

156

11

150

-: HAND WRITTEN NOTES:-

OF

ELECTRICAL ENGINEERING



-: SUBJECT:-

ELECTROMAGNETIC

THEORY

11

321

2

EMT

①

Electromagnetics :-

electrostatics

static electrical fields

$$\vec{E}, \vec{D} \neq f(t)$$

Magnetostatics

static Magnetic fields

$$\vec{B}, \vec{H} \neq f(t)$$

time varying electrical & magnetic fields
 $\vec{E}, \vec{D}; \vec{B}, \vec{H} \neq f(t)$

Static electrical field intensity
 (N/C ; V/m)

electrical field density
 elec. displacement vector (C/m^2)

$$\vec{D} = \epsilon \vec{E}$$

permittivity
 of medium
 (F/m)

$$\epsilon = \epsilon_0 \epsilon_r$$

$\rightarrow \epsilon_0$ --- permittivity
 of free space.

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$= \frac{1}{36\pi} \times 10^{-9} \text{ F/m}$$

$$\epsilon_r \geq 1$$

$\epsilon_r = 1$ --- for free space,
 > 1 --- for any other
 dielectric

ϵ_r = Relative permittivity
 of medium
 (unitless)
 --- dielectric
 constant

$$\text{unless specified}$$

$$\epsilon_r = 1$$

\vec{D} constant irrespect
 of type of medium.

Static Magnetic fields :-

(4)

 \vec{H}

Mag. field intensity --- (A/m)

 \vec{B} Mag. flux density --- (wb/m² = T)

$$\vec{B} = \mu \vec{H}$$

permeability of
medium
(H/m)

$$\mu = \mu_0 \mu_r$$

μ_0 = permeability of free
space

$$= 4\pi \times 10^{-7} \text{ --- H/m}$$

μ_r = relative permeability
of medium

$$\mu_r \leq 1 \text{ --- unit less}$$

$\mu_r < 1$ --- diamag.

$= 1$ --- non-mag.

> 1 --- paramag.

$\gg 1$ --- ferromag.

$\mu_r \leq 1$

non-ferromag.

$\mu_r \gg 1$

ferromag.

unless specified

$$\mu_r \leq 1$$

Water (diamag.) ; $\mu_r = 0.999991$

Air (paramag.) ; $\mu_r = 1.0000006$

Cobalt (ferromag.) ; $\mu_r \approx 250$

Fe (0.5% impure) ; $\mu_r \approx 5000$

Fe (0.05% impure) ; $\mu_r \approx 200000$

pure.

Time Varying elect. & mag. fields :-

(5)

$$\vec{E}, \vec{D}; \vec{B}, \vec{H} = f(t)$$

Imp. points :-

① The maxwell's equations are a set of four eqs.

Which a Relationship b/w time varying ele. & mag. fields

② When ever any wave propagate then the ele. field, mag. field & direction of propagation are mutually perpendicular each other.

$$(\vec{E} \perp \vec{H}) \perp \text{direction of propagation}$$



TEM wave

Transverse em waves (uniform plane waves)

③ When there is large mismatching b/w the length of filament & wavelength of operation at low freq. then entire power dissipation in element.

④ at high freq the length is comparable to the wavelength of operation, then the power is radiated through that element

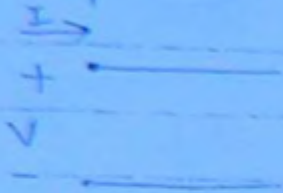
⑤ at low freqs the depth of penetration is high. & therefore we use thick conductor, whereas at high frequencies depth of penetration is low & there for thin conductors are use

the power from the transmitter to the antenna is transported with the help of co axial transmission line & a power wave transmission line.

Transmission line :-

(6)

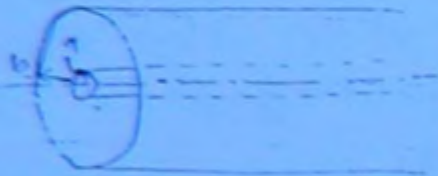
1 parallel wire transmission line



used as:

HT wires (high tension)
telephone wires

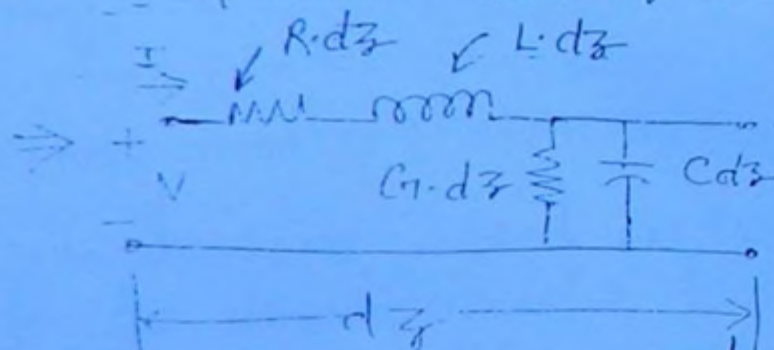
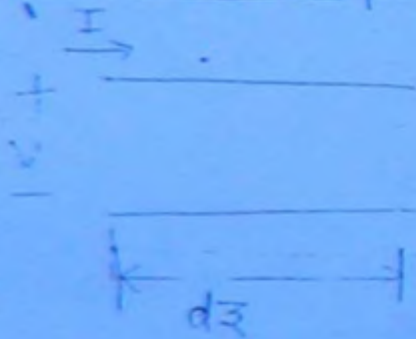
Co-axial T-L



used in

CATV

as CRO load

Distributed Parameters equivalent ckt. of T.L. :-

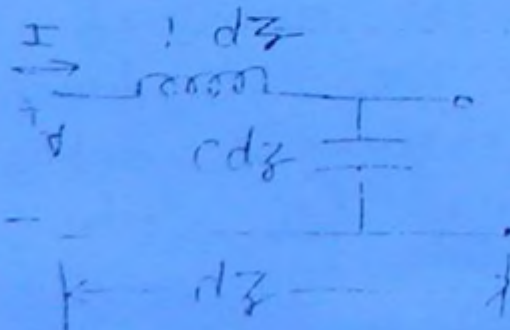
--- for lossy line

primary
constant
of the
line

R	---	Ω/m
L	---	H/m
C	---	F/m
G	---	S/m

lossless line

$$\begin{cases} R=0 \\ G=0 \end{cases}$$



3 imp. points :-

Date _____

① Due to inherent properties of lossy transmission line it is assumed that R, L, G & C are effectively distributed along the entire length of T/L ⑦

② Due to lossy nature of the line & due to finite conductivity of the line

∴ Some losses occur on the T/L due to current flow along the line.

The Resistor R is responsible for total power dissipation taking place due to lossy nature of the line

③ due to current flow & due to mag. fields some mag. energy will exist:-

The Inductor L is responsible for total magnetic storage in the T/L

④ Due to potential difference b/w the two lines, some electrical fields & therefore some ele. energy is finite in the T/L. The capacitor C is responsible for the total electrical energy stored in the transmission line.

⑤ The medium of dielectric b/w the two lines is in general of lossy nature.

Some power dissipation takes place as the current leaks through the lossy dielectric.

G is responsible for total power dissipation taking place due to lossy nature of the dielectric b/w the T/L

- ⑥ for a lossless line R & G are zero
- ⑦ as the voltage or current waveform are introduced on the TL the wave is attenuated exponentially and therefore the magnitude of the voltage & current will decrease as the wave propagates along the line.

(8)

Behaviour of V & I along the line:-

$I \rightarrow$
+ • —————

— • ————— \rightarrow z

$$V = \underbrace{V^+ e^{-\gamma z}}_{\substack{\text{propagating} \\ \text{along } +z \\ \text{Incident wave}}} + \underbrace{V^- e^{+\gamma z}}_{\substack{\text{propagating} \\ \text{along } -z \\ \text{Reflected wave}}}$$

V^+ ----- Amplitude of voltage wave propagating along $+z$ direction.

V^- ----- Amplitude of wave propagating along $-z$

γ ----- propagation const.
(Complex)

$\gamma = \alpha + j\beta$

α ----- attenuation const. (nepers/m)

β ----- phase const. ($^\circ$ / m)

$$I = \frac{1}{Z_0} \left(\underbrace{V^+ e^{-\gamma z}}_{\text{incident wave}} - \underbrace{V^- e^{+\gamma z}}_{\text{reflected wave}} \right)$$

(9)

↓
due to reverse direction of current

Z_0 --- Characteristic Impedance.

$$Z_0 ; \gamma (\equiv \alpha, \beta)$$

--- Secondary Const of the line.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

Basic features: -

- ① as the voltage or the current waveform is impressed on a lossy line it's subjected to attenuation ~~as~~ well as phase change.

Therefore the magnitude of Voltage & current will decrease exponentially as the waves travel along the line.

if $\alpha = 0$ the wave propagation takes place without any attenuation.

therefore the magnitude of Voltage & current will remain const. at all the points along the line.

- ② if $\beta = 0$ there is propagation & line is entirely attenuate.

3) The char. of the line represent the ratio b/w Voltage & current at any point of any infinite long line (10)

4) The propagation const γ & the char. imp. is zero are represented by the secondary const. of the line.

Since depend upon the primary const. of R, L, C & G of the line.

Lossless line :-

$$R=0 \\ G=0 \\ Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} \quad \therefore \text{real; Const.} \\ (\neq f(\omega))$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \\ = j\omega \sqrt{LC} = \alpha + j\beta$$

$$\alpha = 0 \\ \beta = \omega \sqrt{LC}$$

$$\gamma = j\beta$$

$$\beta = \omega \sqrt{LC}$$

$$= \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$v_p = \frac{1}{\sqrt{LC}} \quad \text{--- Const.}$$

Summary:-

(11)

--- lossy line

$$V = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-\gamma z} - V^- e^{+\gamma z})$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

R, L, G, C --- primary const.

$\gamma (= \alpha, \beta); Z_0$ --- sec. const.

--- lossless line

$$R = G = 0$$

$$\alpha = 0$$

$$\gamma = j\beta$$

$$\beta = \omega \sqrt{LC}$$

$$\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$v_p = \frac{1}{\sqrt{LC}} \text{ --- Const.}$$

$$Z_0 = \sqrt{\frac{L}{C}} \text{ --- real; Const; } \neq f(\omega)$$

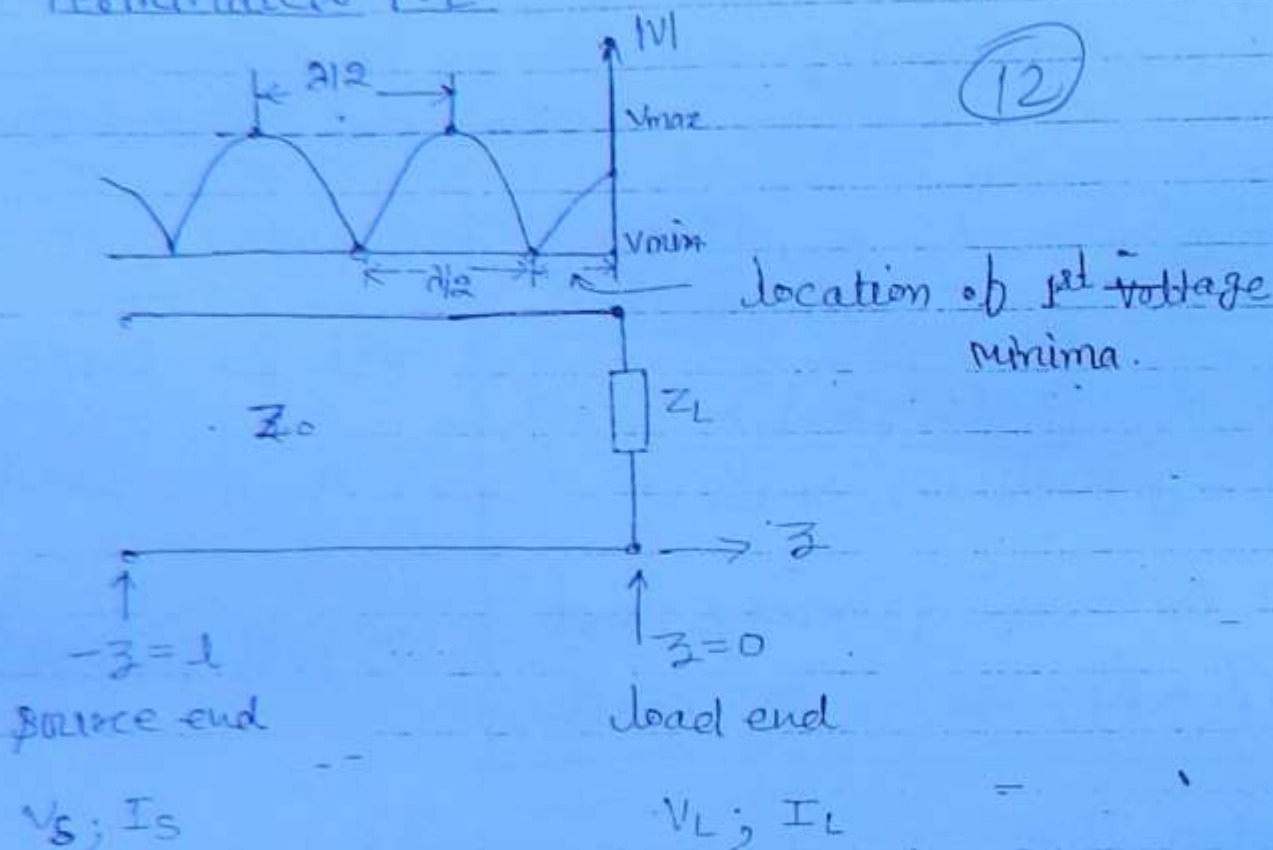
$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z})$$

3m/

Terminated T.L.

(12)

Case 1 : $Z_L \neq Z_0$

--- mismatched line.

- ① Max. power is not transfer to the load.
- ② Incident & Reflected waves will exists.
- ③ Standing wave pattern will exists & there for Maxima & minima will exists along the line. there for there is a ~~an~~ form of Standing wave pattern.
- ④ Coefficient of Reflection has a finite value.
- ⑤ due to V_{max} & V_{min} Voltage standing wave Ratio is finite.

$$VSWR = S = \frac{V_{max}}{V_{min}}$$

Voltage Standing wave Ratio

Case: 2:

$$Z_L = Z_0$$

(13)

---- Matched line.

- ① Maximum power is transferred from source to load.
- ② Reflection coefficient is zero.
- ③ There is no reflected waves, no standing wave pattern, $V_{max} = V_{min}$ & therefore the voltage along the line is const. at all the points.
- ④ The VSWR has a min. value of unity.

$$S = \frac{V_{max}}{V_{min}} = 1$$

---- S_{min}

$$\text{Since } V_{max} = V_{min}$$

In brief:

① Reflection coefficient :- $\Gamma = \frac{V^-}{V^+}$ Complex

$$|\Gamma| e^{j\theta} = \rho e^{j\theta}$$

② Transmission coefficient :-

$$T = \frac{V_L}{V^+}$$

③ $VSWR (S) = \frac{V_{max}}{V_{min}}$

④ $Z_{in} = \frac{V_s}{I_s} = \frac{V}{I} \Big|_{-z=l}$

assume : line is lossless

① $V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z})$$

$$Z = 0$$

$$V_L = V^+ + V^- \quad \text{--- (1)} \quad (14)$$

$$I_L = \frac{1}{Z_0} (V^+ - V^-) \quad \text{--- (2)}$$

$$\frac{V_L}{I_L} = Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$\boxed{\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

if $\frac{Z_L}{Z_0} = \bar{Z}_L$ Normalized load impedance.

$$\Gamma = \frac{\bar{Z}_L / Z_0 - 1}{\bar{Z}_L / Z_0 + 1} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$$

Transmission Coefficient :-

$$T = V_L / V^+$$

from equation (1)

$$\frac{V_L}{V^+} = 1 + \frac{V^-}{V^+}$$

$$\boxed{T = 1 + \Gamma} = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$T = \frac{2 Z_L}{Z_L + Z_0} = \frac{2 \bar{Z}_L}{\bar{Z}_L + 1}$$

$$S = \frac{V_{max}}{V_{min}} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

$$= \frac{1 + |V^-|/|V^+|}{1 - |V^-|/|V^+|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1+\rho}{1-\rho}$$

(15)

$$\rho_{\min} = 0 ; S = 1$$

$$\rho_{\max} = 1 ; S_{\max} = \infty$$

$$(4) Z_{in} = \frac{V}{I} \Big|_{-z=l}$$

$$Z_{in} = Z_0 \frac{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}{V^+ e^{-j\beta z} - V^- e^{+j\beta z}} \Big|_{-z=l}$$

$$e^{\pm j\beta l} = \cos \beta l + j \sin \beta l$$

$$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

Summary:-

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 1}{Z_L + 1}$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

$$1 - \Gamma = \frac{2Z_L}{Z_L + Z_0} = \frac{2\bar{Z}_L}{\bar{Z}_L + 1}$$

$$S = \frac{1+\rho}{1-\rho} ; \rho = \frac{S-1}{S+1}$$

$$0 \leq \rho \leq 1$$

$$1 \leq S \leq \infty$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

ex: A lossless line of length l & characteristic impedance is zero is terminated by a load impedance Z_L . Calculate:

- ① Input Impedance.
- ② Reflection coefficient.
- ③ VSWR when :-

(16)

$$Z_L = 0 \quad \text{--- SC line}$$

$$Z_L = \infty \quad \text{--- o.c. line}$$

$$Z_L = Z_0 \quad \text{--- Matched line}$$

$$Z_L = jX \quad \text{--- purely reactive load.}$$

Case: 1

$$Z_L = 0$$

--- SC line

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$= jZ_0 \tan \beta l$$

--- Purely reactive

--- S.C. stub line

$$Z_{in} = [Z_{sc} = jZ_0 \tan \beta l]$$

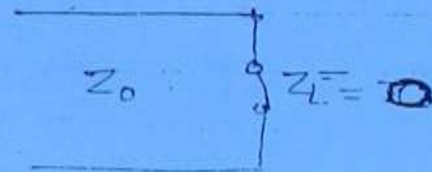
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = \frac{V^-}{V^+}$$

$$\Gamma = \Gamma e^{j\theta} = -1$$

$$\Gamma = 1$$

$$\theta = \pi$$

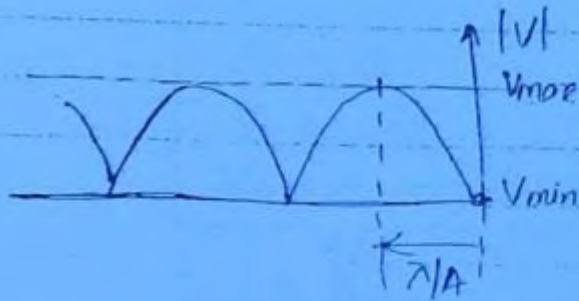
$$\begin{cases} e^{j\theta} = e^{+j\pi} = \cos \pi + j \sin \pi \\ = -1 \end{cases}$$



$$S = \frac{1+\rho}{1-\rho} = \infty = \frac{V_{\max}}{V_{\min}}$$

(17)

Minima is located at the load end.



s.c line :-

$$\begin{aligned} Z_L &= 0 \\ \Gamma &= -1 \\ \theta &= 180^\circ \\ \rho &= 1 \\ S &= \infty \\ Z_{sc} &= j Z_0 \tan \beta l \end{aligned}$$

Features :-

- 1) The input imp. is purely reactive in nature. for the shortest length of the line this imp. is inductive nature.
- 2) A stub line is a portion of line which has been s.c at the load end & has purely reactive app. imp.
- 3) A short circuit stub can be used for matching Transmission line with the load imp. for max. power transfer.
- 4) The reflection voltage & incident voltage are equal in magnitude but are phase shifted by 180° .
- 5) The voltage minima occurs at the load end & the 1st voltage maxima occurs at a distance of $\lambda/4$ from the load end.

Case: 2

$$Z_L = \infty$$

oc line

(18)

$$Z_{in} = Z_0 \frac{-Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$= Z_0 \frac{1 + j(Z_0/Z_L) \tan \beta l}{(Z_0/Z_L) + j \tan \beta l}$$

$$Z_{oc} = -jZ_0 \cot \beta l$$

----- purely reactive.
----- oc stub line.

$$Z_{sc} = jZ_0 \tan \beta l$$

$$Z_{oc} = -jZ_0 \cot \beta l$$

$$Z_{sc} \cdot Z_{oc} = Z_0^2$$

$$\rightarrow Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1 = \frac{V^-}{V^+}$$

$$\Gamma = \rho e^{j\theta} = 1$$

$$\rho = 1$$

$$\theta = 0$$

$$S = \frac{1 + \rho}{1 - \rho} = \infty = \frac{V_{max}}{V_{min}}$$

----- V_{max} occurs at load end.

O.C Line :-

$$Z_L = \infty$$

$$\Gamma = 1$$

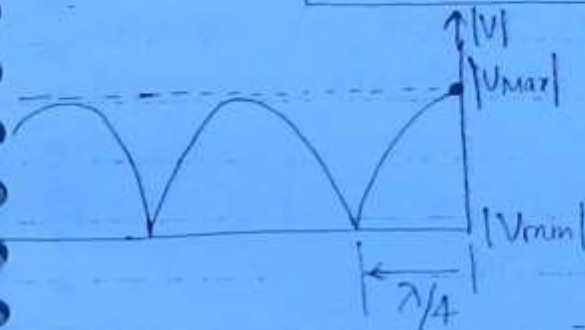
$$\rho = 1$$

$$\theta = 0^\circ$$

$$S = \infty$$

$$Z_{oc} = -jZ_0 \cot \beta l$$

(79)

Features:

- ① The input imp. is purely Reactive & for a shortest length of the line it's capacitive in nature.
- ② The o.c. stub line can be used to match any transmission line with the load imp. for max. power transfer.
- ③ The reflected & incident voltages has same magnitude and are in-phase.
- ④ Voltage maxima occurs at the load end.
- ⑤ When the line is first s.c. & then o.c. the voltage minima shifted by distance $\lambda/4$ from the load end towards a source end.
- ⑥ The charac'tic. imp. of the line is a geometric mean of input imp of the line when it's s.c. & then o.c.
- ⑦ An Impedance Inversion takes place when the line is 1st s.c. & then o.c & vice-versa.

Therefore inductive Imp. is transformed into capacitive inductive & vice-versa.

(20)

case: 3

$$Z_L = Z_0$$

Matched line

$$Z_{in} = Z_0 \frac{\overset{L \rightarrow Z_0}{Z_L + j Z_0 \tan \beta l}}{\underset{L \rightarrow Z_0}{Z_0 + j Z_L \tan \beta l}}$$

$$Z_{in} = Z_0$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 = \frac{V^-}{V^+}$$

$$\Gamma = e^{j\theta} = 0$$

$$\theta = 0$$

$$\theta = \text{indeterminate } (0 \leq \theta \leq 360^\circ)$$

$$= 0^\circ (\text{min})$$

$$S = \frac{1+\Gamma}{1-\Gamma} = 1 = \frac{V_{\max}}{V_{\min}}$$

$$V_{\max} = V_{\min}$$

Matched line :-

$$Z_L = Z_0$$

$$Z_{in} = Z_0$$

$$\Gamma = 0$$

$$\rho = 0$$

$$\theta = 0^\circ (\text{min})$$

$$S = 1$$

feature:

- ① A perfectly matched line behaves as infinity long line. since in each case the o/p imp. of the line is equal to the chg. imp. of the line.
- ② There is no reflected waves, reflection coefficient is zero, VSWR has min. value of unity.
- ③ Maximum pow is transferred to the load so that $V_{max} = V_{min}$, there is no standing waves & there for the voltage along the line is uniform at all the points.

(2)

Case: 4

$$Z_L = jx$$

--- (purely reactive load)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L}$$

\xrightarrow{jx}
 $\xrightarrow{-x}$

$$Z_{in} = j \left[Z_0 \frac{x + Z_0 \tan \beta L}{Z_0 - x \tan \beta L} \right]$$

Real

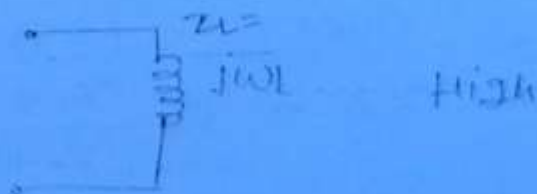
pure reactance

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{jx - Z_0}{jx + Z_0}$$

$$\Gamma = \underbrace{|\Gamma| e^{j\theta}}_{= \rho}$$

$$S = \frac{1 + \rho}{1 - \rho} = \infty = \frac{V_{max}}{V_{min}}$$



$$\theta = -2 \tan^{-1} \left(\frac{x}{z_0} \right)$$

(22)

Features:

- ① if the line is terminated by a purely reactive load then s/p imp. is also purely reactive.
- ② The location of voltage max. & Vmin. on the standing wave pattern will depend upon the type of the reactive load.
for inductive load the Vmax will occur at the load to end whereas minima will occur at the load end if it is capacitive nature.

ex: A lossless TL of length l has char's imp. of Z_0 & is terminated by load imp. Z_L .
find input imp. of the line when:-

Case 1: $l = \lambda$ Case 2: $l = \lambda/2$ Case 3: $l = \lambda/4$ ---- QWTCase 4: $l = \lambda/8$ Quarter wave transformer

To find :- Case 1 : $l = \lambda$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot d\right) = 0$$

(23)

$$Z_{in} = Z_0 \frac{Z_L + j0}{Z_0 + j0}$$

$$Z_{in} = Z_L$$

Case 2: $l = \lambda/2$

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = 0$$

$$Z_{in} = Z_L$$

Case 3:

$$l = \lambda/4$$

QWT

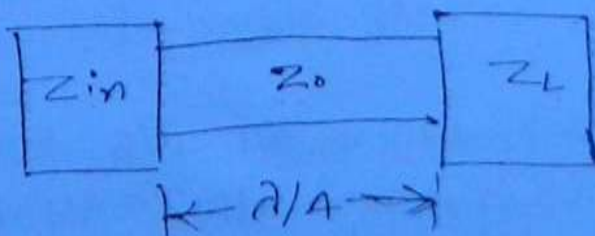
Quarter wave transformer

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \infty$$

$$Z_{in} = Z_0 \frac{(Z_L / \tan \beta l) + jZ_0}{(Z_0 / \tan \beta l) + jZ_L}$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_0 = \sqrt{Z_{in} \cdot Z_L}$$



- ① A QWT exists an Impedance Inverseal. therefore if the load Z_L is inductive then the i/p impedance is capacitive & vice-versa.
- ② A $\lambda/4$ section of the line matched two Imp. Z_L & Z_{in} & perfect matching take place whenever the load Z_L & i/p Imp. is purely resistive Imp.
- ③ A QWT section of the line is used to transform given load Imp. is Z_L to the desired i/p Imp. using a QWT whose charactis Imp. is the geometric mean of the load Z_L & input Imp. Z_L .

Case: 4 :- $l = \lambda/8$

(24)

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot \lambda/8\right) = 1$$

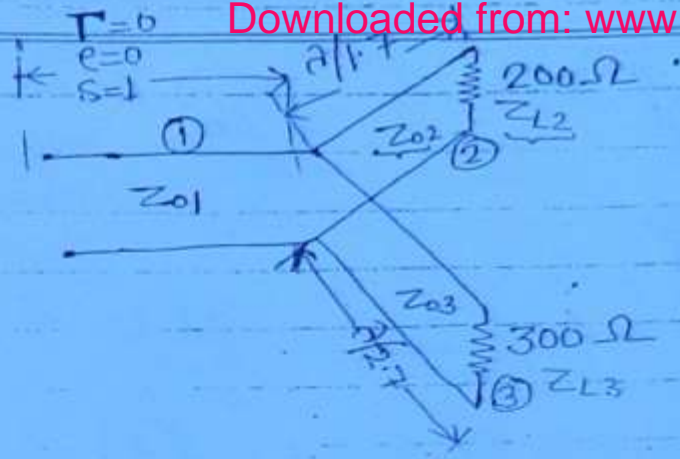
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 1$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0}{Z_0 + jZ_L}$$

--- complex nature.

$$|Z_{in}| = Z_0$$

- ① for $\lambda/8$ section of the line the input impedance is always complex in nature irrespective of the nature of load Imp. Z_L .
- ② The magnitude of input impedance of $\lambda/8$ section of the line is always numerically equal to the characteristics of the line.



