1. Trigonometic Formulae

1. Definitions of Trigonometric Ratios

$$\sin\theta = \frac{y}{r}$$

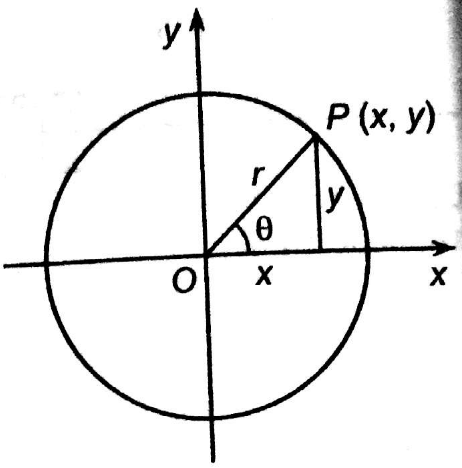
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$



2. Fundamental Trigonometic Identities

(A) $\sin(-\theta) = -\sin\theta,\qquad \cos(-\theta) = \cos\theta$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta,\qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta,$$

$$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta,\qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta,$$

$$\sin(\pi-\theta) = \sin\theta,\qquad \cos(\pi-\theta) = -\cos\theta,$$

$$\sin(\pi+\theta) = -\sin\theta,\qquad \cos(\pi+\theta) = -\cos\theta,$$

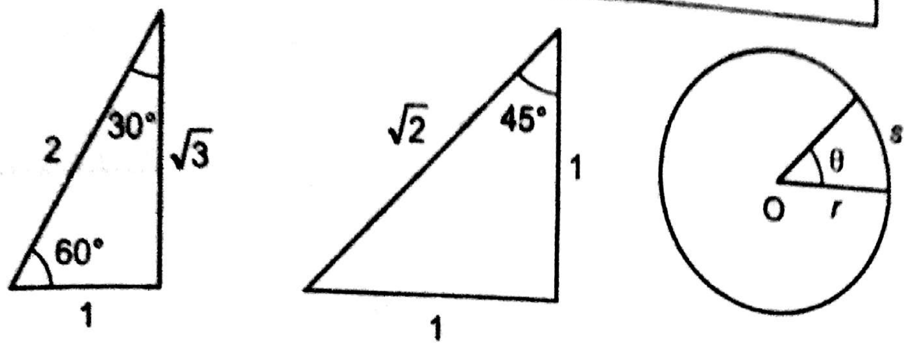
$$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta,\qquad \cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta,$$

$$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta,\qquad \cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta,$$

(B)

Quadrant	sin θ	cos θ	tan θ
I	+	+	+
II	+	–	–
III	–	–	+
IV	–	+	–

θ	0° / 360°	30°	45°	60°	90°	180°	270° / -90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0



$s = r\theta, \ 180^\circ = \pi^c$

(C) $\sin^2\theta + \cos^2\theta = 1, \qquad \sec^2\theta = 1 + \tan^2\theta,$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta.$

(D) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(E) $\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \qquad \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \qquad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \qquad \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

(F) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

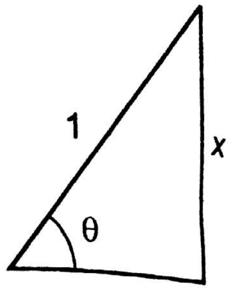
$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$
$2 \cos A \sin B = \sin (A + B) - \sin (A - B)$
$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$
$-2 \sin A \sin B = \cos (A + B) - \cos (A - B)$

(G) $\sin \theta = x, \quad \operatorname{cosec} \theta = \dfrac{1}{x}$

$\therefore \quad \sin^{-1} x = \operatorname{cosec}^{-1} \dfrac{1}{x}, \qquad \cos^{-1} x = \sec^{-1} \dfrac{1}{x}$

$\tan^{-1} x = \cot^{-1} \dfrac{1}{x}, \qquad \sin (\sin^{-1} x) = x,$

$\cos (\cos^{-1} x) = x, \qquad \tan (\tan^{-1} x) = x.$



(H) $\sin h\, x = \dfrac{e^x - e^{-x}}{2}, \qquad \cos h\, x = \dfrac{e^x + e^{-x}}{2},$

