_A = \frac{1}{EI} [A_1 \bar{x}_1 + A_2 \bar{x}_2]$$

Origin  $\rightarrow$  Non zero slope

Ref. point  $\rightarrow$  point of zero slope.

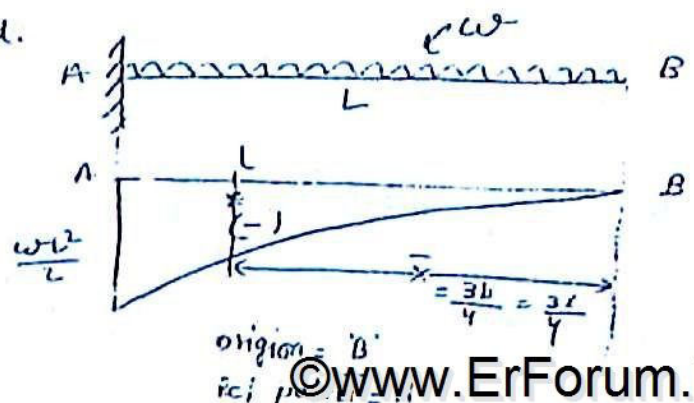
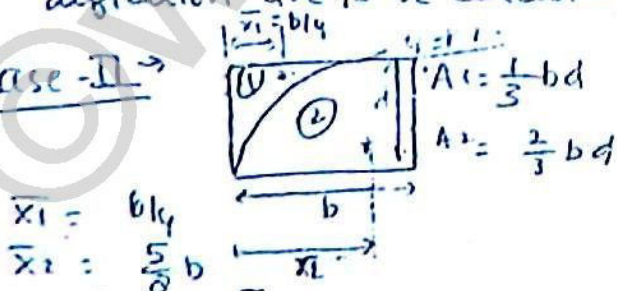
Ist theorem states that  $\rightarrow$  "difference of slope at any two point of beam is equal to  $1/EI$  times the area of B.M.D within those points."

IInd theorem  $\rightarrow$  "Diff. of deflection at any two points of beam is equal to the  $1/EI$  time of the moment of area of B.M.D b/w those points."

Note: 1. select the two point in such a way that one point should be a point of zero slope (reference point) and other point should be point of non-zero slope (origin).

Consider a point of non-zero slope such that where slope & deflection are to be calculated.

Case - II  $\rightarrow$



$$\theta_B - \theta_A = \frac{1}{EI} \left[ \frac{1}{3} \times L \times \frac{WL}{2} \right] \Rightarrow \theta_{\max} = \frac{WL^3}{6EI}$$

$$y_{\max} = \frac{1}{EI} \left[ \frac{WL^3}{6} \times \frac{3L}{4} \right]$$

$$y_{\max} = \frac{WL^4}{8EI}$$

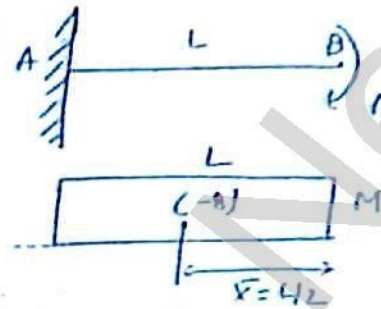
$$\theta_B - \theta_A = \frac{1}{EI} [ML]$$

$$\theta_B = \theta_{\max} = \frac{ML}{EI}$$

$$y_B - y_A = \frac{1}{EI} \left[ ML \times \frac{L}{2} \right]$$

$$y_{\max} = \frac{ML^2}{2EI}$$

Case-III →



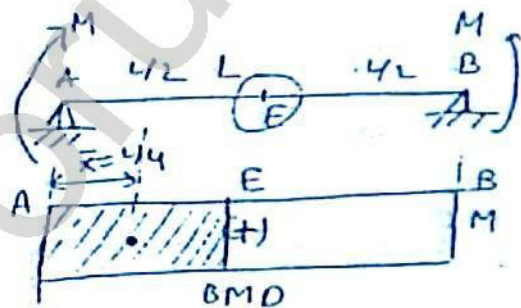
Case IV → Origin = point of non zero slope = A & B

Ref point → point of zero slope = E

$$\theta_A - \theta_E = \frac{1}{EI} M \cdot \frac{L}{2}$$

$$\theta_{\max} = \frac{ML}{2EI}$$

$$y_A - y_E = \frac{1}{EI} \left[ \frac{ML}{2} \times \frac{1}{4} \right] = + \frac{ML^2}{8EI} \quad (\text{downward})$$