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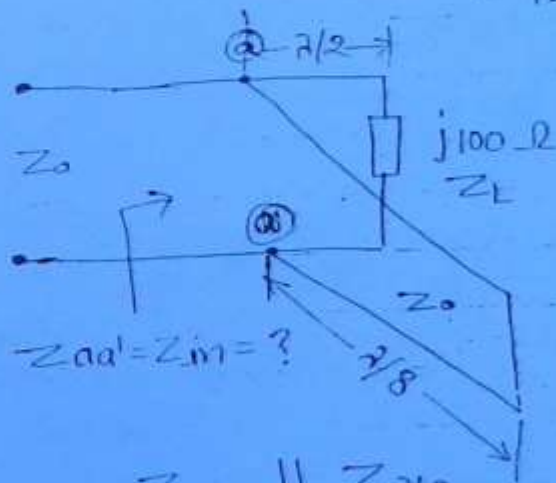
Find the char's impedance Z_{01} of the 1st line so that there is no reflected wave on it.

$$Z_{01} = Z_{L1} = Z_{in2} \parallel Z_{in3}$$

matched line $(= Z_{02} \parallel Z_{03})$ matched line

$$= 200 \parallel 300$$

$$= 120 \Omega$$



$$Z_{01} = Z_{in} = ?$$

$$Z_0 = 200 \Omega$$

to find:
 Z_{in}

$$Z_{in} = Z_{2/2} \parallel Z_{2/8}$$

$$= Z_L$$

$$= j100 \Omega$$

$$= jZ_0 \tan \beta l$$

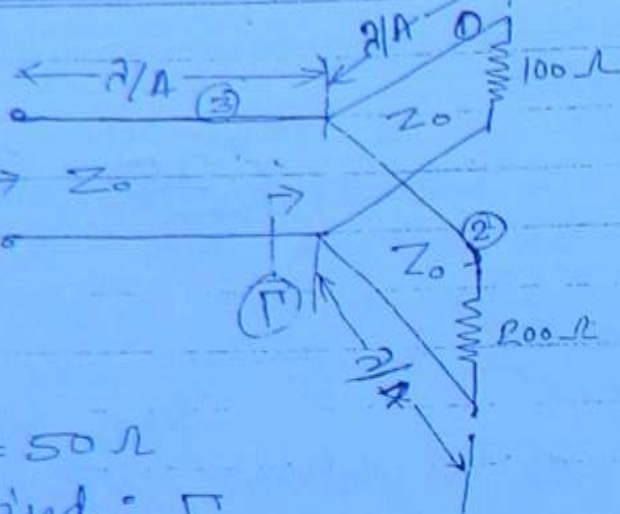
$$= j200 \tan \left(\frac{2\pi}{\lambda} \cdot \frac{2}{8} \right)$$

$$= 1$$

$$Z_{in} = \frac{j100 \times j200}{j100 + j200}$$

$$= \frac{j200}{3} \Omega$$

ex:



(26)

$Z_0 = 50 \Omega$
to find: Γ

$$Z_{L3} = Z_{in1} \parallel Z_{in2}$$

$$= \frac{Z_0^2}{Z_{L1}} \parallel \frac{Z_0^2}{Z_{L2}} \Rightarrow \frac{2500}{100} \parallel \frac{2500}{200}$$

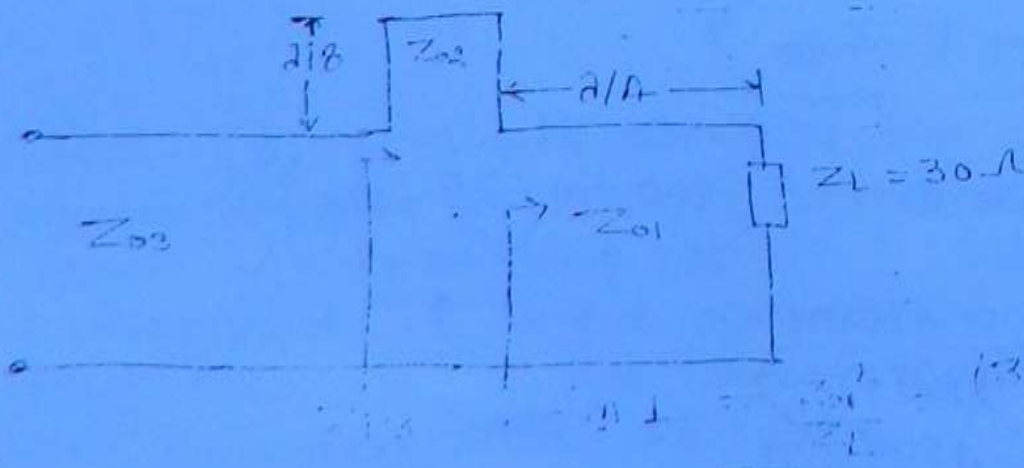
$$\Rightarrow 25 \parallel 25/2 \Rightarrow 25/3 \Omega$$

$$\Gamma \neq \frac{Z_{L3} - Z_0}{Z_{L3} + Z_0} = \frac{25/3 - 50}{25/3 + 50} = -5/7$$

$$Z_{in} = \frac{Z_0^2}{Z_{L3}} = \frac{2500}{25/3} = 300 \Omega$$

$$\Gamma = \frac{Z_{in3} - Z_0}{Z_{in3} + Z_0} = \frac{300 - 50}{300 + 50} = +\frac{5}{7} \quad \text{Ans.}$$

ex:



$$\frac{(30/5)^2}{40} = 60 \Omega$$

$$Z_{01} = 30j2$$

$$Z_{02} = 30$$

$$Z_{03} = 60$$

to find

S on Z_{03} line

$$S = \frac{1+\rho}{1-\rho}$$

(27)

$$\rho = |\Gamma|$$

$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}}$$

$$Z_{L3} = Z_{in} = \underbrace{Z_{02}}_{\lambda/8} + Z_{in1}$$

$$= jZ_{02} \tan \beta l$$

$$= j30 \tan\left(\frac{2\pi}{\lambda} \cdot \lambda/8\right) = j1$$

$$= j30$$

$$Z_{L3} = j30 + 60$$

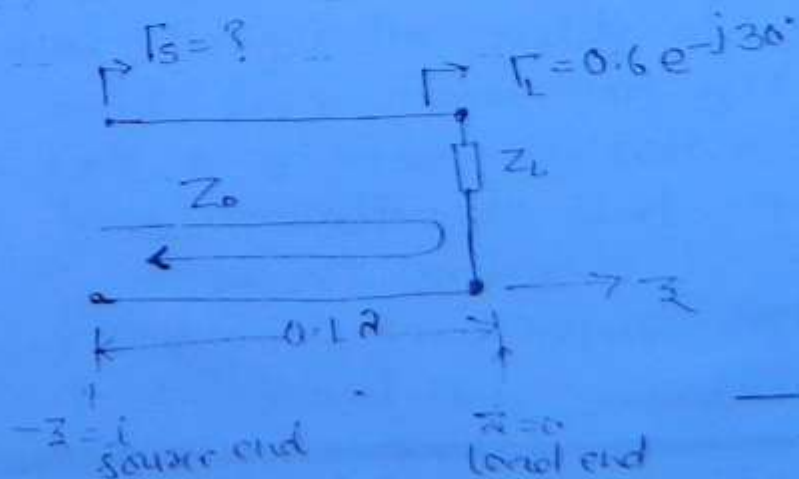
$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}} = \frac{60 + j30 - 60}{60 + j30 + 60} = \frac{j30}{120 + j30}$$

$$= \frac{j1}{4 + j1}$$

find $\rho = |\Gamma|$

find $S = \frac{1+\rho}{1-\rho}$

ex:



$$\Gamma_s = \rho_s \cdot e^{j0}$$

$$= 0.6 e^{j0}$$

$$\theta = \underbrace{\phi}_{\substack{\uparrow \\ \text{due to path} \\ \text{difference}}} + \underbrace{(-30^\circ)}_{\text{due to load}}$$

(28)

$$\phi = (\text{path diff.}) \times \underbrace{\frac{2\pi}{\lambda}}_{\beta} \Rightarrow 2z \cdot \frac{2\pi}{\lambda} \Big|_{-z=l}$$

$$\theta = \phi + (-30^\circ)$$

$$\theta = -72^\circ - 30^\circ$$

$$\theta = -102^\circ$$

$$\Gamma_s = \rho_s \cdot e^{j0}$$

$$= 0.6 e^{-j102^\circ}$$

$$\Gamma_s = 0.6 e^{+j258}$$

Ans:

$$= -2l \cdot \frac{2\pi}{\lambda}$$

$$= -2 \times 0.1\lambda \times \frac{2\pi}{\lambda}$$

$$= -0.4\pi$$

$$= -0.4 \times 180^\circ$$

$$\phi = -72^\circ$$

Distortionless line :-

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

(29)

$$\rightarrow \begin{aligned} \alpha &= f(\omega, \omega^2, \omega^4) \\ \beta &= f(\omega, \omega^2, \omega^4) \end{aligned}$$

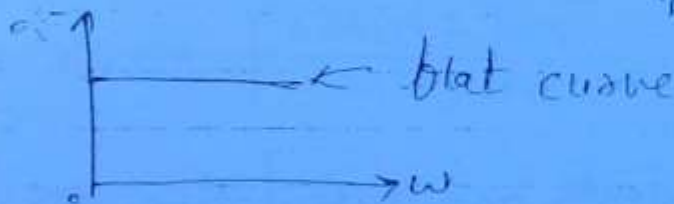
1) frequency distortion :-

$$\alpha = f(\omega, \omega^2, \omega^4)$$

----- Causes freq. distortion.

$$\alpha \neq f(\omega)$$

----- to avoid
freq. distortion.



Equalizer

--- lattice network.

① The frequency distortion occurs since various frequency components are subjected to different amount of attenuation.

this changes the quality of the voice signal.

2) To avoid freq. distortion all the frequency components must be subjected to same amount of attenuation so that quality of voice signal remains same.

therefore freq. response attenuation

constant, α should be constant & independent

of freq. $f(\omega)$

practically the freq. distortion is avoided by using an equaliser which represent a lattice n/w.

(30)

phase or delay distortion :-

$$\beta = f(\omega, \omega^2, \omega^3)$$

----- Causes phase or delay distortion

$\beta = \frac{\omega}{v_p}$ must be constant to avoid Dispersion

$$\beta \propto \omega$$

$$\beta = K\omega$$



① Various frequency components are subjective to diff. amount of phase shift which causes the phase or delay distortion.

To Avoid the phase or delay distortion phase const. β must be directly proportional to freq. of operation so that the phase velocity along the line must remain const.

therefor the freq. response of the phase const. β must have linear variation with the freq. variation.

② Practically freq. distortion is Avoided by using phase or delay equaliser.

for distortion less line

(3)

$$\left[\begin{array}{l} \alpha \neq f(\omega) \\ \beta = K\omega \end{array} \right] \begin{array}{l} \text{to avoid freq. distortion.} \\ \text{to avoid phase of delay} \\ \text{distortion.} \end{array}$$

$$\Rightarrow \boxed{RC = LG} \quad \text{condition for a distortion less line.}$$

★ Characteristic Impedance of distortion less line:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \text{if } RC = LG$$

$$R = \frac{LG}{C}$$

$$\Rightarrow Z_0 = \sqrt{\frac{LG/C + j\omega L}{G + j\omega C}} = \sqrt{\frac{L(G + j\omega C)}{C(G + j\omega C)}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

same as that of lossless line.

case 1: at high frequency

$$\omega L \gg R$$

$$\omega C \gg G$$

$$Z_0 = \sqrt{\frac{\cancel{R} + j\omega L}{\cancel{G} + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

case 2:

At low frequency.

$$\omega L \ll R$$

$$\omega C \ll G$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \cong \sqrt{\frac{R}{G}}$$

(32)

$$Z_0 \cong \sqrt{\frac{L}{C}}$$

$$\therefore RC = LG$$

$$\frac{R}{G} = \frac{L}{C}$$

① The char't's gmp. of a lossless & distortionless line is same & depends only upon the primary constant L & C .

② At high freq. the TL will always behave as a lossless as well as distortionless line irrespective of the condition $RC = LG$ is satisfied or not.

③ At low frequency the line in general does not behave as a distortionless line unless the condition $RC = LG$ is satisfied.

Summary:-

Distortionless line	lossless line
$\alpha \neq f(\omega)$ $\beta = k\omega$ $\beta = \frac{\omega}{v_p}$ $v_p = \frac{1}{\sqrt{LC}}$ $Z_0 = \sqrt{\frac{L}{C}}$ $\boxed{RC = LG}$	$R = G = 0$ $\rightarrow \boxed{\alpha = 0}$ $\beta = \frac{\omega}{v_p}$ $v_p = \frac{1}{\sqrt{LC}}$ $Z_0 = \sqrt{\frac{L}{C}}$

Conclusion :- ① For a distortionless line α must be independent of freq. but may have a finite value.

Therefore in general a distortionless line is a lossy line. (33)

②

For a lossless line R & G are zero & therefore for the condition $RC = LG$ is always satisfied.

Since $\alpha = 0$ for such line it's independent of freq. ω .

Therefore any lossless line is always a distortionless line.

③

Practically any transmission line is always a lossy line & therefore cannot be a distortionless line.

Such line may be made distortionless by suitably selecting the material such that the condition $RC = LG$ is always satisfied.

Ex:

$$Z_0 = 50 \Omega$$

$$R = 0.1 \Omega/m$$

To find α for distortionless line.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = Z_0^2 = 2500$$

$$RC = LG \Rightarrow G = R \cdot \frac{C}{L} = \frac{0.1}{2500} \text{ S/m}$$

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z}) \quad (35)$$

$$V|_{z=1} = V_s = \underline{V^+} e^{+j\beta l} + V^- e^{-j\beta l}$$

$$e^{\pm j\beta l} = \cos \beta l \pm j \sin \beta l$$

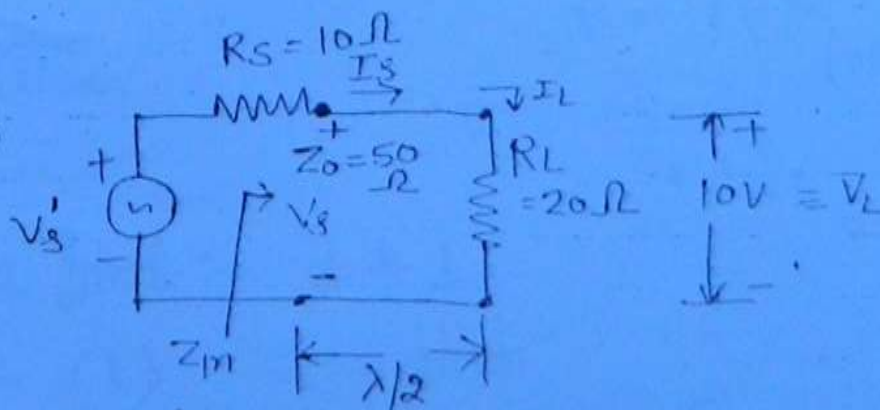
$$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}; \quad Z_L = \frac{V_L}{I_L}$$

$$V^+ \Rightarrow \frac{V_L}{V^+} = T; \quad V^+ = \frac{V_L}{T}$$

$$T = \frac{2Z_L}{Z_L + Z_0}; \quad Z_L = \frac{V_L}{I_L}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cos \beta l & j \sin \beta l \\ (j/Z_0) \sin \beta l & \cos \beta l \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

$$\begin{bmatrix} V_L \\ I_L \end{bmatrix} = \begin{bmatrix} \cos \beta l & -j Z_0 \sin \beta l \\ (-j/Z_0) \sin \beta l & \cos \beta l \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$



$$\text{To find: } \begin{cases} V_s; I_s \\ I_L; Z_{in}; V_s \end{cases}$$

$$\alpha \neq f(\omega)$$

(34)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$\alpha \neq f(\omega)$$

$$\gamma = \alpha + j\beta = \sqrt{RG}$$

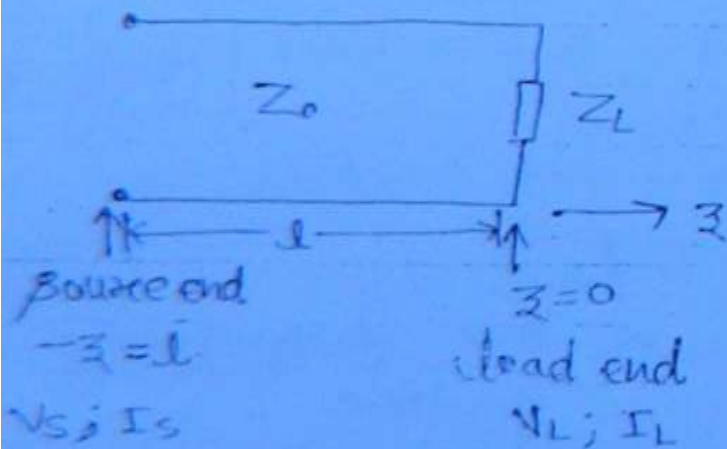
$$\alpha = \sqrt{RG} = \sqrt{\frac{0.1 \times 8.1}{2500}}$$

$$\alpha = 0.002 \text{ nepers/m}$$

$$1 \text{ neper} = 8.686 \text{ dB}$$

$$\alpha = 0.002 \times 8.686 \text{ dB}$$

Transmission matrix :-



$$\begin{Bmatrix} V_s \\ I_s \end{Bmatrix} \leftrightarrow \begin{Bmatrix} V_L \\ I_L \end{Bmatrix}$$

for a lossless line.

$$I_L = \frac{V}{R_L} = \frac{10}{20}$$

(36)

$$I_L = \frac{1}{2} \text{ Amp.}$$

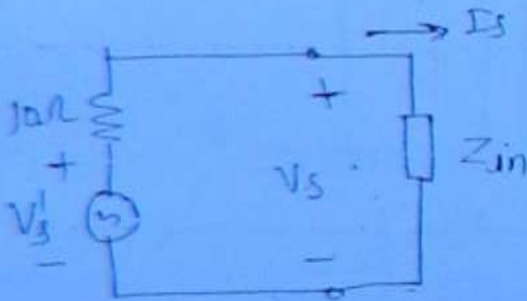
$$V_s = \underbrace{\cos \beta l \cdot V_L}_{\cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right)} + j \underbrace{Z_0}_{50} \underbrace{\sin \beta l \cdot I_L}_{\sin\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = 0} \leftarrow \frac{1}{2}$$

$$V_s = -10V$$

Simple method for I_s

$$Z_{in} = Z|_{\lambda/2} = Z_L = 20 \Omega$$

$$Z_{in} = 20 \Omega$$



$$I_s = \frac{V_s}{Z_{in}} = \frac{-10}{20} = -\frac{1}{2} A$$

$$V'_s = I_s (10 + 20)$$

$$V'_s = -\frac{1}{2} (10 + 20)$$

$$V'_s = -15V$$

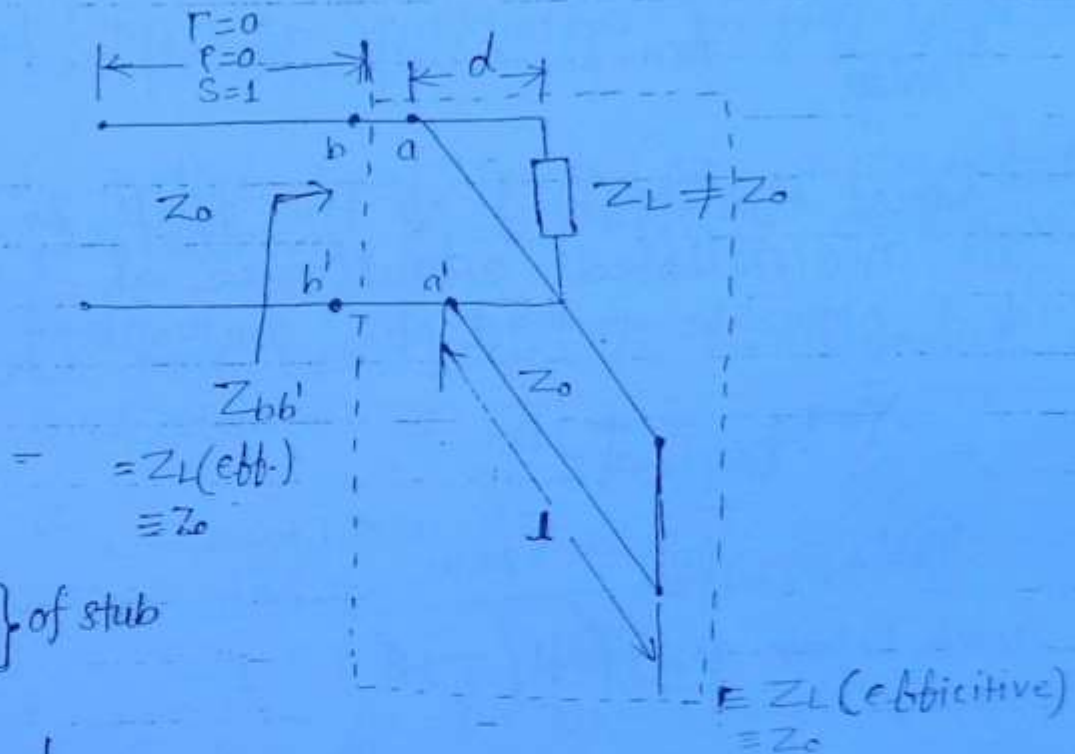
Stub Matching :-

Required when $Z_L \neq Z_0$

(37)

Case 1 :-

Short circuit shunt stub :-



where :-

d - location of stub
 l - length of stub

$$S_{\text{main line}} = 1$$

$$S_{\text{stub line}} = \infty$$

$$S_{\text{load line}} = ?$$

$$S = \frac{1+\Gamma}{1-\Gamma}$$

$$\Gamma = |\Gamma|$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

to find :- d & l

(1) Z_L - given.

(2) find $\left(\frac{Z_L}{Z_0}\right) = \bar{Z}_L$

(3) find $\bar{\Gamma}_L = \frac{1}{\bar{Z}_L}$

(4) Move a distance 'd' such that

$$\bar{Y}_{aa'} = 1 + j\bar{B} \quad (38)$$

Comments : The location of the stub till
—adjust their normalised value of
Real part of admittance at 'aa' becomes a
unity.

(5) adjust the length l of the stub so that
its normalised admittance at 'aa' is
equal & opposite to the imaginary part of

$$\bar{Y}_{aa'} = \pm j\bar{B}$$

$$\begin{aligned} (6) \quad \bar{Y}_{bb'} &= \bar{Y}_{aa'} + \bar{Y}_{\text{stub}} \\ &= 1 + j\bar{B} + (-j\bar{B}) = 1 \end{aligned}$$

$$(7) \quad \bar{Z}_{bb'} = \frac{1}{\bar{Y}_{bb'}} = 1$$

$$(8) \quad \frac{Z_{bb'}}{Z_0} = 1 ; Z_{bb'} = Z_0 \equiv Z_L (\text{effectively})$$

(9) Hence the TL is perfectly matched with the
effectively load imp. at 'bb'

(10) therefore to the left side of 'bb' the line is
perfectly matched, there is no reflected waves,
no standing wave pattern, reflection
coefficient is zero & therefore has a

Minimum value of VSWR is unity.

therefor max. power is transferred from the source to load.

the stub line will act as only the reactive power since the o/p imp. of stub is purely reactive.

this reactive power is not a useful power.

the entire Real power or the useful power is transmitted to the load impedance Z_L .

Hence max. Real power is been transferred to load impedance Z .

Ex:

$$Z_L = 100 + j300 \Omega \quad \rightarrow \text{Real part}$$

$$Z_0 = 100 \Omega \quad \rightarrow \text{Real part}$$

Both are equal then normalised is equal to 1

$$d = 0$$

Stub is connected at the load.

Ex:

$$Z_L = 200 + j300$$

$$Z_0 = 100$$

$$d \neq 0$$

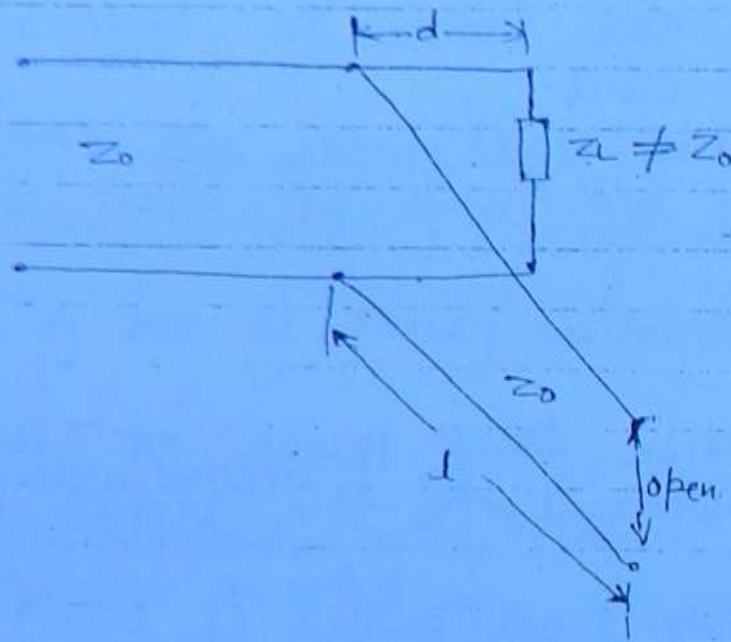
Stub is connected at some specific

Distance from the load.

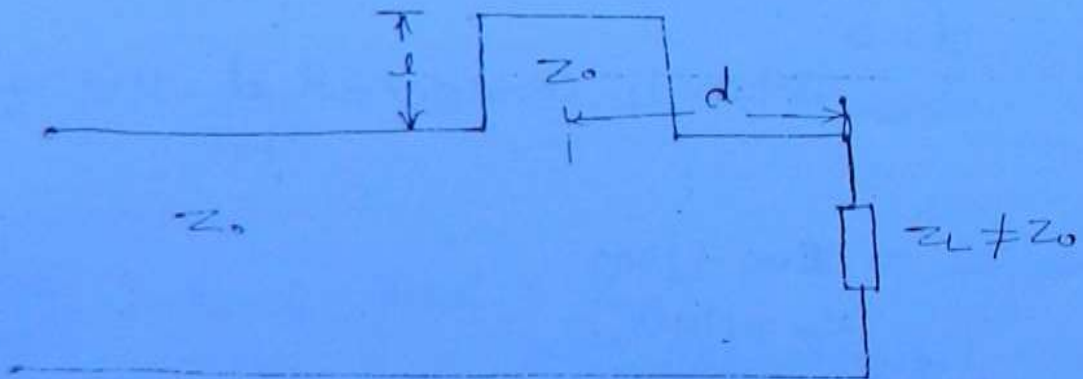
① The stub Matching is used only by a short circuit or open circuit stub line.

discrete components of L & C are not used for stub matching.

case-2 :- open circuit shunt stub :-

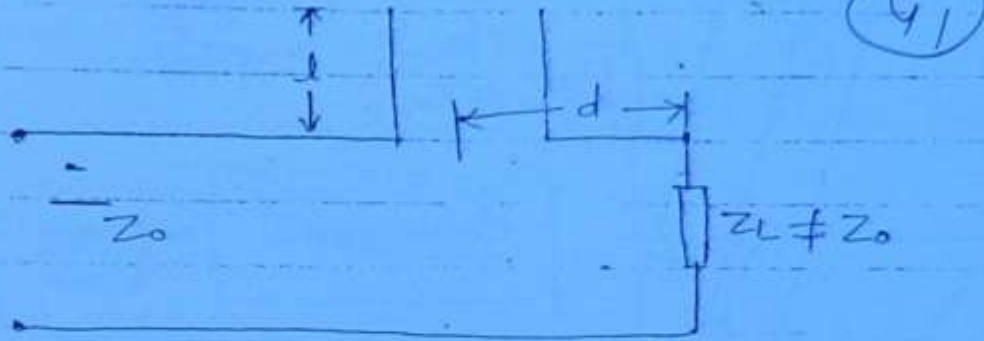


case: 3 - SC series stub:-

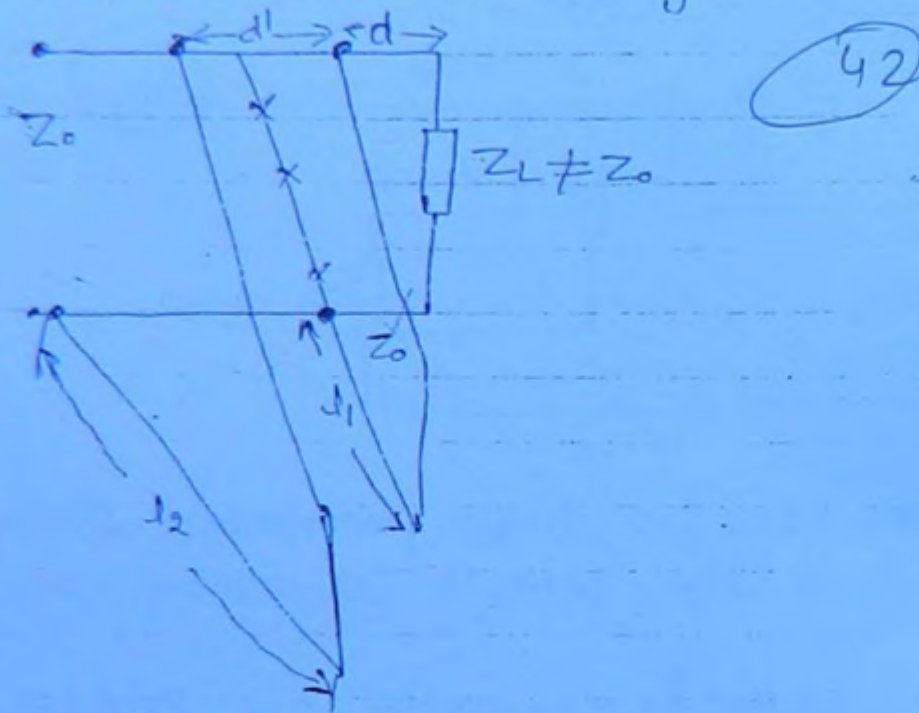


case: 4

o.c. Series stub:-



- ① The s.c. stub is always preferred since the adjustment of the length is more conventional practically.
- ② the o.c. stub is normally not preferred since
 - Ⓐ the adjustment of the length is practically not convenient.
 - Ⓑ an o.c. stub \times has a antenna & em. pow is radiated from it.
- ③ The shunt stub is always preferred since the main line remains unaffected when the load is varied over wide range.
- ④ the series stub is never preferred since the main line is affected if the load Z_L is varied over a wide range.
- ⑤ Therefore for variable load the s.c. shunt stub matching is the best whereas o.c. series stub matching is the worst.

Double Stub matching

① Double stub matching is generally preferred over the single stub matching because of more flexibility in the variation of length of each stub l_1 & l_2 .

using double stub matching we are not able to match all the type of load with the characteristic impedance of the line

Variation of Impedance along the line

$$Z_{\max} = \frac{V_{\max}}{I_{\min}} = \frac{V_{\max}}{V_{\min}/Z_0} = Z_0 S$$

(43)

$$\left(\frac{Z_{\max}}{Z_0} \right) = \boxed{\bar{Z}_{\max} = S}$$

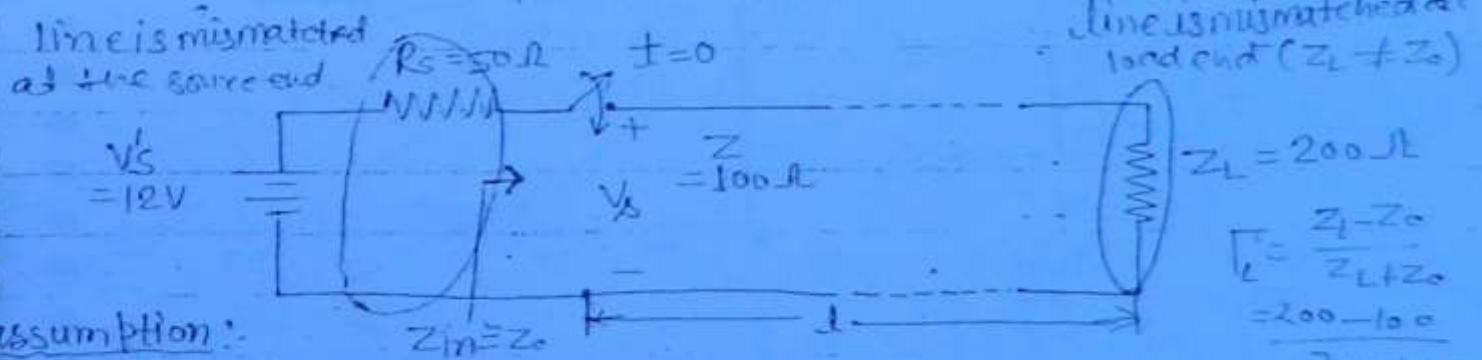
$$Z_{\min} = \frac{V_{\min}}{I_{\max}} = \frac{V_{\min}}{V_{\max}/Z_0} = \frac{Z_0}{S}$$

$$\left(\frac{Z_{\min}}{Z_0} \right) = \boxed{\bar{Z}_{\min} = \frac{1}{S}}$$

$$\boxed{\frac{1}{S} \leq \bar{Z} \leq S}$$

Transient Response in T.L. :-

line is mismatched at the source end



assumption:-

- ① line is lossless.
- ② l is large $Z_{in} \approx Z_0 = 100 \Omega$
- ③ $T = l/v_p$ time taken by voltage wave from source to load end.
- ④

$$T \approx 400 \mu\text{sec}$$

$$\approx 500 \mu\text{sec}$$

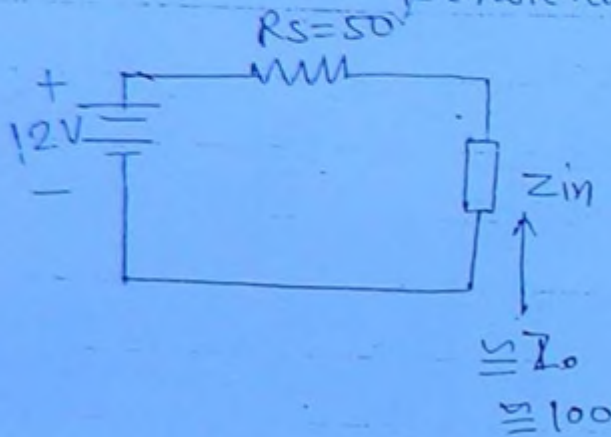
$$\Gamma_s = \frac{R_s - Z_0}{R_s + Z_0} = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$

(44)

To find:

 $V(t)$ vs t

transient response of the line.