$\tan h\, x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}, \qquad \sec h\, x = \dfrac{2}{e^x + e^{-x}},$

$\operatorname{cosec} h\, x = \dfrac{2}{e^x - e^{-x}}, \qquad \cot h\, x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}.$

(I) $\cos h^2 x - \sin h^2 x = 1, \qquad \sec h^2 x = 1 - \tan h^2 x$

$\operatorname{cosec} h^2 x = \cot h^2 x - 1, \qquad \sin h\, 2x = 2 \sin hx \cos hx$

$\cos h\, 2x = 2 \cos h^2 x - 1$

$= 2 \sin h^2 x + 1.$

(J) $\sin h^{-1} x = \operatorname{cosec} h^{-1} \dfrac{1}{x}, \qquad \cos h^{-1} x = \sec h^{-1} \dfrac{1}{x},$

$\tan h^{-1} x = \cot h^{-1} \dfrac{1}{x}.$

2. Algebraic Formulae

(A) $a^m \times a^n = a^{m+n} \qquad \dfrac{a^m}{a^n} = a^{m-n}$

$a^m b^m = (ab)^m \qquad (a^m)^n = a^{mn}$

$a^0 = 1, \quad a^{-r} = \dfrac{1}{a^r} \qquad a^{p/q} = \sqrt[q]{a^p}$

(B) $a^2 - b^2 = (a - b)(a + b)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= (a^3 + b^3) + 3ab(a + b)$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$= (a^3 - b^3) - 3ab(a - b)$$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3b^2c + 3bc^2 + 3c^2a + 3a^2c + 6abc$$
$$= (a^3 + b^3 + c^3) + 3(a + b + c)\cdot(ab + bc + ca) - 3abc.$$

$a^2 + b^2 = (a + b)^2 - 2ab = (a - b)^2 + 2ab$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$$a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b + c)(ab + bc + ca) + 3abc$$
$$= 3abc \text{ if } a + b + c = 0$$

(C) $\log mn = \log m + \log n$, $\quad \log \dfrac{m}{n} = \log m - \log n$

$\log m^n = n \log m \qquad \log_b m = \dfrac{\log a^m}{\log a^b}$

(D) If $ax^2 + bx + c = 0$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of the roots $= -\dfrac{b}{a}$, $\quad$ Product of the roots $= \dfrac{c}{a}$

(E) $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

(a) If two rows or columns are identical then the determinant is zero.

(b) If rows and columns are interchanged the determinant is not changed.

(c) If two rows or columns are interchanged the determinant changes its sign.

(d) If each element of a row or column is multiplied by a constant then the determinant is multiplied by that constant.

(e) The value of a determinant is unchanged if equimultiples of a row or a column are added to the corresponding elements of any other row or column.

Cramer's Rule

If $a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$,

then $x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$

where, $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

and D_x, D_y, D_z are obtained by replacing the coefficients of x, y, z respectively by d_1, d_2, d_3.

3. Differentiation Formulae

(A) $\lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right) = 1$, $\quad \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$,

$\lim_{y \to 0} (1 + y)^{1/y} = e$, $\quad \lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$.