Case V - Origin → A & C

Ref point → B

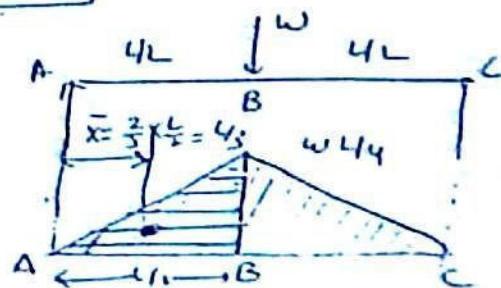
$$\theta_A - \theta_B = \frac{1}{EI} \left[ \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4} \right]$$

$$\theta_{\max} = \frac{WL^2}{16EI}$$

$$y_A - y_B = \frac{1}{EI} \left[ \frac{WL^2}{16} \times \frac{L}{3} \right]$$

$$0 - y_B = \frac{WL^3}{48EI} \Rightarrow$$

$$y_B = \frac{-WL^3}{48EI}$$





Case-VI

origin = A

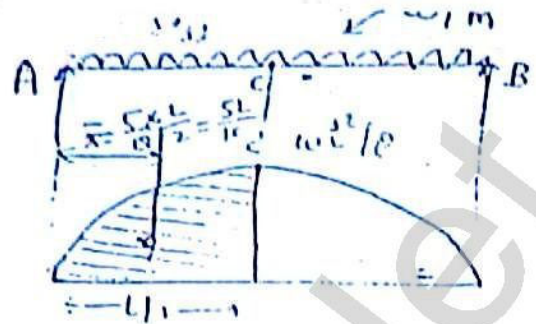
Ref point = C

$$\theta_A - \theta_C = \frac{1}{EI} \left[ \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8} \right]$$

$$\theta_{max} = \frac{wL^3}{24EI}$$

$$y_A - y_C = \frac{1}{EI} \left[ \frac{wL^3}{24} \times \frac{5L}{16} \right]$$

$$y_{max} = -\frac{5wL^4}{384EI}$$



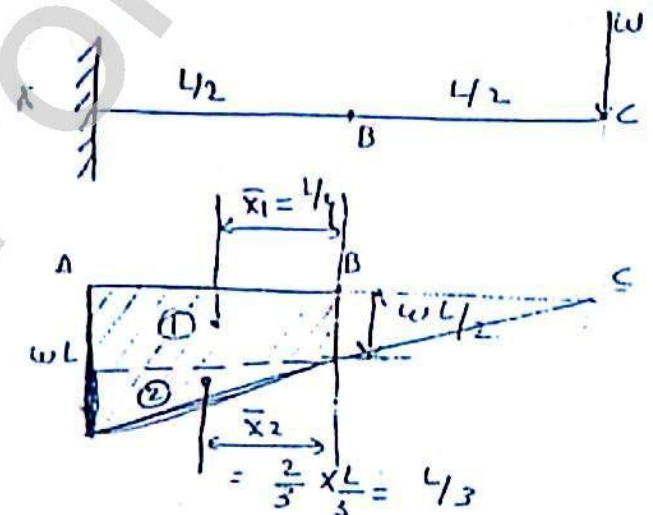
Q. for a cantilever beam shown in fig. Det.  $\theta = ?$ ,  $\delta = ?$  at the mid point of beam.

origin = B

Ref point = A

$$\begin{aligned} \theta_B - \theta_A &= \frac{1}{EI} [A + A_1] \\ &= \frac{1}{EI} \left[ \frac{L}{2} \times \frac{wL}{2} + \frac{1}{2} \times \frac{L}{2} \times \frac{wL}{2} \right] \\ &= \frac{1}{EI} \left[ \frac{wL^2}{4} + \frac{wL^2}{8} \right] \end{aligned}$$