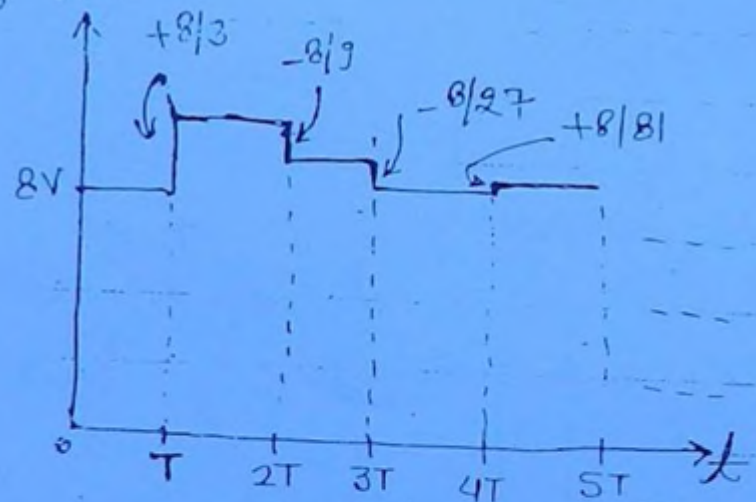
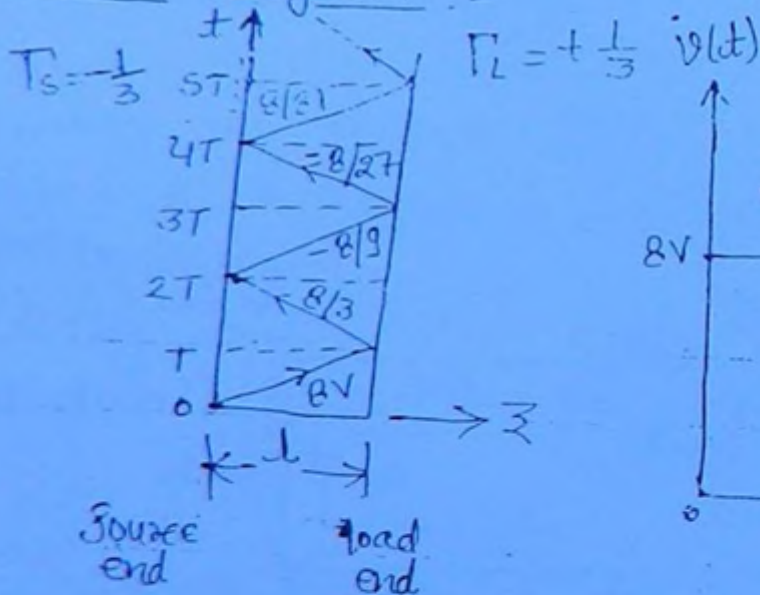


$$V_s = \frac{100}{50 + 100} \times 12$$

$$= \frac{2}{3} \times 12$$

$$= 8V$$

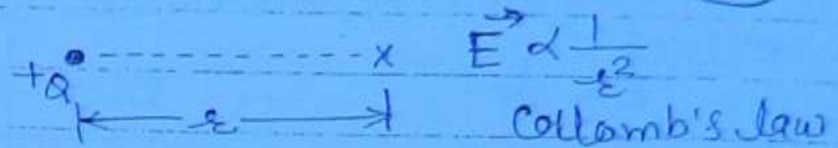
Source diagram



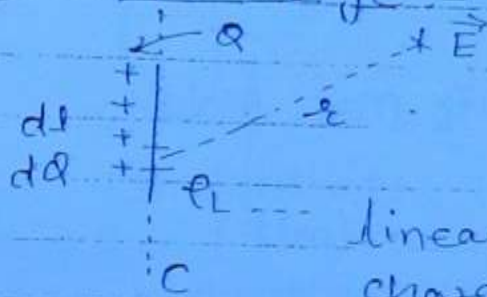
Sources of \vec{E} (electrical field):-

(45)

(1) point charge :-

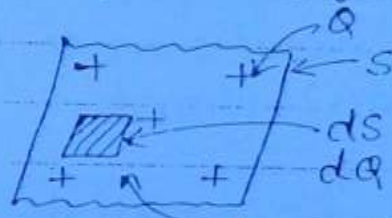


(2) line charge :-



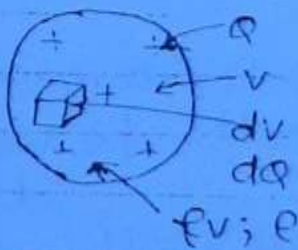
linear charge density
charge per unit length (C/m)

(3) surface charge :-



surface charge density
(C/m²)

(4) Volume charge



Volume charge density (C/m³)

★ Source of \vec{B} (Mag. field)

(46)

1. line current :-

$$\vec{B} \propto \frac{1}{r} \quad \text{Biot-Savart's Law}$$

(A) $I = I_0 \Rightarrow \vec{B} = \vec{B}_0$ static field
 $= I_0 \sin \omega t$

C: $\Rightarrow \vec{B} = \vec{B}_0 \sin \omega t$ time varying mag. field

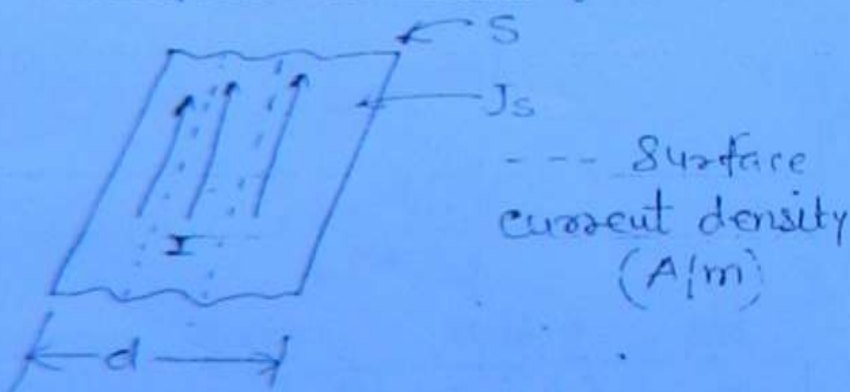
current element

$$\vec{I} d\vec{l} = I d\vec{l}$$

convention:-

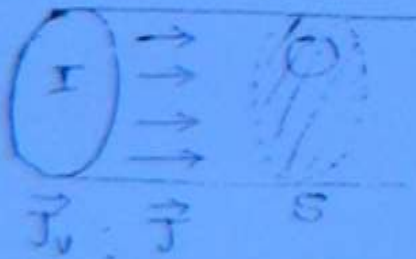
by convention the direction of current I & elementary length $d\vec{l}$ are taken same in all electromag. problems.

2. Surface current :-



$$I = \int_d J_s \cdot d\vec{l}$$

3. Volume current :-



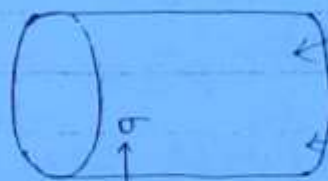
$$I = \iiint_v \vec{J} \cdot d\vec{S}$$

Volume current density
 (A/m^2)

★ Continuity eqn.

Conversion of charge (47)

$$I = - \frac{dQ}{dt}$$

Conductivity
(S/m)

$$Q_i = 5 \mu C$$

$$\Delta t = 1 \text{ m sec.}$$

$$Q_f = 2 \mu C$$

$$I = - \frac{dQ}{dt} = - \frac{Q_f - Q_i}{\Delta t}$$

$$= - \frac{(2-5) \times 10^{-6}}{1 \times 10^{-3}}$$

$$= 3 \text{ mA}$$

$$\nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t}$$

Volume current
density (A/m^2)Volume charge density
(C/m^3)

----- continuity eqn.

where

$$\vec{J} = \sigma \vec{E}$$

----- Ohm's law

$$\text{if } \rho = 0 ; \frac{\partial \rho}{\partial t} = 0$$

 \vec{J} is solenoidal in nature.

$$\nabla \cdot \vec{J} = 0$$

KCL eqns.

Statement:- (1) The divergences of volume current density \vec{J} at any point in the electromagnetic Region is always equal to the rate of decrease of the volume charge density ρ with respect to time t .

therefor there is a continuity b/w the decrement of volume charge density ρ & the corresponding production volume current density \vec{J} .

(2) for a charge free Region the divergence of volume current density \vec{J} at any point is always equal to zero.

and hence volume current density is always solenoidal & forms a closed loop.

$$\rho = 0$$

(48)

--- on perfect cond. ($\sigma = \infty$)

--- on dielect cond. ($\sigma = 0$)

① The charge density ρ is always equal to zero on a perfect conductor or a perfect dielectric conductor.

② Volume charge density is finite on a medium where the conductivity is finite. The decay of volume charge density will depend upon the conductivity of region.

Higher is the conductivity higher is the rate decay of charge & vice-versa.

* Maxwell's eqns. in their general time varying form :-

differential form
(point form)

$$\nabla \cdot \vec{D} = \rho$$

integral form

$$\oint_S \vec{D} \cdot d\vec{S} = q = \iiint_V \rho \, dv \quad \text{Gauss law for elect. fields}$$

electric flux

$$\nabla \cdot \vec{B} = 0$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

--- Gauss law for mag. fields

$$(3) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\underbrace{\oint_C \vec{E} \cdot d\vec{l}}_{\text{e.m.f.}} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad (49)$$

Faraday's law of e.m.
induction.

$$\begin{aligned} \nabla \times \vec{H} &= \vec{J}_c + \vec{J}_d \\ &= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

$$\underbrace{\oint_C \vec{H} \cdot d\vec{l}}_{\text{m.m.f.}} = \iint_S (\vec{J}_c + \vec{J}_d) \cdot d\vec{S}$$

Modified Ampere's
circuit law

Eqn. using displacement
current density concept

$$\vec{J}_c = \sigma \vec{E}$$

Cond. current density (A/m^2)

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

displacement current
density (A/m^2)

Statements :-

① (1a) The divergences of electrical flux density \vec{D} at any point in the electro mag. region is always equal to the volume charge density ρ .

①b: The net electrical flux passing through any closed surface area S is always equal to total charge enclosed within the surface Area S .

② 2(a) The divergences of mag. flux density \vec{B} is always equal to zero since

Reasons: (i) Mag. field lines are always closed in nature.

(ii) The mag. charges in the isolated form do not exist in nature.

2(b) The net mag. flux passing through any closed surface Area S is always equal to zero.

(50)

3) 3(a) : The curve curl of elec. field intensity \vec{E} is always equal to the rate of decrease of mag. flux density \vec{B} w.r. to time t .

3(b) Net emf. produced is always equal to the surface integral of rate of decrease of mag. flux density \vec{B} w.r. to time t .

4(a) The curl of mag. field intensity \vec{H} is always equal to the sum of conduction current density \vec{J}_c & the displacement current density \vec{J}_d .

4(b) Total mmf produced is always equal to the surface integral of the sum of conduction current density \vec{J}_c & displacement current density \vec{J}_d .

Special cases:

Case 1: for static fields:-

$$\begin{array}{l} \frac{\partial \vec{B}}{\partial t} = 0 \\ \frac{\partial \vec{D}}{\partial t} = 0 \end{array}$$

Case 2: - for perfect dielectric ($\sigma = 0$)

or

non-Conducting Medium

or

lossless medium

(5)

or

free space

$$\boxed{\begin{aligned}\vec{J}_c &= \sigma \vec{E} = 0 \\ \rho &= 0\end{aligned}}$$

Case 3: - for Good Conductor

--- σ is High

$$\boxed{\begin{aligned}\vec{J}_b &\neq 0 \\ \rho &\neq 0\end{aligned}}$$

Case 4: - for time-harmonically or sinusoidally varying fields :-

$$\begin{cases} \vec{D} = \vec{D}_0 e^{j\omega t} \\ \vec{B} = \vec{B}_0 e^{j\omega t} \end{cases}$$

$$\frac{\partial \vec{B}}{\partial t} = j\omega \vec{B}_0 e^{j\omega t}$$

$$= j\omega \vec{B}$$

$$\rightarrow j\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow (j\omega)^2 = -\omega^2$$

So, Eqs:

$$(1) \nabla \cdot \vec{D} = \rho$$

$$(2) \nabla \cdot \vec{B} = 0$$

$$(3) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \mu \vec{H}$$

$$(4) \nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

$$= \sigma \vec{E} + j\omega \epsilon \vec{E}$$

$$= (\sigma + j\omega \epsilon) \vec{E}$$

* Loss tangent
($\tan \delta$)

$$\vec{J}_{\text{total}} = \vec{J}_c + \vec{J}_d$$

$$= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

$$\approx \sigma \vec{E} + j\omega \epsilon \vec{E} \quad \text{--- for general medium} \\ (\mu, \epsilon, \sigma)$$

$$\approx \sigma \vec{E} \quad \text{--- for good conductor.}$$

$$\approx j\omega \epsilon \vec{E} \quad \text{--- for good dielectric.}$$

$$\tan \delta = \frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{|\sigma \vec{E}|}{|j\omega \epsilon \vec{E}|} = \frac{\sigma}{\omega \epsilon}$$

$\tan \delta = \frac{\sigma}{\omega \epsilon} \gg 1$	Good Conductor.
$\tan \delta = \frac{\sigma}{\omega \epsilon} \ll 1$	Good dielectric.

imp points:

- 1) for Good Conductors the conductivity is high & therefore the conduction current density is dominant.
- 2) for Good dielectric the conductivity is low, conduction current density is negligible & the displacement current density is more dominated.
- 3) the loss tangent is the ratio of the magnitude of conduction current density & displacement current density.
this is a major of total loss occurring in a material due to finite conductivity at specified frequency.
- 4) depending upon the frequency of operation any medium may behaves as a good conductor

or good dielectric.

53

In general any material may behaves as a good conductor at low freq. whereas some material may behaves as a good dielectric at high frequency.

Conclusion :

Therefore depending upon Application we operate a device at high freq. and low freq. so that it can operate at a good dielectric & good conductor.

★ Poynting's Vector (\vec{P})

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\frac{V}{m}, \frac{A}{m}$$

$$W/m^2$$

Power density at a point

$$Power = \iint_S \vec{P} \cdot d\vec{S}$$

$$if \quad (\vec{E} \perp \vec{H}) \perp \vec{P}$$

Transverse e.m. wave

TEM wave \rightarrow uniform plane wave (plane wave)

$$\vec{P} = \vec{E} \times \vec{H} = EH \sin \alpha \hat{n}$$

$$\sin \alpha = 1$$

$$|\vec{P}| = P = EH$$

$$= E_{rms} \cdot H_{rms}$$

$$\frac{E}{H} = \eta$$

Intrinsic Impedance of the Medium.

$$\frac{\sqrt{\mu}}{\sqrt{\epsilon}} = \eta$$

$$\frac{E}{H} = \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

--- for general lossy medium (μ, ϵ, σ)

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

--- for lossless medium ($\sigma = 0$)

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\cong 120\pi \cong 377 \Omega$$

--- for free space

$$\sigma = 0$$

$$\mu = \mu_0$$

$$\epsilon = \epsilon_0$$

$$|\vec{P}| = P = \begin{aligned} &E \cdot H \\ &= \eta H^2 \\ &= \frac{E^2}{\eta} \end{aligned}$$

E; H

rms Value

$$P = \begin{aligned} &\frac{1}{2} E_m H_m \\ &= \frac{1}{2} \eta H_m^2 \\ &= \frac{1}{2} \frac{E_m^2}{\eta} \end{aligned}$$

Imp. points :- (1) The Poynting vector (P) represent power density at a point.

(2) When integrator over any closed surface Area the total power flow in the specified direction.

The P vectors give the direction of propagation of em waves & is always perpendicular to the plane made by \vec{E} & \vec{H} vectors

- ④ for a transverse e.m. waves \vec{E} , \vec{H} & \vec{P} are mutually perpendicular to each other.
- ⑤ In any e.m. region the ratio b/w the \vec{E} & \vec{H} fields is always constant. & is represented by intrinsic impedance of medium.
- ⑥ This imp. depends only upon the const. of the medium.

for a free space this imp. has a university const. value of $120\pi \Omega$ or 377Ω (SS)

Ex: An e.m. wave is travelling along $-y$ direction & has only x comp. of elect. fields. find the magnetic field intensity associated with e.m. waves.

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{P} = -P_y \hat{a}_y \quad \text{--- direction of prop. of wave.}$$

$$\vec{E} = E_x \hat{a}_x$$

To find:-

$$\vec{H} = ?$$

$$\hat{a}_y \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} = \begin{pmatrix} +\hat{a}_z \\ 0 \\ +\hat{a}_x \end{pmatrix} \quad (\text{cross product})$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$(-P_y \hat{a}_y) = (E_x \hat{a}_x) \times (+H_z \hat{a}_z)$$

$\vec{H} = +H_z \hat{a}_z$

Ex: Find the displacement current at $t=0$ through a $10\mu\text{F}$ capacitor if the voltage across it is given by

$$v(t) = 0.1 \sin 120\pi t \quad \text{--- V}$$

$$C = 10\mu\text{F}$$

$$I_d|_{t=0} = ? = \underline{I_d \cdot A}$$

$$I \cdot A = A \frac{\partial D}{\partial t} = \epsilon A \frac{\partial E}{\partial t} = \underbrace{\frac{\epsilon A}{c}}_C \frac{\partial [V(t)]}{\partial t}$$

$$C \frac{\partial}{\partial t} (0.1 \sin 120 \pi t) = \underbrace{10 \times 0.1 \times 120 \pi}_{10 \times 12 \pi} \underbrace{\cos 120 \pi t}_{=1} \Big|_{t=0}$$

$$\equiv 120 \pi \text{ -- pA}$$

$$\equiv 377 \text{ -- pA}$$

$$\equiv 0.377 \text{ -- nA}$$

(56)

2. An e.m. wave is travelling in a lossless medium is $\mu_r = 1$ & $\epsilon_r = 4$ & has a power density of 4 W/m^2 .

Calculate the max. value of E & H phase.

$$|\vec{P}| = P = 4 \text{ W/m}^2$$

$$\mu_r = 1; \mu = \mu_0 \mu_r = \mu_0$$

$$\epsilon_r = 4; \epsilon = \epsilon_0 \epsilon_r = 4 \epsilon_0$$

to find:

$$E_m; H_m$$

$$P = \frac{1}{2} E_m \cdot H_m$$

$$= \frac{1}{2} \eta H_m^2$$

$$= \frac{1}{2} \frac{E_m^2}{\eta} \leftarrow$$

$$\eta = \frac{E_m}{H_m}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{\eta_0}{\sqrt{4}} = \frac{\eta_0}{2}$$

$$= \frac{120 \pi}{2} = 60 \pi \Omega$$

$$P = \frac{1}{2} \frac{E_m^2}{\eta}$$

$$E_m = \sqrt{2 \eta P} = \sqrt{2 \times 60 \pi \times 4} \text{ V/m}$$

$$\eta = \frac{E_m}{H_m} \Rightarrow \frac{E_m}{60 \pi} = H_m \text{ A/m}$$



Wave Equations

----- Mathematical form of e.m. wave

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

(57)

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (2)}$$

$$\vec{B}(t) \Rightarrow \frac{\partial \vec{B}}{\partial t} \xrightarrow{\text{eq. (1)}} \vec{E}(t) \rightarrow \vec{D}(t)$$

$\vec{H}(t) \xleftarrow{\text{eq. (2)}} \frac{\partial \vec{D}}{\partial t}$



$$w_e = \frac{1}{2} \epsilon E^2 \quad \longleftrightarrow \quad \frac{1}{2} \mu H^2 = w_m$$

$$= \frac{\partial (w_e)}{\partial t} \quad \longleftrightarrow \quad \frac{\partial (w_m)}{\partial t}$$



⇓
power flows in a direction

⇒ e.m. wave propagates.

Conclusion:

(1) due to time vary ele. & mag. field the rate of change of ele. energy is transformed to the rate of change of mag. energy.

2. Vice - Versa.

- 2) Due to this rate of change the por propagates in a particular direction which is given by Poynting vector (P)
- 3) there for e.m. waves propagates in the direction given by Poynting vector (P)

date - 28-07-2010

$$- \log_e \left(\frac{V_o}{V_i} \right) \dots \text{ nepers/m}$$

(58)

$$20 \log_{10} \frac{V_o}{V_i} \dots \text{ dB}$$

$$10 \log_{10} \frac{P_o}{P_i} \dots \text{ dB}$$

Wave eqns

eqn. — ①

eqn. — ②

eliminate \vec{H}

$$\nabla^2 \vec{E} = \underbrace{\mu_0}_{\text{loss factor}} \frac{\partial \vec{E}}{\partial t} + \underbrace{\mu \epsilon}_{\text{propagation factor}} \frac{\partial^2 \vec{E}}{\partial t^2}$$

loss factor

Wave eqns on \vec{E}
propagation factor

Similarly

$$\nabla^2 \vec{H} = \mu_0 \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Wave eqns
on \vec{H}

Comments :

① Hence as the e.m. waves propagate in a general medium it is subjected to attenuation as well as phase change.

Therefore the elec. field strength decrease as the wave propagate in a particular direction which is given by Poynting vector.

(2) The behaviour of elec. & mag. field are exactly same except that the ele. & mag. fields are perpendicular to each other.

(59)

Special case:

--- for sinusoidally varying fields.

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$$

$$\therefore \nabla^2 \vec{E} = \mu\sigma \cdot j\omega \vec{E} + \mu\epsilon (-\omega^2 \vec{E})$$

$$\rightarrow \boxed{\nabla^2 \vec{E} = \gamma^2 \vec{E}}$$

$$\gamma^2 = \mu\sigma \cdot j\omega - \omega^2 \mu\epsilon$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \alpha + j\beta$$

$$\text{if } \sigma = 0$$

--- lossless medium.

$$\gamma = \sqrt{j\omega\mu(0 + j\omega\epsilon)}$$

$$= \boxed{j\omega\sqrt{\mu\epsilon} = \alpha + j\beta}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{v_p}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}}$$

if $\mu = \mu_0$
 $\epsilon = \epsilon_0$ } --- for free space

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \times 10^8 \text{ m/sec.}$$

(60)

$$\gamma = 0 + j\beta$$

$$\gamma^2 = -\beta^2$$

$$\rightarrow \boxed{\nabla^2 \vec{E} = -\beta^2 \vec{E}} \quad \text{--- wave eqn for lossless medium } (\sigma = 0)$$

Ex: An e.m. wave is propagating in a general lossy medium has been const of μ, ϵ & σ in $+\hat{z}$ direction and has only x-components of electrical field.

assuming sinusoidal variation find the value of the elect. field components.

prop. --- along $+\hat{z}$

$$\vec{E} = E_x \hat{a}_x$$

Medium: μ, ϵ, σ

Sinusoidal Variation

To find

E_x

$$\nabla^2 \vec{E} = \gamma^2 \vec{E}$$

$$(\nabla^2 E_x) \hat{a}_x + (\nabla^2 E_y) \hat{a}_y + (\nabla^2 E_z) \hat{a}_z = \gamma^2 (E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z)$$

$$\nabla^2 E_x = \gamma^2 E_x$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \quad (6)$$

characteristic eqn $m^2 = \gamma^2$
 $m = \pm \gamma$

$$E_x = A e^{+\gamma z} + B e^{-\gamma z}$$

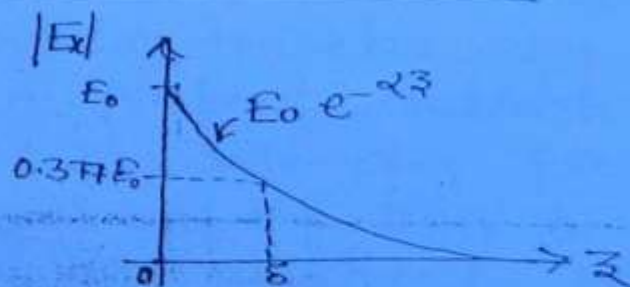
along +z

$$E_x = E_0 e^{-\gamma z} \quad \text{Ans.}$$

★ Depth of penetration :

$$\begin{aligned} E_x &= E_0 e^{-\gamma z} \\ &= E_0 e^{-(\alpha + j\beta)z} \\ &= E_0 e^{-\alpha z} \cdot e^{-j\beta z} \end{aligned}$$

$$|E_x| = E_0 \cdot e^{-\alpha z}$$



If $\alpha z = 1$ for $z = \delta$ depth of penetration

$$\alpha \cdot \delta = 1$$

$$\delta = \frac{1}{\alpha}$$

$$E_x = E_0 \cdot e^{-\alpha z} = E_0 e^{-1} \text{ at } z = \delta$$

$$\cong 0.37 E_0$$

① As the wave enters in a lossy medium having finite conductivity, the electric field strength decreases exponentially. (62)

② The depth of penetration or the skin depth represents the total distance traveled by electromagnetic waves where its electric field strength decreases to 37% of its initial value.

③ The depth of penetration is inversely equal to the attenuation constant α .

④ Higher is the conductivity of medium, higher is the value of attenuation constant α , & therefore lower is the value of depth of penetration & vice-versa.

Case I :-

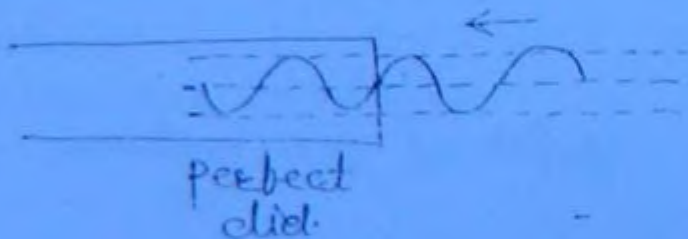
Perfect diel.
($\sigma = 0$)

$$\gamma = \alpha + j\beta$$

$$= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\Rightarrow \alpha = 0$$

$$\boxed{S = \infty} = \frac{1}{\alpha}$$



Case : 2. perfect conductor

$$(\sigma \rightarrow \infty)$$

$$\Rightarrow \lambda \rightarrow \infty$$

$$\delta = \frac{1}{\lambda} \rightarrow 0$$

(63)

E.M. wave = 0

$$\vec{E} = 0$$

$$\vec{H} = 0$$

perfect Cond.

$$(\sigma \rightarrow \infty)$$

① for a perfect dielectric $\sigma = 0$, $\alpha = 0$ therefore depth of penetration is infinite.

In such medium the wave travels without any attenuation & therefore electric field strength remains constant at all points.

② for a perfect conductor the depth of penetration is zero, the wave cannot enter in such medium therefore any perfect conductor behaves as a perfect reflector.

③ Inside any perfect conductor E.M.W, electric field and the magnetic field do not exist.

④ any perfect conductor behaves as an electromagnetic mirror.

§ for a good conductor :-

$$\left[\frac{\sigma}{\omega \epsilon} \gg 1 \right]$$

$$\delta = \frac{1}{\alpha}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \sqrt{j\omega\mu\sigma \left(1 + \frac{j\omega\epsilon}{\sigma}\right)}$$

$$\approx \sqrt{\omega\mu\sigma} \angle 45^\circ$$

$$= \sqrt{\omega\mu\sigma} \left(\underbrace{\cos 45^\circ}_{=1/\sqrt{2}} + j \underbrace{\sin 45^\circ}_{=1/\sqrt{2}} \right)$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} (1+j) = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta \propto \frac{1}{\sqrt{f}}$$

$$f \uparrow \rightarrow \delta \downarrow$$

$$f \downarrow \rightarrow \delta \uparrow$$

Comments :- for a specified material if the freq. of operation is high then the depth of penet. is high & these for thin conductor are preferable.

at low freq. since the depth of penetration is high we have to use thick conductor.

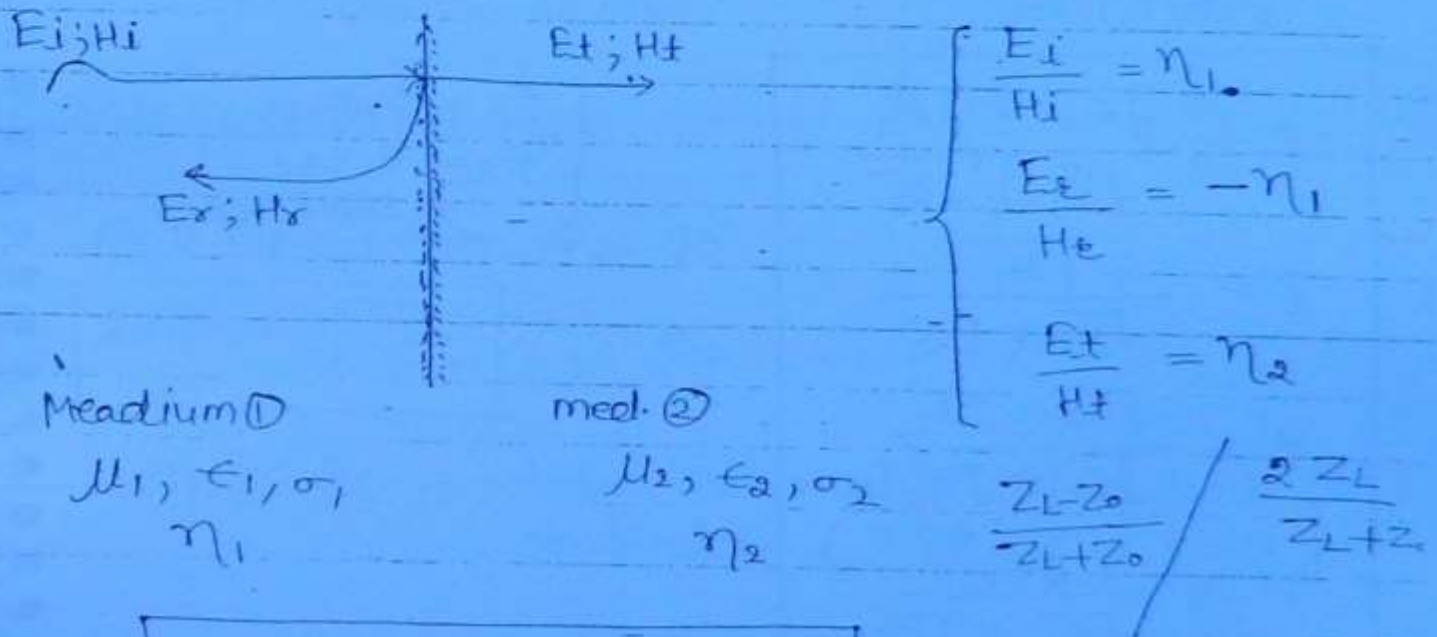
(64)

$\begin{cases} j = \sqrt{-1} \\ \sqrt{j} = \sqrt{\sqrt{-1}} \end{cases}$
 complicated for
 phase angle taken.

so that entire power is combined within the conducting region.