(B) **1.** If $y = x^n$, $\quad \frac{dy}{dx} = n x^{n-1}$

2. If $y = \sin x$, $\quad \frac{dy}{dx} = \cos x$

3. If $y = \cos x$, $\quad \frac{dy}{dx} = -\sin x$

4. If $y = \tan x$, $\quad \frac{dy}{dx} = \sec^2 x$

5. If $y = \operatorname{cosec} x$, $\quad \frac{dy}{dx} = -\operatorname{cosec} x \cot x$

6. If $y = \sec x$, $\quad \frac{dy}{dx} = \sec x \tan x$

7. If $y = \cot x$, $\quad \frac{dy}{dx} = -\operatorname{cosec}^2 x$

8. If $y = e^x$, $\quad \frac{dy}{dx} = e^x$

9. If $y = a^x$, $\frac{dy}{dx} = a^x \log a$

10. If $y = \log_e x$, $\frac{dy}{dx} = \frac{1}{x}$

11. If $y = \log_a x$, $\frac{dy}{dx} = \frac{1}{x \log a}$

12. If $y = \sin^{-1} x$, $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

13. If $y = \cos^{-1} x$, $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

14. If $y = \tan^{-1} x$, $\frac{dy}{dx} = \frac{1}{1+x^2}$

15. If $y = \sec^{-1} x$, $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

16. If $y = \operatorname{cosec}^{-1} x$, $\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$

17. If $y = \cot^{-1} x$, $\frac{dy}{dx} = -\frac{1}{1+x^2}$

18. If $y = \sin hx$, $\frac{dy}{dx} = \cos h x$

19. If $y = \cos hx$, $\frac{dy}{dx} = \sin h x$

20. If $y = \tan hx$, $\frac{dy}{dx} = \sec h^2 x$

21. If $y = \operatorname{cosec} hx$, $\frac{dy}{dx} = -\operatorname{cosec} h x \cot x$

22. If $y = \sec hx$, $\frac{dy}{dx} = \sec h x \tan h x$

23. If $y = \cot hx$, $\frac{dy}{dx} = -\operatorname{cosec} h^2 x$

24. If $y = \sin h^{-1} x$, $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

25. If $y = \cos h^{-1} x$, $\frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}}$

26. If $y = \tan h^{-1} x$, $\frac{dy}{dx} = \frac{1}{1-x^2}$

27. If $y = \sec h^{-1} x$, $\frac{dy}{dx} = -\frac{1}{x\sqrt{1-x^2}}$

28. If $y = \operatorname{cosec} h^{-1} x$, $\quad \dfrac{dy}{dx} = -\dfrac{1}{|x|\sqrt{1+x^2}}$

29. If $y = \cot h^{-1} x$, $\quad \dfrac{dy}{dx} = \dfrac{1}{1-x^2}$

(C) 1. If $y = u \pm v$, $\quad \dfrac{dy}{dx} = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

2. If $y = uv$, $\quad \dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$

3. If $y = \dfrac{u}{v}$, $\quad \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

4. If $y = x^x$, $\quad \dfrac{dy}{dx} = x^x(1+\log x)$

5. If $x = f(t)$, $y = \Phi(t)$, $\quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

4. Integration Formulae

$$\int \mathrm{I}\cdot\mathrm{II}\cdot dx = \mathrm{I}\cdot\int \mathrm{II}\ dx - \int\left[\int \mathrm{II}\ dx\right]\cdot\frac{d\mathrm{I}}{dx}\cdot dx$$

$$\int e^x\,[f(x) + f'(x)]\,dx = e^x f(x)$$

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1}$ if $n \neq -1$

2. $\int \dfrac{dx}{x} = \log x$

3. $\int \sin x\,dx = -\cos x$

4. $\int \cos x\,dx = \sin x$

5. $\int \sec^2 x\,dx = \tan x$

6. $\int \operatorname{cosec}^2 x\,dx = -\cot x$

7. $\int \sec x \tan x\,dx = \sec x$

8. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x$

9. $\int \tan x\,dx = \log\sec x$

10. $\int \cot x\,dx = -\log\operatorname{cosec} x = \log\sin x$

11. $\int \sec x\,dx = \log\left\{\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\} = \log(\sec x + \tan x)$

12. $\int \operatorname{cosec} x\,dx = \log\left(\tan\dfrac{x}{2}\right) = \log(\operatorname{cosec} x - \cot x)$

13. $\int e^x\,dx = e^x$

14. $\int a^x dx = \dfrac{a^x}{\log a}$

15. $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\dfrac{x}{a}$

16. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left(x + \sqrt{x^2 - a^2}\right)$

17. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left(x + \sqrt{x^2 + a^2}\right)$

18. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$

20. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$

21. $\int \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1} x$

22. $\int e^{ax} \sin bx\, dx = \frac{1}{a^2 + b^2} \cdot e^{ax}(a \sin bx - b \cos bx)$

23. $\int e^{ax} \cos bx\, dx = \frac{1}{a^2 + b^2} \cdot e^{ax}(a \cos bx + b \sin bx)$

24. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

25. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left(x + \sqrt{x^2 + a^2}\right)$

26. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left(x + \sqrt{x^2 - a^2}\right)$