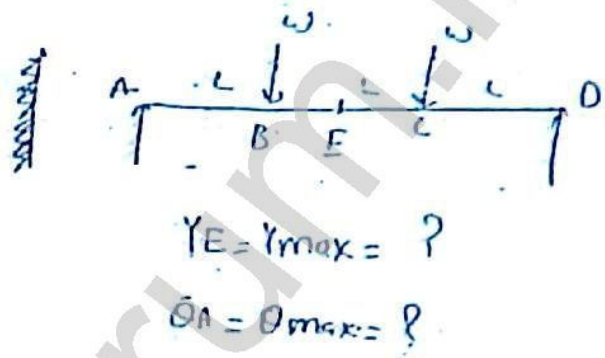
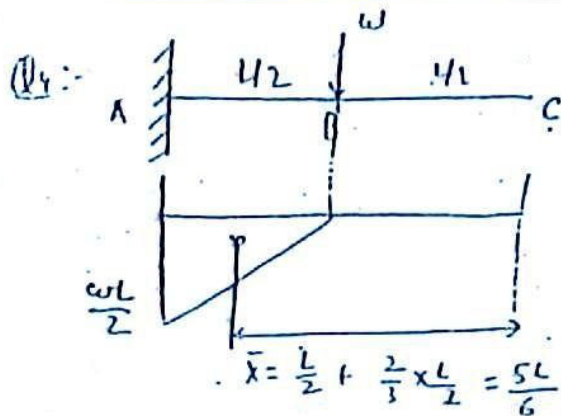
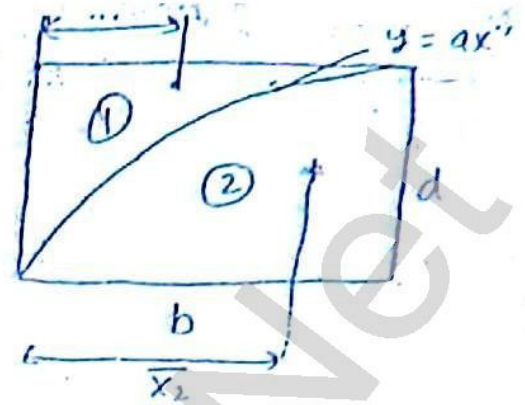
$$\theta_{max} = \frac{3wL^2}{8EI}$$



$$\begin{aligned} y_B - y_A &= \frac{1}{EI} \left[ \frac{5wL^3}{40} \times \frac{L}{4} + \frac{wL^2}{8} \times \frac{L}{3} \right] \\ &= \frac{1}{EI} \left[ \frac{wL^3}{16} + \frac{wL^3}{24} \right] \end{aligned}$$

$$y_{max} = y_B = \frac{5wL^3}{40EI}$$

$$\begin{aligned}
 A_1 &= \left( \frac{1}{n+1} \right) bd \\
 \bar{x}_1 &= \left( \frac{1}{n+1} \right) b \\
 A_2 &= \left( \frac{n}{n+1} \right) bd \\
 \bar{x}_2 &= \left[ \frac{n+3}{n(n+1)} \right] b
 \end{aligned}$$



D.T. Method:-

$$M = EI \frac{d^2 y}{dx^2} \rightarrow \text{①}$$

more careful.

$$M_{int} + C_1 = EI \left( \frac{dy}{dx} \right)$$

$C_1 = ?$  gives  $\theta_{max} = ?$






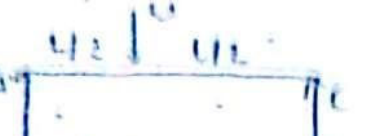
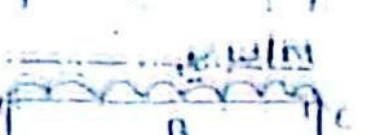

$$\int M_{int} + C_2 = EI y$$

$C_2 = ?$   $y_{max} = ?$

$$F = \frac{dM}{dx} = EI \frac{d^3 y}{dx^3} \rightarrow \text{②}$$

$$W = \frac{dF}{dx} = EI \frac{d^4 y}{dx^4} \rightarrow \text{③}$$



Type of Beam	$\theta_{max}$	$y_{max}$
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$
	$\frac{WL^2}{6EI}$	$\frac{WL^4}{8EI}$
	$\frac{WL^3}{24EI}$	$\frac{WL^4}{30EI}$
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
	$\theta_A \text{ or } \theta_B = \frac{ML}{2EI}$	$y_C = \frac{ML^2}{6EI}$
	$\theta_{A \text{ or } C} = \frac{WL^2}{16EI}$	$y_B = \frac{WL^3}{48EI}$
	$\theta_{A \text{ or } C} = \frac{WL^3}{24EI}$	$y_B = \frac{5}{384} \frac{WL^4}{EI}$
	$\theta_C = \frac{Wb}{3EIL} [a^2 - ab]$	$y_B = \frac{Wb^2}{3EIL}$