(65)

★ Reflection & Refraction of EMW --- normal incidence



$$\left(\frac{E_r}{E_i} \right) = \rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\left(\frac{E_t}{E_i} \right) = T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\left(\frac{H_r}{H_i} \right) = \rho' = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = -\rho$$

$$\left(\frac{H_t}{H_i} \right) = T' = \frac{2\eta_1}{\eta_1 + \eta_2}$$

for General Media

perfect diel.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{\mu_0}{\epsilon}}$$

(66)

$$\eta_1 \propto \sqrt{\frac{1}{\epsilon_1}}$$

$$\eta_2 \propto \sqrt{\frac{1}{\epsilon_2}}$$

$$r = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$T = \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$r' = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

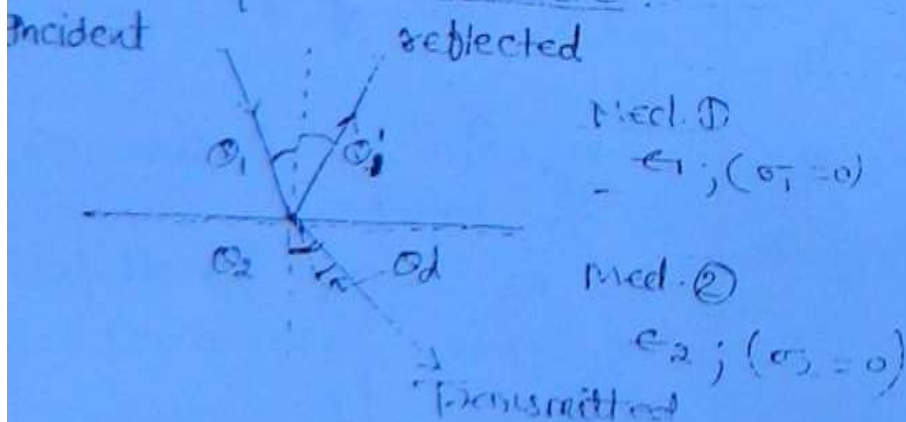
$$T' = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$\begin{cases} \epsilon_1 = \epsilon_0 \epsilon_{r1} \\ \epsilon_2 = \epsilon_0 \epsilon_{r2} \end{cases}$$

is same result

--- for perfect diel.

oblique incidence:-



θ_d = angle of deviation

$$\theta_d = \theta_2 + \theta_1$$

(67)

$\theta_1' = \theta_1$ ---- law of reflection

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

---- Snell's law

$$v_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

Case 1: Critical angle

$$\theta_c = \theta_1 \text{ when } \theta_2 = 90^\circ$$

$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} ; \epsilon_2 < \epsilon_1$$

$$\theta_1 \geq \theta_c \text{ --- total internal reflection (TIR) } \left\{ \begin{array}{l} R=1 \\ T=0 \end{array} \right.$$

Case 2: Brewster's angle

$$\theta_1 = \theta_B$$

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} ; \left\{ \begin{array}{l} R=0 \\ T=1 \end{array} \right.$$

① The critical angle represent the angle of incident for which the angle of transmission is 90° .

② total internal reflection occurs whenever the angle of incident is more than the critical angle.

this phenomenon is important for the propagation of waves in boundary media. Such as propagation through

optical fibres.

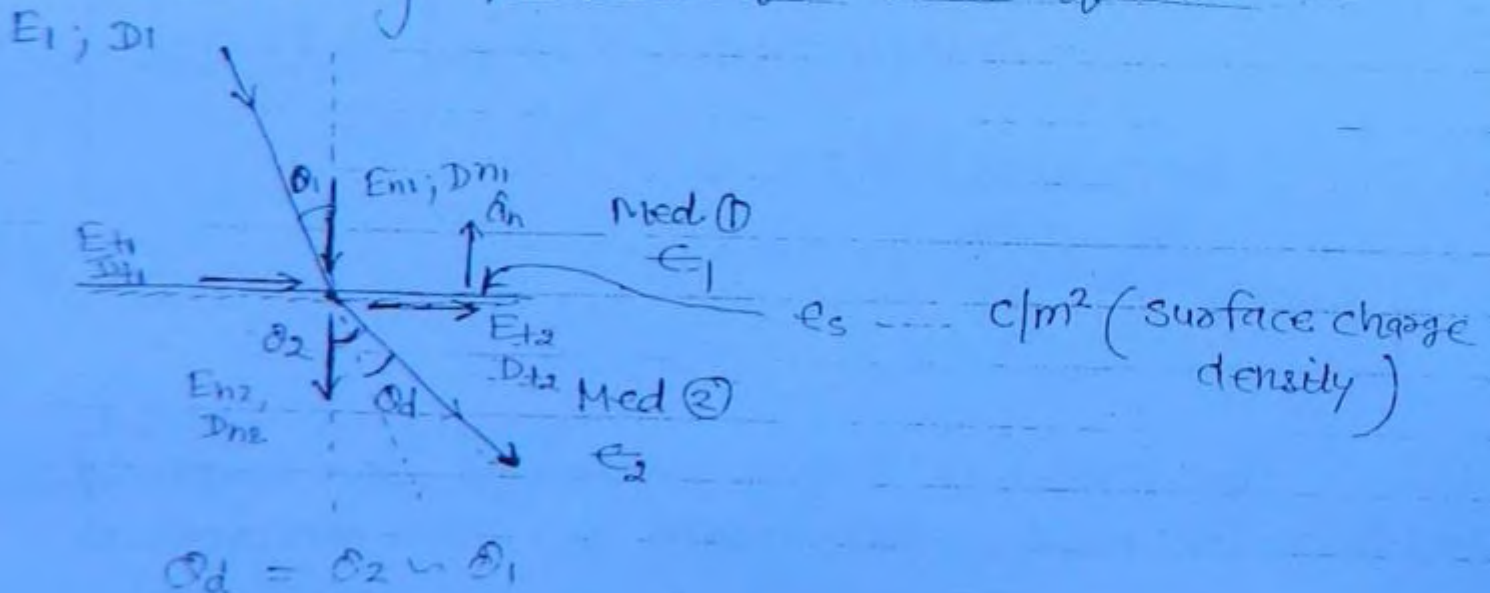
(68)

The Brewster's angle depends the angle of incidence for which the wave is not reflected & the entire wave is transmitted in 2nd medium.

This phenomenon is for the propagation for em.waves in unbound medium.

Such as the propagation through free space. or the transmission of waves through antenna.

* Boundary Relation for elect. fields :-



Tafind :-

$E_{t1} \longleftrightarrow E_{t2}$ tangential
Comp. relation.

$$E_{n1} \longleftrightarrow E_{n2} \quad \text{----- Normal Comp. relation.}$$

$$\left. \begin{matrix} \theta_1 \\ \theta_2 \end{matrix} \right\} \longleftrightarrow \left\{ \begin{matrix} \epsilon_1 \\ \epsilon_2 \end{matrix} \right. \quad \text{----- Oblique incidence of electric fields.} \quad (69)$$

Tangential Comp. Relations :-

$$\begin{aligned} \rightarrow & \boxed{ \begin{aligned} E_{t1} &= E_{t2} \\ \frac{D_{t1}}{\epsilon_1} &= \frac{D_{t2}}{\epsilon_2} \\ \hat{a}_n \times (\vec{E}_1 - \vec{E}_2) &= 0 \end{aligned} } \end{aligned}$$

$$|\hat{a}_n \times \vec{E}_1| = E_{t1}$$

$$|\hat{a}_n \times \vec{E}_2| = E_{t2}$$

$$|\hat{a}_n \times \vec{E}_2| = E_{t2}$$

\hat{a}_n ---- unit normal vector \perp to common boundary.

Case 1: Med. ① is conduct.

$$E_{t1} = 0$$

$$E_{t1} = E_{t2} = 0$$

- ① The tangential comp. of elect. field intensity E is continuous across a common boundary separating two diff. die. medium.
- ② the result is independent of surface charge density present on the common boundary.
- ③ The tangential comp. of elect. field intensity E will not exist across a common boundary separating two diff. media. when one of them or both media are conducting media.

Normal comp. relation

(70)

$$\rightarrow \begin{cases} D_{n1} - D_{n2} = \epsilon_s \\ -\epsilon_1 E_{n1} - \epsilon_2 E_{n2} = \epsilon_s \\ \hat{a}_n \cdot (\vec{D}_1 - \vec{D}_2) = \epsilon_s \end{cases} \quad \begin{aligned} \hat{a}_n \cdot \vec{D}_1 &= D_{n1} \\ \hat{a}_n \cdot \vec{D}_2 &= D_{n2} \end{aligned}$$

Case 1:

$$\epsilon_s = 0$$

$$D_{n1} = D_{n2}$$

Case 2:

Med 2 is cond.

$$D_{n2} = 0$$

$$D_{n1} - D_{n2} = \epsilon_s$$

$$D_{n1} = \epsilon_s$$

Case 3:

Med 1 is cond.

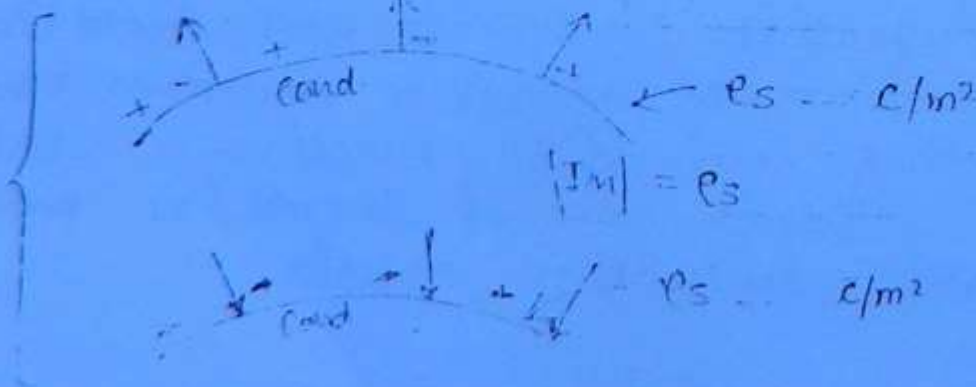
$$D_{n1} - D_{n2} = \epsilon_s$$

$$-D_{n2} = \epsilon_s$$

from case (2) & (3)

$$|D_n| = \epsilon_s$$

$$|D_n| = \epsilon_s$$



① The normal comp. of elect. flux density D is discontinuous by a factor ϵ_s where ϵ_s is const. Surface charge density on the common boundary separating two diff. dielel. media. (71)

② for a charge free common boundary the normal comp. of elect. flux density D is always continuous.

③ Any conducting surface always supports normal comp. of elect. field such that the normal comp. of elect. flux density D is numerically equal to magnitude of surface charge density present on the conducting surface.

Oblique Incidence

$$\text{if } E_3 = 0$$

$$\begin{cases} E_{t1} = E_{t2} \\ D_{n1} = D_{n2} \end{cases}$$

$$\begin{cases} E_{t1} = E_1 \sin \theta_1 \\ E_{t2} = E_2 \sin \theta_2 \end{cases}$$

$$\begin{cases} E_{n1} = E_1 \cos \theta_1 \\ E_{n2} = E_2 \cos \theta_2 \end{cases}$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\epsilon_2}{\epsilon_1}$$

--- Snell's law

(72)

Ex:

$$\hat{a}_n = +\hat{a}_z$$

Med ① ; $\boxed{3 > 0}$
 $\epsilon_1 = 4\epsilon_0$

$$\vec{E}_1 = (3\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z)$$

$$\rho_s = 3 \text{ } \epsilon_0$$

--- C/m^2

$$\vec{E}_2$$

Med ② ; $\boxed{3 < 0}$
 $\epsilon_2 = 6\epsilon_0$

To find

$$\vec{E}_2 = ?$$

$$= E_{x2}\hat{a}_x$$

$$+ E_{y2}\hat{a}_y$$

$$+ E_{z2}\hat{a}_z$$

$$\vec{E}_2$$

tangential comp.
 $E_{t1} = E_{t2}$

$$\rightarrow x \Rightarrow E_{x2} = E_{x1} = 3$$

$$\rightarrow y \Rightarrow E_{y2} = E_{y1} = -6$$

normal comp.

$$D_{n1} - D_{n2} = \rho_s$$

$$\} \Rightarrow D_{z1} - D_{z2} = \rho_s$$

$$\vec{E}_2 = 3\hat{a}_x - 6\hat{a}_y - \frac{35}{6}\hat{a}_z$$

$$\epsilon_1 E_{z1} - \epsilon_2 E_{z2} = \rho_s$$

$$= 4\epsilon_0(-8) - 6\epsilon_0 E_{z2} = 3\epsilon_0$$

$$E_{z2} = \frac{-32-3}{6} = -\frac{35}{6}$$

★ Boundary relations for Mag. fields

tangential comp. relation

(23)

$$\left. \begin{aligned} H_{t1} - H_{t2} &= J_s \\ \frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} &= J_s \\ \hat{a}_n \times (\vec{H}_1 - \vec{H}_2) &= \vec{J}_s \end{aligned} \right\} \begin{aligned} &\text{; } J_s \text{ - Surface current density} \\ &\text{on common boundary} \\ &\text{--- A/m} \end{aligned}$$

$$|\hat{a}_n \times \vec{H}_1| = H_{t1}$$

$$|\hat{a}_n \times \vec{H}_2| = H_{t2}$$

$$H_{t1} = H_{t2} \quad \text{--- } J_s = 0$$

$$\begin{cases} H_{t1} = J_s & \text{--- if med. ② is cond.} \\ -H_{t2} = J_s & \text{--- if med. ① is cond.} \end{cases}$$

$$|H_t| = J_s$$

① The tangential comp of mag. field intensity H is discontinuous by a factor of J_s . J_s represent the surface current density present on the common boundary separating two diff. boundary media.

② on a ~~current~~ current free common boundary the tangential comp of the H field is always continuous.

Any conducting surface will support only the tangential comp of H field such that the magnitude mag. field intensity H is numerically equal to the surface current density present on the conducting surface.

Normal comp. relation

$$\begin{aligned}
 B_{n1} &= B_{n2} \\
 \mu_1 H_{n1} &= \mu_2 H_{n2} \\
 \hat{a}_n \cdot (\vec{B}_1 - \vec{B}_2) &= 0
 \end{aligned}$$

(74)

$$\hat{a}_n \cdot \vec{B}_1 = B_{n1}$$

$$\hat{a}_n \cdot \vec{B}_2 = B_{n2}$$

$B_{n1} = B_{n2} = 0$ if the medium ① & med. ②
or both are conductors.

① The normal comp. of mag. flux density B is continuous across a common boundary separating two different mag. media.

② The result is independent of the surface current density present on the common boundary.

③ The normal comp. of mag. flux density B will not exist across a common boundary separating two different media, when one of them or both media are conducting media.

Oblique Incidence

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{z1}}{\mu_{z2}}$$

75

Med. ① ; $x > 0$
 $\mu_1 = 2\mu_0$

Ex.

$$\vec{B}_1 = 4\hat{a}_x + 5\hat{a}_y - 6\hat{a}_z$$

$$\hat{a}_n = \hat{a}_z$$

$$J_s = 0$$

$$\vec{B}_2 = ?$$

Med. ② ; $x < 0$
 $\mu_2 = 7\mu_0$

Normal
 $B_{n1} = B_{n2} \rightarrow B_{x2} = B_{x1} = \textcircled{4}$

Tangential

$$H_{t1} - H_{t2} = J_s$$

$$H_{t1} = H_{t2}$$

$$H_{y1} = H_{y2} \Rightarrow \frac{B_{y2}}{\mu_2} = \frac{B_{y1}}{\mu_1} ; B_{y2} = \frac{\mu_2}{\mu_1} \cdot B_{y1}$$

$$H_{z2} = H_{z1}$$

$$\Rightarrow B_{z2} = \frac{\mu_2}{\mu_1} \cdot B_{z1}$$

$$= \frac{7}{2} \times 5 = \frac{35}{2}$$

$$\Rightarrow \frac{1}{2} \times (-6) = -21$$

$$\vec{B}_2 = 4\hat{a}_x + \frac{35}{2}\hat{a}_y - 21\hat{a}_z$$

✱

Wave polarization

(76)

(1) The wave polarization is related to the orientation of elec. field vector associated with the EMW.

(2) if the vertical antenna installed, the electrical field vector is also vertical & EMW is vertically polarised.

(3) if any antenna is horizontally polarised the elec. field vector is parallel to the surface at the earth & the EMW is horizontally polarised.

(4) the orientation of the elec. field vector at the transmitting & receiving end must be same so that max. induced emf. is obtained at the receiving antenna.

therefor the polarization the transmitting & Receiving antenna must be identical.

$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_2 \sin(\omega t - \beta z + \alpha)$$

↑
time phase angle b/w
 E_x & E_y

To find out

effect of $E_1, E_2; \alpha$

Where $z=0$ plane

$$E_x = E_1 \sin \omega t$$

$$E_y = E_2 \sin(\omega t + \alpha)$$

$\alpha = 0^\circ$ ----- in same phase.
 $\alpha = 90^\circ$ ----- quadrature phase.

(77)

Case 1:- Linear Polarization

$$\begin{array}{c}
 E_1 \neq E_2 \\
 \alpha = 0^\circ \text{ or } 180^\circ
 \end{array}$$

if $\alpha = 0^\circ$

$$E_x = E_1 \sin \omega t$$

$$E_y = E_2 \sin \omega t$$

$$\frac{E_y}{E_x} = \frac{E_2}{E_1} = m$$

$$E_y = m E_x$$

--- eqn of st. line.

(a) if $E_1 = 0$

$$\begin{cases}
 E_x = 0 \\
 E_y = E_2 \sin \omega t
 \end{cases}$$

--- E.M.W is linearly polarised along y-direction.

--- Wave is Vertically polarised

(b) if $E_2 = 0$

$$\begin{cases}
 E_x = E_1 \sin \omega t \\
 E_y = 0
 \end{cases}$$

--- EMW is linearly polarised along the x-direction.

--- Wave is Horizontally polarised.

Case 2:

Circular Polarization

$$E_1 = E_2 = E_0$$

$$\alpha = \pm 90^\circ$$

(78)

$$E_x = E_0 \sin \omega t$$

$$E_y = E_0 \sin(\omega t + 90^\circ) ; \alpha = \pm 90^\circ$$

$$= E_0 \cos \omega t$$

$$E_x^2 + E_y^2 = E_0^2$$

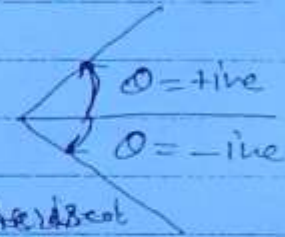
--- eqn of a circle

$$\alpha = +90^\circ$$

--- LCP wave

$$\alpha = -90^\circ$$

--- RCP wave



Right circular polarization

Left circular polarization

Case 3:

Elliptical Polarization

$$E_1 \neq E_2$$

$$\alpha = \pm 90^\circ$$

$$\alpha = +90^\circ$$

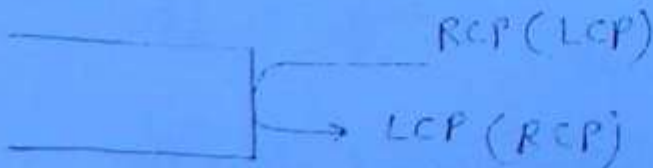
--- LEP wave

$$= -90^\circ$$

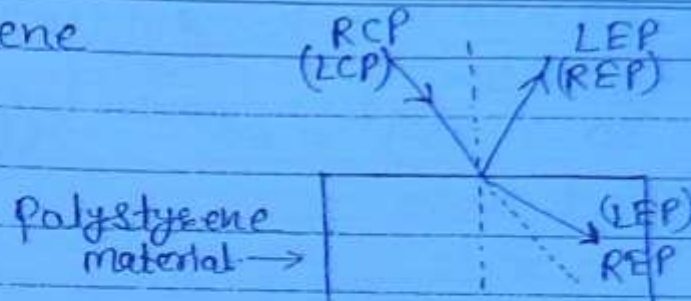
--- REP wave

Main points :- ① When RCP wave is incident

on a perfect conductor perpendicularly, the reflected wave is LCP wave & vice-versa



(2) Polystyrene



(29)

if an RCP wave incidence on polystyrene then:

- Transmitted wave ^{is} REP wave
- Reflected wave is LEP wave & vice-versa

(3) if ^{Brewster's} is a circularly polarized wave or elliptically polarized wave incidence at Brewster's angle on any interface then the reflected wave & transmitted wave are linearly polarized.

(Ex:) $\vec{E} = (5) \sin(\omega t - \beta z) \hat{a}_x + (10) \sin(\omega t - \beta z) \hat{a}_y$
 $\alpha = 0$

$$E_1 \neq E_2$$

-- linearly polarized

(Ex:) $\vec{E} = (5) \sin(\omega t - \beta z) \hat{a}_x + (5) \sin(\omega t - \beta z - 90^\circ) \hat{a}_y$

$$E_1 = E_2$$

$$\alpha = -90^\circ$$

-- RCP wave

(Ex:) $\vec{E} = 5 \sin(\omega t - \beta z - 30^\circ) \hat{a}_x + 10 \sin(\omega t - \beta z + 60^\circ) \hat{a}_y$

$$E_1 \neq E_2$$

$$\alpha = 60^\circ - (-30^\circ) = +90^\circ$$

-- LEP wave

Ex: $(5 \sin(\omega t - \beta z - 30^\circ) \hat{a}_x + 5 \sin(\omega t - \beta z + 30^\circ) \hat{a}_y)$

$$\begin{cases} E_1 = E_2 = 5 \\ \alpha = 30 - (-30) = 60^\circ \end{cases} \quad (80)$$

 --- unpolarized wave

Imp
example:- An emw. having following x-compts. of
 elec. field is travelling in a lossless medium
 with $\mu_r = 1$ & $\epsilon_r = 9$.
 find:

- ① All the parameters associated with emw.
- ② the mag. field intensity associated with em.w.

$$\vec{E} = 10 \cos(6\pi \times 10^3 t - \beta z) \hat{a}_x$$

$$\sigma = 0$$

$$\mu_r = 1$$

$$\epsilon_r = 9$$

To find:

1. Various parameters
2. \vec{H}

$$\vec{E} = E_x \hat{a}_x$$

$$E_x = E_m \cos(\omega t - \beta z)$$

↑ +z direction

- ① $\vec{E} = E_x \hat{a}_x$
- ② $E_m = 10 \text{ V/m}$
- ③ wave is linearly polarized along x-direction
- ④ direction of propagation along +z direction
- ⑤ intrinsic impedance $\eta = \sqrt{\frac{\mu}{\epsilon}}$

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{3} = 40\pi \Omega$$

$$(6) \quad v_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \cdot \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{3} = 10^8 \text{ m/sec.}$$

$$(7) \quad \omega = 6\pi \times 10^8 = 600\pi \text{ --- Mead/sec.} \quad (8)$$

$$f = \frac{\omega}{2\pi} = 300 \text{ --- MHz.}$$

$$(8) \quad \beta = \frac{\omega}{v_p} = \frac{6\pi \times 10^8}{10^8} = 6\pi \text{ --- rad/m}$$

$$(9) \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6\pi} = \frac{1}{3} \text{ --- m}$$

$$(10) \quad \vec{n} = \vec{E} \times \vec{H}$$

$$= \frac{1}{2} \eta H_m^2 \hat{a}_z$$

$$= \frac{1}{2} \frac{E_m^2}{\eta} \cdot \hat{a}_z = \frac{1}{2} \times \frac{100}{40} \hat{a}_z \text{ --- W/m}^2$$

$$(11) \quad \eta = \frac{E_m}{H_m} \quad ; \quad H_m = \frac{E_m}{\eta} = \frac{10}{40\pi} = \frac{1}{4\pi} \text{ --- A/m}$$

$$\vec{n} = \vec{E} \times \vec{H}$$

$$\hat{a}_z = \hat{a}_x * (+\hat{a}_y)$$

$$\vec{H} = H_y \hat{a}_y$$

$$= H_m \cos(\omega t - \beta z) \hat{a}_y$$

$$\frac{1}{4\pi} \cos(6\pi \times 10^8 t - 6\pi z) \hat{a}_y$$

--- A/m

Ans.

Ex: $\vec{H} = 10 \sin(6\pi \times 10^8 t + 6\pi x) \hat{a}_z$

Note \rightarrow Wave is linearly polarized along y-direction.

$$\vec{H} = H_z \cdot \hat{a}_z$$

propag. \rightarrow $-x$ direction

(82)

$$\vec{P} = -\rho_z \hat{a}_x$$

$$\vec{P} = \vec{E} \times \vec{H}$$

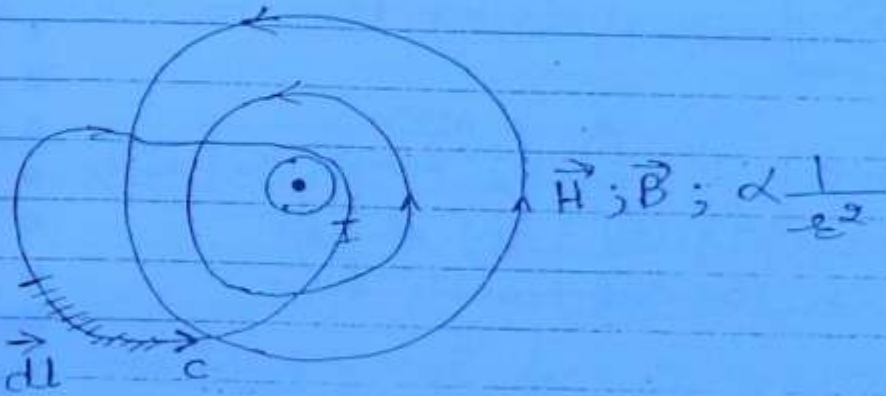
$$-\hat{a}_x = (\hat{a}_y) \times (\hat{a}_z)$$

$$\vec{E} = -E_y \hat{a}_y$$

Static Magnetic fields.
 $\vec{B}, \vec{H} \neq f(t)$

(83)

Ampere's circuital law :-



bind: \vec{H}
 $\cdot d\vec{l}$
 $\cdot \vec{H} \cdot d\vec{l}$

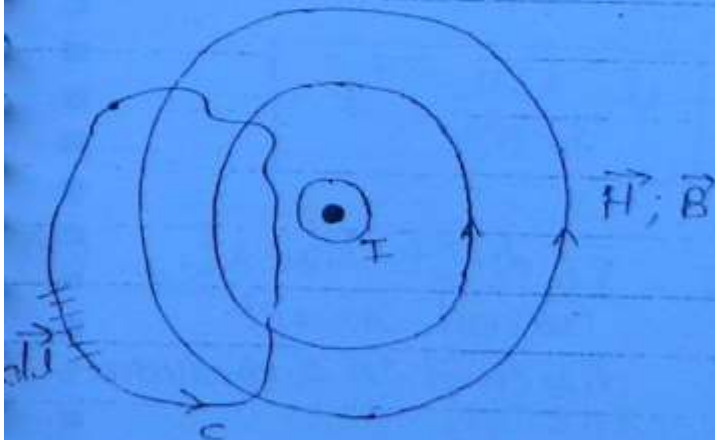
$$\oint_C \vec{H} \cdot d\vec{l} = I$$

(current enclosed)

Where

$$\begin{aligned} I &= I \\ &= \int dJ_s \cdot dl \\ &= \iiint_s \vec{J} \cdot d\vec{\epsilon} \end{aligned}$$

----- Ampere's
 circuital law in
 integral form
 = line current
 = surface current
 = volume current.



$$\oint_C \vec{H} \cdot d\vec{l} = 0$$

* Ampere's circuital law in differential or point form :-

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \quad (84)$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_c = \sigma \vec{E}}$$

Statements :-

(1) The circulation of static ^{mag.} field intensity \vec{H} along any closed curve (C) equal to ~~E~~ is total current I enclosed with the enclosed current C .

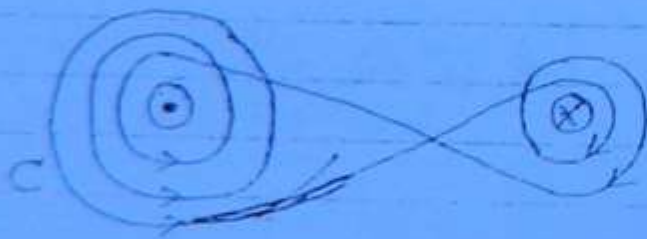
(2) if no current is enclosed the circulation of \vec{H} field along the closed curve is zero although at each point $\vec{H} \cdot d\vec{l}$ has finite value.

(3) The law is applicable irrespective of the shape of the closed curve C .

(4)

The curl of static mag. field intensity \vec{H} at any point is always equal to conduction current density present at that point.

ex:



To find :-

$$\oint_C \vec{H} \cdot d\vec{l} = ?$$

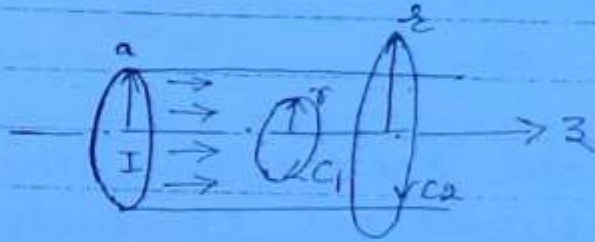
$$= 3I - \left(-2I \right) = 5I$$

due to opposition of circulation of current

due to flow of current in oppo direction in 2 conductor.

ex: A long solid cylindrical conductor radius 'a' carries a uniform current 'I' through out of cross-section of conductor. find the value of mag. flux density \vec{B} at all points/regions. -

(85)



to find: \vec{B} for

- sub
1. $r < a$
 2. $r > a$
 3. $r = a$

① $r < a$

$$\oint_{C_1} \vec{H} \cdot d\vec{l} = I'$$

$$H_\phi \cdot 2\pi r = \frac{r^2 \cdot I}{a^2}$$

$$H_\phi = \frac{I r}{2\pi a^2}$$

$$\therefore \vec{H} = \frac{I r}{2\pi a^2} \hat{a}_\phi$$

$$\frac{I'}{I} = \frac{\pi r^2}{\pi a^2}$$

$$I' = \frac{r^2 \cdot I}{a^2}$$

$$\boxed{\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi}$$

② $r > a$

$$\oint_{C_2} \vec{H} \cdot d\vec{l} = I$$

$$H_\phi \cdot 2\pi r = I$$

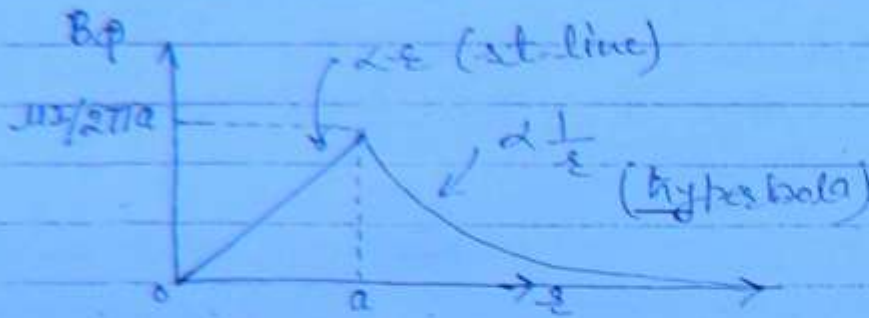
$$\boxed{\vec{B} = \frac{\mu I}{2\pi r} \hat{a}_\phi}$$

$$\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi \quad \dots \quad r < a$$

$$= \frac{\mu I}{2\pi r} \hat{a}_\phi \quad \dots \quad r > a$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{\phi} \quad \text{--- } r=a$$

(86)



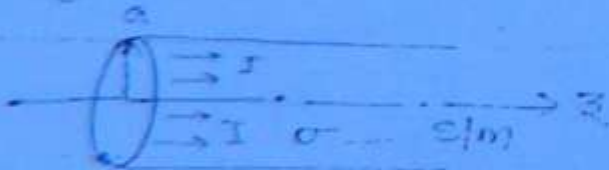
Imp points :- (1) The mag. field increases linearly in a solid cylindrical conductor whereas it decreases in the hyperbolic outside the cylindrical conductor.