27. $\int \sin h\, x\, dx = \cos h\, x$

28. $\int \cos h\, x\, dx = \sin h\, x$

29. $\int \tan h\, x\, dx = \log(\cos h\, x)$

30. $\int \sec h\, x\, dx = \sin^{-1}(\tan h\, x)$

31. $\int \operatorname{cosec} h\, x\, dx = \tan\left|\tan h \frac{x}{2}\right|$

32. $\int \cot h\, x\, dx = \log|\sin h\, x|$

Definite Integrals

1. $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

2. $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

Scanned by CamScanner

1. Trigonometric Limits

(i) $\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{x}{\sin x}$

(ii) $\lim_{x\to 0} \frac{\tan x}{x} = 1 = \lim_{x\to 0} \frac{x}{\tan x}$

(iii) $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1 = \lim_{x\to 0} \frac{x}{\sin^{-1} x}$

(iv) $\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1 = \lim_{x\to 0} \frac{x}{\tan^{-1} x}$

(v) $\lim_{x\to 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180^\circ}$

(vi) $\lim_{x\to 0} \cos x = 1$

(vii) $\lim_{x\to a} \frac{\sin(x-a)}{x-a} = 1$

(viii) $\lim_{x\to a} \frac{\tan(x-a)}{x-a} = 1$

(ix) $\lim_{x\to a} \sin^{-1} x = \sin^{-1} a, |a| \le 1$

(x) $\lim_{x\to a} \cos^{-1} x = \cos^{-1} a, |a| \le 1$

(xi) $\lim_{x\to a} \tan^{-1} x = \tan^{-1} a, -\infty < a < \infty$

(xii) $\lim_{x\to \infty} \frac{\sin x}{x} = \lim_{x\to \infty} \frac{\cos x}{x} = 0$

(xiii) $\lim_{x\to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = 1$

(xiv) $\lim_{x\to 0} \frac{(1+x)^n - 1}{x} = n$

2. Exponential Limits

We use the series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$

(i) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(ii) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

(iii) $\lim_{x \to 0} \frac{e^{\lambda x} - 1}{x} = \lambda$, where $(\lambda \neq 0)$.

3. Logarithmic Limits

We use the series $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty$,

where $-1 < x \leq 1$ and expansion is true only, if base is e.

(i) $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

(ii) $\lim_{x \to e} \log_e x = 1$

(iii) $\lim_{x \to 0} \frac{\log_e (1 - x)}{x} = -1$

(iv) $\lim_{x \to 0} \frac{\log_a (1 + x)}{x} = \log_a e, a > 0, \neq 1$

(v) If $\lim_{x \to a} f(x)$ exists and positive, then

$$\lim_{x \to a} [f(x)]^{\phi(x)} = e^{\lim_{x \to a} \phi(x) \log f(x)}$$

4. Based on the Form 1^{∞}

To evaluate the exponential form 1^{∞}, we use following results.

If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0,$

then, $\lim_{x \to a} \{1 + f(x)\}^{1/g(x)} = e^{\lim_{x \to a} \frac{f(x)}{g(x)}}$

or when $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty.$

Then, $\lim_{x \to a} \{f(x)\}^{g(x)} = \lim_{x \to a} \{1 + f(x) - 1\} g(x) = e^{\lim_{x \to a} \{f(x) - 1\} g(x)}$

(i) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

(ii) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$

(iii) $\lim_{x \to 0} (1 + \lambda x)^{\frac{1}{x}} = e^{\lambda}$

(iv) $\lim_{x \to \infty} \left(1 + \frac{\lambda}{x}\right)^{x} = e^{\lambda}$

(v) $$\lim_{x \to \infty} a^{x} = \begin{cases} 0, & 0 \le a < 1 \\ 1, & a = 1 \\ \infty, & a > 1 \\ \text{does not exist}, & a < 0 \end{cases}$$

Important formulae:

$$(1+x)^{-1} = 1-x+x^2-x^3+ \ ....$$

$$(1-x)^{-1} = 1+x+x^2+x^3+....$$

$$(1+x)^{-2} = 1-2x+3x^2-4x^3+ \ ....$$

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+....$$