2) The mag. field is always constant on the surface of any conductor.

3) any cylindrical conductor behaves as an infinite line current since the mag. field due to both configuration is same.

ex: A long solid cylindrical cond. of radius 'a' & conductivity ' σ ' carries uniform current ' I '.

find the pointing vector on the surface of cylindrical conductor.



$$\vec{H}_{r=a} = ? = \vec{r} \times \vec{J}$$

$$\vec{J} = \vec{J} = \sigma \vec{E}$$

$$\vec{E} = \frac{I}{2\pi a} \hat{\phi}$$

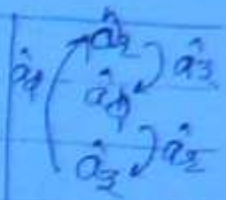
$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{1}{\sigma} \left(\frac{I}{\pi a^2} \right) \hat{a}_z$$

$\underbrace{\left(\frac{I}{\pi a^2} \right)}_{\vec{J} \dots A/m^2}$

$$\vec{P} = \frac{I}{\pi \sigma a^2} \hat{a}_z \times \frac{I}{2\pi a} \hat{a}_\phi$$

$$= \frac{I^2}{2\pi^2 \sigma a^3} \left(\underbrace{\hat{a}_z \times \hat{a}_\phi}_{= -\hat{a}_\theta} \right) = -\hat{a}_\theta$$

(87)

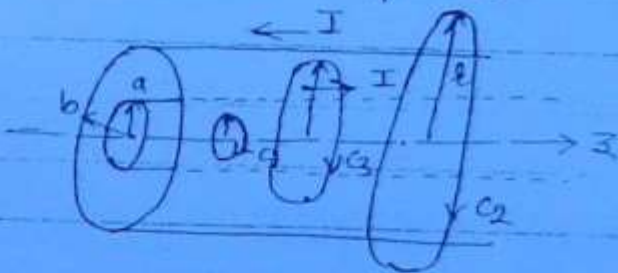


$$\vec{P} = \frac{-I^2 \cdot \hat{a}_\theta}{2\pi^2 \sigma a^3}$$

Ex:

A long co-axial TL of radii 'a' & 'b' with $b > a$ carries uniform current $\pm I$ on the surface of the 2-cylindrical conductor of the TL.

calculate mag. field intensity \vec{H} at all points.



To find:

 \vec{H} for

1. $z < a$

2. $a < z$

3. $z > b$

① $z < a$

$\oint_{C_1} \vec{H} \cdot d\vec{l} = 0$

$\Rightarrow \boxed{H=0}$

② $z > b$

$\oint_{C_2} \vec{H} \cdot d\vec{l} = +I + (-I) = 0$

$\boxed{\vec{H}=0}$

(3) $a < z < b$

(88)

$$\oint_{C3} \vec{H} \cdot d\vec{l} = I$$

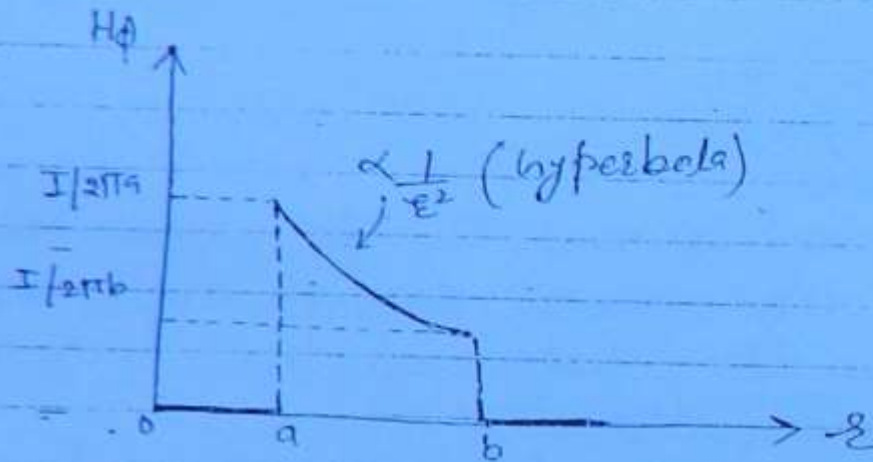
$$H\phi \cdot 2\pi\epsilon = I$$

$$\vec{H} = \frac{I}{2\pi\epsilon} \hat{\phi}$$

$$\vec{H} = 0 \quad \text{--- } z < a$$

$$= \frac{I}{2\pi\epsilon} \hat{\phi} \quad \text{--- } a < z < b$$

$$= 0 \quad \text{--- } z > b$$



★ Magnetic energy density

Date _____

$$w_m \text{ --- (J/m}^3\text{)}$$

$$w_m = \lim_{\Delta V \rightarrow 0} \left(\frac{\Delta w_m}{\Delta V} \right)$$

(89)

$$\boxed{w_m = \frac{1}{2} \mu H^2} \\ = \frac{1}{2} \vec{B} \cdot \vec{H} \text{ --- (J/m}^3\text{)}$$

① The mag. energy density represent the mag. energy store per unit volume & gives Mag. energy per at a point in any electromag. region.

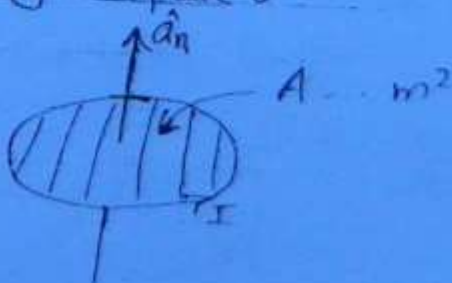
② This depends upon mag. field intensity H & the const. of the medium μ .

★ Magnetic energy stored

$$\boxed{w_m = \frac{1}{2} L I^2} \text{ --- J}$$

★ Magnetic dipole moment

Mag. dipole:



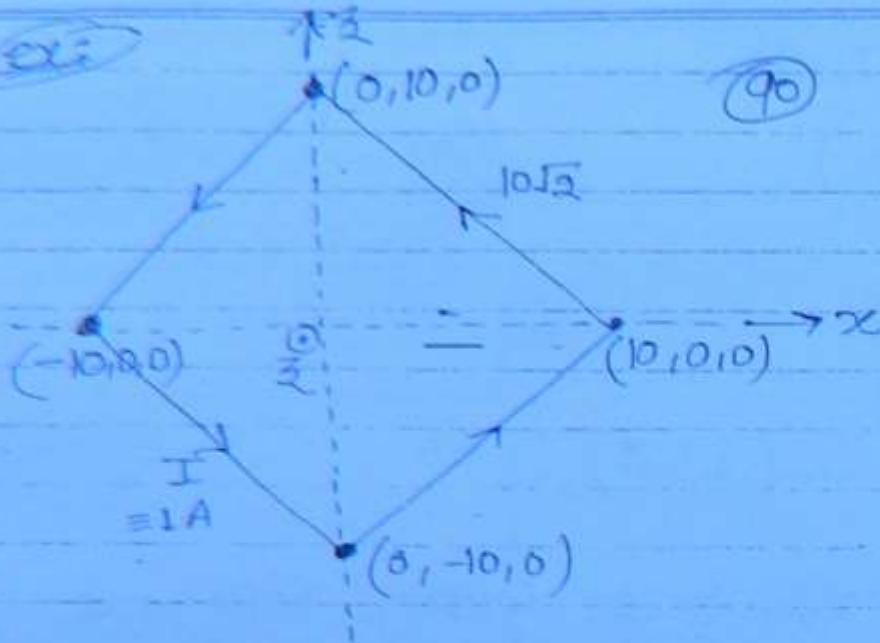
$$\boxed{\vec{m}_m = I A \cdot \hat{n}} \text{ --- A.m}^2$$

(ex)

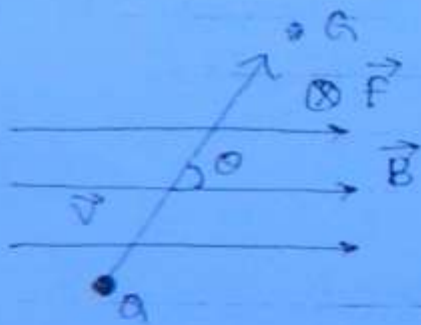
(90)

To find :-

$$\begin{aligned}\vec{m} &= I A \hat{a}_3 \\ &= 1 \times (10\sqrt{2})^2 \hat{a}_3 \\ &= 200 \hat{a}_3 \text{ A/m}^2\end{aligned}$$



Important main points :-

(1) Moving charge in \vec{B} :-

$$\vec{F} = q \vec{v} \times \vec{B} = q v B \sin \theta \hat{a}_n$$

$$\begin{aligned}|F| &= 0 & ; & \theta = 0^\circ \\ &= q v B & ; & \theta = 90^\circ\end{aligned}$$

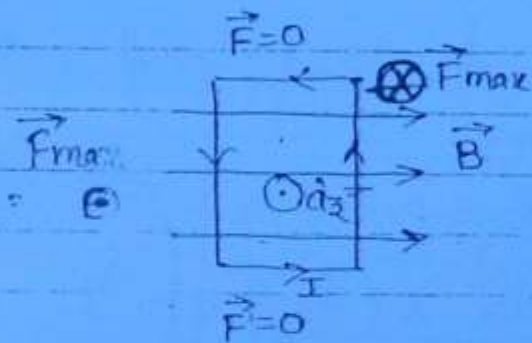
(2) current carrying cond. in \vec{B} :-

$$\vec{F} = \int_c (\vec{I} \times \vec{B}) dl$$

(91)

$$\left(\frac{\vec{F}}{l} \right) = \vec{I} \times \vec{B} \quad - \text{ N/m}$$

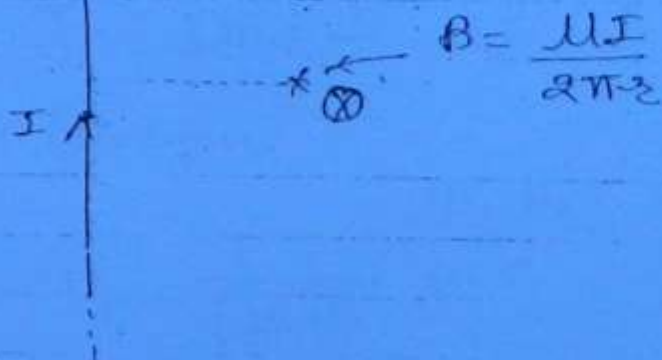
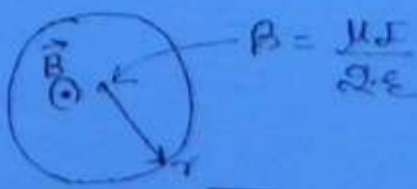
--- force per unit length

current(3) Carrying loop in \vec{B} :-

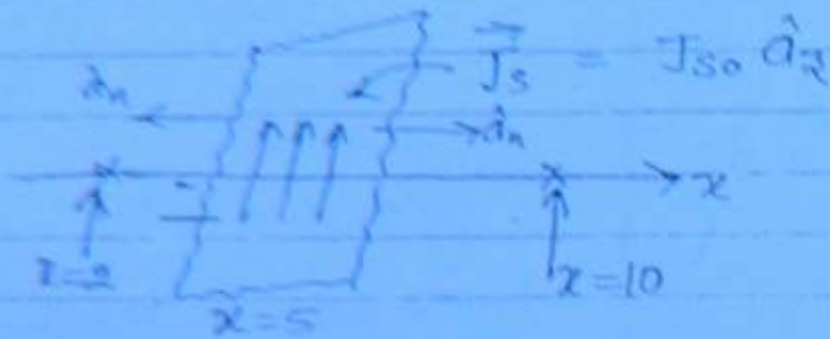
$$\vec{T} = \vec{m} \times \vec{B}$$

\uparrow
 $IA\hat{a}_3$

--- Torque is applicable.

(4) Infinite line current :-(5) Circular loop :-

⑥ \vec{H} or \vec{B} due to infinite current sheet:



(92)

$$\vec{H} = \frac{1}{2} \vec{J}_s \times \hat{a}_n$$

$\hat{a}_n \rightarrow$ unit normal vector \perp to current sheet.

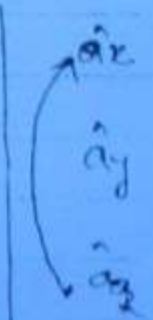
at $x=10$

$$\hat{a}_n = +\hat{a}_x$$

$$\vec{H} = \frac{1}{2} \vec{J}_s \times \hat{a}_n$$

$$= \frac{1}{2} J_{s0} \hat{a}_y \times (\hat{a}_x)$$

$$= \frac{1}{2} J_{s0} \hat{a}_z$$



at $x=2$

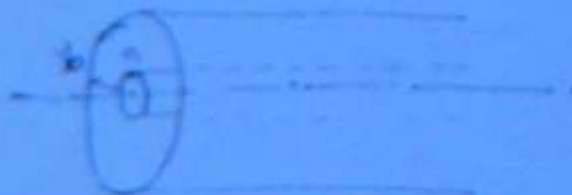
$$\hat{a}_n = -\hat{a}_x$$

$$\vec{H} = \frac{1}{2} \vec{J}_s \times \hat{a}_n = \frac{1}{2} J_{s0} \hat{a}_y \times (-\hat{a}_x)$$

$$\vec{H} = -\frac{1}{2} J_{s0} \hat{a}_z$$

⑦

Co-axial T.L. :-



$$\left(\frac{L}{d} \right) = \begin{bmatrix} L' = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \\ C' = \frac{2\pi \epsilon}{\ln(b/a)} \end{bmatrix} \begin{matrix} \leftarrow H/m \\ \leftarrow F/m \end{matrix}$$

(93)

$$Z_0 = \sqrt{\frac{L'}{C'}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \quad ; \quad \Omega$$

$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

for free space :-

$$\eta = 120\pi$$

$$Z_0 = 60 \ln\left(\frac{b}{a}\right) \quad \Omega$$

$$v_p = \frac{1}{\sqrt{L' C'}}$$

$$v_p = \frac{1}{\sqrt{\mu \epsilon}}$$

for free space:

$$\mu = \mu_0; \quad \epsilon = \epsilon_0$$

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

Vector Mag. potential (\vec{A})

$$\underbrace{\nabla \cdot \vec{B}}_{\text{vector}} = 0 \quad \text{--- ① ---} \quad \text{always Valid.}$$

(94)

$$\underbrace{\nabla \cdot (\nabla \times \vec{A})}_{\text{vector}} = 0 \quad \text{--- ② ---} \quad \begin{array}{l} \text{always Valid.} \\ \text{vector identity.} \end{array}$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

$$\vec{B} \perp \vec{A}$$

relation b/w \vec{A} & \vec{B} Mathematical definition of \vec{A} :-

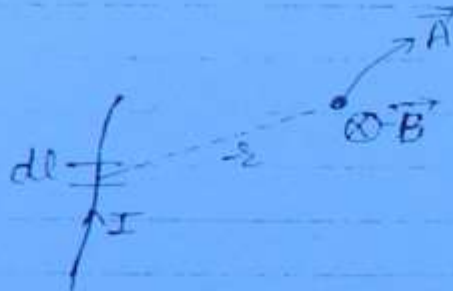
$$\boxed{\vec{A} = \frac{\mu}{4\pi} \int_C \frac{\vec{I}}{r} dl}$$

$$\Rightarrow \vec{A} \parallel \vec{I}$$

Direction-wise

$$\begin{array}{|c|} \hline \vec{A} \parallel \vec{I} \\ \hline \vec{B} \parallel \vec{I} \\ \hline \vec{B} \perp \vec{A} \Rightarrow \\ \hline \end{array}$$

$$\vec{B} = \nabla \times \vec{A}$$



① There is no physical significance of vector mag. potential A since the mag. charges in the isolated form do not exist in nature.

② Calculation of vector A simply gives

to calculate the value of mag. using the eqn:

flux density B , using the relation

$$\vec{B}' = \nabla \times \vec{A}. \quad (95)$$

ex:-

$\vec{A} = \underbrace{2x^2y}_{\equiv \vec{A}_3} \hat{a}_3$

$$Ax = 0$$

$$A_d = 0$$

$$A_2 = 2x^2y$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z \end{vmatrix} = \hat{a}_x \frac{\partial}{\partial y} (2x^2y) + \hat{a}_y \left[-\frac{\partial}{\partial x} (2x^2y) \right] + \hat{a}_z (0)$$

$$\nabla \times \vec{B} = 0 \quad \dots \text{irrotational vector.}$$

$\neq 0$ --- not an "..."

units of \vec{A}

$$\vec{B} = \nabla \times \vec{A}$$

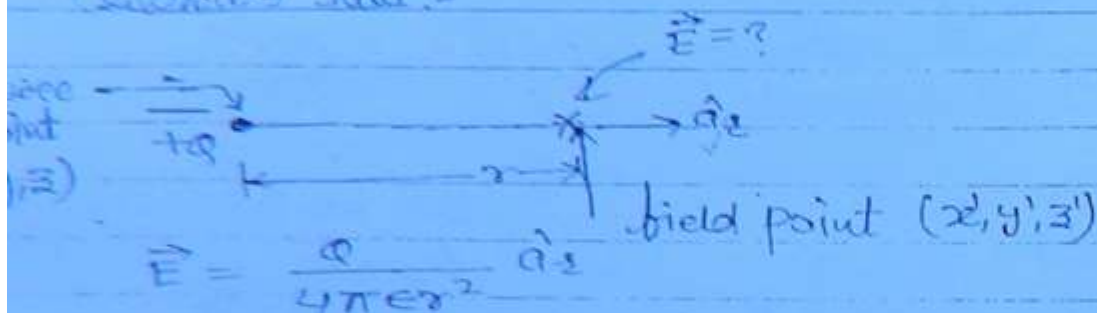
\uparrow \uparrow \Rightarrow Wb/m
 $\frac{\text{Wb}}{\text{m}^2}$ $\frac{1}{\text{m}}$

----- Static electric fields

$$\vec{E}; \vec{D} \neq f(t)$$

(96)

Coulomb's law:-



$$\vec{E} = \frac{Q}{4\pi\epsilon r^2} \hat{a}_r$$

$$\vec{r} = r \hat{a}_r$$

$$\hat{a}_r = \frac{\vec{r}}{r}$$

$$\vec{E} = \frac{Q\vec{r}}{4\pi\epsilon r^3}$$

Where

$$\vec{r} = (x'-x)\hat{a}_x + (y'-y)\hat{a}_y + (z'-z)\hat{a}_z$$

$$|\vec{r}| = \sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}$$

A point charge of +3 nC is placed at a point (2, 1, -2) in a medium whose dielect. const. is 3.

Find the elect. field intensity at point (1, 3, -1).

P --- source point (2, 1, -2)

P' --- field point (1, 3, -1)

$$Q = +3 \text{ nC}$$

$$\epsilon_r = 3$$

To find :- $\vec{E} = \frac{Q\vec{r}}{4\pi\epsilon_0\epsilon_r r^3}$

$$\vec{r} = (1-2)\hat{a}_x + (3-1)\hat{a}_y + (-1-(-2))\hat{a}_z$$

$$\vec{e} = -\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$

$$|\vec{e}| = \sqrt{1+4+1} = \sqrt{6}$$

(92)

$$\vec{E} = \frac{3 \times 10^{-9}}{4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 3(\sqrt{6})^3} (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$$\vec{E} = \frac{27}{3 \times 6\sqrt{6}} (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$$\vec{E} = K(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z) \quad \dots \text{V/m}$$

$$K = \frac{3}{2\sqrt{6}}$$

$$|\vec{E}| = K\sqrt{6} = \frac{3}{2} \text{ V/m}$$

$$\vec{E} = E\hat{a}_1$$

$$\begin{cases} \hat{a}_1 = \frac{\vec{E}}{E} = \frac{K}{3/2} (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z) \\ \hat{a}_1 = \frac{1}{\sqrt{6}} (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z) \end{cases}$$

$$\alpha = \cos^{-1} \frac{E_x}{E} = \cos^{-1} \left(\frac{1}{\sqrt{6}} \right) \quad \dots \text{w.r.t. } x\text{-axis.}$$

$$\beta = \cos^{-1} \frac{E_y}{E} = \cos^{-1} \left(\frac{2}{\sqrt{6}} \right) \quad \dots \text{w.r.t. } y\text{-axis}$$

$$\gamma = \cos^{-1} \frac{E_z}{E} = \cos^{-1} \left(\frac{1}{\sqrt{6}} \right) \quad \dots \text{w.r.t. } z\text{-axis.}$$



Gauss law :-

Date _____

$$\underbrace{\oint_S \vec{D} \cdot d\vec{s}}_{\text{net elct. flux}} = Q$$

in integral form

(98)

Where

$$Q = q$$

Point charge

$$= \int_C \rho_l \cdot dl$$

line charge

$$= \iint_S \rho_s \cdot ds$$

surface charge

$$= \iiint_V \rho_v \cdot dV$$

Volume charge.

$$\nabla \cdot \vec{D} = \rho$$

differential form or point form.

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

irrotational static elct. field.

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

points :-

① The net elct. flux passing through any closed surface area is always equal to total charge enclosed by the closed surface Area S.

The eqn. is valid for any type of the closed surface Area S.

The direction of elct. flux density \vec{D}

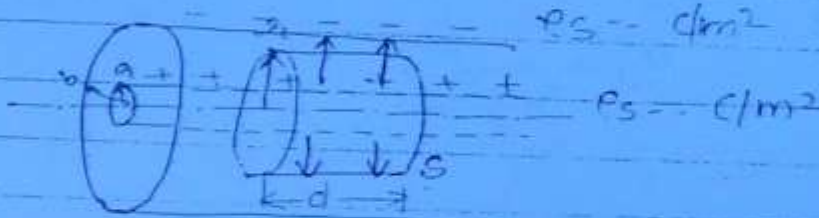
at any point if the elec. mag. Region is always equal to the volume charge density at that point.

(99)

④ The curl of static elec. field intensity \vec{E} at any point is always zero & therefore such field is always irrotational.

therefore the circulation of the static elec. field along any closed curve C is always equal to zero.

ex:



To find \vec{E} ; \vec{D}
for $a < z < b$

$$\vec{E} = \frac{\rho_s \cdot a}{\epsilon_0} \hat{a}_z$$

$$\vec{D} = \epsilon \vec{E}$$

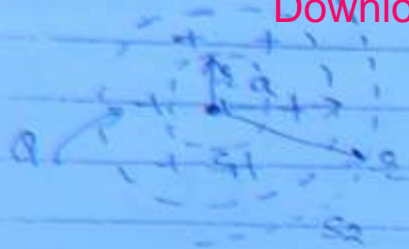
$$\vec{D} = \frac{\rho_s \cdot a}{\epsilon} \cdot \hat{a}_z$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q = \iint_S \rho_s \cdot dS$$

$$\epsilon E_z \cdot 2\pi r d = \rho_s \cdot 2\pi r d$$

$$E_z = \frac{\rho_s \cdot a}{\epsilon}$$

ex: A charge $+Q$ is distributed throughout the volume of Region of radius 'a'.
Find the value of electric field intensity \vec{E} at all point.



To find

 \vec{E} for

(100)

1. $r < a$
2. $r > a$
3. $r = a$

① $r < a$

$$\oint_{S_2} \vec{D} \cdot d\vec{S} = Q'$$

$$\epsilon E_r \cdot 4\pi r^2$$

$$\frac{Q'}{Q} = \frac{4/3 \pi r^3}{4/3 \pi a^3}$$

$$Q' = \frac{Q r^3}{a^3}$$

$$E_r = \frac{Q r}{4\pi \epsilon a^3}$$

$$\vec{E} = \frac{Q r}{4\pi \epsilon a^3} \hat{a}_r$$

② $r > a$

$$\oint_{S_2} \vec{D} \cdot d\vec{S} = Q$$

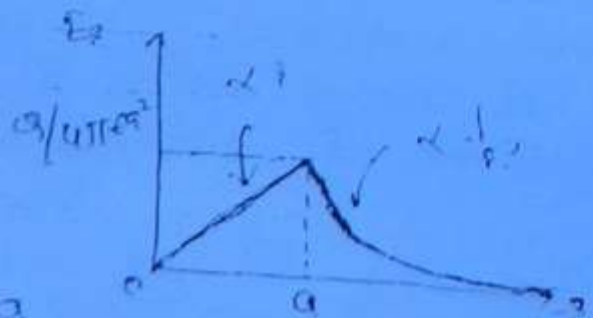
$$\epsilon E_r \cdot 4\pi r^2$$

$$\vec{E} = \frac{Q}{4\pi \epsilon r^2} \hat{a}_r$$

$$\vec{E} = \frac{Q r}{4\pi \epsilon a^3} \hat{a}_r \quad \dots \quad r < a$$

$$= \frac{Q}{4\pi \epsilon r^2} \hat{a}_r \quad \dots \quad r > a$$

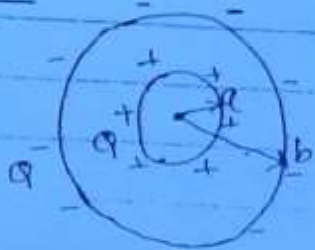
$$= \frac{Q}{4\pi \epsilon a^2} \hat{a}_r \quad \dots \quad r = a$$



ex: Two spherical shells of radius 'a' & 'b'. With b greater than a & equal & opposite charges $\pm Q$ on their surfaces.

find. el. field intensity \vec{E} at all points.

(101)



To find:

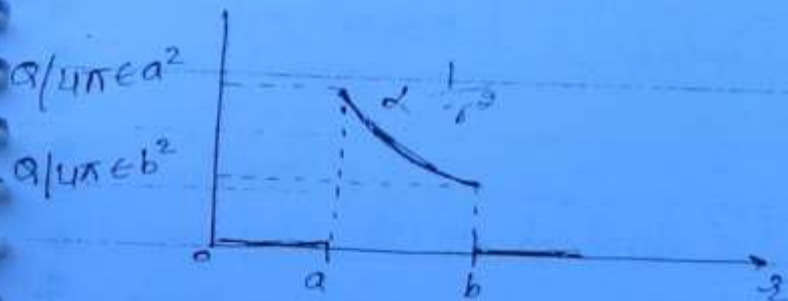
\vec{E} for

1. $r < a$

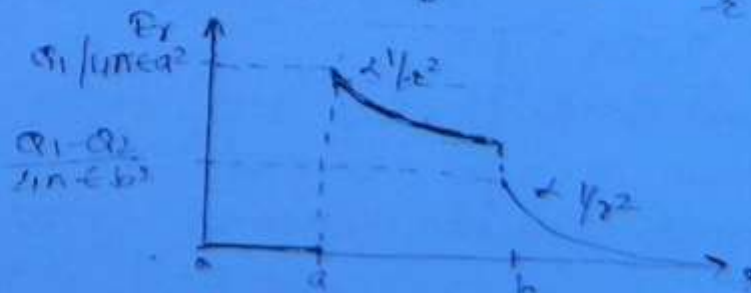
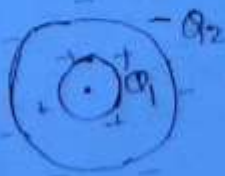
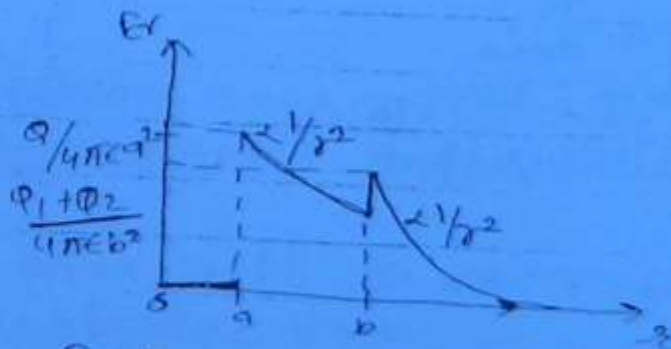
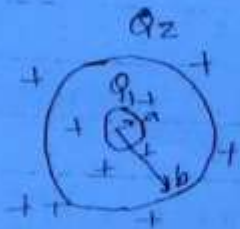
2. $a < r < b$

3. $r > b$

$$\begin{cases} r < a ; \vec{E} = 0 \\ a < r < b ; \vec{E} = \frac{Q}{4\pi\epsilon r^2} \hat{r} \\ r > b ; \vec{E} = 0 \end{cases}$$



Notice



Electric energy density

Date _____

$$W_e \text{ --- } J/m^3$$

$$W_e = \lim_{\Delta V \rightarrow 0} \left(\frac{\Delta W_e}{\Delta V} \right)$$

(102)

$$\begin{aligned} W_e &= \frac{1}{2} \epsilon E^2 \\ &= \frac{1}{2} \vec{D} \cdot \vec{E} \end{aligned} \text{ --- } J/m^3$$

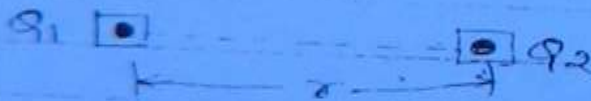
Electric energy stored

$$\begin{aligned} W_e &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} QV \\ &= \frac{1}{2} \frac{Q^2}{C} \end{aligned} \text{ --- } J$$

Since $Q = C \cdot V$

Electric energy stored in a system of 2 charges

$$W_1 = \frac{1}{2} Q_1 V_2 = \frac{1}{2} Q_1 \frac{Q_2}{4\pi\epsilon r}$$



$$\frac{1}{2} \frac{Q_1 Q_2}{4\pi\epsilon r}$$

$$W_{e2} = \frac{1}{2} Q_2 V_1$$

$$= \frac{1}{2} Q_2 \frac{Q_1}{4\pi\epsilon r}$$

$$W_e = W_{e1} = W_{e2}$$

$$= \frac{1}{2} \frac{Q_1 Q_2}{4\pi\epsilon r}$$

$$W_e = \frac{Q_1 Q_2}{4\pi\epsilon r}$$

Poisson's & Laplace Equns

Date _____

$$\nabla \cdot \vec{D} = \rho \quad \text{--- Gauss' law}$$

$$\downarrow \vec{D} = \epsilon \vec{E}$$

$$\Rightarrow \epsilon \nabla \cdot \vec{E} = \rho$$

$$\downarrow$$

$$= -\nabla V$$

$$= -\text{Grad } V$$

$$- \epsilon \nabla \cdot \nabla V = \rho$$

$$\equiv \nabla^2 V$$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\epsilon}}$$

--- Poisson's eqns.

$$\text{if } \rho = 0$$

--- charge free region

$$\boxed{\nabla^2 V = 0}$$

--- Laplace eqns.

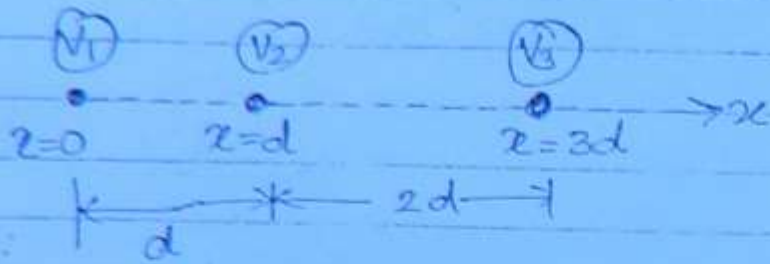
find V

$$\text{find } \vec{E} = -\nabla V$$

points:

- ① The Poisson's eqns represent 2nd order 3-dimen non-homogeneous differential eqn.
- ② The Laplace eqns represent 2nd order 3-dimen homogeneous differential eqns.
- ③ using these eqns the potential V & Elec field \vec{E} can be found due to a specified volume charge distribution or a charge free region.

ex:



104

To find :-

Relation b/w
 V_1, V_2, V_3

$$\nabla^2 V = 0$$

$$V = f(x, y, z)$$

In general

$$V = f(x), \text{ only}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial x} = A; \quad V = Ax + B$$

Boundary conditions :-

$$1. \quad x=0; \quad V=V_1$$

$$2. \quad x=d; \quad V=V_2$$

$$3. \quad x=3d; \quad V=V_3$$

$$(1) \quad V_1 = A \cdot 0 + B \Rightarrow B = V_1$$

$$(2) \quad V_2 = Ad + B = Ad + V_1 \Rightarrow A = \frac{V_2 - V_1}{d}$$

$$V = Ax + B$$

$$V = \frac{V_2 - V_1}{d} \cdot x + V_1$$

(105)

$$\textcircled{3} \quad V_3 = \frac{V_2 - V_1}{d} \cdot 3d + V_1$$

$$V_3 = 3V_2 - 2V_1 \quad \text{Ans}$$

ex:

$$V = 3x^4y^4$$

To find:

$$\rho(1,1,1)$$

$$\nabla^2 V = -\rho/\epsilon$$

$$\Rightarrow \rho = -\epsilon \nabla^2 V$$

$$\rho = -\epsilon \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right]$$

$$\frac{\partial V}{\partial x} = 12x^3y^4 \quad ; \quad \frac{\partial^2 V}{\partial y^2} = 36x^4y^2$$

$$\frac{\partial^2 V}{\partial x^2} = 36x^2y^4$$

$$\rho = -\epsilon [36x^2y^4 + 36x^4y^2]$$

$$\rho(1,1,1) = -\epsilon [72]$$

$$\dots \text{C/m}^3$$

$$\vec{E} = 2x^2y^2\hat{a}_3 \rightarrow E_3\hat{a}_3$$

to find

$$\rho(1,1,1)$$

$$\text{so } E_x = 0$$

$$E_y = 0$$

$$E_z = 2x^2y^2$$

$$\vec{D} = \epsilon \vec{E}$$

$$\rho = \nabla \cdot \vec{D}$$

$$\vec{D} = \epsilon \cdot 2x^2y^2 \hat{a}_z \equiv D_3 \hat{a}_z$$

$$\rho = \nabla \cdot \vec{D}$$

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial D_z}{\partial z} = \frac{\partial}{\partial z} [\epsilon \cdot 2x^2y^2] = 0$$

$$\nabla \cdot \vec{D} = \rho = 0$$

--- Charge free Region

--- \vec{D} is solenoidal

Ex:

$$V = 4x^2y$$

∴ To find \vec{E}

$$\vec{E} = -\nabla V = -\text{Grad } V$$

$$= - \left[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right]$$

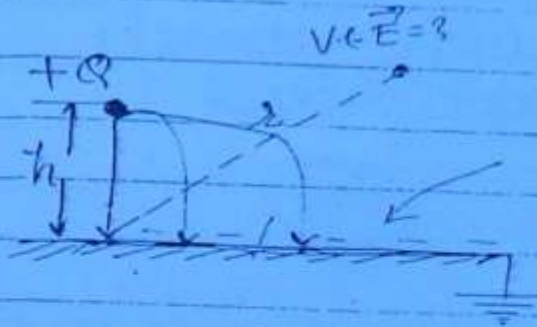
$$= - \left[8xy \hat{a}_x + 4x^2 \hat{a}_y \right]$$

★ Method of Image

----- Another method to find

V & \vec{E}

(107)



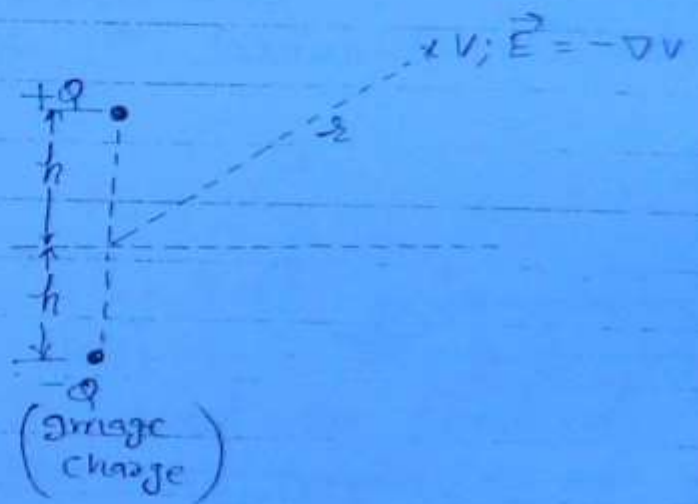
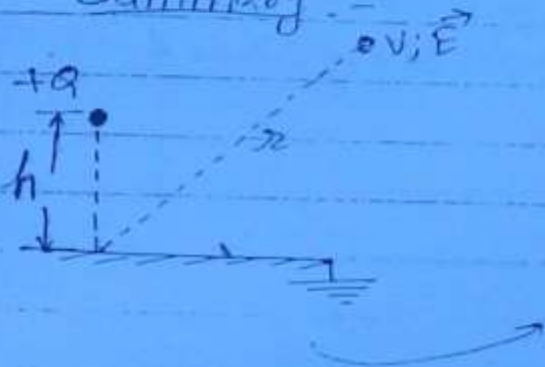
$$Q_{ind} = -Q$$

$E_s(ind.)$ is unknown

Infinite Cond.

or grounded Cond.

Summary:-



Theory of images

points:-

(1) The method of images is applicable to the e.m. problems where any point charge is placed at some height above an infinite cond. or a grounded cond.

(2) Any perfect conductor behaves as a perfect reflector & therefore acts as electromag. mirror.

The entire theory of optics is an applicable.

(3) The theory of images is applicable only

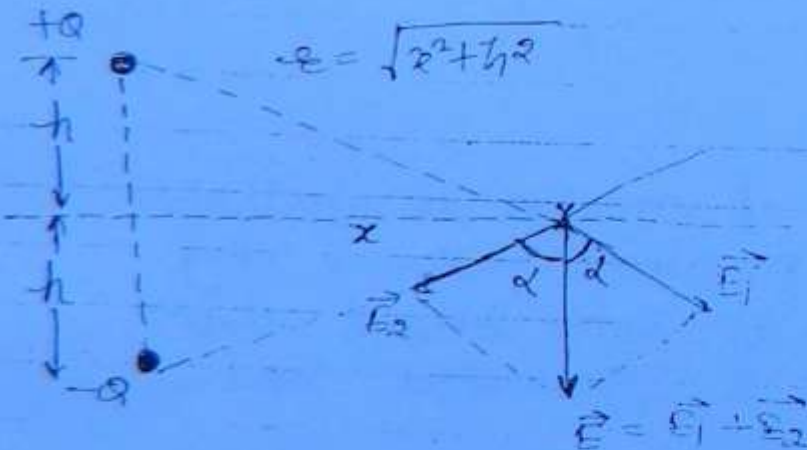
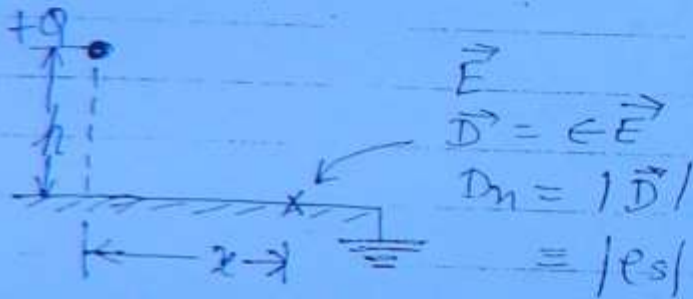
to the e.m. problems.

for mag. field the theory is not applicable. Since the mag. charges in the isolated form do not exist in nature.

(108)

Ex: A point charge $+Q$ is placed at a height H above a grounded conductor.

Calculate the surface charge density induced on the conducting sheet.



$$|\vec{E}_1| = |\vec{E}_2| = E_0 = \frac{Q_0}{4\pi\epsilon r^2}$$

$$E_n = |\vec{E}_1| \cos \alpha + |\vec{E}_2| \cos \alpha$$

$$= 2 E_0 \cos \alpha$$

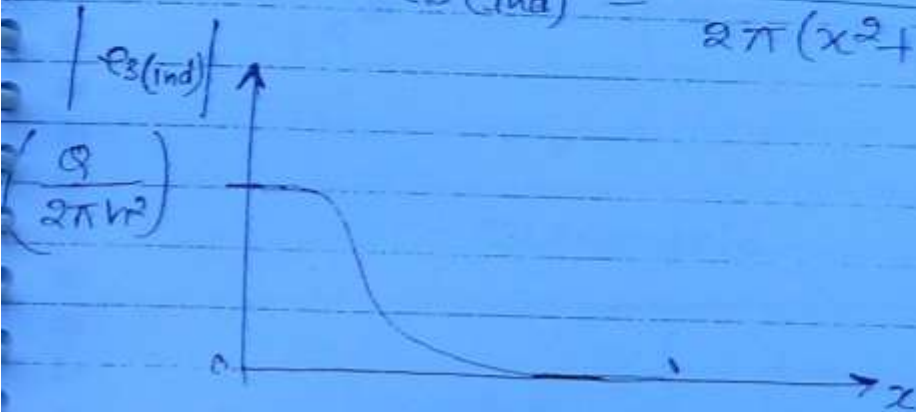
$$= 2 \frac{Q}{4\pi\epsilon r^2} \cdot \frac{h}{r}$$

$$E_n = \frac{Qh}{2\pi\epsilon(x^2+h^2)^{3/2}}$$

(Q)

$$D_n = \epsilon E_n \equiv |e_s| = \frac{Qh}{2\pi(x^2+h^2)^{3/2}}$$

$$e_s(\text{ind}) = \frac{-Qh}{2\pi(x^2+h^2)^{3/2}}$$

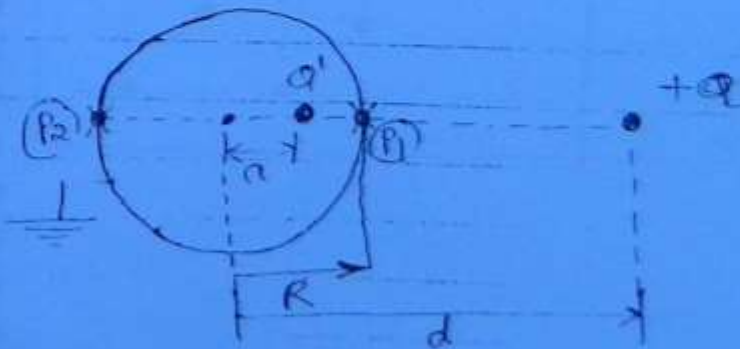


(ex:)

A point charge $+Q$ is placed in front of a grounded spherical conductor of radius R as shown.

calculate the magnitude & the location of image charge.

Images in spheres



Q' = Magnitude } of image
 a = location } charge

(110)

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$V_{P1} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{d-R} + \frac{Q'}{R-a} \right) = 0$$

$$V_{P2} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{d+R} + \frac{Q'}{R+a} \right) = 0$$

$$\frac{Q}{Q'} = - \frac{d-R}{R-a}$$

$$\frac{Q}{Q'} = - \frac{d+R}{R+a}$$

$$\Rightarrow \frac{d-R}{R-a} = \frac{d+R}{R+a}$$

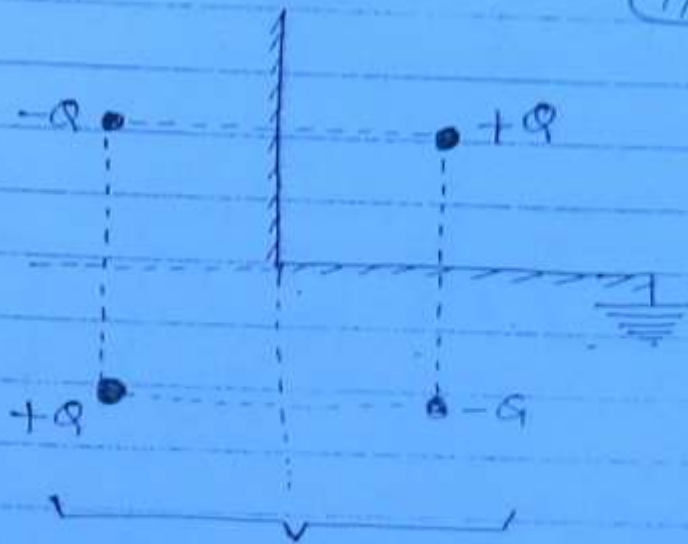
$$\Rightarrow \boxed{a = \frac{R^2}{d}}$$

$$\frac{Q}{Q'} = - \frac{d-R}{R-a}$$

$$\Rightarrow \boxed{Q' = - \frac{QR}{d}}$$

ex:

(III)




No. of Images

$$n = \frac{360^\circ}{\theta} - 1$$

$$n = \frac{360^\circ}{90^\circ} - 1$$

$$= 3$$

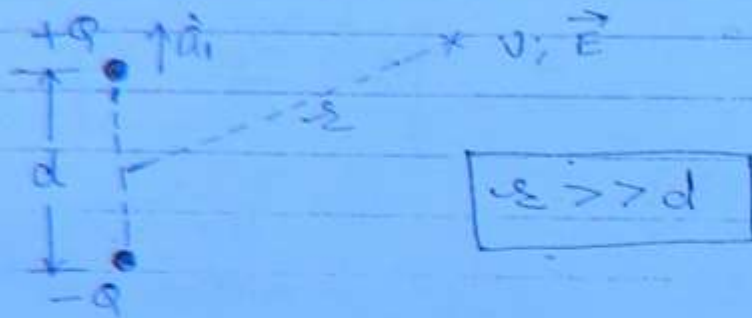
--- Quadrupole

type of configuration	\vec{V}	\vec{E}
Q • monopole	$1/r$	$1/r^2$
$+Q$ • • $-Q$ dipole	$1/r^2$	$1/r^3$
$-$ • • $+$ quadrupole $+$ • • $-$	$1/r^3$	$1/r^4$
 octopole	$1/r^4$	$1/r^5$

Main points :-

(1/2)

(1) elect. dipole



dipole moment

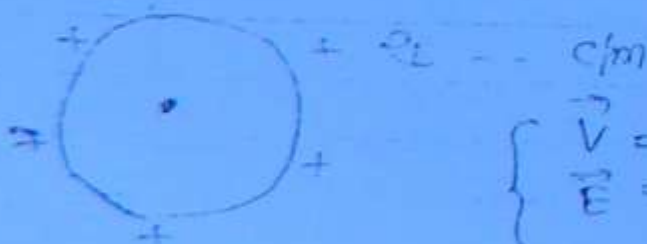
$$\vec{m} = qd \cdot \hat{a}_1$$

— cm

(2) \vec{E} due to infinite line charge

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0} \cdot \hat{a}_\rho$$

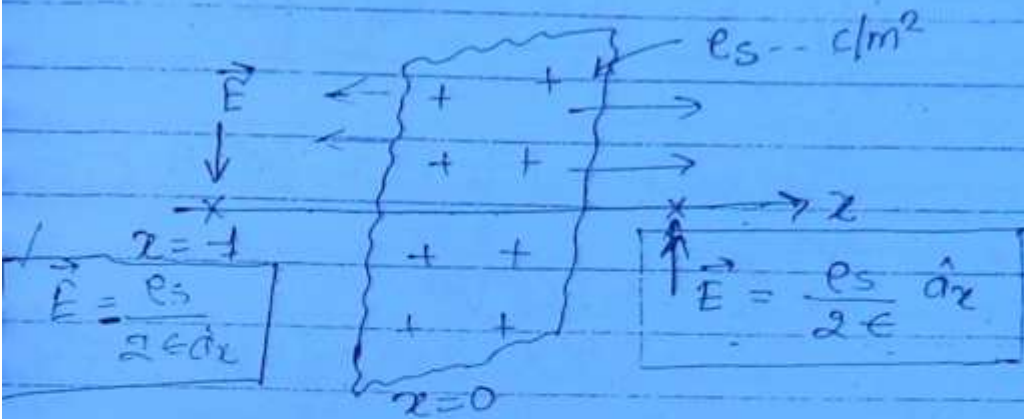
(3) circular loop



$$\begin{cases} \vec{V} = \text{const.} \\ \vec{E} = 0 \end{cases}$$

④ Infinite charge sheet :-

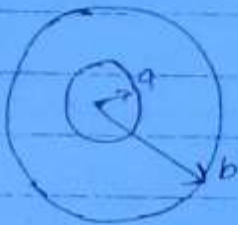
(113)



$$\rho_s = D_n = \epsilon E_n$$

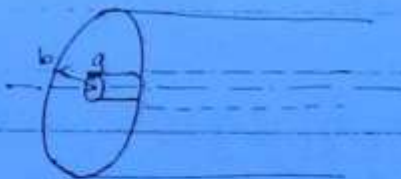
$$E_n = \frac{\rho_s}{2\epsilon}$$

⑤ Concentrating spherical shells :-

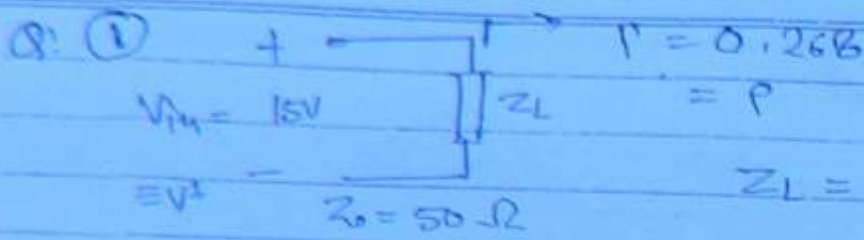


$$c = \frac{4\pi \epsilon ab}{b-a}$$

⑥ cyl. Transmission line :-



$$c = \frac{2\pi \epsilon}{\ln(b/a)} \quad f/m$$



(114)

$$P = \frac{V_L^2}{2Z_L}$$

$$= 2.08 W$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 86.6 \Omega$$

$$\frac{V_L}{V+} = T = 1 + \Gamma = 1.268$$

$$\Rightarrow V_L = 1.268 V+ = 15 \times 1.268 = 19V$$

②

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$V_p = \frac{1}{\sqrt{LC}}$$

$$Z_0 V_p = \frac{1}{C}$$

$$V_p = \frac{Z}{\sqrt{\epsilon_r}}$$

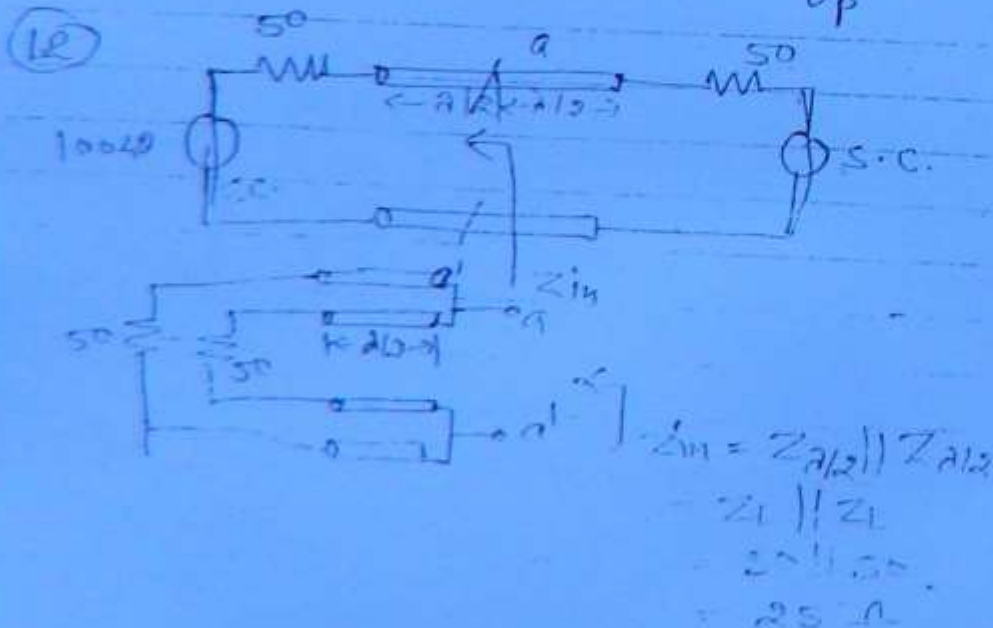
$$Z_0 \times \frac{V}{\sqrt{\epsilon_r}} = \frac{1}{C}$$

$$Z_0 = \frac{\sqrt{\epsilon_r}}{V \cdot C}$$

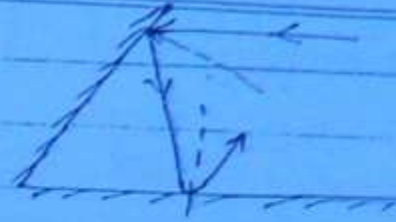
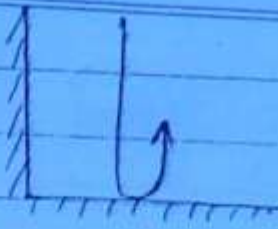
⑦

$f = 10 \text{ GHz}$
 $d = 3 \times 10^{-3} \text{ m}$
 $\theta = 90^\circ = \pi/2$

θ — phase shift — rad
 β — phase const — rad/m
 $\theta = \beta \cdot d$
 $\theta = \frac{\omega}{v_p} \cdot d = \frac{2\pi \cdot f \cdot d}{c/\sqrt{\epsilon_r}}$



(20)



(115)

(21)

$$\lambda/2 = \frac{27.5}{-12.5} = 15 \text{ cm}$$

$$\lambda = 30 \text{ cm}$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{10}}{30} = 1 \text{ GHz}$$

(24)

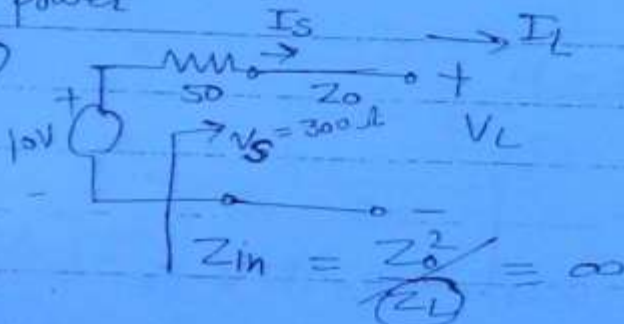
$$S = 3$$

$$\Gamma = \frac{V^-}{V^+} = \frac{E_e}{E_i} = \frac{S-1}{S+1} = 1/2$$

$$\Gamma^2 = 1/4 \Rightarrow 25\%$$

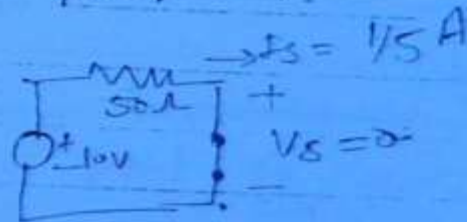
reflection coeff of power

(26)



$$Z_{in} = 0$$

$$|V_L| = ?$$



$$V_L = \cos \beta l \cdot V_S = j Z_0 \sin \beta l \cdot I_S$$

$$|V_L| = \left| -j \frac{300}{5} \right|$$

$$= 60 \text{ V}$$

(28)

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$

$$Z_{sc} = \frac{-Z_0^2}{Z_{oc}} = \frac{-Z_0^2}{100 + j150}$$

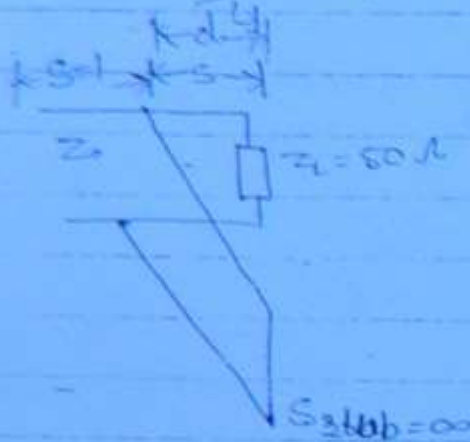
Inductive

(22) $S = \frac{V_{max}}{V_{min}} = \frac{4}{1} = 4$

$Z_{min} = \frac{Z_0}{S} ; Z_{max} = Z_0 \cdot S$

$= \frac{50}{4} = 12.5 \Omega$

(16)



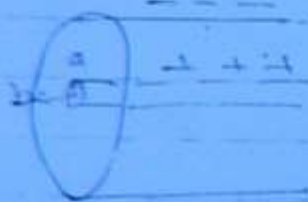
$S = \frac{1+\Gamma}{1-\Gamma} = \frac{1+1/3}{1-1/3} = \frac{4/3}{2/3} = 2$

$\Gamma = |\Gamma| = 1/3$

$T = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 10}{50 + 10} = \frac{40}{60} = \frac{2}{3}$

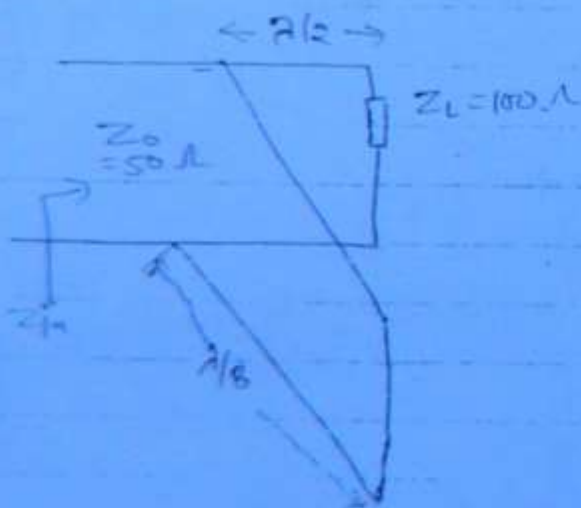
$S_{slab} = \infty$

(15)



$a < e < b$

(25)



$Z_{in} = Z_{2/2} \parallel Z_{1/6}$
 $= Z_L \parallel jZ_0 \tan \beta l$
 $= 1$

$Z_{in} = Z_L \parallel jZ_0$
 $= 100 \parallel 50$

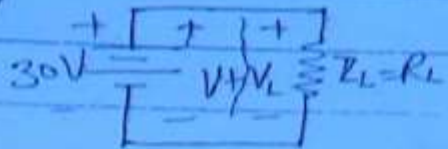
$Y_{in} = Y_L + Y_{2/6}$
 $= \frac{1}{100} + \frac{1}{j50}$

(27)

$\vec{H} = \cos(\omega t + \phi) \hat{z}$

$v_p = \frac{\omega}{\beta}$

(30)



$$V_L = V^+ + V^- = 40$$

$$V^+ = 30$$

$$V^- = 10$$

$$I_{ss} = \frac{1}{2} (I_{max} + I_{min})$$

(112)

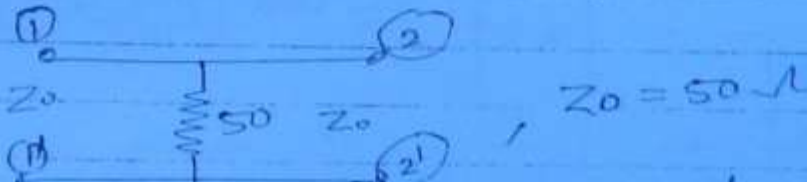
$$\frac{V^-}{V^+} = \frac{10}{30} = \frac{1}{3} ; Z_L = R_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 100 \Omega$$

$$I_{max} = \frac{V_{max}}{R_L} = \frac{V^+ + V^-}{R_L} = \frac{40}{100} = 0.4 A$$

$$I_{min} = \frac{V_{min}}{R_L} = \frac{V^+ - V^-}{R_L} = \frac{20}{100} = 0.2 A$$

$$I_{ss} = \frac{1}{2} (0.4 + 0.2) = 0.3 A$$

(31)



$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

S_{11} = total power sent from port 11' & recd. back by port 11' $\equiv 0$

S_{12} = the total power sent from port 11' & received by port 22' $\equiv 1$

$$[S] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Q: 33.

$$\frac{\sigma}{\omega\epsilon} \ll 1$$

Good dielect

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \left(1 + j \frac{\sigma}{2\omega\epsilon} \right)$$

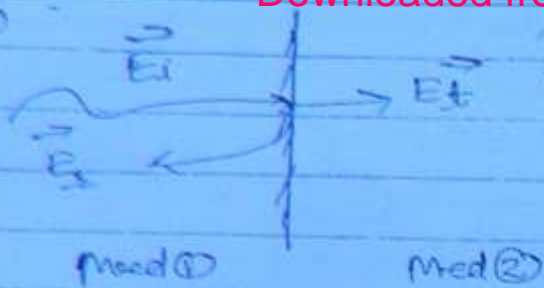
Complex Inductance

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} \sqrt{\frac{1}{1 + \frac{\sigma}{j\omega\epsilon}}}$$

$$= (1+x)^{-1/2}$$

$$= 1 - \frac{1}{2} \frac{\sigma}{j\omega\epsilon} = 1 - \frac{1}{2} \frac{\sigma}{j\omega\epsilon}$$

(38)



(1/8)

$$E_i = E_0 \cos(\omega t - \beta z) \hat{a}_y$$

$$\vec{E}_i = E_0 \hat{a}_y$$

$$= E_y \hat{a}_y$$

propagation
+ direction
 \vec{E} has only
y comp.

$$\omega = 3 \times 10^9 \pi \text{ rad/sec.}$$

$$\beta = 10\pi \text{ rad/m}$$

$$v_p = \frac{\omega}{\beta} = 3 \times 10^8 \text{ m/sec.} = c$$

Med ① is free space.

$$\vec{E}_i = \dots \hat{a}_y \left. \begin{array}{l} \text{prop.} \\ + z \end{array} \right\}$$

Option (C)

$$\frac{1}{2} E_0 \cos(\omega t - \beta z) \hat{a}_y$$

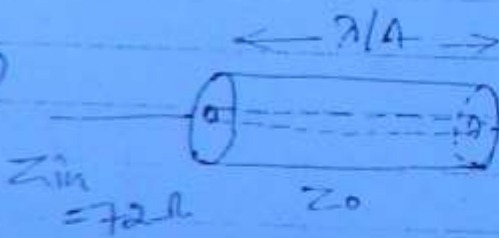
$$(d) \left(\frac{E_0}{2} \cos(\omega t - 3\beta z) \right) \hat{a}_y$$

(40)

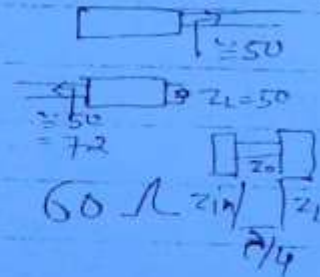
$$\text{skin depth } \delta = \frac{1}{\sqrt{f\mu\sigma}}$$

↑
 $\mu = \mu_0$

(42)



$$Z_L = 50 \Omega$$



$$Z_0 = \sqrt{Z_{in} \cdot Z_L} = \sqrt{72 \times 50} = 60 \Omega$$

$$Z_0 = 60 \ln(b/a) = 60$$

$$Z_0 = \ln(b/a) \Rightarrow b/a = e^1 \approx 2.7$$

$$\frac{2b}{2a} \approx 2.7 ; 2b \approx (2.7) 2a$$

$$2b = 2.7 \text{ mm} \quad 10 \text{ mm}$$

(44)

$$\gamma = \underbrace{0.0005}_{\alpha} + j \frac{\pi}{10} \underbrace{1}_{\beta}$$

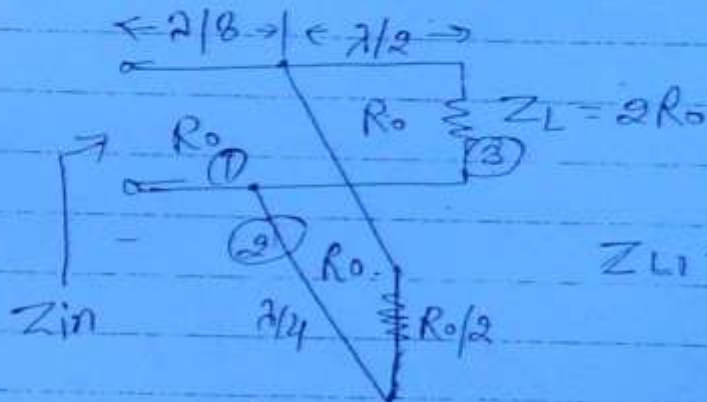
(119)

$$\alpha l = \underbrace{(0.0005 \times 50)}_{\times 8.686} \dots \text{ nepers}$$

$$\downarrow$$

$$\dots \text{ dB}$$

(45)



$$Z_{in} \Big|_{\lambda/8} = Z_0 = R_0$$

perfectly Matched

$$\begin{aligned} Z_{L1} &= Z_{\lambda/2} \parallel Z_{\lambda/4} \\ &= Z_{L3} \parallel \frac{Z_0^2}{Z_{L2}} \\ &= 2R_0 \parallel \frac{R_0^2}{R_0/2} \\ &= 2R_0 \parallel 2R_0 = R_0 \end{aligned}$$

(46)

$R, G \ll \text{small}$

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\left[\begin{matrix} R \ll j\omega L \\ G \ll j\omega C \end{matrix} \right] = \sqrt{j\omega L} \cdot \sqrt{j\omega C} \left[\left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right) \right]^{\frac{1}{2}}$$

$$j\omega \sqrt{LC} \left[\underbrace{1 + \left(\frac{R}{j\omega L} + \frac{G}{j\omega C} \right)}_{(1+x)} - \frac{RG}{\omega^2 LC} \right]^{\frac{1}{2}}$$

$$= \left(1 + \frac{1}{2}x\right)$$

$$\gamma = j\omega \sqrt{LC} \left[1 + \frac{1}{2} \left(\frac{R}{j\omega L} + \frac{G}{j\omega C} \right) \right]$$

Real part

$$\alpha = \omega \sqrt{LC} \left[\frac{R}{2\omega L} + \frac{G}{2\omega C} \right] \quad (120)$$

$$\alpha = \frac{1}{2} \left[R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right]$$

$\underbrace{\hspace{1.5cm}}_{= Y_{Z_0}} \quad \underbrace{\hspace{1.5cm}}_{= Z_0}$

$$\alpha = \frac{1}{2} \left(\frac{R}{Z_0} + G Z_0 \right) \quad \text{Ans.}$$

(46)

$$l = 0.5 \text{ m}$$

$$L =$$

$$C =$$

$$f = 25 \text{ MHz}$$

$$\beta l = \frac{\omega}{v_p} l = \frac{2\pi f l}{1/\sqrt{LC}} = 2\pi \sqrt{LC} \cdot f \cdot l$$

(51)

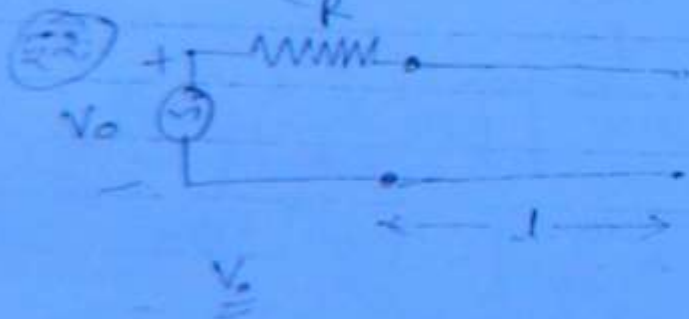
$$Z_0 = 50 \Omega$$

$$\bar{Z} = 0.5 - j0.3 = \frac{Z}{Z_0}$$

$$Z = (0.5 - j0.3) Z_0$$

in Smith chart
all the imp. are
taken in normalized
form

$$Z = \bar{Z} \quad \text{form}$$



chpt : 2.

Date: _____

$$(2) \quad d = \frac{\lambda}{4} = \frac{1}{4} \frac{v_p}{f} = \frac{1}{4f} \frac{c}{\sqrt{\epsilon_r}} \\ \approx 0.28$$

(21)

$$(3) \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \quad v_p = \frac{1}{\sqrt{\mu\epsilon}}$$

(6) free space ... perfect diel.

$$\sigma = 0$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\mu = \mu_0 \mu_r; \mu_r = 1; \mu = \mu_0$$

$$\epsilon = \epsilon_0$$

$$\vec{J}_e = \sigma \vec{E} = 0; \quad \vec{J}_c = 0$$

$$\boxed{\rho = 0}$$

$$v_p = c = 3 \times 10^8 \text{ m/sec.}$$

$$\eta = \eta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi \Omega$$

$$(11) \quad \vec{H} = 0.1 \sin(10^8 \pi t + \beta y) \hat{a}_z \\ \vec{P} = \frac{1}{2} \eta_0 H_m^2 \quad \xrightarrow{\text{prop.}} -\hat{a}_y \\ \frac{1}{2} \times 120\pi \times 0.1$$

$$(16) \quad \sigma = 5 \text{ mho/m}$$

$$\epsilon_r = 80$$

$$f = 25 \text{ KHz}$$

low freq.

$$E_z = E_0 e^{-\alpha z}$$

$$e^{-\alpha z} = \frac{E_z}{E_0} = 0.1$$

$$e + \alpha z = 10$$

$$\boxed{\alpha = \frac{1}{z} \ln(10)}$$

$$\frac{\sigma}{\omega\epsilon} \gg \text{Good Conductor}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma} \approx \mu_0$$

(19) E_z

--- sinusoidal time variation

--- prop $\Rightarrow +z$

lossless medium

$$\sigma = 0$$

$$\Rightarrow \alpha = 0$$

$$E_z = E_0 e^{-\gamma z}$$

$$E_z = E_0 e^{-(\alpha + j\beta)z} \quad (22)$$

$$E_z = E_0 e^{-j\beta z}$$

$$= E_0 e^{-jkz}$$

$$(20) \quad \vec{E} = 50 \sin(10^7 t + Kz) \hat{a}_y = E_y \hat{a}_y$$

 \uparrow
 ω
prop $= -z \leftarrow$

$$\vec{P} = -Pz \hat{a}_z$$

$$K = \beta = \frac{\omega}{v_p} = \frac{10^7}{3 \times 10^8} = \frac{1}{30}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi \times 30 = 60\pi \approx 188.5 \text{ m}$$

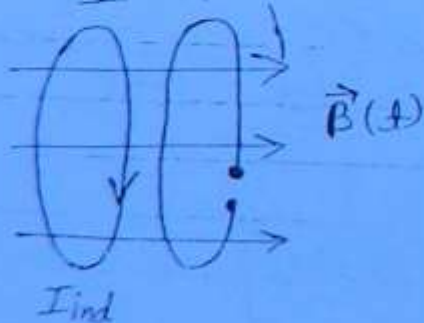
$$(21) \quad u = \epsilon ; \eta_1 = \eta_0 = 377 \Omega$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = 1 \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - 377}{1 + 377} = -\frac{376}{378} \approx -1$$

$$|\Gamma| \approx 1$$

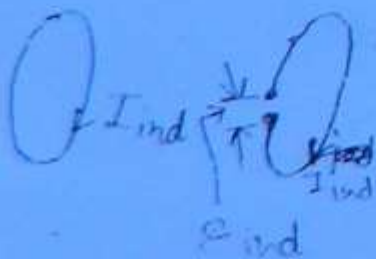
(22)



$$\vec{B}(t) \rightarrow \psi_m(t) \rightarrow \frac{\partial \psi_m}{\partial t} \rightarrow \vec{E}_{ind}$$

\downarrow
 I_{ind}
 given by Ampere Law

\uparrow
 Faraday's Law of emf



Comments :- using Faraday's laws of e.m induction, due to rate of change of Mag flux induced current in the loop which will rise to

Induced emf. when the loop is open-circuited.

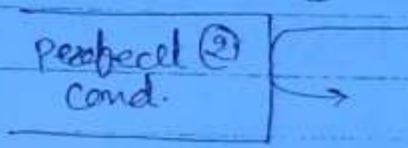
Therefore induced emf equivalent effect of I_{ind} is always present on the loop when ever mag. field is time variant.

$$(24) \quad \vec{A} = \frac{\mu}{4\pi} \int_C \frac{\vec{I}}{r} dl \quad (23)$$

due to line current.

$$\text{as } r \rightarrow \infty ; \vec{A} = 0$$

$$(26) \quad \left. \begin{matrix} v_p \\ \eta \end{matrix} \right\} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 377 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

(27)  perfect cond. (2)

$$\begin{aligned} E_{t1} &= E_{t2} = 0 \\ H_{t1} - H_{t2} &= \vec{J}_s \\ H_{t1} &= H_{t2} = 0 \end{aligned}$$

$$(29) \quad \frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{\omega \epsilon} = 1$$

$$\sigma = \omega \epsilon = 2\pi f \epsilon$$

$$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r}$$

① Surface wave propagation :-

--- used at medium band.

--- AM broadcast

$f: 535 \text{ KHz} - 1605 \text{ KHz}$

(24)

② space wave prop.

Terrestrial prop

LoS (line-of-sight) prop

--- used at VHF;

microwave range

$30 \text{ GHz} - 300 \text{ GHz}$

--- preferred for

- TV broadcast

- microwave link.

③ sky-wave prop

Ionospheric

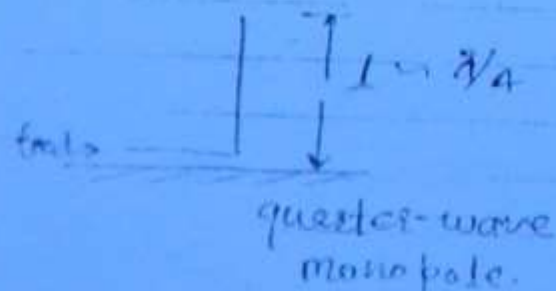
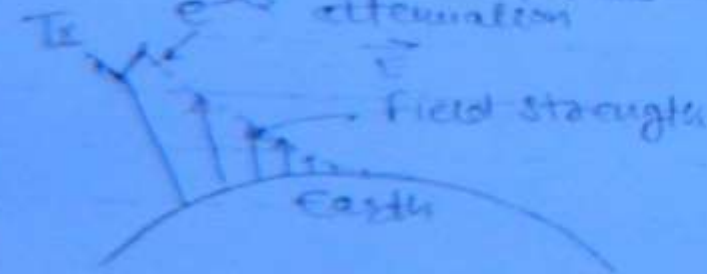
--- used at

HF range (short-wave range)

preferred for FM broadcast

($f: 88 \text{ MHz} - 108 \text{ MHz}$)

① Surface wave propagation



features :-

① The e-m waves travel along the surface of the earth.

- (2) The transmitting antenna is always vertically installed & therefore it is always vertically polarized. (25)
- (3) The electric field associated with the wave is perpendicular to the surface of earth.
- (4) The earth behaves as a good conductor & therefore as the wave travels the electric field strength decreases exponentially.
- (5) The length of the antenna depends upon the freq. or wavelength of operation. Quarterwave monopoles are always preferred for the transmission of such signals.
- (6) As the frequency of operation increases the height of antenna decreases.
- (7) Preferred for AM broadcast on the freq. range 535 to 1605 KHz.
The range of transmission can be increased only by increasing the power of the transmitter.
- (8) Such propagation takes place when the transmitting & receiving antennas are very close to each other & close to the surface of earth.

(3)

limited range of transmission

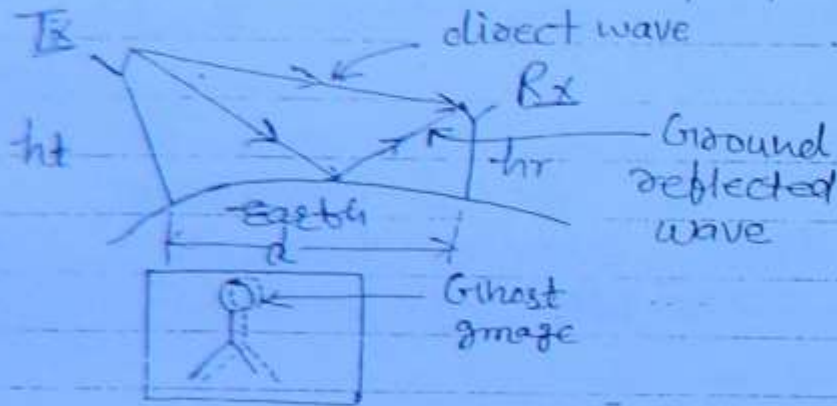
(2)

Space wave prop.

--- Tropospheric prop.

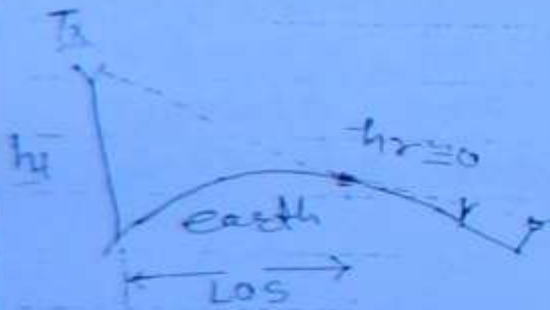
--- LOS prop.

(126)



$$d \approx 3550 (\sqrt{h_t} + \sqrt{h_r})$$

----- m



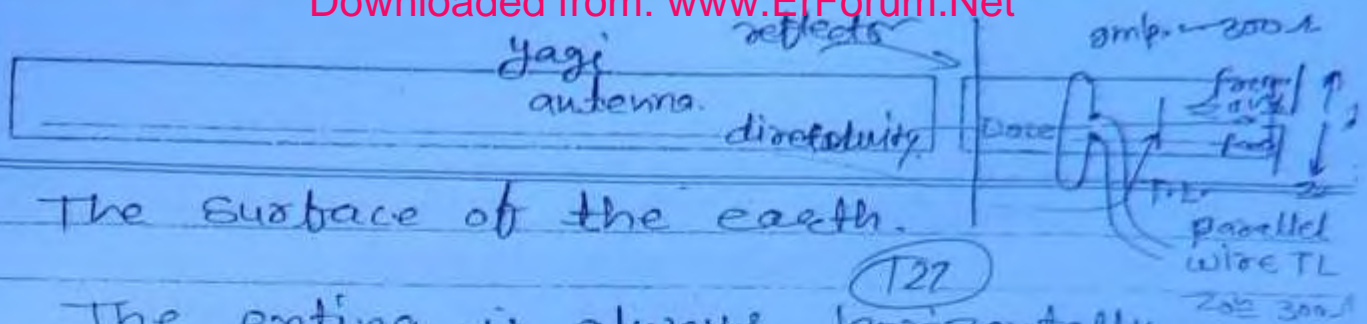
$$d \approx 3550 \sqrt{h_t}$$

Features :-

- (1) The space wave constitutes :-
 - (a) direct wave
 - (b) ground reflected wave.

(2) Suggested for freq. greater than 30 MHz for TV broadcast in the VHF range.

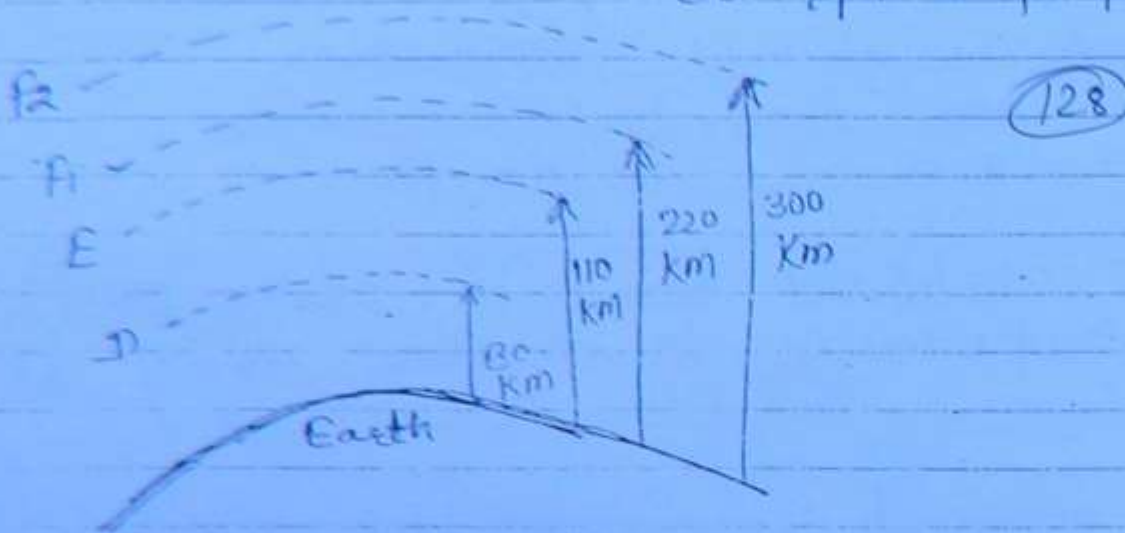
(3) The em. wave travel from transmitter to receiver in the earth atmosphere at a height of 10 to 15 km above



The surface of the earth.

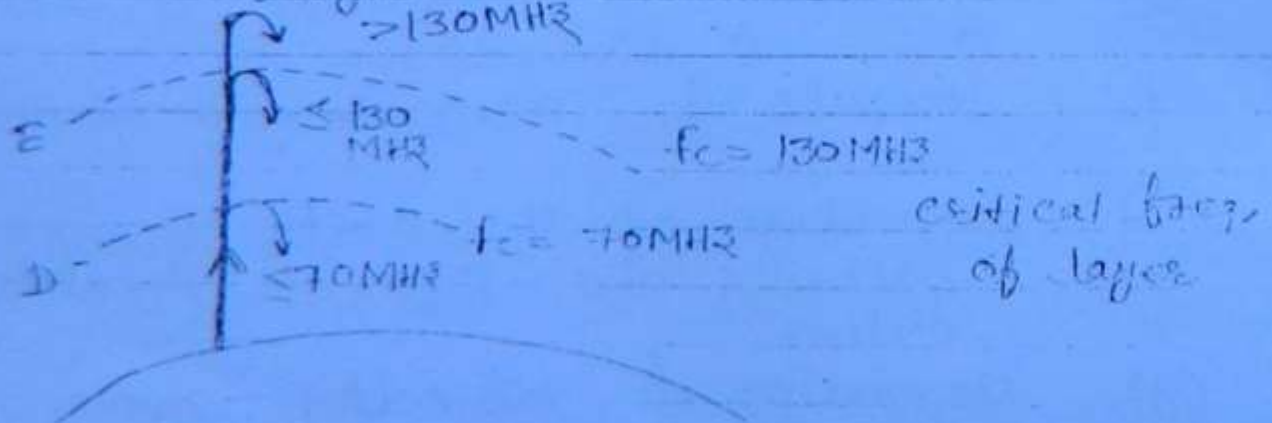
- (3) The antenna is always horizontally polarised so that electric field vector is parallel to the surface of the earth.
- (4) ferris pole is preferred for such purpose so that its impedance is matched with that of T.L.
- (5) The total range of Transmission depends upon:
 - (1) power of the transmitter.
 - (2) Height of Transmitting & Receiving Antenna.
 - (3) The range of Transmission can be increased by increasing the height of the receiving antenna.
- (6) Factors which control the magnitude of space wave:-
 - (a) Conductivity of earth.
 - (b) permittivity & permeability of earth.
 - (c) freq. of the wave.
 - (d) Heights of Transmitting & receiving antenna.
 - (e) curvature of the earth.
 - (f) Distance b/w T/M & Receiving antenna.
 - (g) Variation of reflecting index of earth with Height.

⑤ Sky wave prop. Ionospheric prop.



$$\theta = \text{T to A}$$

take of angle $> 130 \text{ MHz}$



h_p h_o of layer.
S skip distance.

① used in the HFR & preferred for FM broadcast in the freq. range 88 MHz to 200 MHz.

(29)

② long range Transmission is possible.

③ The range of Transmission depends:

(a) Take of angle.

(b) freq. of the signal.

④ The D layer has minimum electron density whereas f₂ layer has Max. electron density.

⑤ The critical freq. of a layer depends upon the electron density of the layer.

$$f_c = \sqrt{81 N}$$

critical
freq.
MHz.

electron
density
--- /m³

Therefore the critical freq. of the D layer is minimum whereas this freq. has a Max. value for f₂ layer.

⑥ During night time d-layer is missing, f₁ & f₂ layer merge together to form a single layer.

⑦ The E-layer is the most stable layer & FM broadcast uses this layer for the T/M of the signal.

f_c critical freq. angle:- This is the maximum freq. of a layer so that the wave is reflected by that layer at vertical incidence.

The e.m. wave of freq. less than or equal to critical freq. will be reflected from the layer irrespective of the angle of incidence.

as the height of layer increases, its critical freq. increases

S) Skip distance:- This is the minimum distance from the Tx at the sky wave of given freq. it return to earth by the ionosphere.

The skip distance depends upon:-

- (a) freq. of the wave.
- (b) f_c (critical freq.)
- (c) Height of the layer.
- (d) charge carried concentration + N.

MUF) --- Maximum usable frequency.

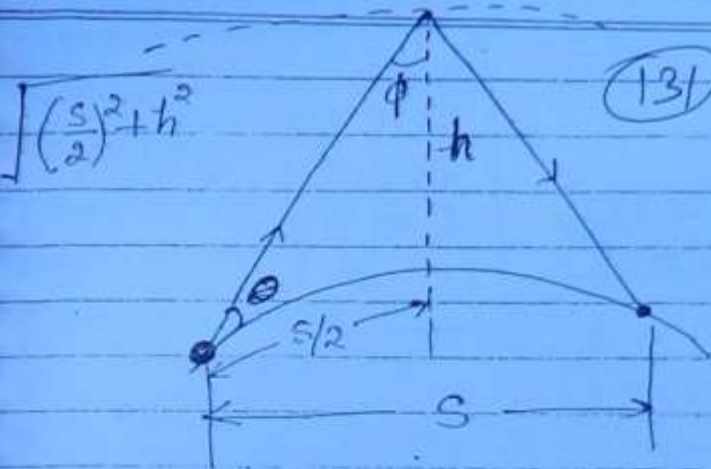
$$MUF = f_c \cdot \sec \phi$$

$$MUF > f_c$$

Tx = Transmitter

Rx = Receiver

Date _____



(13)

 $\theta = \text{TOA}$ $\phi = \text{angle of incidence at the layer}$

$$\theta + \phi = 90^\circ$$

$$\phi = 90^\circ - \theta$$

$$\begin{aligned} \text{MUF} &= f_c \cdot \sec \phi \\ &= f_c \cdot \frac{\sqrt{\left(\frac{S}{2}\right)^2 + h^2}}{h} \end{aligned}$$

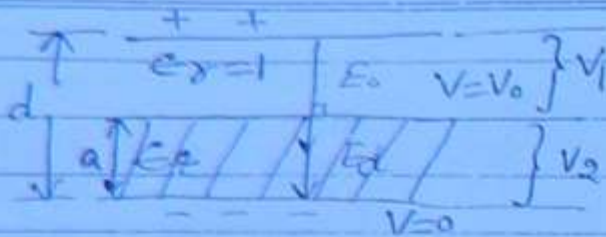
$$\text{MUF} = f_c \cdot \sqrt{\left(\frac{S}{2h}\right)^2 + 1}$$

(1) for fixed location of the Tx & Rx, MUF is the freq. which makes the distance to the receiving point equal to the skip distance.

(2) MUF is the freq. that gives strongest sky wave signal at the received point.

(3) The skip distance increases with the freq. of operation.

(ex)

To find E_0 ; E_d

(132)

$$E = \frac{V}{d}$$

$$V_1 + V_2 = 0$$

$$E_0(d-a) + E_d \cdot a = V_0 \quad \text{--- (1)}$$

$$D_{n1} - D_{n2} = \rho_s^{\uparrow}$$

$$D_1 = D_2$$

$$\epsilon_0 E_0 = \epsilon_0 \epsilon_r E_d$$

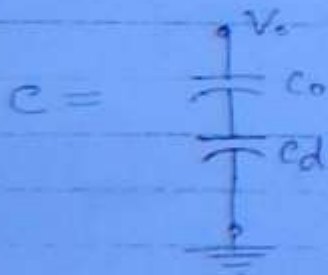
$$E_0 = \epsilon_r E_d \quad \text{--- (2)}$$

$$\epsilon_r E_d(d-a) + E_d a = V_0$$

$$E_d [\epsilon_r d - \epsilon_r a + a] = V_0$$

$$E_d = \frac{V_0}{\epsilon_r(d-a) + a}$$

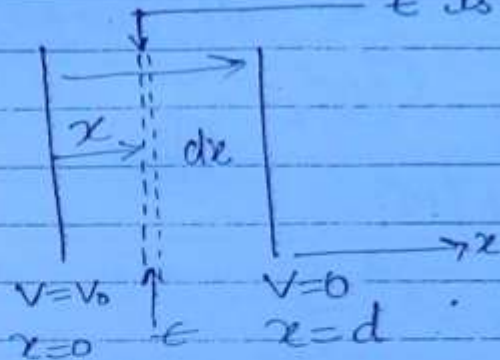
$$E_0 = \epsilon_r E_d = \frac{\epsilon_r V_0}{\epsilon_r(d-a) + a}$$



$$C = \frac{C_0 C_d}{C_0 + C_d} ; \quad C_0 = \frac{\epsilon_0 A}{d-a}$$

$$C_d = \frac{\epsilon_0 \epsilon_r A}{a}$$

ex

 ϵ is varying linearly w.r.t. x at $x=0$; $\epsilon = \epsilon_1$ -- min. $x=d$; $\epsilon = \epsilon_2$ -- max

(133)

To find Capacitance

$$\epsilon = \epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x$$

$$\begin{aligned} \epsilon_s &= D_n \\ V &= - \int_{x=d}^0 \vec{E} \cdot d\vec{l} \end{aligned} \quad \left| \quad \vec{E} = - \nabla V \right.$$

$$= - \int_{x=d}^0 E_n \cdot dx \Rightarrow - \int_{x=d}^0 \frac{D_n}{\epsilon} \cdot dx$$

$$= - \int_{x=d}^0 \frac{\epsilon_s}{\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x} dx$$

$$= - \epsilon_s \int_{x=d}^0 \frac{1}{t} \cdot \frac{d}{\epsilon_2 - \epsilon_1} dt$$

$$= - \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln t \Big|_{x=d}^0$$

$$= - \frac{\epsilon_s d}{\epsilon_2 - \epsilon_1} \ln \left[\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x \right]_{x=d}^0$$

$$= - \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \frac{\epsilon_1}{\epsilon_2}$$

$$= + \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \left(\frac{\epsilon_2}{\epsilon_1} \right)$$

$$C = \frac{Q}{V} = \frac{QS}{V} \quad \text{--- Cap. per unit surface}$$

F/m^2

$$C = \frac{\epsilon_2 - \epsilon_1}{d \ln\left(\frac{\epsilon_2}{\epsilon_1}\right)} \quad \text{--- } F/m^2 \quad (134)$$

30. $\tan \delta = \frac{\sigma}{\omega \epsilon}$

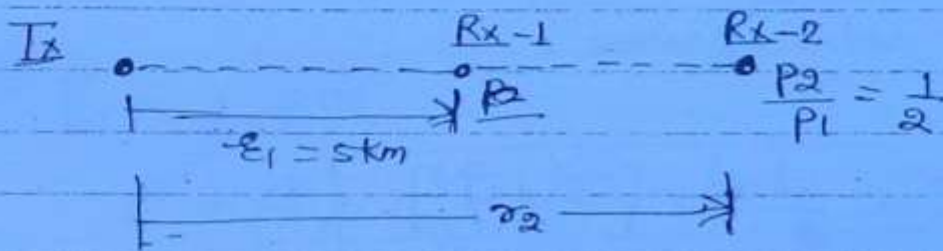
\uparrow \uparrow
 $2\pi f$ $\epsilon_0 \epsilon_r$

(135)

31. $\epsilon_s = D_n = \epsilon E_n$

\uparrow $\leftarrow 2V/m$
 $80 \epsilon_0$

32.



$$d = r_2 - r_1 = (r_2 - 5) \text{ km}$$

$$|\vec{P}| = \frac{\text{power}}{4\pi r^2}$$

$$|\vec{P}| \propto \frac{1}{r^2} = P$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow r_2 = \sqrt{2} r_1 \Rightarrow 5\sqrt{2} \text{ km}$$

$$\Rightarrow r_2 - r_1 = d = (5\sqrt{2} - 5) = 5(\sqrt{2} - 1) = 2.070 \text{ km}$$

$$P_{dB} = 10 \log_{10} P$$

$$\left(\frac{P_2}{P_1}\right) = -3 \text{ dB} \equiv 10 \log_{10} \left(\frac{P_2}{P_1}\right)$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{1}{2}$$

(37)

$$\epsilon = \epsilon_0$$

$$A$$

$$d$$

$$V = 0.5 \text{ V}$$

$$f = 3.6 \text{ GHz}$$

$$I_d = \underset{\substack{\uparrow \\ A/m^2}}{J_d} \cdot A = \frac{\partial D}{\partial t} \cdot A = j\omega \epsilon E A = j\omega \epsilon_0 \frac{V}{d}$$

$$|I_d| = 2\pi f \epsilon_0 \frac{V}{d} A$$

(136)

Tensor:

$$\vec{P} = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*]$$

$$\vec{P} = \frac{1}{2} \text{Re} \left[(\hat{a}_z + j\hat{a}_y) e^{j\vec{k} \cdot \vec{r} - j\omega t} \times \left(\frac{\kappa}{\omega \mu} \right) (\hat{a}_y + j\hat{a}_x) e^{-j\vec{k} \cdot \vec{r} + j\omega t} \right]$$

$$\kappa' \text{Re} [\hat{a}_z - \hat{a}_x]$$

$$= \vec{0} \text{ --- null vector.}$$

$$\left. \begin{array}{l} \hat{a}_x \rightarrow \hat{a}_1 \\ \hat{a}_y \rightarrow \hat{a}_2 \\ \hat{a}_z \end{array} \right\}$$

$$\textcircled{1} \vec{P} = \vec{E} \times \vec{H}$$

Poynting's vector

$$\textcircled{2} \vec{P} = \frac{1}{2} \vec{E} \times \vec{H}$$

average Poynting vector

$$\textcircled{3} \vec{P} = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*]$$

average Poynting vector
when \vec{E} and \vec{H} are phasors.

ex. 1

$$\vec{E} = 2 \hat{a}_x$$

$$\vec{H} = 4 \hat{a}_y$$

(137)

$$\vec{P} = \vec{E} \times \vec{H} = 8 \hat{a}_z$$

$$\vec{E} = 4 \cos(\omega t - \beta z) \hat{a}_x$$

$$\vec{H} = 2 \cos(\omega t - \beta z) \hat{a}_y$$

$$\vec{P} = (\vec{E} \times \vec{H}) = 8 \underbrace{\cos^2(\dots)}_{\text{average value}} \hat{a}_z$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\vec{P}_{av} = \frac{1}{2} 8 \hat{a}_z$$

$$\vec{E} = (\hat{a}_x + j \hat{a}_y) e^{jKz - j\omega t}$$

phasor.

$$\vec{H} = \dots$$

Q: 39

Med ① -- free space
 $\eta_0 ; \epsilon_0$ Med ② ----- perfect die. ($\sigma=0 ; \mu_r=1$)
 $\underline{\eta_2} ; \epsilon_2 : \eta ; \epsilon$

$$\boxed{\epsilon > \epsilon_0}$$

$$S = \frac{V_{max}}{V_{min}} = \frac{E_{max}}{E_{min}} = 5$$

$$\rho = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$= \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$\rho = \frac{\eta_0 - \eta}{\eta_0 + \eta} \leftarrow \frac{\sqrt{\epsilon} - \sqrt{\epsilon_0}}{\sqrt{\epsilon} + \sqrt{\epsilon_0}} \leftarrow \frac{\sqrt{\epsilon_0} - \sqrt{\epsilon}}{\sqrt{\epsilon_0} + \sqrt{\epsilon}}$$

$\uparrow 120\pi$

$$\eta = 24 \pi \text{ AW.}$$

Q: 41.

$$P_i = P_r + P_t$$

(138)

$$1 = \frac{P_r}{P_i} + \frac{P_t}{P_i}$$

$$\left(\frac{P_t}{P_i}\right) = 1 - \left(\frac{P_r}{P_i}\right)$$

← reflection coefficient of power.

$$\frac{P_t}{P_i} = 1 - \left(\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right)^2$$

$$\epsilon_1 = \epsilon_0 \epsilon_{r1} = \epsilon_0$$

$$\epsilon_2 = \epsilon_0 \epsilon_{r2} = 4 \epsilon_0$$

$$= \frac{8}{9}$$

Q: 42

$$E_z = E_0 e^{-\gamma z}$$

along +z direction

$$= E_0 e^{-(\alpha + j\beta)z}$$

$$= E_0 e^{-j\beta z}$$

lossless med.

$$E_0 e^{-j\beta z} e^{j\omega t}$$

for sinusoidally
varying field.

$$E_z = E_0 e^{j(\omega t - \beta z)}$$

+z direction prop.

→ Wave propagates along some
arbitrary direction

$$E_z = E_0 e^{j(\omega t - \vec{r} \cdot \vec{\beta})}$$

To be
inward.

$$\vec{\beta} \cdot \vec{r} = (\beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z) \cdot (x \hat{a}_x + y \hat{a}_y + z \hat{a}_z)$$

$$\vec{\beta} \cdot \vec{r} = \beta_x \cdot x + \beta_y \cdot y + \beta_z \cdot z \quad (139)$$

$$= \beta \cos \theta_1 + \beta \cos \theta_2 + \beta \cos \theta_3$$

$$\frac{2\pi}{\lambda} \cos 30^\circ \quad \uparrow \quad 90^\circ \quad \beta \cos 60^\circ$$

$$= \frac{\sqrt{3} \cdot \pi}{\lambda}$$

$$= \frac{2\pi}{\lambda} \cdot \frac{1}{2} = \pi/\lambda$$

$$E_z = E_0 e^{j(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - 0 - \frac{\pi}{\lambda} \cdot z)}$$

(44)

incident -- RCP

reflected -- LP

↓

$$\theta = \theta_B$$

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \rightarrow \epsilon_{r2} \cdot \epsilon_0$$

$$\sqrt{3} \rightarrow \epsilon_0 \epsilon_{r1} = \epsilon_0$$

$$\epsilon_{r2} = 3$$

(46)

$$\epsilon_{r1} = 1$$

$$\vec{E}_1 = 1 \cdot \hat{a}_x$$

$$\leftarrow \epsilon_s =$$

$$D_{n1} - D_{n2}$$

$$= \epsilon_0 \epsilon_{r1} E_{n1} - \epsilon_0 \epsilon_{r2} E_{n2}$$

$$= \epsilon_0 [1 \times 1 - 2 \times 2]$$

$$= -3\epsilon_0$$

$$\epsilon_{r2} = 2 \quad \vec{E}_2 = 2 \hat{a}_x$$

(49)

ind --- both coil



perfect cond.



$$losses = 0$$



$$Heat\ dissipation = 0$$

(40)

(50)

$$\vec{H} = 0.1 \cos(4 \times 10^7 t - \beta z) \hat{a}_z$$

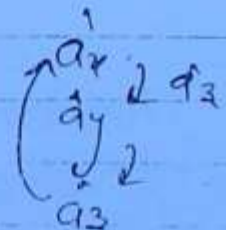
$$\vec{H} = H_z \hat{a}_z$$

prop. $\Rightarrow +z$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\hat{a}_z = (-\hat{a}_y) \times (\hat{a}_x)$$

$$\vec{P} = P_z \hat{a}_z$$



$$\vec{E} = -E_y \hat{a}_y$$

Notice

$$= \eta_0 H_m = 377 \times 0.1$$

(51)

$$\vec{E} = 50 \cos(\dots) \hat{a}_x$$

$$\vec{H} = \frac{5}{12\pi} \cos(\dots) \hat{a}_y$$

direct of prop. $\Rightarrow +z$

$$Power = P_z \times area$$

$$\frac{1}{2} \epsilon_m H_m (\pi r^2)$$

\uparrow \uparrow \uparrow \uparrow
 50 5/12π 24 m

$$(53) \quad \vec{H} = (\quad) \hat{a}_3$$

$$\vec{E} \neq \hat{a}_2$$

(141)

-- wave is not polarized in z direction

$$(54) \quad \left(\frac{\vec{F}}{l} \right) = \vec{I} \times \vec{B}$$

$$(63) \quad E_{ind} \begin{cases} \text{Cond. is moving } (\vec{v}) ; \vec{B} \neq f(t) \\ \vec{v} = 0 ; \vec{B} = f(t) \\ \vec{v} ; \vec{B} = f(t) \end{cases}$$

Case 1:- moving cond. ; \vec{v}
 $\vec{B} \neq f(t)$

$$E_{ind} = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l} \quad \text{----- Motional emf.}$$

Case 2: $\vec{v} = 0$
 $\vec{B} = f(t)$

$$E_{ind} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- due to time varying field}$$

Case 3: moving cond. ----- emf due to transformer action

$$\vec{B} = f(t)$$

As

$$E_{ind} = E_{ind}(\text{case 1}) + E_{ind}(\text{case 2})$$

(67)

 $\sigma ; \rho_s$

$$|D_n| = \rho_s$$

$$p = we = \frac{1}{2} \epsilon E_m^2$$

$$= \frac{\epsilon}{2} \left(\frac{D_n}{\epsilon} \right)^2$$

(142)

$$= \frac{1}{2} \epsilon \cdot \frac{\rho_s^2}{\epsilon^2} = \frac{1}{2} \epsilon \rho_s^2$$

$$= \frac{1}{2} \epsilon \sigma^2$$

(68)

$$PE = KE$$

Potential energy = Kinetic energy

$$eV = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2eV}{m}} \text{ --- Known}$$

$$F = \frac{m v^2}{r} = e v B$$

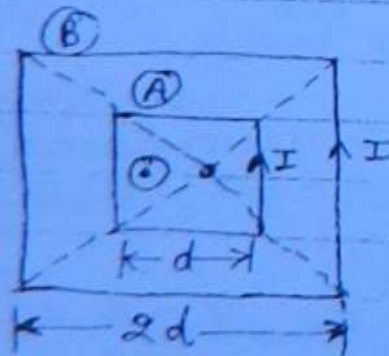
$$\boxed{r = \frac{m v}{e B}}$$

result.

250 Gauss \rightarrow to convert,
 250×10^{-4}

$$= 0.025 \text{ Wb/m}^2$$

(69)



$$\frac{H_A}{H_B} = 1 = \frac{r_B}{r_A}$$

$$H \propto \frac{1}{r}$$

$$\frac{\epsilon_B}{\epsilon_A} = \frac{d}{d/2} = 2$$

(143)

→ phase change in \vec{H}

(74) $\vec{E} = \hat{a}_y \cdot A \cos \omega \left(t - \frac{z}{c} \right)$

$$\vec{E} = E_y \hat{a}_y$$

prop. $\Rightarrow -t \hat{z}$

$$\vec{P} = P_z \hat{a}_z$$

$$\vec{H} = ?$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\hat{a}_z = \hat{a}_y \times (-\hat{a}_x)$$

$$\vec{H} = -H_z \hat{a}_x$$

$$\frac{E}{\eta} = E_y \sqrt{\frac{\epsilon_0}{\mu_0}}$$

option 8
(c) $\hat{a}_x \left[-\hat{j} \sqrt{\frac{\epsilon_0}{\mu_0}} A \cos \left(- \right) \right]$
 $\equiv \sin(-)$

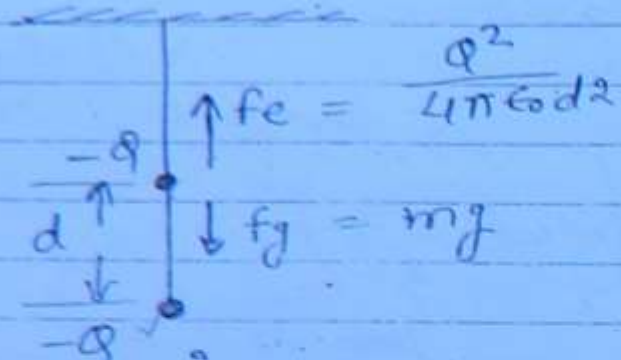
(d) $\hat{a}_x \left[-\hat{j} \sqrt{\frac{\epsilon_0}{\mu_0}} A \sin \left(- \right) \right]$ (105)
 $\equiv \cos(-)$

(75) $\vec{D} = 2(\hat{a}_x - \sqrt{3} \hat{a}_z)$

$$\rho_s = |D_n| = 2\sqrt{1+3} = 4 \text{ C/m}^2$$

$|D_n| = \rho_s = +4 \text{ C/m}^2$

Q: 2



$f_e = \frac{Q^2}{4\pi\epsilon_0 d^2}$ (144)
 $f_g = mg$
 $mg = \frac{Q^2}{4\pi\epsilon_0 d^2}$ find $d = 8.57 \text{ cm}$

Q: 3

$$V = -\frac{6\epsilon^5}{\epsilon_0}$$

I-method

II-method

$$\text{find } \vec{E} = -\nabla V$$

$$\text{find } \vec{D} = \epsilon \vec{E}$$

$$\text{find } \rho = \nabla \cdot \vec{D}$$

$$\text{find } Q = \iiint \rho \, dV$$

$$\nabla^2 V = -\rho/\epsilon$$

$$\text{find } \rho = -\epsilon \nabla^2 V$$

$$\text{find } Q = \iiint \rho \, dV$$

$$\rho = -\epsilon_0 \nabla^2 V; \quad V = f(\epsilon)$$

$$= -\epsilon_0 \cdot \frac{1}{\epsilon^2 \sin \theta} \frac{\partial}{\partial \epsilon} \left(\epsilon^2 \sin \theta \frac{\partial V}{\partial \epsilon} \right)$$

$$= -\epsilon_0 \frac{1}{\epsilon^2} \frac{\partial}{\partial \epsilon} \left[\frac{\epsilon^2}{\partial \epsilon} \left(-\frac{6\epsilon^5}{\epsilon_0} \right) \right]$$

$$= +\epsilon_0 \frac{\partial}{\partial \epsilon} \left[\epsilon^2 \frac{30}{\epsilon_0} \epsilon^4 \right]$$

$$= \frac{1}{\epsilon^2} \times 30 \times 6 \times 5$$

$$= \underline{180 \epsilon^3}$$

$$Q = \iiint_V \rho \, dv$$

$$= 180 \iiint \epsilon^3 \cdot \epsilon^2 \sin \theta \, d\epsilon \cdot d\theta \cdot d\phi$$

(145)

$$= 180 \underbrace{\int_0^1 \epsilon^5 \, d\epsilon}_{=1/6} \underbrace{\int_0^\pi \sin \theta \, d\theta}_{=2} \underbrace{\int_0^{2\pi} d\phi}_{=2\pi}$$

$$= 180 \times \frac{1}{6} \times 4\pi$$

$$= 120\pi \text{ Ans}$$

Q: 7 $\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\nabla \times \vec{A} = 0$ ---- irrotational vector

$\nabla \cdot \vec{A} \neq 0$ ---- not solenoidal

Divergence less

Q: 8 $W_e = \frac{Q_1 Q_2}{4\pi\epsilon\epsilon_0}$

$W_e \propto \frac{1}{\epsilon}$

$W_{e2} = \frac{1}{2} W_{e1}$

Q: 11 Line charge : ($y=3$; $z=5$)

$\vec{E}(0, 6, 1) =$

$\vec{E}(5, 6, 1) = ?$

remains same

Q:17 $\vec{E} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ $V = - \int_y^x \vec{E} \cdot d\vec{l}$

$$d\vec{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

$$\vec{E} \cdot d\vec{l} = x \cdot dx + y \cdot dy + z \cdot dz$$

$$V = - \left[\int_1^2 x dx + \int_2^0 y dy + \int_3^0 z dz \right]$$

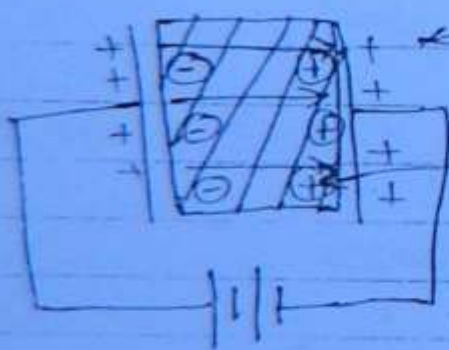
$$V = +5 \text{ Volt}$$

Q:18 $V = 3x^2y - yz$

$$V(1,0,-1) = 0$$

$$\begin{cases} \vec{E} = -\nabla V = - \left[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right] \\ \neq 0 \end{cases}$$

19) Polarization on dielectric materials



free charge; $\epsilon_s \text{ C/m}^2$
-- present on conductor

Bound charges; ϵ_{sp} — surface charge density due to polarization

$$D = \epsilon_0 E \text{ --- for free space}$$

$$D = \epsilon_d \epsilon_0 E \text{ --- for dielectric}$$

$$D = \epsilon_0 E + P \text{ --- for dielect.}$$

$$D = \epsilon_0 E + P$$

(147)

$$P = D - \epsilon_0 E = D - \epsilon_0 \frac{D}{\epsilon} = D - \epsilon_0 \frac{D}{\epsilon_0 \epsilon_r}$$

$$P = D \left(1 - \frac{1}{\epsilon_r} \right) = 2 \left(1 - \frac{1}{5} \right) = 1.6 \text{ C/m}^2$$

Imp. ① The polarization in the dielect. materials is present whenever it is subjected to some externally applied elect. field.

② The charges are induced with in the dielect. due to dipole combination so that net charge induced on the dielectric slab is zero.

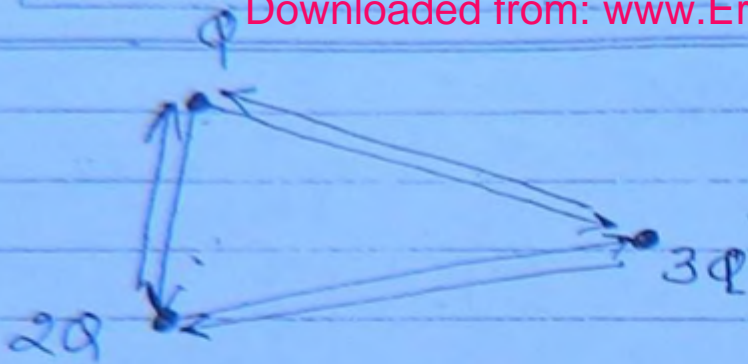
③ Polarization represents total dipole movement combination per unit volume of the dielect.

④ Due to induced charges, overall field distribution on the dielect. is modified.

⑤ it is a temporary phenomenon. so long as externally applied elect. field is present the dielect. will remain in the polarized stage.

as soon as this elect. field with drawn the dielect. returns to its unpolarized stage & the induced charges are no longer present.

(21)



(148)

$$\vec{F}_{13} + \vec{F}_{23} = 3\vec{F} \quad \text{--- (1)}$$

$$\vec{F}_{12} + \vec{F}_{32} = 2\vec{F} \quad \text{--- (2)}$$

$$\vec{F}_{21} + \vec{F}_{31} = ?$$

$$-\vec{F}_{31} - \vec{F}_{32} = 3\vec{F}$$

$$-\vec{F}_{21} + \vec{F}_{32} = 2\vec{F}$$

$$\vec{F}_{31} + \vec{F}_{21} = -5\vec{F} \quad \text{Ans.}$$

Q: 22

$$q\vec{E} = q\vec{v} \times \vec{B}$$

$$\vec{E} = \vec{v} \times \vec{B}$$

Smith chart

① cart. coord system :
(x, y, z)

$$d\vec{r} = \begin{cases} \pm dx \hat{a}_x \\ \pm dy \hat{a}_y \\ \pm dz \hat{a}_z \end{cases}$$

(149)

$$d\vec{s} = \begin{cases} \pm dx dy \cdot \hat{a}_z \\ \pm dy dz \cdot \hat{a}_x \\ \pm dx dz \cdot \hat{a}_y \end{cases}$$

$$dV = dx \cdot dy \cdot dz$$

$$\begin{cases} -\infty < x < +\infty \\ -\infty < y < +\infty \\ -\infty < z < +\infty \end{cases}$$

② cyl. coord. system: (r, φ, z)

$$d\vec{r} = \begin{cases} \pm dr \hat{a}_r \\ \pm r \cdot d\phi \hat{a}_\phi \\ \pm dz \hat{a}_z \end{cases}$$

$$d\vec{s} = \begin{cases} \pm r dr d\phi \cdot \hat{a}_z \\ \pm r d\phi dz \cdot \hat{a}_r \\ \pm dr dz \cdot \hat{a}_\phi \end{cases}$$

$$dV = r \cdot dr \cdot d\phi \cdot dz$$

$$\begin{cases} 0 \leq z < \infty \\ 0 \leq \phi < 2\pi \\ -\infty < r < +\infty \end{cases} \quad (150)$$

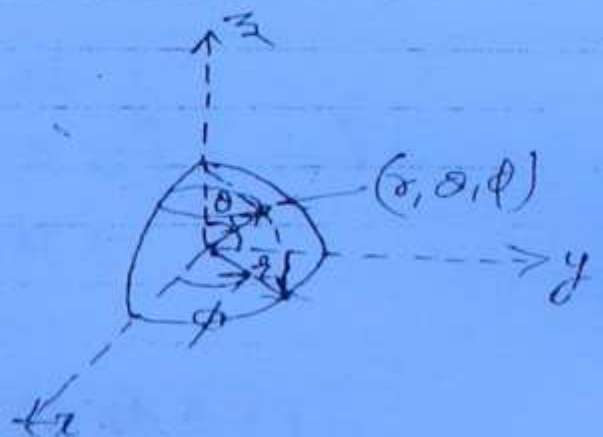
③ Sph. coord. system :- (r, θ, ϕ)

$$d\vec{l} = \begin{cases} + dr \cdot \hat{a}_r \\ + r d\theta \cdot \hat{a}_\theta \\ + r \sin\theta d\phi \cdot \hat{a}_\phi \end{cases}$$

$$d\vec{s} = \begin{cases} + r dr d\theta \cdot \hat{a}_\phi \\ + r^2 \sin\theta d\theta d\phi \cdot \hat{a}_r \\ + r \sin\theta dr d\phi \cdot \hat{a}_\theta \end{cases}$$

$$dV = r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi$$

$$\rightarrow \begin{cases} 0 \leq r < \infty \\ 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{cases}$$



(u, v, w)

$$1. \nabla V = \sum \frac{1}{h_i} \frac{\partial V}{\partial u} \hat{a}_i \quad (15)$$

$$2. \nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u} (h_2 h_3 A_u)$$

$$3. \nabla^2 V = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial V}{\partial u} \right)$$

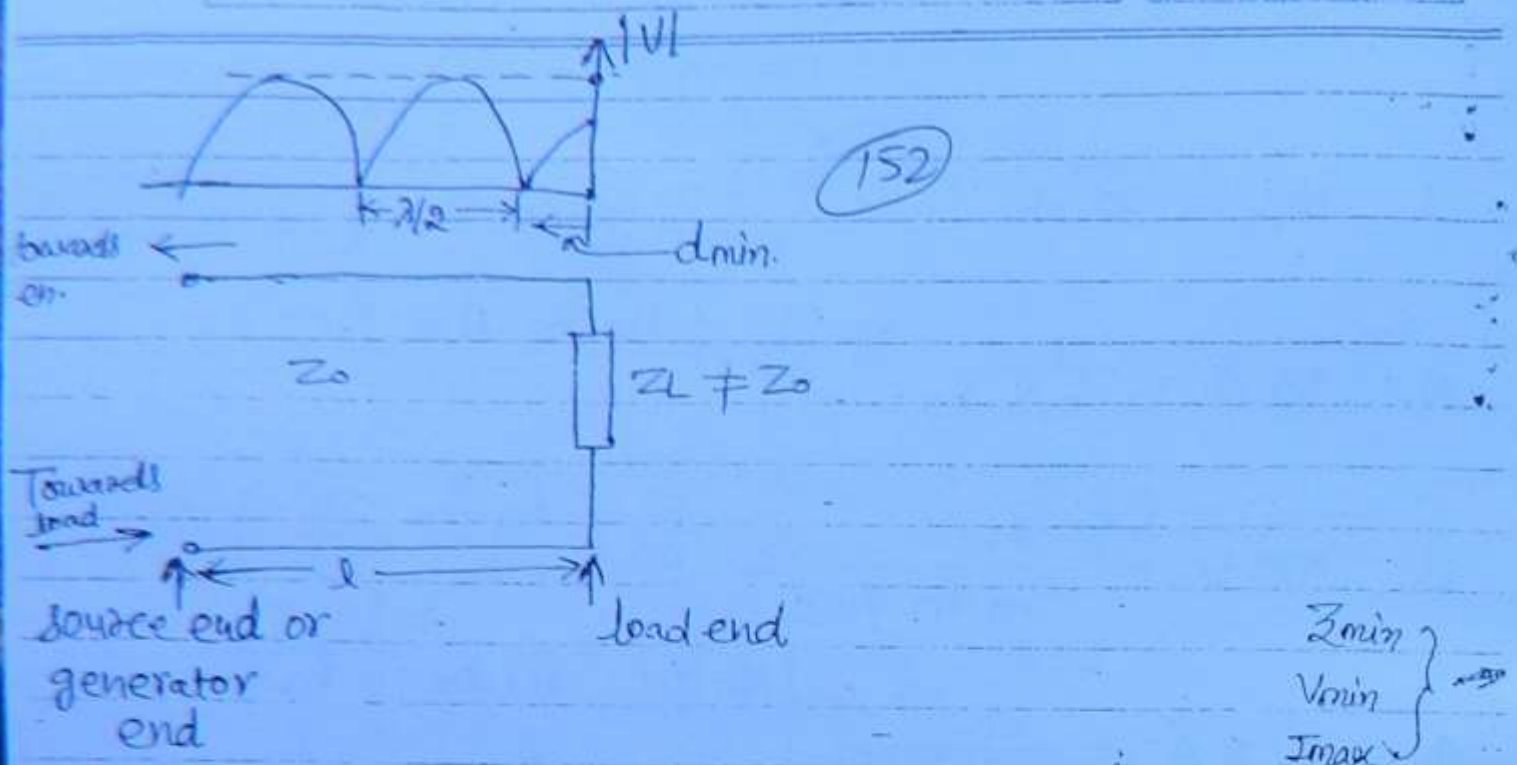
$$4. \nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a}_u & h_2 \hat{a}_v & h_3 \hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

$$\vec{A} = A_u \hat{a}_u + A_v \hat{a}_v + A_w \hat{a}_w$$

	u	v	w	h_1	h_2	h_3
cart.	x	y	z	1	1	1
cyl.	ρ	ϕ	z	1	ρ	1
sph.	ρ	θ	ϕ	1	ρ	$\rho \sin \theta$

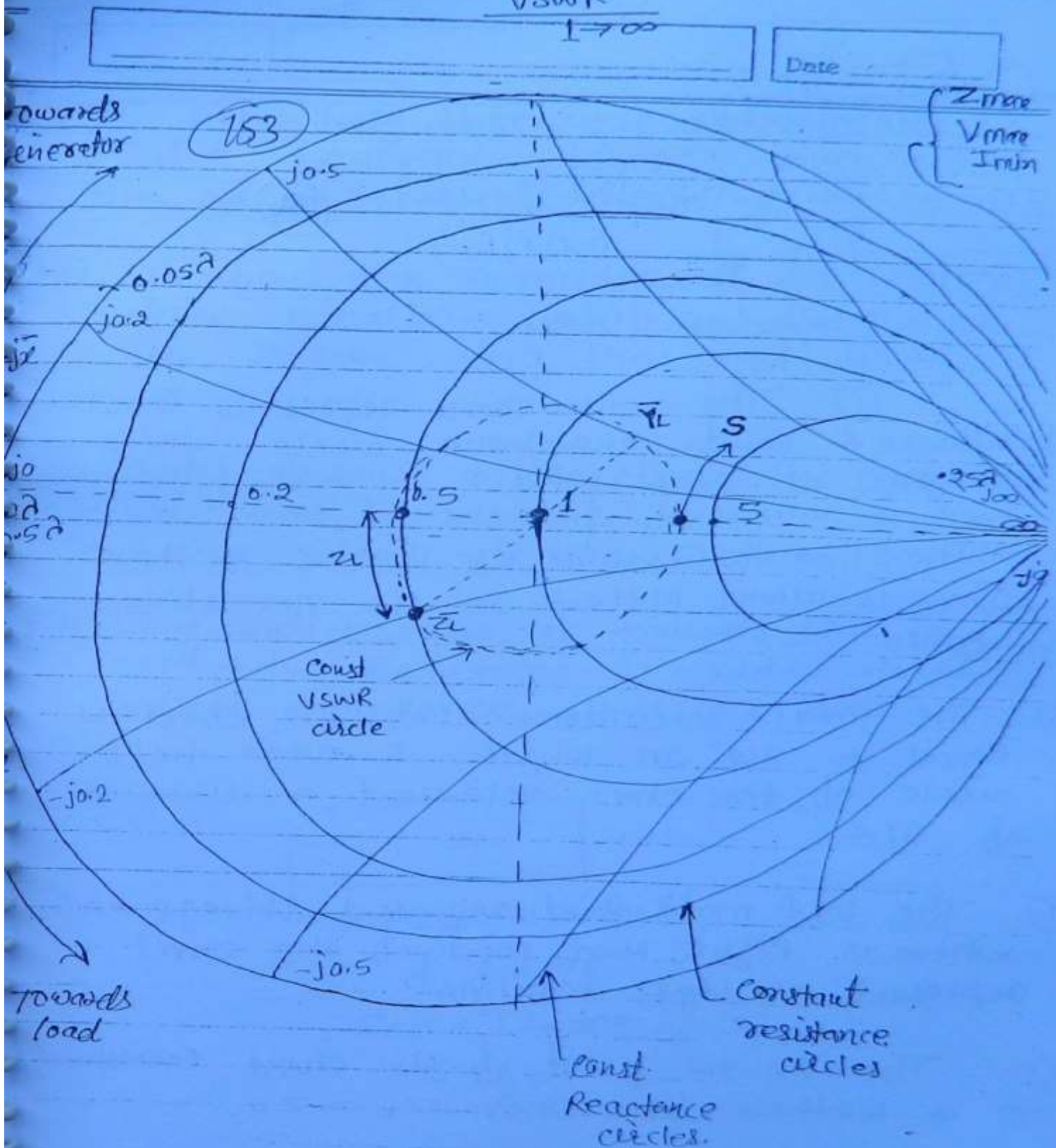
Smith chart :-

T.L Calculate



- (1) lossless line
- (2) normalized impedance

$$\left(\frac{Z}{Z_0} \right) \equiv \bar{Z} = \bar{R} + j\bar{X}$$



Imp. points

 $Z_L = \text{Given}$; $Z_0 = \text{given}$

$$\bar{Z}_L = 0.5 - j0.2$$

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$$\bar{Y}_L = \frac{1}{\bar{Z}_L} = (\text{rec}) + j(\text{---})$$

 S ; d_{\min} ; d_{\max} .

- ① The smith chart represents const. Resistance & const. reactance circle. which are orthogonal at each point.
- ② The line is assume as lossless & the imp. is always plotted in its normalized form.
- ③ The total circumference of the chart is equal to $\lambda/2$ in length & each half circle of the chart represent a distance of $\lambda/4$.
- ④ The left most point represent Voltage minima whereas right most point of the chart represent voltage maxima.
- ⑤ The centre point of the chart corresponds to a matched line where $Z_L = Z_0$.
- ⑥ The total distance b/w the centre of the chart to the right most of point represent total range of VSWR from $1 \rightarrow \infty$

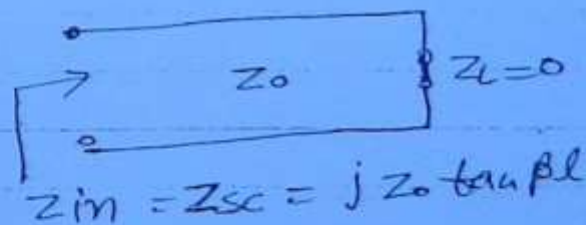
⑦ upper half the circle corresponds to +ive Reactance. Whereas lower half of the circle represents negative reactance

⑧ To find the normalized ⁽⁵⁵⁾ Admittance from normalized imp. a distance of $\lambda/4$ is move along the const. VSWR circle.

⑨ Going clockwise in the chart the impedance ~~are~~ moves towards the generator & induct ~~are~~ added to the initial impedance

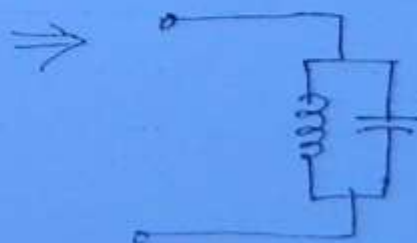
⑩ going anticlockwise or towards the load capacitive Reactance is added to the initial impedance along the line.

ex:



$$\text{if } l = \lambda/4$$

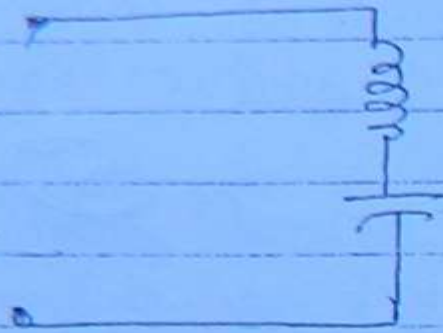
$$Z_{sc} = j Z_0 \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = j\infty$$



parallel LC
Resonance

$$Z_{oc} = -j Z_0 \tan \beta l \Big|_{l = \lambda/4} = 0$$

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Series LC

Resonance circuit

Conclusion :

① for $\lambda/4$ section of the line a short circuited line represent a parallel LC resonant ckt. Where as an open circuited line represent a series LC resonant circuit.